南方科技大学
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Chapter 2: Lexical Analysis

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The chapter numbering in lecture notes does not follow that in the textbook.

Outline

- The Role of the Lexical Analyzer
- Specification of Tokens (Regular Expressions)
- Recognition of Tokens (Transition Diagrams)
- The Lexical-Analyzer Generator (Lab Content)
- Finite Automata

The Lexical-Analyzer Generator `Lex`

- `Lex`, or a more recent tool `Flex`, allows one to specify a lexical analyzer by specifying regexps to describe patterns for tokens

- Often used with Yacc/Bison to create the frontend of compiler

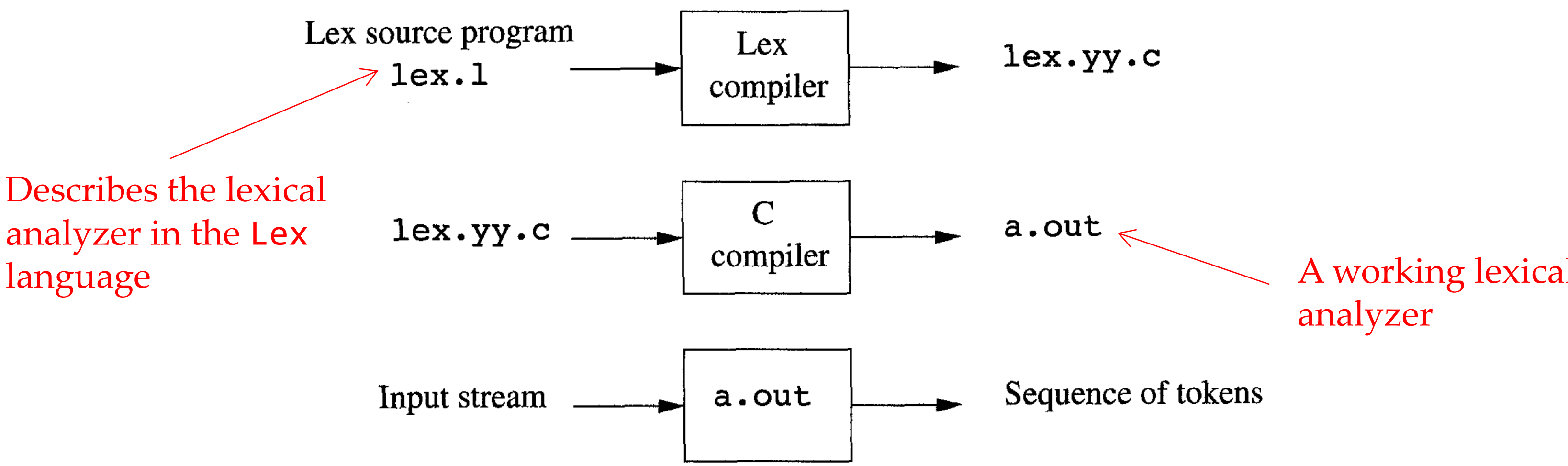


Figure 3.22: Creating a lexical analyzer with Lex

Structure of Lex Programs

- **A Lex program has three sections separated by %%**
  - Declaration (声明)
    - Variables, constants (e.g., token names)
    - Regular definitions
  - Translation rules (转换规则) in the form "Pattern {Action}"
    - Each pattern (模式) is a regexp (may use the regular definitions of the declaration section)
    - Actions (动作) are fragments of code, typically in C, which are executed when the pattern is matched
  - Auxiliary functions section (辅助函数)
    - Additional functions that can be used in the actions

Lex Program Example

```
%{
    /* definitions of manifest constants
    LT, LE, EQ, NE, GT, GE,
    IF, THEN, ELSE, ID, NUMBER, RELOP */
%}

/* regular definitions */
delim       [ \t\n]
ws          {delim}+
letter      [A-Za-z]
digit       [0-9]
id          {letter}({letter}|{digit})*
number      {digit}+(\.{digit}+)?(E[+-]?{digit}+)?

%%
```

Anything in between %{ and }% is copied directly to `lex.yy.c`.

In the example, there is only a comment, not real C code to define manifest constants

Regular definitions that can be used in translation rules

Section separator

Lex Program Example Cont.

Continue to recognize other tokens

Return token name to the parser

Place the lexeme found in the symbol table

Literal strings*

```
{ws}        {/* no action and no return */}
if          {return(IF);}
then        {return(THEN);}
else        {return(ELSE);}
{id}        {yylval = (int) installID(); return(ID);}
{number}    {yylval = (int) installNum(); return(NUMBER);}
"<"         {yylval = LT; return(RELOP);}
"<="        {yylval = LE; return(RELOP);}
"="         {yylval = EQ; return(RELOP);}
"<>"        {yylval = NE; return(RELOP);}
">"         {yylval = GT; return(RELOP);}
">="        {yylval = GE; return(RELOP);}

%%
```

A global variable that stores a pointer to the symbol table entry for the lexeme. Can be used by the parser or a later component of the compiler.

* The characters inside have no special meaning (even if it is a special one such as *).

Lex Program Example Cont.

- Everything in the auxiliary function section is copied directly to the file `lex.yy.c`
- Auxiliary functions may be used in actions in the translation rules

```
int installID() {/* function to install the lexeme, whose
                    first character is pointed to by yytext,
                    and whose length is yyleng, into the
                    symbol table and return a pointer
                    thereto */
}

int installNum() {/* similar to installID, but puts numer-
                    ical constants into a separate table */
}
```

Variables defined and set automatically by the lexical analyzer `Lex` generates

Conflict Resolution

- When the generated lexical analyzer runs, it analyzes the input looking for prefixes that match any of its patterns. *

- **Rule 1:** If it finds multiple such prefixes, it takes the longest one
  - The analyzer will treat <= as a single lexeme, rather than < as one lexeme and = as the next

- **Rule 2:** If it finds a prefix matching different patterns, the pattern listed first in the `Lex` program is chosen.
  - If the keyword patterns are listed before identifier pattern, the lexical analyzer will not recognize keywords as identifiers

* See Flex manual for details (Chapter 8: How the input is matched) at http://dinosaur.compilertools.net/flex/

Outline

- The Role of the Lexical Analyzer
- Specification of Tokens (Regular Expressions)
- Recognition of Tokens (Transition Diagrams)
- The Lexical-Analyzer Generator
- Finite Automata →
  - NFA & DFA
  - NFA → DFA
  - Regexp → NFA
  - Combining NFA's
  - DFA Minimization (Self-Study Materials)

Finite Automata (有穷自动机)

- Finite automata are the simplest machines to recognize patterns
- They are essentially graphs like transition diagrams. They simply say "yes" or "no" about each possible input string.
  - **Nondeterministic finite automata (NFA, 非确定有穷自动机):** A symbol can label several edges out of the same state (allowing multiple target states), and the empty string ϵ is a possible label.
  - **Deterministic finite automata (DFA, 确定有穷自动机):** For each state and for each symbol in the input alphabet, there is exactly one edge with that symbol leaving that state.
- NFA and DFA recognize the same languages, **regular languages**, which regexps can describe.

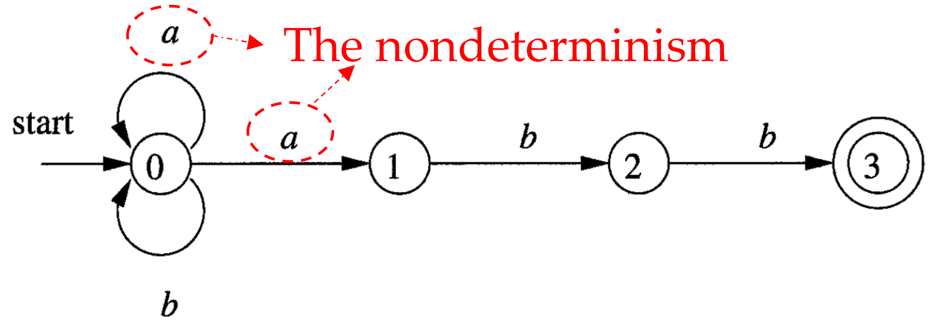
Nondeterministic Finite Automata

- An **NFA** is a 5-tuple, consisting of:
  1. A finite set of states S
  2. A set of input symbols Σ, the *input alphabet*. We assume that the empty string ϵ is never a member of Σ
  3. A *transition function* that gives, for each state, and for each symbol in $\Sigma \cup \{\epsilon\}$ a set of *next states*
  4. A *start state* (or initial state) s_0 from S
  5. A set of *accepting states* (or *final states*) F, a subset of S

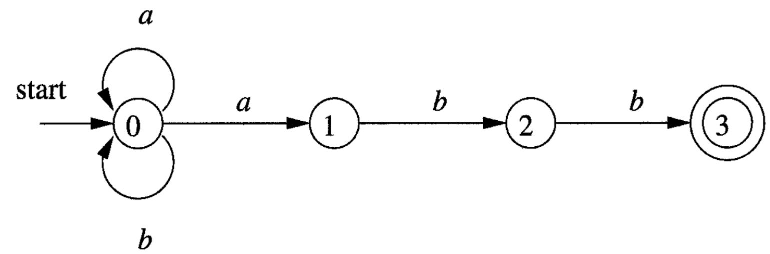
NFA Example

- S = {0, 1, 2, 3}
- Σ = {a, b}
- Start state: 0
- Accepting states: {3}
- Transition function
  - (0, a) → {0, 1} (0, b) → {0}
  - (1, b) → {2} (2, b) → {3}

The NFA can be represented as a Transition Graph:



Transition Table

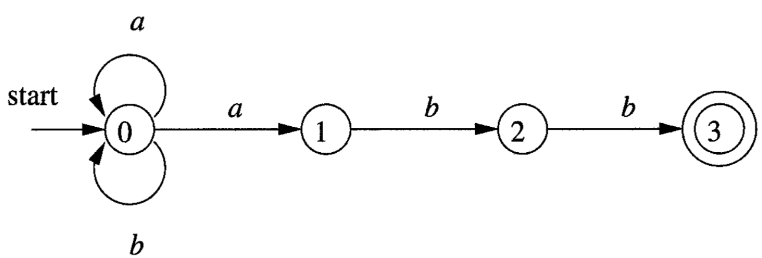


- Another representation of an NFA
  - Rows correspond to states
  - Columns correspond to the input symbols or ϵ
  - The table entry for a state-input pair lists the set of next states
  - $\emptyset$: the transition function has no info about the state-input pair

STATE	a	b	ϵ
0	$\{0, 1\}$	$\{0\}$	$\emptyset$
1	$\emptyset$	$\{2\}$	$\emptyset$
2	$\emptyset$	$\{3\}$	$\emptyset$
3	$\emptyset$	$\emptyset$	$\emptyset$

Acceptance of Input Strings

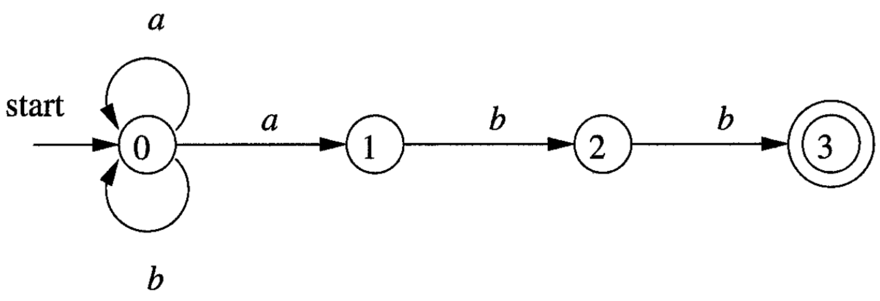
- An NFA accepts an input string x if and only if
  - There is a path in the transition graph from the start state to one accepting state, such that the symbols along the path form x (ϵ labels are ignored).



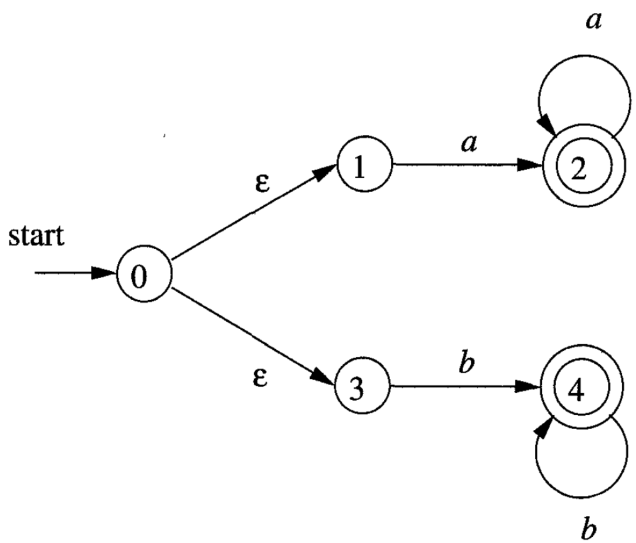
accepts the string *aabb*

- The language defined or accepted by an NFA
  - The set of strings labelling some path from the start state to an accepting state

NFA and Regular Languages



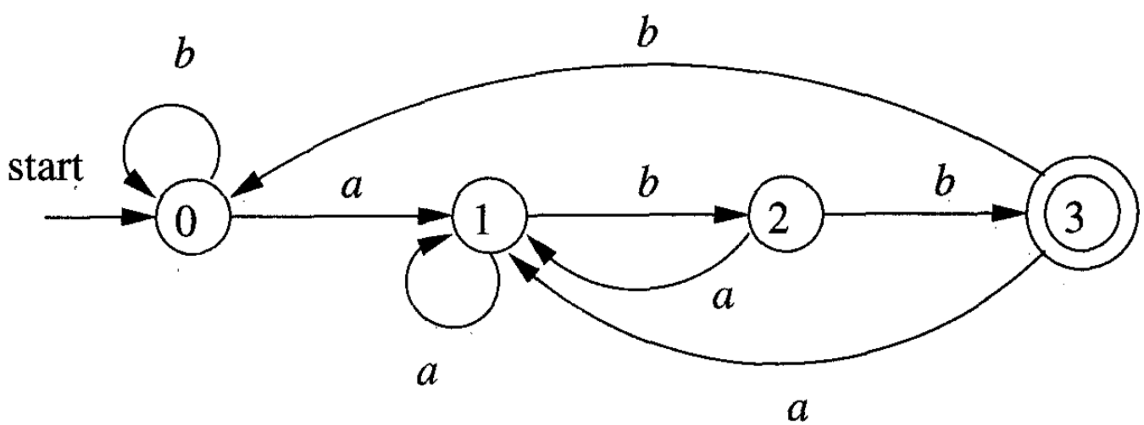
$L((a|b)^*abb)$



$L(aa^*|bb^*)$

Deterministic Finite Automata (DFA)

- A **DFA** is a special NFA where:
  - There are no moves on input ϵ
  - For each state s and input symbol a, there is exactly one edge out of s labeled a (i.e., exactly one target state)



Simulating a DFA

- **Input:**
  - String x terminated by an end-of-file character **eof**.
  - DFA D with *start state* s_0, *accepting states* F, and transition function *move*
- **Output:** Answer "yes" if D accepts x; "no" otherwise

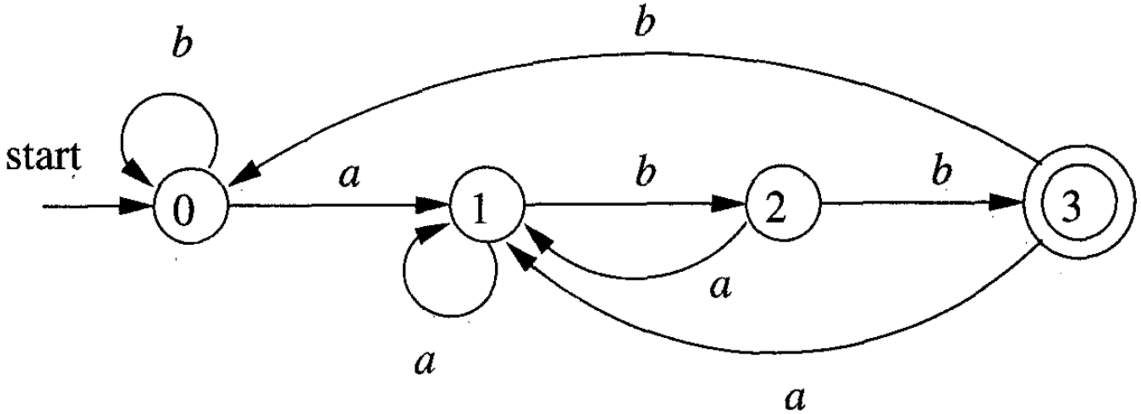
```
s = s0;
c = nextChar();
while ( c != eof ) {
        s = move(s, c);
        c = nextChar();
}
if ( s is in F ) return "yes";
else return "no";
```

We can see from the algorithm:

- DFA can efficiently accept/reject strings (i.e., recognize patterns)

DFA Example

- Given the input string *ababb*, the DFA below enters the sequence of states 0, 1, 2, 1, 2, 3 and returns "yes"



What's the language defined by this DFA?

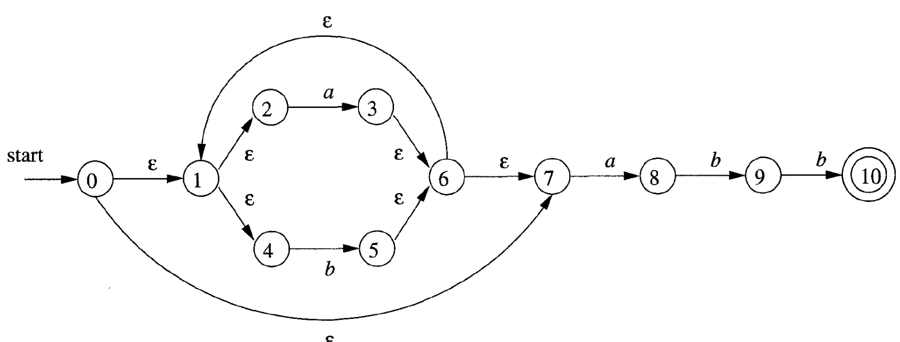
From Regular Expressions to Automata

- Regexps concisely & precisely describe the patterns of tokens
- DFA can efficiently recognize patterns (comparatively, the simulation of NFA is less straightforward*)
- When implementing lexical analyzers, regexps are often converted to DFA:
  - Regexp → NFA → DFA
  - Algorithms: Thompson's construction + subset construction

* There may be multiple transitions at a state when seeing a symbol

Conversion of an NFA to a DFA

- The **subset construction** algorithm (子集构造法)
  - **Insight:** Each state of the constructed DFA corresponds to a set of NFA states
    - Why? Because after reading the input $a_1a_2\ldots a_n$, the DFA reaches one state while the NFA may reach multiple states
  - **Basic idea:** The algorithm simulates "in parallel" all possible moves an NFA can make on a given input string to map a set of NFA states to a DFA state.

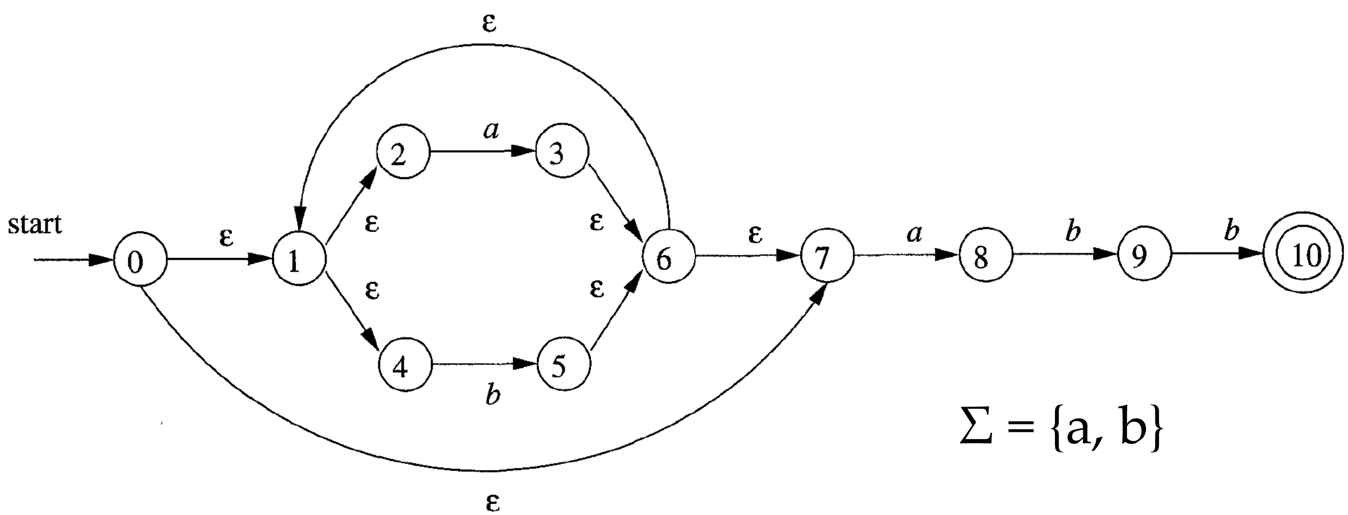


After reading "a", the NFA may reach any of these states:

3, 6, 1, 7, 2, 4, 8

Example for Algorithm Illustration

- The NFA below accepts the string *babb*
  - There exists a path from the start state 0 to the accepting state 10, on which the labels on the edges form the string *babb*



$\Sigma = \{a, b\}$

Subset Construction Technique

- Operations used in the algorithm:
  - ***ϵ-closure*(*s*):** Set of NFA states reachable from NFA state *s* on *ϵ*-transitions alone
  - ***ϵ-closure*(*T*):** Set of NFA states reachable from some NFA state *s* in set *T* on *ϵ*-transitions alone
    - That is, $\bigcup_{s\ in\ T} \epsilon\text{–}closure(s)$
  - ***move*(*T, a*):** Set of NFA states to which there is a transition on input symbol *a* from some state *s* in *T* (i.e., the target states of those states in *T* when seeing *a*)

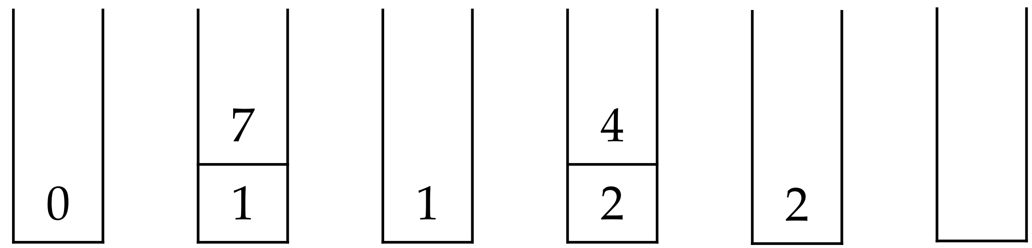
Subset Construction Technique

- **Computing ϵ-*closure*(T)**
  - It is a graph traversal process (only consider ϵ edges)
  - Computing ϵ-*closure*(s) is the same (when T has only one state)

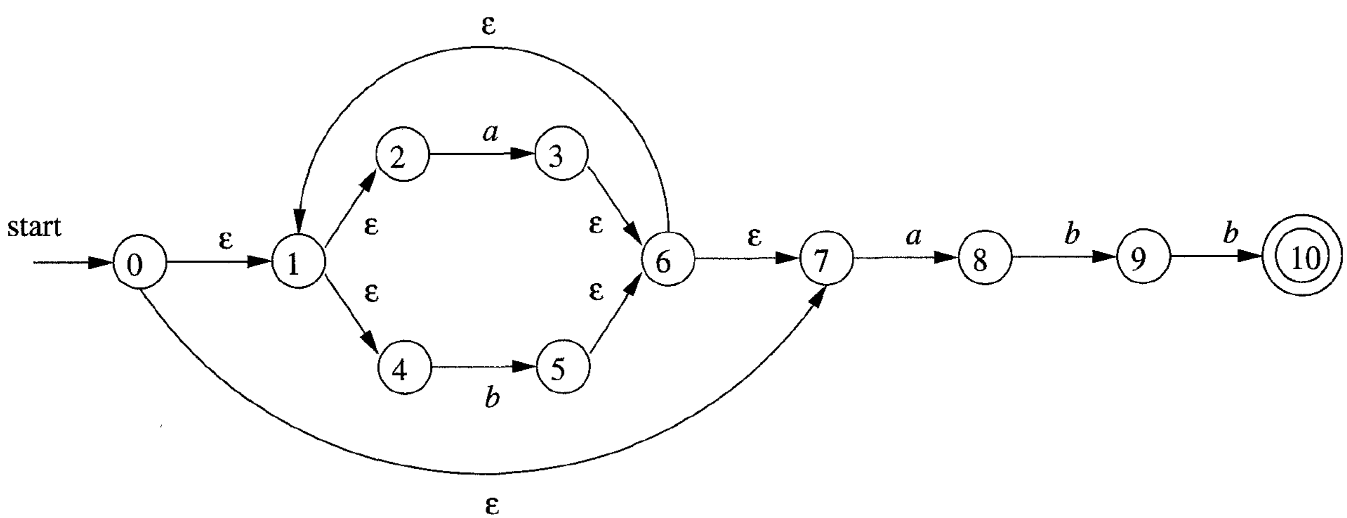
```
push all states of T onto stack;
initialize ε-closure(T) to T;
while ( stack is not empty ) {
        pop t, the top element, off stack;
        for ( each state u with an edge from t to u labeled ε )
                if ( u is not in ε-closure(T) ) {
                        add u to ε-closure(T);
                        push u onto stack;
                }
}
```

Illustrative Example

- ϵ-*closure*(0) = ?

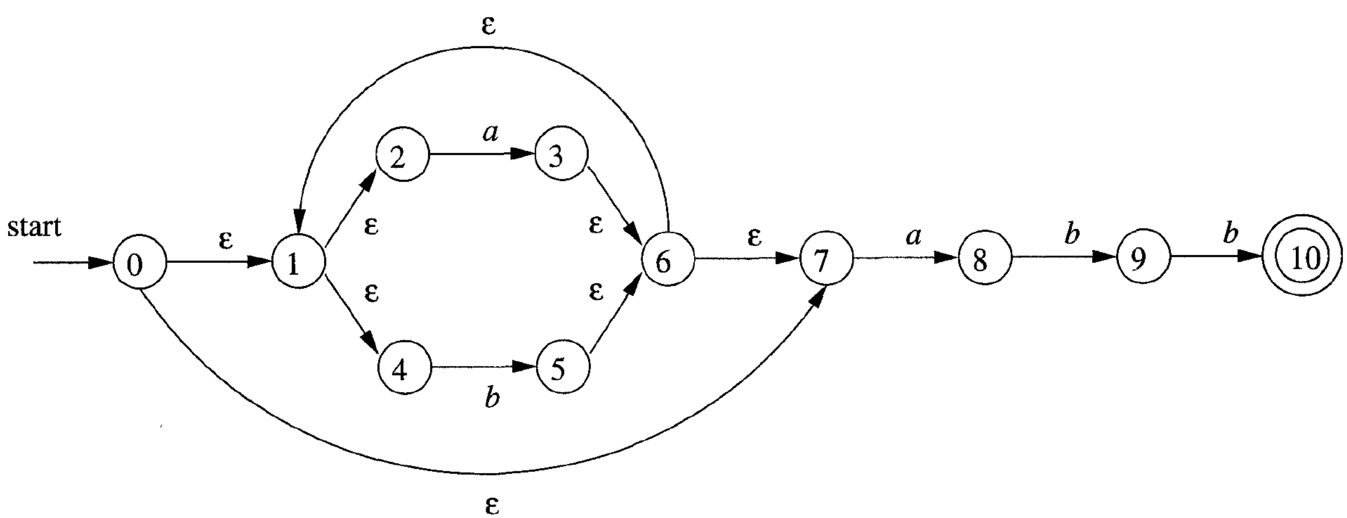


Push 0	Pop 0, Push 1, 7	Pop 7	Pop 1, Push 2, 4	Pop 4	Pop 2
{0}	{0, 1, 7}		{0, 1, 7, 2, 4}		



Exercise

- ϵ-*closure*({3, 8}) = ?



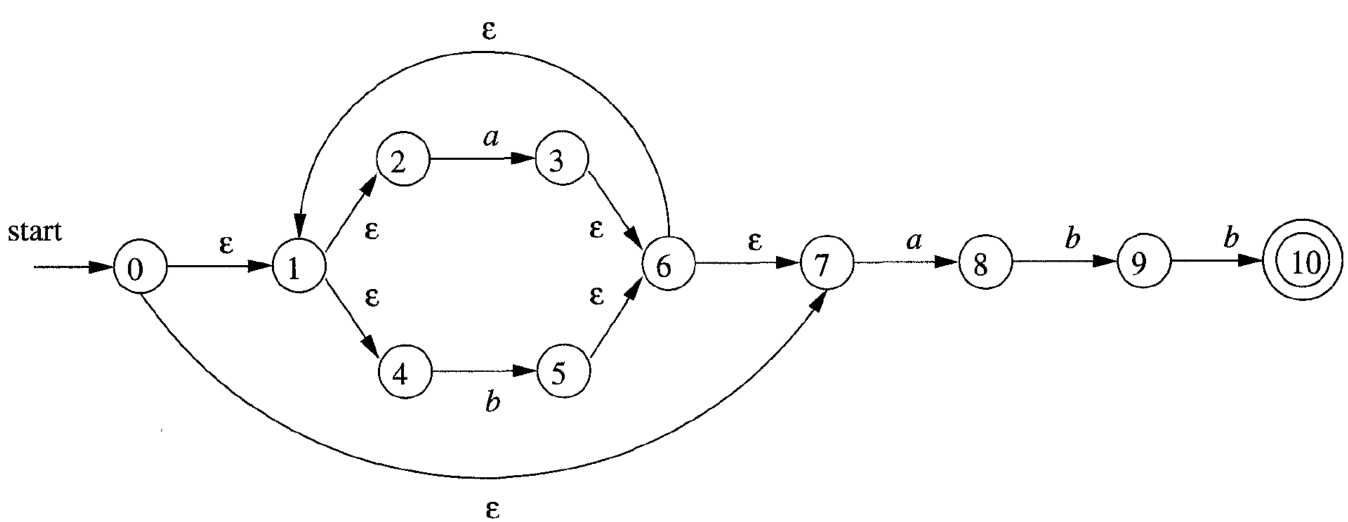
Subset Construction Technique Cont.

- The construction of the DFA *D*'s states, *Dstates*, and the transition function *Dtran* is also a search process
  - Initially, the only state in *Dstates* is ϵ-*closure*(s_0) and it is unmarked
    - Unmarked state means that its next states have not been explored

```
while ( there is an unmarked state T in Dstates ) {
        mark T;
        for ( each input symbol a ) {  // find the next states of T
                U = ϵ-closure(move(T, a));
                if ( U is not in Dstates )
                        add U as an unmarked state to Dstates;
                Dtran[T, a] = U;
        }
}
```

Illustrative Example

- Initially, Dstates only has one unmarked state:
  - ϵ-*closure*(0) = {0, 1, 2, 4, 7} -- A
- Dtran is empty



Illustrative Example

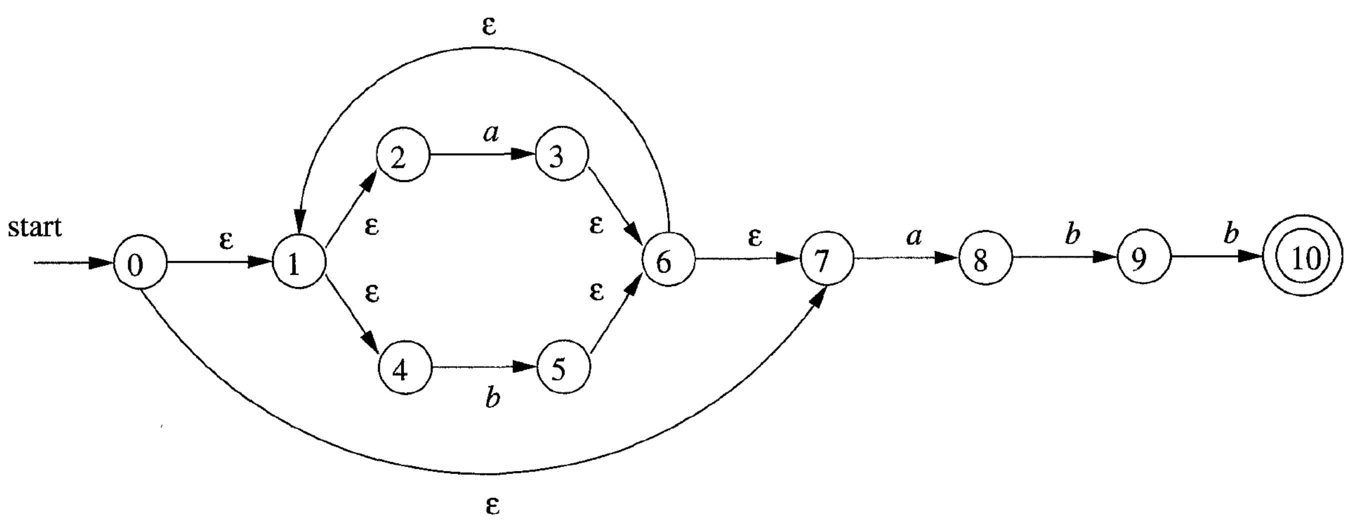
{0, 1, 2, 4, 7} -- A

ϵ-*closure*(*move*[A, a])

= ϵ-*closure*({3, 8})

= {1, 2, 3, 4, 6, 7, 8}

- We get an unseen state {1, 2, 3, 4, 6, 7, 8} -- B
- Update Dstates: {A, B}
- Update Dtran: {[A, a] ➔ B}



Illustrative Example

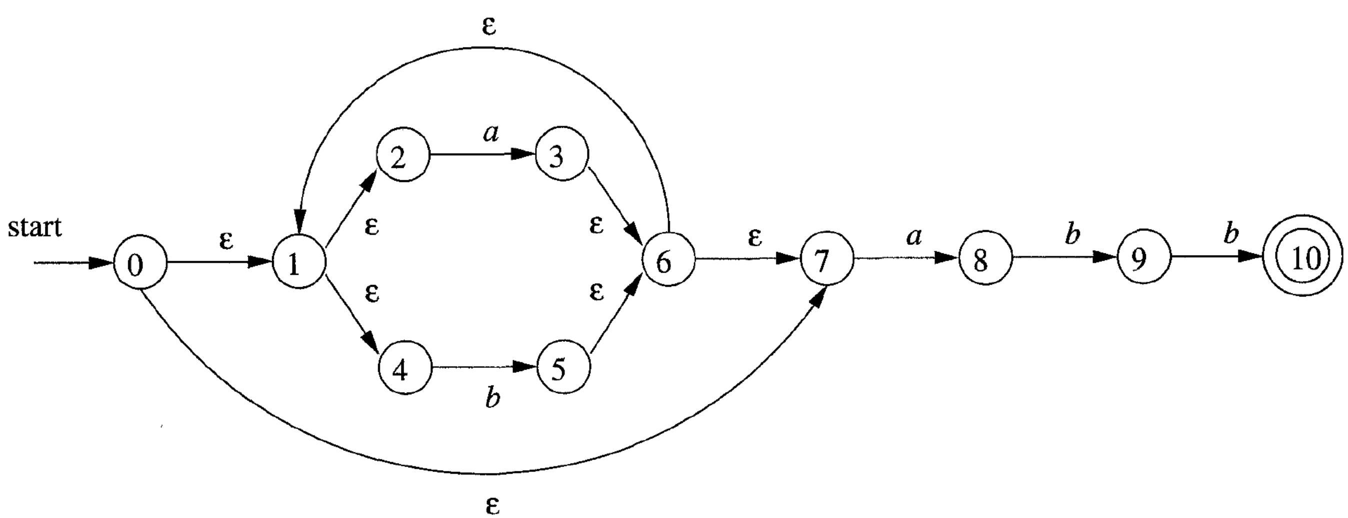
{0, 1, 2, 4, 7} -- A

ϵ-*closure*(*move*[A, b])

= ϵ-*closure*({5})

= {1, 2, 4, 5, 6, 7}

- We get an unseen state {1, 2, 4, 5, 6, 7} -- C
- Update Dstates: {A, B, C}
- Update Dtran: {[A, a] ➔ B, [A, b] ➔ C}



Illustrative Example

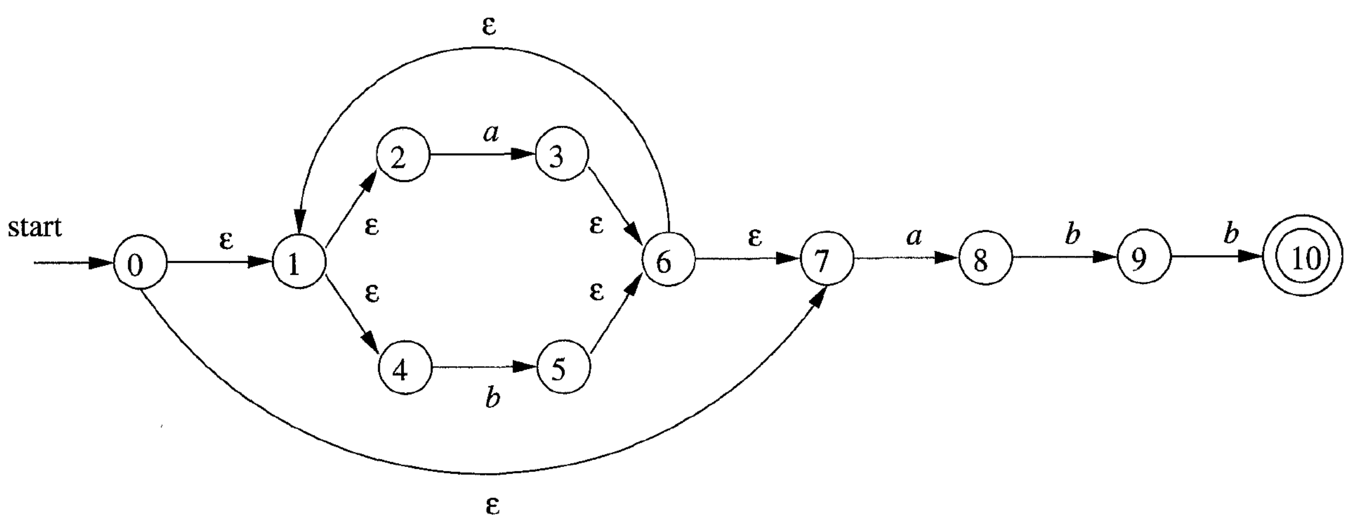
{1, 2, 3, 4, 6, 7, 8} -- B

ϵ-*closure*(*move*[B, a])

= ϵ-*closure*({3, 8})

= {1, 2, 3, 4, 6, 7, 8}

- The state {1, 2, 3, 4, 6, 7, 8} already exists (B)
- No need to update Dstates: {A, B, C}
- Update Dtran: {[A, a] ➔ B, [A, b] ➔ C, [B, a] ➔ B}



Illustrative Example

- Eventually, we will get the following DFA:
  - Start state: A; Accepting states: {E}

NFA STATE	DFA STATE	a	b
$\{0, 1, 2, 4, 7\}$	A	B	C
$\{1, 2, 3, 4, 6, 7, 8\}$	B	B	D
$\{1, 2, 4, 5, 6, 7\}$	C	B	C
$\{1, 2, 4, 5, 6, 7, 9\}$	D	B	E
$\{1, 2, 4, 5, 6, 7, 10\}$	E	B	C

This DFA can be further minimized: A and C have the same moves on all symbols and can be merged.

Regular Expression to NFA

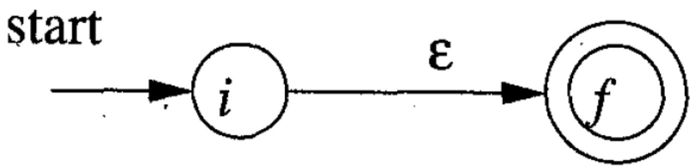
Thompson's construction algorithm (Thompson构造法)

- The algorithm works recursively by splitting a regular expression into subexpressions, from which the NFA will be constructed using the following rules:
  - **Two basis rules (基本规则):** handle subexpressions with no operators
  - **Three inductive rules (归纳规则):** construct larger NFA's from the smaller NFA's for subexpressions

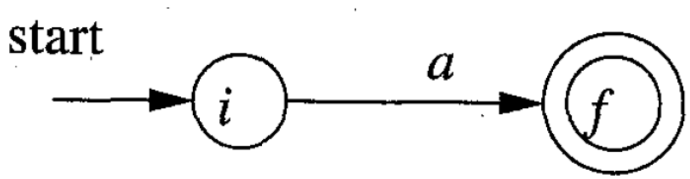
Thompson's Construction Algorithm

Two basis rules:

1. The empty expression ϵ is converted to



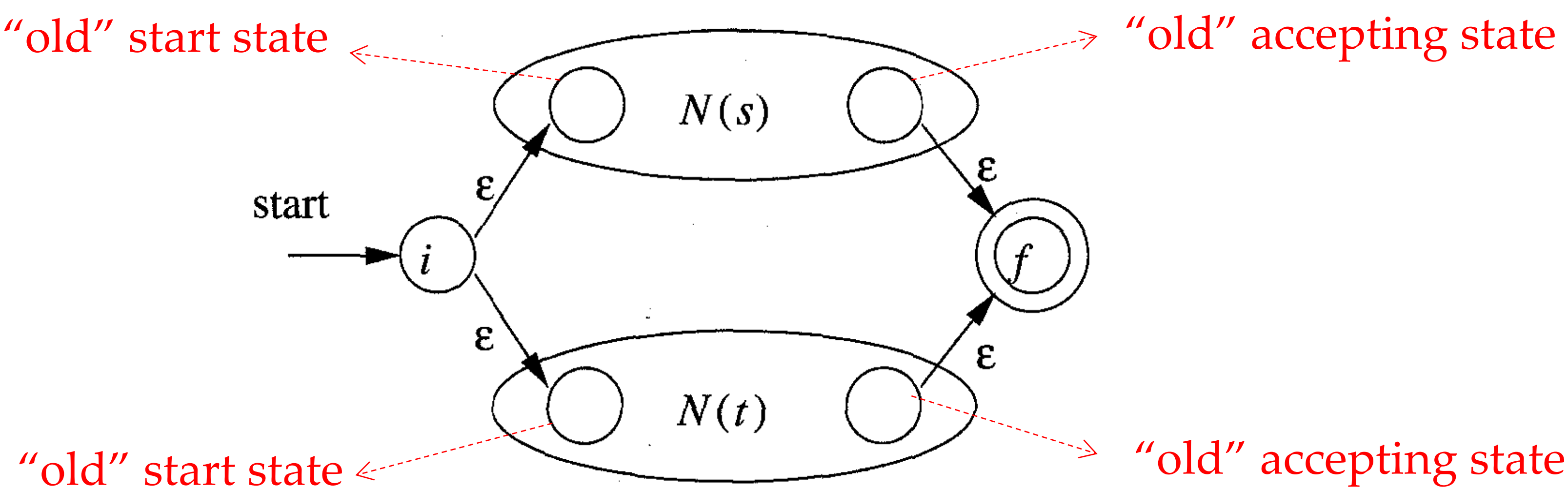
2. Any subexpression a (a single symbol in input alphabet) is converted to



Thompson's Construction Algorithm

The inductive rules: the union case

- ***s* | *t* :** $N(s)$ and $N(t)$ are NFA's for subexpressions s and t

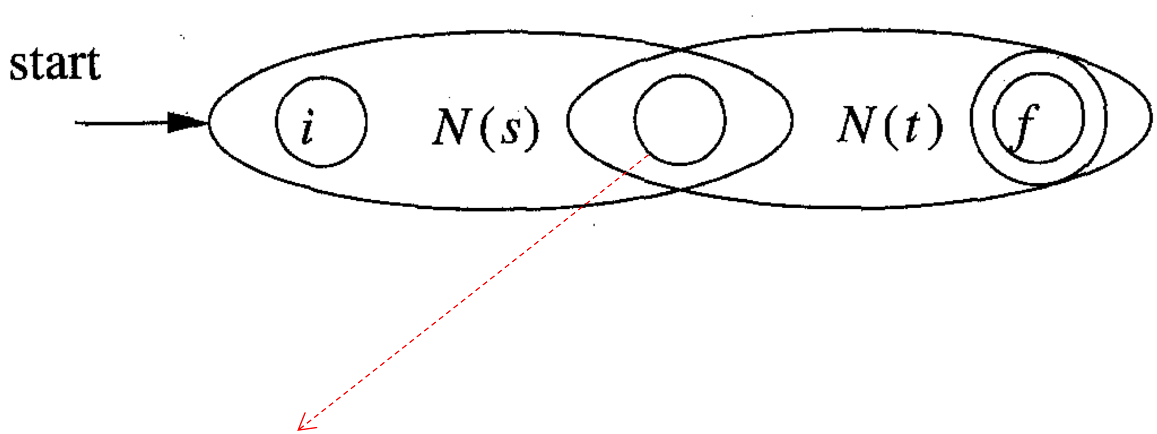


By construction, the NFA's have only one start state and one accepting state

Thompson's Construction Algorithm

The inductive rules: the concatenation case

- ***st*** **:** *N*(*s*) and *N*(*t*) are NFA's for subexpressions *s* and *t*

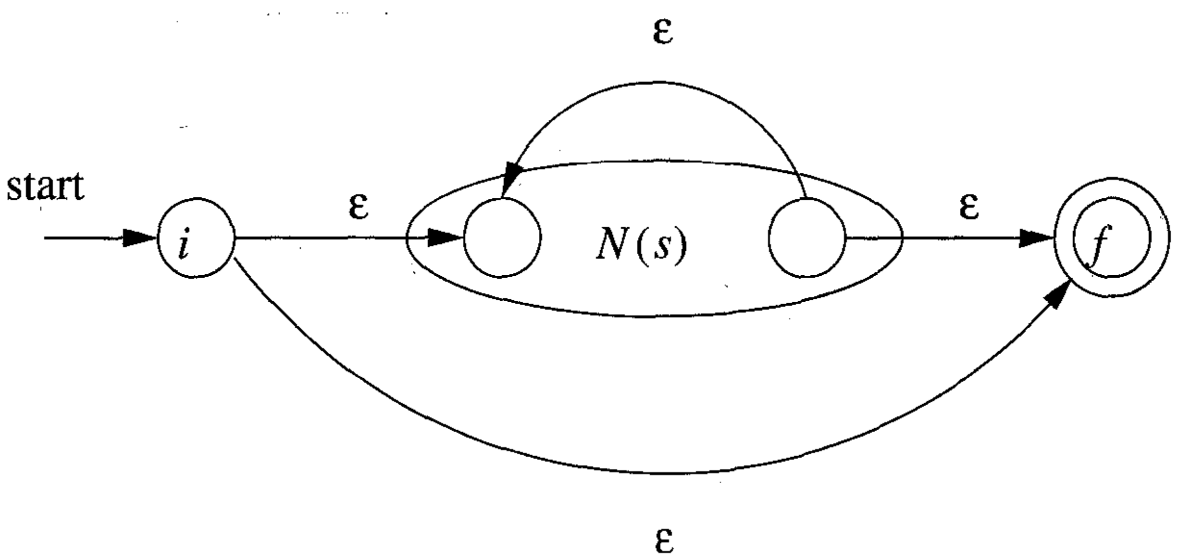


Merging the accepting state of *N*(*s*) and the start state of *N*(*t*)

Thompson's Construction Algorithm

The inductive rules: the Kleene star case

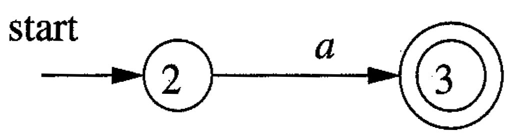
- s^* : $N(s)$ is the NFA for subexpression s



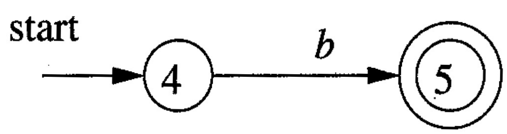
Example

Use Thompson's algorithm to construct an NFA for the regexp r = **(a | b)*a**

1. NFA for the first **a** (apply basis rule #1)



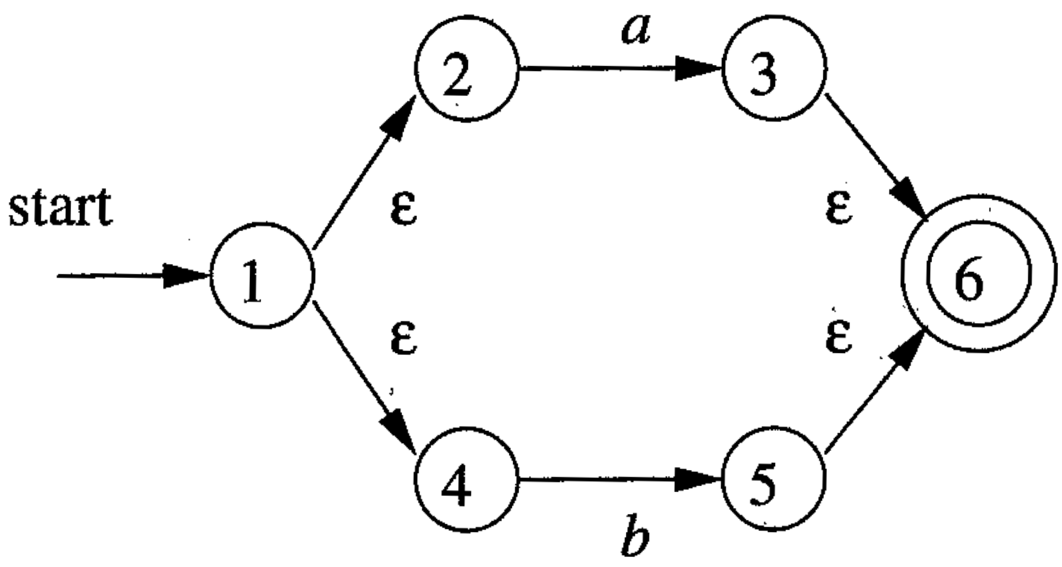
2. NFA for the first **b** (apply basis rule #1)



Example

r = **(a | b)*a**

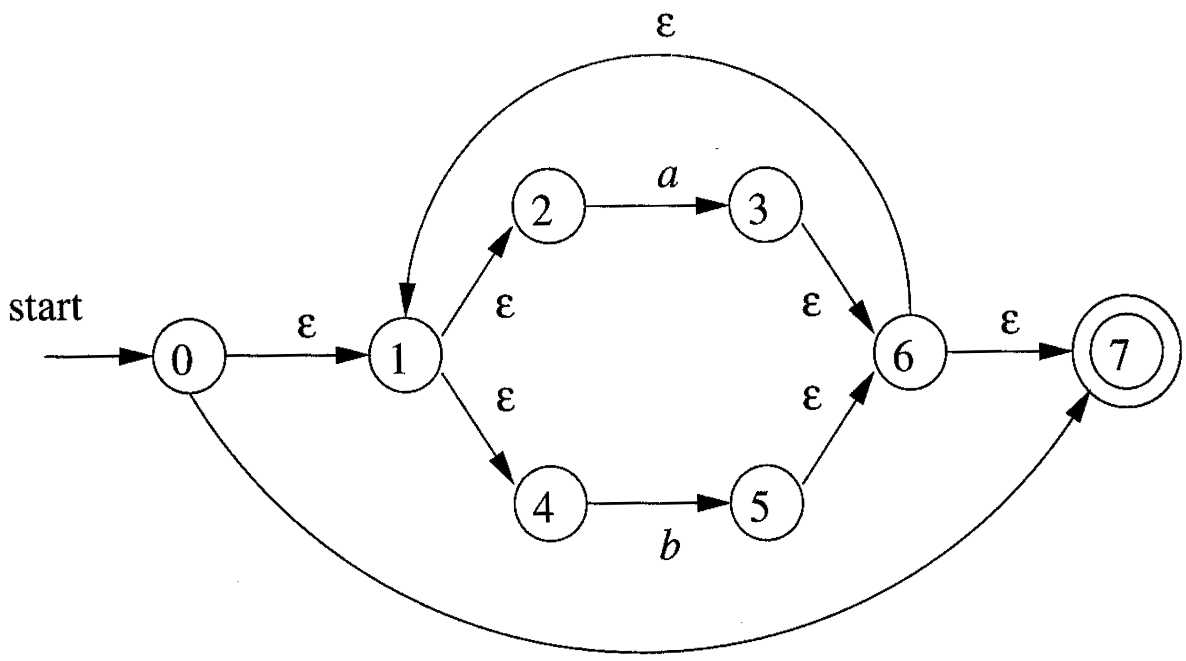
3. NFA for **(a | b)** (apply inductive rule #1)



Example

$r = (\mathbf{a} \mid \mathbf{b})^* \mathbf{a}$

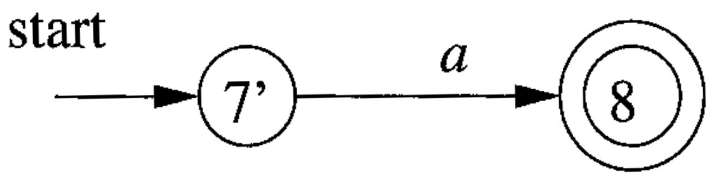
4. NFA for $(\mathbf{a} \mid \mathbf{b})^*$ (apply inductive rule #3)



Example

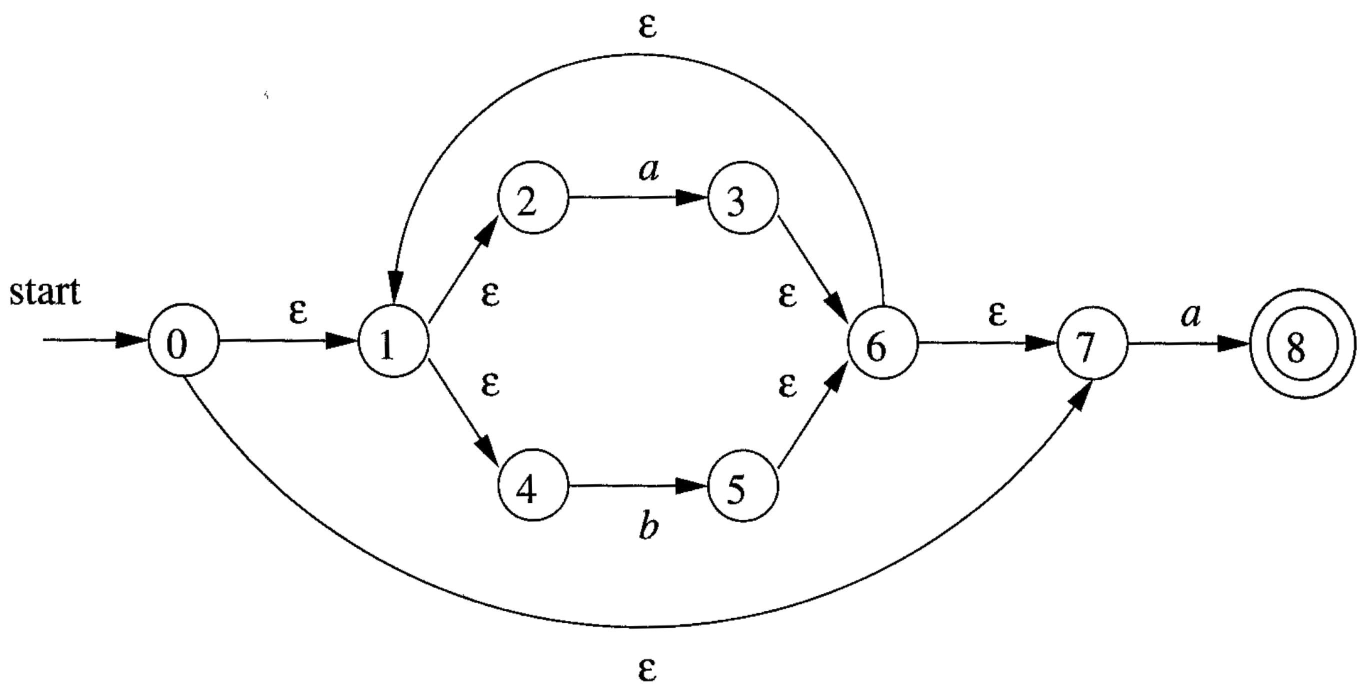
r = **(a | b)*a**

5. NFA for the second **a** (apply basic rule #1)



Example r = $\mathbf{(a|b)^*a}$

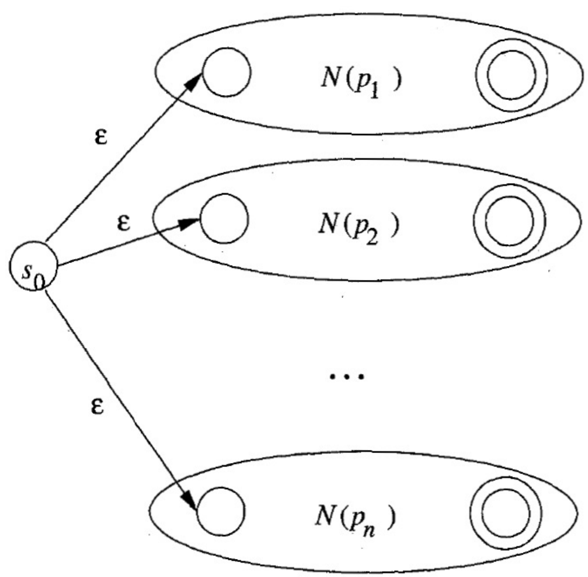
6. NFA for $\mathbf{(a|b)^*a}$ (apply inductive rule #2)



Combining NFA's

- **Why?** In the lexical analyzer, we need a single automaton to recognize lexemes matching any pattern (in the `lex` program)

- **How?** Introduce a new start state with ϵ-transitions to each of the start states of the NFA's for pattern p_i



- The language that can be accepted by the big NFA is the union of the languages that can be accepted by the small NFA's

- Different accepting states correspond to different patterns

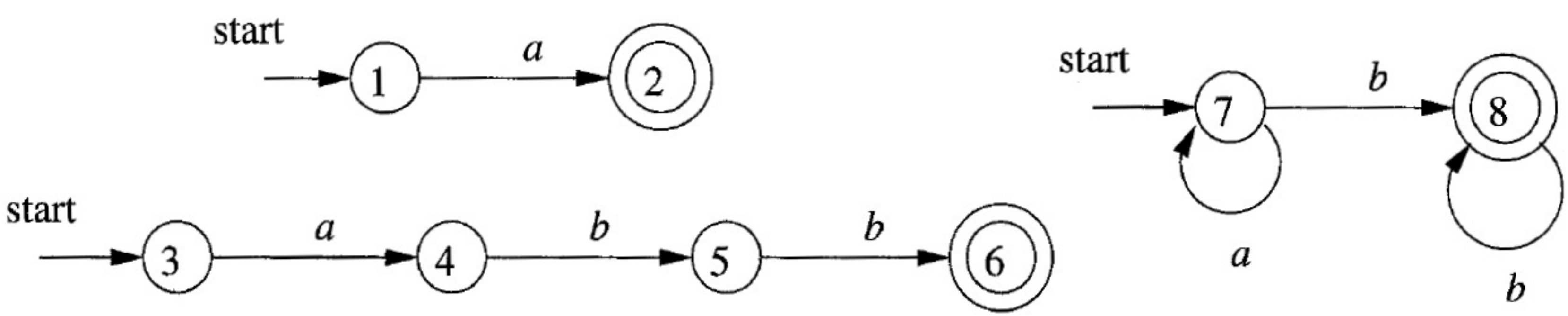
DFA's for Lexical Analyzers

- Convert the NFA for all the patterns into an equivalent DFA, using the subset construction algorithm

- An accepting state of the DFA corresponds to a subset of the NFA states, in which at least one is an accepting NFA state
  - If there are more than one accepting NFA state, this means that conflicts arise (the prefix of the input string matches multiple patterns)
  - Upon conflicts, find the first pattern whose accepting state is in the set and make that pattern the output of the DFA state

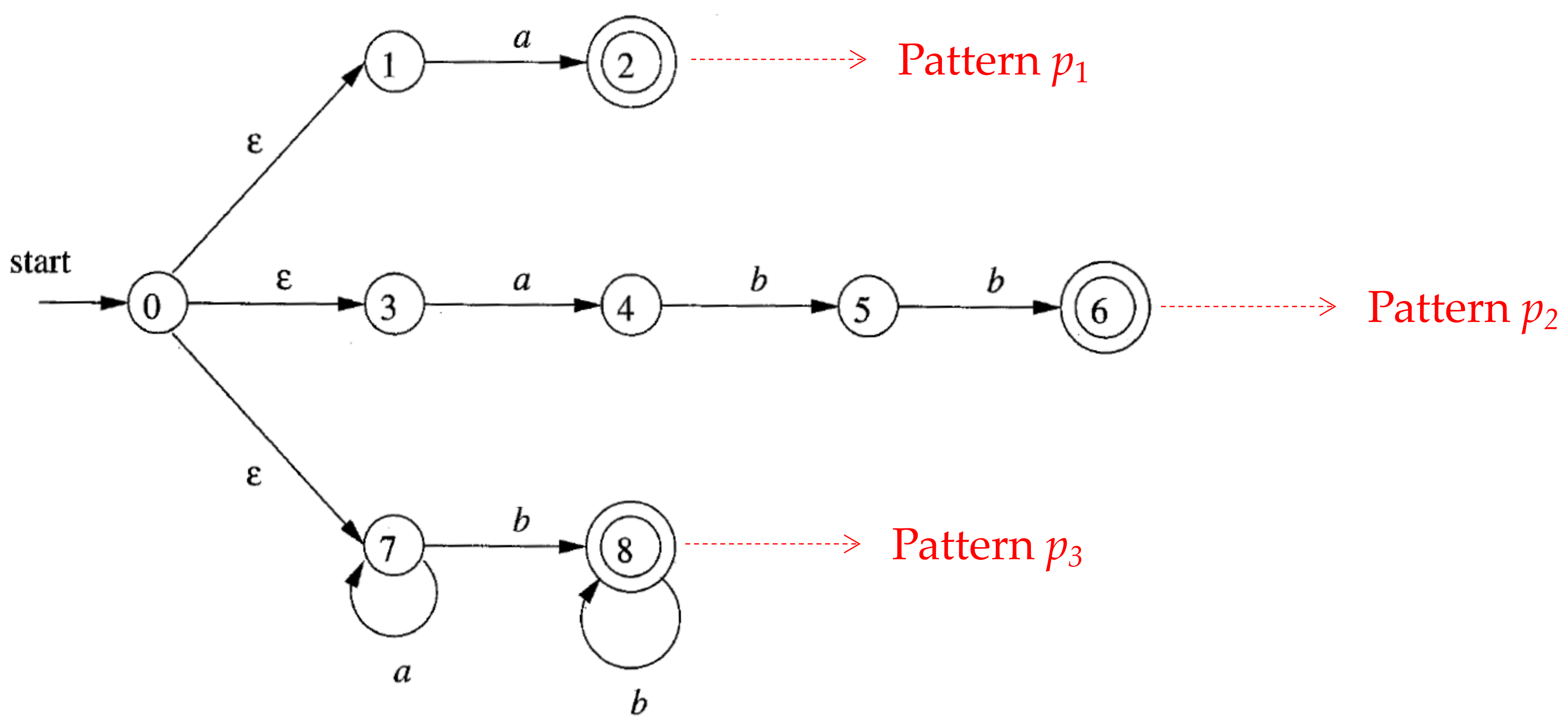
Example

- Suppose we have three patterns: p_1, p_2, and p_3
  - **a** {action A_1 for pattern p_1}
  - **abb** {action A_2 for pattern p_2}
  - $\mathbf{a}^*\mathbf{b}^+$ {action A_3 for pattern p_3}
- We first construct an NFA for each pattern

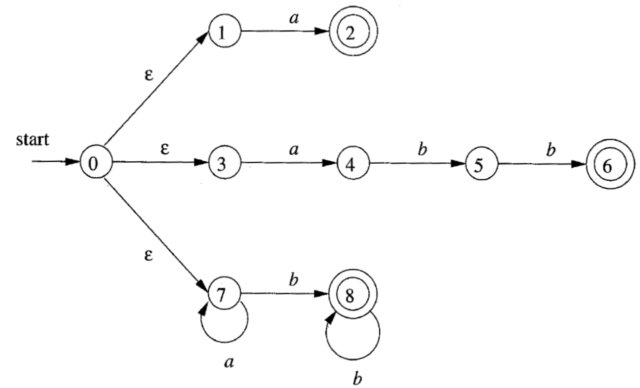


Example

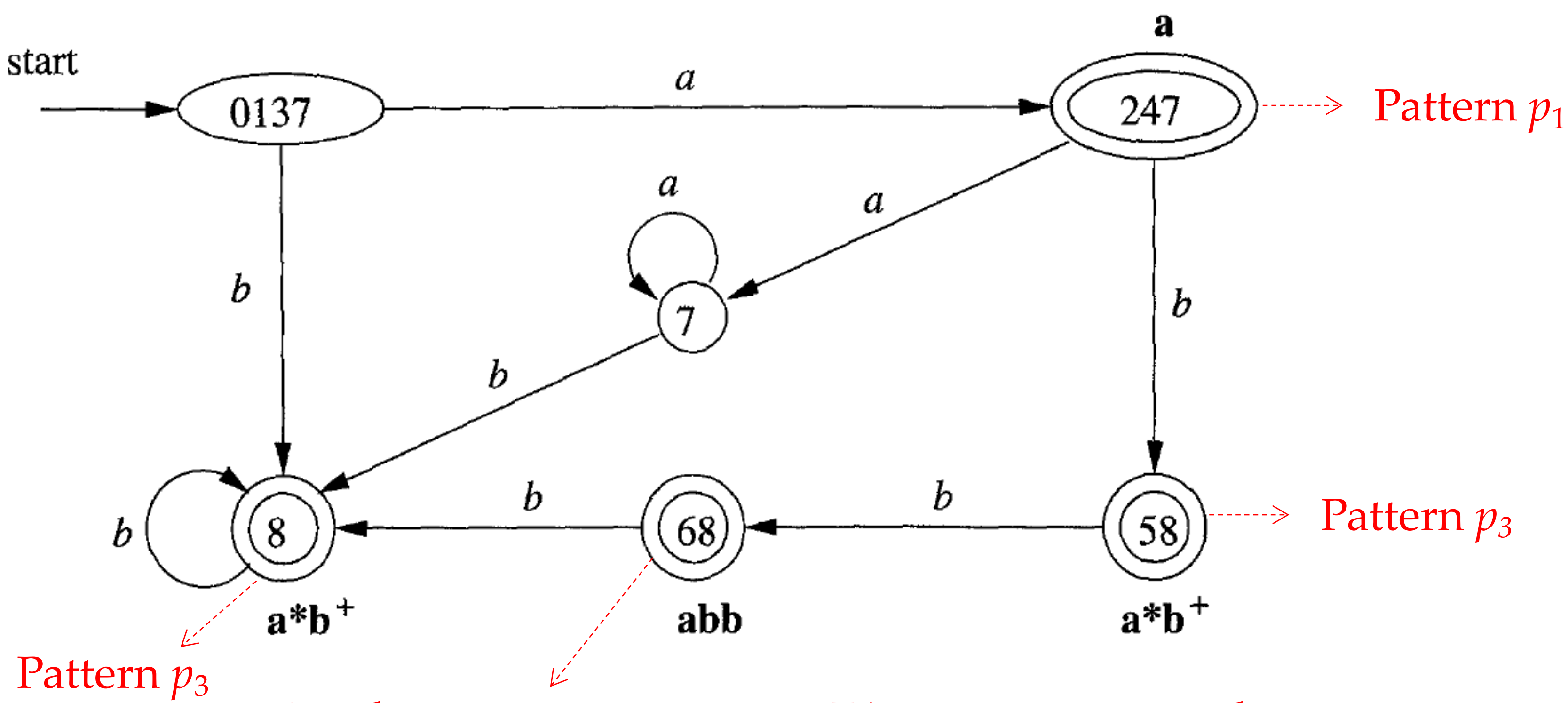
- Combining the three NFA's



Example



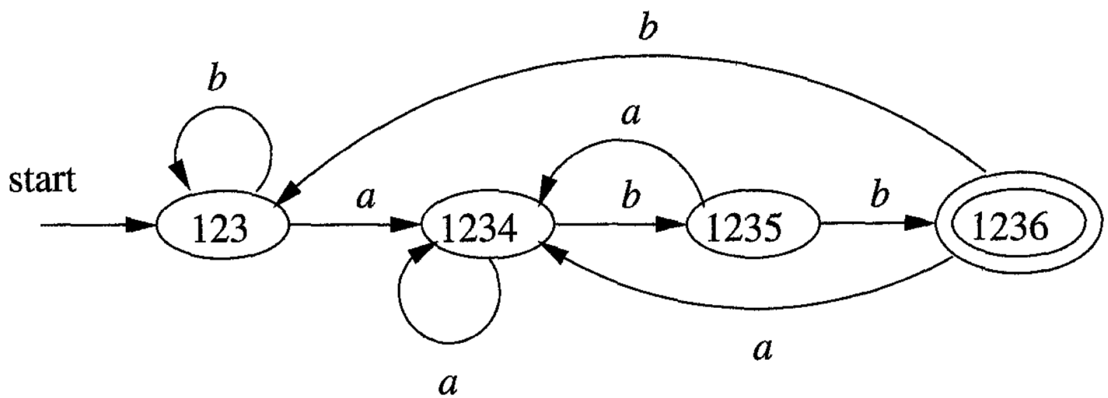
- Converting the big NFA to a DFA



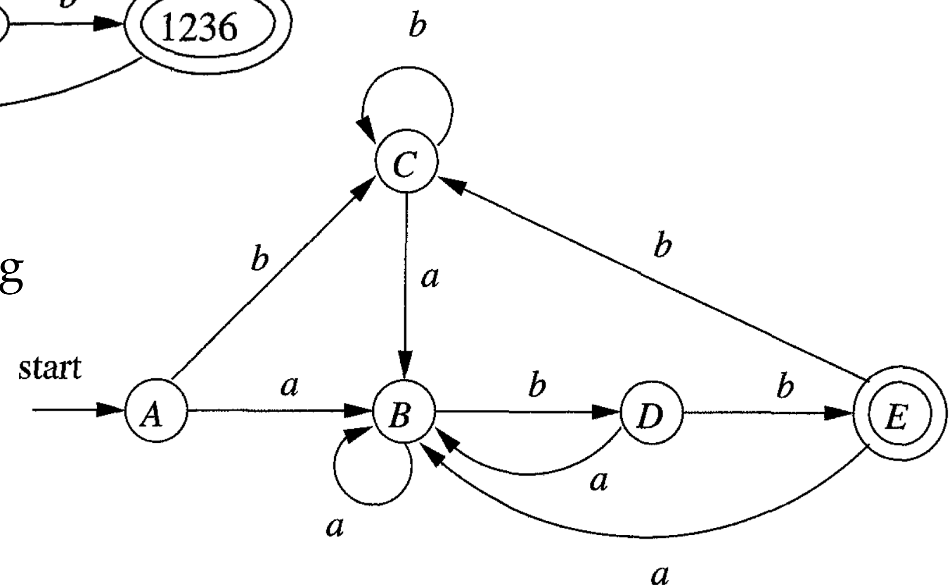
6 and 8 are two accepting NFA states corresponding to two patterns. We choose Pattern p2, which is specified before p3

Minimizing # States of a DFA*

- There can be many DFA's recognizing the same language



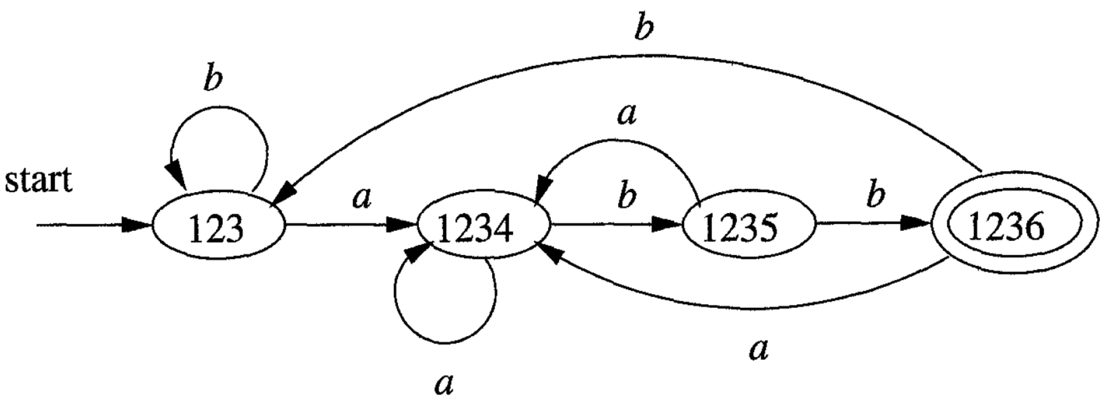
Two equivalent DFA's, both recognizing regular language $L((\mathbf{a}|\mathbf{b})^*\mathbf{abb})$



*Self-study materials

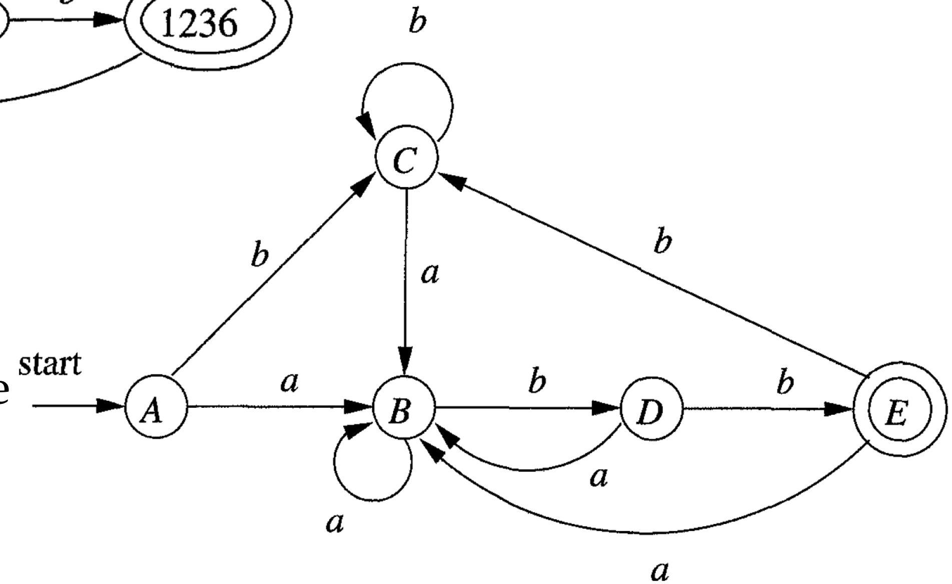
Minimizing # States of a DFA Cont.

- There can be many DFA's recognizing the same language



States *A* and *C* are equivalent:
Neither is accepting, and on any symbol they transfer to the same state

A and *C* behave like 123

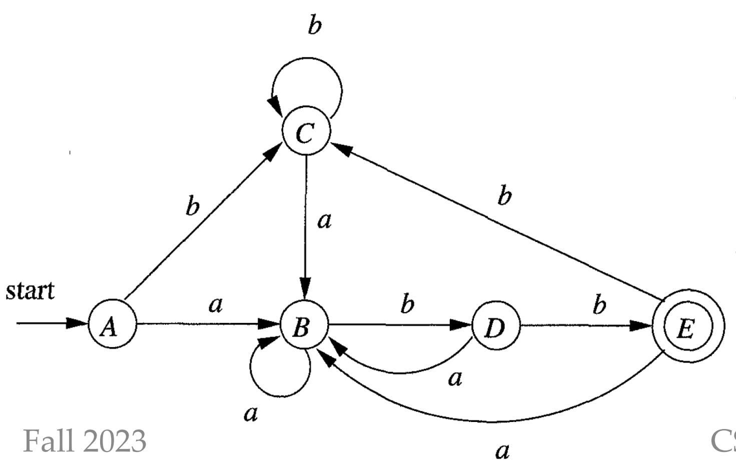


Minimizing # States of a DFA Cont.

- There is always a unique minimum-state DFA for any regular language (state name does not matter)
- The minimum-state DFA can be constructed from any DFA for the same language by grouping sets of equivalent states

Distinguishing States

- **Distinguishable states**
  - We say that string x distinguishes state s from state t if exactly one of the states reached from s and t by following the path with label x is an accepting state
  - States s and t are distinguishable if there exists some string that distinguishes them
- **For two indistinguishable states,** scanning any string will lead to the same state. Such states are equivalent and should be merged.



- The empty string ϵ distinguishes any accepting state from any nonaccepting state
- The string bb distinguishes state A from B, since bb takes A to a nonaccepting state C, but takes B to an accepting state E

DFA State-Minimization Algorithm

Works by partitioning the states of a DFA into groups of states that cannot be distinguished (an iterative process)

- The algorithm maintains a partition (划分), whose groups are sets of states that have not yet been distinguished
- Any two states from different groups are known to be distinguishable
- When the partition cannot be refined further by breaking any group into smaller groups, we have the minimum-state DFA

The Partitioning Part

1. Start with an initial partition $\prod$ with two groups, F and S - F, the accepting and nonaccepting states of D
2. Apply the procedure below to construct a new partition $\prod_{\text{new}}$

```
initially, let Π_new = Π;
for ( each group G of Π ) {
        partition G into subgroups such that two states s and t
                are in the same subgroup if and only if for all
                input symbols a, states s and t have transitions on a
                to states in the same group of Π;
        /* at worst, a state will be in a subgroup by itself */
        replace G in Π_new by the set of all subgroups formed;
}
```

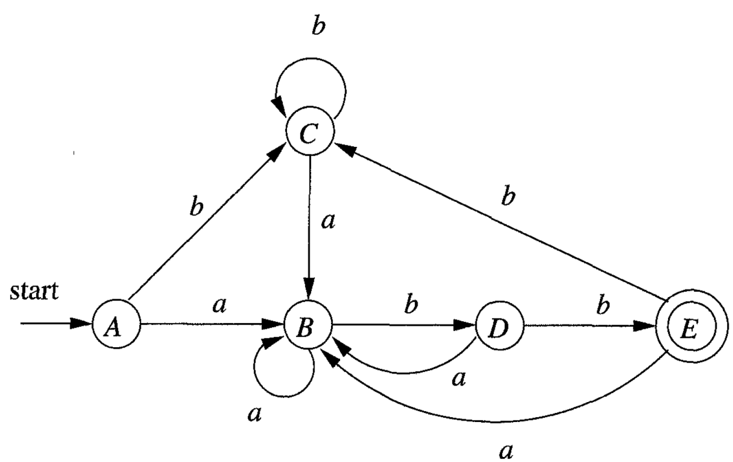
3. If $\prod_{\text{new}} == \prod$, let $\prod_{\text{final}} = \prod$ and the partitioning finishes; Otherwise, $\prod = \prod_{\text{new}}$ and repeat step 2

The Construction Part

- Choose one state in each group of $\prod_{\text{final}}$ as the *representative* for that group. The representatives will be the states of the minimum-state DFA D'
  - The start state of D' is the representative of the group containing the start state of D
  - The accepting states of D' are the representatives of those groups that contain an accepting state of D
  - Establish transitions:
    - Let s be the representative of group G in $\prod_{\text{final}}$; Let the transition of D from s on input a be to state t; Let r be the representative of t's group H
    - Then in D', there is a transition from s to r on input a

Example

- Initial partition: {*A*, *B*, *C*, *D*}, {*E*}
- Handling group {*A*, *B*, *C*, *D*}: *b* splits it to two subgroups {*A*, *B*, *C*} and {*D*}
- Handling group {*A*, *B*, *C*}: *b* splits it to two subgroups {*A*, *C*} and {*B*}
- Picking A, B, D, E as representatives to construct the minimum-state DFA



STATE	a	b
A	B	A
B	B	D
D	B	E
E	B	A

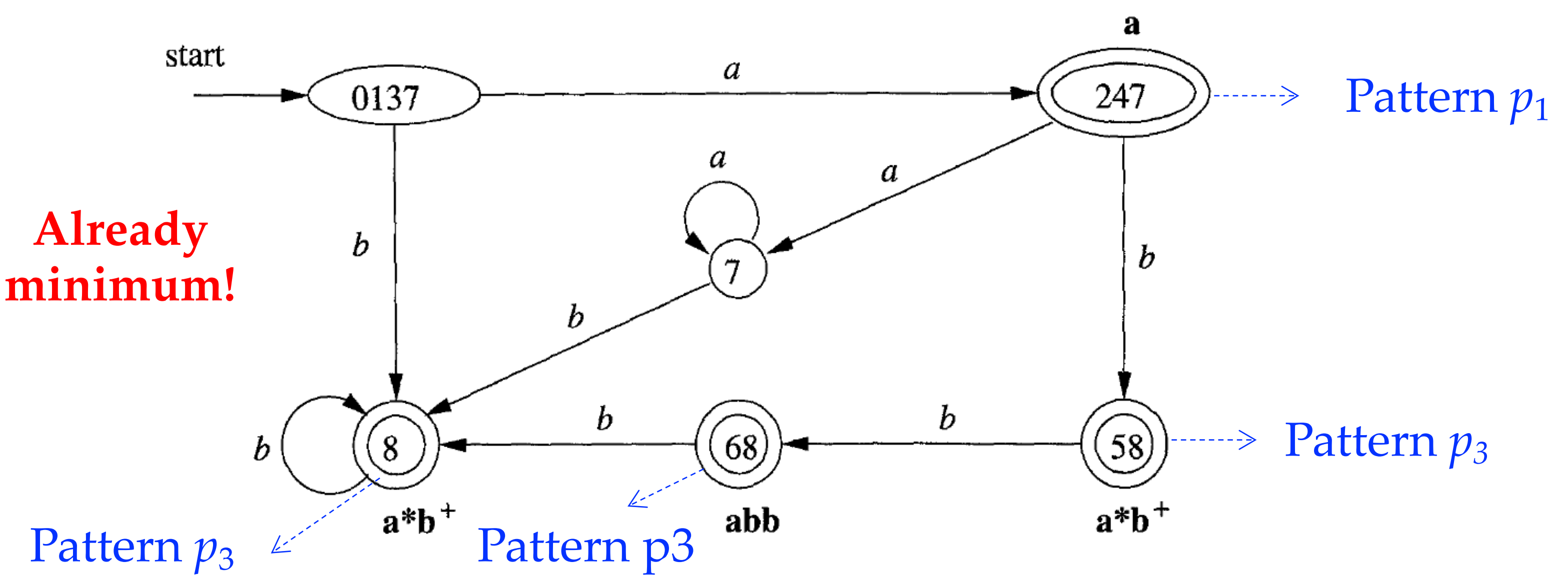
State Minimization in Lexical Analyzers

- The basic idea is the same as the state-minimization algorithm for DFA.

- Differences are:
  - Each accepting state in the lexical analyzer's DFA corresponds to a different pattern. These states are not equivalent.
  - So, the initial partition should be: one group of non-accepting states + groups of accepting states for different patterns

Example

- Initial partition: {0137, 7}, {247}, {68}, {8, 58}, {Ø}
  - We add a dead state Ø: we suppose has transitions to itself on inputs *a* and *b*. It is also the target of missing transitions on *a* from states 8, 58, and 68.



Reading Tasks

- Chapter 3 of the dragon book
  - 3.1 The role of the lexical analyzer
  - 3.3 Specification of tokens
  - 3.4 Recognition of tokens
  - 3.5 The lexical-analyzer generator Lex
  - 3.6 Finite automata
  - 3.7 From regular expressions to automata
  - 3.8 Design of a lexical analyzer generator
    - 3.8.1 – 3.8.3, the remaining can be skipped