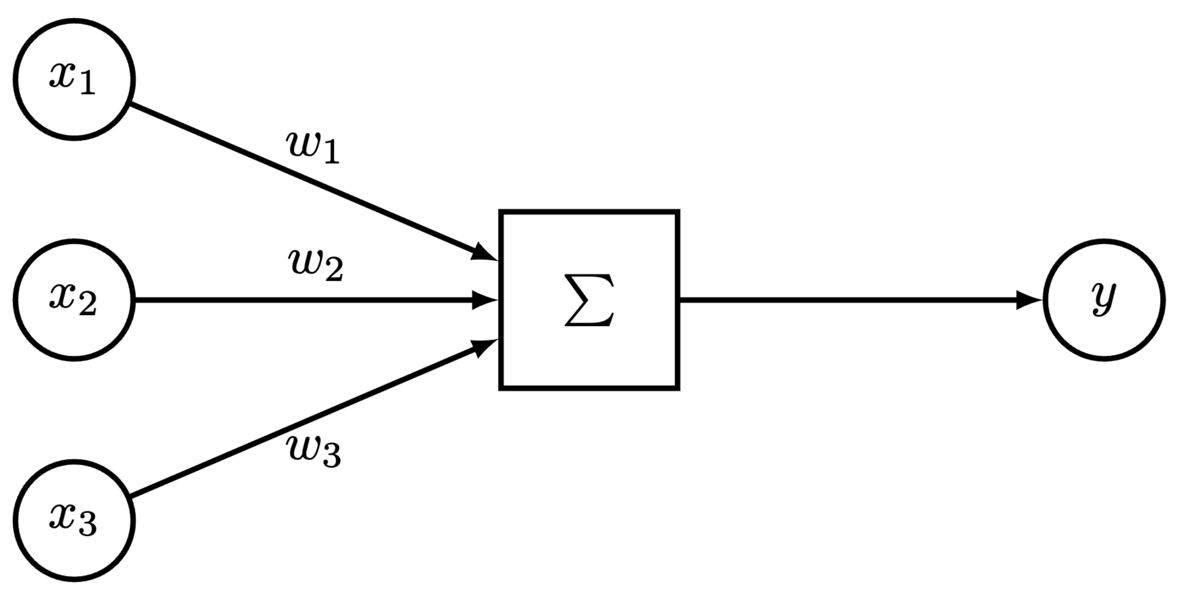
AI for Creativity: Worksheet 1A - Perceptrons

A perceptron is the most basic form of neural network. The output y is the sum of each input x_n multiplied by its corresponding weight w_n. Given by the equation:

$$y = \sum x_n w_n$$

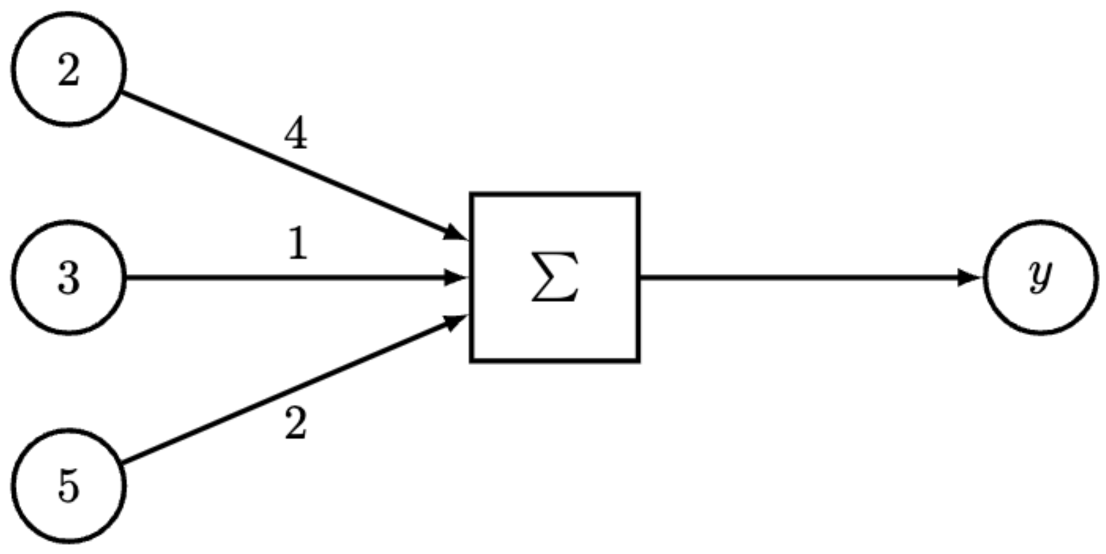
A diagram and corresponding equation for a perceptron with three inputs is show below:



$$y = x_1 w_1 + x_2 w_2 + x_3 w_3$$

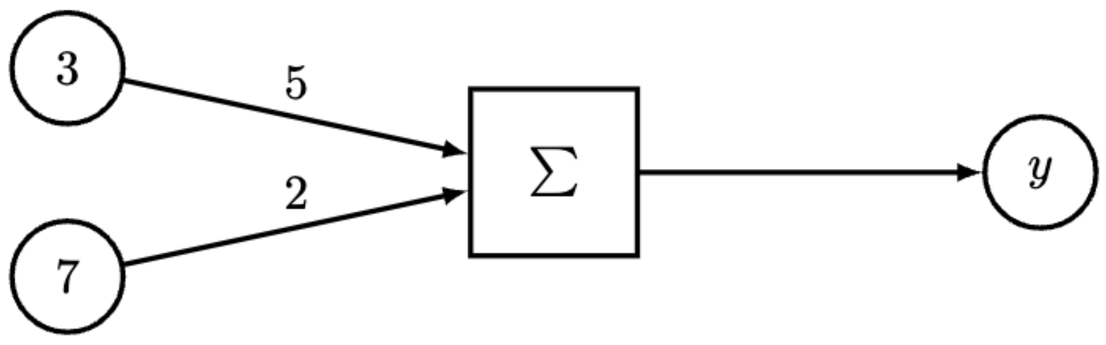
Now calculate the output y for the perceptrons given in the following diagrams:

(a)



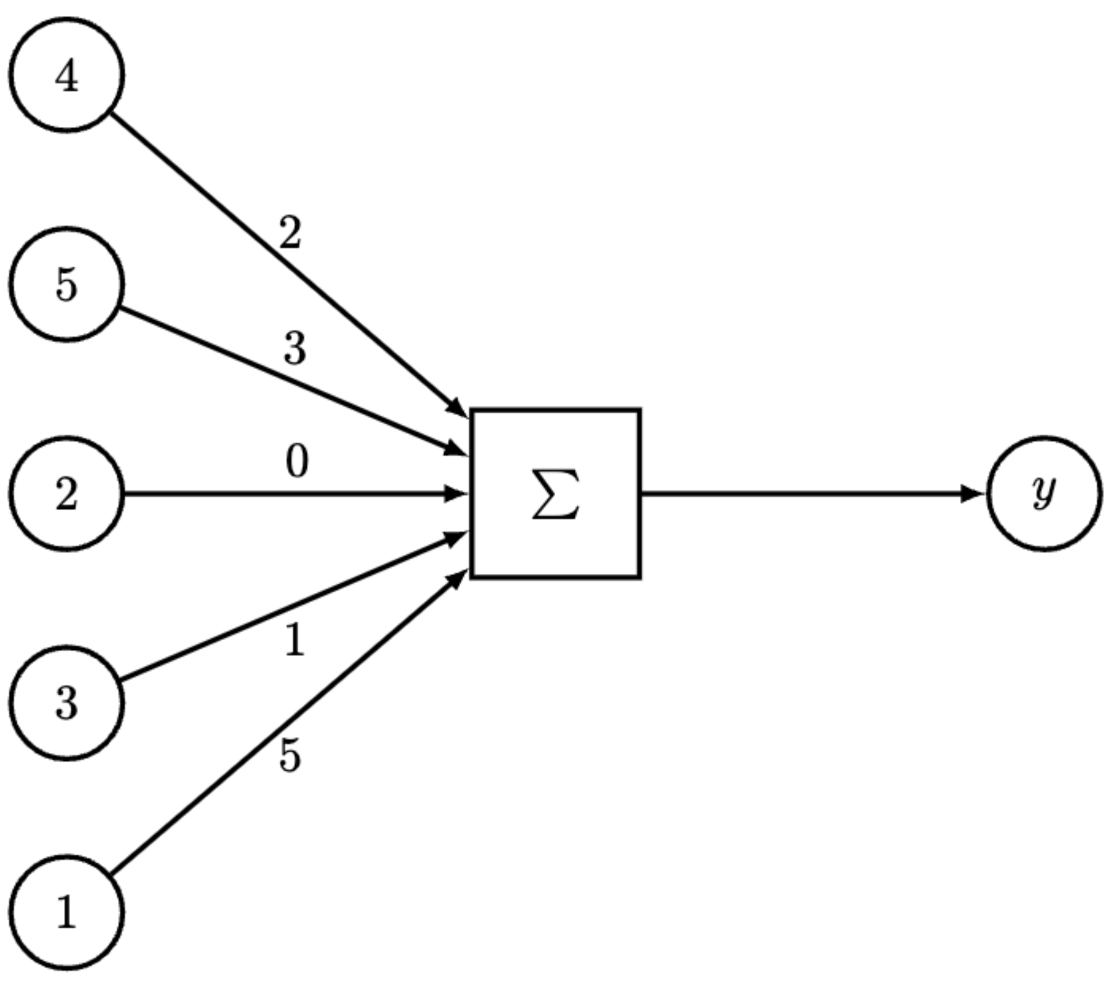
$y =$

(b)



$y =$

(c)



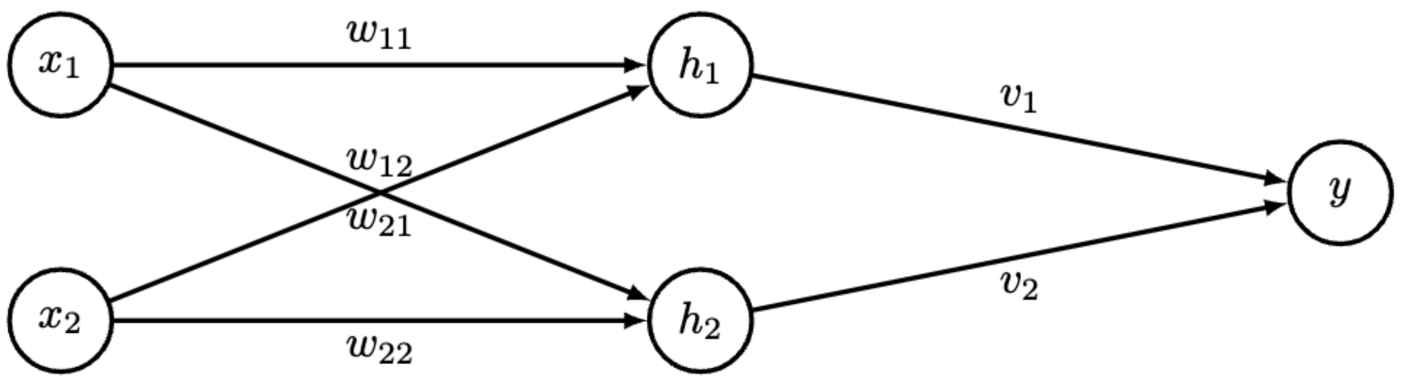
$y =$

Now flip over the worksheet to complete part b.

AI for Creativity: Worksheet 1B - Multilayered Perceptrons

In a multilayered perceptron (MLP), multiple perceptron units are stacked into layer. Each layer can have multiple perceptron units. The outputs of perceptron units fed as data to the perceptrons in the next layer. Each connection between each node (data or perceptron output) in this network has its own weight.

Below is a diagram for an MLP neural network, with one hidden layer and one output layer. There are two perceptron units in the first (hidden) layer: h_1 & h_2. The output layer has 1 unit given by y. The weights between the inputs and the first hidden layer are given by w, in weights between the hidden and output layer are given by v:

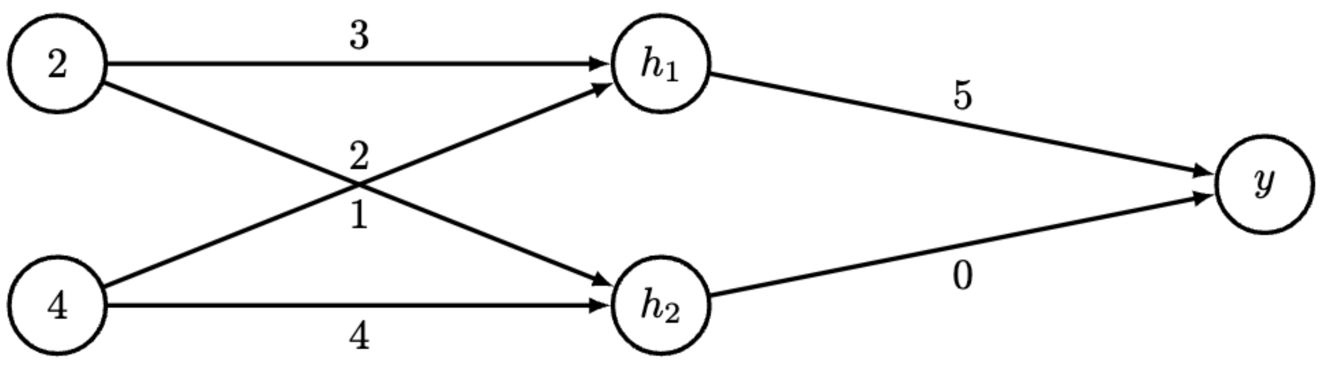


Here are the equations for calculating the outputs of the hidden units h_1 & h_2 and output unit y of this MLP network:

$h_1 = x_1 w_{11} + x_2 w_{21}$ $\qquad$ $h_2 = x_1 w_{12} + x_2 w_{22}$ $\qquad$ $y = h_1 v_1 + h_2 v_2$

Now calculate the value of the output y for the following MLP networks:

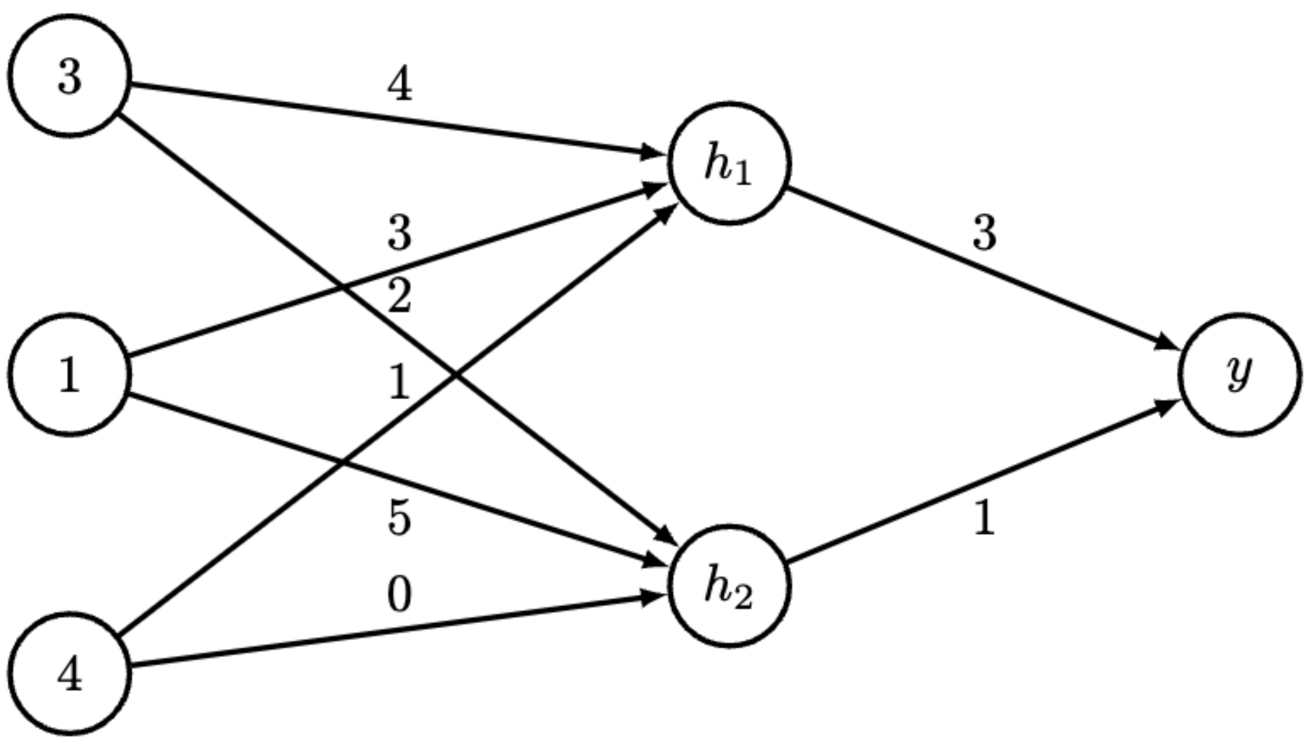
(a)



$y =$

Where $w_{11} = 3, w_{12} = 2, w_{21} = 1, w_{22} = 4$

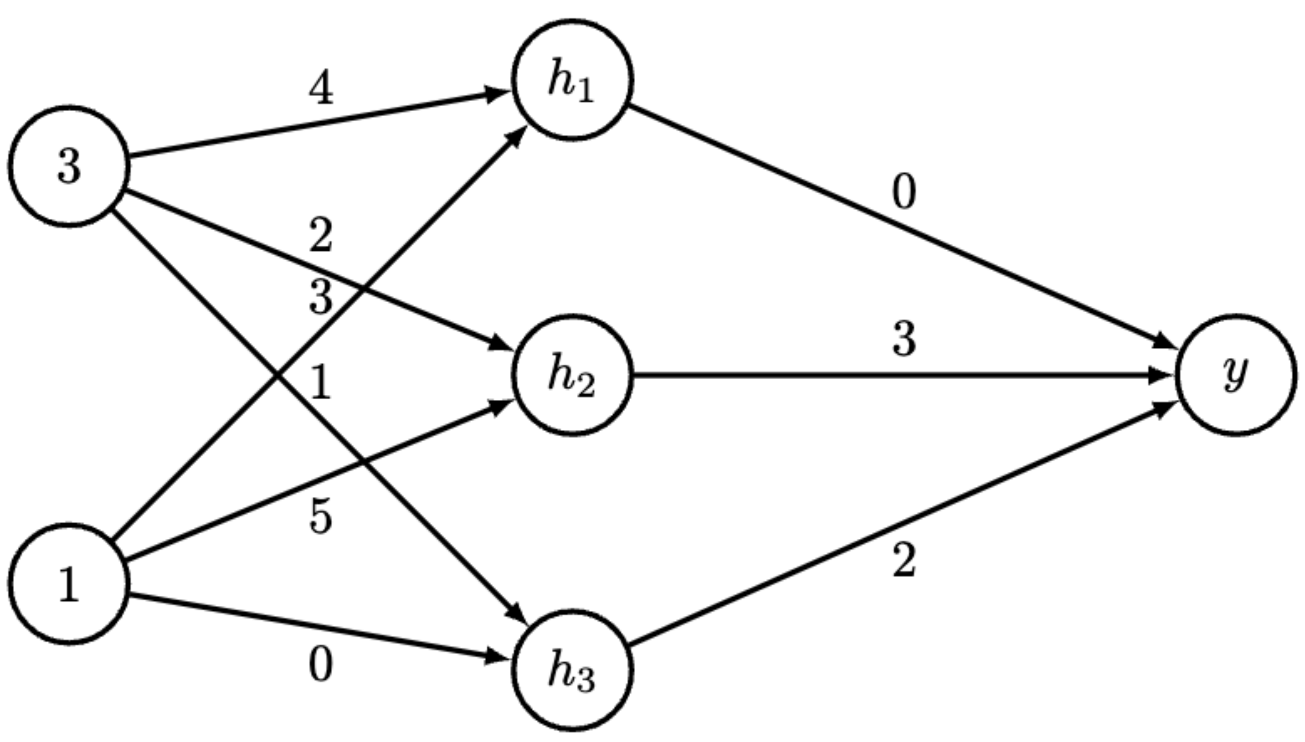
(b)



$y =$

Where $w_{11} = 4, w_{12} = 2, w_{21} = 3, w_{22} = 5, w_{31} = 1, w_{32} = 0$

(c)



$y =$

Where $w_{11} = 4, w_{12} = 2, w_{13} = 1, w_{21} = 3, w_{22} = 5, w_{23} = 0,$

AI for Creativity: Worksheet 2A - Vector dot product

A vector is a number that has more than numerical (scalar) component in it. A vector can be made up of any number n components, which is equivalent to a *list* or *array* of numbers. The following are all examples of vectors:

$$\begin{bmatrix} 1 \\ 5 \\ -4 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 0 & 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 0 & 3 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.2 \\ 0.01 \\ -0.3 \\ 0.2 \\ 0.1 \end{bmatrix}$$

The dot product of two vectors $\vec{a}$ & $\vec{b}$ is given as the sum of each component of the vector multiplied by it's corresponding component. Assuming that vectors $\vec{a}$ & $\vec{b}$ have the same number of components:

$$\vec{a} \cdot \vec{b} = \sum a_n b_n$$

In the case that the vectors $\vec{a}$ & $\vec{b}$ have three components each, then the formula for calculating them is given below:

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Given this, calculate the dot product for the following pairs of vectors:

(a) $\vec{a} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \qquad \vec{a} \cdot \vec{b} =$

(b) $\vec{a} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \qquad \vec{a} \cdot \vec{b} =$

(c) $\vec{a} = \begin{bmatrix} 4 \\ 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \\ 5 \end{bmatrix} \qquad \vec{a} \cdot \vec{b} =$

Now flip over the worksheet to complete part b.

AI for Creativity: Worksheet 2B - Vector matrix multiplication

A matrix is an array of numbers that has two dimensions two it. Just like a vector, each dimension can be made up of any n components. A matrix is equivalent to a *2D array* or table of numbers. The following are all examples of vectors:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 12 & 7 & 9 \\ 5 & 14 & 3 \end{bmatrix} \quad \begin{bmatrix} 8 & 11 \\ 15 & 2 \\ 6 & 19 \end{bmatrix} \quad \begin{bmatrix} 4 & 18 & 7 \\ 21 & 13 & 10 \\ 3 & 9 & 17 \end{bmatrix} \quad \begin{bmatrix} 0.1 & -1.2 \\ -3.2 & 2.41 \end{bmatrix}$$

A matrix W can be multiplied by a vector $\vec{a}$, as along as the number of columns in the matrix A matches the number of rows in the vector $\vec{a}$.

$$W = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

If we multiply a matrix W by a vector $\vec{x}$, we get a new vector $\vec{h}$, where each row of the vector h_n is the sum of the products of the components each row w_n of the Matrix W with the components of the vector $\vec{x}$.

$$W \cdot \vec{x} = \vec{h} = \begin{bmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \\ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 \end{bmatrix}$$

This is equivalent to treating each row from the matrix W as a vector w_n and taking the dot product of each column in the matrix with the vector $\vec{x}$:

$$\vec{h} = \begin{bmatrix} \overrightarrow{w_1} \cdot \vec{x} \\ \overrightarrow{w_2} \cdot \vec{x} \\ \overrightarrow{w_3} \cdot \vec{x} \end{bmatrix}$$

Now work out the following tasks. For each task you will have to work out the value of the vector matrix multiplications $M \cdot \vec{w}$ to get the vector $\vec{h}$. Then calculate the dot product of your answer for $\vec{h}$ with the vector $\vec{v}$ to get a final answer y. if you run out of space please ask for some plain paper to do your working out on:

(a)

$$W = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad M \cdot \vec{w} = \vec{h} =$$

$$\vec{v} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \qquad \vec{h} \cdot \vec{v} = y =$$

(b)

$$W = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & 0 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \qquad M \cdot \vec{w} = \vec{h} =$$

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \vec{h} \cdot \vec{v} = y =$$

(c)

$$W = \begin{bmatrix} 4 & 3 \\ 2 & 5 \\ 1 & 0 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad M \cdot \vec{w} = \vec{h} =$$

$$\vec{v} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \qquad \vec{h} \cdot \vec{v} = y =$$