1 Appendix: Cheat Sheet

This is the cheat sheet that will be provided for every midterm.

1.1 Trigonometry

Pythagoras's theorem $h^2 = x^2 + y^2$ for a right angled triangle where h is the hypotenuse and x and y are the lengths of the two remaining sides.

Sine, Cosine Relation $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos(\theta_C)$ where a, b, c are the lengths of the triangle and θ_A, θ_B and θ_C are the angles of their opposing corner.

1.2 Linear Algebra

For orthogonal matrices $A^{-1} = A^T$

Orthogonality A matrix $\begin{bmatrix}\boldsymbol{v}_1, ..., \boldsymbol{v}_n\end{bmatrix}$ is said to be orthogonal if:

$$\boldsymbol{v}_i^T \boldsymbol{v}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

1.3 Special Operators

Hat

$$\hat{\boldsymbol{\omega}} = \widehat{\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

Wedge

$$\hat{\boldsymbol{\xi}} = \widehat{\begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}} = \begin{bmatrix} \hat{\boldsymbol{\omega}} & \boldsymbol{v} \\ \boldsymbol{0} & 0 \end{bmatrix}$$

1.4 Rotations

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} = e^{\hat{\boldsymbol{x}}\theta}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} = e^{\hat{\boldsymbol{y}}\theta}$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = e^{\hat{\boldsymbol{z}}\theta}$$

1.5 Rodrigues' Formula

$$R(\boldsymbol{\omega}, \theta) = e^{\hat{\boldsymbol{\omega}}\theta} = \mathbb{I}_3 + \hat{\boldsymbol{\omega}} \sin(\theta) + \hat{\boldsymbol{\omega}}^2 (1 - \cos(\theta))$$

1.6 Rigid Body Motion

$$\boldsymbol{g}_{AB} = \begin{bmatrix} \boldsymbol{R}_{AB} & \boldsymbol{p}_{AB} \\ \mathbf{0} & 1 \end{bmatrix} \qquad \boldsymbol{g}_{AB}^{-1} = \begin{bmatrix} \boldsymbol{R}_{AB}^{-1} & -\boldsymbol{R}_{AB}^{-1}\boldsymbol{p}_{AB} \\ \mathbf{0} & 1 \end{bmatrix}$$

1.7 Exponential Notation

$$\boldsymbol{R}_{AB}(\theta_1) = e^{\hat{\omega}_1\theta_1}$$

$$\boldsymbol{g}_{AB}(\theta_1) = e^{\hat{\xi}_1\theta_1}\boldsymbol{g}_{AB}(0)$$

$$\boldsymbol{g}_{ST}(\theta_1, \ldots, \theta_n) = e^{\hat{\xi}_1\theta_1} \ldots e^{\hat{\xi}_n\theta_n}\boldsymbol{g}_{ST}(0)$$

1.7.1 Special Cases

Pure Rotation

$$\boldsymbol{\xi} = \begin{bmatrix} -\boldsymbol{\omega} \times \boldsymbol{q} \\ \boldsymbol{\omega} \end{bmatrix}$$

Pure Translation

$$\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{v} \\ \mathbf{0} \end{bmatrix}$$

Pure Rotations, Screws (Rotation and Translation)

$$e^{\hat{\boldsymbol{\xi}}\theta} = \begin{bmatrix} e^{\hat{\boldsymbol{\omega}}\theta} & \left(\mathbb{I}_3 - e^{\hat{\boldsymbol{\omega}}\theta}\right)(\boldsymbol{\omega} \times \boldsymbol{v}) + \boldsymbol{\omega}\boldsymbol{\omega}^T\boldsymbol{v}\theta \\ \mathbf{0} & 1 \end{bmatrix}$$

Pure Translation

$$e^{\hat{\boldsymbol{\xi}}\theta} = \begin{bmatrix} \mathbb{I}_3 & \boldsymbol{v}\theta \\ \mathbf{0} & 1 \end{bmatrix}$$

1.8 Paden-Kahan

Subproblem 1: Rotation about a single axis

$$e^{\widehat{\xi}\theta}p = q$$

Subproblem 2: Rotation about two subsequent axes

$$e^{\widehat{\xi}_1\theta_1}e^{\widehat{\xi}_2\theta_2}p = q$$

Subproblem 3: Rotation to a distance

$$\left\| e^{\widehat{\xi}\theta}p - q \right\| = \delta$$