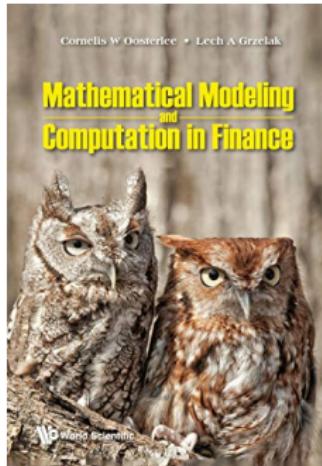


# Materials for the course

The course is based on book “*Mathematical Modeling and Computation in Finance: With Exercises and Python and MATLAB Computer Codes*”, by C.W. Oosterlee and L.A. Grzelak, World Scientific Publishing Europe Ltd, 2019. For more details go [here](#).



- ▶ Youtube Channel with courses can be found [here](#).
- ▶ Slides and the codes can be found [here](#).

# Course Objectives

## ► **Objectives:**

The main focus of this course is the modelling of the interest rates while other asset classes like foreign exchange and inflation are also covered. Ultimately, after completing the course, the student should be able to build a multi-asset portfolio consisting of linear products and perform xVA (CVA, BCVA and FVA) and (H)VaR computations.

## ► **Expected prior knowledge:**

Basic knowledge of stochastic differential equations (SDEs), numerical methods for Monte Carlo simulation, linear algebra. Prior to this course, it is highly recommended to follow the course of [Computational Finance](#).

## ► **Workload:**

The course consists of 14 lectures, and each lecture is divided into 45m-1h blocks. The whole course consists of 32 blocks. Homework assignments are given at the end of each lecture.

## ► **Programming Language:** Python.

## ► **Materials:** Links to slides + codes are in the description of this lecture.



# Course road map



# Understanding of Filtrations and Measures

Length: **2h40m**

- 2.1. Filtration
- 2.2. Conditional Expectations
- 2.3. Conditional Expectations in Python
- 2.4. Option Pricing Using Conditional Expectation
- 2.5. Convergence Experiment in Python
- 2.6. Concept of Numeraire
- 2.7. From P to Q in the Black-Scholes Model
- 2.8. Change of Numeraire: Stock Measure
- 2.9. Change of Numeraire: Dimension Reduction
- 2.10. The T-Forward Measure
- 2.11. The Summary of the Lecture + Homework



# The HJM Framework

Length: **2h**

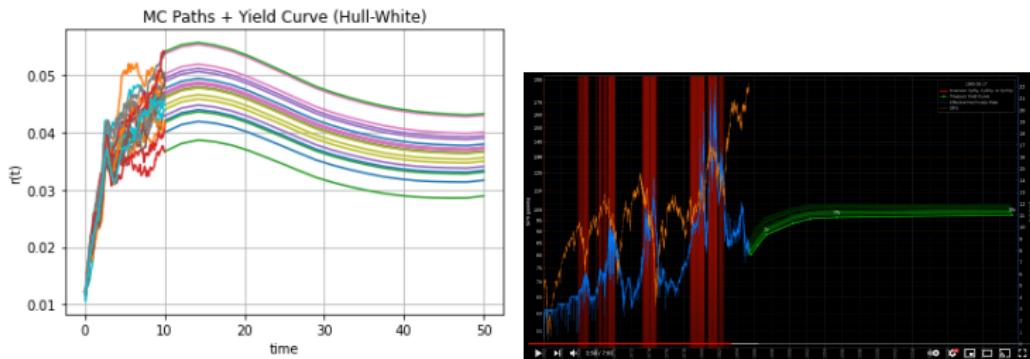
- 3.1. Equilibrium vs. Term-Structure Models
- 3.2. The HJM Framework
- 3.3. The Instantaneous Forward Rate
- 3.4. Arbitrage Free Conditions under HJM
- 3.5. Ho-Lee Model and Python Simulation
- 3.6. Hull-White Model
- 3.7. Hull-White Model and Simulation in Python
- 3.8. The Summary of the Lecture + Homework



# Yield Curve Dynamics under Short Rate

Length: **2h10m**

- 4.1. Exact Solution for the HW Model
- 4.2. Affinity of the Hull-White Model
- 4.3. Brief Introduction to Yield Curves
- 4.4. Limitations of the 1Factor Model and Yield Curve Dynamics
- 4.5. Gaussian 2F Model
- 4.6. The Summary of the Lecture + Homework

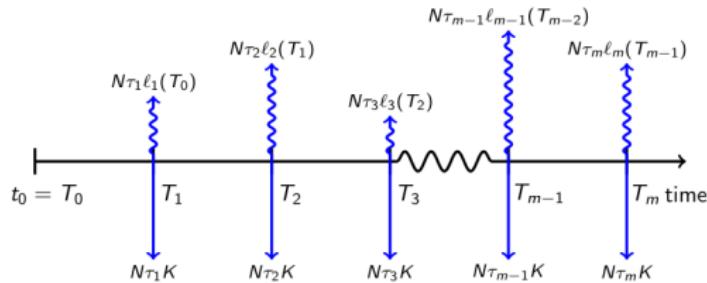


**Figure:** Dynamics of the yield curve for random market scenarios under the 1F Hull-White model.

# Interest Rate Products

Length: **2h**

- 5.1. Simple Compounded Forward Rate
- 5.2. Forward Rate Agreement
- 5.3. Floating Rate Note
- 5.4. Interest Rate Swap
- 5.5. The Hull-White model under the T-Forward Measure
- 5.6. Options on Zero-Coupon Bond
- 5.7. Caplets and Floorlets
- 5.8. Pricing of Caplets/Floorlets Under the HW Model
- 5.9. The Summary of the Lecture + Homework



# Construction of Yield Curve and Multi-Curves

Length: **3h**

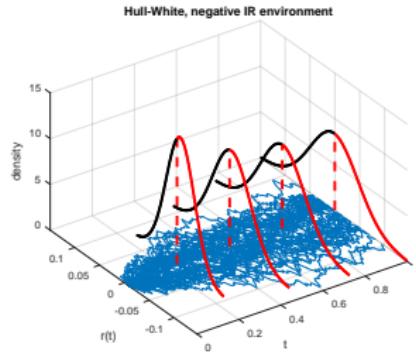
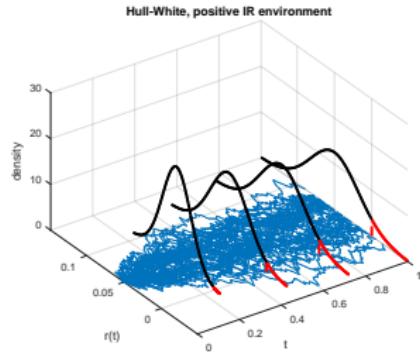
- 6.1. Yield Curve and its Dynamics
- 6.2. Mathematical Formulation
- 6.3. From Implied Volatilities to Building of YC
- 6.4. Spine Points and Optimization Routine
- 6.5. Analytical Example of YC Construction
- 6.6. Python Experiment
- 6.7. Different Interpolations and Impact on Hedging
- 6.8. Introduction to Multi-Curves
- 6.9. Multi-Curves and Connection to Default Probabilities
- 6.10. Python Experiment for Multi-Curves
- 6.11. The Summary of the Lecture + Homework

Date	1 Mo	2 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
06/01/21	0.01	0.01	0.02	0.04	0.04	0.16	0.31	0.81	1.28	1.62	2.22	2.30
06/02/21	0.01	0.01	0.02	0.04	0.05	0.13	0.30	0.80	1.26	1.59	2.21	2.28
06/03/21	0.00	0.01	0.02	0.04	0.04	0.16	0.34	0.84	1.30	1.63	2.22	2.30
06/04/21	0.01	0.02	0.02	0.04	0.05	0.14	0.32	0.78	1.23	1.56	2.16	2.24
06/07/21	0.01	0.02	0.02	0.04	0.05	0.16	0.33	0.79	1.24	1.57	2.17	2.25
06/08/21	0.01	0.02	0.02	0.04	0.05	0.14	0.32	0.77	1.20	1.53	2.13	2.21
06/09/21	0.01	0.02	0.03	0.04	0.05	0.16	0.31	0.75	1.17	1.50	2.10	2.17

# Pricing of Swaptions and Negative Interest Rates

Length: **2h**

- 7.1. Pricing of Caplets/Floorlets
- 7.2. Pricing of Interest Rate Swaps
- 7.3. Pricing of Swaptions under the Black-Scholes Model
- 7.4. Jamshidian's Trick
- 7.5. Swaptions under the Hull-White Model
- 7.6. Negative Interest Rates
- 7.7. Shifted Lognormal, Shifted Implied Volatility
- 7.8. The Summary of the Lecture + Homework



# Mortgages and Prepayments

Length: **4h**

- 8.1 Introduction to Mortgage Contracts
- 8.2 Bullet Mortgage
- 8.3 Bullet Mortgage: Python Experiment
- 8.4 Annuity Mortgage
- 8.5 Annuity Mortgage: Python Experiment
- 8.6 Prepayment determinants
- 8.7 CPR: Constant Prepayment Rate
- 8.8 Index Amortizing Swap
- 8.9 Inclusion of Refinancing Incentive
- 8.10 Stochastic Prepayment: Python Experiment
- 8.11 Stochastic Prepayment and Swaptions
- 8.12 Pipeline Risk
- 8.13 The Summary of the Lecture + Homework

# Hybrid Models and Stochastic Interest Rates

Length: **2h20m**

- 9.1 Hybrid Models for xVA and VaR
- 9.2 The Black-Scholes Hull-White Model
- 9.3 Implied Volatility for Models with Stochastic Interest Rates
- 9.4 Stochastic Vol Models with Stochastic Interest Rates
- 9.5 Example of a Hybrid Payoff: Diversification Product
- 9.6 The Heston Hull-White Hybrid Model
- 9.7 Monte Carlo for Hybrid Models
- 9.8 Monte Carlo for the Heston-Hull-White Model
- 9.9 The Summary of the Lecture + Homework

# Foreign Exchange (FX) and Inflation

Length: **2h50m**

- 10.1 Introduction to Foreign Exchange
- 10.2 Forward FX Contract
- 10.3 Cross-Currency Swaps
- 10.4 Pricing of FX Options, the Black-Scholes Case
- 10.5 The Heston FX Model
- 10.6 Pricing of FX Options with Stochastic Interest Rates
- 10.7 Introduction to Inflation
- 10.8 Pricing of Inflation Forwards and Swaps
- 10.9 Modeling of Inflation with SDEs
- 10.10 The Summary of the Lecture + Homework

# Market Models and Convexity Adjustments

Length: **2h30m**

- 11.1 A bit of History
- 11.2 Libor Market Model Specifications
- 11.3 Libor Rate Dynamics, from  $\mathbb{P} \rightarrow \mathbb{Q}^T$
- 11.4 Lognormal Libor Market Model and Measure Changes
- 11.5 LMM Under the Terminal and Spot Measures
- 11.6 Stochastic Volatility LMM
- 11.7 Smile and Skew in the LMM (Displaced Diffusion)
- 11.8 Freezing Technique
- 11.9 Convexity Correction
- 11.10 Convexity and Inclusion of Volatility Smile and Skew
- 11.11 The Summary of the Lecture + Homework



# Valuation Adjustments- xVA (CVA, BCVA and FVA)

Length: **3h**

- 12.1. Introduction and Basics of CVA
- 12.2. Exposures and Potential Future Exposure
- 12.3. Expected Exposures
- 12.4. Expected Exposures and Closed Form Solutions
- 12.5. Generation of Exposures with Python (1D Case)
- 12.6. Exposure Generation for Portfolio of Assets
- 12.7. Unilateral Credit Value Adjustment (CVA)
- 12.8. Approximations in Calculation of CVA
- 12.9. Bilateral Credit Value Adjustment (BCVA)
- 12.10. Funding Value Adjustment (FVA)
- 12.11. Trade Attributions in (B)CVA
- 12.12. The Summary of the Lecture + Homework

# Value-at-Risk and Expected Shortfall

Length: **2h**

- 13.1. Value at Risk (VaR), Stressed VaR (SVaR)
- 13.2. Coherent Risk Measures
- 13.3. Expected Shortfall
- 13.4. Historical VaR (HVaR) and Python Experiment
- 13.5. Missing Data, Arbitrage and Re-Gridding
- 13.6. VaR Computation with Monte Carlo
- 13.7. Backtesting
- 13.8. The Summary of the Lecture + Homework

# The Summary of the Course

## 14.1. The Summary of the Course.

