091M4041H - Assignment 4

胡鹏飞 计控学院 201828007329009

1 Linear-inequality feasibility

Given a set of m linear inequalities on n variables x1; x2; …… ; xn, the linear-inequality feasibility problem asks if there is a setting of the variables that simultaneously satisfies each of the inequalities.

Show that if we have an algorithm for linear programming, we can use it to solve the linear-inequality feasibility problem. The number of variables and constraints that you use in the linear-programming problem should be polynomial in n and m.

The formulation of an LP

根据题意，现在存在一个线性规划的算法以及一组变量，需要解决的是如何利用这个算法来判断这组变量是否 linear-inequality。

假设线性规划的算法为

$$\begin{aligned} \min \quad & c^T X \\ s.t. \quad & \sum_j a_{ij} x_j <= b_j \\ & x_j >= 0 \end{aligned}$$

其中，i<=n, j<=m

参照老师在课堂上讲述的内敛法技巧，我们将算法中目标函数改变. 即

$$\begin{aligned} \min \quad & x_0 \\ s.t. \quad & \sum_j a_{ij} x_j - x_0 <= b_j \\ & x_j >= 0 \\ & x_0 >= 0 \end{aligned}$$

其中，i<=n, j<=m

如果算法能够求得最优解 $x_0=0$, 则 linear-inequality 是可行的，否则为不可行。

3 Gas Station Placement

Let's consider a long, quiet country road with towns scattered very sparsely along it. Sinopec, largest oil refiner in China, wants to place gas stations along the road. Each gas station is assigned to a nearby town, and the distance between any two gas stations being as small as possible. Suppose there are n towns with distances from one endpoint of the road being d1; d2; ……; dn. n gas stations are to be placed along the road, one station for one town. Besides, each station is at most r far away from its correspond town. d1;……; dn and r have been given

and satisfied d1 < d2 < …… < dn, 0 < r < d1 and di + r < di+1 - r for all i. The objective is to find the optimal placement such that the maximal distance between two successive gas stations is minimized.
Please formulate this problem as an LP..

The formulation of an LP

根据题意，假设两个相邻的加油站之间的最大距离为 z，第 i 个加油站的位置为 x_i，x_i 距 d_i 不能超过 r, 所以可将上述实际问题建模为:

$$
\begin{aligned}
\min \quad & z \\
s.t. \quad & x_i \quad <= \quad d_i + r \\
& -x_i \quad <= \quad -d_i + r \\
& x_{i+1} - x_i - z \quad <= \quad 0 \\
& x_i \quad >= \quad 0 \\
& z \quad >= \quad 0
\end{aligned}
$$

6 Dual Simplex Algorithm

For the problem
minimize

$$-7x1 + 7x2 - 2x3 - x4 - 6x5$$

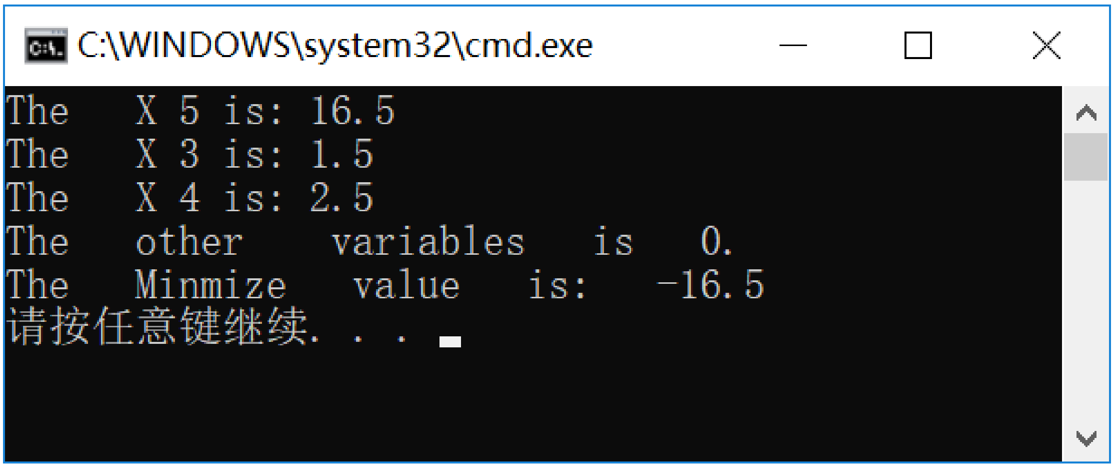
subject to:

$$3x1 - x2 + x3 - 2x4 = -3$$
$$2x1 + x2 + x4 + x5 = 4$$
$$-x1 + 3x2 - 3x4 + x6 = 12$$
$$xi > 0; (i = 1;\cdots; 6)$$

Implement dual simplex algorithm with your favorate language to solve this problem, and make comparison result with GLPK or Gurobi or other similar tools.

4.1 The result of C++

代码见附录，结果如下



4.2 The result of GLPK

```
/* Variables */
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;

/* Object function */
minimize z: -7*x1 + 7*x2 - 2*x3 - x4 - 6*x5;

/* Constrains */
s.t. con1: 3*x1 - x2 + x3 -2*x4 = -3;
s.t. con2: 2*x1 + x2 + x4 + x5 = 4;
s.t. con3: -x1 + 3*x2 - 3*x4 + x6 = 12;

end;
```

运行结果为

```
Problem:    DualSimplex
Rows:       4
Columns:    6
Non-zeros:  17
Status:     OPTIMAL
Objective:  z = -16.5 (MINimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 z            B          -16.5
     2 con1         NS            -3            -3             =          -2.5
     3 con2         NS             4             4             =            -6
     4 con3         NS            12            12             =         < eps

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 x1           NL             0             0                      12.5
     2 x2           NL             0             0                      10.5
     3 x3           NL             0             0                       0.5
     4 x4           B            1.5             0
     5 x5           B            2.5             0
     6 x6           B           16.5             0
```

同样，求得的解为[0, 0, 0, 1. 5, 2. 5, 16. 5],最终结果为 z=-16. 5，与刚才计算结果一致。

附录：

```
#include <iostream>
#include <conio.h>
#include <math.h>
#include <stdio.h>
using namespace std;

typedef int BOOL;
#define TRUE 1
#define FALSE 0
typedef double REAL;
#define ZERO 1e-10

//矩阵求逆
BOOL Inv(REAL ** a, int n);
BOOL Inv(REAL * a, int n);
//矩阵相乘
void Damul(REAL * a, REAL * b, size_t m, size_t n, size_t
k, REAL * c);
//线形规划
BOOL Line_Optimize(REAL * A, REAL * B, REAL * C, int m,
int n,
    REAL * Result, REAL * X, int * Is);

template <class T>
inline void ExChange(T& a, T& b)
{
    T temp = a;
    a = b;
    b = temp;
}
BOOL Inv(REAL ** a, int n)
{
    REAL d;
    int i, j, k;
    int success = FALSE;
    int * is = new int[n];
    int * js = new int[n];

    for (k = 0; k <n; k++) {
        d = 0.0;
        for (i = k; i <n; i++) {
            for (j = k; j <n; j++) {
                if (fabs(a[i][j])> d) {
```

```
                    d = fabs(a[i][j]);
                    is[k] = i;
                    js[k] = j;
                }
            }
        }
        if (d <ZERO)  goto  Clear;
        for (j = 0; j <n; j++)ExChange(a[k][j], a[is[k]][j]);
        for (i = 0; i <n; i++)ExChange(a[i][k], a[i][js[k]]);
        a[k][k] = 1 / a[k][k];
        for (j = 0; j <n; j++) {
            if (j != k)a[k][j] *= a[k][k];
        }
        for (i = 0; i <n; i++) {
            if (i != k) {
                for (j = 0; j <n; j++)
                    if (j != k)a[i][j] -= a[i][k] * a[k][j];
            }
        }
        for (i = 0; i <n; i++) {
            if (i != k) {
                a[i][k] *= ((-1.0)*a[k][k]);
            }
        }
    }  //end  for
    for (k = (n - 1); k >= 0; k--)
    {
        for (j = 0; j <n; j++)
            ExChange(a[k][j], a[js[k]][j]);
        for (i = 0; i <n; i++)
            ExChange(a[i][k], a[i][is[k]]);
    }
    success = TRUE;
Clear:
    delete[]  is;
    delete[]  js;
    return  success;
}
BOOL  Inv(REAL  *  a, int  n)
{
    REAL  **kma = new  REAL*[n];
    for (int i = 0; i <n; i++) {
        kma[i] = a + i*n;
    }
```

```
    BOOL  ret = Inv(kma, n);
    delete[]  kma;
    return  ret;
}
void  Damul(REAL  *  a, REAL  *  b, size_t  m, size_t  n, size_t
k, REAL  *  c)
{
    unsigned int  i, j, l, u;
    for (i = 0; i <= (m - 1); i++)
    {
        for (j = 0; j <= (k - 1); j++)
        {
            u = i*k + j;
            c[u] = 0.0;
            for (l = 0; l <= n - 1; l++)
            {
                c[u] += a[i*n + l] * b[l*k + j];
            }
        }
    }
    return;
}
BOOL  Line_Optimize(REAL  *  A, REAL  *  B, REAL  *  C, int  m,
int  n,
    REAL  *  Result, REAL  *  X, int  *  Is)
{
    REAL  r;
    int  i, j, k;
    int  Success = FALSE;
    REAL*  b = new  REAL[m*m];
    REAL*  MatTmp = new  REAL[m*m];
    REAL*  Mat1 = new  REAL[m];
    REAL*  Mat2 = new  REAL[m];
    REAL*  E = new  REAL[m*m];
    for (i = 0; i <m; i++) {
        for (j = 0; j <m; j++) {
            b[i*m + j] = A[i*n + Is[j]];
        }
    }
    if (!Inv(b, m)) {
        goto  Release;
    }
    Damul(b, B, m, m, 1, X);
    for (;;) {
```

```
for (i = 0; i <m; i++) {
    Mat2[i] = C[Is[i]];
}
Damul(Mat2, b, 1, m, m, Mat1);
for (i = 0; i <n; i++) {
    for (j = 0; j <m; j++) {
        Mat2[j] = A[j*n + i];
    }
    Damul(Mat1, Mat2, 1, m, 1, &r);
    r = C[i] - r;
    if (r <-ZERO) {
        break;
    }
}
if (i >= n)
{
    *Result = 0;
    for (i = 0; i <m; i++) {
        *Result += C[Is[i]] * X[i];
    }
    Success = TRUE;
    goto  Release;
}
Damul(b, Mat2, m, m, 1, Mat1);
r = 1E10;
j = -1;
for (k = 0; k <m; k++) {
    if (Mat1[k]> ZERO) {
        REAL  temp = X[k] / Mat1[k];
        if (temp <r) {
            r = temp;
            j = k;
        }
    }
}
if (j  <  0) {
    Success = FALSE;
    goto  Release;
}
for (k = 0; k <m*m; k++) {
    E[k] = 0;
}
for (k = 0; k <m; k++) {
    E[k*m + k] = 1;
```

```
        }
        for (k = 0; k <m; k++) {
            E[k*m + j] = -Mat1[k] / Mat1[j];
        }
        E[j*m + j] = 1 / Mat1[j];
        Is[j] = i;
        Damul(E, b, m, m, m, MatTmp);
        Damul(E, X, m, m, 1, Mat2);
        for (i = 0; i <m*m; i++) {
            b[i] = MatTmp[i];
        }
        for (i = 0; i <m; i++) {
            X[i] = Mat2[i];
        }
    }
Release:
    delete[]  E;
    delete[]  Mat2;
    delete[]  Mat1;
    delete[]  MatTmp;
    delete[]  b;
    return  Success;
}
void  main()
{
    REAL  A[] = {
        3, -1, 1, -2, 0, 0,
        2,  1, 0,  1, 1, 0,
        -1, 3, 0, -3, 0, 1,
    };
    REAL  B[] = {
        -3,4,12
    };
    REAL  C[] = {
        -7, 7, -2, -1, -6, 0
    };
    REAL  RESULT;
    REAL  x[3];
    int  Is[] = {
        0,1,3
    };
    if (!Line_Optimize(A, B, C, 3, 6, &RESULT, x, Is)) {
        cout << "Calculate  Wrong! " << endl;
        return;
```

```
    }
    for (int i = 0; i <3; i++) {
        cout << "The  X " << Is[i] << " is: " << x[i] << endl;
    }
    cout << "The  other  variables  is  0. " << endl;
    cout << "The  Minmize  value  is:  " << RESULT << endl;
}
```