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Fungal Decomposition Modeling via Stochastic Gradient Descent and Cellular Automata

Fungus plays an important role as decomposers in the ecosystem. However, the efficiency of fungal growth and decomposition is influenced by the environment and the interaction among different species of fungi. Therefore, the study of factors affecting fungal activity is important.

In this paper, we establish relation models among the factors that affect the extension rate and decomposition rate of the fungi. Based on the relation models, we introduce the cellular automata model to simulate the growth and decomposition of multiple species of fungi.

Specifically, we first establish the extension rate model. The extension rate is modeled as Arrhenius relation to the temperature and the humidity. Based on the extension rate model, we establish the decomposition rate model. The decomposition rate is determined by extension rate and moisture trade-off, where the relation is nonlinear. To estimate the parameters in the nonlinear decomposition rate model, we establish an optimization model that fits the collected decomposition rate data. The stochastic gradient descent (SGD) algorithm is utilized to solve the nonlinear optimization problem. The stable convergence behavior shows the effectiveness of the algorithm.

Based on the decomposition rate model, we establish the cellular automata model to simulate the growth and decomposition of the fungi. The extension rate and decomposition rate are utilized to determine the rules of cellular automata. The simulation results show that under the same environment, the number of different species of fungi increased significantly at the first $5-10$ days. Then, after about 10 days of extension, the number of fungi decreases to a certain extent and then stabilized after about 40 days. The decomposition rate after 122 days is 0.68.

Next, we describe the interactions among different species of fungi by competitive relationship. To describe the competitive relationship, we propose a definition to measure the reduction of the extension rate. Based on this definition, we establish the cellular automata model under fungal interactions to simulate the fungal activity under interactions. The results show that after considering the interaction, the number of different types of fungi is considerably decreased by an average percentage of 33.65%. And the decomposition rate after 122 days is 0.39, which is 42.69% lower than that of the situation without competition.

Afterward, we analyze the influence of fungal interactions. In the short term, the interaction between different types of fungi is not obvious. In the long-term, the influence is significant due to the shortage of nutrients. Afterward, the sensitivity of our models is studied. We show that our models are sensitive to 8% change of temperature and humidity. Then, the analysis of combinations of fungi and different environments are included. The dominant and disadvantaged species in different situations are obtained, e.g., Trichoderma becomes the dominant species in the tropical rain forest after 122 days with the decomposition rate of 68.29%. At last, we analyze the importance of biodiversity. The results show that biodiversity is important to the efficiency of fungal decomposition, where the decomposition rate of fungi under multi-species is 2.89 times higher than that of single species.

Keywords: Fungal decomposition, Stochastic gradient descent, Nonlinear, Cellular automata

Contents

1 Introduction

1.1 Problem Background

The material cycle in a forest ecosystem includes two relative processes: biosynthesis and biodegradation [1]. The former is the organic process of inorganic matter, which is mainly completed by green plants and some inorganic nutrient microorganisms. The latter is the inorganic process of organic matter, which is mainly completed by forest microorganisms, which is the decomposition process of forest litter.

Forest litter is not only an important part of the ecosystem but also the main source of energy required by plants and microorganisms. Forest litter contains different chemical substances: cellulose, hemicellulose, lignin, water-soluble, ether soluble and alcohol soluble parts, protein, and minerals [2]. Fungi are one of the main members involved in the decomposition of various organic substances in soil. Many fungi can decompose cellulose, hemicellulose, and other similar compounds. At the same time, fungi can also decompose nitrogen-containing protein compounds and have a strong ability to decompose some refractory substances (such as lignin). Therefore, fungi play an important role in material transformation [3].

1.2 Restatement of Problem

Fungi play a key role in the decomposition of woody fibers and ground litter. Fungi is unique amongst the soil biota in their ability to translocate nutrients, a trait enabling them to establish extensive hyphal networks in a heterogeneous environment and to utilize spatially separated nutrient resources that may differ significantly in nutrient quality [4].

Hyphae are the branch cells of fungi, which constitute the filaments and structures of fungi. Different kinds of hyphae play different roles in the life cycle of fungi. Mycelial elongation is essentially the growth rate of fungi [5]. We need to focus on several characteristics of fungi: the growth rate of the fungi, the fungi' tolerance to moisture, and hyphal extension rate. Based on the above understanding, we need to solve the following problems:

- A mathematical model for the decomposition of litter and woody fibers by fungi needs to be established in the presence of a variety of fungi.
- In the model, we need to combine the interaction among different kinds of fungi, which have different growth rates and different moisture tolerance.
- Analyze the model and describe the interaction among different types of fungi. The dynamic characteristics and description of interactions should include short-term and long-term trends. Model analysis needs to account for sensitivity to rapid fluctuations in the environment. Meanwhile, we need to determine the overall impact of changes in atmospheric trends to assess the impact of changes in local weather patterns.
- Forecast the relative strengths and weaknesses of each species and possible persistent species combination, as well as their status in different environments, including arid, semi-arid, temperate, arboreal, and tropical rainforests.
- Describe how the diversity of fungal communities in the system affects the overall efficiency of the system in decomposing ground litters. Predict the importance and role of biodiversity.
- Write a two-page article. This article needs to be suitable as an introductory textbook for university biology to discuss the latest progress in our understanding of the role of fungi in ecosystems.

1.3 Literature Review

The most obvious identity of fungi is decomposer. According to Chen M R [6], litter decomposition generally includes three stages: leaching stage, crushing stage, and organic matter catabolism stage. Among them, the leaching stage refers to that the soluble matter in the litter is washed away by water. Physical comminution, for instance, animal chewing, alternation of soil drying and wetting, freezing and thawing, precipitation, and other factors can crush the tree residues in varying degrees [6, 7]. Metabolism refers to the transformation of more complex organic compounds to simple salt and water molecules.

The catabolism of organic matter is generally accomplished by microorganisms. Yu shuling [8] proposed that fungi, especially filamentous fungi, play an important role in litter decomposition. Findlay et al. [1] pointed out the hyphae of fungi can penetrate into plant debris and secrete extracellular enzymes to degrade recalcitrant substances (such as lignin) contained in the litter, so as to soften plant debris, change the structure and chemical composition of litter, which is conducive to the further colonization of bacteria and other microorganisms and the feeding of soil animals [9, 10].

The complete decomposition of litter is completed under the action of various litter decomposing enzymes [11]. Dick [12] showed that the activities of litter decomposing enzymes were significantly positively correlated with the microbial biomass. Zhang Ruiqing et al. [13] measured a variety of enzymes related to the carbon cycle in the process of litter decomposition in the tropical rain forest, and pointed out that enzymes related to the carbon cycle can be used as an important indicator of organic matter decomposition.

Wang Qibing et al. [14] claimed that the decomposition rate of litter will increase with the increasing temperature [15]. The research results of Liu Qiang and Vitousek [14, 15] showed that the temperature will begin to decrease with the increasing altitude, and the decomposition rate will be significantly reduced. Moore et al. [16] believed that on a large spatial scale, the annual average temperature was considered to be the dominant climate factor affecting forest litter decomposition. The decomposition rate of litter in each climate zone was tropical $>$ subtropical $>$ temperate $>$ cold temperate [17, 18]. Under the action of high temperature and high humidity, the soil microbial activity in low latitude and low altitude areas is generally higher, the decomposition rate of litter is faster, and the accumulation rate of forest surface organic matter is slow. On the contrary, limited by low temperature, microbial activity in high latitude and high altitude areas is generally low, litter decomposition rate is slow, and forest surface organic matter accumulation is large [1, 16, 17].

1.4 Our Work

In this paper, we first study the relationship among fungal extension rate, moisture trade-off, and decomposition rate. According to [5], the relationship among diffusion rate and decomposition rate is determined as $1/2$ order. In addition, the relationship among moisture trade-off and log decomposition rate is linear. Then we unify these three factors into a single nonlinear equation. A multivariate nonlinear optimization model is established and is solved by SGD. In that case, the quantitative relationship among extension rate, moisture trade-off, and decomposition rate is established.

Then, we introduce cellular automata to simulate the growth and decomposition of fungi. We introduce three definitions to quantify decomposition rate and extension rate into cellular automata. Meanwhile, we consider the interaction among different species of fungi and the impact of different environments. Numerous simulation experiments are conducted to test the robustness, sensitivity, and effectiveness of our models.

In summarize, our models are as follows:

- The relationship among extension rate, moisture trade-off, and decomposition rate (7).
- The nonlinear optimization model to fix the undetermined factors of the relationship (8).
- The cellular automata model under multiple species of fungi (20).
- The cellular automata model under interactions among different fungi (22).

It is worth pointing out that the answers to the five questions are respcetively listed in Sec. 3.2 (the first question), Sec. 3.3 (the second question), Sec. 3.4 (the third and fourth question), and Sec. 3.5 (the fifth question).

2 Assumptions and Notations

2.1 Assumptions

Through the full analysis of the problem, in order to simplify our model, we make the following reasonable assumptions.

Assumption 1 *Assuming that the effects of forest soil fertility, tree species, and forest age on litter decomposition are negligible.*

The effect and influence on fungal decomposition are mainly limited to the environment and its own characteristic [10].

Assumption 2 *Ignoring the impacts of external factors such as forest pests and animal activities on the matrix quality and decomposition rate of forest litter.*

Assumption 3 *The effect of light intensity on fungal decomposition was not considered.*

The intensity of light can lead to the difference of leaf structure and material composition [19]. Specifically, the intensity of light will affect the content of nutrients, cellulose, and lignin in leaves. This paper focuses on the influence of temperature and humidity, so the influence of light intensity is ignored in this paper.

Assumption 4 *The phenomenon of alternation of drying and wetting, freezing and thawing caused by light and rainfall is not considered.*

Assumption 5 *Assuming that the decomposition of litter and woody fiber has gone through the leaching stage and comminution stage, the model is aimed at the catabolism stage of organic matter.*

The nature of leaching and comminution stages requires additional analysis [6].

Assumption 6 *It is assumed that due to the limitation of the actual situation, the fungi would never surpass its theoretical extension rate.*

The theoretical maximum growth rate of the microorganism is based on ideal condition [5, 20]. Thus, it is not likely for the fungi to surpass the theoretical maximum growth rate.

Assumption 7 *In reality, the growth of fungi has a lower limit of humidity. In order to simplify the model, we assume that the higher the humidity, the higher the extension rate.*

There are lower limits of the humidity for the growth of the fungi [21]. However, as far as we know, there are no upper limits. Thus, it is reasonable to make this assumption.

2.2 Notations

The primary notations used in this paper are listed in Table 1. In addition, a set containing element $\cdot$ is expressed as $\{\cdot\}$. Different types of fungi are distinguished by subscripts i, e.g., F_i, V_{E_i}. Generally, symbols with the same capital letter but different subscripts represent the variables with similar properties, e.g., V_E and V_D both represent the rate, while V_E denotes the extension rate and V_D denotes the decomposition rate.

Table 1: Notations

Symbol	Definition
T	temperature (The unit is $^{\circ}C$)
T_{max}	The most suitable temperature of a type of fungi
H_l	The lower limit of the humidity of a type of fungi
λ, a, K	The undetermined parameters in optimization model (3.3)
L	The square loss of the groud-truth data and the predict data
V_E	Extension rate (The scope is $[0, 10]$ according to [5])
$V_{E_{max}}$	The theoretical maximum extension rate
V_D	Decomposition rate (The scope is $[0, 1]$)
M	The moisture trade-off (The scope is $[-1, 1]$ [5])
H	Humidity (The scope is $[0, 1]$)
G	The map in cellular automata ($\in \mathbb{N}^{500\times 500}$)
F_i	The set containing multiple cells, representing a type of fungi ($\subset \mathbb{N}^2$)
N_i	The number of cells infected by a type of fungi ($N_i = \lvert F_i \rvert$)
P_{E_i}	The extension probability defined in Definition 1
P_{C_i}	The competitive-extension probability defined in Definition 3
T_{D_i}	The decomposition time defined in Definition 2

3 Models and Results

This section describes the main models and results of our work. Specifically, Sec. 3.1 establishes the relationship among extension rate, moisture trade-off, and the decomposition rate, where the SGD algorithm is utilized to determine the model parameters. Sec. 3.2 establishes the cellular automata model to describe the growth and decomposition process of the fungi. In Sec. 3.3, we add the interactions of different fungi and re-formulate the cellular automata model. In Sec. 3.4, we analyze the proposed models from the perspective of the connections of different factors, and the sensitivity to environmental change. Finally in Sec. 3.5, we analyze the importance of biodiversity to fungal growth and decomposition.

In order to better carry out our work, it is expected to collect multiple species of fungi and their parameters. Generally, the main components of ground litter and lignocellulose include lignin, cellulose, and hemicellulose. Thus, the types of fungi we select should be able to favorably decompose these elements. Based on the information on common types of fungi [5, 26], we collected 6 types of fungi, as listed in Table 2.

3.1 Decomposition Rate Model and Parameters Estimation

This section establishes the fungal decomposition rate model, which is considered via two aspects: the extension rate and the moisture trade-off. Our model is a general framework and could be applied to all types of fungi. Specifically, we first qualitatively establish the relationship among decomposition rate and extension rate and moisture trade-off. Then, we establish

Table 2: The type of fungi selected and the corresponding parameters (Some parameters are selected from the appendix of [5]).

Fungal types	T_{max}	$V_{E_{max}}$	H_l	M
Trichoderma	$28°C$	8.23	80%	−0.60
Penicillium	$18°C$	6.46	90%	−0.80
Aspergillus	$32°C$	5.17	92%	−0.84
Rust fungi	$25°C$	1.56	80%	−0.60
Gibberella	$27°C$	7.26	90%	−0.80
Fusarium	$28°C$	4.68	70%	−0.40

the relationship among extension rate and humidity and temperature. For the uncertain parameters in the model, we use the data in the relevant literature [5] to determine these parameters by SGD, and finally we can determine the decomposition rate model.

3.1.1 Decomposition Rate Based on Extension Rate and Moisture Trade-off

The decomposition rate, which is defined as the mass loss of the wood over 122 days [5], has high relations with the extension rate and the moisture trade-off. For the extension rate, we can observe that the decomposition rate increased with the increase of the extension rate. The magnitude of this increase is approximated by the power of $1/2$, and the temperature T affects the decomposition rate with a constant ratio. Thus, the univariate function of decomposition rate with respect to the extension rate can be formulated as

$$V_D = T \times V_E^{\frac{1}{2}} + V_{D_0}^1, \tag{1}$$

where V_D denotes the decomposition rate, T denotes the temperature, V_E denotes the extension rate, and $V_{D_0}^1$ is a constant, which is undetermined.

The moisture trade-off M is defined as the difference of each isolate's competitive ranking and their moisture niche width [5]. It is believed that the fungi with a bigger moisture trade-off are likely to have a higher decomposition rate [5]. In fact, the linear correlation can be found among the moisture trade-off and $\log V_D$, i.e.,

$$\log V_D = \alpha \times M + V_{D_0}^2, \tag{2}$$

where α is the linear coefficient and $V_{D_0}^2$ is a constant. Taking exponential function on both sides and let $\lambda = 10^{V_{D_0}}$ and $a = 10^{\alpha}$, we have

$$V_D = \lambda a^M. \tag{3}$$

Here, λ, a become two undetermined parameters of the model.

Note that (1) and (3) are two independent functions that consider the influence of the extension rate and the moisture trade-off, respectively. In fact, these two factors consistently affect the decomposition rate with a synchronous manner. Thus, it is expected to unify these two factors into a single model, which derives

$$V_D = T \times V_E^{\frac{1}{2}} + \lambda a^M + K, \tag{4}$$

where K is an undetermined constant.

In the fungi decomposition rate model (4), the independent variables are the extension rate V_E and the moisture trade-off M. For M, the lower limit of the humidity of a specific specie of fungi can be used to determine M [21]. However, for V_E, it is depends on both the temperature and the humidity. Thus, it is of needs to establish a particular model to describe the relations among temperature, humidity, and the extension rate V_E.

3.1.2 Extension Rate Based on Temperature and Humidity

Fungi provide energy for themselves by decomposing organic matter. Since the decomposition mainly depends on the function of the enzyme, the activity of the decomposing enzyme greatly affects the decomposing efficiency, thus affecting the growth rate of fungi [22]. Generally speaking, the activity of decomposing enzymes is highly related to temperature and humidity. Therefore, the growth rate of fungi depends on temperature and humidity.

According to [22], the temperature rise can be linearized by the Arrhenius diagram. The slope of the Arrhenius diagram gives the activation energy, which determines V_E. Generally, the effect of humidity on the growth of fungi is positively correlated. However, for the temperature, too high temperature will inactivate the enzyme in the fungi. Therefore, we employ T_{max} to express the temperature which causes inactivation. Beyond this temperature, the extension rate would go down as the temperature still increase.

Based on the above analysis, we formulate the extension rate of the fungi with respect to the temperature T and the humidity H as the following Arrhenius diagram [23]:

$$\begin{cases} V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{T \times H}}, \quad 0 \leq T \leq T_{max} \\ V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{(2T_{max}-T) \times H}}, \quad T_{max} \leq T \leq 2T_{max} \\ V_E = 0, \quad \text{other}. \end{cases} \tag{5}$$

where $V_{E_{max}}$ denotes the theoretical maximum extension rate (According to Assumption 6, due to the limitation of the actual situation, the fungi would never surpass its theoretical extension rate). For different species of fungi, the $V_{E_{max}}$ and T_{max} are different. They are depend on the essential characteristics of the fungi.

In order to illustrate the rationality of the above model (5), we display the value of V_E with respect to different T and H in Fig. 1. We can see that when the maximum extension rate $V_{E_{max}}$ of the fungi is large, the fungi are sensitive to the change of environment (temperature and humidity). For example, the yellow line in the Fig. 1 represents a maximum extension rate of 10. When the temperature or humidity changes a lot, the extension rate of these fungi is even lower than that of the fungi with a maximum extension rate of 2.5, which is represented by the blue line. That is to say, the fungi with a higher extension rate are not robust to the change of environment. This phenomenon is consistent with the analysis in the literature [5].

3.1.3 Moisture Trade-off Based on Lower Limit of Humidity

For the moisture trade-off M (whose scope is $[-1,1]$ [5]), we directly use the lower limit data of the humidity to induce M. Specifically, according to [21], the lower limit of humidity H_l of a species of fungi is definite (The scope is $[0,1]$). Besides, the lower this limit is, the higher the moisture trade-off of the fungi species would be, as expounded in Assumption 7. Thus, we directly formulate M with linear correlation to H_l, i.e.,

$$M = -2H_l + 1. \tag{6}$$

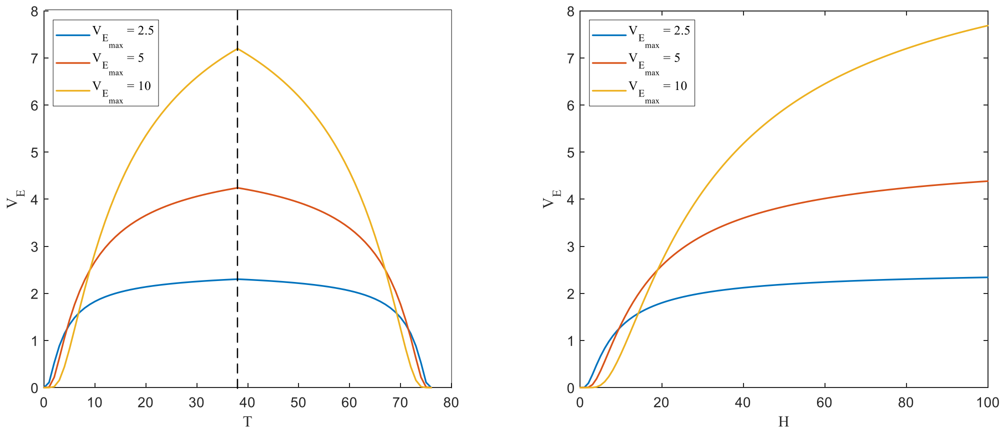

Figure 1: The test of the extension rate model (5). Left: the value of V_E with respect to different temperature T with fixed $H = 0.8$. Right: the value of V_E with respect to different humidity H with fixed $T = 38$. The optimum temperature T_{max} is set as 38.

3.1.4 Fungal Decomposition Rate Model

Based on the above analysis, the final model of the fungal decomposition rate is

$$\begin{cases} V_D = T \times V_E^{\frac{1}{2}} + \lambda a^M + K \\ \lambda, a, K \in \mathbb{R} \\ M = -2H_l + 1 \\ V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{T \times H}}, \quad 0 \le T \le T_{max} \\ V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{(2T_{max} - T) \times H}}, \quad T_{max} \le T \le 2T_{max} \\ V_E = 0, \quad \text{other.} \end{cases} \tag{7}$$

3.1.5 Parameters Estimation Using SGD

In order to establish a definite relationship of extension rate, moisture trade-odd, and the decomposition rate, we need to estimate the undetermined parameters λ, a, K in model (7). Specifically, these parameters should be determined according to the data. Our data come from the appendix of [5]. The equation (4) is a multi-parameters undetermined nonlinear equation. Therefore, it is expected to determine these parameters via minimizing the error among the decomposition rate data at hand and the predicted decomposition rate, which could be described as the following optimization model:

$$\begin{aligned} \{\lambda^*, a^*, K^*\} &= \arg \min_{\{\lambda, a, K\}} \sum_i (V_D^i - V_{D_{data}}^i)^2, \\ V_D^i &= T_{data}^i \times V_{E_{data}}^i{}^{\frac{1}{2}} + \lambda a^{M_{data}^i} + K. \end{aligned} \tag{8}$$

Here, $\{T_{data}^i, V_{E_{data}}^i, M_{data}^i, V_{D_{data}}^i\}$ are the set of data obtained from [5]. The above optimization model aims at minimizing the square error of the ground-truth data $V_{D_{data}}$ and the predicted data V_D using (4).

It is hard to find the closed-form solution of model (8) . Hence, it is of necessity to use iterative methods. In solving unconstrained optimization problems, the iterative gradient descent is one of the most commonly used methods. Since there is multiple data, we introduce the SGD algorithm [24], which is highly efficient under multiple data, to tackle model (8). Specifically, we update $\theta = \{\lambda, a, K\}$ via

$$\theta^{t+1} = \theta^t - r\nabla_\theta L, \tag{9}$$

where $L = (V_D^i - V_{D_{data}}^i)^2$ is the energy function, r is the updating rate [24], and $t = 1, 2, \cdots, I$ denotes the iteration step. Concretely, we have the following updating rules:

λ-updating:

$$\lambda^{t+1} = \lambda^t - r\frac{\partial L}{\partial \lambda} = \lambda^t - 2r(V_D^i - V_{D_{data}}^i) \times a^{t M_{data}^i}. \tag{10}$$

a-updating:

$$a^{t+1} = a^t - r\frac{\partial L}{\partial a} = a^t - 2r(V_D^i - V_{D_{data}}^i) \times \lambda M_{data}^i a^{t(M_{data}^i - 1)}. \tag{11}$$

K-updating:

$$K^{t+1} = K^t - r\frac{\partial L}{\partial K} = K^t - 2r(V_D^i - V_{D_{data}}^i). \tag{12}$$

The optimization process is described in Table 3.

Table 3: The solution of the proposed model (8).

Algorithm 1: Parameters estimation using SGD
input: r, I, dataset $\{T_{data}^i, V_{E_{data}}^i, M_{data}^i, V_{D_{data}}^i\}_{i=1}^N$
Initialize $\lambda^0 = 1, a^0 = 1, K^0 = 1$;
for $k = 1 : I$
t=mod(k,N);
Update λ via (10);
Update a via (11);
Update K via (12);
end for
output: λ^I, a^I, K^I

The loss value (L) and λ with respect to the iteration is shown in Fig. 2. We can see that during the SGD iteration, the loss gradually converges to a relatively small value, and the λ converges to 31.9 stably. Although we use SGD, where the loss value is locally unstable. However, the SGD algorithm can accelerate the iteration and convergence speed [24].

Finally, we have $\lambda = 31.90, a = 1.26, K = -64.80$, which deduces

$$V_D = T \times V_E^{\frac{1}{2}} + 31.90 \times 1.26^M - 64.80. \tag{13}$$

Based on th above solution, the calculation model of decomposition rate and extension rate are simultaneously described as follows:

$$\begin{cases} V_D = T \times V_E^{\frac{1}{2}} + 31.90 \times 1.26^M - 64.80 \\ M = -2H_l + 1 \\ V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{T \times H}}, \quad 0 \le T \le T_{max} \\ V_E = V_{E_{max}} e^{-\frac{V_{E_{max}}}{(2T_{max} - T) \times H}}, \quad T_{max} \le T \le 2T_{max} \\ V_E = 0, \quad \text{other.} \end{cases} \tag{14}$$

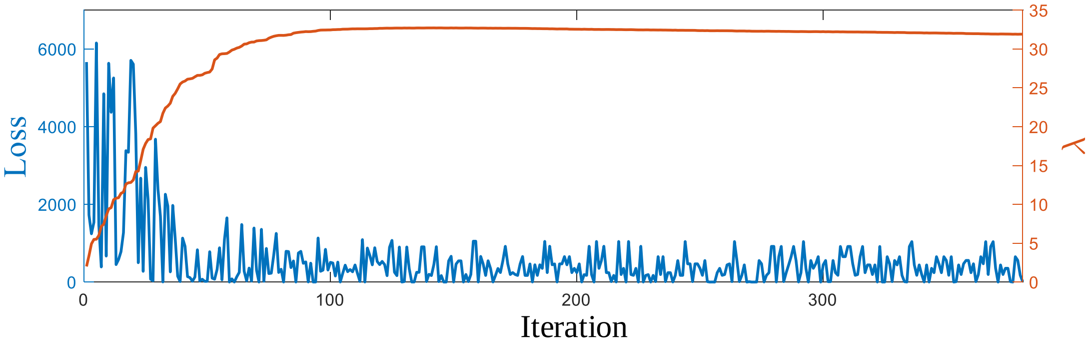

Figure 2: The loss value (L) and λ with respect to the iteration using algorithm in Table 3.

Specifically, given the temperature T and humidity H, we can compute the extension rate V_E of different species of fungi with different $V_{E_{max}}, T_{max}$. Next, the decomposition rate V_D is computed by combining the extension rate V_E and the moisture trade-off M (determined by the tolerance of different fungi to water). This model provides the basis for our cellular automata model to simulate the decomposition of organic matter by fungi.

3.2 Cellular Automata Model for Breakdown Process

The first question is to model the breakdown process by the fungi. In practice, the rate of decomposition by fungi is not constant due to various conditions, so the decomposition time is dynamic. In order to better simulate the decomposition process, we suggest the cellular automata to address this problem.

Specifically, the set (containing multiple cells) that represents the fungi is denoted by $F_i \subset \mathbb{N}^2$. The whole map grid (represents the woody fibers) is $G \in \mathbb{N}^{500\times 500}$. At the beginning stage, every grid is set to 1, i.e., $G(p,q) = 1, \forall s,t$. Then, for each grid, there is a initial possibility to be infected by the fungi F_i (if (p,q) is infected by F_i, then we update $G(p,q) = i+1$). After the initialization, F_i begins to extend to other grids. Once F_i extends to a new grid, it begins to decompose this grid, which takes some time. After the decomposition, the wood disappears, and the fungi on the grid die for lack of hosts. The whole simulation process of cellular automata describes the dynamic process of fungi growth and decomposition on wood. However, some parameters in cellular automata, such as the rules of invading adjacent cells and the decomposition time, need to be further determined.

Here, we use the extension rate V_E and decomposition rate V_D that we calculated before to determine the parameters in the cellular automata.

3.2.1 Relation Between Cellular and Fungal Extension

For the extension process of fungi, we use the infection of adjacent cells in cellular automata as the simulation of growth. Naturally, this process is parameterized by the extension rate V_E. However, we can not directly use V_E in the cellular automata since the simulation process is a discrete process. In such a condition, not only the number but also the position of the cells should be studied. Thus, we propose to transform V_E into a probability factor to determine whether an uninfected grid would be infected by the fungi in the adjacent grid.

According to [25], the *sigmoid* function can commendably describe the process of biological growth. For fungi, it is also a kind of organism, so some of its behaviors and characteristics can be simulated by *sigmoid* function. Given the extension rate V_E, we propose to use *sigmoid* function to transform V_E into a controllable probability factor. The probability (determined via the extension rate V_E) of a fungus is used to determine whether the fungi cell F_i could extend into adjacent lattices. Specifically, we induce the following definition.

Definition 1 (*Extension probability*) *Given the fungi cell F_i with extension rate V_{E_i}, the extension probability of F_i is defined as*

$$P_{E_i} = sigmoid(\frac{1}{V_{E_i}}) = \frac{1}{1+e^{-\frac{1}{V_{E_i}}}}. \tag{15}$$

This probability is used to determine the extent of the growth of the fungi cell. Specifically, for each uninfected cell, if it is adjacent to an infected fungi F_i, then the grid may be infected with probability P_{E_i}, which is determined by the extension rate V_E. When the extension rate increases, P_{E_i} increases (with upper limit 1). Then the probability of cell infection increases, which on the other hand describes the rate of the fungal extension. Thus, the validity of Definition 1 is justified.

Based on the above analysis, for each grid (p,q) that is not being infected, we do the following operation in the time t:

$$\begin{cases} \overline{F_i^{t+1}} = F_i^t \cup \{(p,q)\}, \ \xi \le P_{E_i} \\ \overline{F_i^{t+1}} = F_i^t, \ \xi > P_{E_i}, \end{cases} \tag{16}$$

where $\xi \sim U(0,1)$. The above formula is the growth simulation process of fungi. Note that there are grids $\{p_D^t, q_D^t\}$ that being fully decomposed by the fungi F_i, thus we do

$$F_i^{t+1} = \overline{F_i^{t+1}} \backslash \{(p_D^t, q_D^t)\}. \tag{17}$$

For the total number of fungi N_i (the number of grids infected by F_i), we have $N_i = |F_i|$, where $|\cdot|$ denotes the number of elements in the set. For the variation of the number of cells infected by fungi, we have the following differential form:

$$\frac{dN_i}{dt} = (N_i^{t+1} - N_i^t) - N_{D_i}^t, \tag{18}$$

where $N_{D_i}^t$ denotes the number of cells that fully decomposed by the fungi F_i at time t. The equation (18) is crucial since we can use it to observe the variation of the number of fungi in the simulation process.

3.2.2 Relation Between Cellular and Fungal Decomposition

Similar to the extension rate, the decomposition rate can not be directly used in the cellular automata model. Therefore, it is of necessity to convert the decomposition rate calculated previously into the decomposition time of a fungus to fully decompose a cell that being infected. Naturally, when the decomposition rate is high, the decomposition time is relatively low [26]. Therefore, we propose the following definition of decomposition time.

Definition 2 (*Decomposition time*) *The decomposition time of fungi F_i with decomposition rate V_{D_i}, which is the time of a fungus in a cell to fully decompose the woody fiber, is defined as*

$$T_{D_i} = c\frac{122}{V_{D_i} \times 500^2}, \tag{19}$$

where c is a correction factor. The unit of T_{D_i} is day.

122 is the number of days in the exepirments in [5], and 500 is the size of G. In our work, c is set as 5×10^6. Given the decomposition time T_{D_i}, we can simulate the decomposition process of the fungi in a cell. Consequently, according to Eq. (17), the decomposition time is a factor that influences the cellular automata model.

3.2.3 Cellular Automata Model

Based on the above analysis, let n denots the number of fungal species, the proposed cellular automata model to simulate the decomposition process is formulated as:

$$\begin{cases} F_i^{t+1} = (F_i^t \cup \{(p,q)\}) \backslash \{(p_D^t, q_D^t)\}, \ \xi \leq P_{E_i} \\ F_i^{t+1} = F_i^t \backslash \{(p_D^t, q_D^t)\}, \ \xi > P_{E_i} \\ \xi \sim U(0,1), \ i = 1, \cdots, n \\ P_{E_i} = sigmoid(\dfrac{1}{V_{E_i}}) = \dfrac{1}{1+e^{-\frac{1}{V_{E_i}}}} \\ T_{D_i} = c\dfrac{122}{V_{D_i} \times 500^2}. \end{cases} \tag{20}$$

3.2.4 Model Solution

The model (20) is a simulation framework based on the cellular automata. Thus, the solution can be obtained by stimulating the decomposition process via time iteration. The algorithm of the simulation process is described in Table 4. Note that during the simulation, we consider the falling leaves which provide uninfected cells (We change the cells that are fully decomposed into uninfected cells, i.e., $G(p,q) = 1$ to simulate the falling leaves). In this section, we select 3 types of fungi to conduct the experiments, i.e., Trichoderma, Penicillium, and Aspergillus (see details in Table 2). Also, we choose the environment in tropical rain forest for question 1, 2, and 3 (The details of different areas can be seen in Table 5). The simulation results are shown in Fig. 3.

From the left figure of Fig. 3, we can see that the wood fiber is infected and decomposed by different fungi. This kind of infection and decomposition happen all over woody fiber, i.e., fungi can use all the nutrients in woody fiber when they grow without restriction. From the middle figure of Fig. 3, we can see that in the initial stage, which is about $5-10$ days, fungi infect the woody fiber at an approximately exponential rate. After 10 days, however, the number of fungi began to decline. This is because the nutrient content of wood (that is, the number of uninfected wood in the cell map) is decreasing, while the decomposed wood is increasing. In addition, the existence of several fungi led to the rapid consumption of nutrients. Therefore, the growth environment of fungi became worse, and the number of fungi began to decrease due to the lack of nutrients. After about 40 days, the number of fungi tends to be more stable around 6×10^4. The decomposition rate after 122 days is 0.68.

From the right figure of Fig. 3, the change rate of the number of fungi is very high at first and then tends to be stable. This is consistent with the results in most laboratories [26, 10, 16]. Because in the beginning, the nutrients are sufficient and the growth rate is high. After that, the nutrients decreased and most of the wood was decomposed, and at the same time there are fallen leaves that provide extra nutrients (not enough for the fungi for further extension), so the rate of change of the number of fungi tends to be 0.

3.3 Cellular Automata Model Under Interactions Among Fungi

In this section, we address the second problem. In order to describe the interaction among different species of fungi, we mainly consider the changes of the extension rate, where the interaction among organisms mainly comes from the perspective of growth [25]. Specifically, we should consider whether there are mutually beneficial and mutually exclusive effects among

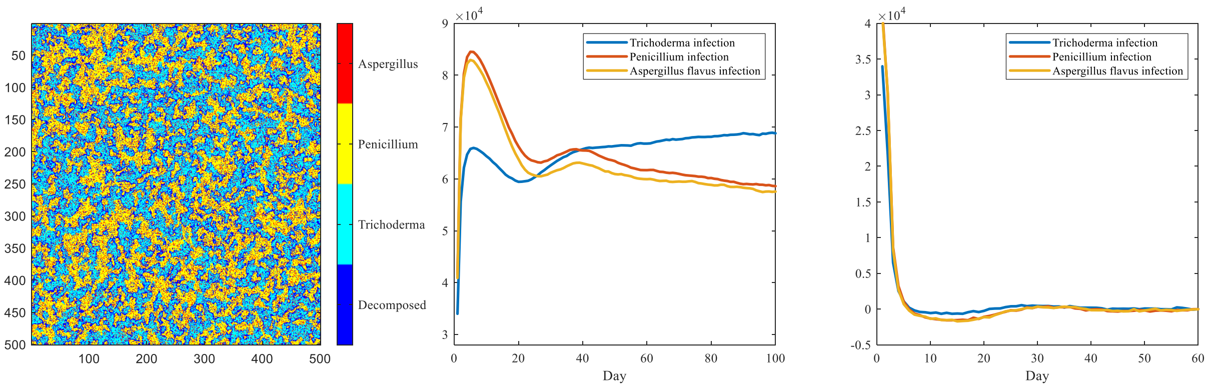

Figure 3: The simulation results of question 1. Left: The illustration of woody fiber infected by different species of fungi and decomposed by the fungi. Middle: The number of different fungi N_i with respect to the time. Right: The difference of the number of different fungi $\frac{dN_i}{dt}$ with respect to the time.

Table 4: The algorithm corresponding to model (20).

Algorithm 1: The fungal extension and decomposition simulation process

input: $G \in \mathbb{N}^{500\times500}$, t_{max}, $\{T_{max_i}\}$, $\{V_{E_{max_i}}\}$, $\{M_i\}$
Initialize $F_i \in \mathbb{N}^2, i = 1, \cdots, n$;
for $t = 1 : t_{max}$
  for $(p,q) = (1,1) : (500,500)$
    Find adjacent points $\{(p_j, q_j)\}$;
    if $G(p_j, q_j) \neq 1$
      Compute P_{E_i} via (15);
      Compute T_{D_i} via (19);
      Update F_i via (16) and (17);
      $G(p_j, q_j) = i + 1$;
    end if
  end for
  $N_i^t = |F_i^t|$; Compute $\frac{dN_i}{dt}$ via (18);
end for
output: $\{N_i\}, \{\frac{dN_i}{dt}\}$

fungi. This effect should be quantified as the effect on extension rate. Then, cellular automata was used to re simulate the activity of fungi under intrinsic interactions.

3.3.1 Model Establishment

When modeling the relationship among fungi of different species, we should have considered two aspects: mutual benefit and mutual exclusion. However, as far as we konw, the mutual benefit exists only among fungi and other organisms [27, 28]. Therefore, when we consider the relationship among fungi, we mainly consider the competitive relationship. That is to say, the extension rate V_E would decrease under interactions of different fungi. Following our previous work (Definition 1), the extension probability would change. We induce the following definition of the extension probability under fungal interactions.

Definition 3 (*Competitive-extension probability*) *Given the fungi cell F_i with extension rate V_{E_i} and the competitive fungi $\{F_j\}$ with quantity $\{N_j\}$, the competitive-extension probability of F_i*

is defined as

$$P_{C_i} = sigmoid(\frac{1}{V_{E_i} \times (1 - \frac{\sum_j |N_j|}{500^2})}) = \frac{1}{1 + e^{-\frac{1}{V_{E_i} \times (1 - \frac{\sum_j |N_j|}{500^2})}}}. \tag{21}$$

Evidently, P_{C_i} is lower than P_{E_i} due to the percentage factor $(1 - \frac{\sum_j |N_j|}{500^2})$. This factor considers that the extent of the impact of the competing fungi on the fungi F_i is related to the total number $\sum_j |N_j|$. When the number of the competing fungi is large, the impact is considerable. This is consistent with the analysis in [27]. Thus, the validity of Definition 3 is justified.

Based on the above analysis, the cellular automata model with fungi interactions can be formulated by re-written model (20) as

$$\begin{cases} F_i^{t+1} = (F_i^t \cup \{(p,q)\}) \backslash \{(p_D^t, q_D^t)\}, \ \xi \leq P_{C_i} \\ F_i^{t+1} = F_i^t \backslash \{(p_D^t, q_D^t)\}, \ \xi > P_{C_i} \\ \xi \sim U(0,1), \ i = 1, \cdots, n \\ P_{C_i} = sigmoid(\frac{1}{V_{E_i} \times (1 - \frac{\sum_j |N_j|}{500^2})}) = \frac{1}{1 + e^{-\frac{1}{V_{E_i} \times (1 - \frac{\sum_j |N_j|}{500^2})}}} \\ T_{D_i} = c \frac{122}{V_{D_i} \times 500^2}. \end{cases} \tag{22}$$

3.3.2 Model Solution

The algorithm to solve model (22) is similar to that of model (20). Thus, the algorithm can be also described as Table 4. The only difference is the computation of the probability. In Table 4, the probability is computed via (15). Here, the probability is computed via (21).

The results are shown in Fig. 4. According to the left figure of Fig. 4, the infection and decomposition of wood are overall and random. Among them, the number of decomposed wood is significantly reduced compared with the result of the first question. Specifically, the number of fungi after stabilization is around 4.5×10^4, which is concretely 33.65% lower than that of the situation without fungal competition. According to the middle figure of Fig. 4, similar to the result of the first question, the number of fungi first increased and then decreased. According to the right figure of Fig. 4, the rate of change in the number of fungi is very large at first and then tends to 0. The decomposition rate after 122 days is 0.39, which is 42.69% lower than that of the situation without fungal competition.

Unlike the first question, we consider the competition among different types of fungi in this section. Under such consideration, our experimental results are quite different from the first question. Specifically, according to the middle figure of Fig. 4, although the overall growth trend of fungi increased and then decreased, the overall number of fungi decreased significantly compared with the result of the first question. This is because we consider the competition among fungi. Based on Definition 3, fungi are less likely to infect wood. Therefore, the general number of fungi has decreased significantly.

3.4 Analysis and Experimental Results

We answer the third and fourth questions in this section. First, we conduct an overall analysis of the previous models. Then, we analyze the competitive relationship among different

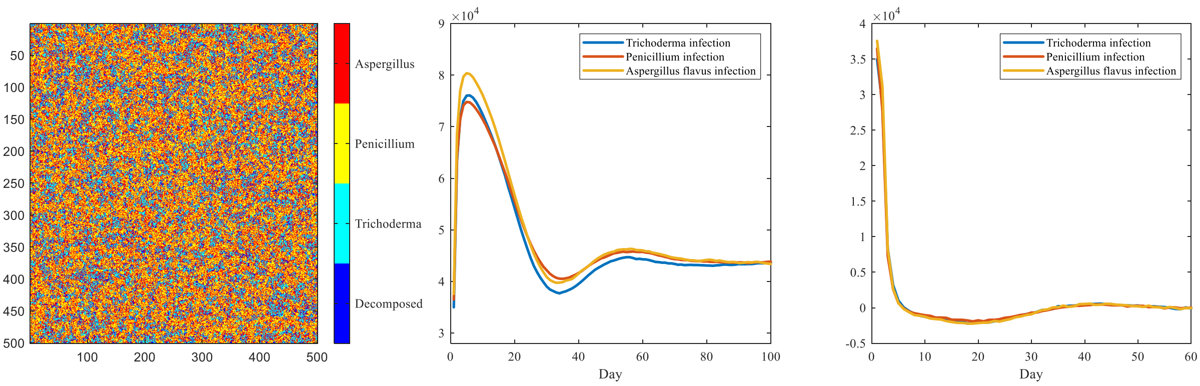

Figure 4: The simulation results of question 2 (Fungal decomposition and growth simulation under interactions of different species of fungi). Left: The illustration of woody fiber infected by different species of fungi and decomposed by the fungi. Middle: The number of different fungi N_i with respect to the time. Right: The difference of the number of different fungi $\frac{dN_i}{dt}$ with respect to the time.

species. Then, we analyze the sensitivity of environmental changes (mainly the changes of temperature and humidity). Finally, we simulate the extension and decomposition of different fungal species in different environments and analyze the advantages and disadvantages of different fungal species in different environments.

3.4.1 Overall Analysis

The models we established are intrinsically linked. This connection is quantified by various parameters. The specific connection can be described as Fig. 5. Started with the fungal type, the lower limit of the humidity and the maximum extension rate of this type of fungi are determined. Then, the extension rate and the moisture trade-off are computed. Using model (7) and (8), the decomposition rate is determined. Next, we induce three definitions which respectively consider the extension probability, the decomposition time, and the competitive extension probability. Finally, the cellular automata models (20) and (22) are established according to these factors.

Based on the above analysis, our model has good flexibility and a robust relationship structure. In addition, our consideration is based on the actual situation of fungal decomposition, so it has good credibility. We formulate the decomposition rate according to the moisture trade-off and the extension rate, which is inspired by [5]. Our experimental results demonstrate that our motivations are correct, where the growth process of fungi and the decomposition process of woody fibers depend on the types and parameters of fungi, as well as the changes of environment (to be discussed in the next part).

3.4.2 Interactions Among Different fungi

In model (22), we summarize the internal interaction among different types of fungi as the competition among different types of fungi. We put forward a definition (Definition 3). This definition considers the effect of another species on a type of fungi mainly based on the extension rate. A percentage factor is used to reduce the original extension rate. This competitive relationship is relative, so it will affect the whole process of decomposition and growth.

From the perspective of the short run, according to Fig 3 and Fig. 4, the additional consideration of fungal competition does not have much impact. This is because in the initial stage, the remaining amount of wood, that is, nutrients, is sufficient. Even if the growth rate of fungi is limited, their growth process takes place in a similar exponential form. Therefore, in the short term, the impact of competition is not obvious.

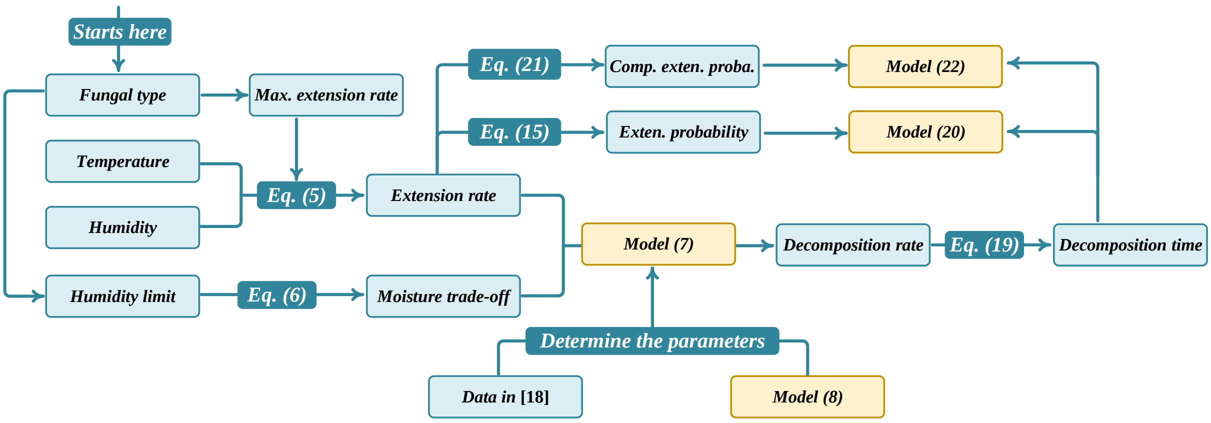

Figure 5: The overall analysis of the relationship of different parameters and models of this paper.

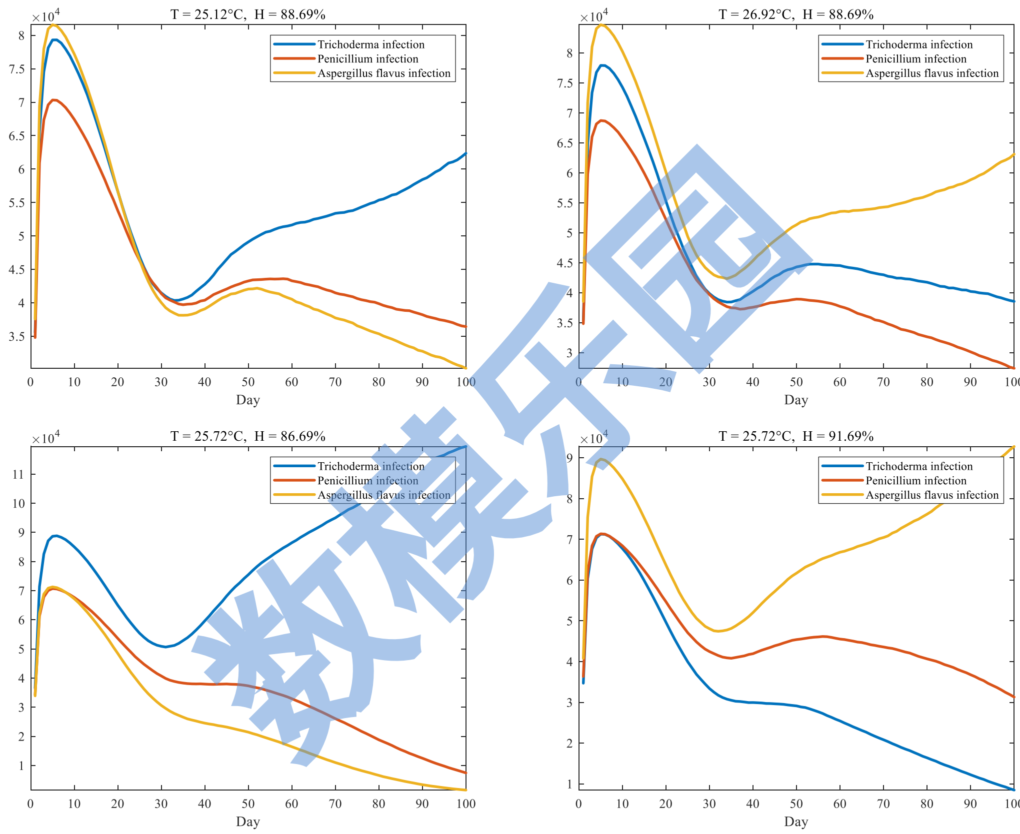

Figure 6: The sensitivity analysis of different temperature and humidity.

However, from the perspective of the long run, the impact of considering the fungal competition is huge. From the middle figures of Fig. 3 and 4, we can find that when we do not consider the interaction, every species of fungi grows at an independent pace, where the intensity of the number N_i is much higher than that of Fig. 4. After considering the interaction, the general number N_i decreased a lot. This is because when the woody fiber being decomposed, the nutrients are becoming more and more limited. In such conditions, competition among fungi makes their growth extremely slow. Thus, their quantities N_i are very low.

3.4.3 Sensitive Analysis of Environment

In order to prove that our model has a good internal relationship, especially can make a good response to environmental changes, we carry out the sensitivity analysis of environmental changes (temperature and humidity). Specifically, we use three fungal types, i.e., Trichoderma,

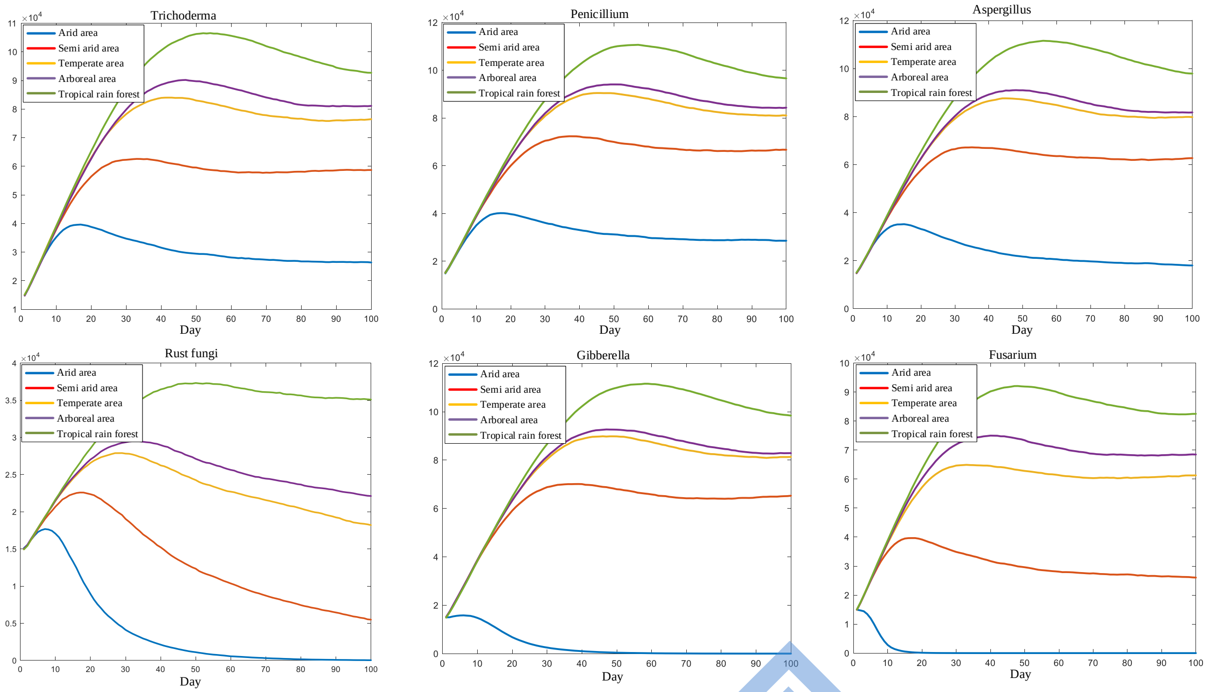

Figure 7: The number of fungi N_i with respect to the time under different environments of different types of fungi.

Penicillium, Aspergillus to simulate the decomposition process under different environments with slightly different temperatures and humidities. The results of the sensitivity analysis are shown in Fig. 6.

In Fig. 6, we can observe that the proposed models are sensitive to considerable changes in the environments. Specifically, an 8% change of the temperature would lead to an impact on the final determination of the dominant species. Meanwhile, there is an impact of the change of the humidity on the final dominant species. The sensitivity of our models is mainly because of the connection of the environmental factors and the extension rate as well as the competitive interactions. Also, different species of fungi have different T_{max} and H_l, which leads to the sensitiveness of the proposed model with different temperatures and humidity.

From our sensitivity test results, we can see that the temperature change near the optimal temperature will have a greater impact on the activity of fungi. If the local atmosphere changes abnormally, the temperature or humidity will change greatly and may deviate from the optimal temperature or humidity limit of fungi, so the growth of fungi and the decomposition efficiency of woody fiber will be reduced. Specifically, for the local tropical rain forest area, if the temperature decreases or increases in the future (deviates from the optimal temperature of fungi), the extension and decomposition rate of fungi will decrease. In terms of humidity, the activity of fungi will be greatly limited when the humidity drops and becomes relatively dry in autumn and winter.

3.4.4 Simulations Under Different Combinations and Regions

Next, we conduct more experiments to verify the effectiveness of our models. Specifically, we conduct the simulation on different conditions, i.e., the arid area, the semi-arid area, the temperate area, and the tropical rain forest area. Specific temperature and humidity data is shown in Table 5 (refered from https://gis.ncdc.noaa.gov/maps/ncei/summaries/daily). Meanwhile, we employ different species of fungi in these areas to see the corresponding results.

First, we simulate each selected species of fungi (they have different model parameters) in

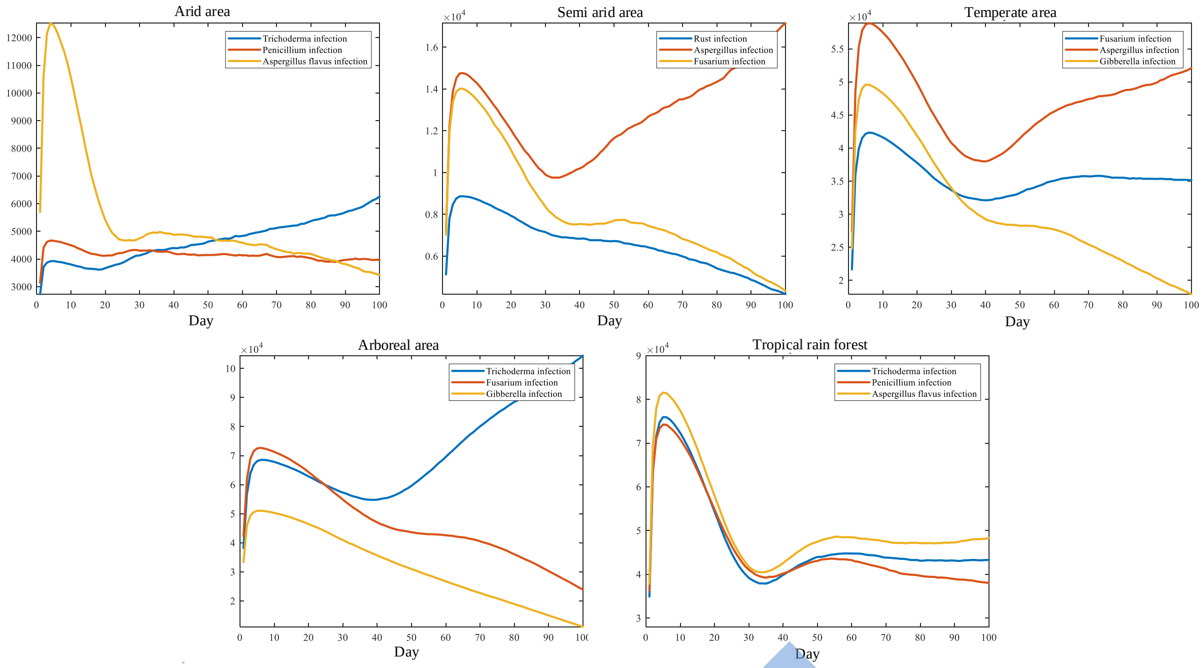

Figure 8: The number of fungi N_i with respect to the time under different environments with the competitive combination of different fungi.

Table 5: The types of environments and the corresponding parameters (We use the temperature and humidity data from a specific observation point whose longitude and latitude are displayed).

Areas	T	H	longitude	latitude
Arid area	$20.4°C$	0.16	$87.7°E$	$40.6°N$
Semi arid area	$22.12°C$	0.39	$118.8°E$	$42.3°N$
Temperate area	$27.63°C$	0.56	$102.2°W$	$39.2°N$
Arboreal area	$26.07°C$	0.69	$82.2°W$	$29.2°N$
Tropical rain forest	$28.72°C$	0.88	$61.2°W$	$5.8°S$

five different environments. The results are shown in Fig. 7. We can see that the growth of each fungus in different environments is similar, but their number N_i is not consistent, which is due to their maximum extension rate $V_{E_{max}}$ is not the same. The tropical rain forest occupied the highest number of growth in all simulation results. This is because in the tropical rain forest, the humidity is very sufficient, and the temperature is more appropriate. In such conditions, all types of fungi can favorably grow and decompose the woody fiber. However, some types of fungi are difficult to grow in arid environments. Especially for Rust fungi, Gibberella, and Fusarium, the number decreased to 0 in an arid area, which reflected that these three kinds of fungi were not suitable for living in this environment.

Then, we simulate the growing and decomposition process of fungal species combinations (under their competitive interactions) in different areas. The results are shown in Fig. 8. The simulation results show that in arid areas, the studied species are Trichoderma, Aspergillus, and Penicillium, in which the dominant species is Trichoderma and the inferior species is Aspergillus. In the semi-arid area, the studied species are Rust fungi, Aspergillus and Fusarium, in which the dominant species is Aspergillus and the inferior species is Rust. In the temperate zone, Fusarium, Aspergillus, and Gibberella are the studied species, in which Aspergillus is the dominant species and Gibberella is the inferior species. In arboreal areas, the studied species are Trichoderma, Hyphomycetes, and Gibberella, in which the dominant species is Trichoderma and the inferior species is Gibberella. In the tropical rain forest area, the studied species are Tri-

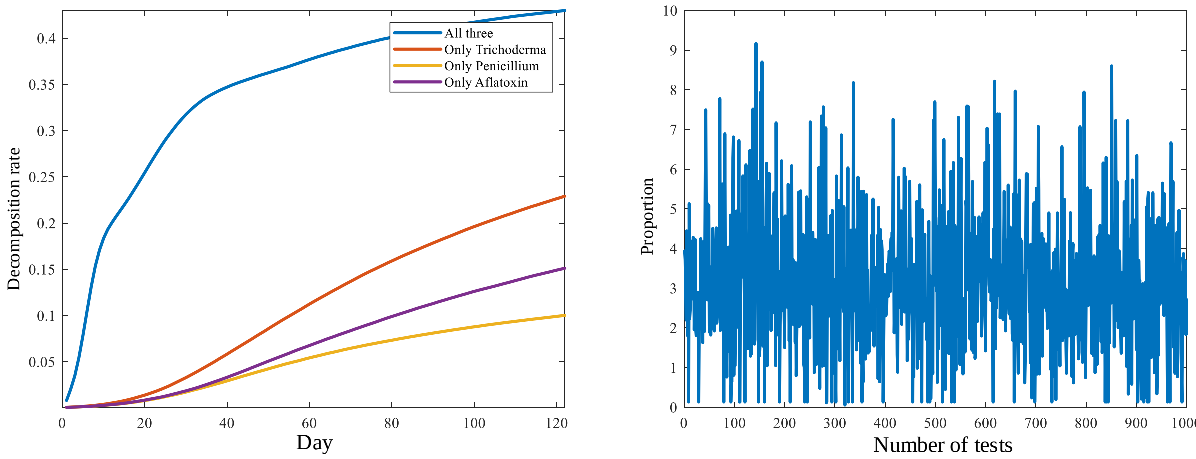

Figure 9: Left: The decomposition rate with respect to the time under different combinations of the species of fungi. Right: The proportion between the decomposition rate of "All three" and the decomposition rate of a single species of fungi with respect to the number of tests.

choderma, Penicillium, and Aspergillus, in which the dominant species is Aspergillus and the inferior species is Penicillium. In these simulation results, the decomposition rate is different, e.g., the decomposition rate of fungi in the tropical rain forest area is 68.29% after 122 days. Here, we do not elaborate on all the decomposition rates due to the page limit.

It is notable that the growth process of different fungi combinations in the five types of environments is very different. This is due to two reasons. First, the temperature and humidity in different environments are very different, which changes the growth process of fungi. Secondly, different combinations of fungi have different optimal temperatures and moisture trade-off, which also change the growth process. Specifically, we can find that in two areas, the Trichoderma obtains the highest number N_i. This is due to high $V_{E_{max}}$ value ($V_{E_{max}} = 8.23$) of Trichoderma, and the value of the most suitable temperature T_{max} ($T_{max} = 28°C$) is also around the most common temperature at earth. Thus, Trichoderma could gain superiority against other species of fungi under competitive interactions.

3.5 Influence of Species Diversity

This section responds to the fifth question, i.e., we study the importance of biodiversity on the decomposition of the woody fibers. Specifically, we simulate the decomposition process with three single types of fungi and their combinations, respectively. The results are shown in Fig. 9. Here, we plot the decomposition rate with respect to the time in the left figure. Then, we re-conduct the experiments 1000 times and compute the proportion between the decomposition rate of "All three" and the decomposition rate of a single species, where the proportion with respect to the number of the experiment is shown in the right figure of Fig. 9.

From the left figure of Fig. 9, we can clearly observe that when the three fungal species are combined (that is, the biodiversity increases), the decomposition rate is much higher than that of a single species. Also in the right figure of Fig. 9, we can find that at the most time the decomposition rate of "All three" combination is considerably larger than that of a single species of fungi. Specifically, the average proportion is 2.89. Therefore, in this environment, the impact of species diversity is huge. This is basically because multiple species not only bring the high quantity of the fungi but also increase the robustness of multiple species, i.e., if the environment (temperature and humidity) is not adapted to one of the fungal species, there may be another fungal species adapted to this environment. Therefore, the decomposition rate under biodiversity can be guaranteed. That is to say, biodiversity is very important to the effectiveness of fungal decomposition in the ecosystem.

4 Strengths and Weaknesses

In this paper, we propose to model the relation of the extension rate, moisture trade-off, and the decomposition rate via SGD. The final relationship is described as (14). Then, we employ the cellular automata to simulate the decomposition process. Three definitions are subsequently induced to guide the simulation process. Extensive experiments on multiple types of fungi and different environments forcefully demonstrate the effectiveness and robustness of the proposed models. The strengths of the proposed model are as follows.

4.1 Strengths

- **Robust relation description** Our relational model (7) is nonlinear and can describe relationships among extension rate, moisture trade-off, and decomposition rate robustly. Meanwhile, the parameters of this model are adaptively fitted according to the literature [5] data, which is accurate.
- **Efficient algorithm** We employ the SGD algorithm to tackle the non-convex and nonlinear optimization problem (8). According to the good generalization property of SGD [24], the iterative process can commendably converge to some critical points.
- **Accurate simulation** We employ the discrete cellular automata to adaptively and precisely simulate the decomposition and extension process of the fungi, with a concise and efficient implementation. The cellular automata model (20) can be addressed via simple time iteration, which is computationally efficient and has high consistency with the real situation.
- **Intrinsic connections** Three definitions (Definition 1, 2, and 3) are introduced to robustly characterize the extension process in the cellular automata, the decomposition process of the fungi, and the extension rate under interactions of different fungi, respectively. Thus, the influence of all factors is connected and reflected simultaneously in cellular automata, which leads to precise modeling of the activity of fungi.
- **High flexibility and portability** Our model has multiple parameters, e.g., $V_{E_{max}}, M, T$, et al. These parameters represent the essential characteristics of fungi and environmental parameters. Therefore, our model is considerably flexible and sensitive to different fungal types and environments (basically temperature and humidity). Hence, it is easy to apply our model under different conditions.

However, there are weaknesses of the proposed models:

4.2 Weaknesses

- **Difficulty in parameter adjustment** Although the multiple parameters in our model ensure the flexibility of the model, the adjustment and setting of parameters require additional data collection, which would be time-consuming and difficult if the studied objects have no open access data.
- **Lack of theoretical explanation** Our model gives the empirical relationship among various factors and stimulates the growth and decomposition of fungi in different situations. However, the theoretical biological mechanism of these phenomena has not been identified and needs further study.

5 Article

Recent Advances in the Role of Fungi in Ecosystems

Any ecosystem has the basic functions of energy flow, material circulation, and information transmission.

The material cycle in the ecosystem includes two relative processes: biosynthesis and biodegradation. The former is the organic process of inorganic matter, which is mainly completed by green plants and some inorganic nutrient microorganisms. The latter is the inorganic process of organic matter, which is mainly completed by forest microorganisms, that is, the decomposition process of forest litter. Litter decomposition generally includes three stages: leaching stage, crushing stage, and organic matter catabolism stage.

There are two groups of soil organisms related to material decomposition, namely soil animals and soil microorganisms. They are not only different in morphology and structure, but also play different roles in the process of material decomposition. From the process of litter decomposition, the decomposition of animals is mainly mechanical fragmentation, while microorganisms participate in the biochemical process of litter decomposition.

Soil microorganism is the general term of microorganisms living in the soil. As decomposers, they are an indispensable part of the forest ecosystem. The diversity of nutrient substrates in the litter layer determines the diversity of decomposers. Bacteria, actinomycetes, fungi (including yeasts), algae, and protozoa are all involved in the decomposition of organic matter. Fungi, especially filamentous fungi, are thought to play a major role in litter decomposition.

The decomposition of litter by fungi is an enzymatic process based on the level of decomposer cells. The hyphae of fungi can penetrate into the plant debris and secrete extracellular enzymes to degrade the recalcitrant substances (such as lignin) contained in the litter, so as to soften the plant debris and change the structure and chemical composition of the litter. Based on this, fungi play the following important roles in the ecosystem.

- Participate in the carbon cycle and maintain ecosystem balance. The CO_2 produced by microbial respiration accounts for about 80% of the total CO_2 required by plant photosynthesis, and fungi account for about 13%. Therefore, fungi are closely related to maintaining the balance of forest ecosystems and play a crucial role in the global terrestrial carbon cycle.

- Participate in soil nutrient cycle and enhance forest productivity. Nitrogen and phosphorus are essential nutrients for life activities. Fungi participate in the decomposition of organic nitrogen in litter and transform it into mineral ions NH_4^+ and NO_3^-, which can be absorbed and utilized by forest plants. Phosphorus is temporarily enriched and then released by microorganisms, which increases the availability of phosphorus and improves the utilization efficiency of phosphorus by forest plants.

- Participate in phytoremediation to improve plant stress resistance. In soils contaminated by metals or organic compounds, mycorrhiza may reduce the bioabsorbable concentration of pollutants in the soil by increasing the degradation of organic pollutants in the mycorrhizal zone, thus contributing to plant resistance.

- Decomposition of plant litter, promote the formation of humic acid. Soil fungi can promote the formation of humic acid from the organic matter around roots. Humic acid salt contains a large number of functional groups, which can not only improve the soil but also stimulate crop growth. The increase of humic acid in the soil directly promotes the growth and development of plants.

- Provide a barrier to reduce pathogen invasion. In order to protect the root growth of plants from pathogens and insects, soil microorganisms such as fungi in the rhizosphere will parasitize and colonize around the roots of soil plants, forming a physical protective layer.

However, the efficiency of fungal growth and decomposition is influenced by the environment and the interaction among different species of fungi. In our study, we come to the following conclusion.

- The influence of fungal species interaction

From the perspective of the short run, there was no obvious effect on the interaction between different kinds of fungi. This is because in the initial stage, the remaining amount of wood, that is, nutrients, is sufficient. Even if the growth rate of fungi is limited, their growth process takes place in a similar exponential form. Therefore, in the short term, the impact of competition is not obvious. However, from the perspective of the long run, the impact of considering the fungal competition is huge. When we do not consider the interaction, every species of fungi grows at an independent pace, where the intensity of the number N_i is much higher. After considering the interaction, the general number N_i decreased a lot. This is because when the woody fiber being decomposed, the nutrients are becoming more and more limited. In such conditions, competition among fungi makes their growth extremely slow. Thus, their quantities N_i are very low.

- The influence of environment on fungi

From our sensitivity experimental results, the temperature change near the optimal temperature will have a greater impact on the fungal activity. If the local weather changes abnormally, the growth of fungi and the decomposition of woody fiber will have lower efficiency and more unpredictable changes. For humidity, if it is too dry, the activity of fungi would be greatly limited as well.

In a word, fungi, as decomposers, play an important role in the ecosystem. In abstract terms, the vital significance of fungi decomposition is to release these chemical substances combined with organisms into nature again. There are limited chemical substances that can be used by living organisms. If these chemical substances are always bound to the dead organisms indefinitely and cannot be utilized by living organisms, then the life of organisms will end ultimately.

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