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## 1 Basic

### 1.1 Default code

```

1 #include<bits/stdc++.h>
1 #define int long long
1 #define mod 1000000007
1 #define endl '\n'
1 #define pii pair<int,int>
1 using namespace std;
2
2 signed main(){
2     ios::sync_with_stdio(0),cin.tie(0);
2 }
3

```

### 1.2 Linux 對拍

```

3 set -e
4 for ((i=0;i<300;i++))
4 do
5     echo "$i"
5     python gen.py > input
5     ./ac < input > ac.out
5     ./wa < input > wa.out
6     diff ac.out wa.out || break
6 done
6

```

### 1.3 Windows 對拍

```

7 @echo off
7 :loop
7     echo %x
8     python gen.py > input
9     ./ac.exe < input > ac.out
9     ./wa.exe < input > wa.out
9     fc ac.out wa.out
9     if not errorlevel 1 goto loop
9

```

### 1.4 builtin 函數

```

11 // 右邊第一個 1 的位置
11 int __builtin_ffs(unsigned int);
11 int __builtin_ffsl(unsigned long);
11 int __builtin_ffsll(unsigned long long);
11 // 左邊第一個 1 之前 0 的數量
11 int __builtin_clz(unsigned int);
11 int __builtin_clzl(unsigned long);
11 int __builtin_clzll(unsigned long long);
11 // 右邊第一個 1 之後 0 的數量
11 int __builtin_ctz(unsigned int);
11 int __builtin_ctzl(unsigned long);
11 int __builtin_ctzll(unsigned long long);
11 // 1 的數量
11 int __builtin_popcount(unsigned int);
11 int __builtin_popcountl(unsigned long);
11 int __builtin_popcountll(unsigned long long);
11 // 1 的數量 mod 2
11 int __builtin_parity(unsigned int);
11 int __builtin_parityl(unsigned long);
11 int __builtin_parityll(unsigned long long);
11 // 二進制表示數字
11 int a = 0b101101;
11

```

### 1.5 輸入輸出

```

15 // 開讀檔
15 fopen("input_file_name","r",stdin);
15 fopen("output_file_name","w",stdout);
16

```

### 1.6 Python 輸入輸出

```

16 a = list(map(int,input().split()))
16
16 # 開讀檔
16 import sys, os.path
16 if(os.path.exists('input_file.txt')):
16     sys.stdin = open("input_file.txt","r")
16     sys.stdout = open("output_file.txt","w")
17

```

## 2 Data Structure

### 2.1 Link-Cut Tree

```

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {

```

```

    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}
}

```

### 2.2 持久化線段樹

```

struct Seg{
    struct Node{
        int v;
        Node* l,*r;
    };
    vector<Node*> version;
    Node* build(int l,int r){
        Node* node=new Node;
        if(l==r){
            node->v=l;
            return node;
        }
        int mid=(l+r)/2;
        node->l=build(l,mid);
        node->r=build(mid+1,r);
        return node;
    }
    int query(Node* cur,int l,int r,int x){
        if(l==r){
            return cur->v;
        }
        int mid=(l+r)/2;
        if(x<=mid) return query(cur->l,l,mid,x);
        else return query(cur->r,mid+1,r,x);
    }
    Node* update(Node* cur,int l,int r,int x,int y){
        Node* node=new Node;
        if(l==r){
            node->v=y;
            return node;
        }
        int mid=(l+r)/2;
        if(x<=mid){
            node->l=update(cur->l,l,mid,x,y);
            node->r=cur->r;
        }
        else{
            node->l=cur->l;
            node->r=update(cur->r,mid+1,r,x,y);
        }
        return node;
    }
};

```

### 2.3 Treap

```

struct Treap {
    struct Node {
        int k, p, sz = 1;
        Node *l = 0, *r = 0;
    };
};

```

```

    bool tag = 0;
    Node(int kk) {
        k = kk;
        p = rand();
    }
};
Node *root = 0;
int size(Node *x) {return x ? x->sz : 0;}
void push(Node *x) {
    if(x->tag) {
        if(x->l) x->l->tag ^= true;
        if(x->r) x->r->tag ^= true;
        x->tag = false;
    }
}
void pull(Node* x) {
    x->sz = size(x->l) + size(x->r) + 1;
}
Node* merge(Node *a, Node *b) {
    if(!a || !b) return a ? b;
    if(a->p > b->p) {
        push(a);
        a->r = merge(a->r, b);
        pull(a);
        return a;
    }
    else{
        push(b);
        b->l = merge(a, b->l);
        pull(b);
        return b;
    }
}
void splitKey(Node* x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(x->k <= k) {
        a = x;
        splitKey(a->r, k, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKey(b->l, k, a, b->l);
        pull(b);
    }
}
void splitKth(Node *x, int k, Node *&a, Node *&b) {
    if(!x) {a = b = 0; return;}
    push(x);
    if(size(x->l) < k) {
        a = x;
        splitKth(a->r, k - size(x->l) - 1, a->r, b);
        pull(a);
    }
    else{
        b = x;
        splitKth(b->l, k, a, b->l);
        pull(b);
    }
}
void insert(int id) {
    Node *l, *r;
    splitKey(root, id, l, r);
    Node *m = new Node(id);
    root = merge(l, merge(m, r));
}
void erase(int x) {
    Node *a, *b, *c;
    splitKey(root, x, b, c);
    splitKey(b, x - 1, a, b);
    root = merge(a, c);
}
};

```

## 2.4 線段樹

```

struct Seg{
    vector<int> seg,tag;
    #define cl (i<<1)+1
    #define cr (i<<1)+2

```

```

    void push(int i,int l,int r){
        if(tag[i]!=0){
            seg[i]+=tag[i]; // update by tag
            if(l==r){
                tag[cl]+=tag[i]; // push
                tag[cr]+=tag[i]; // push
            }
            tag[i]=0;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl,l,mid);push(cr,mid+1,r);
        seg[i]=max(seg[cl],seg[cr]); // pull
    }
    void build(int i,int l,int r,vector<int>&arr){
        if(l==r){
            seg[i]=arr[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl,l,mid,arr);
        build(cr,mid+1,r,arr);
        pull(i,l,r);
    }
    Seg(vector<int>& arr){
        seg.resize(arr.size()*4);
        tag.resize(arr.size()*4);
        build(0,0,arr.size()-1,arr);
    }
    void update(int i,int l,int r,int nl,int nr,int x){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            tag[i]+=x;
            return;
        }
        int mid=(l+r)>>1;
        if(nl<=mid) update(cl,l,mid,nl,nr,x);
        if(nr>mid) update(cr,mid+1,r,nl,nr,x);
        pull(i,l,r);
    }
    int query(int i,int l,int r,int nl,int nr){
        push(i,l,r);
        if(nl<=l&&r<=nr){
            return seg[i];
        }
        int mid=(l+r)>>1;
        int ans=0;
        if(nl<=mid) ans=max(ans,query(cl,l,mid,nl,nr));
        if(nr>mid) ans=max(ans,query(cr,mid+1,r,nl,nr));
        return ans;
    }
};

```

## 3 Flow

### 3.1 Dinic

```

struct Dinic {
    #define SZ(x) (int)(x.size())
    struct Edge {
        int v, f, re;
    };
    vector<vector<Edge>> E;
    vector<int> level;
    int n, s, t;
    Dinic(int nn, int ss, int tt) {
        n = nn; s = ss; t = tt;
        E.resize(n);
        level.resize(n);
    }
    void addEdge(int u, int v, int w) {
        E[u].push_back({v, w, SZ(E[v])});
        E[v].push_back({u, 0, SZ(E[u]) - 1});
    }
    bool bfs() {
        level.assign(n, 0);
        queue<int> q;
        q.push(s);
        level[s] = 1;
        while(!q.empty()) {

```

```

    int u = q.front(); q.pop();
    for(auto& it : E[u]) {
        int v = it.v;
        if(it.f > 0 && !level[v]) {
            level[v] = level[u] + 1;
            q.push(v);
        }
    }
}
return level[t];
}
int dfs(int u, int nf) {
    if(u == t) return nf;
    int ret = 0;
    for(auto& it : E[u]) {
        int v = it.v;
        if(it.f > 0 && level[v] == level[u] + 1) {
            int tem = dfs(v, min(nf, it.f));
            ret += tem; nf -= tem;
            it.f -= tem; E[v][it.re].f += tem;
            if(!nf) return ret;
        }
    }
    if(!ret) level[u] = 0;
    return ret;
}
int flow() {
    int ret = 0;
    while(bfs()) ret += dfs(s, 0x3f3f3f3f);
    return ret;
}
};

```

### 3.2 匈牙利

```

#define NIL -1
#define INF 100000000
int n, matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN], ylabel[MAXN];
int xy[MAXN], yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
    int i, delta=INF;
    for(i=0; i<n; i++) if(!sett[i]) delta=min(slack[i], delta);
    for(i=0; i<n; i++) if(sets[i]) xlabel[i]-=delta;
    for(i=0; i<n; i++) {
        if(sett[i]) ylabel[i]+=delta;
        else slack[i]-=delta;
    }
}
inline void add_sets(int x) {
    int i;
    sets[x]=1;
    for(i=0; i<n; i++) {
        if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {
            slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
            prev[i]=x;
        }
    }
}
inline void augment(int final) {
    int x=prev[final], y=final, tmp;
    matched++;
    while(1) {
        tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
        if(y==NIL) return;
        x=prev[y];
    }
}
inline void phase() {
    int i, y, root;
    for(i=0; i<n; i++) { sets[i]=sett[i]=0; slack[i]=INF; }
    for(root=0; root<n && xy[root]!=NIL; root++);
    add_sets(root);
    while(1) {
        relabel();

```

```

        for(y=0; y<n; y++) if(!sett[y] && slack[y]==0) break;
        if(yx[y]==NIL) { augment(y); return; }
        else { add_sets(yx[y]); sett[y]=1; }
    }
}
inline int hungarian() {
    int i, j, c=0;
    for(i=0; i<n; i++) {
        xy[i]=yx[i]=NIL;
        xlabel[i]=ylabel[i]=0;
        for(j=0; j<n; j++) xlabel[i]=max(cost[i][j], xlabel[i]);
    }
    for(i=0; i<n; i++) phase();
    for(i=0; i<n; i++) c+=cost[i][xy[i]];
    return c;
}

```

### 3.3 KM

```

struct KM { // max weight, for min negate the weights
    int n, mx[MAXN], my[MAXN], pa[MAXN];
    ll g[MAXN][MAXN], lx[MAXN], ly[MAXN], sy[MAXN];
    bool vx[MAXN], vy[MAXN];
    void init(int _n) { // 1-based
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) { g[x][y] = w; }
    void augment(int y) {
        for(int x, z; y; y = z)
            x = pa[y], z = mx[x], my[y] = x, mx[x] = y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]) {
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0) {
                        pa[y]=x;
                        if(!my[y]) { augment(y); return; }
                        vy[y]=1, q.push(my[y]);
                    } else if(sy[y]>t) pa[y]=x, sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y] && cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j) {
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y] && sy[y]==0) {
                if(!my[y]) { augment(y); return; }
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve() {
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    }
} graph;

```

### 3.4 MCMF

```

struct MCMF {
    #define SZ(x) (int)(x.size())
    struct Edge {
        int v, f, re, c;
    };
    vector<vector<Edge>> E;
    vector<int> dis, x, y;
    int n, s, t;
    MCMF(int nn, int ss, int tt) {
        n = nn; s = ss; t = tt;
    }

```

```

    E.resize(n);
    x.resize(n);
    y.resize(n);
}
void addEdge(int u, int v, int w, int c) {
    E[u].push_back({v, w, SZ(E[v]), c});
    E[v].push_back({u, 0, SZ(E[u]) - 1, -c});
}
bool spfa() {
    dis.assign(n, 0x3f3f3f3f);
    x.assign(n, -1);
    y.assign(n, -1);
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(s);
    inq[s] = true;
    dis[s] = 0;
    while(q.size()) {
        int u = q.front(); q.pop();
        inq[u] = false;
        for(int i = 0; i < E[u].size(); i++) {
            auto& it = E[u][i];
            int v = it.v;
            if(it.f > 0 && dis[v] > dis[u] + it.c) {
                dis[v] = dis[u] + it.c;
                x[v] = u;
                y[v] = i;
                if(!inq[v]) {
                    q.push(v);
                    inq[v] = true;
                }
            }
        }
    }
    return x[t] != -1;
}
pii solve() {
    int mf = 0, mc = 0;
    while(spfa()) {
        int nf = 0x3f3f3f3f;
        for(int i = t; i != s; i = x[i]) {
            nf = min(nf, E[x[i]][y[i]].f);
        }
        for(int i = t; i != s; i = x[i]) {
            auto& it = E[x[i]][y[i]];
            it.f -= nf;
            E[it.v][it.re].f += nf;
        }
        mf += nf;
        mc += nf * dis[t];
    }
    return {mf, mc};
}
};

```

## 4 幾何

### 4.1 點宣告

```

typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
    if(abs(x) < eps) return 0;
    else return x < 0 ? -1 : 1;
}
struct Pt {
    ld x, y;
    Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}
    Pt operator+(const Pt &a) const {
        return Pt(x+a.x, y+a.y);
    }
    Pt operator-(const Pt &a) const {
        return Pt(x-a.x, y-a.y);
    }
    Pt operator*(const ld &a) const {
        return Pt(x*a, y*a);
    }
    Pt operator/(const ld &a) const {
        return Pt(x/a, y/a);
    }
    ld operator*(const Pt &a) const {
        return x*a.x + y*a.y;
    }
    ld operator^(const Pt &a) const {
        return x*a.y - y*a.x;
    }
    bool operator<(const Pt &a) const {

```

```

        return x < a.x || (x == a.x && y < a.y);
    }
    //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
    //    dcmp(y-a.y) < 0);
    bool operator==(const Pt &a) const {
        return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0;
    }
    ld norm2(const Pt &a) {
        return a*a;
    }
    ld norm(const Pt &a) {
        return sqrt(norm2(a));
    }
    Pt perp(const Pt &a) {
        return Pt(-a.y, a.x);
    }
    Pt rotate(const Pt &a, ld ang) {
        return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y*cos(ang));
    }
    struct Line {
        Pt s, e, v; // start, end, end-start
        ld ang;
        Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v = e-s; ang = atan2(v.y, v.x); }
        bool operator<(const Line &L) const {
            return ang < L.ang;
        }
    };
    struct Circle {
        Pt o; ld r;
        Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
    };
};

```

### 4.2 矩形面積

```

struct AreaofRectangles{
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
    ll n, id, sid;
    pair<ll,ll> tree[MXN<<3]; // count, area
    vector<ll> ind;
    tuple<ll,ll,ll,ll> scan[MXN<<1];
    void pull(int i, int l, int r){
        if(tree[i].first) tree[i].second = ind[r+1] - ind[l];
        else if(l != r){
            int mid = (l+r)>>1;
            tree[i].second = tree[cl(i)].second + tree[cr(i)].second;
        }
        else tree[i].second = 0;
    }
    void upd(int i, int l, int r, int ql, int qr, int v){
        if(ql <= l && r <= qr){
            tree[i].first += v;
            pull(i, l, r); return;
        }
        int mid = (l+r) >> 1;
        if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
        if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
        pull(i, l, r);
    }
    void init(int _n){
        n = _n; id = sid = 0;
        ind.clear(); ind.resize(n<<1);
        fill(tree, tree+(n<<2), make_pair(0, 0));
    }
    void addRectangle(int lx, int ly, int rx, int ry){
        ind[id++] = lx; ind[id++] = rx;
        scan[sid++] = make_tuple(ly, 1, lx, rx);
        scan[sid++] = make_tuple(ry, -1, lx, rx);
    }
    ll solve(){
        sort(ind.begin(), ind.end());
        ind.resize(unique(ind.begin(), ind.end()) - ind.begin());
        sort(scan, scan + sid);
        ll area = 0, pre = get<0>(scan[0]);
        for(int i = 0; i < sid; i++){
            auto [x, v, l, r] = scan[i];
            area += tree[l].second * (x-pre);
            upd(1, 0, ind.size()-1, lower_bound(ind.begin(), ind.end(), l)-ind.begin(), lower_bound(ind.begin(), ind.end(), r)-ind.begin()-1, v);
            pre = x;
        }
    }
};

```

```

    }
    return area;
} }rect;

```

### 4.3 最近點對

```

#include<bits/stdc++.h>
#define int long long
using namespace std;
using ld = long double;
const int mod = 1e9+7;
struct pt{
    int x,y;
    int id;
    ld dis(const pt& rhs){
        return sqrt((x-rhs.x)*(x-rhs.x)+(y-rhs.y)*(y-rhs.y));
    }
};
signed main(){
    int n;
    cin>>n;
    vector<pt> a(n);
    for(int i=0;i<n;i++){
        cin>>a[i].x>>a[i].y;
        a[i].id=i;
    }
    ld ans = 1e19;
    sort(a.begin(),a.end(),[](const pt&a,const pt&b){
        if(a.x==b.y)return a.y<b.y;
        return a.x<b.x;
    });
    pt ans2;
    function<void(int,int)> dng = [&](int l,int r){
        if(r-l<4){
            for(int i=l;i<=r;i++){
                for(int j=i+1;j<=r;j++){
                    ld temans = a[i].dis(a[j]);
                    if(temans<ans){
                        ans=temans;
                        ans2 = {a[i].id,a[j].id};
                    }
                }
            }
            sort(a.begin()+l,a.begin()+r+1,[](const pt&a,const pt&b){return a.y<b.y;});
            return;
        }
        int mid = (l+r)/2;
        int midx = a[mid].x;
        dng(l,mid);dng(mid+1,r);
        inplace_merge(a.begin()+l,a.begin()+mid+1,a.begin()+r+1,[](const pt&a,const pt&b){return a.y<b.y;});
        vector<int> c;c.reserve(r-l+1);
        for(int i=l;i<=r;i++){
            if(abs(a[i].x-midx)<ans){
                for(int j=c.size()-1;j>=0&&a[i].y-a[c[j]].y<ans;j--){
                    ld temans = a[i].dis(a[c[j]]);
                    if(temans<ans){
                        ans=temans;
                        ans2 = {a[i].id,a[c[j]].id};
                    }
                }
            }
        }
        c.push_back(i);
    };
    dng(0,n-1);
    cout<<min(ans2.x,ans2.y)<<' '<<max(ans2.x,ans2.y)<<
    ' '<<fixed<<setprecision(6)<<ans<<'\n';
}

```

### 4.4 凸包

```

double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){

```

```

    sort(pt.begin(),pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2],stk[top-1],pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

### 4.5 兩直線交點

```

Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1),f2 = (p2-p1)^(p1-q2),f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}

```

### 4.6 兩線段交點

```

int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 , const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
               ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
               ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
               ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1 , p2 , q1 ) * ori( p1 , p2 , q2 )<=0) &&
           (ori( q1 , q2 , p1 ) * ori( q1 , q2 , p2 )<=0);
}

```

### 4.7 李超線段樹

```

struct LiChao_min{
    struct line{
        ll m,c;
        line(ll _m=0,ll _c=0){ m=_m; c=_c; }
        ll eval(ll x){ return m*x+c; } // overflow
    };
    struct node{
        node *l,*r; line f;
        node(line v){ f=v; l=r=NULL; }
    };
    typedef node* pnode;
    pnode root; ll sz,ql,qr;
#define mid ((l+r)>>1)
    void insert(line v,ll l,ll r,pnode &nd){
        /* if(ql<=l&&r<=qr){
            if(!nd) nd=new node(line(0,INF));
            if(ql<=mid) insert(v,l,mid,nd->l);
            if(qr>mid) insert(v,mid+1,r,nd->r);
            return;
        } */
        // used for adding segment */
        if(!nd){ nd=new node(v); return; }
        ll trl=nd->f.eval(l),trr=nd->f.eval(r);
        ll vl=v.eval(l),vr=v.eval(r);
        if(trl<=vl&&trr<=vr) return;
        if(trl>vl&&trr>vr) { nd->f=v; return; }
        if(trl>vl) swap(nd->f,v);
        if(nd->f.eval(mid)<v.eval(mid))
            insert(v,mid+1,r,nd->r);
        else swap(nd->f,v),insert(v,l,mid,nd->l);
    }
    ll query(ll x,ll l,ll r,pnode &nd){

```



```

if(!nd) return INF;
if(l==r) return nd->f.eval(x);
if(mid>=x)
    return min(nd->f.eval(x), query(x, l, mid, nd->l));
return min(nd->f.eval(x), query(x, mid+1, r, nd->r));
}
/* -sz<=ll query_x<=sz */
void init(ll _sz){ sz=_sz+1; root=NULL; }
void add_line(ll m, ll c, ll l=-INF, ll r=INF){
    line v(m, c); ql=l; qr=r; insert(v, -sz, sz, root);
}
ll query(ll x) { return query(x, -sz, sz, root); }
};

```

## 4.8 最小包覆圓

```

/* minimum enclosing circle */
int n;
Pt p[ N ];
const Circle circumcircle(Pt a, Pt b, Pt c){
    Circle cir;
    double fa, fb, fc, fd, fe, ff, dx, dy, dd;
    if( iszero( ( b - a ) ^ ( c - a ) ) ){
        if( ( ( b - a ) * ( c - a ) ) <= 0 )
            return Circle((b+c)/2, norm(b-c)/2);
        if( ( ( c - b ) * ( a - b ) ) <= 0 )
            return Circle((c+a)/2, norm(c-a)/2);
        if( ( ( a - c ) * ( b - c ) ) <= 0 )
            return Circle((a+b)/2, norm(a-b)/2);
    } else {
        fa=2*(a.x-b.x);
        fb=2*(a.y-b.y);
        fc=norm2(a)-norm2(b);
        fd=2*(a.x-c.x);
        fe=2*(a.y-c.y);
        ff=norm2(a)-norm2(c);
        dx=fc*fe-ff*fb;
        dy=fa*ff-fd*fc;
        dd=fa*fe-fd*fb;
        cir.o=Pt(dx/dd, dy/dd);
        cir.r=norm(a-cir.o);
        return cir;
    }
}
inline Circle mec(int fixed, int num){
    int i;
    Circle cir;
    if(fixed==3) return circumcircle(p[0], p[1], p[2]);
    cir=circumcircle(p[0], p[0], p[1]);
    for(i=fixed; i<num; i++) {
        if(cir.inside(p[i])) continue;
        swap(p[i], p[fixed]);
        cir=mec(fixed+1, i+1);
    }
    return cir;
}
inline double min_radius() {
    if(n<=1) return 0.0;
    if(n==2) return norm(p[0]-p[1])/2;
    scramble();
    return mec(0, n).r;
}

```

## 4.9 最小包覆球

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius, tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i, j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res, outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for (j=0; j<2; ++j) m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);

```

```

if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
    return;
L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
res=outer[0]+q[0]*L[0]+q[1]*L[1];
radius=norm2(res, outer[0]);
break;
case 4:
    for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol[i]=(q[i] * q[i]);
    for (i=0; i<3; ++i) for (j=0; j<3; ++j) m[i][j]=(q[i] * q[j])*2;
    det= m[0][0]*m[1][1]*m[2][2]
        + m[0][1]*m[1][2]*m[2][0]
        + m[0][2]*m[1][0]*m[2][1]
        - m[0][2]*m[1][1]*m[2][0]
        - m[0][1]*m[1][0]*m[2][2]
        - m[0][0]*m[1][2]*m[2][1];
    if ( fabs(det)<eps ) return;
    for (j=0; j<3; ++j) {
        for (i=0; i<3; ++i) m[i][j]=sol[i];
        L[j]=( m[0][0]*m[1][1]*m[2][2]
            + m[0][1]*m[1][2]*m[2][0]
            + m[0][2]*m[1][0]*m[2][1]
            - m[0][2]*m[1][1]*m[2][0]
            - m[0][1]*m[1][0]*m[2][2]
            - m[0][0]*m[1][2]*m[2][1]
            ) / det;
        for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
    } res=outer[0];
    for (i=0; i<3; ++i) res = res + q[i] * L[i];
    radius=norm2(res, outer[0]);
}
void minball(int n){ ball();
    if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
        if( norm2(res, pt[i]) - radius > eps ){
            outer[ nouter ++ ] = pt[ i ]; minball(i); --
            nouter;
        }
    if(i>0){ Pt Tt = pt[i];
        memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt;
    }
}
double solve(){
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0; i<n; i++) if(norm2(res, pt[i])-radius>eps)
        nouter=1, outer[0]=pt[i], minball(i);
    return sqrt(radius);
}

```

## 4.10 旋轉卡尺

```

int FarthestPair(vector<Pt>& arr){
    int ret=0;
    for(int i = 0, j = i+1; i<arr.size(); i++){
        while(distance(arr[i], arr[j]) < distance(arr[i], arr[(j+1)%arr.size()])) {
            j = (j+1) % arr.size();
        }
        ret = max(ret, distance(arr[i], arr[j]));
    }
    return ret;
}

```

## 4.11 Circle Cover

```

#define N 1021
#define D long double
struct CircleCover{
    int C; Circ c[ N ]; //填入C(圓數量), c(圓陣列)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a, Circ& b, Pt& p1, Pt& p2 ){
        Pt o1 = a.o, o2 = b.o;
        D r1 = a.R, r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return false;
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return true;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);

```

## 4.12 Convex Hull Trick

```

int sign( LL x ){ // fixed when changed to double
    return x < 0 ? -1 : x > 0; }
pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
    for( ; l + 1 < r; ){
        int mid = (l + r) / 2;
        if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
        else l = mid;
    }
    return max(make_pair(det(vec, conv[r]), r),
               make_pair(det(vec, conv[0]), 0));
}

void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
}

void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
        if (smid == sl) l = mid;
        else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
}

int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid = sign(det(v - u, a[mid % n] - u));
        if (smid == sl) l = mid;
        else r = mid;
    }
    return l % n;
}

// 1. whether a given point is inside the CH
bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
        if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
        if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
}

// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}

// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}

// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i. next(i))

```



```
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
};
```

#### 4.13 Half Plane Intersection

```
// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
} // segment should add Counterclockwise
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end()); // sort by angle
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir=las=0] = L[0];
    for(int i = 1; i < n; i++) {
        while(fir < las && !onleft(L[i], p[las-1])) las--;
        while(fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if(fir < las) p[las-1] = LLIntersect(q[las-1], q[las]);
    }
    while(fir < las && !onleft(q[fir], p[las-1])) las--;
    if(las-fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las-fir+1);
    for(int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}
```

## 5 圖論

### 5.1 BCC

```
struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            }else
                low[u] = min(low[u],dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
```

```
        dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc) res.PB(sccv[i]);
        return res;
    }
}graph;
```

### 5.2 重心剖分

```
struct CentroidDecomposition {
    int n;
    vector<vector<int>> G, out;
    vector<int> sz, v;
    CentroidDecomposition(int _n) : n(_n), G(_n), out(_n), sz(_n), v(_n) {}
    int dfs(int x, int par){
        sz[x] = 1;
        for (auto &&i : G[x]) {
            if(i == par || v[i]) continue;
            sz[x] += dfs(i, x);
        }
        return sz[x];
    }
    int search_centroid(int x, int p, const int mid){
        for (auto &&i : G[x]) {
            if(i == p || v[i]) continue;
            if(sz[i] > mid) return search_centroid(i, x, mid);
        }
        return x;
    }
    void add_edge(int l, int r){
        G[l].PB(r); G[r].PB(l);
    }
    int get(int x){
        int centroid = search_centroid(x, -1, dfs(x, -1)/2);
        v[centroid] = true;
        for (auto &&i : G[centroid]) {
            if(!v[i]) out[centroid].PB(get(i));
        }
        v[centroid] = false;
        return centroid;
    }
};
```

### 5.3 輕重鍊剖分

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruLi
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfssz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfssz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]), [&](int a, int b){return sz[a] > sz[b];});
```

```

bool flag = 1;
for(int& v:g[u]) if(v != prt[u][0]){
    if(flag) head[v] = head[u], flag = 0;
    dfshl(v);
    tr[u] = tr[v];
}
}
inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
        b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
        a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
}
void init( int _n ){
    n = _n; REP( i , 1 , n ) g[ i ].clear();
}
void addEdge( int u , int v ){
    g[ u ].push_back( v );
    g[ v ].push_back( u );
}
void yutruli(){ //build function
    dfssz(1, 0);
    ts = 0;
    dfshl(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
        prt[i][k] = prt[prt[i][k-1]][k-1];
}
vector< PII > getPath( int u , int v ){
    vector< PII > res;
    while( tid[ u ] < tid[ head[ v ] ] ){
        res.push_back( PII(tid[ head[ v ] ], tid[ v ]) );
        v = prt[ head[ v ] ][ 0 ];
    }
    res.push_back( PII( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res;
}
/* res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< PII >& path = tree.getPath( u , v )
 * for( PII tp : path ) {
 *     int l , r;tie( l , r ) = tp;
 *     upd( l , r );
 *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;

```

## 5.4 歐拉路徑

```

#define FOR(i,a,b) for(int i=a;i<=b;i++)
int dfs_st[1000050],dfn=0;
int ans[1000050],cnt=0,num=0;
vector<int>G[1000050];
int cur[1000050];
int ind[1000050],out[1000050];
void dfs(int x){
    FOR(i,1,n)sort(G[i].begin(),G[i].end());
    dfs_st[++dfn]=x;
    memset(cur,-1,sizeof(cur));
    while(dfn>0){
        int u=dfs_st[dfn];
        int complete=1;
        for(int i=cur[u]+1;i<G[u].size();i++){
            int v=G[u][i];
            num++;
            dfs_st[++dfn]=v;
            cur[u]=i;
            complete=0;
            break;
        }
        if(complete)ans[++cnt]=u,dfn--;
    }
}

```

```

}
bool check(int &start){
    int l=0,r=0,mid=0;
    FOR(i,1,n){
        if(ind[i]==out[i]+1)l++;
        if(out[i]==ind[i]+1)r++,start=i;
        if(ind[i]==out[i])mid++;
    }
    if(l==1&&r==1&mid==n-2)return true;
    l=1;
    FOR(i,1,n)if(ind[i]!=out[i])l=0;
    if(l){
        FOR(i,1,n)if(out[i]>0){
            start=i;
            break;
        }
        return true;
    }
    return false;
}
int main(){
    cin>>n>>m;
    FOR(i,1,m){
        int x,y;scanf("%d%d",&x,&y);
        G[x].push_back(y);
        ind[y]++,out[x]++;
    }
    int start=-1,ok=true;
    if(check(start)){
        dfs(start);
        if(num!=m){
            puts("What a shame!");
            return 0;
        }
        for(int i=cnt;i>=1;i--)
            printf("%d ",ans[i]);
        puts("");
    }
    else puts("What a shame!");
}

```

## 5.5 極大團

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i ++){
            lnk[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    Int ans , stk[N] , id[N] , di[N] , deg[N];
    void dfs(int elem_num, Int candi, Int ex){
        if(candi.none()&&ex.none()){
            cans.reset();
            for(int i = 0 ; i < elem_num ; i ++){
                cans[id[stk[i]]] = 1;
                ans = elem_num; // cans is a maximal clique
                return;
            }
            int pivot = (candilex)._Find_first();
            Int smaller_candi = candi & (~lnk[pivot]);
            while(smaller_candi.count()){
                int nxt = smaller_candi._Find_first();
                candi[nxt] = smaller_candi[nxt] = 0;
                ex[nxt] = 1;
                stk[elem_num] = nxt;
                dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i ++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i ++){
            di[id[i]] = i;
        }
        for(int i = 0 ; i < n ; i ++){

```

```

    for(int j = 0 ; j < n ; j ++){
        if(v[i][j]) lnk[di[i]][di[j]] = 1;
        ans = 1; cans.reset(); cans[0] = 1;
        dfs(0, Int(string(n,'1')), 0);
        return ans;
    } }solver;

```

## 5.6 最大團

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i ++){
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[N];
    int id[N] , di[N] , deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num; cans.reset();
            for(int i = 0 ; i < elem_num ; i ++){
                cans[id[stk[i]]] = 1;
            }
            int potential = elem_num + popcount(candi);
            if(potential <= ans) return;
            int pivot = lowbit(candi);
            Int smaller_candi = candi & (~linkto[pivot]);
            while(smaller_candi.count() && potential > ans){
                int next = lowbit(smaller_candi);
                candi[next] = !candi[next];
                smaller_candi[next] = !smaller_candi[next];
                potential --;
                if(next == pivot || (smaller_candi & linkto[next]
                    ).count()){
                    stk[elem_num] = next;
                    maxclique(elem_num + 1, candi & linkto[next]);
                }
            }
        }
        int solve(){
            for(int i = 0 ; i < n ; i ++){
                id[i] = i; deg[i] = v[i].count();
            }
            sort(id , id + n , [&](int id1, int id2){
                return deg[id1] > deg[id2]; });
            for(int i = 0 ; i < n ; i ++){ di[id[i]] = i; }
            for(int i = 0 ; i < n ; i ++){
                for(int j = 0 ; j < n ; j ++){
                    if(v[i][j]) linkto[di[i]][di[j]] = 1;
                }
            }
            Int cand; cand.reset();
            for(int i = 0 ; i < n ; i ++){ cand[i] = 1; }
            ans = 1;
            cans.reset(); cans[0] = 1;
            maxclique(0, cand);
            return ans;
        }
    } }solver;

```

## 5.7 SCC

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear(), rE[i].clear();
        }
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v: E[u]) if (!vst[v]) DFS(v);
    }
}

```

```

        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++){
            if (!vst[i]) DFS(i);
            reverse(vec.begin(),vec.end());
            FZ(vst);
            for (auto v : vec)
                if (!vst[v]){
                    rDFS(v); nScc++;
                }
        }
    }
};

```

## 5.8 SPFA

```

#define MXN 200005
struct SPFA{
    int n;
    LL inq[MXN], len[MXN];
    vector<LL> dis;
    vector<pair<int, LL>> edge[MXN];
    void init(int _n){
        n = _n;
        dis.clear(); dis.resize(n, 1e18);
        for(int i = 0; i < n; i++){
            edge[i].clear();
            inq[i] = len[i] = 0;
        }
    }
    void addEdge(int u, int v, LL w){
        edge[u].push_back({v, w});
    }
    vector<LL> solve(int st = 0){
        deque<int> dq; //return {-1} if has negative cycle
        dq.push_back(st); //otherwise return dis from st
        inq[st] = 1; dis[st] = 0;
        while(!dq.empty()){
            int u = dq.front(); dq.pop_front();
            inq[u] = 0;
            for(auto [to, d] : edge[u]){
                if(dis[to] > d+dis[u]){
                    dis[to] = d+dis[u];
                    len[to] = len[u]+1;
                    if(len[to] > n) return {-1};
                    if(inq[to]) continue;
                    (!dq.empty()&&dis[dq.front()] > dis[to]?
                        dq.push_front(to) : dq.push_back(to));
                    inq[to] = 1;
                }
            }
        }
        return dis;
    }
} }spfa;

```

## 5.9 差分約束

約束條件  $V_j - V_i \leq W$  addEdge( $V_i, V_j, W$ ) and run bellman-ford or spfa

# 6 數論

## 6.1 離散根號

```

void calcH(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        LL t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
            } while (mypow(b, p / 2, p) != p - 1);
            pb = mypow(b, h, p);
        }
    }
}

```

```

    } int s = mypow(a, h / 2, p);
    for (int step = 2; step <= t; step++) {
        int ss = (((LL)(s * s) % p) * a) % p;
        for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
        if (ss + 1 == p) s = (s * pb) % p;
        pb = ((LL)pb * pb) % p;
    } x = ((LL)s * a) % p; y = p - x;
    } return true;
}

```

## 6.2 ex-crt

```

typedef __int128 ll;
void exgcd(ll a,ll b,ll &g,ll &x,ll &y) {
    if (b == 0) {
        g = a;
        x = 1;
        y = 0;
        return;
    }
    exgcd(b,a%b,g,y,x);
    y-=(a/b)*x;
}
bool flag = false;
ll a1,a2,n1,n2;
ll abs(ll x) {
    return x>0?x:-x;
}
void china() {
    ll d = a2 - a1;
    ll g,x,y;
    exgcd(n1,n2,g,x,y);
    if (d % g == 0) {
        x = ((x*d/g)%(n2/g)+(n2/g))%(n2/g);
        a1 = x*n1 + a1;
        n1 = (n1*n2)/g;
    }
    else
        flag = true;
}
int n;
long long as[100001]; //算式答案 x
long long ns[100001]; //模数 MOD
ll realchina() {
    a1 = as[0];
    n1 = ns[0];
    for (ll i = 1;i<n;i++) {
        a2 = as[i];
        n2 = ns[i];
        china();
        if (flag)
            return -1;
    }
    return a1;
}
int main() {
    cin>>n;
    flag = false;
    for (ll i = 0;i<n;i++)
        cin>>ns[i]>>as[i];
    cout<<(long long)realchina()<<endl;
}

```

## 6.3 ex-gcd

```

int exgcd(int a,int b,int&x,int&y){
    if(b==0)return x=1,y=0,a;
    int d = exgcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}

```

## 6.4 FFT

```

// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0,1);
cplx omega[MAXN+1];

```

```

void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^= k); k >= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if(inv) for (i = 0; i < n; i++) a[i] /= n;
    }
    cplx arr[MAXN+1];
    inline void mul(int _n,ll a[],int _m,ll b[],ll ans[]){
        int n=1,sum=_n+_m-1;
        while(n<sum)
            n<=<=1;
        for(int i=0;i<n;i++) {
            double x=(i<_n?a[i]:0),y=(i<_m?b[i]:0);
            arr[i]=complex<double>(x+y,x-y);
        }
        fft(n,arr);
        for(int i=0;i<n;i++)
            arr[i]=arr[i]*arr[i];
        fft(n,arr,true);
        for(int i=0;i<sum;i++)
            ans[i]=(long long int)(arr[i].real()/4+0.5);
    }
}

```

## 6.5 高斯消去法

```

const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a , int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD ,
            k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD ,
            k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(i , 0 , n){
            REP(i , now , n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[i] , v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now] , GAUSS_MOD - 2);
            REP(i , 0 , n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j , now , n + 1) (v[i][j] +=
                    GAUSS_MOD - tmp * v[now][j] %
                    GAUSS_MOD) %= GAUSS_MOD;
            }
        }
        REP(i , 0 , n) ans[i] = v[i][n + 1] * ppow(v[i][i] , GAUSS_MOD - 2) % GAUSS_MOD;
        return ans;
    }
} gs;
// gs.v.clear() , gs.v.resize(n , vector<int>(n + 1 , 0));
} gs;

```

## 6.6 喬瑟夫問題

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

## 6.7 定理

- Lucas's Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod C(m_i, n_i)$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling approximation :  
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem :  $A = i + b/2 - 1$   
 $A$ : Area  $i$ : grid number in the inner  $b$ : grid number on the side
- Catalan number :  $C_n = \binom{2n}{n} / (n+1)$   
 $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)! \frac{n-m+1}{n+1}$  for  $n \geq m$   
 $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$   
 $C_0 = 1$  and  $C_{n+1} = 2 \binom{2n+1}{n+2} C_n$   
 $C_0 = 1$  and  $C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$  for  $n \geq 0$
- Euler Characteristic:  
planar graph:  $V - E + F - C = 1$   
convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$
- Polya' theorem ( $c$  is number of color  $m$  is the number of cycle size):  
 $(\sum_{i=1}^m c^{gcd(i, m)}) / m$
- Burnside lemma:  
 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- 錯排公式: ( $n$  個人中 · 每個人皆不再原來位置的組合數):  
 $dp[0] = 1; dp[1] = 0;$   
 $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell 數 (有  $n$  個人, 把他們拆組的方法總數) :  
 $B_0 = 1$   
 $B_n = \sum_{k=0}^n s(n, k)$  (second - stirling)  
 $B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$
- Wilson's theorem :  
 $(p-1)! \equiv -1 \pmod{p}$
- Fermat's little theorem :  
 $a^p \equiv a \pmod{p}$
- Euler's totient function:  
 $A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p-1)) \bmod p$
- 歐拉函數降冪公式:  
 $A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$
- 6 的倍數:  
 $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

## 6.8 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383      6 : pirms <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[] = {}
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x = mypow(a, u, n);
    for(int i=0; i<t; i++) {
        LL nx = mul(x, x, n);
```

```
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(LL n) {
    int s=(magic number size)
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=magic[s]%n;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

## 6.9 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
   n   2^n      p      a      root
   16  65536    65537    1      3
   20 1048576   7340033   7      3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1) return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >>= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^= k); k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n, P);
            reverse(a+1, a+n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

## 6.10 Pollard's Rho

```
// does not work when n is prime 0(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

## 6.11 質數

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
 * 999983, 1097774749, 1076767633, 100102021, 999997771
 * 1001010013, 1000512343, 987654361, 999991231
 * 999888733, 98789101, 987777733, 999991921, 1010101333
 * 1010102101, 1000000000039, 100000000000037
 * 2305843009213693951, 4611686018427387847
 * 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}
```

## 6.12 phi

```
ll phi(ll n){ // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n; // 0(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
        }
    }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

## 6.13 矩陣快速冪

```
LL len,mod;
vector<vector<LL>> operator*(vector<vector<LL>> x,
    vector<vector<LL>> y){
    vector<vector<LL>> ret(len,vector<LL>(len,0));
    for(int i=0;i<len;i++){
        for(int j=0;j<len;j++){
            for(int k=0;k<len;k++){
                ret[i][j]=(ret[i][j]+x[i][k]*y[k][j])%
                    mod;
            }
        }
    }
    return ret;
}
```

```
}
struct Martix_fast_pow{ //0(len^3 lg k)
    LL init(int _len,LL m=9223372036854775783LL){
        len=_len, mod=m;
    }
    // mfp.solve(k,{0, 1}, {1, 1}) k'th fib {值,係數} // 0-base
    LL solve(LL n,vector<vector<LL>> poly){
        if(n<len) return poly[n][0];
        vector<vector<LL>> mar(len,vector<LL>(len,0)),x
            (len,vector<LL>(len,0));
        for(int i=0;i<len;i++) mar[i][i]=1;
        for(int i=0;i+1<len;i++) x[i][i+1]=1;
        for(int i=0;i<len;i++) x[len-1][i]=poly[i][1];
        while(n){
            if(n&1) mar=mar*x;
            n>>=1, x=x*x;
        }
        LL ans=0;
        for(int i=0;i<len;i++) ans=(ans+mar[len-1][i]
            *poly[i][0]%mod)%mod;
        return ans;
    }
}mfp;
```

## 7 字串

### 7.1 KMP

/\* len-failure[k]:  
在k結尾的情況下，這個子字串可以由開頭  
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴  
如果我們不只想求最多，而且以0-base做為考量  
，那可能的長度由大到小會是

failuer[k]、failure[failuer[k]-1]  
、failure[failure[failuer[k]-1]-1]..  
直到有值為0為止 \*/

```
int failure[MXN];
vector<int> KMP(string& t, string& p){
    vector<int> ret;
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i){
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i){
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1){
            ret.push_bck( i - p.size() + 1 );
            j = failure[j];
        }
    }
}
```

### 7.2 馬拉車

```
void manacher(char *s,int len,int *z){
    len=(len<<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>>1]:'@';
    }
    z[0]=1;
    for(int i=1,l=0,r=0;i<len;i++){
        z[i]=i<r?min(z[l+l-i],r-i):1;
        while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
            ++z[i];
        if(i+z[i]>r) l=i,r=i+z[i];
    }
}
```

### 7.3 回文樹

// len[s]是對應的回文長度  
// num[s]是有幾個回文後綴  
// cnt[s]是這個回文子字串在整個字串中的出現次數  
// fail[s]是他長度次長的回文後綴，aba的fail是a  
const int MXN = 1000010;
struct PalT{
 int nxt[MXN][26],fail[MXN],len[MXN];
 int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
};



```

int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
char s[MXN] = {-1};
int newNode(int l, int f){
    len[tot] = l, fail[tot] = f, cnt[tot] = num[tot] = 0;
    memset(nxt[tot], 0, sizeof(nxt[tot]));
    diff[tot] = (l > 0 ? l - len[f] : 0);
    sfail[tot] = (l > 0 && diff[tot] == diff[f] ? sfail[f] : f);
    return tot++;
}
int getfail(int x){
    while(s[n-len[x]-1] != s[n]) x = fail[x];
    return x;
}
int getmin(int v){
    dp[v] = fac[n-len[sfail[v]]-diff[v]];
    if(diff[v] == diff[fail[v]])
        dp[v] = min(dp[v], dp[fail[v]]);
    return dp[v]+1;
}
int push(){
    int c = s[n] - 'a', np = getfail(lst);
    if(!(lst = nxt[np][c])){
        lst = newNode(len[np]+2, getfail(fail[np]))[c];
        nxt[np][c] = lst; num[lst] = num[fail[lst]]+1;
    }
    fac[n] = n;
    for(int v = lst; len[v] > 0; v = sfail[v])
        fac[n] = min(fac[n], getmin(v));
    return ++cnt[lst], lst;
}
void init(const char *_s){
    tot = lst = n = 0;
    newNode(0, 1), newNode(-1, 1);
    for(; _s[n];) s[n+1] = _s[n], ++n, state[n-1] = push();
    for(int i = tot-1; i > 1; i--) cnt[fail[i]] += cnt[i];
}
}palt;

```

## 7.4 SA

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i > 0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] == true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[xs[sa[i]-1]]++ = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--xs[sa[i]-1]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) {REP(i,n) sa[--c[s[i]]] = i; return; }

```

```

    for(int i = n - 2; i >= 0; i--) t[i] = (s[i] == s[i+1] ? t[i+1] : s[i] < s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--xs[s[i]]] = p[q[i] = nn++] = i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq = lst < 0 || memcmp(s+sa[i], s+lst, (p[q[sa[i]]+1]-sa[i])*sizeof(int));
        ns[q[lst=sa[i]]] = nmzx += neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--xs[p[nsa[i]]]] = p[nsa[i]]);
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

## 7.5 SAM

```

// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]
// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-|P|+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
    int nxt[MXM][33]; //cnt[MXM], ds[MXM], dsl[MXM], fp[MXM]
    // bool v[MXM]
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = 0; //cnt=ds=ds1=fp=v=0
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode(); //cnt[np]=1
        mx[np] = mx[p]+1; //fp[np]=mx[np]-1
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode(); //fp[nq]=fp[q]
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void calc(){
        calc(root);
        iota(ind, ind+tot, 1);
    }
}

```

```

    sort(ind, ind+tot, [&](int i, int j){return mx[i]<mx[j];});
    for(int i=tot-1; i>=0; i--){
        cnt[mom[ind[i]]] += cnt[ind[i]];
    }
    void calc(int x){
        v[x]=ds[x]=1; dsl[x]=0; //rmom[mom[x]].push_back(x);
        for(int i=1; i<=26; i++){
            if(nxt[x][i]){
                if(!v[nxt[x][i]]) calc(nxt[x][i]);
                ds[x] += ds[nxt[x][i]];
                dsl[x] += ds[nxt[x][i]] + dsl[nxt[x][i]];
            }
        }
    }
    void push(const string& str){
        for(int i=0; i<str.size(); i++){
            push(str[i]-'a'+1);
        }
    }
} sam;

```

## 7.6 樹哈希

```

ll dfs(int u){
    vector<ll> h;
    subtree_sz[u] = 1;
    for(ll child : edge[u]){
        h.push_back(dfs(child));
        subtree_sz[u] += subtree_sz[child];
    }
    sort(h.begin(), h.end());
    ll ret = subtree_sz[u];
    for(ll v : h){
        ret = (ret * base + v) % MOD;
    }
    return ret;
}

```

## 7.7 trie

```

//01 bitwise trie
struct trie{
    trie *nxt[2]; // 差別
    int cnt; //紀錄有多少個數字以此節點結尾
    int sz; //有多少數字的前綴包括此節點
    trie():cnt(0),sz(0){
        memset(nxt,0,sizeof(nxt));
    }
};
//創建新的字典樹
trie *root;
void insert(int x){
    trie *now = root; // 每次從根節點開始
    for(int i=22; i>=0; i--){ // 從最高位元開始往低位元走
        now->sz++;
        //cout<<(x>>i&1)<<endl;
        if(now->nxt[x>>i&1] == NULL){ //判斷當前第 i 個位元是 0 還是 1
            now->nxt[x>>i&1] = new trie();
        }
        now = now->nxt[x>>i&1]; //走到下一個位元
    }
    now->cnt++;
    now->sz++;
}

```

## 7.8 Z-value

```

int z[MAXN];
void Z_value(const string& s) { //z[i] = lcp(s[1...],s[i...])
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1; i<len; i++){
        j=max(min(z[i-left],right-i),0);
        for(; i+j<len&&s[i+j]==s[j]; j++);
        z[i]=j;
        if(i+z[i]>right){
            right=i+z[i];
            left=i;
        }
    }
}

```

## 7.9 minRotation

```

//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    }
    return a;
}

```

## 8 DP

### 8.1 數位 dp

```

ll dp[MXN_BIT][PRE_NUM][LIMIT][F0];
ll dfs(int i, int pre, bool lim, bool f0, const string& str){
    if(v[i][pre][f0][lim]) return dp[i][pre][f0][lim];
    v[i][pre][f0][lim] = true;

    if(i == str.size())
        return dp[i][pre][f0][lim] = 1;

    ll ret = 0, h = lim ? str[i] : '9';

    for(int j='0'; j<=h; j++){
        if(abs(j-pre)>=2 || f0){
            ret += dfs(i+1, j, j==h && lim, f0 && j=='0', str);
        }
    }
    return dp[i][pre][f0][lim] = ret;
}

```

### 8.2 SOS dp

```

for(int i=0; i<(1<<N); ++i)
    F[i] = A[i];
for(int i=0; i<N; ++i) for(int mask=0; mask<(1<<N); ++mask){
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}

```

### 8.3 p-median

```

void p_Median(){
    for (int i=1; i<=N; ++i)
        for (int j=i; j<=N; ++j){
            m = (i+j)/2, d[i][j] = 0; // m是中位數 · d[i][j]為距離的總和
            for (int k=i; k<=j; ++k) d[i][j] += abs(arr[k] - arr[m]);
        }
    for (int p=1; p<=P; ++p)
        for (int n=1; n<=N; ++n){
            dp[p][n] = 1e9;
            for (int k=p; k<=n; ++k)
                if (dp[p-1][k-1] + d[k][n] < dp[p][n]){
                    dp[p][n] = dp[p-1][k-1] + d[k][n];
                    r[p][n] = k; // 從第k個位置往右到第j個位置
                }
        }
}

```

## 9 Other

### 9.1 黑魔法

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int, int> umap_t;
typedef priority_queue<int> heap;
#include <ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
}

```

```

set_t s; s.insert(12); s.insert(505);
// The order of the keys should be: 12, 505.
assert(*s.find_by_order(0) == 12);
assert(*s.find_by_order(3) == 505);
// The order of the keys should be: 12, 505.
assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);
// Erase an entry.
s.erase(12);
// The order of the keys should be: 505.
assert(*s.find_by_order(0) == 505);
// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);

heap h1 , h2; h1.join( h2 );
rope<char> r[ 2 ];
r[ 1 ] = r[ 0 ]; // persistenet
string t = "abc";
r[ 1 ].insert( 0 , t.c_str() );
r[ 1 ].erase( 1 , 1 );
cout << r[ 1 ].substr( 0 , 2 );
}

```

## 9.2 CDQ

## 9.3 DLX

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=R[i]; j!=i; j=R[j] ){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
}
void resume(int c){
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=L[i]; j!=i; j=L[j] ){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    L[R[c]]=R[L[c]]=c;
}
int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for( int i=R[0]; i!=0; i=R[i] )
        if(S[i]<md){ md=S[i]; c=i; }
    if(md==0) return 0;
    remove(c);
    for( int i=D[c]; i!=c; i=D[i] ){
        used[ROW[i]]=1;
        for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
        if(dfs()) return 1;
        for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n,int m){
    for( int i=0; i<=m; i++){
        R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
        S[i]=0; C[i]=i;
    }
    R[m]=0; L[0]=m;
    int t=m+1;
    for( int i=0; i<n; i++){
        int k=-1;
        for( int j=0; j<m; j++){
            if(!A[i][j]) continue;
            if(k==-1) L[t]=R[t]=t;
            else{ L[t]=k; R[t]=R[k]; }
            k=t; D[t]=j+1; U[t]=U[j+1];
        }
    }
}

```

```

L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
} }
for( int i=0; i<n; i++) used[i]=0;
return dfs();
}

```

## 9.4 Hiber Curve

```

long long hilbert(int n,int x,int y){
    long long res=0;
    for(int s=n/2;s>=1){
        int rx=(x&s)>0,ry=(y&s)>0; res+=s*1ll*s*((3*rx)^ry);
        if(ry==0){ if(rx==1) x=s-1-x,y=s-1-y; swap(x,y); }
    }
    return res;
}

```

## 9.5 模擬退火

```

mt19937 rng((unsigned long long)(new char));
auto rnd = [&]() -> double {
    return 2 * ((double)rng() / rng.max()) - 1;
};

auto run = [&](int l, int r, int u, int d) -> double {
    double x = (l+r)/2., y = (u+d)/2., s = cal(x, y);
    double nx, ny;
    for (double t = hypot(l-r, u-d); t >= 1e-8; t
        *= 0.99995) {
        do {
            nx = x + t * rnd();
            ny = y + t * rnd();
        } while (!safe(nx, ny));
        if (chmax(s, cal(nx, ny)))
            x = nx, y = ny;
    }
    return s;
};

```





