Master Thesis Notes

Xiangwen Guan

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Contents

Mon, Apr 29

1 Dp/D$(6-p)$ bound states

references The notes from Sebastien [**RRT-draft**]. The $\mathrm{AdS}_1 \times \mathrm{S}^1$ paper [**SKT22**]. A review of duality in supergravity solution [**DKL94**]. String theory discussion Polchinski. The review of brane-intersections and corresponding susy [**Dou03**].

terminology: dyon. the particle that carries electric and magnetic charges. electric charge. the Noether charge associated with the equation of motion. magnetic charge. the topological charge associated with the Bianchi identity. Montonen-Olive duality. A strong-weak duality for the gauge field theory living on the D3-brane. It can be understood as a consequence of $SL(2,\mathbb{Z})$ duality (but in our context, it's not the $SL(2,\mathbb{Z})$. brane-probe condition. a brane probe (without backreaction) the susy (BPS) background generated by the same type of branes should feel no force. moduli space.

The bosonic truncation of type IIA/B supergravity (for a single R-R field, Einstein frame)

$$S = \int \sqrt{|g|} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\frac{1}{(p+2)!} \mathrm{e}^{\frac{(3-p)}{2}\phi} F_{p+2}^2 \right) \tag{1}$$

to understand: F^2 term leads to a dilaton gradient, diverge near the "would-be horizon"?; electric charge and magnetic charge contribute to F^2, with opposite sign? Vanish for D1/D5? Near horizon or overall? Constraints on charge relations?

Understand more about the electric-magnetic duality for Dp-branes.

The R-R charges carried by Dp-brane is the magnetic dual to that carried by D$(6-p)$-brane. The R-R field strength is F_{p+2} for Dp-brane. $p=3$ as a special case (self-dual?)

p-brane and F_{p+2} gauge field Some notions of gauge field F_{p+2} of p-branes. In particular, how the Dp-brane carries electric charge, while D$(6-p)$-brane carries magnetic charge.

Consider a p-brane that couples to a $(p+1)$-form gauge potential A. The gauge transformation is $A \to A + \mathrm{d}\lambda$ (maybe add a factor $(p+1)$ before $\lambda_{\mu_2\cdots\mu_{p+1}}$?)

$$A_{\mu_1\mu_2\cdots\mu_{p+1}} \to A_{\mu_1\mu_2\cdots\mu_{p+1}} + \partial_{[\mu_1}\lambda_{\mu_2\cdots\mu_{p+1}]}.$$

The field strenght $F = \mathrm{d}A$ is defined as

$$F_{\mu_1\mu_2\cdots\mu_{p+2}} = (p+2)\partial_{[\mu_1}A_{\mu_2\cdots\mu_{p+2}]}.$$

By definition, the Bianchi identity is satisfied.

$$\mathrm{d}F = 0.$$

The p-brane is the source of A, a $p+1$ form J. The equation of motion is

$$\mathrm{d} * F = *J.$$

The Hodge dual $*$ is defined as

$$(*J)^{\mu_1\cdots\mu_{9-p}} \equiv \frac{1}{(p+1)!}\varepsilon^{\mu_1\cdots\mu_{10}}J_{\mu_{10-p}\cdots\mu_{10}}.$$

with $\varepsilon^{01\cdots 9} = 1$. The equation of motion tells us that there is a conserved "electric charge". The Bianchi identity tells us that there is no dual "magnetic charge". The way to introduce the "magnetic charge" is to add a source term to the Bianchi identity. This can be done by modifying the definition of F as follows

$$F = \mathrm{d}A + \omega.$$

ω could give a source term to the Bianchi identity

$$\mathrm{d}F = \mathrm{d}\omega = X.$$

X is a $(p+3)$ form. $*X$ is a $(7-p)$ form, and is interpreted as the magnetic charge density carried by a $(6-p)$-brane. The electric and magnetic charge is obtained from the integration

$$e \equiv \int_{S^{8-p}} *F = \int_{M^{9-p}} *J.$$

$$g \equiv \int_{S^{p+2}} F = \int_{M^{p+3}} X.$$

Let's put this into the context of type IIB supergravity. D5-brane is a magnetic source of the R-R 2-form potential. D1-brane is the electric source.

fluxes Flux is important for keeping supersymmetries? How to understand this point?

supersymmetry of the bound states Supersymmetry: a single Dp-brane preserve half of the supersymmetry (out of 32 real supercharges). The bound states preserve less. (more details... how it matches with that of supergravity solution? There are methods to determine the preserved susy in some brane intersection configurations. For example, by checking the projection conditions for both brane, whether some susy generators survive.

Two susy: world-volume and space-time. Pull-back of gamma-matrices on the world-volume: $\gamma_\mu \equiv (\partial_{\sigma^\mu} X^M)\Gamma_M$

harmonic rules? Basic requirements for Dp-brane intersections(?): the harmonic function rules.

Open string boundary condition: Dirichlet, Neumann, Mixed. Supersymmetric bound state: 4 mixed boundary conditions? (where this condition comes from?)

D-brane profile Configuration: wrap the D$(6-p)$ brane over a torus to compactify the dimension $10 \to 4$? and same time put it on the same footing as the Dp-brane in the non-compactified directions? Wrap D7 brane over T^8. Dissolve it in the flux of F_5. Smear $D(-1)$ branes over T^8.

D1/D5 geometry: $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$. What's the relation between the geometry and the Dp-brane configuration?

Lorentz v.s. Euclidean: the supergravity solution of D(-1)/D7; D(-1) only exist in Euclidean? (Is it really the case? Why it's the case? Implication on supergravity solution?) Be careful about the Hodge duality of F_n, the G_{D-n}, the sign of kinetic term of the field strength: wrong sign in Euclidean signature (generally even number of time directions)

Tha metric ansatz $\mathrm{AdS}_1 \times \mathrm{S}^1 \times \mathrm{T}^8$

$$\mathrm{d}s^2 = L_y^2 \mathrm{d}y^2 + L_x^2 \mathrm{d}x^2 + \sum_{i=1}^{8} L_i^2 (\mathrm{d}\theta^i)^2 \tag{2}$$

In [**SKT22**], the y-direction is understood as "Euclidean time": $y = it$ with t is the Lorentz time. This corresponds to the AdS_1 factor.

Just like identifying the supergravity solution of extremal p-brane as the geometry generated by the Dp-brane in string theory, there are also solutions that carry both electric and magnetic charges could be identified as the geometry of Dp/D$(6-p)$ brane bound states. example here for D1/D5 bound state Tue, Apr 30

the ansatz for D(-1)**/D**7

$$F_1 = \alpha(y)\mathrm{d}x + i\beta(y)\mathrm{d}y$$

The magnetic piece of F_1: $\alpha(y)\mathrm{d}x$? The electric piece of F_1: $i\beta(y)\mathrm{d}y$?

$F_1^2 = g^{\mu\nu}(F_1)_\mu (F_1)_\nu = g^{xx}\alpha^2 - g^{yy}\beta^2$. g^{xx} and g^{yy} are positive definite in Euclidean signature? Note that electirc and magnetic contributes oppositely.

$F_9 = F_1 = \beta(\cdots)\mathrm{d}x \wedge \mathrm{d}\theta + i\alpha(\cdots)\mathrm{d}y \wedge \mathrm{d}\theta$. The imaginary number i?

D7-branes are wrapped over $\mathbb{T}^8$. F_9, the dual of F_1, has "legs" along $\mathbb{T}^8$ and transverse. The source of F_1 or F_9? Magnetic source and electric source, D(-1) or D7?

the D1/**D**5 **example** The metric in string frame (appearing in [**Dou03**] section 4.2.1)

$$\mathrm{d}s^2 = \frac{1}{\sqrt{H_1 H_5}}(-\mathrm{d}x_0^2 + \mathrm{d}x_1^2) + \sqrt{H_1 H_5}(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_3^2) + \sqrt{\frac{H_1}{H_5}}(\mathrm{d}x_6^2 + \cdots + \mathrm{d}x_9^2) \quad (3)$$

x_0, x_1: directions along the world-volume of D1 and D5; $x_6, \cdots, x_9$: directions along the world-volume of D5, transverse to D1; r, Ω_3: transverse directions. Note a pattern: longitudinal direction has metric factor $1/\sqrt{H}$; transversal direction has metric factor $\sqrt{H}$. Interpretation: D1 smeared over D5?

RR-fluxes:

field theory, D1/D5 CFT? supergravity solution preserves 8 susy.