Review on Frequency-Modulated Continuous Wave Radar (FM-CW)

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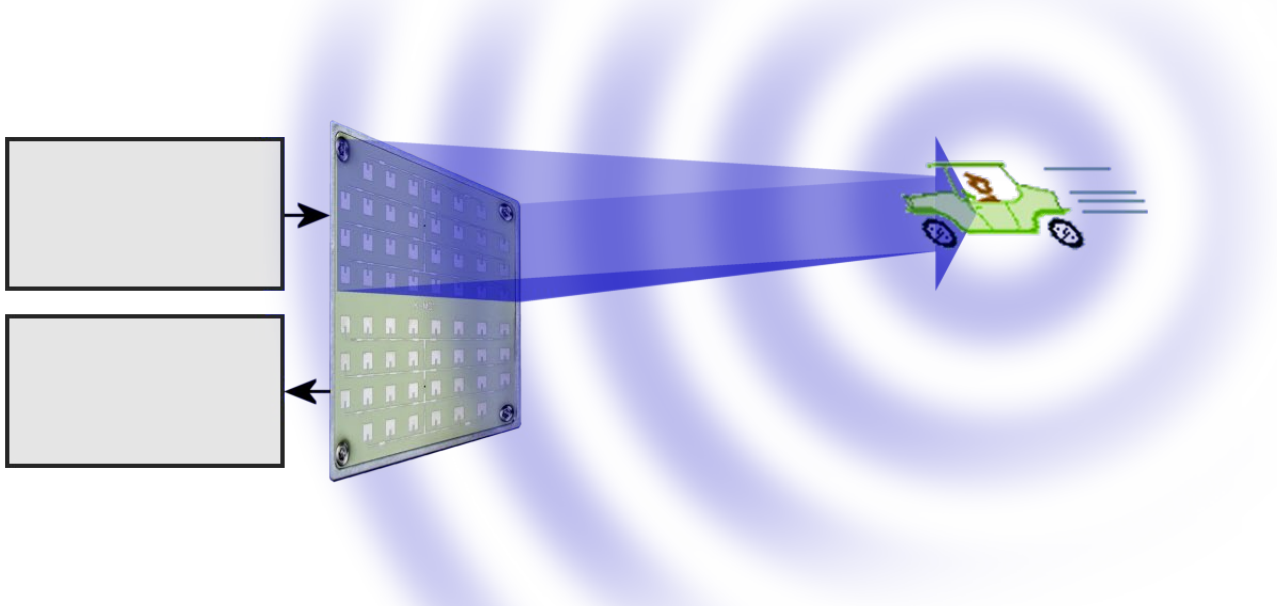


FM and CW

Frequency Modulated

Change its operating frequency during the measurement. That is, the transmission signal is modulated in frequency(or in phase)

Time & Frequency-domain Signal

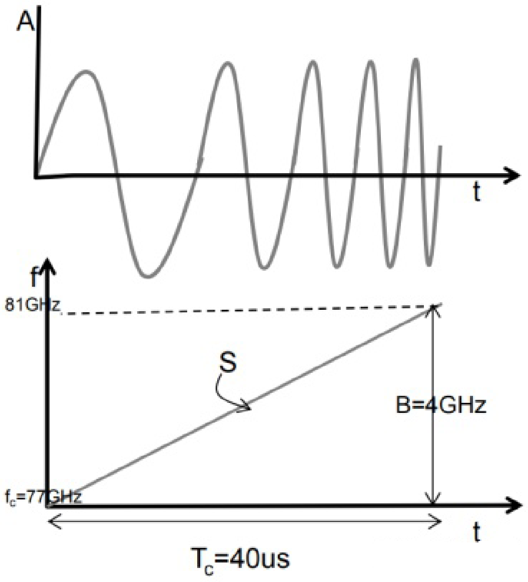


Figure 1

Continuous Wave Radar

- Radar that continuously emits electromagnetic waves
- No frequency modulation
- Cannot determine target range

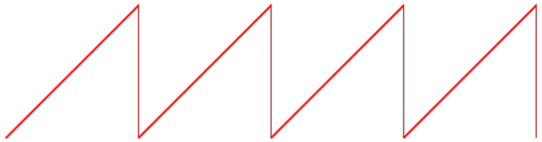
Variant:

Non-Modulated Single-Frequency Continuous Wave Radar

Multi-Frequency Continuous Wave Radar

Frequency-Modulated Continuous Wave Radar

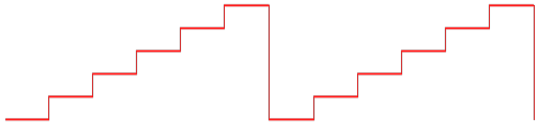
Modulation Pattern



Sawtooth modulation

Triangular modulation

Square-wave modulation



Stepped modulation

Block Diagram

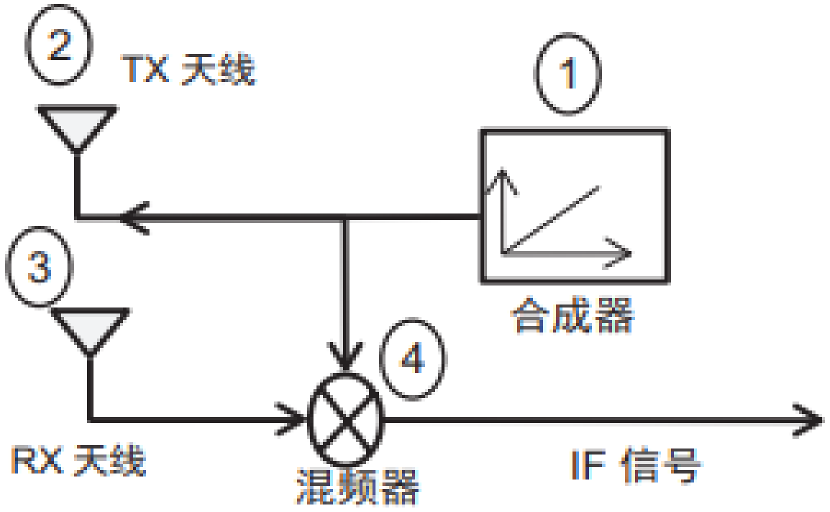


Figure 2

- Synthesizer(合成器) generates a chirp.
- The chirp is transmitted by the transmitting antenna (TX 天线).
- The object's reflection of the chirp generates a reflected chirp captured by the receiving antenna (RX 天线).
- Mixer(混频器) combines the RX and TX signals to generate an intermediate frequency (IF) signal.

Mixer

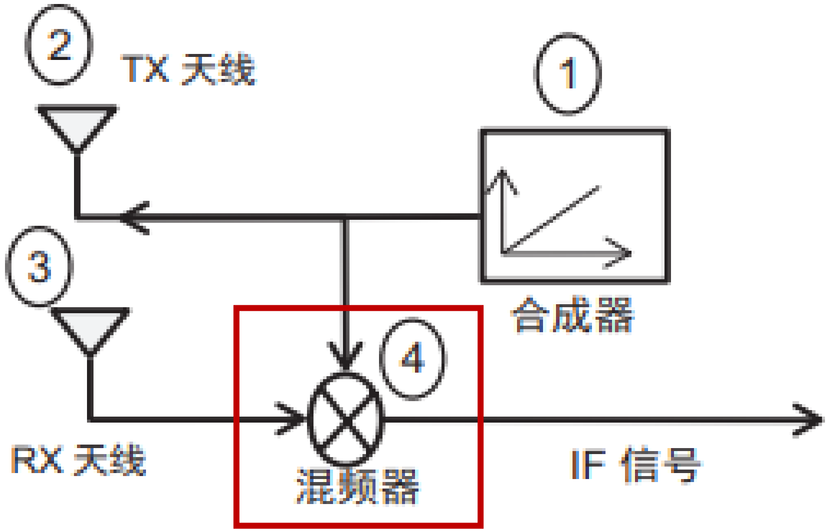


Figure 3

Mixer is an electronic component that combines two signals together to generate a new signal with a new frequency.

For 2 sinusoidal input $x1$ and $x2$ (Equation 1 and 2):

$$x1 = \sin(\omega 1 t + \Phi 1) \tag{1}$$

$$x2 = \sin(\omega 2 t + \Phi 2) \tag{2}$$

The output $xout$ has an instantaneous frequency equal to the difference between the instantaneous frequencies of the two input sine functions. The phase of the output $xout$ is equal to the difference between the phases of the two input signals (Equation 3):

$$xout = \sin[(\omega 1 - \omega 2)\, t + (\Phi 1 - \Phi 2)] \tag{3}$$

Antenna & IF Signal

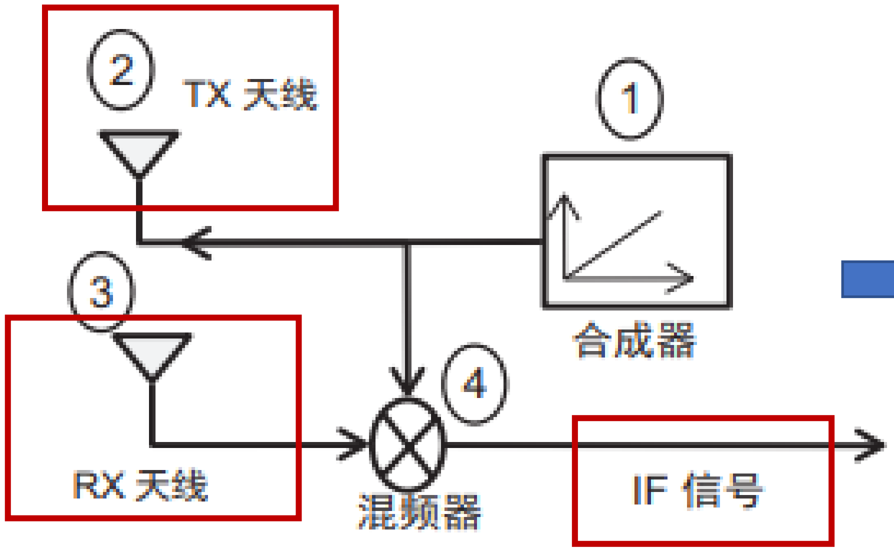


Figure 4

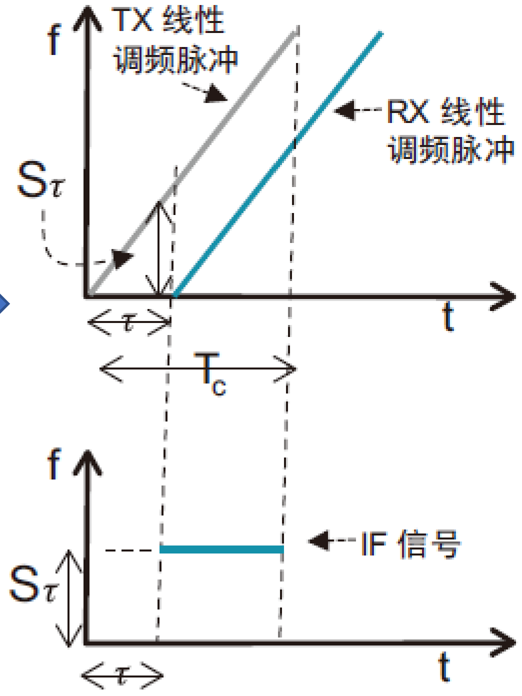


Figure 5

The IF signal is only valid during the overlap period of the TX chirp and RX chirp (the period between the vertical dashed lines in Figure 5).

The output signal of the mixer is a sine wave as a function of the amplitude of time because it has a constant frequency

The initial phase (ϕ_0) of the IF signal is the difference between the phase of the TX chirp and the phase of the RX chirp at the time point corresponding to the starting point of the IF signal (the time point indicated by the vertical dashed line on the left in Figure 5).

$$\tau = \frac{2d}{c} \qquad (4)$$

$$\phi_0 = 2\pi f_c \tau \qquad (5)$$

$$\phi_0 = \frac{4\pi d}{\lambda} \qquad (6)$$

For objects at a distance of *d* from the radar, the IF signal will be a sine wave

$$\text{IF Signal} = A\sin(2\pi f_0 t + \phi_0) \qquad (7)$$

$$\left(f_0 = \frac{S2d}{c},\ \phi_0 = \frac{4\pi d}{\lambda}\right)$$

Several Objects Detection

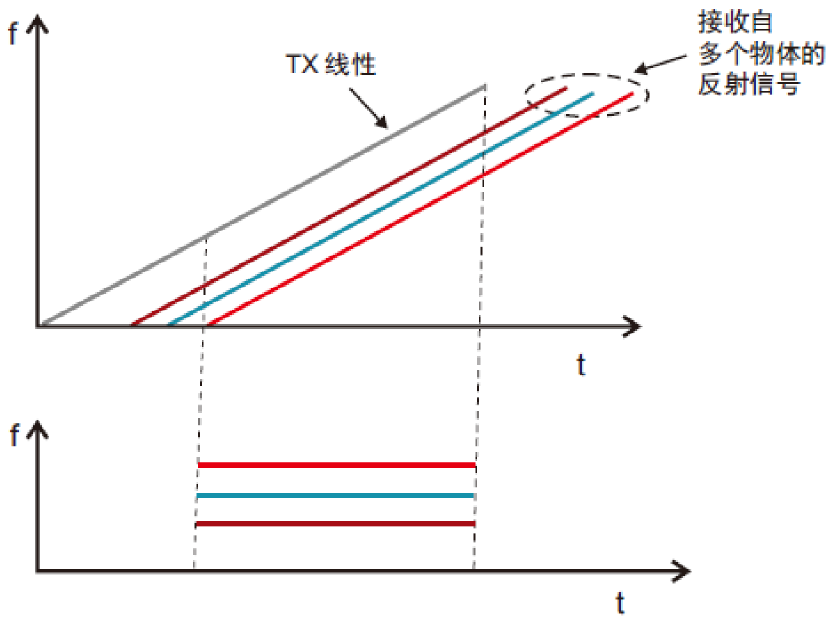


Figure 6

- The delay of each chirp is different, and the delay is proportional to the distance to the object.
- Different RX chirp pulses are converted into multiple IF tone signals, each of which has a constant frequency.
- This IF signal containing multiple tones must be processed by **Fourier transform** in order to separate different tones. Fourier transform processing will produce a spectrum with different separated peaks, each peak representing the presence of an object at a specific distance.

Range Resolution

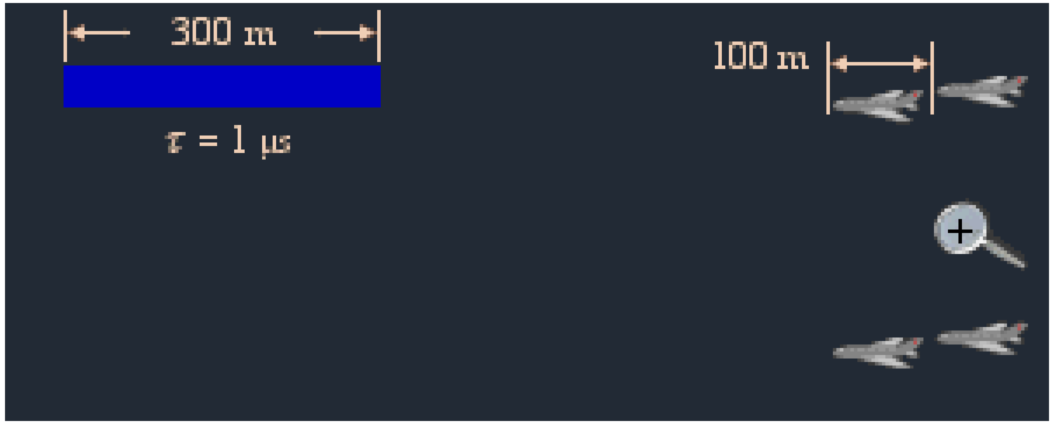


Figure 7

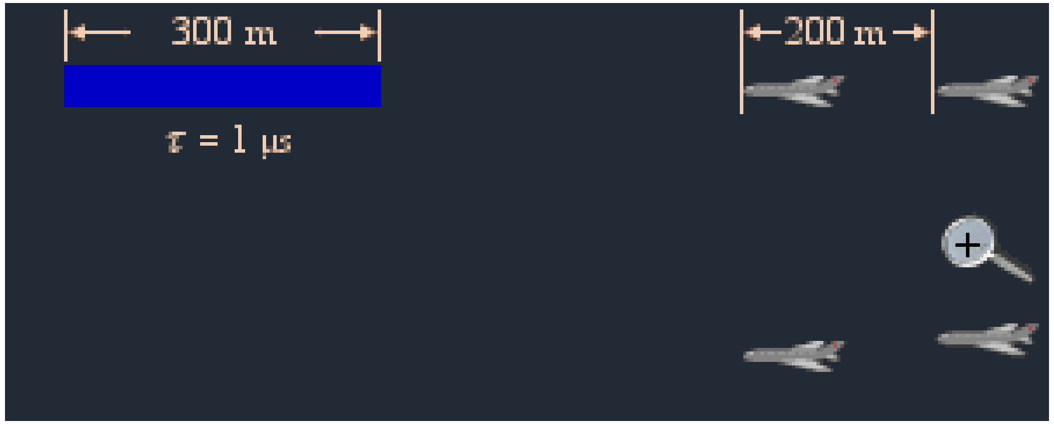


Figure 8

If the spacing between two aircrafts is too small, then the radar "see" only one target as shown in Figure 7 above.

Figure 8 is the other example when the spacing is large enough.

Range Resolution

Intrapulse Modulation

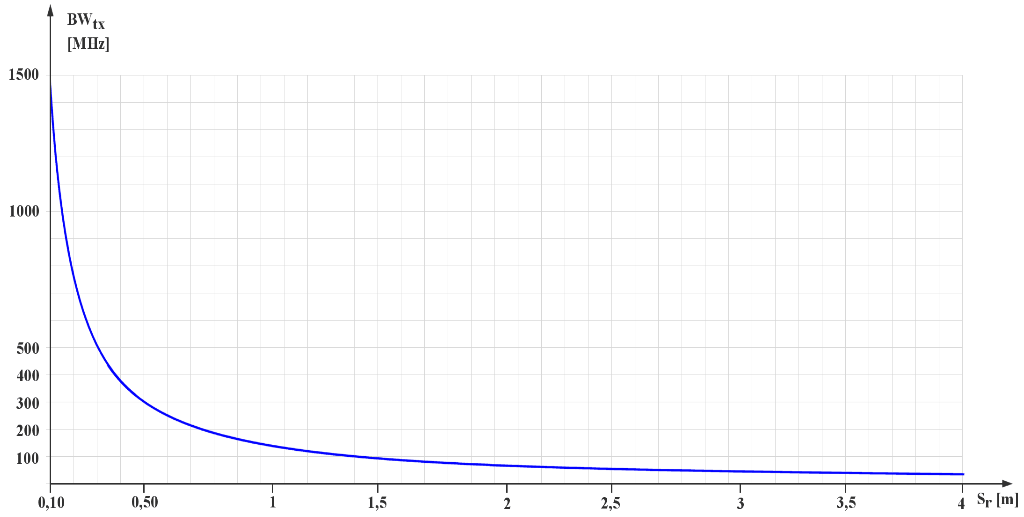


Figure 9

- The range-resolution of the radar is given by the length of the pulse at the output-jack of the pulse compressing stage.
- The ability to compress the pulse depends on the *bandwidth* of the transmitted pulse (BW_{tx}) not by its *pulse width*.
- As a matter of course the receiver needs at least the same bandwidth to process the full spectrum of the echo signals.

$$S_r \geq \frac{c_0}{2BW_{tx}} \tag{8}$$

Velocity measurement

Velocity measurement using two chirp pulses

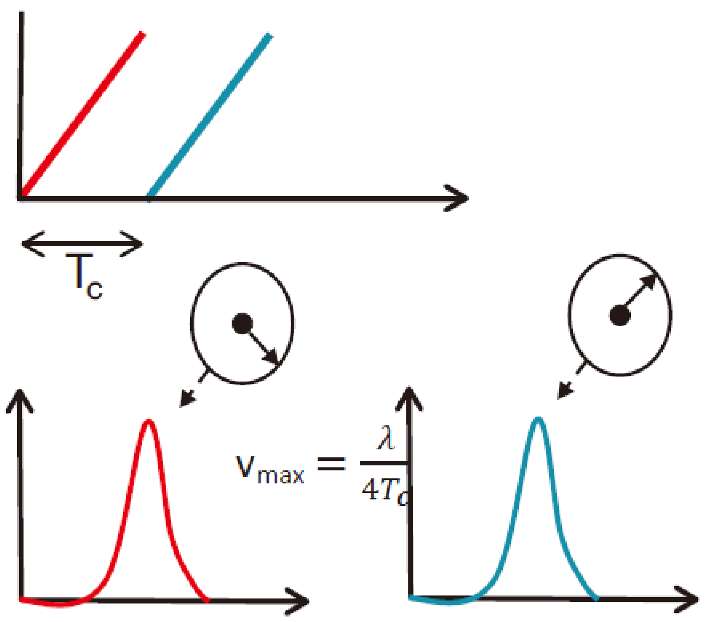


Figure 10

- In order to measure the speed, the FMCW radar will emit two chirp pulses separated by T_c.
- Each reflected chirp is processed by FFT to detect the distance of the object.
- The distance FFT corresponding to each chirp pulse will have a peak at the same position, but with a different phase.

$$\Delta\phi = \frac{4\pi v T_c}{\lambda} \tag{9}$$

$$v = \frac{\lambda \Delta\phi}{4\pi T_c} \tag{10}$$

$$v_{max} = \frac{\lambda}{4T_c} \tag{11}$$

Velocity measurement

Velocity measurement using multiple physics located at the same distance

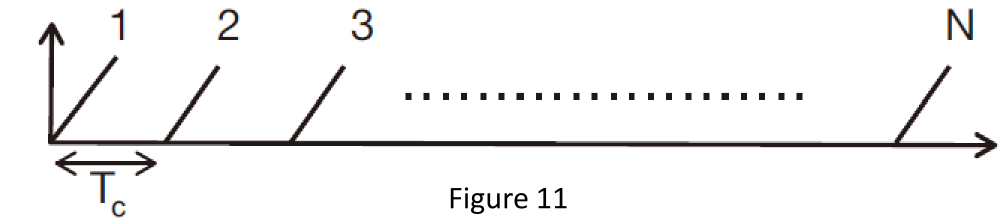


Figure 11

The frequency of a chirp frame over time.

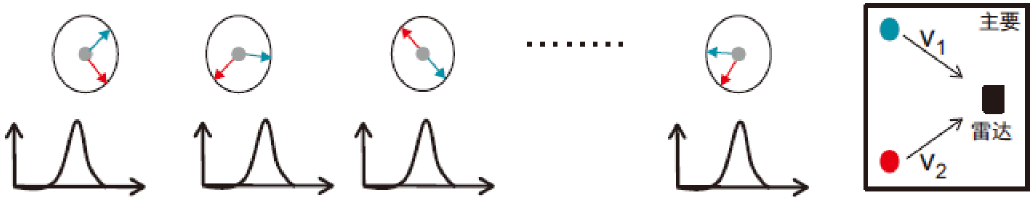


Figure 12

The distance of the reflected chirp frame FFT will generate N phasors.

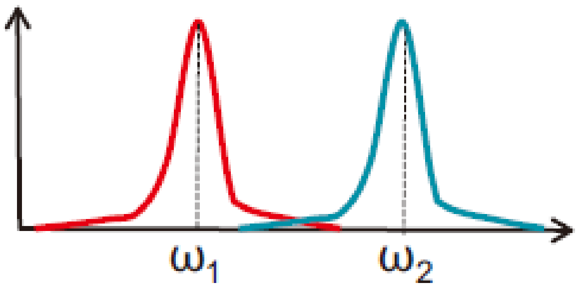


Figure 13

Doppler FFT can distinguish these 2 objects.

ω_1 and ω_2 correspond to the phase difference between successive chirps of each object:

$$V_1 = \frac{\lambda \omega_1}{4\pi T_c},\ V_2 = \frac{\lambda \omega_2}{4\pi T_c} \tag{12}$$

Velocity measurement

Velocity resolution

The theory of discrete Fourier transform points out that the two discrete frequencies w_1 and w_2 can be distinguished when $\Delta w = \omega_2 - \omega_1 > \frac{2\pi}{N}$ (13)

We have $\Delta V = \frac{\lambda \Delta\omega}{4\pi T_c}$(according to (12)) (14)

And $\Delta w = \omega_2 - \omega_1 > \frac{2\pi}{N}$

We get $\Delta V > \frac{\lambda}{2NT_c}$ (15)

Therefore, $V_{res} = \frac{\lambda}{2T_f}$ (16)

The velocity resolution of the radar is inversely proportional to the frame time(T_f).

Angle measurement

Angle estimation

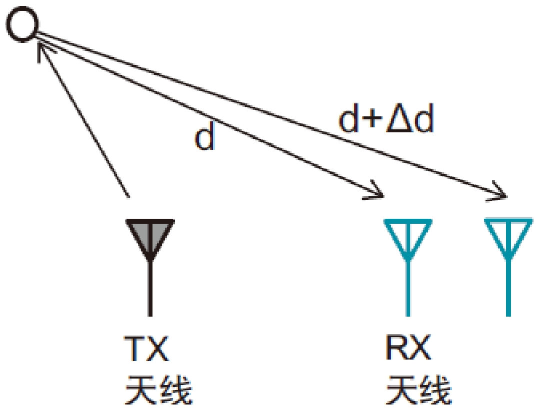
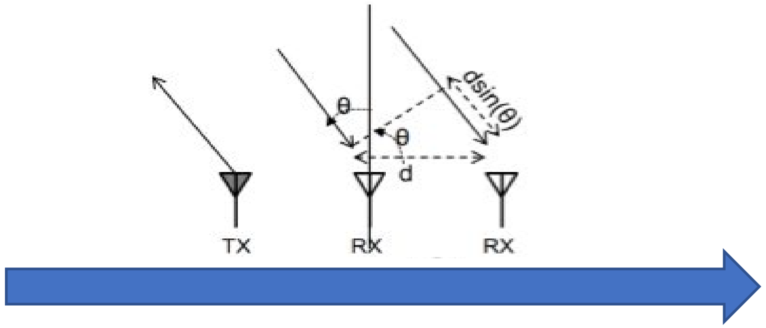


Figure 14



$$\Delta\phi = \frac{2\pi\Delta d}{\lambda} \quad (17)$$

$$\theta = \arcsin\left(\frac{\lambda\Delta\phi}{2\pi d}\right) \quad (18)$$

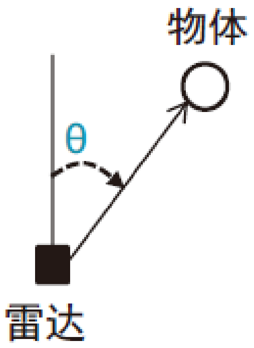


Figure 15

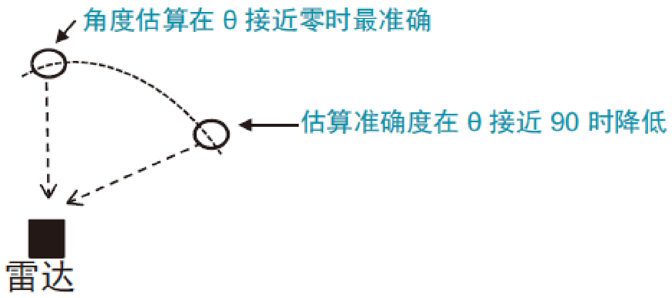


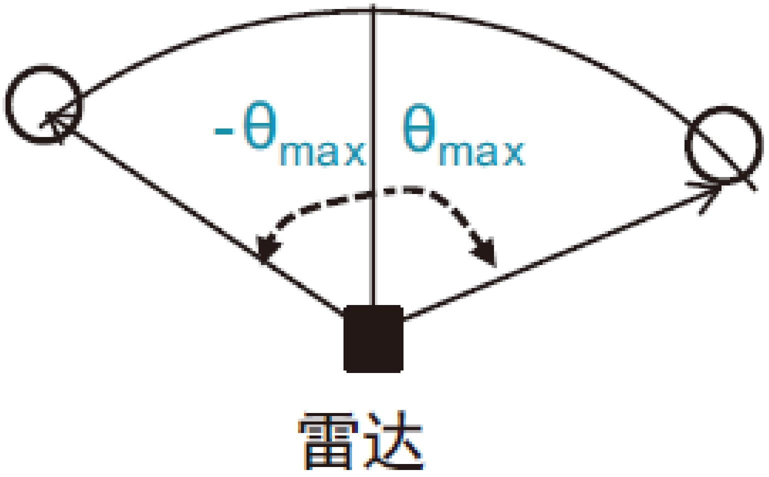
Figure 16

The distance difference between the object and the two antennas will cause the phase change of the FFT peak. Phase change enables estimation of AoA.

The estimation accuracy depends on AoA, and it is more accurate when the value of θ is small.

Speed measurement

Maximum angular field of view



The maximum angular field of view of the radar is defined by the maximum AoA that the radar can estimate.

Angle measurement is inseparable from $|\Delta\phi| < 180°$, which is correspond to $\frac{2\pi \mathrm{d} sin(\theta)}{\lambda} < \pi$.

Equation 17 shows the maximum field of view that can be served by two antennas separated by l.

$$\theta_{max} = arcsin(\frac{\lambda}{2d}) \quad (19)$$

Conclusion

Description	▪ **Typical sawtooth wave** ▪ **Bandwidth 100-150MHz**
Advantages	• High ranging accuracy • Easy to calculate relative velocity and range • Low transmit power, small size and low cost • Measure Doppler frequency shift and static target probability directly
Disadvantages	○ Long calculation time for multiple chirps

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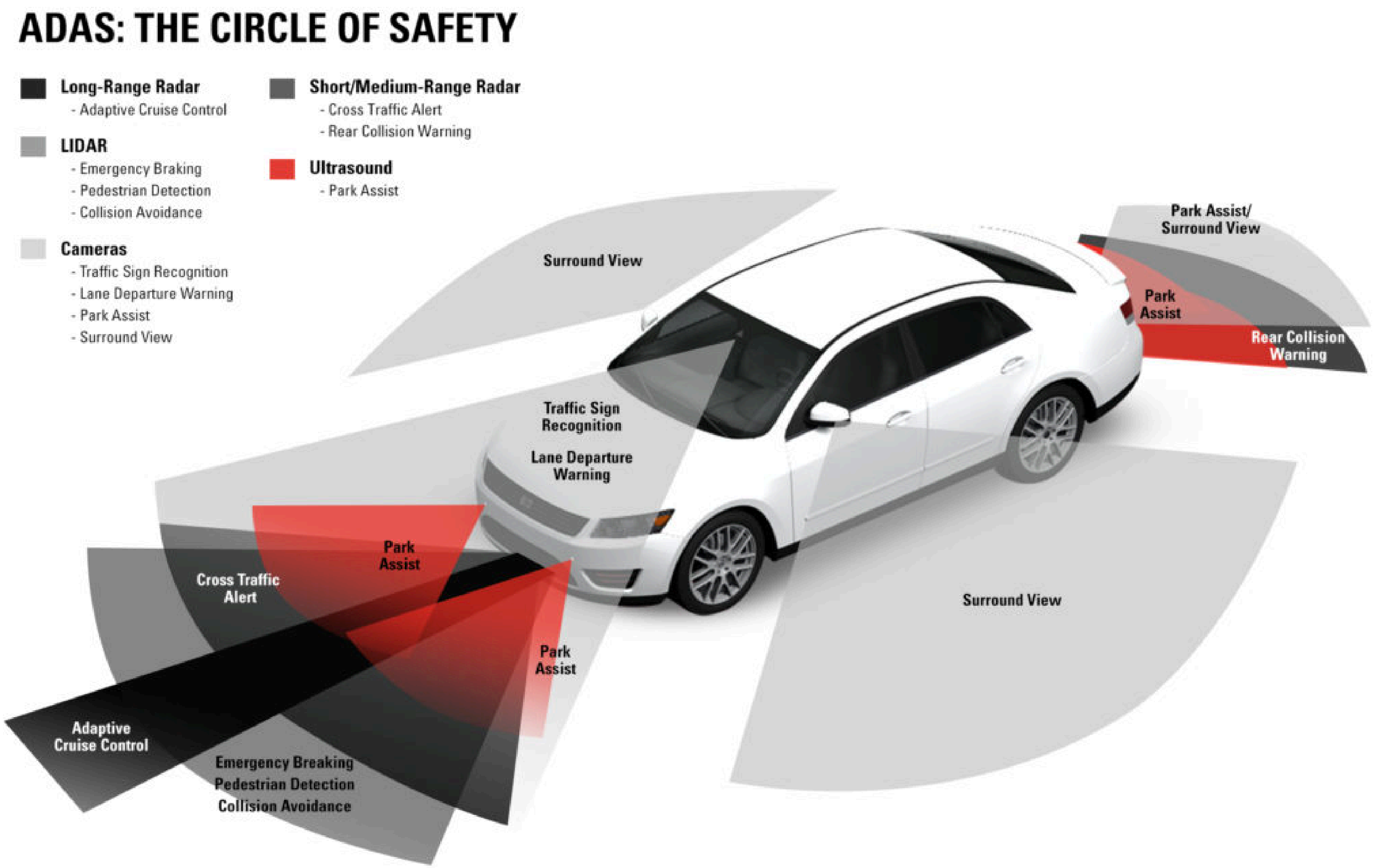
Application

Advanced Driving Assistance System(ADAS)



- Proximity Sensing
- Gesture Recognition

Application

Proximity Sensing

Application

Gesture Recognition