Contents

1 Basic

1.1 Default code

```cpp
#include<bits/stdc++.h>
#define int long long
#define mod 1000000007
#define endl '\n'
#define pii pair<int,int>
using namespace std;

signed main(){
  ios::sync_with_stdio(0),cin.tie(0);
}
```

1.2 Linux 對拍

```sh
set -e
for ((i=0;i<300;i++))
do
    echo "$i"
    python gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

1.3 Windows 對拍

```bat
@echo off
:loop
    echo %%x
    python gen.py > input
    ./ac.exe < input > ac.out
    ./wa.exe < input > wa.out
    fc ac.out wa.out
if not errorlevel 1 goto loop
```

1.4 builtin 函數

```cpp
// 右邊第一個 1 的位置
int __builtin_ffs(unsigned int);
int __builtin_ffsl(unsigned long);
int __builtin_ffsll(unsigned long long);
// 左邊第一個 1 之前 0 的數量
int __builtin_clz(unsigned int);
int __builtin_clzl(unsigned long);
int __builtin_clzll(unsigned long long);
// 右邊第一個 1 之後 0 的數量
int __builtin_ctz(unsigned int);
int __builtin_ctzl(unsigned long);
int __builtin_ctzll(unsigned long long);
// 1 的數量
int __builtin_popcount(unsigned int);
int __builtin_popcountl(unsigned long);
int __builtin_popcountll(unsigned long long);
// 1 的數量 mod 2
int __builtin_parity(unsigned int);
int __builtin_parityl(unsigned long);
int __builtin_parityll(unsigned long long);
// 二進制表示數字
int a = 0b101101;
```

1.5 輸入輸出

```cpp
// 開讀檔
fropen("input_file_name","r",stdin);
fropen("output_file_name","w",stdout);
```

1.6 Python 輸入輸出

```py
a = list(map(int,input().split()))

# 開讀檔
import sys, os.path
if(os.path.exists('input_file.txt')):
    sys.stdin = open("input_file.txt","r")
    sys.stdout = open("output_file.txt","w")
```

2 Data Structure

2.1 Link-Cut Tree

```
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
  int val, rev, size;
  Splay (int _val=-1) : val(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d){
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void push(){
    if( !rev ) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
    if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
  }
  void pull(){
    size = ch[0]->size + ch[1]->size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
  splayVec.clear();
  for (Splay *q=x;; q=q->f){
    splayVec.push_back(q);
    if (q->isr()) break;
  }
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir()==x->f->dir())
      rotate(x->f),rotate(x);
    else rotate(x),rotate(x);
  }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
  Splay *q = nil;
  for (;x!=nil;x=x->f){
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void chroot(Splay *x){
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y){
  access(x);
  splay(x);
  chroot(y);
  x->setCh(y, 1);
}
void cut_p(Splay *y) {
  access(y);
  splay(y);
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
  chroot(x);
  cut_p(y);
}
Splay* get_root(Splay *x) {
  access(x);
  splay(x);
  for(; x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  x = get_root(x);
  y = get_root(y);
  return x == y;
}
Splay* lca(Splay *x, Splay *y) {
  access(x);
  access(y);
  splay(x);
  if (x->f == nil) return x;
  else return x->f;
}
```

2.2 持久化線段樹

```
struct Seg{
    struct Node{
        int v;
        Node* l,*r;
    };
    vector<Node*> version;
    Node* build(int l,int r){
        Node* node=new Node;
        if(l==r){
           node->v=l;
           return node;
        }
        int mid=(l+r)/2;
        node->l=build(l,mid);
        node->r=build(mid+1,r);
        return node;
    }
    int query(Node* cur,int l,int r,int x){
        if(l==r){
            return cur->v;
        }
        int mid=(l+r)/2;
        if(x<=mid) return query(cur->l,l,mid,x);
        else return query(cur->r,mid+1,r,x);
    }
    Node* update(Node* cur,int l,int r,int x,int y){
        Node* node=new Node;
        if(l==r){
           node->v=y;
           return node;
        }
        int mid=(l+r)/2;
        if(x<=mid){
            node->l=update(cur->l,l,mid,x,y);
            node->r=cur->r;
        }
        else{
            node->l=cur->l;
            node->r=update(cur->r,mid+1,r,x,y);
        }
        return node;
    }
};
```

2.3 Treap

```
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count()); // C++ randomizer
struct Node {
    int k, p, sz = 1;
```

```
    Node *l = 0, *r = 0;
    bool tag = 0;
    Node(int kk) {
        k = kk;
        p = gen();
    }
};
Node *root = 0;
int size(Node *x) {return x ? x->sz : 0;}
void push(Node *x) {
    if(x->tag) {
        if(x->l) x->l->tag ^= true;
        if(x->r) x->r->tag ^= true;
        x->tag = false;
    }
}
void pull(Node* x) {
    x->sz = size(x->l) + size(x->r) + 1;
}
Node* merge(Node *a, Node *b) {
    if(!a || !b) return a ?: b;
    if(a->p > b->p) {
        push(a);
        a->r = merge(a->r, b);
        pull(a);
        return a;
    }
    else{
        push(b);
        b->l = merge(a, b->l);
        pull(b);
        return b;
    }
}
void splitKey(Node* x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(x->k <= k) {
        a = x;
        splitKey(a->r, k, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKey(b->l, k, a, b->l);
        pull(b);
    }
}
void splitKth(Node *x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(size(x->l) < k) {
        a = x;
        splitKth(a->r, k - size(x->l) - 1, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKth(b->l, k, a, b->l);
        pull(b);
    }
}
void insert(int id) {
    Node *l, *r;
    splitKey(root, id, l, r);
    Node *m = new Node(id);
    root = merge(l, merge(m, r));
}
void erase(int x) {
    Node *a, *b, *c;
    splitKey(root, x, b, c);
    splitKey(b, x - 1, a, b);
    root = merge(a, c);
}
```

2.4 線段樹

```
struct Seg{
    vector<int> seg,tag;
    #define cl (i<<1)+1
    #define cr (i<<1)+2
    void push(int i,int l,int r){
        if(tag[i]!=0){
            seg[i]+=tag[i]; // update by tag
            if(l!=r){
                tag[cl]+=tag[i]; // push
                tag[cr]+=tag[i]; // push
            }
            tag[i]=0;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl,l,mid);push(cr,mid+1,r);
        seg[i]=max(seg[cl],seg[cr]); // pull
    }
    void build(int i,int l,int r,vector<int>&arr){
        if(l==r){
            seg[i]=arr[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl,l,mid,arr);
        build(cr,mid+1,r,arr);
        pull(i,l,r);
    }
    Seg(vector<int>& arr){
        seg.resize(arr.size()*4);
        tag.resize(arr.size()*4);
        build(0,0,arr.size()-1,arr);
    }
    void update(int i,int l,int r,int nl,int nr,int x){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            tag[i]+=x;
            return;
        }
        int mid=(l+r)>>1;
        if(nl<=mid) update(cl,l,mid,nl,nr,x);
        if(nr>mid) update(cr,mid+1,r,nl,nr,x);
        pull(i,l,r);
    }
    int query(int i,int l,int r,int nl,int nr){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            return seg[i];
        }
        int mid=(l+r)>>1;
        int ans=0;
        if(nl<=mid) ans=max(ans,query(cl,l,mid,nl,nr));
        if(nr>mid) ans=max(ans,query(cr,mid+1,r,nl,nr))
            ;
        return ans;
    }
};
```

3 Flow

3.1 Dinic

```
const int inf = 1e8;
struct Dinic{
  #define SZ(x) (int)(x.size())
  struct Edge{
    int v,f,re;
  };
  vector<vector<Edge>> E;
  vector<int> level;
  int n,s,t;
  Dinic(int nn,int ss,int tt){
    n=nn;s=ss;t=tt;
    E.resize(n);
    level.resize(n);
  }
  void addEdge(int u,int v,int w){
    E[u].push_back({v,w,SZ(E[v])});
    E[v].push_back({u,0,SZ(E[u])-1});
  }
  bool bfs(){
    level.assign(n,0);
    queue<int> q;
    q.push(s);
    level[s]=1;
    while(q.size()){
```

```
      int u=q.front();q.pop();
      for(auto&it:E[u]){
        int v=it.v;
        if(it.f>0 && !level[v]){
          level[v]=level[u]+1;
          q.push(v);
        }
      }
    }
    return level[t];
  }
  int dfs(int u,int nf){
    if(u==t)return nf;
    int ret=0;
    for(auto&it:E[u]){
      int v=it.v;
      if(it.f>0&&level[v]==level[u]+1){
        int tem = dfs(v,min(nf,it.f));
        ret+=tem;nf-=tem;
        it.f-=tem;E[v][it.re].f+=tem;
        if(!nf)return ret;
      }
    }
    if(!ret)level[u]=0;
    return ret;
  }
  int flow(){
    int ret=0;
    while(bfs()) ret+=dfs(s,inf);
    return ret;
  }
};
```

3.2 匈牙利

```
#define NIL -1
#define INF 100000000
int n,matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN],ylabel[MAXN];
int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-
    cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
  int i,delta=INF;
  for(i=0;i<n;i++) if(!sett[i]) delta=min(slack[i],
      delta);
  for(i=0;i<n;i++) if(sets[i]) xlabel[i]-=delta;
  for(i=0;i<n;i++) {
    if(sett[i]) ylabel[i]+=delta;
    else slack[i]-=delta;
  }
}
inline void add_sets(int x) {
  int i;
  sets[x]=1;
  for(i=0;i<n;i++) {
    if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {
      slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
      prev[i]=x;
    }
  }
}
inline void augment(int final) {
  int x=prev[final],y=final,tmp;
  matched++;
  while(1) {
    tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
    if(y==NIL) return;
    x=prev[y];
  }
}
inline void phase() {
  int i,y,root;
  for(i=0;i<n;i++) { sets[i]=sett[i]=0; slack[i]=INF; }
  for(root=0;root<n&&xy[root]!=NIL;root++);
  add_sets(root);
  while(1) {
    relabel();
    for(y=0;y<n;y++) if(!sett[y]&&slack[y]==0) break;
    if(yx[y]==NIL) { augment(y); return; }
    else { add_sets(yx[y]); sett[y]=1; }
  }
}
inline int hungarian() {
  int i,j,c=0;
  for(i=0;i<n;i++) {
    xy[i]=yx[i]=NIL;
    xlabel[i]=ylabel[i]=0;
    for(j=0;j<n;j++) xlabel[i]=max(cost[i][j],xlabel[i
        ]);
  }
  for(i=0;i<n;i++) phase();
  for(i=0;i<n;i++) c+=cost[i][xy[i]];
  return c;
}
```

3.3 KM

```
struct KM{ // max weight, for min negate the weights
  int n, mx[MXN], my[MXN], pa[MXN];
  ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
  bool vx[MXN], vy[MXN];
  void init(int _n) { // 1-based
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
  }
  void addEdge(int x, int y, ll w) {g[x][y] = w;}
  void augment(int y) {
    for(int x, z; y; y = z)
      x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
  }
  void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
      while(q.size()) {
        int x=q.front(); q.pop(); vx[x]=1;
        for(int y=1; y<=n; ++y) if(!vy[y]){
          ll t = lx[x]+ly[y]-g[x][y];
          if(t==0){
            pa[y]=x;
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
          }else if(sy[y]>t) pa[y]=x,sy[y]=t;
      } }
      ll cut = INF;
      for(int y=1; y<=n; ++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
      for(int j=1; j<=n; ++j){
        if(vx[j]) lx[j] -= cut;
        if(vy[j]) ly[j] += cut;
        else sy[j] -= cut;
      }
      for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){augment(y);return;}
        vy[y]=1, q.push(my[y]);
  } } }
  ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
} }graph;
```

3.4 MCMF

```
struct MCMF {
    #define SZ(x) (int)(x.size())
    struct Edge {
        int v, f, re, c;
    };
    vector<vector<Edge>> E;
    vector<int> dis, x, y;
    int n, s, t;
    MCMF(int nn, int ss, int tt) {
        n = nn; s = ss; t = tt;
```

```
        E.resize(n);
        x.resize(n);
        y.resize(n);
    }
    void addEdge(int u, int v, int w, int c) {
        E[u].push_back({v, w, SZ(E[v]), c});
        E[v].push_back({u, 0, SZ(E[u]) - 1, -c});
    }
    bool spfa() {
        dis.assign(n, 0x3f3f3f3f);
        x.assign(n, -1);
        y.assign(n, -1);
        vector<bool> inq(n, false);
        queue<int> q;
        q.push(s);
        inq[s] = true;
        dis[s] = 0;
        while(q.size()) {
            int u = q.front(); q.pop();
            inq[u] = false;
            for(int i = 0; i < E[u].size(); i++) {
                auto& it = E[u][i];
                int v = it.v;
                if(it.f > 0 && dis[v] > dis[u] + it.c)
                    {
                    dis[v] = dis[u] + it.c;
                    x[v] = u;
                    y[v] = i;
                    if(!inq[v]) {
                        q.push(v);
                        inq[v] = true;
                    }
                }
            }
        }
        return x[t] != -1;
    }
    pii solve() {
        int mf = 0, mc = 0;
        while(spfa()) {
            int nf = 0x3f3f3f3f;
            for(int i = t; i != s; i = x[i]) {
                nf = min(nf, E[x[i]][y[i]].f);
            }
            for(int i = t; i != s; i = x[i]) {
                auto& it = E[x[i]][y[i]];
                it.f -= nf;
                E[it.v][it.re].f += nf;
            }
            mf += nf;
            mc += nf * dis[t];
        }
        return {mf, mc};
    }
};
```

4 幾何

4.1 宣告

```
typedef long double ld;
const ld eps = 1e-10;
#define sign(x) (((x) > eps) - ((x) < -eps))
#define sq(p) ((p)*(p))
#define len(p) (sqrt(sq(p)))
#define r90(p) Pt(-(p).y, (p).x)

struct Pt {
  ld x, y;
  Pt(ld x = 0, ld y = 0) : x(x), y(y) {}
  Pt operator + (const Pt &a) const {
    return Pt(x + a.x, y + a.y);  }
  Pt operator - (const Pt &a) const {
    return Pt(x - a.x, y - a.y);  }
  Pt operator * (const ld &a) const {
    return Pt(x * a, y * a);  }
  Pt operator / (const ld &a) const {
    return Pt(x / a, y / a);  }
  ld operator * (const Pt &a) const {
    return x * a.x + y * a.y;  }
  ld operator ^ (const Pt &a) const {
    return x * a.y - y * a.x;  }
  bool operator < (const Pt &a) const {
    return sign(x - a.x) < 0 || sign(x - a.x) == 0 &&
        sign(y - a.y) < 0; }
  bool operator == (const Pt &a) const {
    return sign(x - a.x) == 0 && sign(y - a.y) == 0; }
};

struct Line {
  Pt s, e, v; // start, end, end-start
  ld rad;
  Line(Pt s = Pt(), Pt e = Pt())
    : s(s), e(e), v(e - s), rad(atan2(v.y, v.x)) {}
  bool operator < (const Line &L) const {
    return rad < L.rad;
} };

struct Circle {
  Pt o; ld r2;
  Circle(Pt o = Pt(), ld r = 0) : o(o), r2(sq(r)) {}
  Circle(const Pt &p1, const Pt &p2)
    : o((p1 + p2) / 2), r2(sq(p1 - p2) / 4.0) {}
  Circle(const Pt &p1, const Pt &p2, const Pt &p3) {
    Pt va = r90(p1 - p2), vb = r90(p1 - p3);
    if (sign(va ^ vb) == 0) {
      *this = Circle(p1, p2);
      Circle t(p1, p3);
      if (r2 < t.r2) *this = t;
      t = Circle(p2, p3);
      if (r2 < t.r2) *this = t; }
    else {
      Pt p12 = (p1 + p2) / 2, p13 = (p1 + p3) / 2;
      ld t = ((p13 - p12) * (p1 - p3)) / (va ^ vb);
      o = p12 + va*t;
      r2 = sq(o - p1);
    }
  }
  bool contain(const Pt &a) {
    return sign(sq(a - o) - r2) <= 0; }
};
```

4.2 矩形面積

```
struct AreaofRectangles{
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
    ll n, id, sid;
    pair<ll,ll> tree[MXN<<3];   // count, area
    vector<ll> ind;
    tuple<ll,ll,ll,ll> scan[MXN<<1];
    void pull(int i, int l, int r){
        if(tree[i].first)  tree[i].second = ind[r+1] -
            ind[l];
        else if(l != r){
            int mid = (l+r)>>1;
            tree[i].second = tree[cl(i)].second + tree[
                cr(i)].second;
        }
        else    tree[i].second = 0;
    }
    void upd(int i, int l, int r, int ql, int qr, int v
        ){
        if(ql <= l && r <= qr){
            tree[i].first += v;
            pull(i, l, r); return;
        }
        int mid = (l+r) >> 1;
        if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
        if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
        pull(i, l, r);
    }
    void init(int _n){
        n = _n; id = sid = 0;
        ind.clear(); ind.resize(n<<1);
        fill(tree, tree+(n<<2), make_pair(0, 0));
    }
    void addRectangle(int lx, int ly, int rx, int ry){
        ind[id++] = lx; ind[id++] = rx;
        scan[sid++] = make_tuple(ly, 1, lx, rx);
        scan[sid++] = make_tuple(ry, -1, lx, rx);
    }
    ll solve(){
        sort(ind.begin(), ind.end());
```

```
        ind.resize(unique(ind.begin(), ind.end()) - ind
            .begin());
        sort(scan, scan + sid);
        ll area = 0, pre = get<0>(scan[0]);
        for(int i = 0; i < sid; i++){
            auto [x, v, l, r] = scan[i];
            area += tree[1].second * (x-pre);
            upd(1, 0, ind.size()-1, lower_bound(ind.
                begin(), ind.end(), l)-ind.begin(),
                lower_bound(ind.begin(),ind.end(),r)-
                ind.begin()-1, v);
            pre = x;
        }
        return area;
}   }rect;
```

4.3 最近點對

```
#include<bits/stdc++.h>
#define int long long
using namespace std;
using ld = long double;
const int mod = 1e9+7;
struct pt{
    int x,y;
    int id;
    ld dis(const pt& rhs){
        return sqrt((x-rhs.x)*(x-rhs.x)+(y-rhs.y)*(y-
            rhs.y));
    }
};
signed main(){
    int n;
    cin>>n;
    vector<pt> a(n);
    for(int i=0;i<n;i++){
        cin>>a[i].x>>a[i].y;
        a[i].id=i;
    }
    ld ans = 1e19;
    sort(a.begin(),a.end(),[](const pt&a,const pt&b){
        if(a.x==b.y)return a.y<b.y;
        return a.x<b.x;
    });
    pt ans2;
    function<void(int,int)> dnq = [&](int l,int r){
        if(r-l<4){
            for(int i=l;i<=r;i++){
                for(int j=i+1;j<=r;j++){
                    ld temans = a[i].dis(a[j]);
                    if(temans<ans){
                        ans=temans;
                        ans2 = {a[i].id,a[j].id};
                    }
                }
            }
            sort(a.begin()+l,a.begin()+r+1,[](const pt&
                a,const pt&b){return a.y<b.y;});
            return;
        }
        int mid = (l+r)/2;
        int midx = a[mid].x;
        dnq(l,mid);dnq(mid+1,r);
        inplace_merge(a.begin()+l,a.begin()+mid+1,a.
            begin()+r+1,[](const pt&a,const pt&b){
            return a.y<b.y;});
        vector<int> c;c.reserve(r-l+1);
        for(int i=l;i<=r;i++){
            if(abs(a[i].x-midx)<ans){
                for(int j=c.size()-1;j>=0&&a[i].y-a[c[j
                    ]].y<ans;j--){
                    ld temans = a[i].dis(a[c[j]]);
                        if(temans<ans){
                            ans=temans;
                            ans2 = {a[i].id,a[c[j]].id
                                };
                        }
                }
            }
            c.push_back(i);
        }
    };
    dnq(0,n-1);
    cout<<min(ans2.x,ans2.y)<<' '<<max(ans2.x,ans2.y)<<
        ' '<<fixed<<setprecision(6)<<ans<<'\n';
}
```

4.4 凸包

```
double cross(Pt o, Pt a, Pt b){
  return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<Pt> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){
    while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  stk.resize(top-1);
  return stk;
}
```

4.5 兩直線交點

```
pair<int, Pt> L_intersect(const Line &a, const Line &b)
    {
  ld f1 = a.v ^ (b.s - a.s), f2 = a.v ^ (a.s - b.e), f;
  if(sign(f = f1 + f2) == 0)
    return sign(f1) ? {0, a.s} : {2, a.s};
  return {1, b.s * (f2 / f) + b.e * (f1 / f)};
}
```

4.6 兩線段交點

```
inline int ori(const Pt &o, const Pt &a, const Pt &b) {
  return sign((a - o) ^ (b - o)); }
// 0:out, 1:ontop, 2:in
inline int btw(const Pt &a, const Pt &b, const Pt &c) {
  if (ori(a, b, c)) return 0;
  int s = sign((c - a) * (c - b));
  return (s < 0) + (s <= 0); }
// 0:no, 1:1pt(parallel), -1:1pt, 2:inf pt
int banana(const Pt &p1, const Pt &p2,
           const Pt &p3, const Pt &p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if (a123 == 0 && a124 == 0) {
    return btw(p1, p2, p3) + btw(p1, p2, p4) +
           btw(p3, p4, p1) + btw(p3, p4, p2) >> 1;
  }
  return -(a123 * a124 <= 0 && a341 * a342 <= 0);
}
pair<int, Pt> S_intersect(const Pt &p1, const Pt &p2,
                          const Pt &p3, const Pt &p4) {
    int b = banana(p1, p2, p3, p4);
    if (b != -1) return {b, btw(p1, p2, p3) ? p3 : p4};
    ld a123 = (p2 - p1) ^ (p3 - p1);
    ld a124 = (p2 - p1) ^ (p4 - p1);
    return {1, (p4 * a123 - p3 * a124) / (a123 - a124)
        };
}
```

4.7 李超線段樹

```
struct LiChao_min{
  struct line{
    ll m,c;
    line(ll _m=0,ll _c=0){ m=_m; c=_c; }
    ll eval(ll x){ return m*x+c; } // overflow
  };
  struct node{
```

```
    node *l,*r; line f;
    node(line v){ f=v; l=r=NULL; }
  };
  typedef node* pnode;
  pnode root; ll sz,ql,qr;
#define mid ((l+r)>>1)
  void insert(line v,ll l,ll r,pnode &nd){
    /* if(!(ql<=l&&r<=qr)){
      if(!nd) nd=new node(line(0,INF));
      if(ql<=mid) insert(v,l,mid,nd->l);
      if(qr>mid) insert(v,mid+1,r,nd->r);
      return;
    } used for adding segment */
    if(!nd){ nd=new node(v); return; }
    ll trl=nd->f.eval(l),trr=nd->f.eval(r);
    ll vl=v.eval(l),vr=v.eval(r);
    if(trl<=vl&&trr<=vr) return;
    if(trl>vl&&trr>vr) { nd->f=v; return; }
    if(trl>vl) swap(nd->f,v);
    if(nd->f.eval(mid)<v.eval(mid))
      insert(v,mid+1,r,nd->r);
    else swap(nd->f,v),insert(v,l,mid,nd->l);
  }
  ll query(ll x,ll l,ll r,pnode &nd){
    if(!nd) return INF;
    if(l==r) return nd->f.eval(x);
    if(mid>=x)
      return min(nd->f.eval(x),query(x,l,mid,nd->l));
    return min(nd->f.eval(x),query(x,mid+1,r,nd->r));
  }
  /* -sz<=ll query_x<=sz */
  void init(ll _sz){ sz=_sz+1; root=NULL; }
  void add_line(ll m,ll c,ll l=-INF,ll r=INF){
    line v(m,c); ql=l; qr=r; insert(v,-sz,sz,root);
  }
  ll query(ll x) { return query(x,-sz,sz,root); }
};
```

4.8 最小包覆圓

```
// remember to shuffle!!!
Circle encircle(const vector<Pt> &pts) {
  Circle ans(Pt(), 0);
  for (int i = 0; i < pts.size(); ++i) {
    if (ans.contain(pts[i])) continue;
    ans = {pts[i], 0};
    for (int j = 0; j < i; ++j) {
      if (ans.contain(pts[j])) continue;
      ans = {pts[i], pts[j]};
      for (int k = 0; k < j; ++k) {
        if (ans.contain(pts[k])) continue;
        ans = {pts[i], pts[j], pts[k]}; } } }
  return ans;
}
```

4.9 最小包覆球

```
// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
  Pt q[3]; double m[3][3], sol[3], L[3], det;
  int i,j; res.x = res.y = res.z = radius = 0;
  switch ( nouter ) {
    case 1: res=outer[0]; break;
    case 2: res=(outer[0]+outer[1])/2; radius=norm2(res
        , outer[0]); break;
    case 3:
      for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
      for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q
          [i] * q[j])*2;
      for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
      if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps
          ) return;
      L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
      L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
      res=outer[0]+q[0]*L[0]+q[1]*L[1];
      radius=norm2(res, outer[0]);
      break;
    case 4:
      for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
          [i]=(q[i] * q[i]);
      for (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=(q[i]
          * q[j])*2;
      det= m[0][0]*m[1][1]*m[2][2]
        + m[0][1]*m[1][2]*m[2][0]
        + m[0][2]*m[2][1]*m[1][0]
        - m[0][2]*m[1][1]*m[2][0]
        - m[0][1]*m[1][0]*m[2][2]
        - m[0][0]*m[1][2]*m[2][1];
      if ( fabs(det)<eps ) return;
      for (j=0; j<3; ++j) {
        for (i=0; i<3; ++i) m[i][j]=sol[i];
        L[j]=( m[0][0]*m[1][1]*m[2][2]
             + m[0][1]*m[1][2]*m[2][0]
             + m[0][2]*m[2][1]*m[1][0]
             - m[0][2]*m[1][1]*m[2][0]
             - m[0][1]*m[1][0]*m[2][2]
             - m[0][0]*m[1][2]*m[2][1]
           ) / det;
        for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
      } res=outer[0];
      for (i=0; i<3; ++i ) res = res + q[i] * L[i];
      radius=norm2(res, outer[0]);
}}
void minball(int n){ ball();
  if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
    if( norm2(res, pt[i]) - radius > eps ){
      outer[ nouter ++ ] = pt[ i ]; minball(i); --
          nouter;
      if(i>0){ Pt Tt = pt[i];
        memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
            ;
}}}
double solve(){
  // n points in pt
  random_shuffle(pt, pt+n); radius=-1;
  for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
    nouter=1, outer[0]=pt[i], minball(i);
  return sqrt(radius);
}
```

4.10 旋轉卡尺

```
int FarthestPair(vector<Pt>& arr){
    int ret=0;
    for(int i = 0, j = i+1; i<arr.size(); i++){
        while(distance(arr[i], arr[j]) < distance(arr[i
            ], arr[(j+1)%arr.size()]) ){
            j = (j+1) % arr.size();
        }
        ret = max(ret, distance(arr[i],arr[j]));
    }
    return ret;
}
```

4.11 Circle Cover

```
#define N 1021
#define D long double
struct CircleCover{
  int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
  bool g[ N ][ N ], overlap[ N ][ N ];
  // Area[i] : area covered by at least i circles
  D Area[ N ];
  void init( int _C ){ C = _C; }
  bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
    Pt o1 = a.O , o2 = b.O;
    D r1 = a.R , r2 = b.R;
    if( norm( o1 - o2 ) > r1 + r2 ) return {};
    if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
        return {};
    D d2 = ( o1 - o2 ) * ( o1 - o2 );
    D d = sqrt(d2);
    if( d > r1 + r2 ) return false;
    Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
  }
  struct Teve {
```

```
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[ N * 2 ];
  // strict: x = 0, otherwise x = -1
  bool disjuct( Circ& a, Circ &b, int x )
  {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
  bool contain( Circ& a, Circ &b, int x )
  {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
  bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
                contain(c[i], c[j], -1);
  }
  void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
      Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
      int E = 0, cnt = 1;
      for( int j = 0 ; j < C ; j ++ )
        if( j != i && overlap[j][i] )
          cnt ++;
      for( int j = 0 ; j < C ; j ++ )
        if( i != j && g[i][j] ){
          Pt aa, bb;
          CCinter(c[i], c[j], aa, bb);
          D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E ++] = Teve(bb, B, 1);
          eve[E ++] = Teve(aa, A, -1);
          if(B > A) cnt ++;
        }
      if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
      else{
        sort( eve , eve + E );
        eve[E] = eve[0];
        for( int j = 0 ; j < E ; j ++ ){
          cnt += eve[j].add;
          Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
          D theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2.0 * pi;
          Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}}};
```

4.12 Convex Hull Trick

```
/* Given a convexhull, answer querys in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
  int n;
  vector<Pt> a;
  vector<Pt> upper, lower;
  Conv(vector<Pt> _a) : a(_a){
    n = a.size();
    int ptr = 0;
    for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
    for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
    for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
    upper.push_back(a[0]);
  }
  int sign( LL x ){ // fixed when changed to double
    return x < 0 ? -1 : x > 0; }
  pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
    for( ; l + 1 < r; ){
      int mid = (l + r) / 2;
      if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
      else l = mid;
    }
    return max(make_pair(det(vec, conv[r]), r),
               make_pair(det(vec, conv[0]), 0));
  }
  void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
  }
  void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
      if (smid == sl) l = mid;
      else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
  }
  int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid = sign(det(v - u, a[mid % n] - u));
      if (smid == sl) l = mid;
      else r = mid;
    }
    return l % n;
  }
  // 1. whether a given point is inside the CH
  bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
      if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
      if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
  }
  // 2. Find 2 tang pts on CH of a given outside point
  // return true with i0, i1 as index of tangent points
  // return false if inside CH
  bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
  }
  // 3. Find tangent points of a given vector
  // ret the idx of vertex has max cross value with vec
  int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
  }
  // 4. Find intersection point of a given line
  // return 1 and intersection is on edge (i, next(i))
  // return 0 if no strictly intersection
  bool get_intersection(Pt u, Pt v, int &i0, int &i1){
   int p0 = get_tang(u - v), p1 = get_tang(v - u);
   if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
     if (p0 > p1) swap(p0, p1);
     i0 = bi_search(u, v, p0, p1);
     i1 = bi_search(u, v, p1, p0 + n);
     return 1;
```

```
  }
  return 0;
} };
```

4.13 Half Plane Intersection

```
// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
  return dcmp(L.v^(p-L.s)) > 0;
} // segment should add Counterclockwise
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
  sort(L.begin(), L.end()); // sort by angle
  int n = L.size(), fir, las;
  Pt *p = new Pt[n];
  Line *q = new Line[n];
  q[fir=las=0] = L[0];
  for(int i = 1 ; i < n ; i++) {
    while(fir < las && !onleft(L[i], p[las-1])) las--;
    while(fir < las && !onleft(L[i], p[fir])) fir++;
    q[++las] = L[i];
    if(dcmp(q[las].v^q[las-1].v) == 0) {
      las--;
      if(onleft(q[las], L[i].s)) q[las] = L[i];
    }
    if(fir < las) p[las-1] = LLIntersect(q[las-1], q[
        las]);
  }
  while(fir < las && !onleft(q[fir], p[las-1])) las--;
  if(las-fir <= 1) return {};
  p[las] = LLIntersect(q[las], q[fir]);
  int m = 0;
  vector<Pt> ans(las-fir+1);
  for(int i = fir ; i <= las ; i++) ans[m++] = p[i];
  return ans;
}
```

5 圖論

5.1 BCC

```
struct BCC {
  vector<vector<int>> g;
  vector<int> dfn,low;
  vector<vector<int>> bcc;
  vector<int> stk;
  int nbcc=0;
  int cur=1;
  BCC(int n){
    g.resize(n);
    dfn.resize(n,0);
    low.resize(n);
  }
  void addEdge(int u,int v){
    g[u].push_back(v);
    g[v].push_back(u);
  }
  void dfs(int x,int f){
    if(!g[x].size()){
      bcc.push_back({x});
      nbcc++;
      return;
    }
    dfn[x]=low[x]=cur++;
    stk.push_back(x);
    for(int y:g[x]){
      if(y==f)continue;
      if(dfn[y]){
        low[x]=min(low[x],dfn[y]);
      }
      else{
        dfs(y,x);
        low[x]=min(low[x],low[y]);
        if(low[y]>=dfn[x]){
          bcc.push_back({});
          int b;
          do{
            bcc[nbcc].push_back(b=stk.back());
            stk.pop_back();
          }while(b!=y);
          bcc[nbcc++].push_back(x);
        }
      }
    }
  }
  void solve(){
    for(int i=0;i<g.size();i++){
      if(!dfn[i]){
        dfs(i,-1);
      }
    }
  }
};
```

5.2 重心剖分

```
struct CentroidDecomposition {
    int n;
    vector<vector<int>> G, out;
    vector<int> sz, v;
    CentroidDecomposition(int _n) : n(_n), G(_n), out(
        _n), sz(_n), v(_n) {}
    int dfs(int x, int par){
        sz[x] = 1;
        for (auto &&i : G[x]) {
            if(i == par || v[i]) continue;
            sz[x] += dfs(i, x);
        }
        return sz[x];
    }
    int search_centroid(int x, int p, const int mid){
        for (auto &&i : G[x]) {
            if(i == p || v[i]) continue;
            if(sz[i] > mid) return search_centroid(i, x
                , mid);
        }
        return x;
    }
    void add_edge(int l, int r){
        G[l].PB(r); G[r].PB(l);
    }
    int get(int x){
        int centroid = search_centroid(x, -1, dfs(x,
            -1)/2);
        v[centroid] = true;
        for (auto &&i : G[centroid]) {
            if(!v[i]) out[centroid].PB(get(i));
        }
        v[centroid] = false;
        return centroid;
} };
```

5.3 輕重鍊剖分

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG  = 19;
struct HLD{
  int n;
  vector<int> g[MAXN];
  int sz[MAXN], dep[MAXN];
  int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
  //  ts : timestamp , useless after yutruli
  //  tid[ u ] : pos. of node u in the seq.
  //  tdi[ i ] : node at pos i of the seq.
  //  tl , tr[ u ] : subtree interval in the seq. of
      node u
  int prt[MAXN][LOG], head[MAXN];
  // head[ u ] : head of the chain contains u
  void dfssz(int u, int p){
    dep[u] = dep[p] + 1;
    prt[u][0] = p; sz[u] = 1; head[u] = u;
    for(int& v:g[u]) if(v != p){
      dep[v] = dep[u] + 1;
      dfssz(v, u);
      sz[u] += sz[v];
    }
  }
  void dfshl(int u){
    ts++;
    tid[u] = tl[u] = tr[u] = ts;
    tdi[tid[u]] = u;
    sort(ALL(g[u]),
```

```
        [&](int a, int b){return sz[a] > sz[b];});
    bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
      if(flag) head[v] = head[u], flag = 0;
      dfshl(v);
      tr[u] = tr[v];
    }
  }
  inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
      b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
      a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
  }
  void init( int _n ){
    n = _n; REP( i , 1 , n ) g[ i ].clear();
  }
  void addEdge( int u , int v ){
    g[ u ].push_back( v );
    g[ v ].push_back( u );
  }
  void yutruli(){ //build function
    dfssz(1, 0);
    ts = 0;
    dfshl(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
      prt[i][k] = prt[prt[i][k-1]][k-1];
  }
  vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){
      res.push_back( PII(tid[ head[ v ] ] , tid[ v ]) )
        ;
      v = prt[ head[ v ] ][ 0 ];
    }
    res.push_back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
    /* res : list of intervals from u to v
     * u must be ancestor of v
     * usage :
     * vector< PII >& path = tree.getPath( u , v )
     * for( PII tp : path ) {
     *   int l , r;tie( l , r ) = tp;
     *   upd( l , r );
     *   uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
     *   uu ~> vv is a heavy path on tree
     * }
     */
  }
} tree;
```

5.4 歐拉路徑

```
#define FOR(i,a,b) for(int i=a;i<=b;i++)
int dfs_st[10000500],dfn=0;
int ans[10000500],cnt=0,num=0;
vector<int>G[1000050];
int cur[1000050];
int ind[1000050],out[1000050];
void dfs(int x){
    FOR(i,1,n)sort(G[i].begin(),G[i].end());
    dfs_st[++dfn]=x;
    memset(cur,-1,sizeof(cur));
    while(dfn>0){
        int u=dfs_st[dfn];
        int complete=1;
        for(int i=cur[u]+1;i<G[u].size();i++){
            int v=G[u][i];
            num++;
            dfs_st[++dfn]=v;
            cur[u]=i;
            complete=0;
            break;
        }
        if(complete)ans[++cnt]=u,dfn--;
    }
}
bool check(int &start){
    int l=0,r=0,mid=0;
    FOR(i,1,n){
        if(ind[i]==out[i]+1)l++;
        if(out[i]==ind[i]+1)r++,start=i;
        if(ind[i]==out[i])mid++;
    }
    if(l==1&&r==1&&mid==n-2)return true;
    l=1;
    FOR(i,1,n)if(ind[i]!=out[i])l=0;
    if(l){
        FOR(i,1,n)if(out[i]>0){
            start=i;
            break;
        }
        return true;
    }
    return false;
}
int main(){
    cin>>n>>m;
    FOR(i,1,m){
        int x,y;scanf("%d%d",&x,&y);
        G[x].push_back(y);
        ind[y]++,out[x]++;
    }
    int start=-1,ok=true;
    if(check(start)){
        dfs(start);
        if(num!=m){
            puts("What a shame!");
            return 0;
        }
        for(int i=cnt;i>=1;i--)
            printf("%d ",ans[i]);
        puts("");
    }
    else puts("What a shame!");
}
```

5.5 極大團

```
#define N 80
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int lnk[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      lnk[i].reset(); v[i].reset();
  } }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int ans , stk[N], id[N] , di[N] , deg[N];
  Int cans;
  void dfs(int elem_num, Int candi, Int ex){
    if(candi.none()&&ex.none()){
      cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
      ans = elem_num; // cans is a maximal clique
      return;
    }
    int pivot = (candi|ex)._Find_first();
    Int smaller_candi = candi & (~lnk[pivot]);
    while(smaller_candi.count()){
      int nxt = smaller_candi._Find_first();
      candi[nxt] = smaller_candi[nxt] = 0;
      ex[nxt] = 1;
      stk[elem_num] = nxt;
      dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
  } }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
```

```
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) lnk[di[i]][di[j]] = 1;
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n,'1')), 0);
    return ans;
} }solver;
```

5.6 最大團

```
#define N 111
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int linkto[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      linkto[i].reset(); v[i].reset();
  } }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int popcount(const Int& val)
  { return val.count(); }
  int lowbit(const Int& val)
  { return val._Find_first(); }
  int ans , stk[N];
  int id[N] , di[N] , deg[N];
  Int cans;
  void maxclique(int elem_num, Int candi){
    if(elem_num > ans){
      ans = elem_num; cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
    }
    int potential = elem_num + popcount(candi);
    if(potential <= ans) return;
    int pivot = lowbit(candi);
    Int smaller_candi = candi & (~linkto[pivot]);
    while(smaller_candi.count() && potential > ans){
      int next = lowbit(smaller_candi);
      candi[next] = !candi[next];
      smaller_candi[next] = !smaller_candi[next];
      potential --;
      if(next == pivot || (smaller_candi & linkto[next
          ]).count()){
        stk[elem_num] = next;
        maxclique(elem_num + 1, candi & linkto[next]);
  } } }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) linkto[di[i]][di[j]] = 1;
    Int cand; cand.reset();
    for(int i = 0 ; i < n ; i ++) cand[i] = 1;
    ans = 1;
    cans.reset(); cans[0] = 1;
    maxclique(0, cand);
    return ans;
} }solver;
```

5.7 SCC

```
struct Scc{
    vector<vector<int>> g,rg;
    vector<int> scc;
    vector<bool> vis;
    vector<int> stk;
    int nscc=0;
    Scc(int n){
        g.resize(n);
        rg.resize(n);
        scc.resize(n);
        vis.resize(n);
    }
    void addEdge(int u,int v){
        g[u].push_back(v);
        rg[v].push_back(u);
    }
    void dfs1(int x){
        vis[x]=true;
        for(int y:g[x]){
            if(!vis[y])dfs1(y);
        }
        stk.push_back(x);
    }
    void dfs2(int x){
        vis[x]=true;
        scc[x]=nscc;
        for(int y:rg[x]){
            if(!vis[y])dfs2(y);
        }
    }
    void solve(){
        vis.assign(vis.size(),false);
        stk.clear();
        for(int i=0;i<vis.size();i++){
            if(!vis[i])dfs1(i);
        }
        reverse(stk.begin(),stk.end());
        vis.assign(n,false);
        for(int i:stk){
            if(!vis[i]){
                dfs2(i);
                nscc++;
            }
        }
    }
};
```

5.8 SPFA

```
#define MXN 200005
struct SPFA{
  int n;
  LL inq[MXN], len[MXN];
  vector<LL> dis;
  vector<pair<int, LL>> edge[MXN];
  void init(int _n){
    n = _n;
    dis.clear(); dis.resize(n, 1e18);
    for(int i = 0; i < n; i++){
      edge[i].clear();
      inq[i] = len[i] = 0;
  } }
  void addEdge(int u, int v, LL w){
    edge[u].push_back({v, w});
  }
  vector<LL> solve(int st = 0){
    deque<int> dq; //return {-1} if has negative cycle
    dq.push_back(st); //otherwise return dis from st
    inq[st] = 1; dis[st] = 0;
    while(!dq.empty()){
      int u = dq.front();  dq.pop_front();
      inq[u] = 0;
      for(auto [to, d] : edge[u]){
        if(dis[to] > d+dis[u]){
          dis[to] = d+dis[u];
          len[to] = len[u]+1;
          if(len[to] > n) return {-1};
          if(inq[to]) continue;
          (!dq.empty()&&dis[dq.front()] > dis[to]?
              dq.push_front(to) : dq.push_back(to));
          inq[to] = 1;
    } } }
    return dis;
} }spfa;
```

5.9 樹哈希

```
const ull mask = mt19937_64(time(nullptr))();
ull shift(ull x) {
  x ^= mask;
  x ^= x << 13;
  x ^= x >> 7;
  x ^= x << 17;
  x ^= mask;
  return x; }
```

```
void dfs(int u) { // edge[父] = {子}
  tree_hash[u] = 1;
  for (int v : edge[u]) {
    dfs(v);
    tree_hash[u] += shift(tree_hash[v]);
} }
```

5.10 差分約束

約束條件 $V_j - V_i \leq W$ addEdge(V_i, V_j, W) and run bellman-ford or spfa

6 數論

6.1 離散根號

```
int discrete_sqrt(int y, int p) { // find x s.t. x^2 =
    y (mod p)
    if (!y) return 0;
    if (p == 2) return (y & 1) == 1 ? 1 : -1;
    if (fpow(y, p - 1 >> 1, p) != 1) return -1;
    int Q = p - 1, S = 0;
    while (~Q & 1) { Q >>= 1; S++; }
    if (S == 1) return fpow(y, p + 1 >> 2, p);
    int z;
    while (1) {
        z = 1 + rand() % (p - 1);
        if (fpow(z, p - 1 >> 1, p) != 1) break;
    }
    int c = fpow(z, Q, p), R = fpow(y, Q + 1 >> 1, p);
    int t = fpow(y, Q, p), M = S, b, i;
    while (1) {
        if (t % p == 1) break;
        for (i = 1; i < M && fpow(t, 1LL << i, p) != 1;
             i++);
        b = fpow(c, 1LL << M - i - 1, p);
        R = R * b % p;
        c = fpow(b, 2, p);
        t = t * c % p; M = i;
    }
    return (R % p + p) % p;
}
```

6.2 離散對數

```
int BSGS(int start, int x, int y, int m) {
    unordered_map<int, int> mp;
    int big = 1, STEP = sqrt(m);
    for (int i = 0; i < STEP; i++) {
        mp[y] = i;
        y = y * x % m;
        big = big * x % m;
    }
    for (int step = 0; step < m + 10; step += STEP) {
        start = start * big % m;
        if (mp.count(start))
            return (step + STEP) - mp[start];
    }
    return -1;
}
int discrete_log(int x, int y, int m) { // find min k s
    .t. x^k = y (mod m)
    if (m == 1) return 0;
    int start = 1;
    for (int i = 0; i < 100; i++) {
        if (start == y) return i;
        start = start * x % m;
    }
    int pred = 100 + BSGS(start, x, y, m);
    if (fpow(x, pred, m) != y) return -1;
    return pred;
}
```

6.3 ex-crt

```
typedef __int128 ll;
void exgcd(ll a,ll b,ll &g,ll &x,ll &y) {
    if (b == 0) {
        g = a;
        x = 1;
        y = 0;
        return;
    }
    exgcd(b,a%b,g,y,x);
    y-=(a/b)*x;
}
bool flag = false;
ll a1,a2,n1,n2;
ll abs(ll x) {
    return x>0?x:-x;
}
void china() {
    ll d = a2 - a1;
    ll g,x,y;
    exgcd(n1,n2,g,x,y);
    if (d % g == 0) {
        x = ((x*d/g)%(n2/g)+(n2/g))%(n2/g);
        a1 = x*n1 + a1;
        n1 = (n1*n2)/g;
    }
    else
        flag = true;
}
int n;
long long as[100001];  //算式答案 x
long long ns[100001];  //模數 MOD
ll realchina() {
    a1 = as[0];
    n1 = ns[0];
    for (ll i = 1;i<n;i++) {
        a2 = as[i];
        n2 = ns[i];
        china();
        if (flag)
            return -1;
    }
    return a1;
}
int main() {
    cin>>n;
    flag = false;
    for (ll i = 0;i<n;i++)
        cin>>ns[i]>>as[i];
    cout<<(long long)realchina()<<endl;
}
```

6.4 ex-gcd

```
int exgcd(int a,int b,int&x,int&y){
    if(b==0)return x=1,y=0,a;
    int d = exgcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
```

6.5 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
  for(int i=0; i<=MAXN; i++)
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {
      cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                         : i*theta%MAXN];
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
    } }
    theta = (theta * 2) % MAXN;
  }
```

```
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
  if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n,ll a[],int _m,ll b[],ll ans[]){
  int n=1,sum=_n+_m-1;
  while(n<sum)
    n<<=1;
  for(int i=0;i<n;i++) {
    double x=(i<_n?a[i]:0),y=(i<_m?b[i]:0);
    arr[i]=complex<double>(x+y,x-y);
  }
  fft(n,arr);
  for(int i=0;i<n;i++)
    arr[i]=arr[i]*arr[i];
  fft(n,arr,true);
  for(int i=0;i<sum;i++)
    ans[i]=(long long int)(arr[i].real()/4+0.5);
}
```

6.6 高斯消去法

```
const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a , int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD ,
            k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD ,
            k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now , 0 , n){
            REP(i , now , n) if(v[now][now] == 0 && v[i
                ][now] != 0)
                swap(v[i] , v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now] , GAUSS_MOD - 2)
                ;
            REP(i , 0 , n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j , now , n + 1) (v[i][j] +=
                    GAUSS_MOD - tmp * v[now][j] %
                    GAUSS_MOD) %= GAUSS_MOD;
            }
        }
        REP(i , 0 , n) ans[i] = v[i][n + 1] * ppow(v[i
            ][i] , GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
    // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1
        , 0));
} gs;
```

6.7 喬瑟夫問題

```
int josephus(int n, int m){ //n人 每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

6.8 定理

- Lucas's Theorem :
  For $n, m \in \mathbb{Z}^*$ and prime P, $C(m,n) \mod P = \Pi(C(m_i, n_i))$ where m_i is the i-th digit of m in base P.
- Stirling approximation :
  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n e^{\frac{1}{12n}}$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
  $S(n,k) =$ coefficient of x^k in $\Pi_{i=0}^{n-1}(x+i)$
- Stirling Numbers(Partition n elements into k non-empty set):
  $S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$
- Pick's Theorem : $A = i + b/2 - 1$
  A: Area , i: grid number in the inner , b: grid number on the side
- Catalan number : $C_n = \binom{2n}{n}/(n+1)$
  $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)!\frac{n-m+1}{n+1} \quad for \quad n \geq m$
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
  $C_0 = 1 \quad and \quad C_{n+1} = 2(\frac{2n+1}{n+2})C_n$
  $C_0 = 1 \quad and \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad for \quad n \geq 0$
- Euler Characteristic:
  planar graph: $V - E + F - C = 1$
  convex polyhedron: $V - E + F = 2$
  V, E, F, C: number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
  $A_{ii} = deg(i), A_{ij} = (i,j) \in E \ ? -1 : 0$, Deleting any one row, one column, and cal the det(A)
- Polya' theorem (c is number of color , m is the number of cycle size):
  $(\sum_{i=1}^{m} c^{gcd(i,m)})/m$
- Burnside lemma:
  $|X/G| = \frac{1}{|G|}\sum_{g \in G}|X^g|$
- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
  $dp[0] = 1; dp[1] = 0;$
  $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell 數 (有 n 個人, 把他們拆組的方法總數) :
  $B_0 = 1$
  $B_n = \sum_{k=0}^{n} s(n,k) \quad (second - stirling)$
  $B_{n+1} = \sum_{k=0}^{n}\binom{n}{k}B_k$
- Wilson's theorem :
  $(p-1)! \equiv -1 (mod \ p)$
- Fermat's little theorem :
  $a^p \equiv a (mod \ p)$
- Euler's totient function:
  $A^{B^C} mod \ p = pow(A, pow(B, C, p-1)) mod \ p$
- 歐拉函數降冪公式:
  $A^B \mod C = A^{B \mod \phi(c) + \phi(c)} \mod C$
- 6 的倍數:
  $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

6.9 Miller Rabin

```
// {2,7,61} for [1, 2^32)
int magic[] = {2, 325, 9375, 28178, 450775, 9780504,
    1795265022};
bool witness(int a, int n, int u, int t) {
  if(!a) return false;
  int x = powm(a, u, n); // a, u using __int128 in `
      powm`
  for(int i = 0; i < t; i++) {
    int nx = (__int128)x * x % n;
    if(nx == 1 && x != 1 && x != n-1) return true;
    x = nx;
  }
  return x != 1;
}
bool miller_rabin(int n) {
  int s = 7;
  if(n < 2) return false;
  if(~n & 1) return n == 2;
  int u = n - 1, t = 0;
  while(~u & 1) u >>= 1, t++;
  while(s--) {
    int a = magic[s] % n;
    if(witness(a, n, u, t)) return false;
  }
  return true;
}
```

6.10 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
   n    2^n         p          a    root
   16   65536       65537      1    3
   20   1048576     7340033    7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
      if(b&1) res=(res*bs)%P;
    return res;
  }
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  }
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
      omega[i] = (omega[i-1]*r)%P;
  }
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n , theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
      for (int i = 0; i < mh; i++) {
        LL w = omega[i*theta%MAXN];
        for (int j = i; j < n; j += m) {
          int k = j + mh;
          LL x = a[j] - a[k];
          if (x < 0) x += P;
          a[j] += a[k];
          if (a[j] > P) a[j] -= P;
          a[k] = (w * x) % P;
        }
      }
      theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
      for (int k = n >> 1; k > (i ^= k); k >>= 1);
      if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
      LL ni = inv(n,P);
      reverse( a+1 , a+n );
      for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
} } };
const LL P=2013265921,root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

6.11 Pollard's Rho

```
// does not work when n is prime  O(n^(1/4))
int mul(__int128 a, __int128 b, int m) { return (a % m)
     * (b % m) % m; }
int pollard_rho(int p) {
  int x, y, z, c, g, i, j;
  while (1) {
    y = x = rand() % p;  c = rand() % p;
    i = 0, z = j = 1;
    while (++i) {
      x = (mul(x, x, p) + c) % p;
      z = mul(z, abs(y - x), p);
      if (x == y || !z) break;
      if (i % 127 == 0 || i == j) {
        g = __gcd(z, p);
        if (g > 1) return g;
        if (i == j) y = x, j <<= 1;
} } } }
```

6.12 質數

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 1000000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int lpf[N], phi[N], mu[N];
bitset<N + 1> np;
vector<int> primes;
void sieve() {
    np[0] = np[1] = phi[1] = mu[1] = 1;
    for(int i = 2; i < N; i++) {
        if (!np[i]) {
            primes.push_back(i);
            lpf[i] = i;
            phi[i] = i - 1;
            mu[i] = -1;
        }
        for(int p : primes) {
            int j = i * p;
            if (j >= N) break;
            np[j] = 1;
            lpf[j] = p;
            if (i % p == 0) {
                phi[j] = p * phi[i];
                mu[j] = 0;
                break;
            }
            phi[j] = phi[i] * phi[p];
            mu[j] = -mu[i];
} } }
```

6.13 phi

```
ll phi(ll n){   // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n;    // O(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
    }   }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

6.14 矩陣快速冪

```
LL len,mod;
vector<vector<LL>> operator*(vector<vector<LL>> x,
    vector<vector<LL>> y){
    vector<vector<LL>> ret(len,vector<LL>(len,0));
    for(int i=0;i<len;i++){
        for(int j=0;j<len;j++){
            for(int k=0;k<len;k++){
                ret[i][j]=(ret[i][j]+x[i][k]*y[k][j])%
                    mod;
    }   }   }
    return ret;
}
struct Martix_fast_pow{ //O(len^3 lg k)
    LL init(int _len,LL m=9223372036854775783LL){
        len=_len, mod=m;
    }   // mfp.solve(k,{0, 1}, {1, 1}) k'th fib {值,係
        數} // 0-base
    LL solve(LL n,vector<vector<LL>> poly){
        if(n<len)   return poly[n][0];
        vector<vector<LL>> mar(len,vector<LL>(len,0)),x
            (len,vector<LL>(len,0));
        for(int i=0;i<len;i++)    mar[i][i]=1;
        for(int i=0;i+1<len;i++)  x[i][i+1]=1;
        for(int i=0;i<len;i++)    x[len-1][i]=poly[i
            ][1];
        while(n){
            if(n&1) mar=mar*x;
            n>>=1, x=x*x;
        }
        LL ans=0;
        for(int i=0;i<len;i++)   ans=(ans+mar[len-1][i
            ]*poly[i][0]%mod)%mod;
        return ans;
    }
```

```
}mfp;
```

7 字串

7.1 KMP

```
/* len-failure[k]:
在k結尾的情況下，這個子字串可以由開頭
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴
如果我們不只想求最多，而且以0-base做為考量
，那可能的長度由大到小會是
failuer[k]、failure[failuer[k]-1]
、failure[failure[failuer[k]-1]-1]..
直到有值為0為止 */
int failure[MXN];
vector<int>ret;
void KMP(string& t, string& p){
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i){
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i){
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1){
            ret.push_back( i - p.size() + 1 );
            j = failure[j];
}   }   return ;}
```

7.2 馬拉車

```
void manacher(char *s,int len,int *z){
  len=(len<<1)+1;
  for(int i=len-1;i>=0;i--)
    s[i]=i&1?s[i>>1]:'@';
  z[0]=1;
  for(int i=1,l=0,r=0;i<len;i++){
    z[i]=i<r?min(z[l+l-i],r-i):1;
    while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
        ++z[i];
    if(i+z[i]>r) l=i,r=i+z[i];
} }
```

7.3 回文樹

```
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{
  int nxt[MXN][26],fail[MXN],len[MXN];
  int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
  int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
  char s[MXN]={-1};
  int newNode(int l,int f){
    len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
    memset(nxt[tot],0,sizeof(nxt[tot]));
    diff[tot]=(l>0?l-len[f]:0);
    sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
    return tot++;
  }
  int getfail(int x){
    while(s[n-len[x]-1]!=s[n]) x=fail[x];
    return x;
  }
  int getmin(int v){
    dp[v]=fac[n-len[sfail[v]]-diff[v]];
    if(diff[v]==diff[fail[v]])
        dp[v]=min(dp[v],dp[fail[v]]);
    return dp[v]+1;
  }
  int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(!(lst=nxt[np][c])){
      lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
      nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst;len[v]>0;v=sfail[v])
        fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
  }
  void init(const char *_s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
  }
}palt;
```

7.4 SA

```
const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
  bool _t[N*2];
  int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
      hei[N], r[N];
  int operator [] (int i){ return _sa[i]; }
  void build(int *s, int n, int m){
    memcpy(_s, s, sizeof(int) * n);
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
  }
  void mkhei(int n){
    REP(i,n) r[_sa[i]] = i;
    hei[0] = 0;
    REP(i,n) if(r[i]) {
      int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
      while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
      hei[r[i]] = ans;
    }
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
      int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
        ]-1]]++] = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
        ]]]=p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
      neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
          [i])*sizeof(int));
      ns[q[lst=sa[i]]]=nmxz+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]]] = p[nsa[i]]);
  }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
  // should padding a zero in the back
  // ip is int array, len is array length
  // ip[0..n-1] != 0, and ip[len] = 0
  ip[len++] = 0;
  sa.build(ip, len, 128);
  for (int i=0; i<len; i++) {
    H[i] = sa.hei[i + 1];
```

```
    SA[i] = sa._sa[i + 1];
  }
  // resulting height, sa array \in [0,len)
}
```

7.5 SAM

```
struct SAM{
  struct Node{
    array<int,26>next={};
    int link=0;
    int len=0;
  };
  vector<Node> s;
  int n;
  int last=1;
  int sz=2;
  SAM(int nn){
    n=nn*2+10;
    s.resize(n);
  }
  void add(int x){
    int p=last;
    if(s[p].next[x]){
      int q=s[p].next[x];
      if(s[p].len+1==s[q].len){
        last=q;return;
      }
      int r=sz++;
      s[r]=s[q];
      s[r].len=s[p].len+1;
      while(p&&s[p].next[x]==q){
        s[p].next[x]=r;
        p=s[p].link;
      }
      s[q].link=r;
      last=r;
      return;
    }
    int q=sz++;last=q;
    s[q].len=s[p].len+1;
    while(p&&!s[p].next[x]){
      s[p].next[x]=q;
      p=s[p].link;
    }
    if(!p){
      s[last].link=1;
      return;
    }
    q=s[p].next[x];
    if(s[p].len+1==s[q].len){
      s[last].link=q;
      return;
    }
    int r=sz++;
    s[r]=s[q];
    s[r].len=s[p].len+1;
    while(p&&s[p].next[x]==q){
      s[p].next[x]=r;
      p=s[p].link;
    }
    s[last].link=s[q].link=r;
  }
};
```

7.6 trie

```
//01 bitwise trie
struct trie{
    trie *nxt[2];  // 差別
    int cnt;    //紀錄有多少個數字以此節點結尾
    int sz;     //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root;
void insert(int x){
    trie *now = root;  // 每次從根節點開始
    for(int i=22;i>=0;i--){ // 從最高位元開始往低位元走
        now->sz++;
        //cout<<(x>>i&1)<<endl;
        if(now->nxt[x>>i&1] == NULL){ //判斷當前第 i 個位元是 0 還是 1
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1];  //走到下一個位元
    }
    now->cnt++;
    now->sz++;
}
```

7.7 Z-value

```
int z[MAXN];
void Z_value(const string& s) { //z[i] = lcp(s[1...],s[i...])
  int i, j, left, right, len = s.size();
  left=right=0; z[0]=len;
  for(i=1;i<len;i++) {
    j=max(min(z[i-left],right-i),0);
    for(;i+j<len&&s[i+j]==s[j];j++);
    z[i]=j;
    if(i+z[i]>right) {
      right=i+z[i];
      left=i;
}   }   }
```

7.8 minRotation

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
  int a = 0, N = s.size(); s += s;
  rep(b,0,N) rep(k,0,N) {
    if(a+k == b || s[a+k] < s[b+k])
      {b += max(0, k-1); break;}
    if(s[a+k] > s[b+k]) {a = b; break;}
  } return a;
}
```

8 DP

8.1 數位 dp

```
int dp[MXN_LEN][PRE_NUM][LIMIT][F0];
int DP(const string &s, int i = 0, int pre = 0, bool
    lim = 1, bool f0 = 1) {
  if (dp[i][pre][lim][f0]) return dp[i][pre][lim][f0];
  if (i == s.size()) return dp[i][pre][lim][f0] = 1;
  int res = 0, h = lim ? s[i] - '0' : 9;
  for (int j = 0; j <= h; j++)
    if (abs(j - pre) >= 2 || f0)
      res += DP(s, i + 1, j, j == h && lim, f0 && !j);
  return dp[i][pre][lim][f0] = res;
}
```

8.2 SOS dp

```
for(int i = 0; i<(1<<N); ++i) F[i] = A[i];
for(int i = 0;i < N; ++i)
for(int mask = 0; mask < (1<<N); ++mask)
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
```

8.3 p-median

```
void p_Median() {
  for (int i=1; i<=N; ++i)
    for (int j=i; j<=N; ++j) {
      m = (i+j)/2,d[i][j] = 0; // m是中位數，d[i][j]為距離的總和
      for (int k=i; k<=j; ++k) d[i][j] += abs(arr[k] - arr[m]);
    }
  for (int p=1; p<=P; ++p)
    for (int n=1; n<=N; ++n) {
      dp[p][n] = 1e9;
      for (int k=p; k<=n; ++k)
        if (dp[p-1][k-1] + d[k][n] < dp[p][n]) {
          dp[p][n] = dp[p-1][k-1] + d[k][n];
          r[p][n] = k; // 從第k個位置往右到第j個位置
        }
    }
}
```

9 Other

9.1 黑魔法

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );
    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}
```

9.2 DLX

```
// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
  L[R[c]]=L[c]; R[L[c]]=R[c];
  for( int i=D[c]; i!=c; i=D[i] )
    for( int j=R[i]; j!=i; j=R[j] ){
      U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
}   }
void resume(int c){
  for( int i=D[c]; i!=c; i=D[i] )
    for( int j=L[i]; j!=i; j=L[j] ){
      U[D[j]]=D[U[j]]=j; S[C[j]]++;
    }
  L[R[c]]=R[L[c]]=c;
}
int dfs(){
  if(R[0]==0) return 1;
  int md=100000000,c;
  for( int i=R[0]; i!=0; i=R[i] )
    if(S[i]<md){ md=S[i]; c=i; }
  if(md==0) return 0;
  remove(c);
  for( int i=D[c]; i!=c; i=D[i] ){
    used[ROW[i]]=1;
    for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
    if(dfs()) return 1;
    for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
    used[ROW[i]]=0;
  }
  resume(c);
  return 0;
}
int exact_cover(int n,int m){
  for( int i=0; i<=m; i++ ){
    R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
    S[i]=0; C[i]=i;
  }
  R[m]=0; L[0]=m;
  int t=m+1;
  for( int i=0; i<n; i++ ){
    int k=-1;
    for( int j=0; j<m; j++ ){
      if(!A[i][j]) continue;
      if(k==-1) L[t]=R[t]=t;
      else{ L[t]=k; R[t]=R[k]; }
      k=t; D[t]=j+1; U[t]=U[j+1];
      L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
      C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
  } }
  for( int i=0; i<n; i++ ) used[i]=0;
  return dfs();
}
```

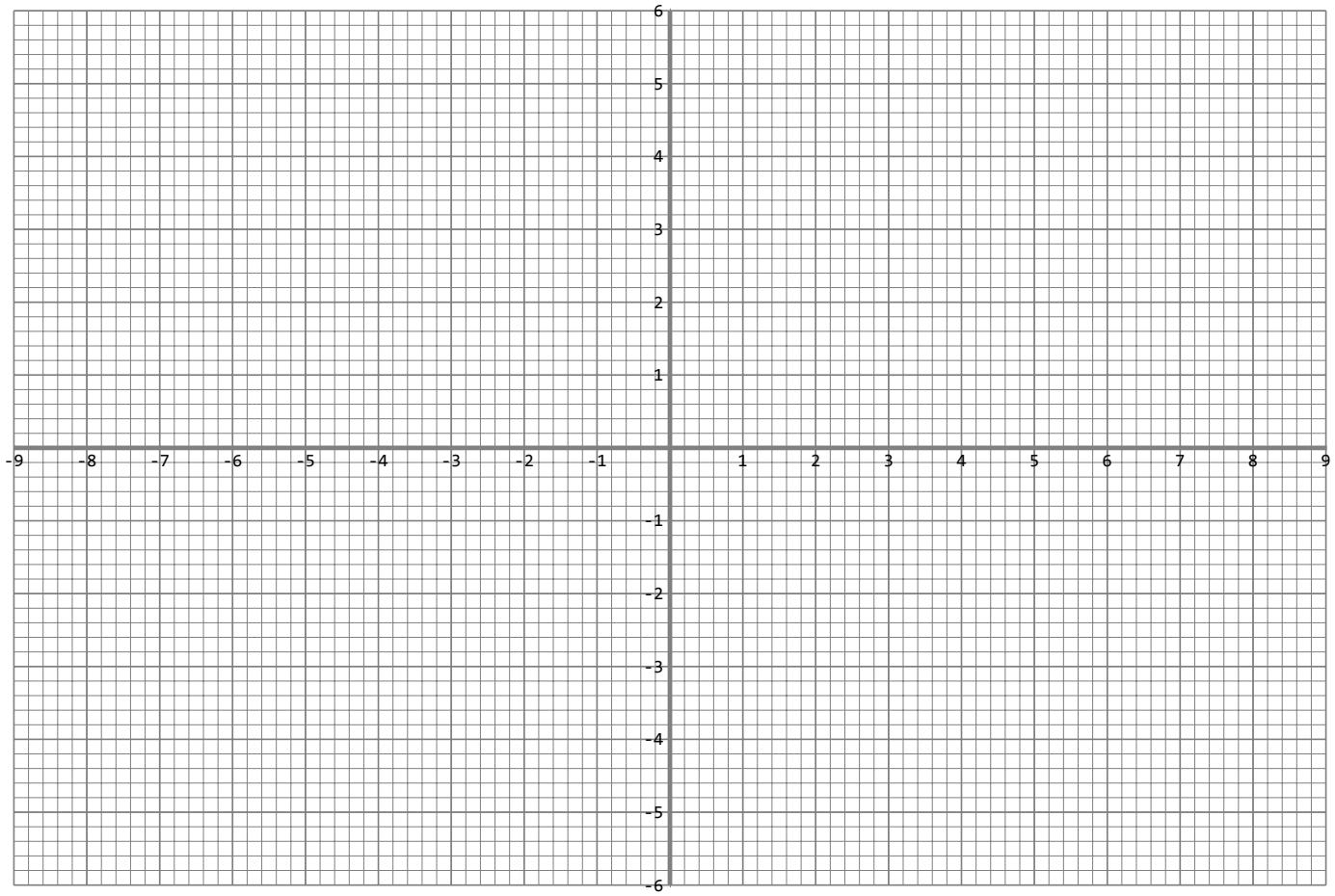
9.3 Hilbert Curve

```
long long hilbert(int n,int x,int y){
  long long res=0;
  for(int s=n/2;s;s>>=1){
    int rx=(x&s)>0,ry=(y&s)>0; res+=s*1ll*s*((3*rx)^ry)
        ;
    if(ry==0){ if(rx==1) x=s-1-x,y=s-1-y; swap(x,y); }
  }
  return res;
}
```

9.4 模擬退火

```
mt19937 rng((unsigned long long)(new char));
auto rnd = [&]() -> double {
    return 2 * ((double)rng() / rng.max()) - 1;
};

auto run = [&](int l, int r, int u, int d) -> double {
        double x = (l+r)/2., y = (u+d)/2., s = cal(x, y
            );
        double nx, ny;
        for (double t = hypot(l-r, u-d); t >= 1e-8; t
            *= 0.99995) {
            do {
                nx = x + t * rnd();
                ny = y + t * rnd();
            } while (!safe(nx, ny));
            if (chmax(s, cal(nx, ny)))
                x = nx, y = ny;
        }
        return s;
    };
```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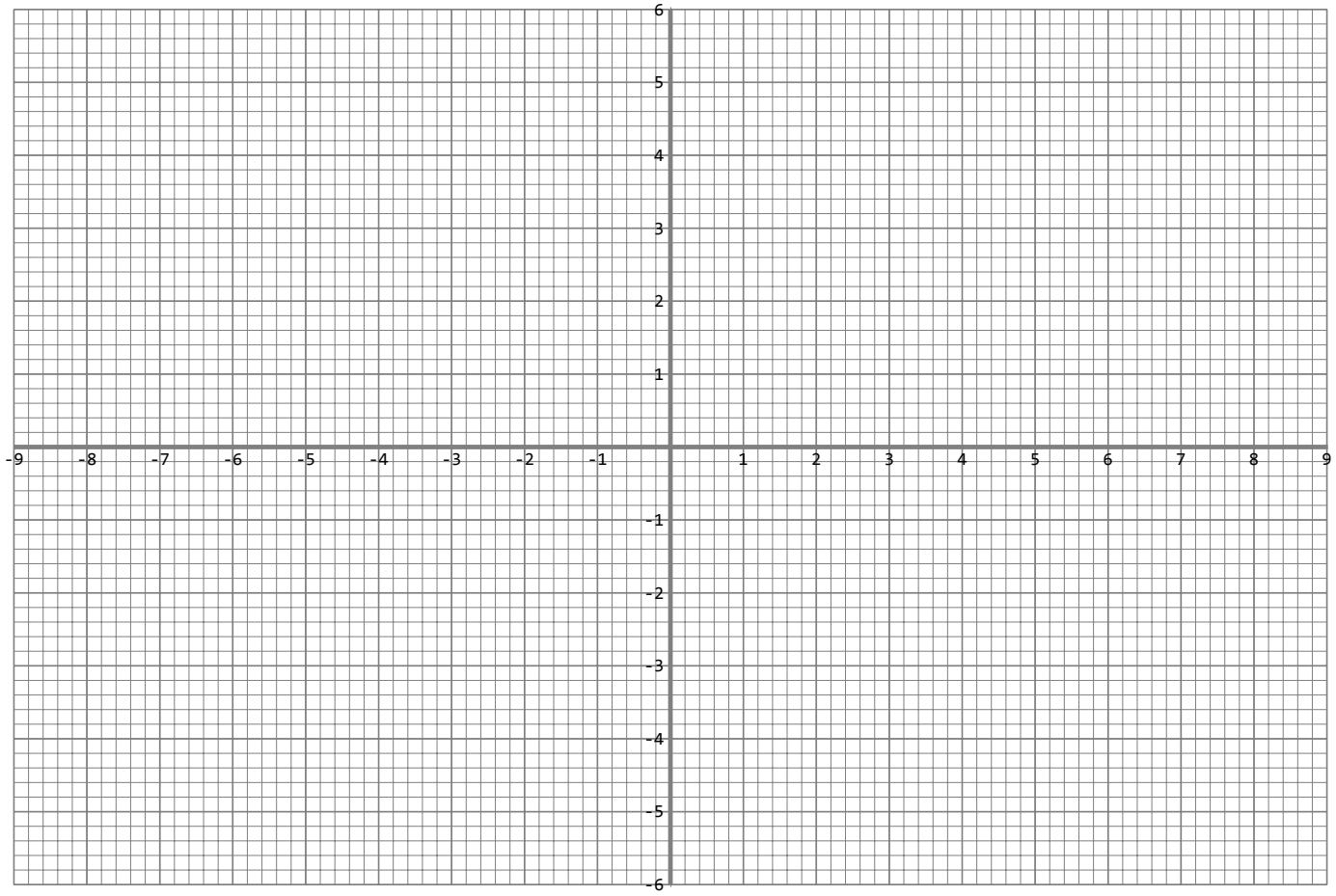
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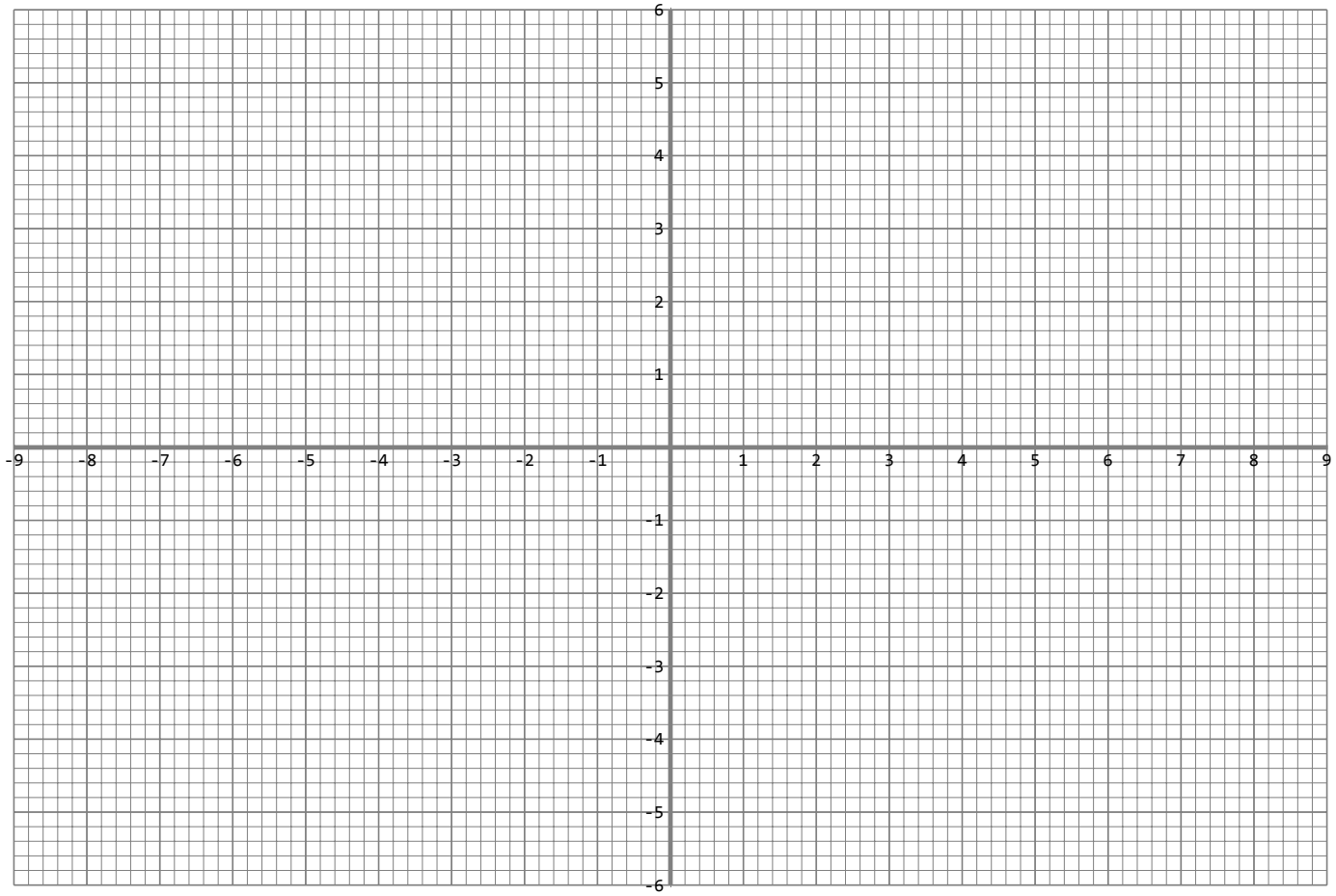
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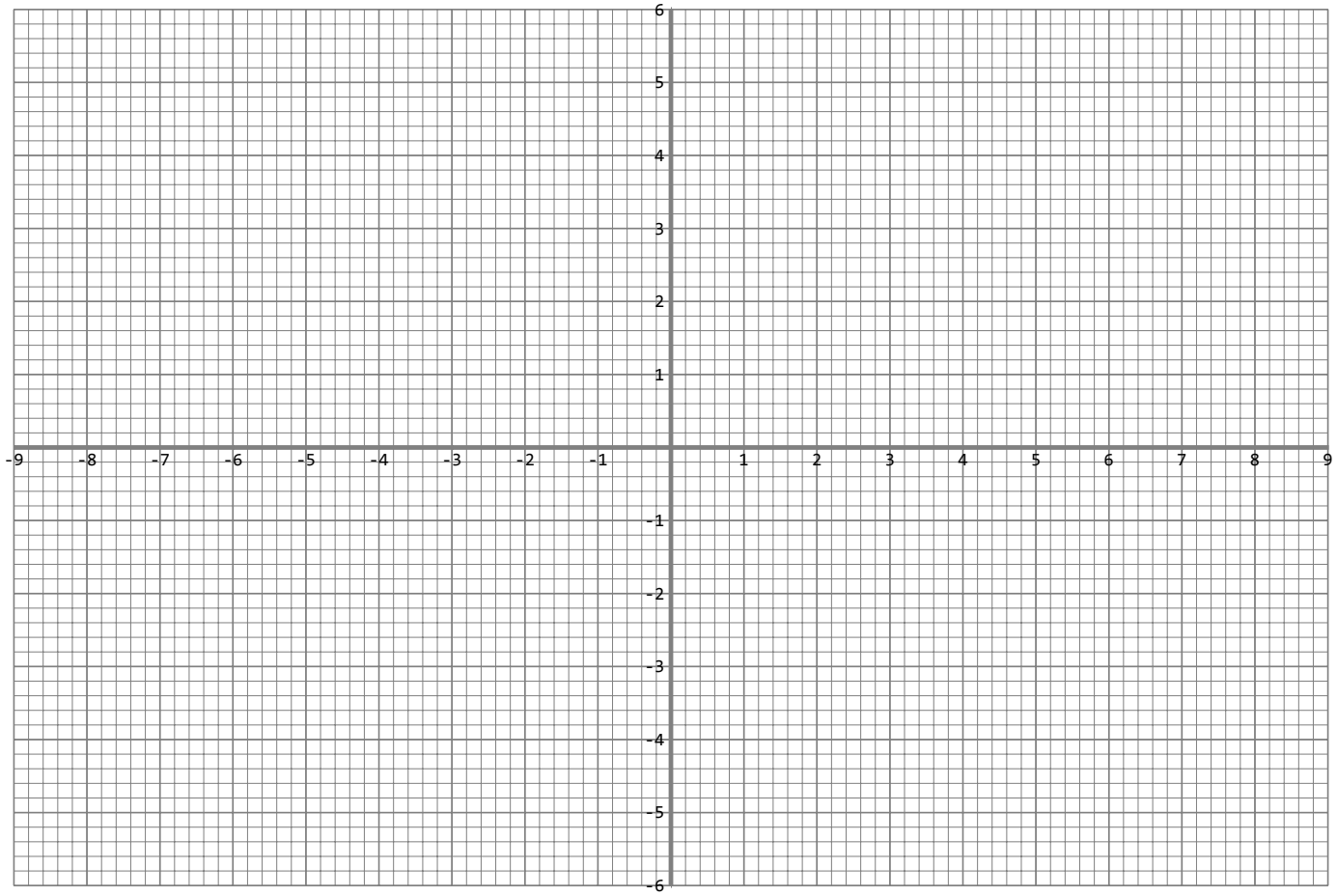
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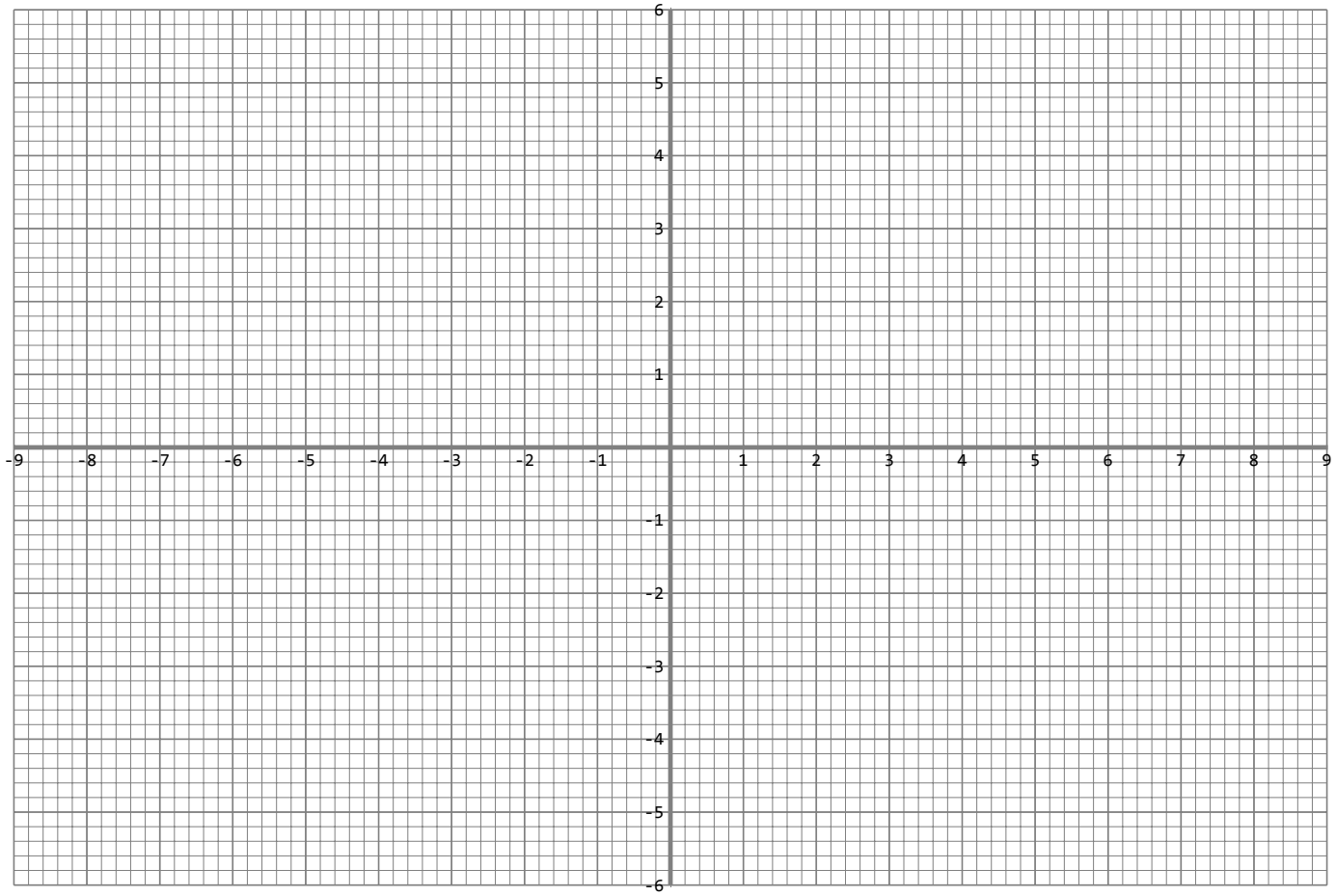
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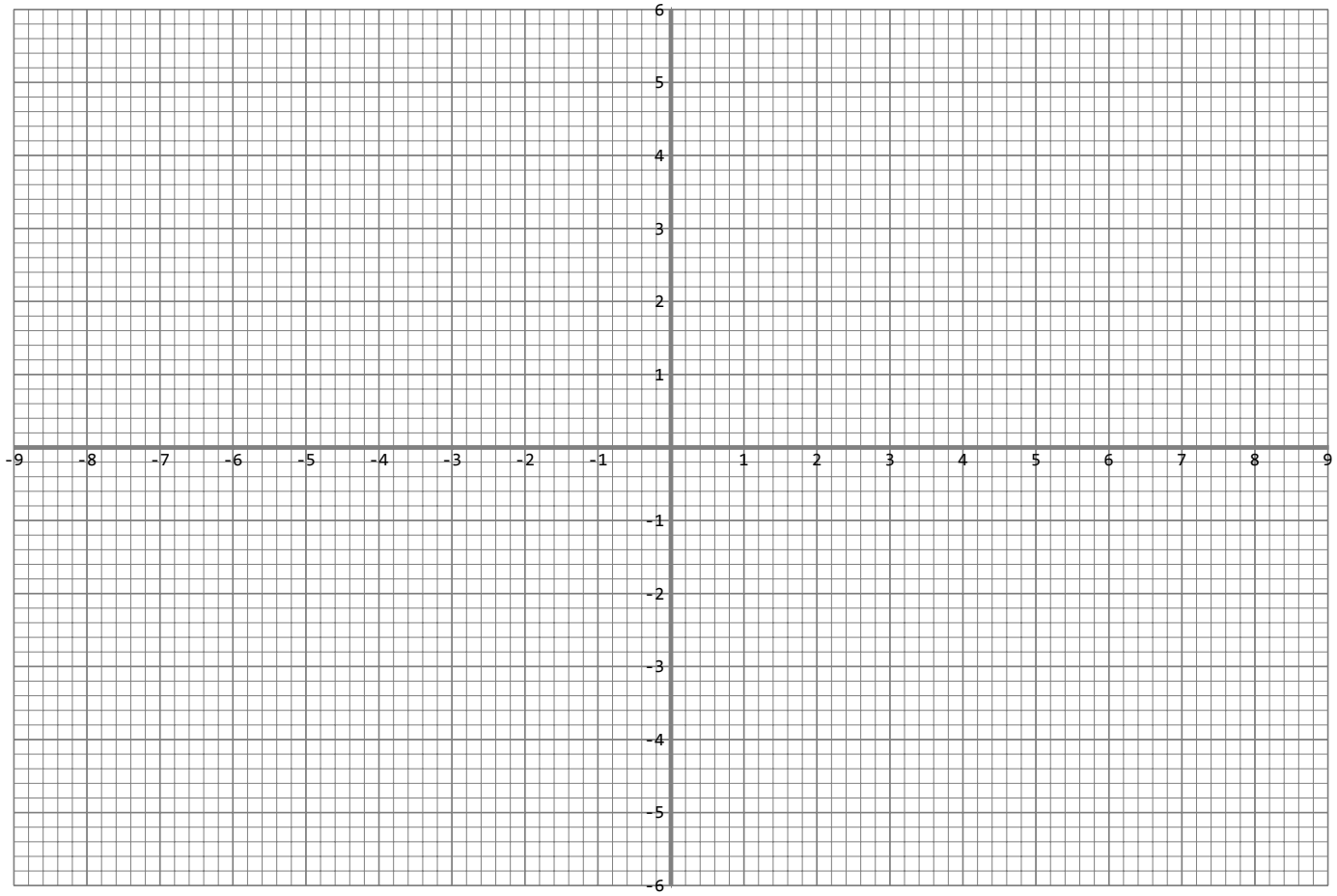
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