Master Thesis Notes

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Contents

One crucial idea for analyzing the matrix model is to reduce the number of variables from N^2 to N. This can be done because of the symmetry of the model. What's the physical insight follows from this reduction? If one interpret the matrix as a Hamiltonian, the reduction is essentially chosing the energy eigen-basis for the physical states. The symmetry is just the fact that the physical observables do not depend on the choice of basis. However, we would like to take about the matrix model arising in the string theory. There, rather than an operator on the space of physical states, the matrix itself represents the physical states: the Chan-Paton degrees of freedom of open string. The number of the Hermitian matrices matches the number of the Chan-Paton degrees of freedom. Wed, Feb 28

The question: imagine a scaling matrix model, in the sense that the matrix M and coupling g scale with N in a simple way. Along this scaling, the physical observables are kept invariant. If such a scaling phenomenon exists, what can we say about the physical observables around such critical point? Thu, Feb 29

Fri, Mar 1

It's not clear that what's the meaning of conformal invariance for a matrix model, but we have the motivation to believe that there exists such a notion. From the point view of AdS/CFT, the matching between the conformal group, for example in $4d$ the conformal group is $SO(4,2)$, and the AdS isometry group, for example in $5d$ the isometry group is $O(4,2)$. However, such a "group argument" does not make sense for AdS1/CFT0.

Let's review that how the AdS/CFT is understood in string theory. The central object is the D-brane whose dynamics can be described by an effective action of field theory. The fields live on the world-volume of D-brane: there are massless scalar fields and vector fields. Their appearance is understood as the massless excitation of open strings with end points on the D-brane. The form of the action is obtained by matching with the calculation from the perturbative string theory. This field theory will become the CFT side.

Question 1. *But why the field theory of D-branes must be a CFT? Does this fact relate to the dynamics of the D-branes? Or is there a "string theory" argument for the appearance of conformal invariance?*

Answer 1. *Because the open string excitation is massless when the two end points locate on the coincident D-branes.*

D-branes carry the R-R charges, which means that it will couple to the higher form gauge fields. Therefore they can be understood physically as the sources of those gauge fields. The D(-1)-brane couples to the 0-form potential in the IIB theory C_0. The energy of that coupling is given by the value of C_0 at the point where the D(-1)-brane locating. Therefore the D(-1)-brane is properly interpreted as an instanton.

We are interested in the D-instanton, a natural question is that how the D-instantons interact with themselves and other objects like higher dimensional D-branes. This question could be answered in the perturbative string theory. One calculates the string amplitudes for exchanging the closed string states (graviton, dilaton and the R-R state) between the D-branes. The fundamental physical properties of the D-branes (the tension and the R-R charge) could be related to the fundamental constants in string theory.

Question 2. *When discussing the interaction and action of D-branes, we have in mind a picture where the D-brane locating at a particular space-time position. In this way, rather than taking them as a fundamental degrees of freedom, we regard them as absolute objects (like black hole in GR). In such a setting, how should we understand the physics under the D-brane action? A clearer name seems to be the "action of D-brane fluctuation", and the fluctuation has its origin at the open string oscillation. Or maybe one should call it the "open string action restricted on the D-branes"?*

There is an interesting explanation of the collective coordinates X^μ of the D-branes: the Goldstone bosons for the spontaneous translation symmetry breaking. But why it's called the "spontaneous"?

Idea 1. *Why not trying to look further at the "Ward identity" of the matrix model and think about the possible implications on our calculations?*

To include supersymmetry in the matrix model, we are forced to consider the Grassmann-valued matrix. This leads us to think about how to build a matrix upon the Grassmann algebra. In particular, think about how the supersymmetry transformation is realized as a transformation inside the Grassmann algerba (interchange odd and even elements). Sun, Mar 3

First, one should difference between the elements of Grassmann algebra and the Grassmann variables. The later are the generators of the Grassmann algebra. The differential and integration are defined for the Grassmann variables. Then

Question 3. *The fermionic matrix has entries of the Grassmann variables or the general odd elements of the Grassmann algebra?*

It seems natural to allow general odd elements and take the number of the Grassmann variables to infinity.

Question 4. *How to define the integration over such a Grassmann-valued matrix?*

The supersymmetry of a matrix model may imply the vanishing of some correlation functions. This may be regarded as the matrix analog of the Ward identity. An example is the superfield model mentioned in ✎ **Ple96**. The reason behind the vanishing is the cancellation between bosonic and fermionic degrees of freedom. However, the prove given in the article based on the diagonalization of the bosonic matrix. One would like to see the cancellation in a direct way: how the cancellation works when calculating the effective action. The first step would be a careful study of the action under a parameterization of the model.

One problem of the RG calculation is that the susy transformation of the reduced matrix involving the variables that being integrated out. This implies that the parameterization is not done in a way that the susy is preserved. To avoid the problem, maybe we should use the formalism where the susy is realized linearly.

Idea 2. *Do the RG calculation for a model where the supersymmetry is realized linearly.*

The supersymmetry Ward identity gives a relation between correlation functions

$$\frac{1}{2}\left\langle \mathrm{Tr}V'(\phi)\phi^{n-1}\right\rangle = \sum_{a+b=n-2}\left\langle \mathrm{Tr}\phi^a\psi\phi^b\overline{\psi}\right\rangle . \tag{1}$$

This identity maybe used to reduce the result of the calculation.

Problem 1. *Try to derive this Ward identity.*

Problem 2. *How does the measure* $\mathrm{d}\phi\mathrm{d}\psi\mathrm{d}\overline{\psi}$ *transforms under the susy?*

First note that the super-determinant must involve $\bar{\epsilon}\epsilon$ such that the integral is bosonic. So the measure is invariant to the first order of $\epsilon, \bar{\epsilon}$. The "superfield" $\Phi = \phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F$ could be the starting point for deriving the Ward identity. Calculate Φ^n

$$\begin{aligned}
\Phi^n &= \left(\phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F\right)^n \\
&= \phi^n + \left(\sum_{a+b=n-1}\phi^a\overline{\psi}\phi^b\right)\theta + \overline{\theta}\left(\sum_{a+b=n-1}\phi^a\psi\phi^b\right) \\
&+ \theta\overline{\theta}\left(\sum_{a+b+c=n-2}\phi^a\overline{\psi}\phi^b\psi\phi^c + \sum_{a+b=n-1}\phi^aF\phi^b\right).
\end{aligned}$$

This leads us to consider the following reparameterization (induced by Φ^n, this reparameterization should be supersymmetric.)

$$\begin{aligned}
&\phi \to \phi' = \phi + \varepsilon\phi^n \\
&\overline{\psi} \to \overline{\psi}' = \overline{\psi} + \varepsilon\sum_{a+b=n-1}\phi^a\overline{\psi}\phi^b \\
&\psi \to \psi' = \psi + \varepsilon\sum_{a+b=n-1}\phi^a\psi\phi^b \\
&F \to F' = F + \varepsilon\left(\sum_{a+b+c=n-2}\phi^a\overline{\psi}\phi^b\psi\phi^c + \sum_{a+b=n-1}\phi^aF\phi^b\right)
\end{aligned}$$

Consider how the measure $\mathrm{d}\phi\mathrm{d}\overline{\psi}\mathrm{d}\psi\mathrm{d}F$ changes to the first order of ε. The usual coordinate transformation formula detJac is generalized to that involving Grassmann variables BerJac. The Berezinian satisfies the formula

$$\mathrm{Ber}\left(\mathbb{1} + \varepsilon M\right) = 1 + \varepsilon\mathrm{STr}M. \tag{2}$$

Here the STr is the super-trace. It can be proved that there is no change of the measure.

Now the following terms arise from the variation of the action

$$
\begin{aligned}
&\varepsilon \mathrm{Tr}[V''(\phi)\phi^n F] - 2\varepsilon \sum_{a+b=n-1} \mathrm{Tr}(\phi^a F \phi^b F) + \varepsilon \sum_{a+b=n-1} \mathrm{Tr}[V'(\phi)\phi^a F \phi^b] \\
&-2\varepsilon \sum_{a+b+c=n-2} \mathrm{Tr}(\phi^a \overline{\psi} \phi^b \psi \phi^c F) + \varepsilon \sum_{a+b+c=n-2} \mathrm{Tr}[V'(\phi)\phi^a \overline{\psi}\phi^b \psi \phi^c] \\
&+\varepsilon \sum_{k=0}^{\infty} k g_k \sum_{a+b=k-2} \Big[a \mathrm{Tr}(\phi^{a+n-1}\overline{\psi}\phi^b \psi) + b \mathrm{Tr}(\phi^a \overline{\psi}\phi^{b+n-1}\psi) \\
&\qquad + 2 \sum_{c+d=n-1} \mathrm{Tr}(\phi^{a+c}\overline{\psi}\phi^{b+d}\psi) \Big]
\end{aligned}
$$

The second line will vanish after integrating over F. The first line will give $-2\varepsilon \sum_{a+b=n-1} \mathrm{Tr}\phi^a \mathrm{Tr}\phi^b + \frac{\varepsilon}{2}\mathrm{Tr}[V''(\phi)\phi^n V'(\phi)]$.

Remark 1. *The above calculation gives a fairly complicate identity. It's also not clear whether it will be useful or not. Also, this does not lead to the Ward identity.*

Study the RG flow of the "superfield" matrix model. This model is mentioned in **Ple96** Mon, Mar 4

The matrix-value superfield is constructed as

$$\Phi = \phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F. \tag{3}$$

$\theta, \overline{\theta}$ are the coordinates of the superspace. ϕ, F are bosonic $N \times N$ matrix, and we assume them to be Hermitian. $\psi, \overline{\psi}$ are fermionic $N \times N$ matrix, whose entries are Grassmann variables. Recall that the complex conjugate of the product of Grassmann variables is defined as $(\xi\eta)^* = \eta^*\xi^*$. To keep Φ Hermitian $\Phi^\dagger = \Phi$, we require that

$$\psi^\dagger = \overline{\psi}, \quad \theta^* = \overline{\theta}.$$

The measure over the superfield will be the usual Berezian integral of the matrix entries

$$\mathrm{d}\Phi = \mathrm{d}\phi \mathrm{d}F \mathrm{d}\overline{\psi}\mathrm{d}\psi. \tag{4}$$

The supersymmetric action can be constructed as

$$S[\Phi] = \mathrm{Tr} \int \mathrm{d}\overline{\theta}\mathrm{d}\theta \left\{ -D_\theta \Phi D_{\overline{\theta}} \Phi + \sum_{k=0}^{\infty} g_k \Phi^k \right\}. \tag{5}$$

The super-derivative acts from the right. The matrix model partition function is

$$Z_\Phi[g_k] = \int [\mathrm{d}\Phi] \exp\left(-NS[\Phi]\right). \tag{6}$$

We have $D_\theta \Phi = \overline{\psi} - \overline{\theta}F$ and $D_{\overline{\theta}}\Phi = -\psi + \theta F$. Their product will contribute to the action only if the measure $\mathrm{d}\theta\mathrm{d}\overline{\theta}$ is saturated. This will give a term $-\mathrm{Tr}F^2$ in the action. We calculate the Φ^k term as

$$\begin{aligned}
\Phi^k &= \left(\phi + \overline{\psi}\theta + \overline{\theta}\psi + \theta\overline{\theta}F\right)^k \\
&= \phi^k + \left(\sum_{a+b=k-1} \phi^a \overline{\psi} \phi^b\right)\theta + \overline{\theta}\left(\sum_{a+b=k-1} \phi^a \psi \phi^b\right) \\
&+ \theta\overline{\theta}\left(\sum_{a+b+c=k-2} \phi^a \overline{\psi} \phi^b \psi \phi^c + \sum_{a+b=k-1} \phi^a F \phi^b\right).
\end{aligned}$$

By taking trace and keeping only the $\theta\overline{\theta}$ term, we get in the action a term

$$\mathrm{Tr}[V'(\phi)F] + \sum_{k=0}^{\infty} k g_k \sum_{a+b=k-2} \mathrm{Tr}(\phi^a \overline{\psi} \phi^b \psi).$$

Let's assume a quartic potential $V(\phi) = \frac{1}{2}\phi^2 + \frac{g}{4}\phi^4$,

$$\begin{aligned}
S[\Phi] &= -\mathrm{Tr}F^2 + \mathrm{Tr}(\phi F) + g\mathrm{Tr}(\phi^3 F) \\
&+ \mathrm{Tr}(\overline{\psi}\psi) + g\left[\mathrm{Tr}(\phi^2\overline{\psi}\psi) + \mathrm{Tr}(\phi\overline{\psi}\phi\psi) + \mathrm{Tr}(\overline{\psi}\phi^2\psi)\right].
\end{aligned}$$

Although it's easy to do the integral over F, we will not do that. Because it will lead to a non-linear supersymmetry transformation rule. However, let's define $F' = F - \frac{1}{2}\phi$ and rewrite the action in terms of F such that a quadratic term in ϕ will appear in the action

$$\begin{aligned}
S[\Phi] &= -\mathrm{Tr}F'^2 + \frac{1}{4}\mathrm{Tr}\phi^2 + \frac{g}{2}\mathrm{Tr}\phi^4 + g\mathrm{Tr}(\phi^3 F') \\
&+ \mathrm{Tr}(\overline{\psi}\psi) + g\left[\mathrm{Tr}(\phi^2\overline{\psi}\psi) + \mathrm{Tr}(\phi\overline{\psi}\phi\psi) + \mathrm{Tr}(\overline{\psi}\phi^2\psi)\right].
\end{aligned}$$

The supersymmetry variation reads

$$\begin{aligned}
\delta\phi &= \overline{\varepsilon}\psi + \overline{\psi}\varepsilon \\
\delta\psi &= -\varepsilon(F' + \frac{1}{2}\phi) \\
\delta\overline{\psi} &= -\overline{\varepsilon}(F' + \frac{1}{2}\phi) \\
\delta F' &= -\frac{1}{2}(\overline{\varepsilon}\psi + \overline{\psi}\varepsilon)
\end{aligned}$$

Think about applying the RG strategy on this action. If we denote $\phi_{i,N} = v_i, \phi_{N,i} = v_i^*$, the following term will appear

$$g v^\dagger F' \phi v + g v^\dagger \phi F' v$$

from the interaction $g\mathrm{Tr}(\phi^3 F')$. It will contribute a $\mathrm{Tr}(\phi F')$ term to the effective action. Why the supersymmetry cancellation does not happen here? This is because the supersymmetry transformation acts on all matrix entries simultaneously: it does not realize solely on the variables that being integrated out.

Idea 3. *Study the effect of a "local" supersymmetry transformation: that is, those only act on a part of the matrix.*

Idea 4. *Carry out the calculation anyway, to see what you get.*

For the RG strategy, let's decompose the matrix

$$F_0' = \begin{pmatrix} F' & f \\ f^\dagger & a \end{pmatrix}, \quad \phi_0 = \begin{pmatrix} \phi & v \\ v^\dagger & \alpha \end{pmatrix}$$
$$\psi_0 = \begin{pmatrix} \psi & \chi \\ \omega^\dagger & \beta \end{pmatrix}, \quad \overline{\psi}_0 = \begin{pmatrix} \overline{\psi} & \omega \\ \chi^\dagger & \overline{\beta} \end{pmatrix}$$

The interactions decompose correspondingly

$$\begin{aligned}
\frac{g}{2}\mathrm{Tr}\phi_0^4 &= \frac{g}{2}\mathrm{Tr}\phi^4 + 2gv^\dagger\phi^2 v + g(v^\dagger v)^2 + 2g\alpha v^\dagger\phi v + 2g\alpha^2 v^\dagger v + \frac{g}{2}\alpha^4 \\
g\mathrm{Tr}(\phi_0^3 F_0') &= g\mathrm{Tr}(\phi^3 F') + g(v^\dagger\phi^2 f + f^\dagger\phi^2 v) + g(v^\dagger\phi F' v + v^\dagger F'\phi v) \\
&\quad + g(\alpha v^\dagger\phi f + \alpha f^\dagger\phi v) + gav^\dagger\phi v + g\alpha v^\dagger F' v + g(v^\dagger v v^\dagger f + v^\dagger v f^\dagger v) \\
&\quad + g(\alpha^2 v^\dagger f + \alpha^2 f^\dagger v) + 2ga\alpha v^\dagger v + ga\alpha^3 \\
g\mathrm{Tr}(\phi_0^2\overline{\psi}_0\psi_0 + \overline{\psi}_0\phi_0^2\psi_0) &= g\mathrm{Tr}(\phi^2\overline{\psi}\psi + \overline{\psi}\phi^2\psi) + g(v^\dagger\overline{\psi}\psi v + v^\dagger\psi\overline{\psi}v) - g(\omega^\dagger\phi^2\omega - \chi^\dagger\phi^2\chi) \\
&\quad + g(\chi^\dagger\psi\phi v + v^\dagger\phi\overline{\psi}\chi - v^\dagger\phi\psi\omega - \omega^\dagger\overline{\psi}\phi v) \\
&\quad + g(\alpha\chi^\dagger\psi v - \alpha v^\dagger\psi\omega + \alpha v^\dagger\overline{\psi}\chi - \alpha\omega^\dagger\overline{\psi}v) \\
&\quad + g(\overline{\beta}\omega^\dagger\phi v + \overline{\beta}v^\dagger\phi\chi - \beta v^\dagger\phi\omega - \beta\chi^\dagger\phi v) \\
&\quad + g(v^\dagger\omega\omega^\dagger v - v^\dagger\chi\chi^\dagger v + v^\dagger v\chi^\dagger\chi - v^\dagger v\omega^\dagger\omega) \\
&\quad + g(\alpha\overline{\beta}\omega^\dagger v + \alpha\overline{\beta}v^\dagger\chi + v^\dagger\omega\beta\alpha + \chi^\dagger v\alpha\beta) \\
&\quad + g(2v^\dagger v\overline{\beta}\beta + \alpha^2\chi^\dagger\chi - \alpha^2\omega^\dagger\omega) + g\alpha^2\overline{\beta}\beta \\
g\mathrm{Tr}(\phi_0\overline{\psi}_0\phi_0\psi_0) &= g\mathrm{Tr}(\phi\overline{\psi}\phi\psi) + g(v^\dagger\overline{\psi}\phi\chi + \chi^\dagger\phi\psi v - v^\dagger\psi\phi\omega - \omega^\dagger\phi\overline{\psi}v) \\
&\quad + g(\alpha\chi^\dagger\phi\chi - \alpha\omega^\dagger\phi\omega + \overline{\beta}v^\dagger\psi v - \beta v^\dagger\overline{\psi}v) + g(\chi^\dagger v\omega^\dagger v + v^\dagger\omega v^\dagger\chi) \\
&\quad + g(\alpha\overline{\beta}\omega^\dagger v - \alpha\beta v^\dagger\omega + \alpha\overline{\beta}v^\dagger\chi - \alpha\beta\chi^\dagger v) + g\alpha^2\overline{\beta}\beta.
\end{aligned}$$

To the first order of g, the non-vanishing contributions are

$$2gv^\dagger\phi^2 v + g(v^\dagger\phi F' v + v^\dagger F'\phi v) - g(\omega^\dagger\phi^2\omega - \chi^\dagger\phi^2\chi).$$

The quadratic terms in the exponential are $-\frac{1}{2}(v^\dagger v)$ and $-\chi^\dagger\chi + \omega^\dagger\omega$. Then the effective interaction to the first order of g is

$$\frac{2g}{N}\mathrm{Tr}\phi^2 + \frac{4g}{N}\mathrm{Tr}(F'\phi).$$

The effective action becomes

$$S_{\text{eff}} = -\mathrm{Tr}F'^2 + \left(\frac{1}{4} + \frac{2g}{N}\right)\mathrm{Tr}\phi^2 + \frac{g}{2}\mathrm{Tr}\phi^4 + \frac{4g}{N}\mathrm{Tr}F'\phi + g\mathrm{Tr}\phi^3 F' + \cdots \tag{7}$$

where $\cdots$ are those involving $\overline{\psi}, \psi$, which does not change to this order. The new term $\mathrm{Tr}F'\phi$ could be absorbed by a redefinition of F'

$$\tilde{F}' = F' - \frac{2g}{N}\phi.$$

Then one could write the effective action as

$$S_{\text{eff}} = -\mathrm{Tr}\tilde{F}'^2 + \left(\frac{1}{4} + \frac{2g}{N}\right)\mathrm{Tr}\phi^2 + \frac{g}{2}\mathrm{Tr}\phi^4 + g\mathrm{Tr}\phi^3\tilde{F}' + \cdots \tag{8}$$

where the correction of the order g^2 is ignored. Is it possible to take it back into the original form by rescaling g, ϕ? It's impossible without a rescaling of $\tilde{F}'$. But then the term $-\mathrm{Tr}\tilde{F}'$ is modified. In this sense, the RG flow is even not well-defined.

Idea 5. *There are two ways through which the supersymmetry could help us in the calculation: 1. build up a model in which the supersymmetry is realized "locally"; 2. develop another RG method which is compatible with the global supersymmetry;*

We focus on susy because we believe it's a crucial ingredient to build a conformal matrix model.

notes about ✎ **BFFGLM21**: Tue, Mar 5

The paper consider a D(-1)/D7-brane system in the Type II B string theory with a magnetic flux along the world-volume of the D7-brane. There are $(N+M)$ D7-branes stacked along the first 8 directions $\mu = 1, \ldots, 8$. A constant background field is introduced on the first N D7-branes in the following way

$$2\pi\alpha' F^{(0)} = \begin{pmatrix} F_1^{(0)} & 0 & 0 & 0 \\ 0 & F_2^{(0)} & 0 & 0 \\ 0 & 0 & F_3^{(0)} & 0 \\ 0 & 0 & 0 & F_4^{(0)} \end{pmatrix} \mathbb{1}_{N\times N}.$$

With $F_i^{(0)} = \begin{pmatrix} 0 & +f_1 \\ -f_1 & 0 \end{pmatrix}$. The initial gauge symmetry group $U(N+M)$ is broken to $U(N)\times U(M)$ by this constant background. It also breaks the Lorentz invariance in the 8-dimensional space in general.

The first N D7-branes are labeled by D7; while the M D7-branes are labeled by D7'.

Then a stack of k D(-1)-branes are added to study some non-perturbative features in the effective theory defined on the world-volume of the D7 and D7' branes. Now we study the physical states of the open strings stretching between two D(-1)-branes. Because they have Dirichlet/Dirchlet boundary conditions in all ten directions they do not carry any momentum and describe non-propagating degrees of freedom.

For the open string, the fermionic world-sheet fields $\psi^\mu(z), \tilde{\psi}^\mu(\bar{z})$ have two sectors: Neveu-Schwarz and Ramond. To be specific, in the coordinate $(\sigma_1, \sigma_2), 0 \le \sigma_1 \le \pi$

the possible boundary conditions are

$$\psi^\mu(0,\sigma_2) = \exp(2\pi i\nu)\tilde{\psi}^\mu(0,\sigma_2) \quad \psi^\mu(\pi,\sigma_2) = \exp(2\pi i\nu')\tilde{\psi}^\mu(\pi,\sigma_2). \tag{9}$$

where ν, ν' take the values 0 and $\frac{1}{2}$. However, ν' can be set to zero $\nu' = 0$ by a redefinition of $\tilde{\psi}^\mu \to \exp(-2\pi i\nu')\tilde{\psi}^\mu$. Therefore, there are two sectors: $\nu = 0$ for Ramond and $\nu = \frac{1}{2}$ For Neveu-Schwarz.

The physical states vertex operators are written down in the following ways. The space-time indices are $I = 1, 2, 3, 4$ for the 8-dimensions in four complex coordinates. The remaining 2-dimensions are just labeled by different letters.

The physical states in the NS sector under consideration are those excited by one fermionic oscillator. The vertex operators are written as

$$V_{B^I} = \frac{B^I}{\sqrt{2}}\overline{\Psi}_I(z)\mathrm{e}^{-\varphi(z)}, \quad V_{\overline{B}_I} = \frac{\overline{B}_I}{\sqrt{2}}\Psi^I(z)\mathrm{e}^{-\varphi(z)}$$
$$V_\xi = \xi\overline{\Psi}_5(z)\mathrm{e}^{-\varphi(z)}, \quad V_\chi = \chi\Psi^5(z)\mathrm{e}^{-\varphi(z)}.$$

Ψs are the fermionic string coordinates; the $\mathrm{e}^{-\varphi(z)}$ is the bosonization of the superconformal ghost operator $\delta(\gamma)$. This form of the vertex operator is called in the (-1)-picture. These vertex operators are conformal fields of dimension 1, and possess an even F-charge; thus they are preserved by the GSO projection.

Problem 3. *The vertex operators are objects that are defined for the world-sheet CFT. The "moduli" B, ξ, χ determine the strength of the vertex operators. The effective action is written down in terms of these "moduli".*

The structure of the effective action is obtained by matching the calculation of the scattering amplitudes using the vertex operators. However, the world-sheet point of view seems quite different from the effective action point of view. Is it worth to review the world-sheet calculation to get some understanding of the structure of the effective action? Maybe first describing the scattering process in string theory?

In the R sector, the only physical state is the vacuum. (What's the meaning of vacuum here?)

Wed, Mar 6

TODO 1. *Learn something about the D1/D5 CFT.*

Such a D5/D1 system is described in ✎ **Mal97** as the following: Consider IIB string theory compactified on M^4 (where $M^4 = T^4$or$K3$) to six spacetime dimensions. Introduce a D5-brane with four dimensions wrapping on M^4 giving a string in six dimensions. Consider a system with Q_5 D5-branes and Q_1 D1-branes, where the D1-branes are parallel to the string in six dimensions arising from the D5-branes. This system is described at low energies by a 1+1 dimensional (4,4) superconformal field theory.

Problem 4. *Why such a D-brane system has the (4,4) supersymmetry? Why the low energy field theory is conformal?*

TODO 2. *Read* ✎ **Wit95** *for the CFT description of p-branes.*

The symmetry of this system is discussed as follows ✎ **AGMOO99**. The D1-branes along one non-compact direction; The D5-branes extend in the same direction; and they are coincident in that direction. The unbroken Lorentz symmetry of this configuration is $SO(1,1) \times SO(4)$ $SO(1,1)$ corresponds to boosts along the string, and $SO(4)$ is the group of rotations in the four non-compact directions transverse to the string. This configuration also preserves eight supersymmetries, actually $\mathcal{N} = (4,4)$ supersymmetry. We decompose the supercharges into left and right moving spinors of $SO(1,1)$.

What's the conformal field theory on this system? ✎ **AGMOO99**. The conformal field theory is the IR fixed point of the field theory living on the D1-D5 branes. The symmetry of this theory before going into the IR fixed point is the $\mathcal{N} = (4,4)$ supersymmetry. This amount of supersymmetry is equivalent to $\mathcal{N} = 2$ in $d = 4$. There is a vector multiplet and a hypermultiplet. In two dimensions both multiplets have the same propagating degrees of freedom, four scalars and four fermions, but they have different properties under the $SU(2)_L \times SU(2)_R$ global R-symmetry. Under this group, the scalars in the hypermultiplets are in the trivial representation, while the scalars in the vector multiplet are in the $(\mathbf{2},\mathbf{2})$. On the fermions these global symmetries act chirally. The left moving vector multiplet fermions are in the $(\mathbf{1},\mathbf{2})$ and the left moving hypermultiplet fermions are in the $(\mathbf{2},\mathbf{1})$. The right moving fermions have similar properties with $SU(2)_L \leftrightarrow SU(2)_R$. The theory can have a Coulomb branch where the scalars in the vector multiplets have expectation values, and a Higgs branch where the scalars in the hypermultiplets have expectation values.

Question 5. *A better understanding of the above discussion.*

- *the supermultiplet structure*
- *the global R-symmetry structure*
- *the IR fixed point*
- *the Coulomb branch and the Higgs branch*

How to interpret this CFT? ✎ **AGMOO99**. The D1-branes can be viewed as instantons of the low energy SYM on the D5-brane. These instantons live on M^4 and are translationally invariant along the time and the x_5 direction, where x_5 is the non-compact direction along the D5-branes. This instanton configuration has moduli, which are the parameters that parameterize a continuous family of solutions. All these solutions have the same energy. Small fluctuation of this configuration (at low energy) are described by fluctuations of the instanton moduli. So the low energy dynamics is given by a 1+1 dimensional sigma model whose target space is the instanton moduli space.

Question 6. *Some questions related to the above paragraph:*

- *What is the instanton of the SYM on D5-brane?*
- *Why the D1-brane can be viewed as those instantons?*
- *What's the meaning of the fluctuation of the moduli?*
- *What's the meaning of a 2d sigma model with the moduli target space?*
- *Is this sigma model the 2d SCFT discussed above?*

TODO 3. *Read the papers that study the D1-D5 system:* **Cos98**, **HW97-a**, **HW97-b**

More on D5-brane bound state: a bound state of two D5-branes wrapped on $S^1 \times T^4$ with coordinates $x^1, \cdots, x^5$. **Cos98** Each D5-brane has winding number N_i along S^1, p_i along the x^2-direction and $\overline{p}_i$ along the x^4-direction. Thus, the worldvolume fields take values on the $U(N_1 p_1 \overline{p}_1 + N_2 p_2 \overline{p}_2)$ Lie algebra. To get a non-trivial D5-brane configuration we turn on the worldvolume gauge field such that the corresponding field strength is diagonal and self-dual on T^4. The non-vanishing components are taken to be (assume $\tan\theta_1 > \tan\theta_2$)

$$G^0_{23} = G^0_{45} = \frac{1}{2\pi\alpha'} \mathrm{diag}\left(\tan\theta_1, \cdots, \tan\theta_1, \tan\theta_2, \cdots, \tan\theta_2\right).$$

where

$$\frac{1}{2\pi\alpha'}\tan\theta_i = \frac{2\pi}{L_2 L_3}\frac{q_i}{p_i} = \frac{2\pi}{L_4 L_5}\frac{\overline{q}_i}{p_i}.$$

with $q_i, \overline{q}_i$ are integers, and $L_{\hat{\alpha}} = 2\pi R_{\hat{\alpha}}$ the length of each T^4 circles.

Question 7. *People are discussing the D5-brane configuration on a compact manifold, with certain background gauge field.*

- *Why the background gauge field is necessary for the discussion?*
- *What is the G^0 field? Why the number of components are related to the D5-brane winding number?*
- *What's special about the value of* $\tan\theta$*?*

The background gauge field breaks the gauge invariance

$$U(N_1 p_1 \overline{p}_1 + N_2 p_2 \overline{p}_2) \to U(N_1 p_1 \overline{p}_1) \otimes U(N_2 p_2 \overline{p}_2).$$

Because the branes are wrapped along the x^1, x^2, x^4 directions, the gauge invariance is further broken to $U(1)^{\cdots}$.

Thu, Mar 7

The Dp-brane is dynamical: oscillating by itself (described by open strings ending on the Dp-brane) and scattering with the closed strings. Let's first review some physics of the Dirichlet brane in the context of the worldsheet string theory.

Problem 5. *Understand the D-brane from the T-duality. Refer to the Polchinski.*

A freely moving open string could be restricted to a hyperplane in the T-dual picture of the toroidal compactification. The oscillation of the open string is then interpreted as the oscillation of the hyperplane in the T-dual picture.

Problem 6. *An example of the T-duality in the toroidal compactification.*

The target-space duality (T-duality) is an equivalence between two string theories with different target spaces. The simplest example is in the context of the toroidal compactification: a certain direction in the target space is periodic

$$X \simeq X + 2\pi R.$$

The closed string and the open string have different dynamics along the periodic direction. An extreme case the $R \to 0$ limit: the closed string behaves exactly the same as $R \to \infty$; the open string has no motion along that direction. One can see this by checking the mass spectrum in terms of the momentum (and other numbers).

The bosonic closed strings with such a target space has the following mass spectrum

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2) \tag{10}$$

Here $n \in \mathbb{Z}$ labels the quantized momentum along the periodic direction; $w \in \mathbb{Z}$ is the closed string winding number. $N, \tilde{N}$ are the levels the left-moving and right-moving string oscillators respectively. The interesting point is that the spectrum is invariant under

$$R \to R' = \frac{\alpha'}{R}, \quad n \to w.$$

In particular, the $R \to 0$ and $R \to \infty$ limits are physically identical.

The bosonic open strings do not have the winding number, but its spectrum can be modified by a flat background for the gauge field (the Wilson line). The Wilson line will have a very interesting explanation in the T-dual picture.

Consider the open strings with $U(n)$ Chan-Paton factors. The background gauge field along the periodic direction is assumed to be

$$A = -\frac{1}{2\pi R}\mathrm{diag}(\theta_1, \cdots, \theta_n).$$

This is diagonal. (It may be interesting to consider non-diagonal case.) Then the open string spectrum has the form

$$m^2 = \frac{(2\pi l - \theta_j + \theta_i)^2}{4\pi^2 R^2} + \frac{1}{\alpha'}(N - 1). \tag{11}$$

Remember the general states of open strings with the Chan-Paton factor transforming under the adjoint representation of the gauge group. The gauge field couples to the Chan-Paton factor as $[A, \lambda]$. This is the reason for the $(\theta_j - \theta_i)$ part of the spectrum.

Now one can see that the open string spectrum has a different $R \to 0$ behavior. There is no counterpart of the winding number wR that leading to a continuum spectrum. So the mass gap between $l = 0$ and $l = 1$ will tend to infinity as $R \to 0$. Roughly speaking, in the $R \to 0$ limit, the open string moves in the 25 spacetime dimension ($l = 0$). This requires that, in the dual picture $R' \to \infty$, to keep the open string spectrum the same, the end points must be fixed along this direction.

Problem 7 (T-duality)**.** *How the T-duality is realized in the worldsheet theory?*

First separate the scalar field $X(z, \bar{z}) = X_L(z) + X_R(\bar{z})$. Then define a new field $X'(z, \bar{z})$

$$X'(z, \bar{z}) = X_L(z) - X_R(\bar{z}).$$

One claims that the worldsheet CFT using X' and X are T-dual to each other. This further means that the Neumann condition and the Dirichlet condition are exchanged under the T-duality

$$\partial_n X = -i\partial_t X'.$$

where n is the normal and t is the tangent at the boundary.

Question 8. *This is not obvious: the T-duality is formulated for the periodicity of the target space; How this can be formulated in an equivalent way in terms of the worldsheet field?*

The string dilaton Φ will change under the T-duality. This bases on the following argument: the gravitational coupling κ is related to the dilaton field

$$\kappa \propto \mathrm{e}^{\Phi}.$$

In the toroidal compactification, the gravitational couplings in different dimensiona are related by

$$\frac{1}{\kappa_d^2} = \frac{2\pi\rho}{\kappa^2}.$$

with $2\pi\rho$ being the volume of the compactified dimension. The point is that the lower dimensional gravitational coupling κ_d should be invariant under the T-duality. It captures the physics that is un-related to the periodic dimension. This implies that κ, and therefore Φ, must change under the T-duality $\rho \to \rho'$.

$$\rho' = \frac{\alpha'}{\rho}, \quad \kappa' = \frac{\alpha'^{1/2}}{\rho}\kappa. \tag{12}$$

TODO 4 (D-brane in type II superstring)**.** *Understand the physics of D-brane and its implication in the superstring theory.*

TODO 5 (the superstring spectrum)**.** *There are many different kinds of superstrings: try to summarize them.*

The worldsheet fields: $X, \psi, \tilde{\psi}$; The worldsheet theory of the superstring starts from a $(N, \tilde{N}) = (1, 1)$ SCFT. It may be useful to keep in mind the space-time interpretation. The center-of-mass modes of the worldsheet current $(\partial X^\mu, \overline{\partial} X^\mu)$ is the space-time momenta p^μ. The center-of-mass modes of the worldsheet fermions $\psi_0^\mu, \tilde{\psi}_0^\mu$ is the gamma matrices Γ^μ. The supercurrents T_F, T_B then has a similar form with the Dirac operator and the Klein-Gordon operator. (Polchinski 10.1)

Question 9. *Why there are two copies* $(\partial X, \overline{\partial} X)$ *and* $\psi, \tilde{\psi}$*: what's the space-time interpretation?*

The superconformal ghosts: b, c, β, γ.

The spectrum of the $X\psi$ SCFT: the R and the NS sector. The vertex operators for the spectrum.

The superconformal ghosts: the spectrum and the vertex operators.

The physical states and the consistent superstring theories. The superconformal constranits and the BRST formalism. Type IIA, Type IIB and Type I $SO(32)$.

Fri, Mar 8

Sat, Mar 9

TODO 6. *Review the geometry of the Dp-branes: focus on the relation between the supergravity solution and the physical properties of the Dp-branes.*

Keep in mind the following questions:

- *How the dilaton field profile depends on the dimension?*
- *The geometry of the solution: the curvature and the area of horizon.*
- *Comment on the validity regime of the solution.*

A particular type IIB supergravity solution is proved to be sourced by Dp-branes, which carry the R-R charge and preserve half of the supersymmetry. Let's try to understand this, starting by looking at the supergravity theory.

The type IIB supergravity provides a field theory description of the massless states of the corresponding superstring theory. The massless states are summarized in the following table: (TODO)

Question 10. *About the Type IIB supergravity:*

- *How to construct the action?*
- *How the couplings are related to the string couplings and the string length scale?*
- *How could it reproduce the string interaction?*
- *What's the supersymmetry algebra?*
- *How the fields transform under the supersymmetry?*
- *Clarify that "the supergravity theory captures the low energy dynamics of the superstrings in the strong coupling regime?"*

The bosonic part of the type IIB supergravity ✎ **Joh03**

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_0^2}\int d^{10}x(-G)^{1/2}\Big\{e^{-2\Phi}\left[R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{12}(H^{(3)})^2\right] \\ -\frac{1}{12}(G^{(3)} + C^{(0)}H^{(3)})^2 - \frac{1}{2}(dC^{(0)})^2 - \frac{1}{480}(G^{(5)})^2\Big\} \\ +\frac{1}{4\kappa_0^2}\int\left(C^{(4)} + \frac{1}{2}B^{(2)}C^{(2)}\right)G^{(3)}H^{(3)}. \tag{13}$$

$H^{(3)} = dB^{(2)}$ and Φ, G are in the NS-NS sector; $G^{(3)} = dC^{(2)}$, $G^{(5)} = dC^{(4)} + H^{(3)}C^{(2)}$ and $C^{(0)}$ are in the R-R sector. The normalization of the kinetic terms of the forms: there is a prefactor of the inverse of $-2\times p!$ for a p-form field strength. There is a self-dual condition that need to be imposed by hand

$$F^{(5)} = dC^{(4)}, \quad F^{(5)} = *F^{(5)}.$$

to keep the correct number of degrees of freedom.

One can split this action into three parts

$$S_{\mathrm{IIB}} = S_{\mathrm{NS}} + S_{\mathrm{R}} + S_{\mathrm{CS}}.$$

The last part is the Chern-Simon action. (Check the gauge invariance.) The S_{NS} is written in the string frame; while there is no dilaton coupling in S_{R}. It's interesting to understand more about the modified strength $G^{(5)} = dC^{(4)} + H^{(3)}C^{(2)}$.

TODO 7. *Write down the equation of motion of this action. (Maybe it's easier to do it in the Einstein's frame?)*

Sun, Mar 10

Question 11. *In* [**DKL94**], *two kinds of solution was discussed: the elementary solution and the topological solution. The elementary p-brane solution carries the "electric" Noether charge e_d; The topological $\tilde{p}$-brane solution carries the "magnetic" charge $g_{\tilde{d}}$. There is a Dirac quantization rule $e_d g_{\tilde{d}} = 2\pi n$. How to understand these two dual solutions? Whether there is a singularity in these two solutions?*

The action used in [**DKL94**]. Consider an antisymmetry tensor potential of rank d, $A_{M_1 M_2 \cdots M_d}$, in D dimensional space-time $M = 0, \cdots, (D-1)$. The field strength is $F_{d+1} = dA_d$. The following action captures the interaction between the gravity g_{MN} and the dilaton ϕ:

$$I_D(d) = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(d+1)!} e^{-a(d)\phi} F_{d+1}^2 \right). \qquad (14)$$

The $a(d)$ is an yet undetermined constant.

Let's compare it with the NS-NS sector of the type IIB supergravity action written above [**Joh03**]

$$S_{\text{IIB,NS-NS}} = \frac{1}{2\kappa_0^2} \int d^{10}x (-G)^{1/2} e^{-2\Phi} \left[R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{12}(H^{(3)})^2 \right].$$

One notices that the dilaton field coupling is different. One may take one form to another by the Weyl transformation on the metric:

Let's consider a Wely transformation $G \to \tilde{G}$

$$\tilde{G}_{\mu\nu}(x) = \exp(2\omega(x)) G_{\mu\nu}(x). \qquad (15)$$

One has the following formula for the Ricci scalar

$$\tilde{R} = \exp(-2\omega) \left[R - 2(D-1)\nabla^2\omega - (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega \right]. \qquad (16)$$

where D is the space-time dimension.

Let's write the $S_{\text{IIB,NS-NS}}$ in terms of the $\tilde{G}$ and $\tilde{R}$.

$$(-G)^{1/2} e^{-2\Phi} R = (-\tilde{G})^{1/2} e^{-2\Phi - D\omega} \left[e^{2\omega}\tilde{R} + 2(D-1)\nabla^2\omega + (D-2)(D-1)\partial_\mu\omega\partial^\mu\omega \right].$$

Then one can choose

$$\omega = \frac{2}{2-D}\Phi.$$

to decouple R with the dilaton Φ. With this choice

$$(-G)^{1/2} e^{-2\Phi} R = (-\tilde{G})^{1/2}\tilde{R} + (-\tilde{G})^{1/2} e^{-\frac{4}{2-D}\Phi} \left[\frac{4(D-1)}{2-D}\nabla^2\Phi - 2(D-1)\partial_\mu\Phi\partial^\mu\Phi \right].$$

$D = 2$ is special in the sense that one can not decouple the dilaton field by choosing the Wely scaling factor $\omega(x)$. Now the dilaton field terms in the action

$$(-\tilde{G})^{1/2} e^{-\frac{4}{2-D}\Phi} \left[(6-2D)\partial_\mu \Phi \partial^\mu \Phi + \frac{4(D-1)}{2-D} \nabla^2 \Phi \right].$$

Then one may use the following identity

$$e^{-\frac{4}{2-D}\Phi} \cdot \frac{4(D-1)}{2-D} \nabla^2 \Phi = \frac{16(D-1)}{(2-D)^2} e^{-\frac{4}{2-D}\Phi} \partial_\mu \Phi \partial^\mu \Phi - (D-1) \nabla^2 e^{-\frac{4}{2-D}\Phi}.$$

to get rid of $\nabla^2 \Phi$ term. The last term on the r.h.s. is a boundary term.

Question 12. *We will get a term*

$$-2 \frac{D^3 - 7D^2 + 8D - 4}{(2-D)^2} e^{-\frac{4}{2-D}\Phi} \partial_\mu \Phi \partial^\mu \Phi.$$

How to get the usual kinetic term $-\frac{1}{2}\partial_\mu \tilde{\Phi} \partial^\mu \tilde{\Phi}$ *out of it?*

It's important to note that the space-time indices μ should be raised using the new metric $\tilde{G}^{\mu\nu}$. $G^{\mu\nu} = e^{2\omega} \tilde{G}^{\mu\nu} = e^{4/(2-D)\Phi} \tilde{G}^{\mu\nu}$. This cancels the exponent factor before $\partial_\mu \Phi \partial^\mu \Phi$. However, for the gauge field term $(H^{(3)})^2$, there are three indices to raise which leads to the following dilaton coupling

$$-\frac{1}{12} e^{\frac{8}{2-D}\Phi} (H^{(3)})^2.$$

The NS-NS sector ϕ, g_{MN} and $A_{M_1 M_2 \cdots M_d}$ could be sourced by a $(d-1)$-brane. One could write down the action that describing how the brane couples to the fields. (TODO)

Question 13. *First, the* $(d-1)$*-brane that coupling to the NS-NS sector is not the Dirichlet brane (confirm this point); because the D-brane sources the R-R sector instead of the NS-NS sector.*

However, in the action that is used in [**DKL94**]*; it's not clear that the form field* $A_{M_1 M_2 \cdots M_d}$ *is in the NS-NS sector or the R-R sector, or could be suitable for both sectors.*

The variation of $I_D(d)$ with respect to $\delta A^{M_1 \cdots M_d}, \delta g^{MN}$ and $\delta \phi$

The convention for the forms:

$$(F_{d+1})_{N M_1 \cdots M_d} = (d+1) \partial_{[N} A_{M_1 \cdots M_d]}.$$

with

$$\partial_{[N} A_{M_1 \cdots M_d]} = \frac{1}{d+1} \left(\partial_N A_{M_1 \cdots M_d} + \partial_{M_1} A_{M_2 \cdots N} + \cdots + \partial_{M_d} A_{N \cdots M_{d-1}} \right).$$

The variation of the form action with respect to $\delta A^{M_1\cdots M_d}$ is

$$\begin{aligned}\delta\left[e^{-a\phi}F_{d+1}^2\right] &= 2e^{-a\phi}(F_{d+1})_{NM_1\cdots M_d}\delta(F_{d+1})^{NM_1\cdots M_d}\\ &= \partial^N\left[e^{-a\phi}F_{NM_1\cdots M_N}\right]\delta A^{M_1\cdots M_N}+\nabla^N(\cdots)_N\end{aligned}$$

The last term contributes to a boundary term. Then the equation of motion reads

$$\partial^N\left(e^{-a\phi}F_{NM_1\cdots M_N}\right)=\cdots \tag{17}$$

The r.h.s. comes from the possible source term.

To do the variation of δg^{MN}, the following formula are necessary

$$\delta\sqrt{-g}=-\frac{1}{2}\sqrt{-g}g_{MN}\delta g^{MN} \tag{18}$$

and

$$\delta R=R_{MN}\delta g^{MN}+\nabla_P(g^{MN}\delta\Gamma^P_{MN}-g^{NP}\delta\Gamma^M_{MN}) \tag{19}$$

Calculations:

$$-\frac{1}{2}\delta\left[\sqrt{-g}(\partial\phi)^2\right]=-\frac{1}{2}\sqrt{-g}\left[-\frac{1}{2}g_{MN}(\partial\phi)^2+\partial_M\phi\partial_N\phi\right]\delta g^{MN}.$$

and

$$\begin{aligned}&-\frac{1}{2(d+1)!}\delta\left(\sqrt{-g}e^{-a(d)\phi}F_{d+1}^2\right)\\ =&-\frac{\sqrt{-g}}{2}\frac{1}{d!}\left(F_{MM_1\cdots M_d}F_N{}^{M_1\cdots M_d}-\frac{1}{2(d+1)}g_{MN}F_{d+1}^2\right)e^{-a(d)\phi}\delta g^{MN}\end{aligned}$$

The variation of the Ricci scalar part simple gives the Einstein tensor. The equation of motion reads then

$$\begin{aligned}\sqrt{-g}\Bigg\{&R_{MN}-\frac{1}{2}g_{MN}R-\frac{1}{2}\left[\partial_M\phi\partial_N\phi-\frac{1}{2}g_{MN}(\partial\phi)^2\right]\\ &-\frac{1}{2(d!)}\left[F_{MM_1\cdots M_d}F_N{}^{M_1\cdots M_d}-\frac{1}{2(d+1)}g_{MN}F_{d+1}^2\right]e^{-a(d)\phi}\Bigg\}=\cdots\end{aligned}$$

The r.h.s. comes from the possible source terms.

Mon, Mar 11

TODO 8. *Get some ideas on the "emergent geometry" from the IKKT model.*

The simplest example is the semicircle law of the Gaussian matrix model. If one take the eigenvalue distribution $\rho(\lambda)$ as the distribution of "points" that constitute the geometry.

Question 14. *Another notion (closely related) of emergent geometry arises from the continuum limit of the matrix model, which is not simply $N \to \infty$ but also requiring the "free energy" develop scaling behavior near certain critical point.*

Clarfiy this.

Questions and Ideas in [**IMSY98**]:

Tue, Mar 12

Question 15. *It's counter-intuitive that a large number of branes corresponds to a small curvature solution of supergravity. Is there any physical interpretation?*

The radial coordinate r transversal to the Dp-branes relates to the energy scale of the field theory on the Dp-brane. From a field theory perspective, the transverse coordinates should be interpreted as the vaccum expectation value of the scalar fields that describing the transverse oscillation of the Dp-branes.

Question 16. *About the scalar fields:*

- *What's the form of the action for those scalar fields?*
- *What's the dimension of those scalar fields?*
- *What's the interpretation of the scalar fields? (Higgs? Goldstone?)*
- *The diagonal and off-diagonal components of the scalar fields (matrix valued in N-coincident Dp-branes) plays different roles?*

The argument given in [**IMSY98**] to relate r with the energy scale:

Consider a D-brane sitting at the position r. This will lead to a gauge symmetry broken $U(N) \to U(N-1) \times U(1)$. Large r will correspond to a large vacuum expectation value of the Higgs field (What's the Higgs field here?). The expectation value has the dimension of energy (clarify this) (recall that the energy dimension of a scalar field is $[\varphi] = (d-2)/2$) Therefore, large r corresponds to a large energy scale.

To have a picture about the couplings that are involved in the discussion, let's review the result of the tension of Dp-branes from string theory. Let's start by looking at the Dp-brane action (Polchinski (8.7.2))

$$S_p = -T_p \int d^{p+1}\xi e^{-\Phi} \left[-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})\right]^{\frac{1}{2}} \tag{20}$$

When there is a constant background dilaton field $\Phi = \Phi_0 + \tilde{\Phi}$, the tension T_p will be modified to $\tau_p = T_p e^{-\Phi_0}$. One could calculate the interaction amplitude (one loop) in string theory, the result is (Polchinski (13.3.1))

$$\mathcal{A}_{\text{NS-NS}} \approx iV_{p+1} 2\pi (4\pi^2\alpha')^{3-p} G_{9-p}(y). \tag{21}$$

The $G(y)$ is the scalar Green's function, y is the separation between two Dp-branes. The R-R amplitude cancels with the NS-NS amplitude $\mathcal{A}_{\text{R-R}} = -\mathcal{A}_{\text{NS-NS}}$. One the other hand, the same thing can be calculated by using the field theory. (I don't know the detail here) The result relates the slope α' with the Dp-brane tension τ_p and the supergravity coupling κ (Polchinski (13.3.4)).

$$\tau_p^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^{3-p}. \tag{22}$$

The supergravity coupling κ is a low energy parameter to describe the closed string interactions. The closed string coupling g_s is defined as a normalization of the closed string vertex operator (Polchinski (3.6.1)). It relates to the gravitational coupling κ in the following way (Polchinski (12.3.47))

$$g_s = \frac{\kappa}{2\pi}.$$

In this way, the Dp-brane tension τ_p relates directly to the closed string coupling g_s. Let's define the low energy (the lowest order of α') field theory on the Dp-brane from the action S_p. It's a Yang-Mills theory, the coupling is (Polchinski (13.3.25))

$$g_{D_p}^2 = \frac{1}{(2\pi\alpha')^2\tau_p}.$$

All of these allow us to relate the Yang-Mills coupling $g_{D_p}^2$ (or g_{YM}^2) to the closed string coupling g_s. The result is the (1) in [**IMSY98**]

$$g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{(p-3)/2} \tag{23}$$

The open string theory on the Dp-brane can be taken as a field theory when $\alpha' \to 0$. One could take the limit in such a way that the g_{YM} keeps fixed. This is called the "field theory limit" in [**IMSY98**].

Question 17. *It seems like that we don't have a fundamental understanding of the closed string coupling g_s in terms of the slope α'. The lack of knowledge about g_s is related to the unkown vacuum value of the dilaton field ϕ_0?*

Question 18. *What's the open string theory on the Dp-branes? Type I or Heterotic?*

Let's take the example of a collection of D2-branes in [**IMSY98**]. The energy scale of the field theory is set by the following value (This is general, not depending on the dimension of the branes)

$$U = \frac{r}{\alpha'}.$$

In terms of the field theory, this is vacuum expectation value of the Higgs.

Question 19. *Why this particular combination $U = r/\alpha'$ appears as the vacuum expectation value of the Higgs? Why this combination does not depend on the dimension of the field theory?*

One needs to make difference between the dimension of the string theory and the dimension of the field theory. For example, in string theory, α' has the unit of $[\text{length}]^2$, here the length should be understood as the space-time length. Then it's obvious that U has nothing to do with the energy of the space-time. Also, let's look at the Yang-Mills coupling g_{YM}^2. We know from the field theory that it's dimension is $[\text{energy}]^{3-p}$. This has nothing to do with the space-time dimension reading from $g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{(p-3)/2}$.

TODO 9. *It's desirable to have a clear discussion of various dimensions: from a space-time perspective and from the worldvolume field theory perspective.*

It is said that the effective dimensionless coupling of the super-Yang-Mills theory at the energy scale U is

$$g_{eff}^2 \approx g_{YM}^2 N U^{p-3}.$$

This is the coupling constant according to which we do the perturbative calculation in field theory. The N appears in the effective coupling because we are interested in such a large N scaling: the coupling g_{YM} changing with N such that $g_{YM}^2 N$ keeps fixed.

It's necessary to clarify the different limits of string theory that appearing in the AdS/CFT duality. First, on both sides of the duality we have field theories, which in general are the low energy limit of string theories $\alpha' \to 0$. However, remember that $[\alpha'] = [L]^2$, one should always compare α' with some other scales.

The AdS side of the duality is essentially a theory in which the string moving in a geometry background. Such a theory is in general described by a non-linear sigma model which generalizes the Polyakov action (Pol (3.76))

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_M d^2\sigma g^{1/2} \left[\left(g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X) \right) \partial_a X^\mu \partial_b X^\nu + \alpha' R \Phi(X) \right] \tag{24}$$

This model could have a field theory description in the following limit: Consider the target space has a characteristic radius of curvature R_c. Then the effective dimensionless coupling in this theory is $\alpha'^{\frac{1}{2}} R_c^{-1}$. If it is small, one can ignore the extended structure of string instead using a field theory description of the string states. This leads to the use of the "low energy effective action". For example (Pol (3.7.20) for the bosonic string)

$$\mathbf{S} = \frac{1}{2\kappa_0^2} \int d^D x (-G)^{1/2} e^{-2\Phi} \left[-\frac{2(D-26)}{3\alpha'} + R - \frac{1}{12} H_{(3)}^2 + 4(\partial\Phi)^2 + O(\alpha') \right] \tag{25}$$