HW1: Rotations

EE106A/206A Fall 2018
Due: Thursday, September 6, 2018 at 11:59 PM on Gradescope

1 Multiple Rotation Matrices

    a) You are given the rotation matrices: $\boldsymbol{R}_{AB}$, $\boldsymbol{R}_{CB}$. Write and expression for $\boldsymbol{R}_{CA}$.

    b) You are given the rotation matrices: $\boldsymbol{R}_{AB}$, $\boldsymbol{R}_{CA}$. Write and expression for $\boldsymbol{R}_{BC}$.

    c) You are given the rotation matrices: $\boldsymbol{R}_{AB}$, $\boldsymbol{R}_{BC}$. Write and expression for $\boldsymbol{R}_{AA}$.

    d) You are given the rotation matrices: $\boldsymbol{R}_{AB}^{-1}$, $\boldsymbol{R}_{BC}^{T}$. Write and expression for $\boldsymbol{R}_{AC}$.

2 Euler Angles

Consider two initially overlapping frames, A and B. Frame B is then rotated about the Z axis by $\pi/4$ radians.

    a) Sketch the coordinate frames A and B after the rotation.

    b) Write the rotation matrix $\boldsymbol{R}_{AB}$ that will take a point from the B frame and represent it in the A frame.

    c) Write the rotation matrix $\boldsymbol{R}_{BA}$.

    d) What are the coordinates in frame A of a point with coordinates $\boldsymbol{p}_B = [0, 0, 1]^T$ given with respect to frame B?

    e) What are the coordinates in frame B of a point with coordinates $\boldsymbol{p}_A = [0, 0, 1]^T$ given with respect to frame A?

3 Multiple Euler Angles

A frame is rotated first about the Z axis by angle $\frac{\pi}{2}$, then about the mobile Y axis by an angle of $\frac{\pi}{2}$, then about the mobile X axis an angle of $\frac{\pi}{2}$.

    a) Draw the frame before and after the rotation. Label all axes.

    b) Write the net rotation matrix.

A frame is rotated first about the Z axis by angle $\frac{\pi}{2}$, then about the original Y axis by an angle of $\frac{\pi}{2}$, then about the original X axis an angle of $\frac{\pi}{2}$.

    c) Draw the frame before and after the rotation. Label all axes.

    d) Write the net rotation matrix.

4 Properties of Rotations

State whether each transformation matrix below is a valid rotation. Justify.

$$\text{a) } \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{b) } \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \text{c) } \begin{bmatrix} \frac{1}{2} & \sqrt{2} \\ -\sqrt{2} & 0 \end{bmatrix}$$

5 Axis Angle Notation

    a) Use the Rodrigues formula to show that a rotation of θ radians about the Y axis results in the same rotation matrix as the Euler Y equation.

    b) Use the Rodrigues formula to find the rotation matrix for a rotation of $\frac{\pi}{4}$ about the axis given by the vector $[1, 2, 3]$.