1. $\max\ \mathcal{L}(\alpha) = \sum \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i^T x_j$

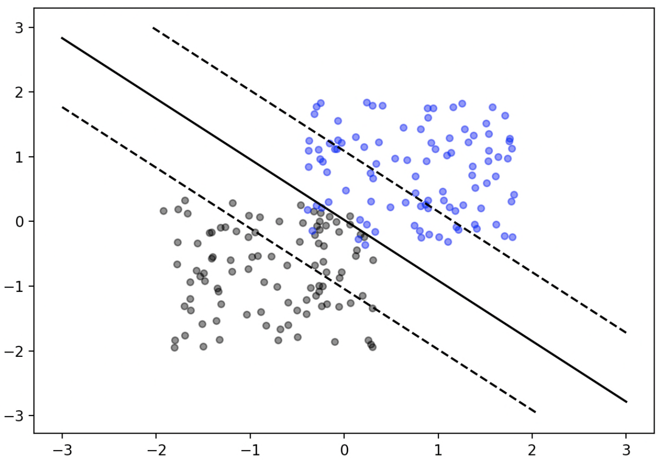
s.t. $\sum \alpha_i y_i = 0$

$0 \le \alpha_i \le C, \quad \forall \alpha_i$

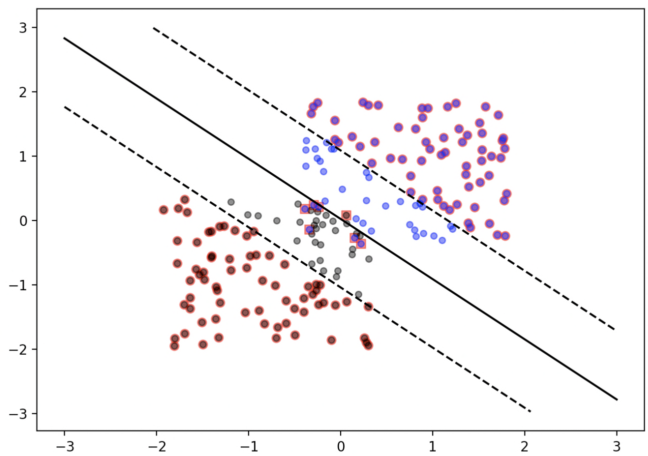
$$\Rightarrow \begin{cases} \alpha_1 = 0.25 \\ \alpha_2 = 0.25 \\ \alpha_3 = 0.5 \\ \alpha_4 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} w = \sum_i \alpha_i y_i x_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ b = y_k - \sum_i \alpha_i y_i x_i^T x_k \ (\alpha_k > 0) = 0. \end{cases}$$

2. original plot:



marked plot:



$\varepsilon_i > 0$: outliers

cooperate with Lin Yuxuan 118010200

3. Original Problem:

$$\min \quad \frac{\|w\|^2}{2} + C\left(\sum_{i=1}^{N} \varepsilon_i\right)^2$$

$$\text{s.t.} \quad y_i(w^T x_i + b) \geq 1 - \varepsilon_i, \quad i = 1, \dots, n$$

$$\varepsilon_i \geq 0, \quad i = 1, \dots, n.$$

Lagrange Multiplier:

$$\mathcal{L} = \frac{\|w\|^2}{2} + C\left(\sum_{i=1}^{n} \varepsilon_i\right)^2 - \sum_{i=1}^{n} \alpha_i (y_i (w^T x_i + b) - 1 + \varepsilon_i) - \sum_{i=1}^{n} r_i \varepsilon_i$$

$$\frac{\partial L}{\partial w} \overset{set}{=} 0 \Rightarrow w = \sum_{i=1}^{n} \alpha_i y_i x_i$$

$$\frac{\partial L}{\partial b} \overset{set}{=} 0 \Rightarrow \sum_{i=1}^{n} \alpha_i y_i = 0.$$

$$\frac{\partial L}{\partial \varepsilon_i} \overset{set}{=} 0 \Rightarrow 2 \times C \sum_{j=1}^{n} \varepsilon_j - \alpha_i - r_i = 0, \quad i = 1, \dots, n.$$

Dual Problem:

$$\max \quad \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i}^{n} \sum_{j}^{n} \alpha_i \alpha_j y_i y_j x_i^T x_j - C \times \sum_{j=1}^{n} \varepsilon_j$$

$$\text{s.t.} \quad \alpha_i + r_i = 2 \times C \times \sum_{j=1}^{n} \varepsilon_j$$

$$\sum_{i=1}^{n} \alpha_i y_i = 0.$$

$$\alpha_i, r_i, C, \sum_{j=1}^{n} \varepsilon_j \geq 0.$$