- The Lagrange function for the generated sub-problem is as follows:

$$\mathrm{L}_{\pi} = \sum_{i=1}^{3}[(P_i + s - Pm)q_i - sy_i - (P_i + s + h)\int_{\xi_i}^{q_i - y_i} F_i(\xi_i)d\xi_i] - (\phi + \varphi r)(r + h) - (c_0 - c_1 l_2)^2 - (c_2 - c_3 l_3)^2 - k' + \lambda_1(P_1 - P_2) + \lambda_2(P_2 - r) + \lambda_3(P_1 - P_3) \quad \text{(A1)}$$

- As a result of KKT optimality conditions, the following nonlinear Equations are obtained:

$$\frac{\partial L_\pi}{\partial q_i} = (P_i + s - Pm) - (P_i + s + h)F_i(q_i - y_i) = 0 \quad \forall i = 1,2,3 \quad \text{(A2)}$$

$$\frac{\partial L_\pi}{\partial P_1} = q_1 + \sum_{i=1}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial P_1} - \int_{\xi_1}^{q_1 - y_1} F_1(\xi_1)d\xi_1 + \lambda_1 + \lambda_3 = q_1 - (P_1 - Pm)(\beta + \delta_1 + \delta_2) + (P_2 - Pm)\delta_1 + (P_3 - Pm)\delta_2 - \int_{\xi_1}^{F_1^{-1}(\rho_1)} F_1(\xi_1)d\xi_1 + \lambda_1 + \lambda_3 = 0 \quad \text{(A3)}$$

$$\frac{\partial L_\pi}{\partial P_2} = q_2 + \sum_{i=1}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial P_2} - \int_{\xi_2}^{q_2 - y_2} F_2(\xi_2)d\xi_2 - \lambda_1 + \lambda_2 = q_2 + (P_1 - Pm)\delta_1 - (P_2 - Pm)(\beta + \delta_1 + \delta_3) + (P_3 - Pm)\delta_3 - \int_{\xi_2}^{F_2^{-1}(\rho_2)} F_2(\xi_2)d\xi_2 - \lambda_1 + \lambda_2 = 0 \quad \text{(A4)}$$

$$\frac{\partial L_\pi}{\partial P_3} = q_3 + \sum_{i=1}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial P_3} - \int_{\xi_3}^{q_3 - y_3} F_3(\xi_3)d\xi_3 - \lambda_3 = q_3 + (P_1 - Pm)\delta_2 + (P_2 - Pm)\delta_3 - (P_3 - Pm)(\beta + \delta_2 + \delta_3) - \int_{\xi_3}^{F_3^{-1}(\rho_3)} F_3(\xi_3)d\xi_3 - \lambda_3 = 0 \quad \text{(A5)}$$

$$\frac{\partial L_\pi}{\partial r} = \sum_{i=2}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial r} - \varphi(r + h) - (\phi + \varphi r) - \lambda_2 = (P_2 - Pm)\theta - \varphi(2r + h) - \phi - \lambda_2 = 0 \quad \text{(A6)}$$

$$\frac{\partial L_\pi}{\partial l_2} = \sum_{i=1}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial l_2} + 2c_1(c_0 - c_1 l_2) = (P_1 - Pm)\frac{\tau_1}{2}\gamma_2 + (P_2 - Pm)(-\gamma_2 - \tau_2) + (P_3 - Pm)\left(\frac{\tau_1}{2}\gamma_2 + \tau_2\right) + 2c_1(c_0 - c_1 l_2) = 0 \quad \text{(A7)}$$

$$\frac{\partial L_\pi}{\partial l_3} = \sum_{i=2}^{3}[(P_i + s + h)F_i(q_i - y_i) - s]\frac{\partial y_i}{\partial l_3} + 2c_3(c_2 - c_3 l_3) = (P_2 - Pm)\tau_2 + (P_3 - Pm)(-\gamma_3 - \tau_2) + 2c_3(c_2 - c_3 l_3) = 0 \quad \text{(A8)}$$

$$\lambda_1(P_1 - P_2) = 0 \quad \text{(A9)}$$

$$\lambda_2(P_2 - r) = 0 \quad \text{(A10)}$$

$$\lambda_3(P_1 - P_3) = 0 \quad \text{(A11)}$$

where

$\frac{\partial y_1}{\partial P_1} = -\beta - \delta_1 - \delta_2$ ، $\frac{\partial y_2}{\partial P_1} = \delta_1$ ، $\frac{\partial y_3}{\partial P_1} = \delta_2$ ، $\frac{\partial y_1}{\partial P_2} = \delta_1$ ، $\frac{\partial y_2}{\partial P_2} = -\beta - \delta_1 - \delta_3$ ، $\frac{\partial y_3}{\partial P_2} = \delta_3$ ، $\frac{\partial y_1}{\partial P_3} = \delta_2$ ، $\frac{\partial y_2}{\partial P_3} = \delta_3$ ، $\frac{\partial y_3}{\partial P_3} = -\beta - \delta_2 - \delta_3$ ، $\frac{\partial y_1}{\partial r} = 0$ ، $\frac{\partial y_2}{\partial r} = \theta$ ، $\frac{\partial y_3}{\partial r} = 0$ ، $\frac{\partial y_1}{\partial l_2} = \frac{\tau_1}{2}\gamma_2$ ، $\frac{\partial y_2}{\partial l_2} = -\gamma_2 - \tau_2$ ، $\frac{\partial y_3}{\partial l_2} = \frac{\tau_1}{2}\gamma_2 + \tau_2$، $\frac{\partial y_1}{\partial l_3} = 0$، $\frac{\partial y_2}{\partial l_3} = \tau_2$، $\frac{\partial y_3}{\partial l_3} = -\gamma_3 - \tau_2$ ، $\frac{\partial R}{\partial r} = \varphi$ ، $F_i(q_i - y_i) = \frac{(P_i + s - Pm)}{(P_i + s + h)} = \rho_i$ ، $\lambda_1, \lambda_2, \lambda_3 \geq 0$ .

**Case (5):** $\lambda_1, \lambda_2 = 0\,, \lambda_3 > 0$; insert these mathematical terms into equations A9 to A11, this case would yield to $P_1 = P_3$, and $\lambda_3$ by using Equation A5 is as follows:

$$\lambda_3 = q_3 + \delta_3(P_2 - P_1) + \beta(Pm - P_1) - \int_{\xi_3}^{{F_3}^{-1}(\rho_3)} F_3(\xi_3)d\xi_3 \tag{A21}$$

This case has a feasible solution if $\lambda_3 \geq 0$. Furthermore, the aforementioned KKT conditions, as detailed in Equations A3 to A6 and A9 to A11, are employed in the calculation of Equation 24.

- Solve the system of Equation 24, obtained from case 5 from the appendix, to determine the third response.

$$\begin{cases} y_1 + {F_1}^{-1}(\rho_1) + y_3 + {F_3}^{-1}(\rho_3) - \int_{\xi_1}^{{F_1}^{-1}(\rho_1)} F_1(\xi_1)d\xi_1 - \int_{\xi_3}^{{F_3}^{-1}(\rho_3)} F_3(\xi_3)d\xi_3 + 2\beta(Pm - P_1) \\ \qquad + (\delta_1 + \delta_3)(P_2 - P_1) = 0 \\ y_2 + {F_2}^{-1}(\rho_2) + (\delta_1 + \delta_3)(P_1 - P_2) + \beta(Pm - P_2) - \int_{\xi_2}^{{F_2}^{-1}(\rho_2)} F_2(\xi_2)d\xi_2 = 0 \\ (P_2 - Pm)\theta - \varphi(2r + h) - \phi = 0 \\ P_1 - P_3 = 0 \end{cases} \tag{24}$$