南方科技大學
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Chapter 2: Lexical Analysis

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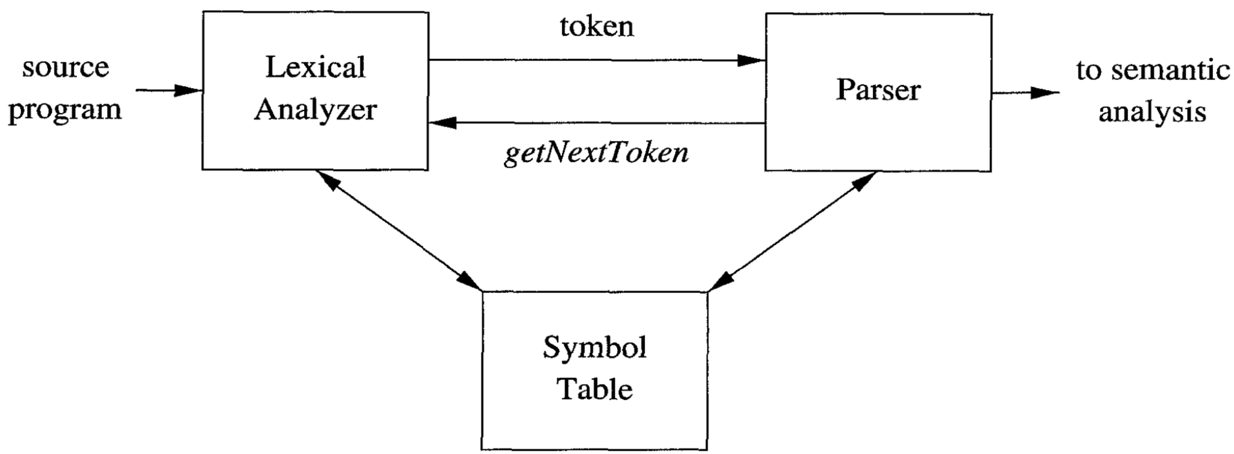
The chapter numbering in lecture notes does not follow that in the textbook.

Outline

- The Role of Lexical Analyzer
- Specification of Tokens (Regular Expressions)
- Recognition of Tokens (Transition Diagrams)
- The Lexical-Analyzer Generator
- Finite Automata

The Role of Lexical Analyzer

- Read the input characters of the source program, group them into lexemes, and produces a sequence of tokens
- Add lexemes into the symbol table when necessary



The Role of Lexical Analyzer

```
position = initial + rate * 60
```


```
<id, 1>
<=>
<id, 2>
<+>
<id, 3>
<*>
<60>
```

SYMBOL TABLE

1	position	...
2	initial	...
3	rate	...

Tokens, Patterns, and Lexemes

- A ***lexeme*** is a string of characters that is a lowest-level syntactic unit in programming languages

- A ***token*** is a syntactic category representing a class of lexemes. Formally, it is a pair <token name, attribute value>
  - Token name: an abstract symbol representing the kind of the token
  - Attribute value (optional) points to the symbol table

- Each token has a particular ***pattern***: a description of the form that the lexemes of the token may take

Examples

TOKEN	INFORMAL DESCRIPTION	SAMPLE LEXEMES
if	characters i, f	if
else	characters e, l, s, e	else
comparison	< or > or <= or >= or == or !=	<=, !=
id	letter followed by letters and digits	pi, score, D2
number	any numeric constant	3.14159, 0, 6.02e23
literal	anything but ", surrounded by "'s	"core dumped"

Consider the C statement: `printf("Total = %d\n", score);`

Lexeme	printf	score	"Total = %d\n"	(	...
Token	**id**	**id**	**literal**	**left_parenthesis**	**...**

Attributes for Tokens

- When more than one lexeme match a pattern, the lexical analyzer must provide additional information, named *attribute values*, to the subsequent compiler phases
  - Token names influence parsing decisions
  - Attribute values influence semantic analysis, code generation etc.

- For example, an **id** token is often associated with: (1) its lexeme, (2) type, and (3) the location at which it is first found. Token attributes are stored in the symbol table.

```
A = B * 2   ──────▶   <id, pointer to symbol-table entry for A>
                      <assign_op>
                      <id, pointer to symbol-table entry for B>
                      <mult_op> <number, integer value 2>
```

Lexical Errors

- When none of the patterns for tokens match any prefix of the remaining input
- Example: `int 3a = a * 3;`

Lexical errors and syntax errors in Java (learn by yourself):

- https://www.javatpoint.com/lexical-error
- https://www.javatpoint.com/syntax-error

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Specification of Tokens

- **Regular expression (正则表达式, regexp for short)** is an important notation for specifying lexeme patterns
- Content of this part
  - Strings and Languages (串和语言)
  - Operations on Languages (语言上的运算)
  - Regular Expressions
  - Regular Definitions (正则定义)
  - Extensions of Regular Expressions

Strings and Languages

- **Alphabet (字母表)**: any finite set of symbols
  - Examples of symbols: letters, digits, and punctuations
  - Examples of alphabets: {1, 0}, ASCII, Unicode
- A **string (串)** over an alphabet is a finite sequence of symbols drawn from the alphabet
  - The length of a string s, denoted $|s|$, is the number of symbols in s (i.e., cardinality)
  - Empty string (空串): the string of length 0, ϵ

Terms (using banana for illustration)

- **Prefix (前缀) of string *s*:** any string obtained by removing 0 or more symbols from the end of s (ban, banana, ϵ)
- **Proper prefix (真前缀):** a prefix that is not ϵ and not s itself (ban)
- **Suffix (后缀):** any string obtained by removing 0 or more symbols from the beginning of s (nana, banana, ϵ).
- **Proper suffix (真后缀):** a suffix that is not ϵ and not equal to s itself (nana)

Terms Cont.

- **Substring (子串) of s:** any string obtained by removing any prefix and any suffix from s (banana, nan, ϵ)
- **Proper substring (真子串):** a substring that is not ϵ and not equal to s itself (nan)
- **Subsequence (子序列):** any string formed by removing 0 or more not necessarily consecutive symbols from s (bnn)

How many substrings & subsequences does banana have?

(Two substrings are different as long as they have different start/end index)

String Operations (串的运算)

- **Concatenation (连接)**: the concatenation of two strings x and y, denoted xy, is the string formed by appending y to x
  - x = dog, y = house, xy = doghouse

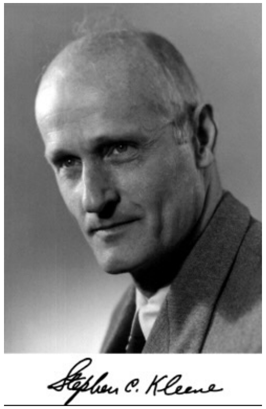
- **Exponentiation (幂/指数运算):** $s^0 = \epsilon$, $s^1 = s$, $s^i = s^{i-1}s$
  - x = dog, $x^0 = \epsilon$, x^1 = dog, x^3 = dogdogdog

Language (语言)

- A **language** is any **countable set**[1] of strings over some fixed alphabet
  - The set containing only the empty string, that is $\{\epsilon\}$, is a language, denoted $\emptyset$
  - The set of all grammatically correct English sentences
  - The set of all syntactically well-formed C programs

[1] In mathematics, a countable set is a set with the same cardinality (number of elements) as some subset of the set of natural numbers. A countable set is either a finite set or a countably infinite set.

Operations on Languages (语言的运算)



- 并，连接，Kleene闭包，正闭包

Operation	Definition and Notation
Union of L and M	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
Concatenation of L and M	$LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L	$L^* = \cup_{i=0}^{\infty} L^i$
Positive closure of L	$L^+ = \cup_{i=1}^{\infty} L^i$

The exponentiation of L can be defined using concatenation. L^n means concatenating L n times.

https://en.wikipedia.org/wiki/Stephen_Cole_Kleene

Examples

- L = {A, B, …, Z, a, b, …, z}
- D = {0, 1, …, 9}

$L \cup D$	{A, B, …, Z, a, b, …, z, 0, 1, …,9}
LD	the set of 520 strings of length two, each consisting of one letter followed by one digit
L^4	the set of all 4-letter strings
L^*	the set of all strings of letters, including ϵ
$L(L \cup D)^*$	?
D^+	?

Note: L, D might seem to be the alphabets of letters and digits. We define them to be languages: all strings happen to be of length one.

Regular Expressions - For Describing Languages/Patterns

Rules that define regexps over an alphabet Σ:

- **BASIS**: two rules form the basis:
  - ϵ is a regexp, $L(\epsilon) = \{\epsilon\}$
  - If a is a symbol in Σ, then a is a regexp, and $L(a) = \{a\}$
- **INDUCTION:** Suppose r and s are regexps denoting the languages $L(r)$ and $L(s)$
  - $(r)|(s)$ is a regexp denoting the language $L(r) \cup L(s)$
  - $(r)(s)$ is a regexp denoting the language $L(r)L(s)$
  - $(r)^*$ is a regexp denoting $(L(r))^*$
  - (r) is a regexp denoting $L(r)$. Additional parentheses do not change the language an expression denotes.

Regular Expressions Cont.

- Following the rules, regexps often contain unnecessary pairs of parentheses. We may drop some if we adopt the conventions:
  - **Precedence:** closure * > concatenation > union |
  - **Associativity:** All three operators are left associative, meaning that operations are grouped from the left, e.g., a | b | c would be interpreted as (a | b) | c
- Example: $(a) \mid ((b)^*(c)) = a \mid b^*c$

Regular Expressions Cont.

- Examples: Let $\Sigma = \{a, b\}$
  - $a|b$ denotes the language $\{a, b\}$
  - $(a|b)(a|b)$ denotes $\{aa, ab, ba, bb\}$
  - a^* denotes $\{\epsilon, a, aa, aaa, \ldots\}$
  - $(a|b)^*$ denotes the set of all strings consisting of 0 or more *a*'s or *b*'s: $\{\epsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$
  - $a|a^*b$ denotes the string *a* and all strings consisting of 0 or more *a*'s and ending in *b*: $\{a, b, ab, aab, aaab, \ldots\}$

Regular Language (正则语言)

- A **regular language** is a language that can be defined by a regexp
- If two regexps r and s denote the same language, they are *equivalent*, written as $r = s$

Regular Language Cont.

- Each algebraic law below asserts that expressions of two different forms are equivalent

LAW	DESCRIPTION
$r\|s = s\|r$	$\|$ is commutative
$r\|(s\|t) = (r\|s)\|t$	$\|$ is associative
$r(st) = (rs)t$	Concatenation is associative
$r(s\|t) = rs\|rt;\ (s\|t)r = sr\|tr$	Concatenation distributes over $\|$
$\epsilon r = r\epsilon = r$	ϵ is the identity for concatenation
$r^* = (r\|\epsilon)^*$	ϵ is guaranteed in a closure
$r^{**} = r^*$	$*$ is idempotent

Is (**a** | **b**)(**a** | **b**) = **aa** | **ab** | **ba** | **bb** true?

| can be viewed as + in arithmetics, concatenation can be viewed as ×, * can be viewed as the power operator.

Regular Definitions (正则定义)

- For notational convenience, we can give names to certain regexps and use those names in subsequent expressions

If Σ is an alphabet of basic symbols, then a ***regular definition*** is a sequence of definitions of the form:

$$d_1 \rightarrow r_1$$
$$d_2 \rightarrow r_2$$
$$\ldots$$
$$d_n \rightarrow r_n$$

where:

- Each d_i is a new symbol not in Σ and not the same as the other d's
- Each r_i is a regexp over the alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

Each new symbol denotes a regular language. The second rule means that you may reuse previously-defined symbols.

Examples

- Regular definition for C identifiers

$$\begin{array}{rcl} letter\_ & \rightarrow & \texttt{A} \mid \texttt{B} \mid \cdots \mid \texttt{Z} \mid \texttt{a} \mid \texttt{b} \mid \cdots \mid \texttt{z} \mid \texttt{\_} \\ digit & \rightarrow & \texttt{0} \mid \texttt{1} \mid \cdots \mid \texttt{9} \\ id & \rightarrow & letter\_ \; (\; letter\_ \mid digit \;)^* \end{array}$$

`_hello` valid?

`3times` valid?

- Regexp for C identifiers

```
(A|B|...|Z|a|b|...|z|_)((A|B|...|Z|a|b|...|z|_)|(0|1|...|9))*
```

Extensions of Regular Expressions

- **Basic operators:** union |, concatenation, and Kleene closure * (proposed by Kleene in 1950s)
- A few **notational extensions**:
  - One of more instances: the unary, postfix operator $^+$
    - $r^+ = rr^*$, $r^* = r^+ \mid \epsilon$
  - Zero or one instance: the unary postfix operator ?
    - $r? = r \mid \epsilon$
  - Character classes: shorthand for a logical sequence
    - $[a_1 a_2 \ldots a_n] = a_1 \mid a_2 \mid \ldots \mid a_n$
    - $[a\text{-}e] = a \mid b \mid c \mid d \mid e$
- The extensions are only for notational convenience, they do not change the descriptive power of regexps

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Recognition of Tokens

- Lexical analyzer examines the input string and finds a prefix that matches one of the tokens

- The first thing when building a lexical analyzer is to define the patterns of tokens using regular definitions

- A special token: `ws` → `(blank | tab | newline)`$^+$
  - When the lexical analyzer recognizes a whitespace token, it does not return it to the parser, but restart from the next character

Example: Patterns and Tokens

$$
\begin{array}{rcl}
\mathit{digit} & \rightarrow & \texttt{[0-9]} \\
\mathit{digits} & \rightarrow & \mathit{digit}^{+} \\
\mathit{number} & \rightarrow & \mathit{digits}\ (.\ \mathit{digits})?\ (\ \texttt{E}\ \texttt{[+-]}?\ \mathit{digits}\ )? \\
\mathit{letter} & \rightarrow & \texttt{[A-Za-z]} \\
\mathit{id} & \rightarrow & \mathit{letter}\ (\ \mathit{letter} \mid \mathit{digit}\ )^{*} \\
\mathit{if} & \rightarrow & \texttt{if} \\
\mathit{then} & \rightarrow & \texttt{then} \\
\mathit{else} & \rightarrow & \texttt{else} \\
\mathit{relop} & \rightarrow & \texttt{<} \mid \texttt{>} \mid \texttt{<=} \mid \texttt{>=} \mid \texttt{=} \mid \texttt{<>}
\end{array}
$$

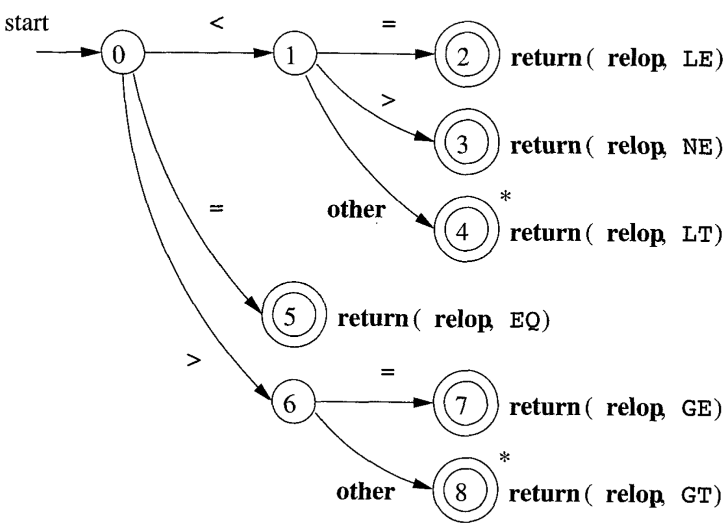
Patterns for tokens

LEXEMES	TOKEN NAME	ATTRIBUTE VALUE
Any *ws*	–	–
`if`	**if**	–
`then`	**then**	–
`else`	**else**	–
Any *id*	**id**	Pointer to table entry
Any *number*	**number**	Pointer to table entry
`<`	**relop**	`LT`
`<=`	**relop**	`LE`
`=`	**relop**	`EQ`
`<>`	**relop**	`NE`
`>`	**relop**	`GT`
`>=`	**relop**	`GE`

Lexemes, tokens, and attribute values

Transition Diagrams (状态转换图)

- An important step in constructing a lexical analyzer is to convert patterns into "transition diagrams"
- Transition diagrams have a collection of nodes, called *states* (状态) and *edges* (边) directed from one node to another

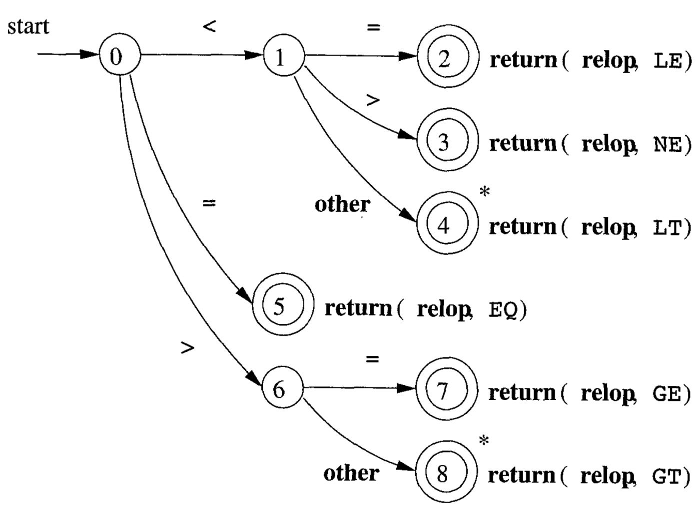


LEXEMES	TOKEN NAME	ATTRIBUTE VALUE
<	relop	LT
<=	relop	LE
=	relop	EQ
<>	relop	NE
>	relop	GT
>=	relop	GE

The transition diagram in the left recognizes **relop** tokens

States

- Represent conditions that could occur during the process of scanning (i.e., what characters we have seen)
- The *start state* (开始状态), or *initial state*, is indicated by an edge labeled "start", which enters from nowhere
- Certain states are said to be *accepting* (接受状态), or *final*, indicating that a lexeme has been found

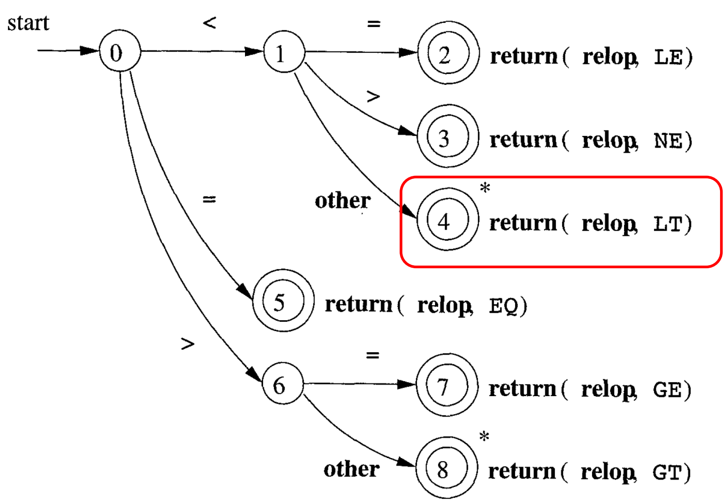


States 2-8 are accepting. They return a pair (token name, attribute value).

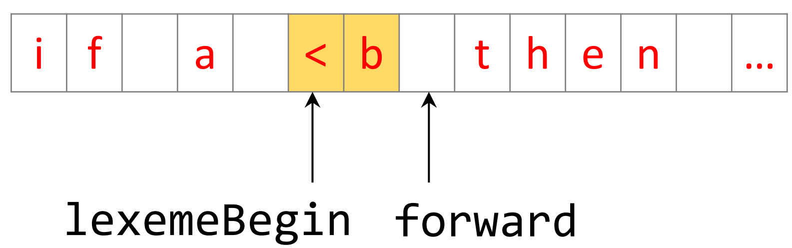
By convention, we indicate accepting states by double circles

The Retract Action

- At certain accepting states, the found lexeme may not contain all characters that we have seen from the start state (such states are annotated with *)
- When entering * states, it is necessary to retract (回退) the `forward` pointer, which points to the next char in the input string



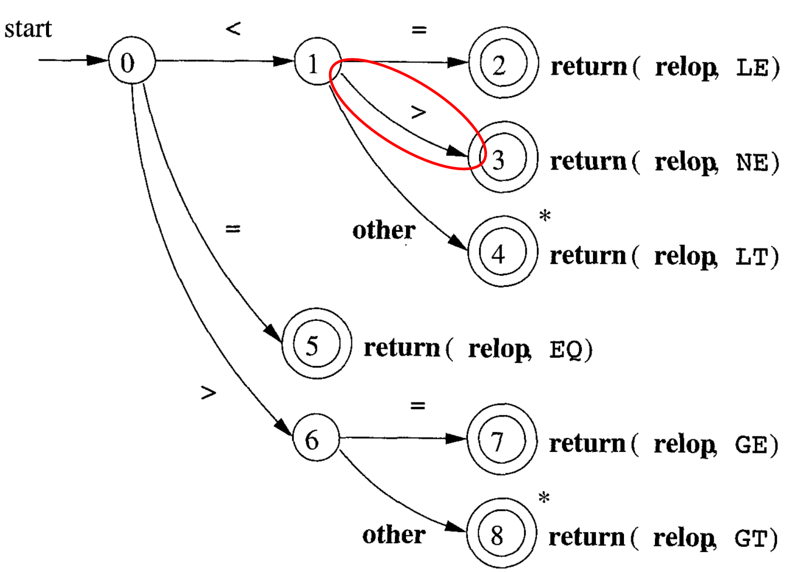
- The found lexeme: <
- The characters we've seen: <b

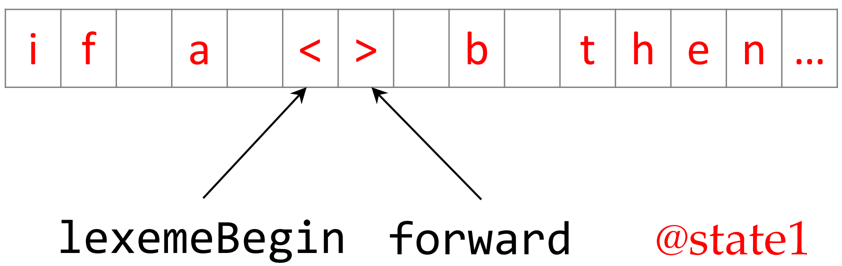


We should retract `forward` one step back

Edges

- *Edges* are directed from one state to another
- Each edge is labeled by a symbol or set of symbols

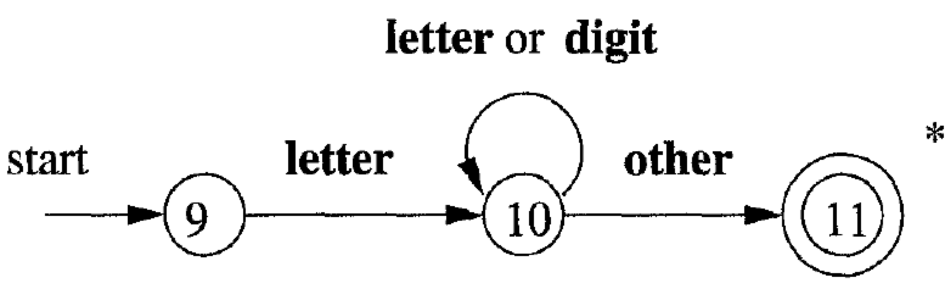




In the above case, we should follow the circled edge to enter state 3 and advance the `forward` pointer

Recognition of Reserved Words and Identifiers (保留字和标识符的识别)

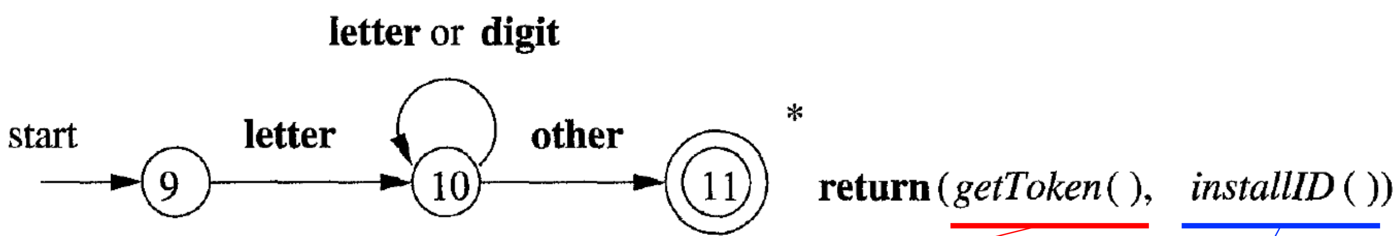
- In many languages, **reserved words** or **keywords** (e.g., `then`) also match the pattern of identifiers
- **Problem:** the transition diagram that searches for identifiers can also recognize reserved words



Is `then` an identifier?

Handling Reserved Words

- **Strategy 1:** Preinstall the reserved words in the symbol table. Put a field in the symbol-table entries to indicate that these strings are not ordinary identifiers (预先存表方案)

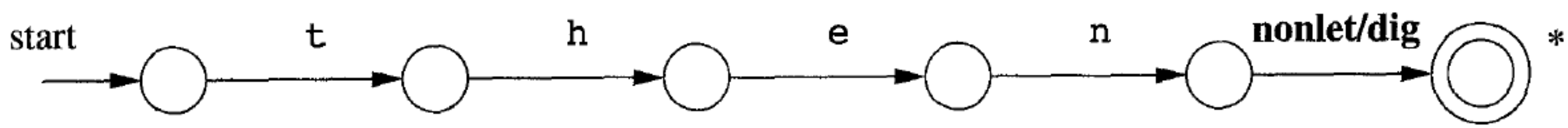


Examine the symbol table and return the token name (identifier or reserved word)

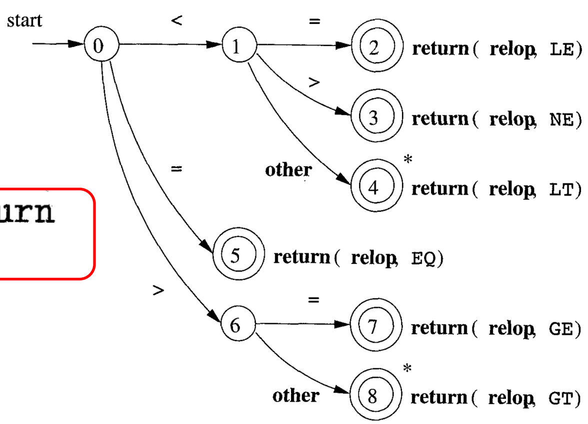
Place a recognized identifier/keyword in the symbol table if it is not already there and return a pointer to the entry

Handling Reserved Words

- **Strategy 2:** Create a separate transition diagram with **a high priority** for each keyword (多状态转移图方案)



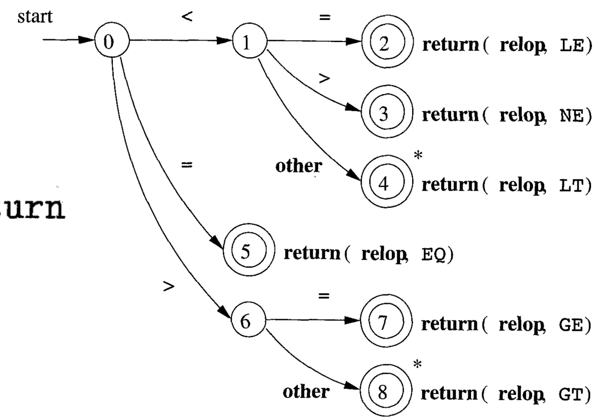
Building a Lexical Analyzer from Transition Diagrams




```
TOKEN getRelop()
{
    TOKEN retToken = new(RELOP);
    while(1) { /* repeat character processing until a return
                  or failure occurs */
        switch(state) {
            case 0: c = nextChar();
                    if ( c == '<' ) state = 1;
                    else if ( c == '=' ) state = 5;
                    else if ( c == '>' ) state = 6;
                    else fail(); /* lexeme is not a relop */
                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```

Sketch implementation of `relop` transition diagram

Building a Lexical Analyzer from Transition Diagrams

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                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
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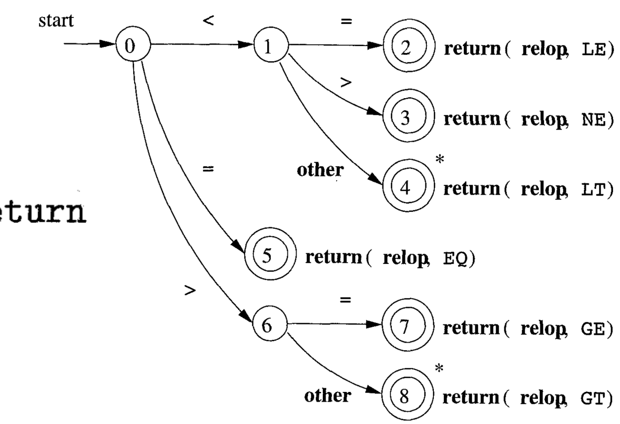
```

Use a variable `state` to record the current state

Sketch implementation of `relop` transition diagram

Building a Lexical Analyzer from Transition Diagrams

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                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```
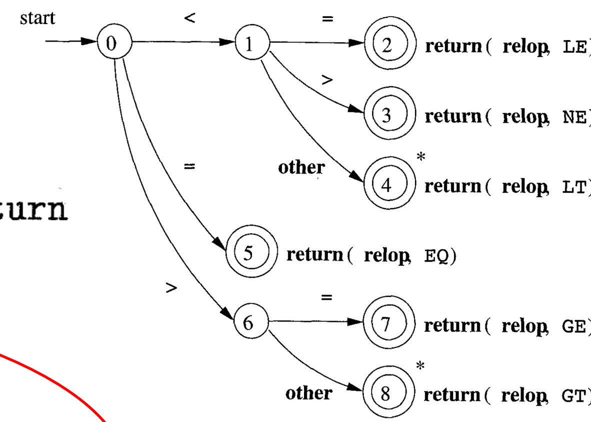

A `switch` statement based on the value of `state` takes us to the processing code

Sketch implementation of `relop` transition diagram

Building a Lexical Analyzer from Transition Diagrams

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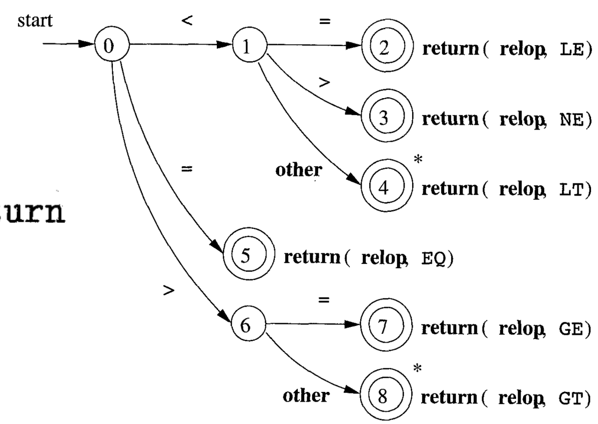
                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```

The code of a normal state:

1. Read the next character
2. Determine the next state
3. If step 2 fails, do error recovery

Sketch implementation of `relop` transition diagram

Building a Lexical Analyzer from Transition Diagrams

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                    break;
            case 1: ...
            ...
            case 8: retract();
                    retToken.attribute = GT;
                    return(retToken);
        }
    }
}
```

The code of an accepting state:

1. Perform retraction if the state has *
2. Set token attribute values
3. Return the token to parser

Sketch implementation of `relop` transition diagram

Building the Entire Lexical Analyzer

- **Strategy 1:** Try the transition diagram for each type of token sequentially
  - `fail()` resets the pointer `forward` and tries the next diagram

- **Problem:** Not efficient
  - May need to try many irrelevant diagrams whose first edge does not match the first character in the input stream

Building the Entire Lexical Analyzer

- **Strategy 2**: Run transition diagrams in parallel
  - Need to resolve the case where one diagram finds a lexeme and others are still able to process input.
  - **Solution:** take the longest prefix of the input that matches any pattern

- **Problem:** Requires special hardware for parallel simulation, may degenerate into the sequential strategy on certain machines

Building the Entire Lexical Analyzer

- **Strategy 3:** Combining all transition diagrams into one
  - Allow the transition diagram to read input until there is no possible next state
  - Take the longest lexeme that matched any pattern

- This is a commonly-adopted strategy in real-world compiler implementation (efficient & requires no special hardware)

How? Be patient ☺, we will talk about this later.