Student Name:

Student ID:

答题内容写在边线外视为无效

Major:

哈尔滨工业大学深圳研究生院

2011 年 秋 季学期期末考试试卷

HIT Shenzhen Graduate School Examination Paper

Course Name: 组合数学 **Lecturer:** 黄荷姣

Question	One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Total
Mark											

The following formula and results may be useful for you in this examination.

$$1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots=\frac{1}{2}(e^x+e^{-x})$$

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots$$

$$e^{ax}=\sum_{n=0}^{\infty}a^n\frac{x^n}{n!}$$

The number of ways to place n non-attacking, indistinguishable rooks on an n-by-n board with forbidden positions equals $n! - r_1(n-1)! + r_2(n-2)! - \dots + (-1)^k r_k(n-k)! + \dots + (-1)^r r_n$.

For a specified grid i in chessboard C, let C_i be a chessboard induced from C by deleting the row and the column of grid i；let C_e be induced from C by deleting grid i from C. Then, $r_k(C)=r_{k-1}(C_i)+r_k(C_e)$，$R(C)= xR(C_i) + R(C_e)$.

Assume that the sequence $h_0, h_1, h_2, \dots, h_n, \dots$ has a difference table whose 0^{th} diagonal equals, $c_0, c_1, c_2, \dots, c_p \neq 0, 0, 0, \dots$ Then, $h_n = c_0 C(n, 0) + c_1 C(n, 1) + c_2 C(n, 2) + \dots + c_p C(n, p)$. and

$$\sum_{k=0}^{n}h_k=c_0\binom{n+1}{1}+c_1\binom{n+1}{2}+\cdots+c_p\binom{n+1}{p+1}.$$

Let $n \ge 2$ be an integer. If there exist n-1 MOLS of order n, then there exists a resolvable BIBD with parameters $b = n^2+n$, $v = n^2$, $k = n$, $r = n+1$, $\lambda = 1$.

$$N(G,C)=\frac{1}{|G|}\sum_{f\in G}|C(f)|.$$

$$P_G(z_1,z_2,\cdots,z_n)=\frac{1}{|G|}\sum_{f\in G}z_1^{e_1}z_2^{e_2}\cdots z_n^{e_n}$$, where type(f) = $(e_1, e_2, \dots, e_n)$.

Question One: Please give the answers for the following 17 questions **without explanation.**

1. Determine an integer m_n such that if m_n points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most $\frac{1}{n}$. m_n = ( )

2. Consider the permutation of {1,2,3,4,5,6,7,8}. The inversion sequence of 83476215 is ( ); The permutation with an inversion sequence 66142100 is ( ).

3. The unique permutation with $\frac{n(n-1)}{2}$ inversions is ( ).

4. Among the combination of $\{x_7, x_6, \dots, x_1, x_0\}$, what is the combination that immediately follow $\{x_7, x_5, x_4, x_3, x_2, x_1, x_0\}$ by using the base 2 arithmetic generating scheme? ( )

5. Let h_n equal the number of different ways in which the squares of a 1-by-n chessboard can be colored, using the colors red, green, and black so that no two squares that are colored red are adjacent. Then, the recurrence relation for h_n is ( ).

6. Determine the generating function for the number h_n of non-negative integral solution of $3e_1+5e_2+e_3+6e_4=n$.

7. Let h_n equal the number of different ways in which the squares of a 1-by-n chessboard can be colored, using the colors red, blue, white, and green, so that the number of squares colored red is even, and the number of squares colored white is odd. Then h_n = ( ).

8. Let the sequence $h_0, h_1, h_2, \dots h_n \dots$ be defined by $h_n = 2n^2 - n + 3, (n \ge 0)$. Then $\sum_{k=0}^{n} h_k$ = ( ).

9. Let

$$f=\begin{pmatrix}1&2&3&4&5&6\\6&4&2&1&5&3\end{pmatrix} \text{ and } g=\begin{pmatrix}1&2&3&4&5&6\\3&5&6&2&4&1\end{pmatrix}$$

Let c=(R, B, B, R, R, R) be a coloring of 1, 2, 3, 4, 5, 6 with the colors R and B. Then $(g\circ f)*c$ = ( ) and $(f\circ g)*c$ = ( )

10. Suppose 20 indistinguishable balls are allocated to 5 persons so that everyone has no less than 3 balls. Then the number of ways is ( ). If the balls are distinguishable, then the number of ways is ( ).

11. Determine the number of ternary groups choosing from the set {1, 2, …, 20}, where there are no two adjacent numbers in each group.

12. In how many ways can 100 students be allocated into A, B and C classes, where class A can accommodate 30 girls, class B can accommodate 32 boys, class C can accommodate 40 students(boys or girls), and the 100 students have 40 girls and 60 boys.

13. In how many ways can 5 red rooks and 3 blue rooks can be placed on 12×12 board so that no two rooks can attack one another, where the first line and the first column should not empty.

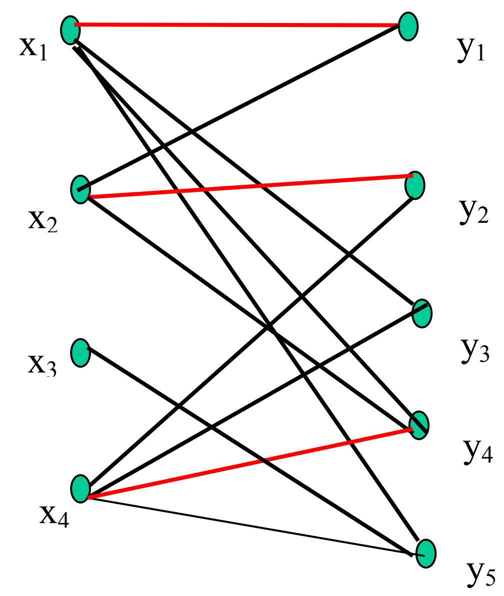
14. There is a party with 4 couples. In how many ways can they choose their partner, where each pair of partners is not couples?

15. Count the permutations $i_1, i_2, i_3, i_4, i_5, i_6$ of $\{1,2,3,4,5,6\}$ where $i_2 \neq 1,4$; $i_3 \neq 2,3,5$, $i_4 \neq 4$, $i_6 \neq 5,6$

16. Apply the deferred acceptance algorithm to obtain a stable complete marriage for the following preferential ranking matrix

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1,3 & 2,3 & 3,2 & 4,3 \\ 1,4 & 4,1 & 3,3 & 2,2 \\ 2,2 & 1,4 & 3,4 & 4,1 \\ 4,1 & 2,2 & 3,1 & 1,4 \end{bmatrix} \end{array}$$

17. Give a max-matching and a min-cover of the following graph by applying the matching algorithm with the initial matching $M_1 = \{(x_1, y_1), (x_2, y_2), (x_4, y_4)\}$.



Note: Please give a detailed explanation for the following three problems.

Question Two: Let $A = \{A_1, A_2, A_3, A_4, A_5, A_6\}$, where $A_1 = \{a, b, c\}$, $A_2 = \{a, b, c, d, e\}$, $A_3 = \{a, b\}$, $A_4 = \{b, c\}$, $A_5 = \{a\}$, $A_6 = \{a, c, e\}$. Does A has SDRs ? Why? What is the largest number of sets of A which can be chosen such that they have an SDR?

Question Three: Let p be a prime number. Determine the number of different necklaces that can be made from p beads of n different colors.

Question Four: Suppose there are 25 varieties of products which need to be tested by 30 consumers. The test is to have a property that each pair of the 25 varieties is compared by exactly one person. Each consumer is responsible for 5 products and each product is tested 6 times. Please give a proper solution.