# Master Thesis Notes

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## Contents



Mon, Feb 19

**how the BRST structure could be helpful?** Let's test that whether the BRST structure is restrictive enough such that no new interaction could be generated from the RG calculation.

Let's get a closer look at the action

$$-\frac{1}{2}\mathrm{Tr}H^2 + ig\mathrm{Tr}\left([\overline{Z}, Z]H\right) + ig\mathrm{Tr}\left([\overline{\Psi}, Z]\eta + [\overline{Z}, \Psi]\eta\right).$$

Let's assume that $g$ is small and try to do the perturbative calculation. However, it's not possible because there is no quadratic term. To solve this, let's deform the action by adding the following term

$$i\kappa\delta\left(\mathrm{Tr}(\overline{Z}\Psi - Z\overline{\Psi})\right) = 2i\kappa\mathrm{Tr}\left(\overline{\Psi}\Psi\right) - 2\kappa\epsilon\mathrm{Tr}\left(\overline{Z}Z\right) - 2i\kappa\mathrm{Tr}\left([\overline{Z}, Z]H\right).$$

Because its $\delta$-exact, the deformation is $\delta$-invariant. $\kappa$ is also a small parameter. To get the usual quadratic term for $Z$, we choose $\epsilon = 1/4\kappa$. Also note that the action can be simplified if $\kappa = g/2$ because the term $\mathrm{Tr}\left([\overline{Z}, Z]H\right)$ being cancelled. Let's keep this choice, then the action reads

$$-\frac{1}{2}\mathrm{Tr}H^2 - \frac{1}{2}\mathrm{Tr}(\overline{Z}Z) + ig\mathrm{Tr}(\overline{\Psi}\Psi) + ig\mathrm{Tr}\left([\overline{\Psi}, Z]\eta + [\overline{Z}, \Psi]\eta\right).$$

Or getting back to the matrices $A, B, \psi, \chi$

$$-\frac{1}{2}\mathrm{Tr}H^2 - \frac{1}{2}\mathrm{Tr}A^2 - \frac{1}{2}\mathrm{Tr}B^2 - 2g\mathrm{Tr}(\psi\chi) - 2g\mathrm{Tr}\left([\psi, B]\eta - [\chi, A]\eta\right).$$

Let's work with $2\times 2$ matrix and use the following parameterization

$$A = \begin{pmatrix} A & \alpha \\ \alpha^* & a \end{pmatrix}, \quad B = \begin{pmatrix} B & \beta \\ \beta^* & b \end{pmatrix}$$
$$\psi = \begin{pmatrix} \psi & y \\ y^* & \varphi \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi & x \\ x^* & \iota \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta & h \\ h^* & \theta \end{pmatrix}$$

For the $(1,1)$ components, we use the same letter to indicate that they will not be integrated. Then the quadratic terms have the form

$$\begin{aligned} -\frac{1}{2}A^2 - \frac{1}{2}B^2 - \frac{1}{2}a^2 - \frac{1}{2}b^2 - \alpha^*\alpha - \beta^*\beta \\ -2g\psi\chi - 2gyx^* - 2gxy^* - 2g\varphi\iota \end{aligned}$$

The interaction term $-2g\mathrm{Tr}([\psi, B]\eta - [\chi, A]\eta)$ contains

$$\begin{aligned} -2g\psi\beta h^* + 2g\beta^*\psi h - 2gy^*Bh + 2gByh^* - 2gy\beta^*\eta + 2g\beta y^*\eta \\ +2g\chi\alpha h^* - 2g\alpha^*\chi h + 2gx^*Ah - 2gAxh^* + 2gx\alpha^*\eta - 2g\alpha x^*\eta \\ -2gy^*bh + 2gbyh^* - 2g\varphi\beta h^* + 2g\beta^*\varphi h - 2gy^*\beta\theta + 2g\beta^*y\theta \\ +2gx^*ah - 2gaxh^* + 2g\iota\alpha h^* - 2g\alpha^*\iota h + 2gx^*\alpha\theta - 2g\alpha^*x\theta \end{aligned}$$

There is an additional term that only involving the $(1,1)$-component. It has the same form with the initial action $-2g\mathrm{Tr}([\psi, B]\eta - [\chi, A]\eta)$.

For the Grassmann integral, to get non-vanishing result, one needs to saturate the measure $\mathrm{d}y\mathrm{d}y^*\mathrm{d}\varphi\mathrm{d}x\mathrm{d}x^*\mathrm{d}\iota\mathrm{d}h\mathrm{d}h^*\mathrm{d}\theta$. Because there is no quadratic term of $\eta$, the non-vanishing terms must include $hh^*\theta$. For other variables, if they appear, they must form the pairs

$$a^2, \quad b^2, \quad \alpha^*\alpha, \quad \beta^*\beta, \quad yx^*, \quad xy^*, \quad \varphi\iota.$$

For fermionic variables $yx^*, xy^*, \varphi\iota$, they must not appear more than one time. These are the basic constraints to get non-vanishing integral result.

First note that $h, h^*, \theta$ appear in each term once. Therefore, to get non zero result, one needs at least the cubic term

$$-\frac{4}{3}g^3\left[\mathrm{Tr}([\psi, B]\eta - [\chi, A]\eta)\right]^3.$$

Let's focus on the terms that containing these combinations: $\psi B, \psi A, \chi B, \chi A$. One would expect that all of them being vanish because they have an odd number of fermionic variables. For $\psi B, \chi A$, to fully contract the variables, one needs terms containing $x\beta^*, y\alpha^*$. There is no such term. For $\psi A$, one selects the following terms

$$(-\psi\beta h^*)(x^*Ah)(\beta^*y\theta) + (\beta^*\psi h)(-Axh^*)(-y^*\beta\theta).$$

By contracting $hh^*, xy^*, yx^*$ one finds that the result being zero. Similar for $\chi B$. So it's in accordance with the expectation. (TODO: analyze other terms like $A^2, AB, B^2, \psi\chi$)

It's easy to understand that $[\mathrm{Tr}([\psi, B]\eta - [\chi, A]\eta)]^5$ always vanishing because one of the $h, h^*, \theta, \eta$ will appear twice. So one only need to consider the quartic term

$$\frac{2}{3} g^4 \left[\mathrm{Tr}([\psi, B]\eta - [\chi, A]\eta)\right]^4 .$$

Out of all possible combinations, it may be interestring to consider those containing $\psi\eta$ and $\chi\eta$ first. To find the terms that containing $\psi\eta$, one first select $(-\psi\beta h^*)$ or $(\beta^*\psi h)$. Let's take the first choice as an example. Then one needs to add a term that containing $h$ but not containing $A, B, \chi$ because we only focus on $\psi\eta$. The possibilities are the following

$$-y^* bh, \quad x^* ah, \quad \beta^*\varphi h, \quad -\alpha^*\iota h.$$

Each of them containing one of $a, b, \varphi, \iota$. To get non-vanishing result, one needs another term that only containing one of them. But there is no such term: they either contain $\varphi b, \iota a$ or containing none of them. Therefore, $\psi\eta$ will not be generated through the perturbative calculation.

It's also easy to see that the terms like $A\psi\chi$ will not be generated because $h, h^*$ would be over-saturate. Then it's interesting to consider $\psi A\eta$ which does not appear in the original action. For example, one starts by selecting the two terms

$$(-\psi\beta h^*)(x^* Ah).$$

Then one must select the term $(-y\beta^*\eta)$ that containing $\eta$. Because otherwise $\beta, x^*$ cannot be contracted. However, then one must include also an extra $\theta$ term to saturate $\mathrm{d}\theta$. Any of them will contain one of $\alpha, \beta, \varphi, \iota, a, b$ that having no partner to contract. Therefore, no $\psi A\eta$ could be generated.

let's consider another possible new term $\psi\chi\eta$. It contains three fermionic variables, therefore being expected to vanish. Let's verify it explicitly. The following terms would contribute to $\psi\chi\eta$

$$(-\psi\beta h^*)(-\alpha^*\chi h)\left[(-y\beta^*\eta)(x^*\alpha\theta) + (-\alpha x^*\eta)(\beta^* y\theta)\right].$$

$$(\beta^*\psi h)(\chi\alpha h^*)\left[(\beta y^*\eta)(\alpha^* x\theta) + (x\alpha^*\eta)(-y^*\beta\theta)\right].$$

It's straightforward to check they vanish because the order of $y, x^*$ and $y^*, x$ are different in the two terms.

It turns out that nothing could be generated from integrating out those variables. Then the question is that whether the same thing hold for general $N$?

Tue, Feb 20

**Extend the previous analysis to other cases.** It's clear that the fermionic matrix $\eta$ plays a special role in this model. It has no "partner", so the only way to get non-vanishing result is to saturate the measure. In the case of $2 \times 2$ matrix, this forces the first non-vanishing perturbative expansion starting from the cubic term (interactions)$^3$. Similarly, for $N \times N$ matrix, the first non-trivial contribution would start from $(\cdots)^{2N-1}$.

There is one natrual question to think about. Given an $N \times N$ fermionic matrix $\psi$, one may wonder what's the largest power $\mathrm{Tr}\psi^k$ for non-zero result. It's obvious that if $k > N^2$, then it must vanish because there are only $N^2$ elements in $\psi$. But what if $k = N^2$? If $N$ is even, the $\mathrm{Tr}\psi^{N^2} = 0$ according to the cyclic identity. (TODO: I don't know how to prove it's non-vanishing for odd $N$. And how to prove that it's non-vanishing for $\mathrm{Tr}\psi^{N^2-1}$ with even $N$.)

(TODO: There is an error in the analyze below: the matrix product arising from the contraction of fermionic variables should not be understood in the usual sense. The same contraction, for example between $x_1, y_1^*$, can not happen twice.)

For a general $N$, one needs to saturate $\mathrm{d}h_1\mathrm{d}h_1^* \cdots \mathrm{d}h_{N-1}\mathrm{d}h_{N-1}^*\mathrm{d}\theta$. The first case that one may consider is that all the $h$s are contributed by $x^*Ah$ and $h^*Ax$. The only way to contract these two terms is by adding $\beta^*\eta y$ and $y^*\eta\beta$ in the following way

$$(h^*Ax)(y^*\eta\beta)(\beta^*\eta y)(x^*Ah).$$

Then one expects that the term that could be generated containing the elements of the matrix $A\eta\eta A$. For example, a typical term could look like

$$(A\eta\eta A)_{1\sigma(1)} \cdots (A\eta\eta A)_{(N-1)\sigma(N-1)}.$$

where $\sigma$ is a certain permutation. However, one still need to saturate the fermionic variable $\theta$. That is, one must replace one of the $\eta$ above by $\theta$. This always accompanies with a same term with opposite sign $(A\eta\theta A)$ and $(A\theta\eta A)$. Or one can expect that by realizing there must be an odd number of fermionic variables. So there is no term that containing purely $A$ or $B$ could be generated.

At this stage one may wonder that how is it possible to get only one $\theta$ in the perturbative expansion while getting non-vanishing result? This leads us to consider the coupling between the bosonic and fermionic matrices. Let's start with $A\psi$, the two structures $x^*Ah$ and $h^*\psi\beta$ can be contracted by using $\beta^*\eta y$ or $\beta^*\theta y$.

$$(h^*\psi\beta)(\beta^*\eta y)(x^*Ah).$$

Then one expect that the elements of matrix $\psi\eta A$ will appear. However, don't forget that whenever such a term exist, there is a corresponding possibility: replacing it with the following

$$(h^*Ax)(y^*\eta\beta)(\beta^*\psi h).$$

Note that they are conjugate to each other

$$[(h^*\psi\beta)(\beta^*\eta y)(x^*Ah)]^\dagger = -(h^*Ax)(y^*\eta\beta)(\beta^*\psi h).$$

The minus sign arises from the interchange between fermionic numbers in each bracket $(\cdots)$. (TODO: So if the action is self-conjugate, one expects that the terms involving $A\eta\psi$ will vanish. But it's not clear that the action is self-conjugate.)

The followings are all possible results of contraction

$$\begin{aligned} A\eta\eta A, \quad B\eta\eta B, \quad AB \\ A\eta\psi, \quad B\eta\chi \\ \psi\psi, \quad \chi\chi, \quad \psi\eta\eta\chi \end{aligned}$$

These forms can be read from the structure of the interaction. A general term in the perturbative expansion will be a multiplication of matrix elements $M_{ij}$ with $M$ choosing from the above matrices and $i$ and $j$ run over all possible values from 1 to $N-1$. This prescription is to ensure the saturation of $h$. For the saturation of $\theta$, one can replace one $\eta$ with $\theta$. But this replacement will lead to cancellation as above. Therefore, it is understood that the fermionic matrix $\eta$ plays an essential role in all the cancelation.

Wed, Feb 21

**rethink the model** In constructing the previous model, the way we introduce the matrix $\eta$ is suspicious. Specifically $\delta^2\eta = 0$, but it should generate the unitary transformation $\delta^2\eta = [H, \eta]$. Remember that the idea behind the BRST construction is "taking the square root of gauge transformation". However, it seems impossible to satisfy $\delta^2 H = 0$ simultaneously. It seems like the only way to solve this problem is to introduce again an auxillary matrix such that $\delta H = [X, \eta]$. That is the gauge transformation is parameterized by the auxillary matrix $X$.

What's the motivation to introduce $\eta$ in our model? I want to introduce a term $\mathrm{Tr}([\overline{Z}, Z]H)$ in the action, from doing the variation $\delta(\cdots)$. To obtaine $H$ from $\delta$, I introduce $\eta$ such that $\delta\eta = H$. This is a bad move as having been said above: this breaks the unitary invariance when the model involving the matrix $\eta$.

**one possible solution to the previous problem?** If we introduce another auxillary matrix $X$ to parameterize the unitary transformation, and modify $\delta H = [X, \eta]$ with $\delta X = 0$. (Remark. $H, X, \eta$ should all be anti-hermitian to keep each term "real" under the hermitian conjugation $\dagger$) The model build may still start from the commutator term

$$-\frac{1}{2}\mathrm{Tr}H^2 + ig\mathrm{Tr}\left([\overline{Z}, Z]H\right).$$

Then one wants to find $\delta$-exact terms to generate these two terms. The candidates are

$$ig\delta\mathrm{Tr}\left([\overline{Z}, Z]\eta\right) - \frac{1}{2}\delta\mathrm{Tr}\left(\eta H\right)$$

Other terms are generated along

$$ig\mathrm{Tr}\left([\overline{\Psi}, Z]\eta + [\overline{Z}, \Psi]\eta\right) + \frac{1}{2}\mathrm{Tr}\left(\eta[X, \eta]\right).$$

Note that $(1/2)\mathrm{Tr}(\eta[X, \eta]) = \mathrm{Tr}(\eta X\eta)$ gives a quadratic term of $\eta$. To deal with the interaction terms $ig\mathrm{Tr}([\overline{Z}, Z]H)$ and $ig\mathrm{Tr}([\overline{\Psi}, Z]\eta + [\overline{Z}, \Psi]\eta)$, it's important to introduce quadratic term for $Z, \overline{Z}$ and $\Psi, \overline{\Psi}$. This is done by adding to following $\delta$-exact term

$$i\kappa\delta\mathrm{Tr}\left(\overline{Z}\Psi - Z\overline{\Psi}\right) = 2i\kappa\mathrm{Tr}(\overline{\Psi}\Psi) - 2\kappa\epsilon\mathrm{Tr}(\overline{Z}Z) - 2i\kappa\mathrm{Tr}\left([\overline{Z}, Z]X\right).$$

Here $X$ is not a dynamical matrix, and it should be anti-hermitian. In summary, let's collect all terms in the action

$$\begin{aligned} & -\frac{1}{2}\mathrm{Tr}H^2 + \mathrm{Tr}\eta X\eta \\ +\frac{i}{2}\kappa\mathrm{Tr}\overline{\Psi}\Psi - \frac{1}{2}\kappa\epsilon\mathrm{Tr}\overline{Z}Z & - \frac{i}{4}\kappa\mathrm{Tr}\left([\overline{Z}, Z]X\right) \\ +ig\mathrm{Tr}\left([\overline{Z}, Z]H\right) & + ig\mathrm{Tr}\left([\overline{\Psi}, Z]\eta + [\overline{Z}, \Psi]\eta\right) \end{aligned}$$

It can be verified all the terms are real under † (It acts on the matrix, will not take $i \to -i$).

Compare to previous wrong model, it has several differences. First is the appearance of the $X$ matrix. It it is taken to be zero, the model will become very similar to what we have studied previously, but with an additional term $ig\mathrm{Tr}\left([\overline{Z}, Z]H\right)$. However, the previous calculation is wrong because $\eta$ should be anti-hermitian, rather than hermitian. This means that there is no $\theta$ variable to be integrated out; also $h^*$ and $h$ terms should have a relative minus sign. Let's list all terms that are relavent to the integration

$$\begin{aligned} -v^*A\beta + \beta^*Av + v^*B\alpha - \alpha^*Bv + \alpha^*H\beta - \beta^*H\alpha \\ x^*Ah - h^*Ax - y^*Bh + h^*By \\ \beta^*\psi h - h^*\psi\beta - \alpha^*\chi h + h^*\chi\alpha \\ -\alpha^*\eta x - x^*\eta\alpha + \beta^*\eta y + y^*\eta\beta \end{aligned}$$

It's interesting to take $X = iz\mathbb{1}$. $z$ is a real constant to be determined later. One consequence is

$$\mathrm{Tr}\left(\eta X\eta\right) \sim -izh_i^*h_i.$$

Remember that $\eta_{(N+1),i} = -h_i^*,\ \eta_{i,(N+1)} = h_i$. Another consequence is

$$\frac{\kappa}{2}\mathrm{Tr}\left([A, B]X\right) \sim \frac{\kappa}{2}iz(\beta_i^*\alpha_i - \alpha_i^*\beta_i).$$

So the quadratic term for $\alpha, \beta$ is

$$-\frac{\kappa\epsilon}{2}(\alpha_i^*\alpha_i + \beta_i^*\beta_i) + \frac{i\kappa z}{2}(\beta_i^*\alpha_i - \alpha_i^*\beta_i).$$

This motivates us to redefine

$$\tilde{\alpha} = \frac{1}{\sqrt{2}}(\alpha + i\beta), \quad \tilde{\beta} = \frac{1}{\sqrt{2}}(\alpha - i\beta).$$

Then the quadratic term becomes

$$-\frac{\kappa}{2}\left[(\epsilon + z)\tilde{\alpha}_i^*\tilde{\alpha}_i + (\epsilon - z)\tilde{\beta}_i^*\tilde{\beta}_i\right].$$

The interaction of $A, B$ with the "bosonic legs" in terms of $\tilde{\alpha}, \tilde{\beta}$ reads

$$\frac{1}{\sqrt{2}}\left(iv^*A\tilde{\alpha} + i\tilde{\alpha}^*Av - iv^*A\tilde{\beta} - i\tilde{\beta}^*Av + v^*B\tilde{\alpha} - \tilde{\alpha}^*Bv + v^*B\tilde{\beta} - \tilde{\beta}^*Bv\right).$$

It's possible to obtain a new operator $\mathrm{Tr}(AB)$ through the contraction. To the lowest order there are four terms that contribute to $\mathrm{Tr}(AB)$:

$$\frac{i}{2}\left(\tilde{\alpha}^* A v v^* B \tilde{\alpha} - v^* A \tilde{\alpha}\tilde{\alpha}^* B v + v^* A \tilde{\beta}\tilde{\beta}^* B v - \tilde{\beta}^* A v v^* B \tilde{\beta}\right).$$

We see that there is actually no $\mathrm{Tr}(AB)$ generated at this order. For the interaction of $A, B$ with the "fermionic legs"

$$x^* A h - h^* A x - y^* B h + h^* B y.$$

There are two terms that contribute to the $\mathrm{Tr}(AB)$

$$x^* A h h^* B y + h^* A x y^* B h.$$

The order of contraction is $yx^*$ and $y^* x$. So there is also no contribution at this order.

(TODO: more discussions on the propagators $\alpha^*\alpha$ and $\beta^*\beta$, and possible cancellation or suppression of new interaction generations.)

Fri, Feb 23

**A comparison between the "dimensional model" and the "dimensionless model**
Let's start with a "dimensionful" model, whose partition function is given by

$$Z_N(g) = \int \exp\left[-N\left(\frac{1}{2}\,\mathrm{Tr}\,M^2 + \frac{g}{4}\,\mathrm{Tr}\,M^4\right)\right] \mathrm{d}M. \tag{1}$$

The scaling of the matrix $M$, and the coupling $g$ under $N \to \lambda N$ is

$$M \to \lambda^{-1/2} M$$
$$g \to \lambda g$$

One could also write the same theory in a "dimensionless" way, by introducing $\tilde{M} = \sqrt{N} M$ and $\tilde{g} = N^{-1} g$. Then in terms of $\tilde{M}$ and $\tilde{g}$ the partition function reads

$$Z_N(g) = N^{N^2/2} \int \exp\left[-\left(\frac{1}{2}\,\mathrm{Tr}\,\tilde{M}^2 + \frac{\tilde{g}}{4}\,\mathrm{Tr}\,\tilde{M}^4\right)\right] \mathrm{d}\tilde{M}. \tag{2}$$

Starting from the "dimensionless model" (2), it's possible to recover the "dimensionful model" (1) by specifying the quadratic term $\frac{N}{2}\,\mathrm{Tr}\,M^2$ first.

**the notions of scaling** First is the scaling from the RG flow. Let's start by thinking about the RG calculation to the lowest order. This will just reproduce the classical scaling: $g \to \lambda g$ and $\tilde{g}$ is invariant. However, they are essentially different. In the language of QFT, one can think of $N$ as a cut-off. Then the RG method allows us to relate theories with different cut-off such that they will produce the same results (the correlation functions). Specifically, to the lowest order, the coupling $g \to \lambda g$ with the change of cut-off $N \to \lambda N$. To the lowest order, these are exactly the same as the classical scaling. However, the higher order corrections are essential for the RG flow.

Conceptually, the RG flow keeps the theory invariant, while the classical scaling not: one knows that the matrix model has a non-trivial $N$ dependence although they share the same form of action. The classical scaling is just a natural way to define how the matrix and coupling depending on the underlying scale such that the form of the action keeping the same. The classical scaling works like "zooming in" or "zooming out". However, the RG flow works like "coarse graining". There is no reason to believe that the "coarse graining" will give a similar result comparing to the "zooming" in general. The Gaussian model is a special example that they giving exactly the same result.

Now what about the notion of "scaling invariance"? In this case, it's more interesting to consider yet another scaling. Let's call it "dynamical scaling". The meaning is that only "dynamical variables" should be rescaled. The matrix is dynamical but the coupling constant is not. Therefore, this scaling should not be understood as a change of dimension; It's a symmetry of the action. The interesting thing about the RG flow is that the dynamical scaling invariance could emerge at certain critical point $g_*$. The existence of such a point $g_* \neq 0$ seems impossible by just looking at the action, because it is written in a form that only the classical scaling is obvious. It's impossible to obtain a dynamical scaling invariance along the classical scaling. While the RG flow could deviate from the classical scaling significantly at some points, along which the scaling of $g$ could be frozen.

Start with the $N+1$-model, and decomposing the matrix $M$ as

$$\begin{pmatrix} M & v \\ v^\dagger & \alpha \end{pmatrix}.$$

The action can be expanded as

$$\begin{aligned} S_{N+1}[M, v, v^\dagger, \alpha; g] =& (N+1) \operatorname{Tr} \left( \frac{1}{2} M^2 + \frac{g}{4} M^4 \right) + (N+1) \left( v^\dagger v + \frac{1}{2} \alpha^2 \right) \\ &+ g(N+1) \left( v^\dagger M^2 v + \alpha v^\dagger M v + \alpha^2 v^\dagger v + \frac{1}{2} (v^\dagger v)^2 + \frac{1}{4} \alpha^4 \right). \end{aligned} \tag{3}$$

To do the integration over $v, v^\dagger$ and $\alpha$, one needs to first identify the quadratic term around which the interaction is treated perturbatively. For $v, v^\dagger$, the quadratic term is Sat, Feb 24

$$(N+1) v^\dagger \left( \mathbb{1} + g M^2 \right) v.$$

For $\alpha$, it's

$$\frac{N+1}{2} \alpha^2.$$

These leads to the following contraction rules

$$\left\langle v_i^\dagger v_j \right\rangle = \frac{1}{N+1} \left( \frac{1}{1 + g M^2} \right)_{ij}$$

$$\langle \alpha \alpha \rangle = \frac{1}{N+1}$$

The interaction terms are

$$(N+1) \left( g \alpha v^\dagger M v + g \alpha^2 v^\dagger v + \frac{g}{2} (v^\dagger v)^2 + \frac{g}{4} \alpha^4 \right).$$

First, at the zeroth order, the partition function has the form

$$Z_{N+1} \sim \int [\mathrm{d}M] \frac{1}{(N+1)^N \mathrm{Det}(\mathbb{1}+gM^2)} \mathrm{e}^{-(N+1)\,\mathrm{Tr}\left(\frac{1}{2}M^2+\frac{g}{4}M^4\right)}.$$

The $\frac{1}{(N+1)^N}$ factor will be compensated by a rescaling of the effective partition function. The determinant can be re-exponented as

$$\frac{1}{\mathrm{Det}(\mathbb{1}+gM^2)} = \mathrm{e}^{-\,\mathrm{Tr}\ln(1+gM^2)}.$$

This leads to the following "effective action"

$$S_{\text{eff}} = (N+1)\,\mathrm{Tr}\left(\frac{1}{2}M^2+\frac{g}{4}M^4\right) + \mathrm{Tr}\ln(1+gM^2). \tag{4}$$

Then at the order $g$, one could write the partition function $Z_{N+1}$ as

$$Z_{N+1} \sim -g(N+1)\int [\mathrm{d}M] \left\langle \alpha v^\dagger M v + \alpha^2 v^\dagger v + \frac{1}{2}(v^\dagger v)^2 + \frac{1}{4}\alpha^4 \right\rangle \mathrm{e}^{-S_{\text{eff}}[M]}.$$

The first term will not contribute, while other terms giving

$$\left\langle \alpha^2 v^\dagger v \right\rangle = \frac{1}{(N+1)^2}\,\mathrm{Tr}\left(\frac{1}{\mathbb{1}+gM^2}\right)$$

$$\frac{1}{2}\left\langle (v^\dagger v)^2 \right\rangle = \frac{1}{2(N+1)^2}\left[\mathrm{Tr}\left(\frac{1}{\mathbb{1}+gM^2}\right)\right]^2 + \frac{1}{2(N+1)^2}\,\mathrm{Tr}\left(\frac{1}{\mathbb{1}+gM^2}\right)^2$$

$$\frac{1}{4}\left\langle \alpha^4 \right\rangle = \frac{3}{4(N+1)}.$$

Let's study the case where $g$ is small and keep only the first order of $g$. The "effective action" becomes

$$S_{\text{eff}}[M] = (N+1)\,\mathrm{Tr}\left(\frac{1}{2}M^2+\frac{g}{4}M^4\right) + g\,\mathrm{Tr}\,M^2.$$

The corrections from the interactions to the partition function are

$$Z_{N+1} \sim -g(N+1)\int [\mathrm{d}M] \left(\frac{3N}{2(N+1)^2} + \frac{N^2}{2(N+1)^2} + \frac{3}{4(N+1)}\right) \mathrm{e}^{-S_{\text{eff}}[M]}.$$

Re-exponentiate these terms will give us a new action, which could reproduce the correlation functions calculated from $S_{N+1}[M]$ to the first order of $g$,

$$S'_{\text{eff}}[M] = (N+1)\,\mathrm{Tr}\left(\frac{1}{2}M^2+\frac{g}{4}M^4\right) + g\,\mathrm{Tr}\,M^2 + \frac{3N}{2(N+1)}g + \frac{N^2}{2(N+1)}g + \frac{3}{4}g.$$

A feature of the RG flow procedure is the appearance of terms with different orders of $N$.

To put the effective action in the canonical form, one defines

$$M' = \left(1+\frac{1+2g}{N}\right)^{\frac{1}{2}} M,$$

$$g' = \frac{N(N+1)}{(N+1+2g)^2}\,g$$

To the first order of $g$

$$M' \approx \left(\frac{N+1}{N}\right)^{\frac{1}{2}} \left(1 + \frac{g}{N+1}\right) M,$$
$$g' \approx \left(\frac{N}{N+1}\right) \left(1 - \frac{4g}{N+1}\right) g$$

such that

$$S'_{\text{eff}}[M] = N\,\text{Tr}\left(\frac{1}{2}M'^2 + \frac{g'}{4}M'^4\right) + \cdots \tag{5}$$

To get a feeling about what kind of terms could be generated through this RG procedure, let's calculate the decomposition of $\frac{1}{6}\text{Tr}M^6$: Sun, Feb 25

$$v^\dagger M^4 v + (v^\dagger v)(v^\dagger M^2 v) + \frac{1}{2}(v^\dagger M v)(v^\dagger M v) + \frac{1}{3}(v^\dagger v)^3$$
$$+\alpha^2 v^\dagger M^2 v + \frac{3}{2}\alpha^2 (v^\dagger v)^2 + \alpha^4 v^\dagger v.$$

Looking at the first line (terms without $\alpha$), one could think about the possible contractions. The first term is quadratic in $v$, while other terms should be treated by perturbation theory. It's important to note that, for the second term, if contracting $(v^\dagger v)$ one gets

$$\frac{1}{N+1}\text{Tr}\left(\frac{1}{\mathbb{1} + \cdots}\right).$$

where $\cdots$ coming from the quadratic terms like $gM^2$. The leading order in $g$ would be

$$\frac{1}{N+1}\text{Tr}\mathbb{1} = \frac{N}{N+1}.$$

One would say that this term has the $N$-dimension 0. This is inconsistent with the dimension of $(v^\dagger v)$, which is the same as $\text{Tr}M^2$. The reason is that $\text{Tr}\mathbb{1}$ has dimension 1. The dimension of Tr should also be taken into account correctly.

To the zeroth order of $g$, contraction of the first line gives

$$\frac{N}{(N+1)^2}\text{Tr}M^2 + \frac{3}{2(N+1)^2}\text{Tr}M^2 + \frac{1}{2(N+1)^2}\left(\text{Tr}M\right)^2 + \frac{1}{3}\left(\frac{N}{N+1}\right)^3.$$

The first term has the "wrong" dimension because of the $v^\dagger v$ contraction. When $N$ is large, however, it gives the leading contribution to $\text{Tr}M^2$. The $N$-dimension essentially tells us how the terms behave when changing $N$. This is an assumption we made for our theory.

One crucial idea for analyzing the matrix model is to reduce the number of variables from $N^2$ to $N$. This can be done because of the symmetry of the model. What's the physical insight follows from this reduction? If one interpret the matrix as a Hamiltonian, the reduction is essentially chosing the energy eigen-basis for the physical states. The symmetry is just the fact that the physical observables do not depend on the choice of basis. However, we would like to take about the matrix model arising in the string theory. There, rather than an operator on the space of Wed, Feb 28 physical states, the matrix itself represents the physical states: the Chan-Paton degrees of freedom of open string. The number of the Hermitian matrices matches the number of the Chan-Paton degrees of freedom.

The question: imagine a scaling matrix model, in the sense that the matrix $M$ and coupling $g$ scale with $N$ in a simple way. Along this scaling, the physical observables are kept invariant. If such a scaling phenomenon exists, what can we say about the physical observables around such critical point? Thu, Feb 29

It's not clear that what's the meaning of conformal invariance for a matrix model, but we have the motivation to believe that there exists such a notion. From the point view of AdS/CFT, the matching between the conformal group, for example in $4d$ the conformal group is $SO(4,2)$, and the AdS isometry group, for example in $5d$ the isometry group is $O(4,2)$. However, such a "group argument" does not make sense for AdS1/CFT0. Fri, Mar 1

Let's review that how the AdS/CFT is understood in string theory. The central object is the D-brane whose dynamics can be described by an effective action of field theory. The fields live on the world-volume of D-brane: there are massless scalar fields and vector fields. Their appearance is understood as the massless excitation of open strings with end points on the D-brane. The form of the action is obtained by matching with the calculation from the perturbative string theory. This field theory will become the CFT side.

**Question 1.** *But why the field theory of D-branes must be a CFT? Does this fact relate to the dynamics of the D-branes? Or is there a "string theory" argument for the appearance of conformal invariance?*

**Answer 1.** *Because the open string excitation is massless when the two end points locate on the coincident D-branes.*

D-branes carry the R-R charges, which means that it will couple to the higher form gauge fields. Therefore they can be understood physically as the sources of those gauge fields. The D(-1)-brane couples to the 0-form potential in the IIB theory $C_0$. The energy of that coupling is given by the value of $C_0$ at the point where the D(-1)-brane locating. Therefore the D(-1)-brane is properly interpreted as an instanton.

We are interested in the D-instanton, a natural question is that how the D-instantons interact with themselves and other objects like higher dimensional D-branes. This question could be answered in the perturbative string theory. One calculates the string amplitudes for exchanging the closed string states (graviton, dilaton and the R-R state) between the D-branes. The fundamental physical properties of the D-branes (the tension and the R-R charge) could be related to the fundamental constants in string theory.

**Question 2.** *When discussing the interaction and action of D-branes, we have in mind a picture where the D-brane locating at a particular space-time position. In this way, rather than taking them as a fundamental degrees of freedom, we regard them as absolute objects (like black hole in GR). In such a setting, how should we understand the physics under the D-brane action? A clearer name seems to be the "action of D-brane fluctuation", and the fluctuation has its origin at the open string oscillation. Or maybe one should call it the "open string action restricted on the D-branes"?*

There is an interesting explanation of the collective coordinates $X^\mu$ of the D-branes: the Goldstone bosons for the spontaneous translation symmetry breaking. But why it's called the "spontaneous"?

**Idea 1.** *Why not trying to look further at the "Ward identity" of the matrix model and think about the possible implications on our calculations?*

To include supersymmetry in the matrix model, we are forced to consider the Grassmann-valued matrix. This leads us to think about how to build a matrix upon the Grassmann algebra. In particular, think about how the supersymmetry transformation is realized as a transformation inside the Grassmann algerba (interchange odd and even elements). Sun, Mar 3

First, one should difference between the elements of Grassmann algebra and the Grassmann variables. The later are the generators of the Grassmann algebra. The differential and integration are defined for the Grassmann variables. Then

**Question 3.** *The fermionic matrix has entries of the Grassmann variables or the general odd elements of the Grassmann algebra?*

It seems natural to allow general odd elements and take the number of the Grassmann variables to infinity.

**Question 4.** *How to define the integration over such a Grassmann-valued matrix?*

The supersymmetry of a matrix model may imply the vanishing of some correlation functions. This may be regarded as the matrix analog of the Ward identity. An example is the superfield model mentioned in ✎ **Ple96**. The reason behind the vanishing is the cancellation between bosonic and fermionic degrees of freedom. However, the prove given in the article based on the diagonalization of the bosonic matrix. One would like to see the cancellation in a direct way: how the cancellation works when calculating the effective action. The first step would be a careful study of the action under a parameterization of the model.

One problem of the RG calculation is that the susy transformation of the reduced matrix involving the variables that being integrated out. This implies that the parameterization is not done in a way that the susy is preserved. To avoid the problem, maybe we should use the formalism where the susy is realized linearly.

**Idea 2.** *Do the RG calculation for a model where the supersymmetry is realized linearly.*

The supersymmetry Ward identity gives a relation between correlation functions

$$\frac{1}{2}\left\langle \mathrm{Tr} V'(\phi)\phi^{n-1}\right\rangle = \sum_{a+b=n-2}\left\langle \mathrm{Tr}\phi^a\psi\phi^b\overline{\psi}\right\rangle . \tag{6}$$

This identity maybe used to reduce the result of the calculation.

**Problem 1.** *Try to derive this Ward identity.*

**Problem 2.** *How does the measure* $\mathrm{d}\phi\mathrm{d}\psi\mathrm{d}\overline{\psi}$ *transforms under the susy?*

First note that the super-determinant must involve $\bar{\epsilon}\epsilon$ such that the integral is bosonic. So the measure is invariant to the first order of $\epsilon, \bar{\epsilon}$. The "superfield" $\Phi = \phi + \overline{\psi}\theta + \bar{\theta}\psi + \theta\bar{\theta}F$ could be the starting point for deriving the Ward identity. Calculate $\Phi^n$

$$\begin{aligned}
\Phi^n &= \left(\phi + \overline{\psi}\theta + \bar{\theta}\psi + \theta\bar{\theta}F\right)^n \\
&= \phi^n + \left(\sum_{a+b=n-1}\phi^a\overline{\psi}\phi^b\right)\theta + \bar{\theta}\left(\sum_{a+b=n-1}\phi^a\psi\phi^b\right) \\
&+ \theta\bar{\theta}\left(\sum_{a+b+c=n-2}\phi^a\overline{\psi}\phi^b\psi\phi^c + \sum_{a+b=n-1}\phi^a F\phi^b\right).
\end{aligned}$$

This leads us to consider the following reparameterization (induced by $\Phi^n$, this reparameterization should be supersymmetric.)

$$\begin{aligned}
&\phi \to \phi' = \phi + \varepsilon\phi^n \\
&\overline{\psi} \to \overline{\psi}' = \overline{\psi} + \varepsilon\sum_{a+b=n-1}\phi^a\overline{\psi}\phi^b \\
&\psi \to \psi' = \psi + \varepsilon\sum_{a+b=n-1}\phi^a\psi\phi^b \\
&F \to F' = F + \varepsilon\left(\sum_{a+b+c=n-2}\phi^a\overline{\psi}\phi^b\psi\phi^c + \sum_{a+b=n-1}\phi^a F\phi^b\right)
\end{aligned}$$

Consider how the measure $\mathrm{d}\phi\mathrm{d}\overline{\psi}\mathrm{d}\psi\mathrm{d}F$ changes to the first order of $\varepsilon$. The usual coordinate transformation formula detJac is generalized to that involving Grassmann variables BerJac. The Berezinian satisfies the formula

$$\mathrm{Ber}\left(\mathbb{1} + \varepsilon M\right) = 1 + \varepsilon\mathrm{STr}M. \tag{7}$$

Here the STr is the super-trace. It can be proved that there is no change of the measure.

Now the following terms arise from the variation of the action

$$\begin{aligned}
\varepsilon \mathrm{Tr}[V''(\phi)\phi^n F] - 2\varepsilon \sum_{a+b=n-1} \mathrm{Tr}(\phi^a F \phi^b F) + \varepsilon \sum_{a+b=n-1} \mathrm{Tr}[V'(\phi)\phi^a F \phi^b] \\
-2\varepsilon \sum_{a+b+c=n-2} \mathrm{Tr}(\phi^a \overline{\psi} \phi^b \psi \phi^c F) + \varepsilon \sum_{a+b+c=n-2} \mathrm{Tr}[V'(\phi)\phi^a \overline{\psi} \phi^b \psi \phi^c] \\
+\varepsilon \sum_{k=0}^{\infty} k g_k \sum_{a+b=k-2} \Big[ a\mathrm{Tr}(\phi^{a+n-1} \overline{\psi} \phi^b \psi) + b\mathrm{Tr}(\phi^a \overline{\psi} \phi^{b+n-1} \psi) \\
+2 \sum_{c+d=n-1} \mathrm{Tr}(\phi^{a+c} \overline{\psi} \phi^{b+d} \psi) \Big]
\end{aligned}$$

The second line will vanish after integrating over $F$. The first line will give $-2\varepsilon \sum_{a+b=n-1} \mathrm{Tr}\phi^a \mathrm{Tr}\phi^b + \frac{\varepsilon}{2} \mathrm{Tr}[V''(\phi)\phi^n V'(\phi)]$.

**Remark 1.** *The above calculation gives a fairly complicate identity. It's also not clear whether it will be useful or not. Also, this does not lead to the Ward identity.*

Study the RG flow of the "superfield" matrix model. This model is mentioned in **Ple96** Mon, Mar 4

The matrix-valude superfield is constructed as

$$\Phi = \phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F. \tag{8}$$

$\theta, \overline{\theta}$ are the coordinates of the superspace. $\phi, F$ are bosonic $N \times N$ matrix, and we assume them to be Hermitian. $\psi, \overline{\psi}$ are fermionic $N \times N$ matrix, whose entries are Grassmann variables. Recall that the complex conjugate of the product of Grassmann variables is defined as $(\xi\eta)^* = \eta^* \xi^*$. To keep $\Phi$ Hermitian $\Phi^\dagger = \Phi$, we require that

$$\psi^\dagger = \overline{\psi}, \quad \theta^* = \overline{\theta}.$$

The measure over the superfield will be the usual Berezian integral of the matrix entries

$$\mathrm{d}\Phi = \mathrm{d}\phi \mathrm{d}F \mathrm{d}\overline{\psi} \mathrm{d}\psi. \tag{9}$$

The supersymmetric action can be constructed as

$$S[\Phi] = \mathrm{Tr} \int \mathrm{d}\theta \mathrm{d}\overline{\theta} \left\{ -D_\theta \Phi D_{\overline{\theta}} \Phi + \sum_{k=0}^{\infty} g_k \Phi^k \right\}. \tag{10}$$

The super-derivative acts from the right. The matrix model partition function is

$$Z_\Phi[g_k] = \int [\mathrm{d}\Phi] \exp\left(-NS[\Phi]\right). \tag{11}$$

We have $D_\theta \Phi = \overline{\psi} - \overline{\theta} F$ and $D_{\overline{\theta}} \Phi = -\psi + \theta F$. Their product will contribute to the action only if the measure $\mathrm{d}\theta \mathrm{d}\overline{\theta}$ is saturated. This will give a term $\mathrm{Tr} F^2$ in the action. We calculate the $\Phi^k$ term as

$$\begin{aligned}
\Phi^k &= \left(\phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F\right)^k \\
&= \phi^k + \left(\sum_{a+b=k-1} \phi^a \overline{\psi} \phi^b\right)\theta + \overline{\theta}\left(\sum_{a+b=k-1} \phi^a \psi \phi^b\right) \\
&+ \theta\overline{\theta}\left(\sum_{a+b+c=k-2} \phi^a \overline{\psi} \phi^b \psi \phi^c + \sum_{a+b=k-1} \phi^a F \phi^b\right).
\end{aligned}$$

By taking trace and keeping only the $\theta\overline{\theta}$ term, we get in the action a term

$$-\mathrm{Tr}[V'(\phi)F] - \sum_{k=0}^{\infty} k g_k \sum_{a+b=k-2} \mathrm{Tr}(\phi^a \overline{\psi} \phi^b \psi).$$

Let's assume a quartic potential $V(\phi) = \frac{1}{2}\phi^2 + \frac{g}{4}\phi^4$,

$$\begin{aligned}
S[\Phi] &= \mathrm{Tr} F^2 - \mathrm{Tr}(\phi F) - g\mathrm{Tr}(\phi^3 F) \\
&- \mathrm{Tr}(\overline{\psi}\psi) - g\left[\mathrm{Tr}(\phi^2 \overline{\psi}\psi) + \mathrm{Tr}(\phi\overline{\psi}\phi\psi) + \mathrm{Tr}(\overline{\psi}\phi^2\psi)\right].
\end{aligned}$$

Although it's easy to do the integral over $F$, we will not do that. Because it will lead to a non-linear supersymmetry transformation rule. However, let's define $F' = F - \frac{1}{2}\phi$ and rewrite the action in terms of $F$ such that a quadratic term in $\phi$ will appear in the action

$$\begin{aligned}
S[\Phi] &= \mathrm{Tr} F'^2 - \frac{1}{4}\mathrm{Tr}\phi^2 - \frac{g}{2}\mathrm{Tr}\phi^4 - g\mathrm{Tr}(\phi^3 F') \\
&- \mathrm{Tr}(\overline{\psi}\psi) - g\left[\mathrm{Tr}(\phi^2 \overline{\psi}\psi) + \mathrm{Tr}(\phi\overline{\psi}\phi\psi) + \mathrm{Tr}(\overline{\psi}\phi^2\psi)\right].
\end{aligned}$$

The supersymmetry variation reads

$$\begin{aligned}
\delta\phi &= \overline{\varepsilon}\psi + \overline{\psi}\varepsilon \\
\delta\psi &= -\varepsilon(F' + \frac{1}{2}\phi) \\
\delta\overline{\psi} &= -\overline{\varepsilon}(F' + \frac{1}{2}\phi) \\
\delta F' &= -\frac{1}{2}(\overline{\varepsilon}\psi + \overline{\psi}\varepsilon)
\end{aligned}$$

Think about applying the RG strategy on this action. If we denote $\phi_{i,N} = v_i, \phi_{N,i} = v_i^*$, the following term will appear

$$-g v^\dagger F' \phi v - g v^\dagger \phi F' v$$

from the interaction $-g\mathrm{Tr}(\phi^3 F')$. It will contribute a $\mathrm{Tr}(\phi F')$ term to the effective action. Why the supersymmetry cancellation does not happen here? This is because the supersymmetry transformation acts on all matrix entries simultaneously: it does not realize solely on the variables that being integrated out.

**Idea 3.** *Study the effect of a "local" supersymmetry transformation: that is, those only act on a part of the matrix.*

**Idea 4.** *Carry out the calculation anyway, to see what you get.*

For the RG strategy, let's decompose the matrix

$$F_0' = \begin{pmatrix} F' & f \\ f^\dagger & a \end{pmatrix}, \quad \phi_0 = \begin{pmatrix} \phi & v \\ v^\dagger & \alpha \end{pmatrix}$$
$$\psi_0 = \begin{pmatrix} \psi & \chi \\ \omega^\dagger & \beta \end{pmatrix}, \quad \overline{\psi}_0 = \begin{pmatrix} \overline{\psi} & \omega \\ \chi^\dagger & \overline{\beta} \end{pmatrix}$$

The interactions decompose correspondingly

$$\begin{aligned}
\frac{g}{2}\mathrm{Tr}\phi_0^4 &= \frac{g}{2}\mathrm{Tr}\phi^4 + 2gv^\dagger\phi^2 v + g(v^\dagger v)^2 + 2g\alpha v^\dagger \phi v + 2g\alpha^2 v^\dagger v + \frac{g}{2}\alpha^4 \\
g\mathrm{Tr}(\phi_0^3 F_0') &= g\mathrm{Tr}(\phi^3 F') + g(v^\dagger\phi^2 f + f^\dagger\phi^2 v) + g(v^\dagger\phi F' v + v^\dagger F'\phi v) \\
&\quad + g(\alpha v^\dagger\phi f + \alpha f^\dagger\phi v) + gav^\dagger\phi v + g\alpha v^\dagger F' v + g(v^\dagger v v^\dagger f + v^\dagger v f^\dagger v) \\
&\quad + g(\alpha^2 v^\dagger f + \alpha^2 f^\dagger v) + 2ga\alpha v^\dagger v + ga\alpha^3 \\
g\mathrm{Tr}(\phi_0^2\overline{\psi}_0\psi_0 + \overline{\psi}_0\phi_0^2\psi_0) &= g\mathrm{Tr}(\phi^2\overline{\psi}\psi + \overline{\psi}\phi^2\psi) + g(v^\dagger\overline{\psi}\psi v + v^\dagger\psi\overline{\psi}v) - g(\omega^\dagger\phi^2\omega - \chi^\dagger\phi^2\chi) \\
&\quad + g(\chi^\dagger\psi\phi v + v^\dagger\phi\overline{\psi}\chi - v^\dagger\phi\psi\omega - \omega^\dagger\overline{\psi}\phi v) \\
&\quad + g(\alpha\chi^\dagger\psi v - \alpha v^\dagger\psi\omega + \alpha v^\dagger\overline{\psi}\chi - \alpha\omega^\dagger\overline{\psi}v) \\
&\quad + g(\overline{\beta}\omega^\dagger\phi v + \overline{\beta}v^\dagger\phi\chi - \beta v^\dagger\phi\omega - \beta\chi^\dagger\phi v) \\
&\quad + g(v^\dagger\omega\omega^\dagger v - v^\dagger\chi\chi^\dagger v + v^\dagger v\chi^\dagger\chi - v^\dagger v\omega^\dagger\omega) \\
&\quad + g(\alpha\overline{\beta}\omega^\dagger v + \alpha\overline{\beta}v^\dagger\chi + v^\dagger\omega\beta\alpha + \chi^\dagger v\alpha\beta) \\
&\quad + g(2v^\dagger v\overline{\beta}\beta + \alpha^2\chi^\dagger\chi - \alpha^2\omega^\dagger\omega) + g\alpha^2\overline{\beta}\beta \\
g\mathrm{Tr}(\phi_0\overline{\psi}_0\phi_0\psi_0) &= g\mathrm{Tr}(\phi\overline{\psi}\phi\psi) + g(v^\dagger\overline{\psi}\phi\chi + \chi^\dagger\phi\psi v - v^\dagger\psi\phi\omega - \omega^\dagger\phi\overline{\psi}v) \\
&\quad + g(\alpha\chi^\dagger\phi\chi - \alpha\omega^\dagger\phi\omega + \overline{\beta}v^\dagger\psi v - \beta v^\dagger\overline{\psi}v) + g(\chi^\dagger v\omega^\dagger v + v^\dagger\omega v^\dagger\chi) \\
&\quad + g(\alpha\overline{\beta}\omega^\dagger v - \alpha\beta v^\dagger\omega + \alpha\overline{\beta}v^\dagger\chi - \alpha\beta\chi^\dagger v) + g\alpha^2\overline{\beta}\beta.
\end{aligned}$$

To the first order of $g$, the non-vanishing contributions are

$$2gv^\dagger\phi^2 v + g(v^\dagger\phi F' v + v^\dagger F'\phi v) - g(\omega^\dagger\phi^2\omega - \chi^\dagger\phi^2\chi).$$

The quadratic terms are $\frac{1}{2}(v^\dagger v)$ and $\chi^\dagger\chi - \omega^\dagger\omega$.

**Idea 5.** *There are two ways through which the supersymmetry could help us in the calculation: 1. build up a model in which the supersymmetry is realized "locally"; 2. develop another RG method which is compatible with the global supersymmetry;*

*We focus on susy because we believe it's a crucial ingredient to build a conformal matrix model.*