On Ambiguity in 6-DoF Magnetic Pose Estimation

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Abstract—**Magnetic pose estimation is an accurate and non-contacting pose tracking method insusceptible to occlusion problems. It has been widely employed to track various incorporeal medical robots, e.g., capsule endoscopes and flexible surgical robots, in minimally invasive interventions. Although extensive research has been conducted on modeling, design, and estimation algorithms of magnetic pose tracking systems, the ambiguity problem in magnetic pose estimation is long overlooked that could result in inaccuracy and even divergence of pose tracking. This paper is devoted to fundamentally investigate the ambiguity issue in 6-DoF magnetic pose estimation. The magnetic ambiguity is formally defined and its existence and impacts on pose estimation are analyzed and discussed for the single-magnet and dual-magnet tracking systems. It is found that there exist different types of ambiguity that have different influence on pose estimation. Efficient algorithms are proposed to systematically identify all ambiguous poses of all types in the workspace. Among typical configurations of dual-magnet systems, the collocated configuration can achieve stable pose estimation with minimum impacts of ambiguity. It is concluded that there should be at least TWO magnetic sources to prevent impacts of ambiguity in pose estimation with appropriate initialization and ambiguity can be eliminated using THREE or more magnetic sources. This paper proposes a theoretical and implementable framework to analyze, categorize and identify ambiguity in magnetic pose estimation, which lays the foundation for optimizing configurations of magnetic sources and designing compact, energy-efficient, but unambiguous magnetic tracking systems with minimum number of magnetic sources.**

Index Terms—**magnetic tracking, pose estimation, ambiguity, localization, medical robots**

I. Introduction

Magnetic field of various magnetic sources has been explored to determine pose (position and orientation) of a moving object. For example, locally uniform geomagnetic field can be measured by a compass to find the orientation of pedestrians and vehicles relative to the north pole of the earth [1]. To determine position along with orientation, magnetic objects with stable magnetization and spatially varying magnetic field, e.g., permanent magnets and electromagnets, are used as magnetic sources with their magnetic field modeled and measured for pose estimation [2][3]. Since magnetic field can penetrate most of non-ferromagnetic materials, magnetic pose tracking method can localize occluded objects such as medical robots moving inside of human body. In addition, magnetic tracking works well in harsh environments with dusts and liquids. Although magnetic field attenuates rapidly with the distance from the magnetic source, magnetic pose tracking is stable and robust because the spatial distribution of the magnetic field does not depend on atmospheric conditions such as temperature, pressure, and air convection. Recently, small-size magnetic sensors (e.g., tunnel magneto-resistive (TMR) sensors) are developed with increased sensitivity and reduced cost, enabling accurate magnetic field measurement and pose estimation in a large range up to tens of centimeters [4].

Compared with other pose tracking methods, e.g., optical tracking, inertial tracking, radio-frequency (RF) tracking, and X-ray tracking, magnetic tracking is accurate, non-contacting, low-cost, and safe with no need of light-of-sight access to the target. Therefore, magnetic tracking has been increasingly integrated in medical robotic systems to track various intracorporeal medical devices including puncture needles [5], catheters [6], endoscopes [7][8], and surgical tools [9]. In robot-assisted minimally invasive interventions, magnetic pose tracking is employed for image registration [10], navigation of medical instruments [11], and feedback control of medical robots [12], which can increase accuracy, improve safety, and achieve a higher level of autonomy [13].

Existing magnetic pose estimation systems can be classified as permanent magnet (PM) based systems [2][14-17] and electromagnet (EM) based systems [3-4][18-20]. In PM based systems, a permanent magnet is attached to the target object and an array of sensors nearby are used to measure its magnetic field for pose estimation. However, since the magnetic field of a single permanent magnet is rotationally symmetric about the axis of its magnetic moment, the rotation of the magnet about this axis does not cause variation of the surrounding magnetic field and thus this degree of freedom of orientation cannot be determined from magnetic measurements. In addition, PM based pose estimation suffers from surrounding ferromagnetic disturbances due to the use of DC magnetic field. To achieve full 6-DoF pose estimation and prevent ferromagnetic disturbances, EM based systems are developed, where a tri-axial sensor is attached to the moving target to measure the AC magnetic field of multiple electromagnetic coils for pose determination. Most commercial magnetic pose tracking systems are EM based systems, e.g., NDI Aurora. Hence, the scope of this paper is focused on EM based pose estimation although its conclusions can be partially generalized to PM based systems.

Extensive research has been performed on key issues of magnetic pose estimation including modeling and calibration of magnetic tracking systems [20][21], design of magnetic sources

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and sensors [22][23], development of nonlinear estimation algorithms [24-27], and improvement of measurement sensitivity and accuracy [28][29]. However, one issue is long overlooked that the 6-DoF pose of the target (sensor) cannot be uniquely determined by the specific magnetic measurement in certain regions of the workspace. This ambiguity can lead to divergence and failure of pose estimation. The ambiguity in magnetic pose estimation is rare in most commercial and prototype systems since they normally use magnetic measurements of many electromagnetic coils, e.g., 9 coils in [7], 12 coils in the NDI tetrahedron field generator [13], etc., where ambiguity is assumed to be eliminated in most of the workspace. However, pose tracking systems using too many electromagnets are bulky, consume excessive power, and need additional cooling. Therefore, there is a clear need of developing a magnetic pose estimation system with a minimum number of electromagnets while preventing the ambiguity problem. On the other hand, little is known about the nature and impacts of ambiguity in magnetic pose estimation. It is necessary to investigate the ambiguity problem fundamentally from a theoretical perspective in order to lay the foundation for optimal configuration design of magnetic sources for accurate and robust pose estimation. Furthermore, an efficient method to identify the distribution of ambiguity is needed to determine the effective (unambiguous) workspace of the magnetic pose estimation system for reliable target tracking.

In this paper, the ambiguity issue in magnetic pose estimation is theoretically investigated and experimentally demonstrated for the single-magnet and dual-magnet pose estimation systems. The fundamental conclusions obtained in this paper can also facilitate analysis of ambiguity in multiple-magnet (≥ 3) systems since they can be considered as multiple dual-magnet systems. Major contributions of this paper are as follows:

(1) The ambiguity in magnetic pose estimation is formally defined.

(2) A categorization of ambiguity problems is proposed with their different impacts on pose estimation explained.

(3) Algorithms are proposed to identify all types of ambiguous poses in the workspace. Specifically, an efficient algorithm is proposed to detect connected ambiguous poses.

(4) The ambiguity in magnetic pose estimation is discussed for different configurations of magnetic sources. An optimal configuration of dual-magnet systems is proposed with minimum impacts of ambiguity.

The paper is organized as follows: In Section II, magnetic field model and estimation algorithms for magnetic pose estimation are generally introduced. The experimental setup for validation of theoretical results is also shown. Section III gives the general definition of ambiguity and discusses ambiguity in single-magnet systems. Section IV discusses different types of ambiguity in dual-magnet systems. In Section V, an efficient algorithm to detect connected ambiguous poses is proposed. Section VI summarizes the ambiguity issue in different configurations of dual-magnet systems and proposes an optimal configuration. The paper is concluded in Section VII.

II. Magnetic Pose Estimation

A. Magnetic Measurement Model

This paper considers the problem of estimating the 6-DoF pose (i.e., 3-DoF position and 3-DoF orientation) of a moving target by measuring the magnetic field of one or more stationary magnetic sources using a tri-axial magnetic sensor attached to the target. Fig. 1 shows the application of magnetic pose estimation system in localization of flexible robotic endoscopes. In this paper, the electromagnet is used as the magnetic source although the permanent magnet can also be used. The magnetic source can be modeled as a magnetic dipole if the workspace of motion tracking is sufficiently far from the source [27]. The magnetic flux intensity $\mathbf{b}$ at position $\mathbf{r}$ is given by a function $\mathrm{b}(\cdot)$,

$$\mathbf{b} = \mathrm{b}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\mathbf{m}\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{m}}{r^3}\right] \tag{1}$$

where μ_0 is the magnetic permeability in vacuum. $\mathbf{m} = NIA\hat{\mathbf{n}}$ is the magnetic moment of an electromagnet, where N is the number of wire turns, I is the electric current, A is the cross section area of the electromagnet and $\hat{\mathbf{n}}$ is the unit normal vector of the cross section area.

Throughout this paper, the right superscript of a vector is used to indicate the coordinate frame in which this vector is represented, e.g., $\mathbf{b}^{\mathrm{s}}$ denotes vector $\mathbf{b}$ represented in the sensor frame. Vectors without a superscript is represented in the world frame, e.g., $\mathbf{r}$ is the same as $\mathbf{r}^{\mathrm{w}}$.

The magnetic measurement $\mathbf{b}^{\mathrm{s}}$ obtained from the triaxial magnetic sensor is given by

$$\mathbf{b}^{\mathrm{s}} = {}^{\mathrm{s}}_{\mathrm{w}}\mathbf{R}\,\mathrm{b}(\mathbf{r}) \tag{2}$$

where rotation matrix ${}^{\mathrm{s}}_{\mathrm{w}}\mathbf{R}$ denotes the relative orientation of the world frame ("w") with respect to the sensor frame ("s"). Since the world frame is fixed, ${}^{\mathrm{s}}_{\mathrm{w}}\mathbf{R}$ can represent the orientation of the sensor. It is shown from (2) that the magnetic sensor measurement is a function of both position ($\mathbf{r}$) and orientation (${}^{\mathrm{s}}_{\mathrm{w}}\mathbf{R}$) of the sensor to be estimated. Thus, we can rewrite (2) as

$$\mathbf{b}^{\mathrm{s}} = \mathrm{B}(\boldsymbol{\xi}) \tag{3}$$

where $\boldsymbol{\xi} = (\mathbf{r}, {}^{\mathrm{s}}_{\mathrm{w}}\mathbf{R}) \in \mathrm{SE}(3)$ and $\mathrm{B}(\cdot)$ denotes the magnetic measurement model, mapping 6-DoF pose elements from the special Euclidean group $\mathrm{SE}(3)$ to the magnetic field in $\mathbb{R}^3$.

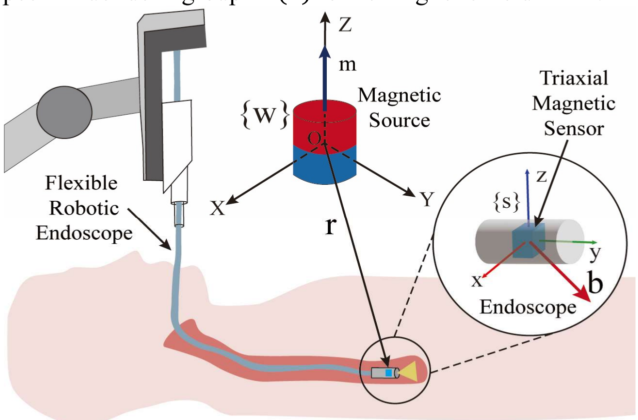

Fig. 1. Schematic of magnetic pose estimation using stationary magnetic source(s) and a triaxial magnetic sensor for localization of a flexible robotic endoscope.