

# Contents

<b>1 Basic</b>	
1.1 Default code	1
1.2 Linux 對拍	1
1.3 Windows 對拍	1
1.4 builtin 函數	1
1.5 輸入輸出	1
1.6 Python 輸入輸出	1
<b>2 Data Structure</b>	
2.1 Link-Cut Tree	2
2.2 持久化線段樹	2
2.3 Treap	2
2.4 線段樹	3
<b>3 Flow</b>	
3.1 Dinic	3
3.2 匈牙利	3
3.3 KM	4
3.4 MCMF	4
<b>4 幾何</b>	
4.1 宣告	5
4.2 矩形面積	5
4.3 最近點對	5
4.4 凸包	6
4.5 兩直線交點	6
4.6 兩線段交點	6
4.7 李超線段樹	6
4.8 最小包圍圓	7
4.9 最小包圍球	7
4.10 旋轉卡尺	7
4.11 Circle Cover	7
4.12 Convex Hull Trick	7
4.13 Half Plane Intersection	7
<b>5 圖論</b>	
5.1 BCC	9
5.2 重心剖分	9
5.3 輕重鍊剖分	9
5.4 歐拉路徑	9
5.5 極大團	10
5.6 最大團	10
5.7 SCC	10
5.8 SPFA	10
5.9 樹哈希	10
5.10 差分約束	10
<b>6 數論</b>	
6.1 離散根號	12
6.2 離散對數	12
6.3 ex-crt	12
6.4 ex-gcd	12
6.5 FFT	12
6.6 高斯消去法	12
6.7 喬瑟夫問題	12
6.8 定理	12
6.9 Miller Rabin	13
6.10 NTT	13
6.11 Pollard's Rho	13
6.12 質數	14
6.13 phi	14
6.14 矩陣快速幂	14
<b>7 字串</b>	
7.1 KMP	15
7.2 馬拉車	15
7.3 回文樹	15
7.4 SA	15
7.5 SAM	15
7.6 trie	16
7.7 Z-value	16
7.8 minRotation	16
<b>8 DP</b>	
8.1 數位 dp	16
8.2 SOS dp	16
8.3 p-median	16
<b>9 Other</b>	
9.1 黑魔法	17
9.2 DLX	17
9.3 Hiber Curve	17
9.4 模擬退火	17

## 1 Basic

### 1.1 Default code

```

1 #include<bits/stdc++.h>
1 #define int long long
1 #define mod 1000000007
1 #define endl '\n'
1 #define pii pair<int,int>
1 using namespace std;
2
2 signed main(){
2     ios::sync_with_stdio(0),cin.tie(0);
2 }
3

```

### 1.2 Linux 對拍

```

3 set -e
4 for ((i=0;i<300;i++))
4 do
5     echo "$i"
5     python gen.py > input
5     ./ac < input > ac.out
5     ./wa < input > wa.out
6     diff ac.out wa.out || break
6 done
6

```

### 1.3 Windows 對拍

```

7 @echo off
7 :loop
7     echo %x
8     python gen.py > input
8     ./ac.exe < input > ac.out
9     ./wa.exe < input > wa.out
9     fc ac.out wa.out
9     if not errorlevel 1 goto loop
9

```

### 1.4 builtin 函數

```

11 // 右邊第一個 1 的位置
11 int __builtin_ffs(unsigned int);
11 int __builtin_ffsl(unsigned long);
11 int __builtin_ffsll(unsigned long long);
12 // 左邊第一個 1 之前 0 的數量
12 int __builtin_clz(unsigned int);
12 int __builtin_clzl(unsigned long);
12 int __builtin_clzll(unsigned long long);
12 // 右邊第一個 1 之後 0 的數量
12 int __builtin_ctz(unsigned int);
12 int __builtin_ctzl(unsigned long);
12 int __builtin_ctzll(unsigned long long);
13 // 1 的數量
13 int __builtin_popcount(unsigned int);
13 int __builtin_popcountl(unsigned long);
13 int __builtin_popcountll(unsigned long long);
14 // 1 的數量 mod 2
14 int __builtin_parity(unsigned int);
14 int __builtin_parityl(unsigned long);
14 int __builtin_parityll(unsigned long long);
14 // 二進制表示數字
15 int a = 0b101101;
15

```

### 1.5 輸入輸出

```

16 // 開讀檔
16 fopen("input_file_name","r",stdin);
16 fopen("output_file_name","w",stdout);
16

```

### 1.6 Python 輸入輸出

```

17 a = list(map(int,input().split()))
17
17 # 開讀檔
17 import sys, os.path
17 if(os.path.exists('input_file.txt')):
17     sys.stdin = open("input_file.txt","r")
17     sys.stdout = open("output_file.txt","w")
17

```

## 2 Data Structure

### 2.1 Link-Cut Tree

```

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {

```

```

    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}
}

```

### 2.2 持久化線段樹

```

struct Seg{
    struct Node{
        int v;
        Node* l,*r;
    };
    vector<Node*> version;
    Node* build(int l,int r){
        Node* node=new Node;
        if(l==r){
            node->v=l;
            return node;
        }
        int mid=(l+r)/2;
        node->l=build(l,mid);
        node->r=build(mid+1,r);
        return node;
    }
    int query(Node* cur,int l,int r,int x){
        if(l==r){
            return cur->v;
        }
        int mid=(l+r)/2;
        if(x<=mid) return query(cur->l,l,mid,x);
        else return query(cur->r,mid+1,r,x);
    }
    Node* update(Node* cur,int l,int r,int x,int y){
        Node* node=new Node;
        if(l==r){
            node->v=y;
            return node;
        }
        int mid=(l+r)/2;
        if(x<=mid){
            node->l=update(cur->l,l,mid,x,y);
            node->r=cur->r;
        }
        else{
            node->l=cur->l;
            node->r=update(cur->r,mid+1,r,x,y);
        }
        return node;
    }
};

```

### 2.3 Treap

```

mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count()); // C++ randomizer
struct Node {
    int k, p, sz = 1;

```

```

Node *l = 0, *r = 0;
bool tag = 0;
Node(int kk) {
    k = kk;
    p = gen();
}
};
Node *root = 0;
int size(Node *x) {return x ? x->sz : 0;}
void push(Node *x) {
    if(x->tag) {
        if(x->l) x->l->tag ^= true;
        if(x->r) x->r->tag ^= true;
        x->tag = false;
    }
}
void pull(Node* x) {
    x->sz = size(x->l) + size(x->r) + 1;
}
Node* merge(Node *a, Node *b) {
    if(!a || !b) return a ? b;
    if(a->p > b->p) {
        push(a);
        a->r = merge(a->r, b);
        pull(a);
        return a;
    }
    else{
        push(b);
        b->l = merge(a, b->l);
        pull(b);
        return b;
    }
}
void splitKey(Node* x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(x->k <= k) {
        a = x;
        splitKey(a->r, k, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKey(b->l, k, a, b->l);
        pull(b);
    }
}
void splitKth(Node *x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(size(x->l) < k) {
        a = x;
        splitKth(a->r, k - size(x->l) - 1, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKth(b->l, k, a, b->l);
        pull(b);
    }
}
void insert(int id) {
    Node *l, *r;
    splitKey(root, id, l, r);
    Node *m = new Node(id);
    root = merge(l, merge(m, r));
}
void erase(int x) {
    Node *a, *b, *c;
    splitKey(root, x, b, c);
    splitKey(b, x - 1, a, b);
    root = merge(a, c);
}

```

## 2.4 線段樹

```

struct Seg{
    vector<int> seg,tag;
    #define cl (i<<1)+1
    #define cr (i<<1)+2
    void push(int i,int l,int r){

```

```

        if(tag[i]!=0){
            seg[i]+=tag[i]; // update by tag
            if(l!=r){
                tag[cl]+=tag[i]; // push
                tag[cr]+=tag[i]; // push
            }
            tag[i]=0;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl,l,mid);push(cr,mid+1,r);
        seg[i]=max(seg[cl],seg[cr]); // pull
    }
    void build(int i,int l,int r,vector<int>&arr){
        if(l==r){
            seg[i]=arr[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl,l,mid,arr);
        build(cr,mid+1,r,arr);
        pull(i,l,r);
    }
    Seg(vector<int>& arr){
        seg.resize(arr.size()*4);
        tag.resize(arr.size()*4);
        build(0,0,arr.size()-1,arr);
    }
    void update(int i,int l,int r,int nl,int nr,int x){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            tag[i]+=x;
            return;
        }
        int mid=(l+r)>>1;
        if(nl<=mid) update(cl,l,mid,nl,nr,x);
        if(nr>mid) update(cr,mid+1,r,nl,nr,x);
        pull(i,l,r);
    }
    int query(int i,int l,int r,int nl,int nr){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            return seg[i];
        }
        int mid=(l+r)>>1;
        int ans=0;
        if(nl<=mid) ans=max(ans,query(cl,l,mid,nl,nr));
        if(nr>mid) ans=max(ans,query(cr,mid+1,r,nl,nr));
        return ans;
    }
};

```

## 3 Flow

### 3.1 Dinic

```

const int inf = 1e8;
struct Dinic{
    #define SZ(x) (int)(x.size())
    struct Edge{
        int v,f,re;
    };
    vector<vector<Edge>> E;
    vector<int> level;
    int n,s,t;
    Dinic(int nn,int ss,int tt){
        n=nn;s=ss;t=tt;
        E.resize(n);
        level.resize(n);
    }
    void addEdge(int u,int v,int w){
        E[u].push_back({v,w,SZ(E[v])});
        E[v].push_back({u,0,SZ(E[u])-1});
    }
    bool bfs(){
        level.assign(n,0);
        queue<int> q;
        q.push(s);
        level[s]=1;
        while(q.size()){

```

```

    int u=q.front();q.pop();
    for(auto&it:E[u]){
        int v=it.v;
        if(it.f>0 && !level[v]){
            level[v]=level[u]+1;
            q.push(v);
        }
    }
    return level[t];
}
int dfs(int u,int nf){
    if(u==t)return nf;
    int ret=0;
    for(auto&it:E[u]){
        int v=it.v;
        if(it.f>0&&level[v]==level[u]+1){
            int tem = dfs(v,min(nf,it.f));
            ret+=tem;nf-=tem;
            it.f-=tem;E[v][it.re].f+=tem;
            if(!nf)return ret;
        }
    }
    if(!ret)level[u]=0;
    return ret;
}
int flow(){
    int ret=0;
    while(bfs()) ret+=dfs(s,inf);
    return ret;
}
};

```

### 3.2 匈牙利

```

#define NIL -1
#define INF 100000000
int n,matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN],ylabel[MAXN];
int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
    int i,delta=INF;
    for(i=0;i<n;i++) if(!sett[i]) delta=min(slack[i],delta);
    for(i=0;i<n;i++) if(sets[i]) xlabel[i]-=delta;
    for(i=0;i<n;i++) {
        if(sett[i]) ylabel[i]+=delta;
        else slack[i]-=delta;
    }
}
inline void add_sets(int x) {
    int i;
    sets[x]=1;
    for(i=0;i<n;i++) {
        if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {
            slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
            prev[i]=x;
        }
    }
}
inline void augment(int final) {
    int x=prev[final],y=final,tmp;
    matched++;
    while(1) {
        tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
        if(y==NIL) return;
        x=prev[y];
    }
}
inline void phase() {
    int i,y,root;
    for(i=0;i<n;i++) { sets[i]=sett[i]=0; slack[i]=INF; }
    for(root=0;root<n&&xy[root]!=NIL;root++);
    add_sets(root);
    while(1) {
        relabel();

```

```

        for(y=0;y<n;y++) if(!sett[y]&&slack[y]==0) break;
        if(yx[y]==NIL) { augment(y); return; }
        else { add_sets(yx[y]); sett[y]=1; }
    }
}
inline int hungarian() {
    int i,j,c=0;
    for(i=0;i<n;i++) {
        xy[i]=yx[i]=NIL;
        xlabel[i]=ylabel[i]=0;
        for(j=0;j<n;j++) xlabel[i]=max(cost[i][j],xlabel[i]);
    }
    for(i=0;i<n;i++) phase();
    for(i=0;i<n;i++) c+=cost[i][xy[i]];
    return c;
}

```

### 3.3 KM

```

struct KM{ // max weight, for min negate the weights
    int n, mx[MAXN], my[MAXN], pa[MAXN];
    ll g[MAXN][MAXN], lx[MAXN], ly[MAXN], sy[MAXN];
    bool vx[MAXN], vy[MAXN];
    void init(int _n) { // 1-based
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;}
    void augment(int y) {
        for(int x, z; y; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x,sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y]&&cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                if(!my[y]){augment(y);return;}
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve(){
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    }
} graph;

```

### 3.4 MCMF

```

struct MCMF {
    #define SZ(x) (int)(x.size())
    struct Edge {
        int v, f, re, c;
    };
    vector<vector<Edge>> E;
    vector<int> dis, x, y;
    int n, s, t;
    MCMF(int nn, int ss, int tt) {
        n = nn; s = ss; t = tt;

```

```

    E.resize(n);
    x.resize(n);
    y.resize(n);
}
void addEdge(int u, int v, int w, int c) {
    E[u].push_back({v, w, SZ(E[v]), c});
    E[v].push_back({u, 0, SZ(E[u]) - 1, -c});
}
bool spfa() {
    dis.assign(n, 0x3f3f3f3f);
    x.assign(n, -1);
    y.assign(n, -1);
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(s);
    inq[s] = true;
    dis[s] = 0;
    while(q.size()) {
        int u = q.front(); q.pop();
        inq[u] = false;
        for(int i = 0; i < E[u].size(); i++) {
            auto& it = E[u][i];
            int v = it.v;
            if(it.f > 0 && dis[v] > dis[u] + it.c) {
                dis[v] = dis[u] + it.c;
                x[v] = u;
                y[v] = i;
                if(!inq[v]) {
                    q.push(v);
                    inq[v] = true;
                }
            }
        }
    }
    return x[t] != -1;
}
pii solve() {
    int mf = 0, mc = 0;
    while(spfa()) {
        int nf = 0x3f3f3f3f;
        for(int i = t; i != s; i = x[i]) {
            nf = min(nf, E[x[i]][y[i]].f);
        }
        for(int i = t; i != s; i = x[i]) {
            auto& it = E[x[i]][y[i]];
            it.f -= nf;
            E[it.v][it.re].f += nf;
        }
        mf += nf;
        mc += nf * dis[t];
    }
    return {mf, mc};
}
};

```

## 4 幾何

### 4.1 宣告

```

typedef long double ld;
const ld eps = 1e-10;
#define sign(x) ((x) > eps) - ((x) < -eps)
#define sq(p) ((p)*(p))
#define len(p) (sqrt(sq(p)))
#define r90(p) Pt(-(p).y, (p).x)

struct Pt {
    ld x, y;
    Pt(ld x = 0, ld y = 0) : x(x), y(y) {}
    Pt operator + (const Pt &a) const {
        return Pt(x + a.x, y + a.y);
    }
    Pt operator - (const Pt &a) const {
        return Pt(x - a.x, y - a.y);
    }
    Pt operator * (const ld &a) const {
        return Pt(x * a, y * a);
    }
    Pt operator / (const ld &a) const {
        return Pt(x / a, y / a);
    }
    ld operator * (const Pt &a) const {
        return x * a.x + y * a.y;
    }
    ld operator ^ (const Pt &a) const {
        return x * a.y - y * a.x;
    }
}

```

```

bool operator < (const Pt &a) const {
    return sign(x - a.x) < 0 || sign(x - a.x) == 0 &&
        sign(y - a.y) < 0;
}
bool operator == (const Pt &a) const {
    return sign(x - a.x) == 0 && sign(y - a.y) == 0;
}

```

```

struct Line {
    Pt s, e, v; // start, end, end-start
    ld rad;
    Line(Pt s = Pt(), Pt e = Pt())
        : s(s), e(e), v(e - s), rad(atan2(v.y, v.x)) {}
    bool operator < (const Line &l) const {
        return rad < l.rad;
    }
};

```

```

struct Circle {
    Pt o; ld r2;
    Circle(Pt o = Pt(), ld r = 0) : o(o), r2(sq(r)) {}
    Circle(const Pt &p1, const Pt &p2)
        : o((p1 + p2) / 2), r2(sq(p1 - p2) / 4.0) {}
    Circle(const Pt &p1, const Pt &p2, const Pt &p3) {
        Pt va = r90(p1 - p2), vb = r90(p1 - p3);
        if (sign(va ^ vb) == 0) {
            *this = Circle(p1, p2);
            Circle t(p1, p3);
            if (r2 < t.r2) *this = t;
            t = Circle(p2, p3);
            if (r2 < t.r2) *this = t;
        }
        else {
            Pt p12 = (p1 + p2) / 2, p13 = (p1 + p3) / 2;
            ld t = ((p13 - p12) * (p1 - p3)) / (va ^ vb);
            o = p12 + va*t;
            r2 = sq(o - p1);
        }
    }
    bool contain(const Pt &a) {
        return sign(sq(a - o) - r2) <= 0;
    }
};

```

### 4.2 矩形面積

```

struct AreaofRectangles {
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
    ll n, id, sid;
    pair<ll, ll> tree[MXN<<3]; // count, area
    vector<ll> ind;
    tuple<ll, ll, ll, ll> scan[MXN<<1];
    void pull(int i, int l, int r) {
        if(tree[i].first) tree[i].second = ind[r+1] - ind[l];
        else if(l != r) {
            int mid = (l+r)>>1;
            tree[i].second = tree[cl(i)].second + tree[cr(i)].second;
        }
        else tree[i].second = 0;
    }
    void upd(int i, int l, int r, int ql, int qr, int v) {
        if(ql <= l && r <= qr) {
            tree[i].first += v;
            pull(i, l, r); return;
        }
        int mid = (l+r) >> 1;
        if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
        if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
        pull(i, l, r);
    }
    void init(int _n) {
        n = _n; id = sid = 0;
        ind.clear(); ind.resize(n<<1);
        fill(tree, tree+(n<<2), make_pair(0, 0));
    }
    void addRectangle(int lx, int ly, int rx, int ry) {
        ind[id++] = lx; ind[id++] = rx;
        scan[sid++] = make_tuple(ly, 1, lx, rx);
        scan[sid++] = make_tuple(ry, -1, lx, rx);
    }
    ll solve() {
        sort(ind.begin(), ind.end());
    }
}

```

```

    ind.resize(unique(ind.begin(), ind.end()) - ind
        .begin());
    sort(scan, scan + sid);
    ll area = 0, pre = get<0>(scan[0]);
    for(int i = 0; i < sid; i++){
        auto [x, v, l, r] = scan[i];
        area += tree[1].second * (x-pre);
        upd(1, 0, ind.size()-1, lower_bound(ind.
            begin(), ind.end(), l)-ind.begin(),
            lower_bound(ind.begin(), ind.end(), r)-
            ind.begin()-1, v);
        pre = x;
    }
    return area;
} }rect;

```

### 4.3 最近點對

```

#include<bits/stdc++.h>
#define int long long
using namespace std;
using ld = long double;
const int mod = 1e9+7;
struct pt{
    int x,y;
    int id;
    ld dis(const pt& rhs){
        return sqrt((x-rhs.x)*(x-rhs.x)+(y-rhs.y)*(y-
            rhs.y));
    }
};
signed main(){
    int n;
    cin>>n;
    vector<pt> a(n);
    for(int i=0;i<n;i++){
        cin>>a[i].x>>a[i].y;
        a[i].id=i;
    }
    ld ans = 1e19;
    sort(a.begin(),a.end(),[](const pt&a,const pt&b){
        if(a.x==b.y)return a.y<b.y;
        return a.x<b.x;
    });
    pt ans2;
    function<void(int,int)> dnq = [&](int l,int r){
        if(r-l<4){
            for(int i=l;i<=r;i++){
                for(int j=i+1;j<=r;j++){
                    ld temans = a[i].dis(a[j]);
                    if(temans<ans){
                        ans=temans;
                        ans2 = {a[i].id,a[j].id};
                    }
                }
            }
            sort(a.begin()+l,a.begin()+r+1,[](const pt&
                a,const pt&b){return a.y<b.y;});
            return;
        }
        int mid = (l+r)/2;
        int midx = a[mid].x;
        dnq(l,mid);dnq(mid+1,r);
        inplace_merge(a.begin()+l,a.begin()+mid+1,a.
            begin()+r+1,[](const pt&a,const pt&b){
                return a.y<b.y;});
        vector<int> c;c.reserve(r-l+1);
        for(int i=l;i<=r;i++){
            if(abs(a[i].x-midx)<ans){
                for(int j=c.size()-1;j>=0&&a[i].y-a[c[j]
                    ]>ans;j--){
                    ld temans = a[i].dis(a[c[j]]);
                    if(temans<ans){
                        ans=temans;
                        ans2 = {a[i].id,a[c[j]].id
                            };
                    }
                }
            }
        }
        c.push_back(i);
    }
}

```

```

};
dnq(0,n-1);
cout<<min(ans2.x,ans2.y)<<' '<<max(ans2.x,ans2.y)<<
    ' '<<fixed<<setprecision(6)<<ans<<'\n';
}

```

### 4.4 凸包

```

double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(),pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i]
            ]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2],stk[top-1],pt[i]
            ]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

### 4.5 兩直線交點

```

pair<int, Pt> L_intersect(const Line &a, const Line &b)
{
    ld f1 = a.v ^ (b.s - a.s), f2 = a.v ^ (a.s - b.e), f;
    if(sign(f = f1 + f2) == 0)
        return sign(f1) ? {0, a.s} : {2, a.s};
    return {1, b.s * (f2 / f) + b.e * (f1 / f)};
}

```

### 4.6 兩線段交點

```

inline int ori(const Pt &o, const Pt &a, const Pt &b) {
    return sign((a - o) ^ (b - o)); }
// 0:out, 1:ontop, 2:in
inline int btw(const Pt &a, const Pt &b, const Pt &c) {
    if (ori(a, b, c)) return 0;
    int s = sign((c - a) * (c - b));
    return (s < 0) + (s <= 0); }
// 0:no, 1:1pt(parallel), -1:1pt, 2:inf pt
int banana(const Pt &p1, const Pt &p2,
    const Pt &p3, const Pt &p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0) {
        return btw(p1, p2, p3) + btw(p1, p2, p4) +
            btw(p3, p4, p1) + btw(p3, p4, p2) >> 1;
    }
    return -(a123 * a124 <= 0 && a341 * a342 <= 0);
}
pair<int, Pt> S_intersect(const Pt &p1, const Pt &p2,
    const Pt &p3, const Pt &p4) {
    int b = banana(p1, p2, p3, p4);
    if (b != -1) return {b, btw(p1, p2, p3) ? p3 : p4};
    ld a123 = (p2 - p1) ^ (p3 - p1);
    ld a124 = (p2 - p1) ^ (p4 - p1);
    return {1, (p4 * a123 - p3 * a124) / (a123 - a124)
        };
}

```

### 4.7 李超線段樹

```

struct LiChao_min{
    struct line{
        ll m,c;
        line(ll _m=0,ll _c=0){ m=_m; c=_c; }
        ll eval(ll x){ return m*x+c; } // overflow
    };
    struct node{

```



```

node *l,*r; line f;
node(line v){ f=v; l=r=NULL; }
};
typedef node* pnode;
pnode root; ll sz,ql,q,r;
#define mid ((l+r)>>1)
void insert(line v,ll l,ll r,pnode &nd){
    /* if(!(ql<=l&&r<=qr)){
        if(!nd) nd=new node(line(0,INF));
        if(ql<=mid) insert(v,l,mid,nd->l);
        if(qr>mid) insert(v,mid+1,r,nd->r);
        return;
    }
    used for adding segment */
    if(!nd) nd=new node(v); return; }
    ll trl=nd->f.eval(l),trr=nd->f.eval(r);
    ll vl=v.eval(l),vr=v.eval(r);
    if(trl<=vl&&trr<=vr) return;
    if(trl>vl&&trr>vr) { nd->f=v; return; }
    if(trl>vl) swap(nd->f,v);
    if(nd->f.eval(mid)<v.eval(mid))
        insert(v,mid+1,r,nd->r);
    else swap(nd->f,v),insert(v,l,mid,nd->l);
}
ll query(ll x,ll l,ll r,pnode &nd){
    if(!nd) return INF;
    if(l==r) return nd->f.eval(x);
    if(mid>=x)
        return min(nd->f.eval(x),query(x,l,mid,nd->l));
    return min(nd->f.eval(x),query(x,mid+1,r,nd->r));
}
/* -sz<=ll query_x<=sz */
void init(ll _sz){ sz=_sz+1; root=NULL; }
void add_line(ll m,ll c,ll l=-INF,ll r=INF){
    line v(m,c); ql=l; qr=r; insert(v,-sz,sz,root);
}
ll query(ll x) { return query(x,-sz,sz,root); }
};

```

#### 4.8 最小包覆圓

```

// remember to shuffle!!!
Circle encircle(const vector<Pt> &pts) {
    Circle ans(Pt(), 0);
    for (int i = 0; i < pts.size(); ++i) {
        if (ans.contains(pts[i])) continue;
        ans = {pts[i], 0};
        for (int j = 0; j < i; ++j) {
            if (ans.contains(pts[j])) continue;
            ans = {pts[i], pts[j]};
            for (int k = 0; k < j; ++k) {
                if (ans.contains(pts[k])) continue;
                ans = {pts[i], pts[j], pts[k]};
            }
        }
    }
    return ans;
}

```

#### 4.9 最小包覆球

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res, outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
            if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
                return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
            res=outer[0]+q[0]*L[0]+q[1]*L[1];
            radius=norm2(res, outer[0]);
            break;
        case 4:

```

```

for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=(q[i] * q[i]);
for (i=0; i<3; ++i) for(j=0; j<3; ++j) m[i][j]=(q[i] * q[j])*2;
det= m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[1][0]*m[2][1]
- m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1];
if ( fabs(det)<eps ) return;
for (j=0; j<3; ++j) {
    for (i=0; i<3; ++i) m[i][j]=sol[i];
    L[j]=( m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[1][0]*m[2][1]
- m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1]
) / det;
    for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
} res=outer[0];
for (i=0; i<3; ++i) res = res + q[i] * L[i];
radius=norm2(res, outer[0]);
}
void minball(int n){ ball();
if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
if( norm2(res, pt[i]) - radius > eps ){
    outer[ nouter ++ ] = pt[ i ]; minball(i); --
    nouter;
if(i>0){ Pt Tt = pt[i];
    memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
    ;
}
}
double solve(){
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
        nouter=1, outer[0]=pt[i], minball(i);
    return sqrt(radius);
}
}

```

#### 4.10 旋轉卡尺

```

int FarthestPair(vector<Pt>& arr){
    int ret=0;
    for(int i = 0, j = i+1; i<arr.size(); i++){
        while(distance(arr[i], arr[j]) < distance(arr[i], arr[(j+1)%arr.size()])){
            j = (j+1) % arr.size();
        }
        ret = max(ret, distance(arr[i],arr[j]));
    }
    return ret;
}

```

#### 4.11 Circle Cover

```

#define N 1021
#define D long double
struct CircleCover{
    int C; Circ c[ N ]; //填入C(圖數量),c(圖陣列)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.o , o2 = b.o;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return false;
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return true;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
} Teve {

```

```
Pt p; D ang; int add;
Teve() {}
Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
bool operator<(const Teve &a)const
{return ang < a.ang;}
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1
bool disjunct( Circ& a, Circ &b, int x )
{return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
bool contain( Circ& a, Circ &b, int x )
{return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
bool contain(int i, int j){
/* c[j] is non-strictly in c[i]. */
return (sign(c[i].R - c[j].R) > 0 ||
(sign(c[i].R - c[j].R) == 0 && i < j) ) &&
contain(c[i], c[j], -1);
}
void solve(){
for( int i = 0 ; i <= C + 1 ; i ++ )
Area[ i ] = 0;
for( int i = 0 ; i < C ; i ++ )
for( int j = 0 ; j < C ; j ++ )
overlap[i][j] = contain(i, j);
for( int i = 0 ; i < C ; i ++ )
for( int j = 0 ; j < C ; j ++ )
g[i][j] = !(overlap[i][j] || overlap[j][i] ||
disjunct(c[i], c[j], -1));
for( int i = 0 ; i < C ; i ++ ){
int E = 0, cnt = 1;
for( int j = 0 ; j < C ; j ++ )
if( j != i && overlap[j][i] )
cnt ++;
for( int j = 0 ; j < C ; j ++ )
if( i != j && g[i][j] ){
Pt aa, bb;
CCinter(c[i], c[j], aa, bb);
D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
eve[E++] = Teve(bb, B, 1);
eve[E++] = Teve(aa, A, -1);
if(B > A) cnt ++;
}
}
if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
else{
sort( eve , eve + E );
eve[E] = eve[0];
for( int j = 0 ; j < E ; j ++ ){
cnt += eve[j].add;
Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
D theta = eve[j + 1].ang - eve[j].ang;
if (theta < 0) theta += 2.0 * pi;
Area[cnt] +=
(theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}}};
```

### 4.12 Convex Hull Trick

```

/* Given a convexhull, answer queries in  $O(\lg N)$ 
CH should not contain identical points, the area should
be  $> 0$ , min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
    }
};

```

```

return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
}

void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
}

void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl = sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid = sign(det(a[mid % n] - p, a[(mid + 1) % n] - p));
        if (smid == sl) l = mid;
        else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
}

int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid = sign(det(v - u, a[mid % n] - u));
        if (smid == sl) l = mid;
        else r = mid;
    }
    return l % n;
}

// 1. whether a given point is inside the CH
bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
        if (lower[id].Y > p.Y) return 0;
    } else if (det(lower[id - 1] - p, lower[id] - p) < 0) return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
        if (upper[id].Y < p.Y) return 0;
    } else if (det(upper[id - 1] - p, upper[id] - p) < 0) return 0;
    return 1;
}

// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}

// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second + (int)lower.size() - 1) % n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}

// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v - u, a[p0] - u)) * sign(det(v - u, a[p1] - u)) < 0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
}

```



```

    }
    return 0;
} };

```

### 4.13 Half Plane Intersection

```

// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
} // segment should add Counterclockwise
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end()); // sort by angle
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir=las=0] = L[0];
    for(int i = 1; i < n; i++) {
        while(fir < las && !onleft(L[i], p[las-1])) las--;
        while(fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if(fir < las) p[las-1] = LLIntersect(q[las-1], q[las]);
    }
    while(fir < las && !onleft(q[fir], p[las-1])) las--;
    if(las-fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las-fir+1);
    for(int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}

```

## 5 圖論

### 5.1 BCC

```

struct BCC {
    vector<vector<int>> g;
    vector<int> dfn, low;
    vector<vector<int>> bcc;
    vector<int> stk;
    int nbcc=0;
    int cur=1;
    BCC(int n){
        g.resize(n);
        dfn.resize(n,0);
        low.resize(n);
    }
    void addEdge(int u,int v){
        g[u].push_back(v);
        g[v].push_back(u);
    }
    void dfs(int x,int f){
        if(!g[x].size()){
            bcc.push_back({x});
            nbcc++;
            return;
        }
        dfn[x]=low[x]=cur++;
        stk.push_back(x);
        for(int y:g[x]){
            if(y==f)continue;
            if(dfn[y]){
                low[x]=min(low[x],dfn[y]);
            }
            else{
                dfs(y,x);
                low[x]=min(low[x],low[y]);
                if(low[y]>=dfn[x]){
                    bcc.push_back({});
                    int b;
                    do{
                        bcc[nbcc].push_back(b=stk.back());
                        stk.pop_back();
                    }while(b!=y);
                    bcc[nbcc++].push_back(x);
                }
            }
        }
    }
}

```

```

    }
}
void solve(){
    for(int i=0;i<g.size();i++){
        if(!dfn[i]){
            dfs(i,-1);
        }
    }
}
};

```

### 5.2 重心剖分

```

struct CentroidDecomposition {
    int n;
    vector<vector<int>> G, out;
    vector<int> sz, v;
    CentroidDecomposition(int _n) : n(_n), G(_n), out(_n), sz(_n), v(_n) {}
    int dfs(int x, int par){
        sz[x] = 1;
        for (auto &i : G[x]) {
            if(i == par || v[i]) continue;
            sz[x] += dfs(i, x);
        }
        return sz[x];
    }
    int search_centroid(int x, int p, const int mid){
        for (auto &i : G[x]) {
            if(i == p || v[i]) continue;
            if(sz[i] > mid) return search_centroid(i, x, mid);
        }
        return x;
    }
    void add_edge(int l, int r){
        G[l].PB(r); G[r].PB(l);
    }
    int get(int x){
        int centroid = search_centroid(x, -1, dfs(x, -1)/2);
        v[centroid] = true;
        for (auto &i : G[centroid]) {
            if(!v[i]) out[centroid].PB(get(i));
        }
        v[centroid] = false;
        return centroid;
    }
};

```

### 5.3 輕重鍊剖分

```

#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfssz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfssz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]),

```

```

    [&](int a, int b){return sz[a] > sz[b];});
    bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
        if(flag) head[v] = head[u], flag = 0;
        dfs1(v);
        tr[u] = tr[v];
    }
}
inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
        b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
        a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
}
void init( int _n ){
    n = _n; REP( i , 1 , n ) g[ i ].clear();
}
void addEdge( int u , int v ){
    g[ u ].push_back( v );
    g[ v ].push_back( u );
}
void yutruli(){ //build function
    dfsz(1, 0);
    ts = 0;
    dfs1(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
        prt[i][k] = prt[prt[i][k-1]][k-1];
}
vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){
        res.push_back( PII(tid[ head[ v ] ] , tid[ v ] ) );
        v = prt[ head[ v ] ][ 0 ];
    }
    res.push_back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
}
/* res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< PII >& path = tree.getPath( u , v )
 * for( PII tp : path ) {
 *     int l , r; tie( l , r ) = tp;
 *     upd( l , r );
 *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;

```

## 5.4 歐拉路徑

```

#define FOR(i,a,b) for(int i=a;i<=b;i++)
int dfs_st[10000500],dfn=0;
int ans[10000500],cnt=0,num=0;
vector<int>G[10000500];
int cur[10000500];
int ind[10000500],out[10000500];
void dfs(int x){
    FOR(i,1,n)sort(G[i].begin(),G[i].end());
    dfs_st[++dfn]=x;
    memset(cur,-1,sizeof(cur));
    while(dfn>0){
        int u=dfs_st[dfn];
        int complete=1;
        for(int i=cur[u]+1;i<G[u].size();i++){
            int v=G[u][i];
            num++;
            dfs_st[++dfn]=v;
            cur[u]=i;
            complete=0;
            break;
        }
        if(complete)ans[++cnt]=u,dfn--;
    }
}

```

```

}
}
bool check(int &start){
    int l=0,r=0,mid=0;
    FOR(i,1,n){
        if(ind[i]==out[i]+1)l++;
        if(out[i]==ind[i]+1)r++,start=i;
        if(ind[i]==out[i])mid++;
    }
    if(l==1&&r==1&&mid==n-2)return true;
    l=1;
    FOR(i,1,n)if(ind[i]!=out[i])l=0;
    if(l){
        FOR(i,1,n)if(out[i]>0){
            start=i;
            break;
        }
        return true;
    }
    return false;
}
int main(){
    cin>>n>>m;
    FOR(i,1,m){
        int x,y;scanf("%d%d",&x,&y);
        G[x].push_back(y);
        ind[y]++,out[x]++;
    }
    int start=-1,ok=true;
    if(check(start)){
        dfs(start);
        if(num!=m){
            puts("What a shame!");
            return 0;
        }
        for(int i=cnt;i>=1;i--){
            printf("%d ",ans[i]);
            puts("");
        }
        else puts("What a shame!");
    }
}

```

## 5.5 極大團

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i ++){
            lnk[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int ans , stk[N], id[N] , di[N] , deg[N];
    Int cans;
    void dfs(int elem_num, Int candi, Int ex){
        if(candi.none()&&ex.none()){
            cans.reset();
            for(int i = 0 ; i < elem_num ; i ++){
                cans[id[stk[i]]] = 1;
                ans = elem_num; // cans is a maximal clique
                return;
            }
            int pivot = (candilex)._Find_first();
            Int smaller_candi = candi & (~lnk[pivot]);
            while(smaller_candi.count()){
                int nxt = smaller_candi._Find_first();
                candi[nxt] = smaller_candi[nxt] = 0;
                ex[nxt] = 1;
                stk[elem_num] = nxt;
                dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i ++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i ++){
            di[id[i]] = i;
        }
    }
}

```

```

for(int i = 0 ; i < n ; i ++){
    for(int j = 0 ; j < n ; j ++){
        if(v[i][j]) lnk[di[i]][di[j]] = 1;
    }
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n, '1')), 0);
    return ans;
} }solver;

```

## 5.6 最大團

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N], v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i ++){
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[N];
    int id[N] , di[N] , deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num; cans.reset();
            for(int i = 0 ; i < elem_num ; i ++){
                cans[id[stk[i]]] = 1;
            }
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans){
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential --;
            if(next == pivot || (smaller_candi & linkto[next]
                ).count()){
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i ++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i ++){
            di[id[i]] = i;
        }
        for(int i = 0 ; i < n ; i ++){
            for(int j = 0 ; j < n ; j ++){
                if(v[i][j]) lnkto[di[i]][di[j]] = 1;
            }
        }
        Int cand; cand.reset();
        for(int i = 0 ; i < n ; i ++){
            cand[i] = 1;
        }
        ans = 1;
        cans.reset(); cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} }solver;

```

## 5.7 SCC

```

struct Scc{
    vector<vector<int>> g,rg;
    vector<int> scc;
    vector<bool> vis;
    vector<int> stk;
    int nsc=0;
    Scc(int n){
        g.resize(n);
        rg.resize(n);
        scc.resize(n);
        vis.resize(n);
    }
    void addEdge(int u,int v){

```

```

        g[u].push_back(v);
        rg[v].push_back(u);
    }
    void dfs1(int x){
        vis[x]=true;
        for(int y:g[x]){
            if(!vis[y])dfs1(y);
        }
        stk.push_back(x);
    }
    void dfs2(int x){
        vis[x]=true;
        scc[x]=nsc;
        for(int y:rg[x]){
            if(!vis[y])dfs2(y);
        }
    }
    void solve(){
        vis.assign(vis.size(),false);
        stk.clear();
        for(int i=0;i<vis.size();i++){
            if(!vis[i])dfs1(i);
        }
        reverse(stk.begin(),stk.end());
        vis.assign(n,false);
        for(int i:stk){
            if(!vis[i]){
                dfs2(i);
                nsc++;
            }
        }
    }
}
};

```

## 5.8 SPFA

```

#define MXN 200005
struct SPFA{
    int n;
    LL inq[MXN], len[MXN];
    vector<LL> dis;
    vector<pair<int, LL>> edge[MXN];
    void init(int _n){
        n = _n;
        dis.clear(); dis.resize(n, 1e18);
        for(int i = 0; i < n; i++){
            edge[i].clear();
            inq[i] = len[i] = 0;
        }
    }
    void addEdge(int u, int v, LL w){
        edge[u].push_back({v, w});
    }
    vector<LL> solve(int st = 0){
        deque<int> dq; //return {-1} if has negative cycle
        dq.push_back(st); //otherwise return dis from st
        inq[st] = 1; dis[st] = 0;
        while(!dq.empty()){
            int u = dq.front(); dq.pop_front();
            inq[u] = 0;
            for(auto [to, d] : edge[u]){
                if(dis[to] > d+dis[u]){
                    dis[to] = d+dis[u];
                    len[to] = len[u]+1;
                    if(len[to] > n) return {-1};
                    if(inq[to]) continue;
                    if(!dq.empty() && dis[dq.front()] > dis[to]){
                        dq.push_front(to) : dq.push_back(to);
                    }
                    inq[to] = 1;
                }
            }
        }
        return dis;
    }
} }spfa;

```

## 5.9 樹哈希

```

const ull mask = mt19937_64(time(nullptr))();
ull shift(ull x) {
    x ^= mask;
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    x ^= mask;
    return x; }

```

```
void dfs(int u) { // edge[父] = {子}
    tree_hash[u] = 1;
    for (int v : edge[u]) {
        dfs(v);
        tree_hash[u] += shift(tree_hash[v]);
    }
}
```

### 5.10 差分約束

約束條件  $V_j - V_i \leq W$  addEdge( $V_i, V_j, W$ ) and run bellman-ford or spfa

## 6 數論

### 6.1 離散根號

```
int discrete_sqrt(int y, int p) { // find x s.t. x^2 = y (mod p)
    if (!y) return 0;
    if (p == 2) return (y & 1) == 1 ? 1 : -1;
    if (fpow(y, p - 1 >> 1, p) != 1) return -1;
    int Q = p - 1, S = 0;
    while (~Q & 1) { Q >>= 1; S++; }
    if (S == 1) return fpow(y, p + 1 >> 2, p);
    int z;
    while (1) {
        z = 1 + rand() % (p - 1);
        if (fpow(z, p - 1 >> 1, p) != 1) break;
    }
    int c = fpow(z, Q, p), R = fpow(y, Q + 1 >> 1, p);
    int t = fpow(y, Q, p), M = S, b, i;
    while (1) {
        if (t % p == 1) break;
        for (i = 1; i < M && fpow(t, 1LL << i, p) != 1; i++);
        b = fpow(c, 1LL << M - i - 1, p);
        R = R * b % p;
        c = fpow(b, 2, p);
        t = t * c % p; M = i;
    }
    return (R % p + p) % p;
}
```

### 6.2 離散對數

```
int BSGS(int start, int x, int y, int m) {
    unordered_map<int, int> mp;
    int big = 1, STEP = sqrt(m);
    for (int i = 0; i < STEP; i++) {
        mp[y] = i;
        y = y * x % m;
        big = big * x % m;
    }
    for (int step = 0; step < m + 10; step += STEP) {
        start = start * big % m;
        if (mp.count(start))
            return (step + STEP) - mp[start];
    }
    return -1;
}

int discrete_log(int x, int y, int m) { // find min k s
    .t. x^k = y (mod m)
    if (m == 1) return 0;
    int start = 1;
    for (int i = 0; i < 100; i++) {
        if (start == y) return i;
        start = start * x % m;
    }
    int pred = 100 + BSGS(start, x, y, m);
    if (fpow(x, pred, m) != y) return -1;
    return pred;
}
```

### 6.3 ex-crt

```
typedef __int128 ll;
void exgcd(ll a, ll b, ll &g, ll &x, ll &y) {
    if (b == 0) {
        g = a;
        x = 1;
        y = 0;
        return;
    }
    exgcd(b, a % b, g, y, x);
```

```
    y -= (a / b) * x;
}
bool flag = false;
ll a1, a2, n1, n2;
ll abs(ll x) {
    return x > 0 ? x : -x;
}
void china() {
    ll d = a2 - a1;
    ll g, x, y;
    exgcd(n1, n2, g, x, y);
    if (d % g == 0) {
        x = ((x * d / g) % (n2 / g) + (n2 / g)) % (n2 / g);
        a1 = x * n1 + a1;
        n1 = (n1 * n2) / g;
    }
    else
        flag = true;
}
int n;
long long as[100001]; // 算式答案 x
long long ns[100001]; // 模數 MOD
ll realchina() {
    a1 = as[0];
    n1 = ns[0];
    for (ll i = 1; i < n; i++) {
        a2 = as[i];
        n2 = ns[i];
        china();
        if (flag)
            return -1;
    }
    return a1;
}
int main() {
    cin >> n;
    flag = false;
    for (ll i = 0; i < n; i++)
        cin >> ns[i] >> as[i];
    cout << (long long)realchina() << endl;
}
```

### 6.4 ex-gcd

```
int exgcd(int a, int b, int&x, int&y) {
    if (b == 0) return x = 1, y = 0, a;
    int d = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return d;
}
```

### 6.5 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; // real(), imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN + 1];
void pre_fft() {
    for (int i = 0; i <= MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv = false) {
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN - (i * theta % MAXN) : i * theta % MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
}
```

```

int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^ k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
}
if (inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n, ll a[], int _m, ll b[], ll ans[]){
    int n=1, sum=_n+_m-1;
    while(n<sum)
        n<<=1;
    for(int i=0; i<n; i++) {
        double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y, x-y);
    }
    fft(n, arr);
    for(int i=0; i<n; i++)
        arr[i]=arr[i]*arr[i];
    fft(n, arr, true);
    for(int i=0; i<sum; i++)
        ans[i]=(long long int)(arr[i].real()/4+0.5);
}

```

## 6.6 高斯消去法

```

const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a, int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD, k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD, k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now, 0, n){
            REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[i], v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            REP(i, 0, n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j, now, n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
            }
        }
        REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][i], GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
} gs;
// gs.v.clear(), gs.v.resize(n, vector<int>(n + 1, 0));

```

## 6.7 喬瑟夫問題

```

int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}

```

## 6.8 定理

- Lucas's Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod (C(m_i, n_i))$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling approximation :  
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x + i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

- Pick's Theorem :  $A = i + b/2 - 1$   
 $A$ : Area ·  $i$ : grid number in the inner ·  $b$ : grid number on the side
- Catalan number :  $C_n = \binom{2n}{n} / (n+1)$   
 $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1}$  for  $n \geq m$   
 $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$   
 $C_0 = 1$  and  $C_{n+1} = 2 \binom{2n+1}{n+2} C_n$   
 $C_0 = 1$  and  $C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$  for  $n \geq 0$
- Euler Characteristic:  
planar graph:  $V - E + F - C = 1$   
convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$
- Polya' theorem ( $c$  is number of color ·  $m$  is the number of cycle size):  
 $(\sum_{i=1}^m c^{gcd(i, m)}) / m$
- Burnside lemma:  
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 錯排公式: ( $n$  個人中 · 每個人皆不再原來位置的組合數):  
 $dp[0] = 1; dp[1] = 0;$   
 $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell 數 (有  $n$  個人, 把他們拆組的方法總數) :  
 $B_0 = 1$   
 $B_n = \sum_{k=0}^n s(n, k)$  (second - stirling)  
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$
- Wilson's theorem :  
 $(p-1)! \equiv -1 \pmod{p}$
- Fermat's little theorem :  
 $a^p \equiv a \pmod{p}$
- Euler's totient function:  
 $A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$
- 歐拉函數降冪公式:  
 $A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$
- 6 的倍數:  
 $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

## 6.9 Miller Rabin

```

// {2,7,61} for [1, 2^32)
int magic[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
bool witness(int a, int n, int u, int t) {
    if(!a) return false;
    int x = powm(a, u, n); // a, u using __int128 in `powm`
    for(int i = 0; i < t; i++) {
        int nx = (__int128)x * x % n;
        if(nx == 1 && x != 1 && x != n-1) return true;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(int n) {
    int s = 7;
    if(n < 2) return false;
    if(~n & 1) return n == 2;
    int u = n - 1, t = 0;
    while(~u & 1) u >>= 1, t++;
    while(s--) {
        int a = magic[s] % n;
        if(witness(a, n, u, t)) return false;
    }
    return true;
}

```

## 6.10 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
n    2^n    p    a    root
16   65536   65537   1    3
20   1048576 7340033   7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if (b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1) return 1;
        return ((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >>= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta*MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^= k); k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n,P);
            reverse(a+1, a+n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

## 6.11 Pollard's Rho

```
// does not work when n is prime O(n^(1/4))
int mul(__int128 a, __int128 b, int m) { return (a % m)
    * (b % m) % m; }
int pollard_rho(int p) {
    int x, y, z, c, g, i, j;
    while (1) {
        y = x = rand() % p; c = rand() % p;
        i = 0, z = j = 1;
        while (++i) {
            x = (mul(x, x, p) + c) % p;
            z = mul(z, abs(y - x), p);
            if (x == y || !z) break;
            if (i % 127 == 0 || i == j) {
                g = __gcd(z, p);
                if (g > 1) return g;
                if (i == j) y = x, j <= 1;
            }
        }
    }
}
```

## 6.12 質數

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int lpf[N], phi[N], mu[N];
bitset<N + 1> np;
vector<int> primes;
void sieve() {
    np[0] = np[1] = phi[1] = mu[1] = 1;
    for (int i = 2; i < N; i++) {
        if (!np[i]) {
            primes.push_back(i);
            lpf[i] = i;
            phi[i] = i - 1;
            mu[i] = -1;
        }
        for (int p : primes) {
            int j = i * p;
            if (j >= N) break;
            np[j] = 1;
            lpf[j] = p;
            if (i % p == 0) {
                phi[j] = p * phi[i];
                mu[j] = 0;
                break;
            }
            phi[j] = phi[i] * phi[p];
            mu[j] = -mu[i];
        }
    }
}
```

## 6.13 phi

```
ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n; // O(sqrt(N))
    for (ll i=2; i*i<=a; i++){
        if (a%i==0){
            res = res/i*(i-1);
            while (a%i==0) a/=i;
        }
    }
    if (a>1) res = res/a*(a-1);
    return res;
}
```

## 6.14 矩陣快速冪

```
LL len, mod;
vector<vector<LL>> operator*(vector<vector<LL>> x,
    vector<vector<LL>> y){
    vector<vector<LL>> ret(len, vector<LL>(len, 0));
    for (int i=0; i<len; i++){
        for (int j=0; j<len; j++){
            for (int k=0; k<len; k++){
                ret[i][j] = (ret[i][j] + x[i][k] * y[k][j]) %
                    mod;
            }
        }
    }
    return ret;
}
struct Martix_fast_pow{ //O(len^3 lg k)
    LL init(int _len, LL m=9223372036854775783LL){
        len=_len, mod=m;
    }
    // mfp.solve(k, {0, 1}, {1, 1}) k'th fib {值, 係數} // 0-base
    LL solve(LL n, vector<vector<LL>> poly){
        if (n<len) return poly[n][0];
        vector<vector<LL>> mar(len, vector<LL>(len, 0)), x
            (len, vector<LL>(len, 0));
        for (int i=0; i<len; i++) mar[i][i]=1;
        for (int i=0; i+1<len; i++) x[i][i+1]=1;
        for (int i=0; i<len; i++) x[len-1][i]=poly[i][1];
        while (n){
            if (n&1) mar=mar*x;
            n>>=1, x=x*x;
        }
        LL ans=0;
        for (int i=0; i<len; i++) ans=(ans+mar[len-1][i]
            *poly[i][0]%mod)%mod;
        return ans;
    }
}
```



```
}mfp;
```

## 7 字串

### 7.1 KMP

```
/* len-failure[k]:
```

在k結尾的情況下，這個子字串可以由開頭  
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴

如果我們不只想求最多，而且以0-base做為考量  
，那可能的長度由大到小會是

failuer[k]、failure[failuer[k]-1]  
、failure[failure[failuer[k]-1]-1]..  
直到有值為0為止 \*/

```
int failure[MXN];
vector<int>ret;
void KMP(string& t, string& p){
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i){
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i){
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1){
            ret.push_back( i - p.size() + 1 );
            j = failure[j];
        }
    } return ;}
}
```

### 7.2 馬拉車

```
void manacher(char *s,int len,int *z){
    len=(len<<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>>1]:'0';
        z[0]=1;
        for(int i=1,l=0,r=0;i<len;i++){
            z[i]=i<r?min(z[l+l-i],r-i):1;
            while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
                ++z[i];
            if(i+z[i]>r) l=i,r=i+z[i];
        }
    }
}
```

### 7.3 回文樹

```
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26],fail[MXN],len[MXN];
    int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
    int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
    char s[MXN]={-1};
    int newNode(int l,int f){
        len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
        memset(nxt[tot],0,sizeof(nxt[tot]));
        diff[tot]=(l>0?l-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v],dp[fail[v]]);
        return dp[v]+1;
    }
    int push(){
        int c=s[n]-'a',np=getfail(lst);
        if(!lst=nxt[np][c]){
            lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
            nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
        }
        fac[n]=n;
        for(int v=lst;len[v]>0;v=sfail[v])
            fac[n]=min(fac[n],getmin(v));
        return ++cnt[lst],lst;
    }
}
void init(const char *s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
}
}
```

### 7.4 SA

```
const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[sa[i]
            ]-1]++ = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
            ]-1) sa[-x[sa[i]-1]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
            +1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[-x[s[i]
            ]]=p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
                [i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmzx+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
            + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[-x[s[p[
            nsa[i]]]] = p[nsa[i]]);
    }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
    }
}
```

```

    SA[i] = sa._sa[i + 1];
}
// resulting height, sa array \in [0,len)
}

```

## 7.5 SAM

```

struct SAM{
    struct Node{
        array<int,26>next={};
        int link=0;
        int len=0;
    };
    vector<Node> s;
    int n;
    int last=1;
    int sz=2;
    SAM(int nn){
        n=nn*2+10;
        s.resize(n);
    }
    void add(int x){
        int p=last;
        if(s[p].next[x]){
            int q=s[p].next[x];
            if(s[p].len+1==s[q].len){
                last=q;return;
            }
            int r=sz++;
            s[r]=s[q];
            s[r].len=s[p].len+1;
            while(p&&s[p].next[x]==q){
                s[p].next[x]=r;
                p=s[p].link;
            }
            s[q].link=r;
            last=r;
            return;
        }
        int q=sz++;last=q;
        s[q].len=s[p].len+1;
        while(p&&!s[p].next[x]){
            s[p].next[x]=q;
            p=s[p].link;
        }
        if(!p){
            s[last].link=1;
            return;
        }
        q=s[p].next[x];
        if(s[p].len+1==s[q].len){
            s[last].link=q;
            return;
        }
        int r=sz++;
        s[r]=s[q];
        s[r].len=s[p].len+1;
        while(p&&s[p].next[x]==q){
            s[p].next[x]=r;
            p=s[p].link;
        }
        s[last].link=s[q].link=r;
    }
};

```

## 7.6 trie

```

//01 bitwise trie
struct trie{
    trie *nxt[2]; // 差別
    int cnt; //紀錄有多少個數字以此節點結尾
    int sz; //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root;
void insert(int x){
    trie *now = root; // 每次從根節點開始
    for(int i=22;i>=0;i--){ // 從最高位元開始往低位元走
        now->sz++;

```

```

        //cout<<(x>>i&1)<<endl;
        if(now->nxt[x>>i&1] == NULL){ //判斷當前第 i 個
            位元是 0 還是 1
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1]; //走到下一個位元
    }
    now->cnt++;
    now->sz++;
}

```

## 7.7 Z-value

```

int z[MAXN];
void Z_value(const string& s) { //z[i] = lcp(s[1...],s[
    i...])
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 7.8 minRotation

```

//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}

```

## 8 DP

### 8.1 數位 dp

```

int dp[MXN_LEN][PRE_NUM][LIMIT][F0];
int DP(const string &s, int i = 0, int pre = 0, bool
    lim = 1, bool f0 = 1) {
    if (dp[i][pre][lim][f0]) return dp[i][pre][lim][f0];
    if (i == s.size()) return dp[i][pre][lim][f0] = 1;
    int res = 0, h = lim ? s[i] - '0' : 9;
    for (int j = 0; j <= h; j++)
        if (abs(j - pre) >= 2 || f0)
            res += DP(s, i + 1, j, j == h && lim, f0 && !j);
    return dp[i][pre][lim][f0] = res;
}

```

### 8.2 SOS dp

```

for(int i = 0; i<(1<<N); ++i) F[i] = A[i];
for(int i = 0; i < N; ++i)
    for(int mask = 0; mask < (1<<N); ++mask)
        if(mask & (1<<i))
            F[mask] += F[mask^(1<<i)];

```

### 8.3 p-median

```

void p_Median() {
    for (int i=1; i<=N; ++i)
        for (int j=1; j<=N; ++j) {
            m = (i+j)/2,d[i][j] = 0; // m是中位數 · d[i][j]為
            距離的總和
            for (int k=i; k<=j; ++k) d[i][j] += abs(arr[k] -
                arr[m]);
        }
    for (int p=1; p<=P; ++p)
        for (int n=1; n<=N; ++n) {
            dp[p][n] = 1e9;
            for (int k=p; k<=n; ++k)
                if (dp[p-1][k-1] + d[k][n] < dp[p][n]) {
                    dp[p][n] = dp[p-1][k-1] + d[k][n];
                    r[p][n] = k; // 從第k個位置往右到第j個位置
                }
        }
}

```

## 9 Other

### 9.1 黑魔法

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include <ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1, h2; h1.join( h2 );
    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0, t.c_str() );
    r[ 1 ].erase( 1, 1 );
    cout << r[ 1 ].substr( 0, 2 );
}
```

### 9.2 DLX

```
// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for( int i=D[c]; i!=c; i=D[i] ){
        for( int j=R[i]; j!=i; j=R[j] ){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
    }
}
void resume(int c){
    for( int i=D[c]; i!=c; i=D[i] ){
        for( int j=L[i]; j!=i; j=L[j] ){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    }
    L[R[c]]=R[L[c]]=c;
}
int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for( int i=R[0]; i!=0; i=R[i] ){
        if(S[i]<md){ md=S[i]; c=i; }
    }
    if(md==0) return 0;
    remove(c);
    for( int i=D[c]; i!=c; i=D[i] ){
        used[ROW[i]]=1;
        for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
        if(dfs()) return 1;
        for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n,int m){
    for( int i=0; i<=m; i++ ){
```

```
R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
S[i]=0; C[i]=i;
}
R[m]=0; L[0]=m;
int t=m+1;
for( int i=0; i<n; i++ ){
    int k=-1;
    for( int j=0; j<m; j++ ){
        if(!A[i][j]) continue;
        if(k==-1) L[t]=R[t]=t;
        else{ L[t]=k; R[t]=R[k]; }
        k=t; D[t]=j+1; U[t]=U[j+1];
        L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
        C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
    }
}
for( int i=0; i<n; i++ ) used[i]=0;
return dfs();
}
```

### 9.3 Hiber Curve

```
long long hilbert(int n,int x,int y){
    long long res=0;
    for(int s=n/2;s>=1){
        int rx=(x&s)>0,ry=(y&s)>0; res+=s*1ll*s*((3*rx)^ry)
        ;
        if(ry==0){ if(rx==1) x=s-1-x,y=s-1-y; swap(x,y); }
    }
    return res;
}
```

### 9.4 模拟退火

```
mt19937 rng((unsigned long long)(new char));
auto rnd = [&]() -> double {
    return 2 * ((double)rng() / rng.max()) - 1;
};

auto run = [&](int l, int r, int u, int d) -> double {
    double x = (l+r)/2., y = (u+d)/2., s = cal(x, y);
    double nx, ny;
    for (double t = hypot(l-r, u-d); t >= 1e-8; t
        *= 0.99995) {
        do {
            nx = x + t * rnd();
            ny = y + t * rnd();
        } while (!safe(nx, ny));
        if (chmax(s, cal(nx, ny)))
            x = nx, y = ny;
    }
    return s;
};
```





