Computer System

- Hardware
  - I/O
  - CPU
    - ALU (Airthmatic Logic Unit)
    - CU (Control Unit)
  - memory
    - Primary
      - RAM
      - ROM
    - Secondary
      - Harddisk
- Software

\# If we run the code the memory storage in RAM.

\# If we save the code the memory storage in Harddisk.

ALU → +, −, *, ÷, And, not, OR
ADDER, MULTIPLEIR, DIV → Logic Gates

CU → Decoder max

Number System

Base 1 {0}
Base 2 {0,1}
Base 4 {0,1,2,3}
Base 8 {0-7}
Base 10 {0-9, A}
Base 11 {0-9, A}
Base 16 {0-9, A, B, C, D, E, F}

$(012)_3$ $\quad x_i < r_i$

$b_1 \longrightarrow b_2$

If express in 2
$b_1 \rightarrow$ binary
binary $\rightarrow b_2$

$b_1 \longrightarrow b_2$
not express in 2
$b_1 \rightarrow 10$
$10 \rightarrow b_2$

Decimal to Binary convert

$$\begin{array}{r|l} 2 & 7 \\ 2 & 3 \quad 1 \rightarrow LSB \\ 2 & 1 \quad 1 \\ & 0 \quad 1 \rightarrow MSB \end{array}$$

$(7)_{10} \rightarrow (111)_2$

MSB $\rightarrow$ most Significant byte

in Decimal convert

$(ABC)_n$

$A \times n^2 + B \times n^1 + C \times n^0$

Decimal	Binary 2^1	Octal 2^3	hexadecimal 2^4
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A

$(1011001011)_2 \longrightarrow (\quad)_8, (\quad)_{16}$

$\Rightarrow (1313)_8$

$\Rightarrow (2CB)_{16}$

$(65)_{10} \longrightarrow (\quad)_2$

$$\begin{array}{r|c|c} 64 & 65 & 1 \\ \hline 64 & 1 & 1 \\ & 0 & 1 \end{array}$$

$(65)_{10} \longrightarrow (11)_{64}$

$(11)_{2^6} \rightarrow (\quad)_2$

$(1000001)_2$

$(12AB)_{16} \rightarrow (\quad)_{10}$

$1 \times 16^3 + 2 \times 16^2 + 10 \times 16^1 + 11 \times 16^0$

$4096 + 512 + 160 + 11$

$\Rightarrow 4779$

$(1056)_8 \longrightarrow (\quad)_2$

$(1000101110)_2$

$(1056)_{16} \longrightarrow (\quad)_2$

$(0001000001010110)_2$

Q What is the minimum number of bits required to represent $(6728)_{10}$ in binary.

$(\_ \_ \_ \; \frac{n-1}{n^2} \frac{n-1}{n^1} \frac{n-1}{n^0})_n$

$(n-1)n^{n-1} + (n-1)n^n \;---\; (n-1)n + (n-1)n^0$

$(n-1)(1 + n + n^2 \;---\; n^n)$

$\cancel{(n-1)}\left(\frac{n^n - 1}{\cancel{n-1}}\right) = n^n - 1$

$n^n - 1 \geq 6728$

$n^n \geq 6729$

$[n = 13]$

$(123)_5 = (x8)_y$ Find all possible value of x & y

$1\times5^2 + 2\times5^1 + 3\times5^0 = xy^1 + 8\cdot y^0$

$25 + 10 + 3 = xy + 8$

$\Rightarrow 38 = xy + 8$

$xy = 30$

$x < y$

$8 < y$

$(x, y) \in$ (1,30) (2,15) (3,10)

Possible x	value y	
1	30	✓
30	1	X
2	15	✓
15	2	X
5	6	X
6	5	X
3	10	✓
10	3	X

Q. How many value of x & y are possible for $(42)_9 = (x3)_y$

$4\times9^1 + 2\times9^0 = x\times y^1 + 3\times y^0$

$36 + 2 = xy + 3$

$35 = xy$

1	35 ✓	1	42 ✓
35	1	42	1 X
5	7 ✓	2	21 ✓
7	5	21	2 X
		3	14 ✓
		14	3 X
		6	7 ✓
		7	6 X

2 possible value

$(123)_n = (12n)_3$

$n^2 + 2n + 3 = 36 + n$ none

$n^2 + n - 6 = 0$

$n^2 + 3n - 2n - 6 = 0$

$n = -3, 2$

In a number System, the range of number are -3 to 3 represented as C, B, A, 0, 1, 2, 3 express $(102)_{10}$ in the number system Base → 7

$(7003)_{16}$

$0\times16^1 + 3\times16$

$$\begin{array}{r|l} 7 & 102 \\ 7 & 14 \quad 4 \\ 7 & 2 \quad 0 \\ & 0 \quad 2 \end{array}$$

$(21C)_7$

1024

$(16^3\times7+3)$ $(\cancel{24195})_{10} = (\quad)_2$

$(28675)_{10} = (\quad)_2$

1110000000000011

2	28675	
2	14337	1
2	7168	1
2	3584	0
2	1792	0
2	896	0
2	448	0
2	224	0
2	112	0
2	56	0
2	28	0
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

$(16^3\times7+3) = (\quad)_2$

$7\times16^3 + 0\times16^2 + 0\times16^1 + 3\times16^0$

$(7003)_{16}$

2^4 10

(1110000000000011)

$(516)_7 = (\quad)_4$ minimum no of bit required

$n^{n}-1 > 517$

$n^{n}-1 \geq 517$

$4^{n}-1 > 517$

$4^{n} > 517$

$\log_{10} 2^{2n} > \log_{10} 517$

$2n \log_{10} 2 > \log_{10} 517$

Complementary Number System

$x + y \rightarrow$ ADDER

$x - y \rightarrow$ Substracter

Base - b

Base - 10

$(b-1)$'s Complement — 9's, 10's

b's complement

$x \rightarrow (b-1)^{s}$ compliment of $x = (b^{n}-1)-x$

b's " " " $= (b^{n}-1) - x + 1$

b's $= (b-1)$'s $+ 1$

Base-2 — 1's $\rightarrow (2^{n}-1)-x$

2's $\rightarrow 2^{n}-1-x+1 = 2^{n}-x$

Base-3 — 2's $\rightarrow 3^{n}-1-x$

3's $\rightarrow 3^{n}-x$

Base-10 — 9's $\rightarrow 10^{n}-1-x$

10's $\rightarrow 10^{n}-x$

$(654) \rightarrow$ 9's $(10^{3}-1)$

$\Rightarrow (10^{3}-1) - 654$

$= 999 - 654$

345

10's $\Rightarrow 345 + 1 = 346$

Base-2 $(1101)_2$

1's $(b^{n}-1)-x \Rightarrow (\underbrace{2^{4}-1}_{15}) - 1101$

$1111 - 1101$

$\Rightarrow 10$

2's 11

$\Rightarrow 1101$

1's compliment $\Rightarrow 0010$

2's compliment

$$\begin{array}{r} 0010 \\ +1 \\ \hline 0011 \end{array}$$

11011

1's 00100

2's

$$\begin{array}{r} 00100 \\ +1 \\ \hline 00101 \end{array}$$

2's compliment ~~11100~~ 00101

$(74A)_{16}$

15's $(b^n-1)-n \Rightarrow FFF - 74A = 9C8$

16's 9C9

Compliment of $n = -n$, $\bar{n}$

$(b-1)$'s

$x + \bar{x} = x + (b^n - 1) - n$

$= b^n - 1 \Rightarrow 0$

Base-2 $\quad 1111 \ldots n$ times, $000000 \ldots$] two representation of zero

Base-3 $\quad 2222$, 0000

Base-4 $\quad 3333$ —, 0000

Base-10 $\quad 9999$ —, 0000 —

b's

$n + \bar{n} = 0$

$\Rightarrow n + b^n - n$

$= b^n = 0$

97 − 23 $\quad (b-1)$'s

1's

$23 \Rightarrow \overset{2}{\cancel{20}}1 - 23$
99

$\Rightarrow 76$

$\boxed{\cancel{97+76} \; \cancel{173}}$

$97 + (99 - 23)$

$(97 - 23) + (100 - 1)$

74 Ans

$$\begin{array}{r} 99 \\ -23 \\ \hline 76 \\ +97 \\ \hline 173 \\ +1 \\ \hline 74 \end{array}$$

23 − 97

$23 + (99 - 97)$

$(23 - 97) + \cdot$

97

$\Rightarrow 99 - 97$

$\Rightarrow 2$

$$\begin{array}{r} 23 \\ +2 \\ \hline 25 \end{array}$$

23 − 97

10111 − 00010

10101

$\Rightarrow \cancel{10101}$

$$\begin{array}{r|rl} 2 & 23 & \\ 2 & 11 & 1 \\ 2 & 5 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & 0 \\ & 0 & 1 \end{array}$$

$(x - y) \geq 0$ carry Generate

↳ Remove carry

ADD +1

23 - 97

-97 → 1's

$$\begin{array}{r} 99 \\ -97 \\ \hline 02 \\ +23 \\ \hline 25 \end{array}$$

1's

$$\begin{array}{r} 99 \\ -25 \\ \hline 74 \end{array}$$

$x - y$

$x + (b^n - 1 - y)$

$(x-y) + (b^n - 1) \rightarrow$ 1's compli.

$b^n - 1 - [(x-y) + (b^n - 1)]$

$\cancel{b^n - 1} - (x-y) - \cancel{(b^n - 1)}$

$-(x-y)$

$\Rightarrow -74$

$x - y < 0$

↳ Result

↳ - Append

Using b's

97 - 23

97 + (100 - 23)

(97 - 23) + 100 77

(97 - 23) ~~77~~

$\Rightarrow$ 74

$$\begin{array}{r} 97 \\ 77 \\ \hline 174 \end{array}$$

$(x-y)$

$x + (b^n - y)$

$(x - y) + b^n$

$x - y \geq 0$ Remove carry

23 - 97 → -23

23 + 3

$\Rightarrow$ 26

100 - 26

$\Rightarrow$ 74

$x - y < 0$

Q. 5 - 4

101 - 100

↳ using 1's

↳ 2's

1's 101 + 011

Carry remo ← 1000

+1

001 Ans

2's 101 + 011

$$\begin{array}{r} 011 \\ +1 \\ \hline 100 \end{array}$$

101 + 100

remove ← 1001

$\Rightarrow$ 001

4 - 5

100 - 101

↳ 1's compliment

↳ 2's compli

1's 100 - 101 small

100 + 010

110

- 001

2's 100 - 101 ← 2's

100 + 011

$\Rightarrow$ 111

↓ 2's

-(001)

Hamming Code

Hamming Distance

Hamming → ~~जितने~~ position पर code word match na ho

Code word में error detect

101101
100110
$\times\times\checkmark\times\checkmark\checkmark$ HD = 3

1 detected $\boxed{101001}$
— — — — 101000

HD=2 $\boxed{101010}$

HD=3 $\boxed{101110}$

To detect d bit error minimum hamming distance should be $d+1$

To correct d bit error minimum h.D should be $2d+1$

Hamming Code

To correct 1 bit error

$$m+p = m+p+1$$

$$2^p \geq (m+p+1)$$

4 bit + 3 bit = (0,1,2,3,4,5,6,7)

$\Rightarrow 2^3 = 8$

m = 0

1 2

P_1 P_2

even

$P_1 \Rightarrow 0$

$P_2 =$ 0

P_3

$P_1 = 0$ P

$\Rightarrow$ 0

P 0

$m = \underline{0101}$ 4 Bit + 3 Bit $\Rightarrow 2^3 \Rightarrow 0$ to 7

(m under 4 Bit, p under 3 Bit)

$2^3 \geq 4+3+1$

$P_1 \; P_2 \; P_3$

even

1	2	3	4	5	6	7
P_1	P_2	0	P_3	1	0	1

$P_1 \Rightarrow 0$ 0 0 1 1

$P_2 \Rightarrow 1$ 1 0 0 1

$P_3 \Rightarrow$ 0 1 0 1

0 1 0 0 1 0 1

error $\rightarrow$ 0 1 0 0 0 0 1

	P_3	P_2	P_1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

$P_1 \Rightarrow \{1, 3, 5, 7\}$

$P_2 \Rightarrow \{2, 3, 6, 7\}$

$P_3 \Rightarrow \{4, 5, 6, 7\}$

$P_3 \; P_2 \; P_1$

$1 \; 0 \; 1 \Rightarrow 4+0+1 \Rightarrow 5^{th}$ position

Ans

0 1 0 0 1 0 1

error 1 1 0 0 1 0 1

$P_1 \; P_2 \; P_3$

$0 \; 0 \; 1 \Rightarrow 1^{st}$ position

Q- 1010

1, 2, 4, 8, 16 $2^P \geq m + p + 1$ (4 under m, 3 under p)

$2^3 \geq \{0, 1, 2, 3, 4\}$

1 0 1 1 0 1 0

error 1 0 1 1 1 1 0

1	2	3	4	5	6	7
P_1	P_2	1	P_3	0	1	0

$P_1 = 1$ 1 1 0 0

$P_2 = 0$ 0 1 1 0

$P_3 \Rightarrow 1$ 0 1 0

$P_3 \; P_2 \; P_1$

$1 \; 0 \; 1 \Rightarrow 5$

Boolean Algebra

$x + y$ → Operand, operator

↓ +, ·, NOT

Real world problem — ckt bnana

Real world Problem → Boolean Expression → Solve/minimize → ckt

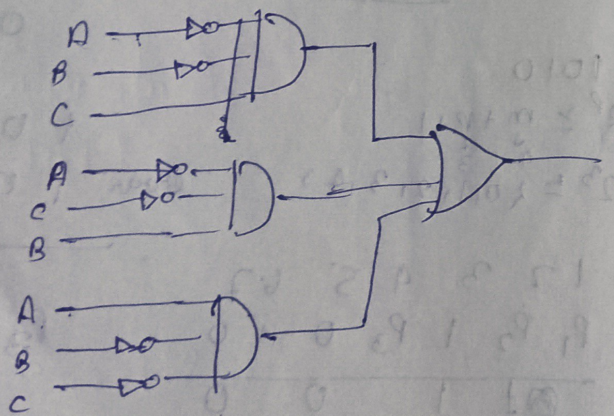
Lift

		ON	OFF
1st	→ A →	1	0
2nd	→ B →	1	0
3rd	→ C →	1	0

A	B	C	F
0	0	0	0
0	0	1	1 ✓
0	1	0	1 ✓
0	1	1	0
1	0	0	1 ✓
1	0	1	0
1	1	0	0
1	1	1	0

0 → Compliment

$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$



Basic Property

- Idempotency

$x + x = x$
$x \cdot x = x$
$x + 1 = x$
$x \cdot 1 = x$
$x + 0 = x$
$x \cdot 0 = 0$

- ~~Distributivity~~ — Commutativity —

$x + y = y + x$
$xy = yz$

- Associativity —

Left Associative

$x + (y + z) = (x + y) + z$

Right associative

$x(yz) = (xy)z$

- Complementation —

$x + \bar{x} = 1$
$x \cdot \bar{x} = 0$

- Distributivity —

$x \cdot (y + z) = xy + xz$

$x + yz = (x + y)(x + z)$

if $z = 0$

$x = (x + y)x$

$\Rightarrow x + xy$

$x(1 + y) \Rightarrow x \cdot 1 \Rightarrow x$

Every boolean expression must satisfy –

- complementation
- duality → $\overline{x+y}$
- Every boolean exp. have dual and dual $f x^n$ hold same property

dual –

$f(x_1, x_2, x_3 - - x_n, 0, 1, +, \cdot)$ ↓ as it is

$f_{dual}(x_1, x_2, x_3 - - x_n, 1, 0, \cdot, +)$

$f = x + yz$ $f = x + 1$

$f_{dual} = x(y+z)$ $= x \cdot 0 = 0$

$x'y'z + yz + xz$

$x'y'z + z(y+x)$

$z(x'y' + (y+x))$

$z((x'+y)(y+y') + x)$

$z(x'+y+x)$

$z(1+y)$

z

Demargan's law

$\overline{(x+y)} = x' \cdot y'$

$(\overline{x \cdot y}) = x' + y'$

$f = (x_1, x_2, x_3 - - x_n, 1, 0, \cdot, +)$

$f_{compl} = (\bar{x}_1, \bar{x}_2 - - \bar{x}_n, 0, 1, +, \cdot)$

$f = x + y$ $(\overline{\bar{x} + \bar{y}})$

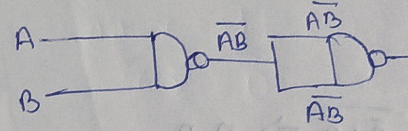
$f_{com} = x' \cdot y'$ $\bar{\bar{x}} \cdot \bar{\bar{y}}$

$f_{dual} = xy$ $\Rightarrow xy$

Universal Gate –

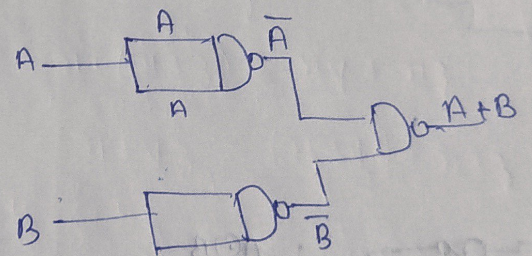
Using NAND

AND

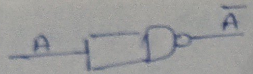


$(\overline{\overline{AB} \cdot \overline{AB}}) \Rightarrow AB$

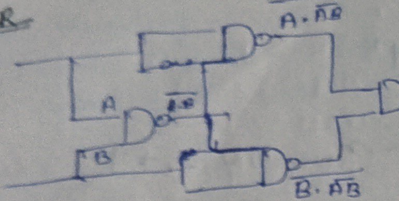
OR



NOT



XOR



$\overline{\overline{A \cdot \overline{AB}} \cdot \overline{B \cdot \overline{AB}}} \Rightarrow A \cdot \overline{AB} + B \cdot \overline{AB}$

$A \cdot (A' + B') + B(\ldots$

$AA' + AB' + BA' + B\ldots$

$\Rightarrow A \oplus B$

X NOR

NAND

$\overline{A \cdot B}$

$\Rightarrow A' + B'$

dual

$A'B'$

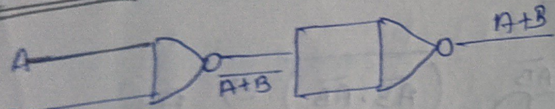
NOR

$\overline{A + B}$

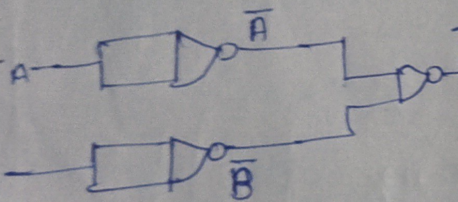
dual $A' \cdot B'$

Using NOR

OR

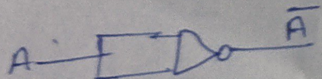


AND

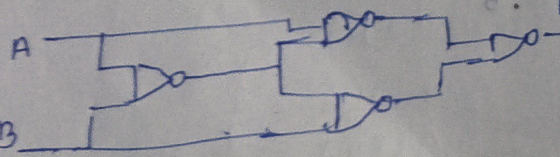


$\overline{\bar{A} + \bar{B}} \Rightarrow A \cdot B$

NOT



XNOR



$\overline{\bar{A} \cdot \bar{B}}$

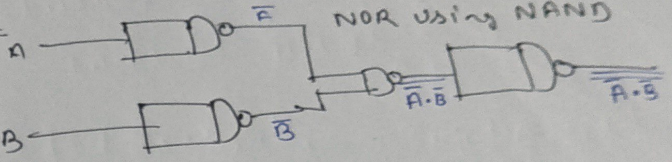
$\bar{A} \cdot \bar{B}$

$A +$

$\bar{A}'$

Q-

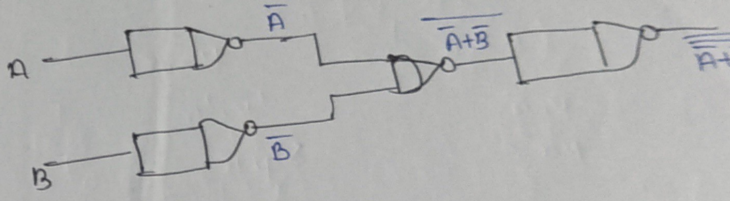
NOR using NAND



$\Rightarrow \overline{A' + B'}$

$\Rightarrow \overline{(A+B)}$

NAND using NOR



$\overline{\overline{\bar{A}+\bar{B}}} \Rightarrow \bar{A} + \bar{B}$

$\Rightarrow \overline{A \cdot B}$