HW#2 Solutions

$\boxed{\#1}$ 1.1

$$g_{AB}(\theta_1) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

1.2

$$g_{BC} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

1.3

$$g_{AC}(\theta_1) = g_{AB}\, g_{BC}$$

$$= \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$g_{AC}(\theta_1) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ -\cos(\theta_1) & \sin(\theta_1) & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ -2\sin(\theta_1) \\ 2\cos(\theta_1)+4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

1.4

- Rotation of π radians about the X-axis.
- Translation of θ_2 in the Y direction.

1.5

$$g_{DE}(\theta_3) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} \theta_3 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

1.6 $$g_{AE}(\theta_1,\theta_2,\theta_3) = \underbrace{g_{AC}(\theta_1)\, g_{CD}(\theta_2)}_{g_{AD}(\theta_1,\theta_2)}\, g_{DE}(\theta_3)$$

$$\Rightarrow \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ -\cos(\theta_1) & \sin(\theta_1) & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ -2\sin(\theta_1) \\ 2\cos(\theta_1)+4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} 0 \\ \theta_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$\Rightarrow g_{AD}(\theta_1,\theta_2) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ \sin(\theta_1) & -\cos(\theta_1) & 0 \\ -\cos(\theta_1) & -\sin(\theta_1) & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ \cos(\theta_1)\theta_2 - 2\sin(\theta_1) \\ \sin(\theta_1)\theta_2 + 2\cos(\theta_1) + 4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$\Rightarrow g_{AE}(\theta_1,\theta_2,\theta_3) = g_{AD}(\theta_1,\theta_2) \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} \theta_3 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$g_{AE}(\theta_1,\theta_2,\theta_3) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ \sin(\theta_1) & -\cos(\theta_1) & 0 \\ -\cos(\theta_1) & -\sin(\theta_1) & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ \sin(\theta_1)\theta_3 + \cos(\theta_1)\theta_2 - 2\sin(\theta_1) \\ -\cos(\theta_1)\theta_3 + \sin(\theta_1)\theta_2 + 2\cos(\theta_1) + 4 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

$$g_{AE}\left(\frac{\pi}{2},4,3\right) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$$

HW2

1.2.1.1) $\xi_1 = \begin{bmatrix} \vec{v} \\ \vec{\omega} \end{bmatrix} = \begin{bmatrix} -\vec{\omega} \times \vec{q} \\ \vec{\omega} \end{bmatrix} = \begin{bmatrix} -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix}$

2) MATLAB EXPM

3) $g_{AB}(\theta_1) = e^{\hat{\xi}_1 \theta_1} g_{AB}(0) = \begin{bmatrix} \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ 0 & 1 & 0 \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} 5\sin(\theta_1) \\ 0 \\ 5(\cos(\theta_1)+1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$

$g_{AB}(0) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$ $\quad g_{AB}(\pi/4) = \begin{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} & \begin{bmatrix} 5\sqrt{2}/2 \\ 0 \\ 5(\sqrt{2}/2+1) \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$

$g_{AB}(\pi/2) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 1 \end{bmatrix}$

HW2

1.2.2.4) $\vec{\xi}_2 = \begin{bmatrix} \vec{v} \\ \vec{\omega} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$

5) Matlab Exam

6) $g_{AB}(\theta_2) = e^{\hat{\xi}_2 \theta_2} g_{AB}(0) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \theta_2 \\ 10 \end{bmatrix} \\ [0 \ 0 \ 0] & 1 \end{bmatrix}$

$g_{AB}(0) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \\ [0 \ 0 \ 0] & 1 \end{bmatrix}$ $g_{AB}(\pi/4) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \pi/4 \\ 10 \end{bmatrix} \\ [0 \ 0 \ 0] & 1 \end{bmatrix}$ $g_{AB}(\pi/2) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \pi/2 \\ 10 \end{bmatrix} \\ [0 \ 0 \ 0] & [1] \end{bmatrix}$

HW2

1.2.3.7)

$$e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} = \begin{bmatrix} \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ 0 & 1 & 0 \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} -5\sin(\theta_1) \\ 0 \\ 5(1-\cos(\theta_1)) \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \theta_2 \\ 0 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ 0 & 1 & 0 \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} -5\sin(\theta_1) \\ \theta_2 \\ 5(1-\cos(\theta_1)) \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

8)

$$g_{AB}(\theta_1,\theta_2) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} g_{AB}(0,0) = \begin{bmatrix} \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ 0 & 1 & 0 \\ -\sin(\theta_1) & 0 & \cos(\theta_1) \end{bmatrix} & \begin{bmatrix} 5\sin(\theta_1) \\ \theta_2 \\ 5(1+\cos(\theta_1)) \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

8, 9, 10)

$$g_{AB}(0,0) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix} \qquad g_{AB}(\pi/4, 0) = \begin{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} & \begin{bmatrix} 5\sqrt{2}/2 \\ 0 \\ 5(1+\sqrt{2}/2) \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix} \qquad g_{AB}(\pi/2, 0) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

$$g_{AB}(0, \pi/4) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \pi/4 \\ 10 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix} \qquad g_{AB}(\pi/4, \pi/4) = \begin{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} & \begin{bmatrix} 5\sqrt{2}/2 \\ \pi/4 \\ 5(1+\sqrt{2}/2) \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

$$g_{AB}(0, \pi/2) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ \pi/2 \\ 10 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix} \qquad g_{AB}(\pi/2, \pi/2) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 5 \\ \pi/2 \\ 5 \end{bmatrix} \\ [0 \quad 0 \quad 0] & 1 \end{bmatrix}$$

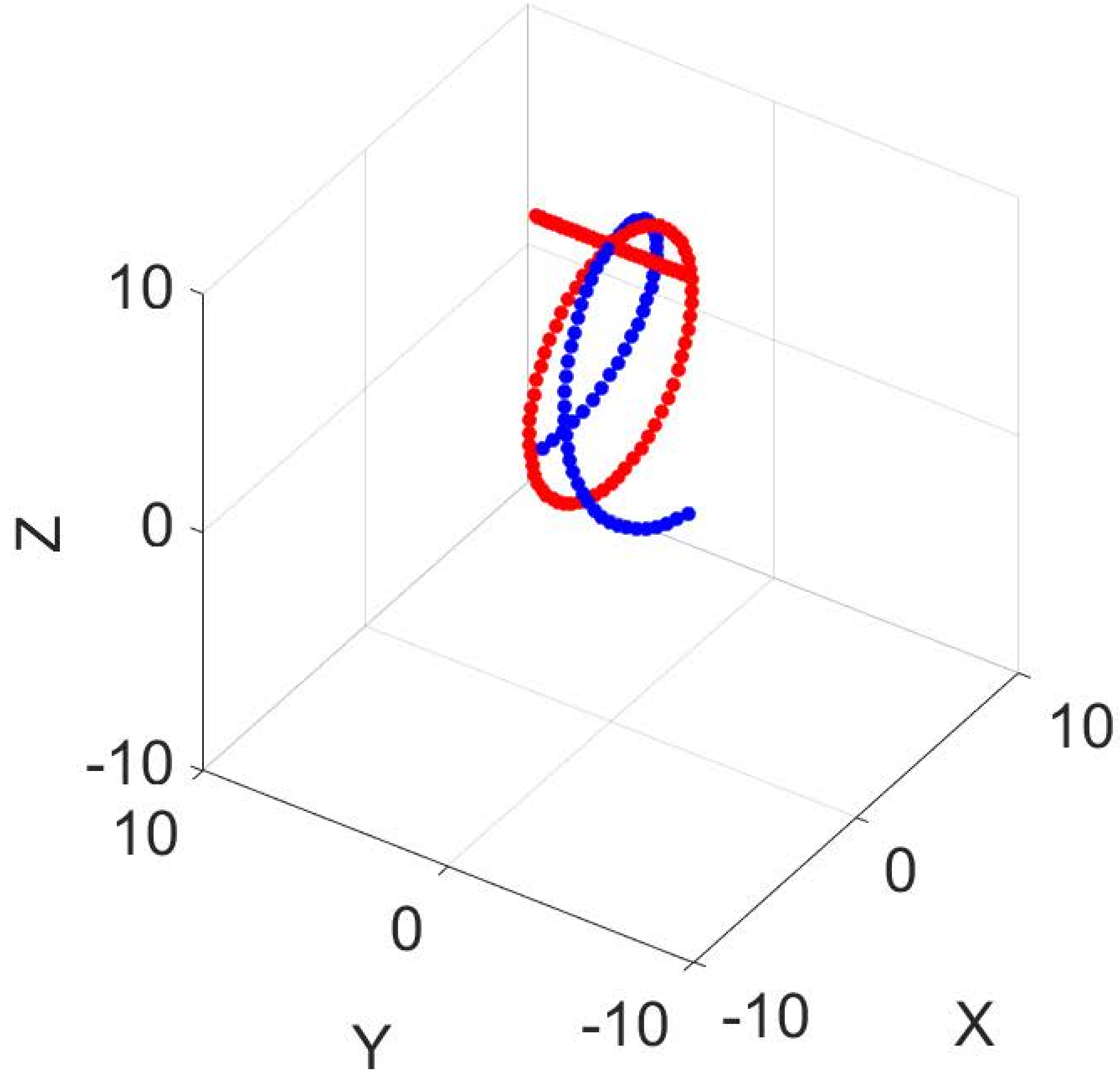
HW2

1.2.4.11) MATLAB EXPM

12) $g_{AB}(\theta_3) = e^{\hat{\xi}_3 \theta_3} g_{AB}(0)$ & MATLAB EXPM

13) $g_{AB}(0) = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \\ [0\ 0\ 0] & 1 \end{bmatrix}$ $g_{AB}(\pi/4) = \begin{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} & \begin{bmatrix} 5\sqrt{2}/2 \\ \pi/4 \\ 5(1+\sqrt{2}/2) \end{bmatrix} \\ [0\ 0\ 0] & 1 \end{bmatrix}$ $g_{AB}(\pi/2) = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 5 \\ \pi/2 \\ 5 \end{bmatrix} \\ [0\ 0\ 0] & 1 \end{bmatrix}$

14) $\xi_3 = \underset{\substack{\uparrow \\ \text{Rotation}}}{\xi_1} + \underset{\substack{\uparrow \\ \text{Translation}}}{\xi_2}$ $\Rightarrow$ Helical Motion



10
0
-10
Z
10
0
-10
Y
10
0
-10
X

Contents

FK Rotation

```
clear all
clc
syms t1 t2 t3
gBC     = [f_rotateZ(pi/2),[0;2;0];[0,0,0,1]]
gAB     = [f_rotateZ(t1),[-2;0;0];[0,0,0,1]]

% Clean matrix
gAC     = simplify(gAB*gBC);
for kounterRow = 1:1:size(gAC,1)
    for kounterCol = 1:1:size(gAC,1)
        [c,t]   = coeffs(gAC(kounterRow,kounterCol));
        d       = double(abs(c))<=1e-8;
        gAC(kounterRow,kounterCol)  = (double(d==0).*c)*transpose(t);
    end
end
gAC

gCD     = [f_rotateY(-pi/2),[t2;0;0];[0,0,0,1]];
for kounterRow = 1:1:size(gCD,1)
    for kounterCol = 1:1:size(gCD,1)
        [c,t]   = coeffs(gCD(kounterRow,kounterCol));
        d       = double(abs(c))<=1e-8;
        gCD(kounterRow,kounterCol)  = (double(d==0).*c)*transpose(t);
    end
end
gCD

gDE     = [eye(3),[0;0;t3];[0,0,0,1]];
for kounterRow = 1:1:size(gDE,1)
    for kounterCol = 1:1:size(gDE,1)
        [c,t]   = coeffs(gDE(kounterRow,kounterCol));
        d       = double(abs(c))<=1e-8;
        gDE(kounterRow,kounterCol)  = (double(d==0).*c)*transpose(t);
    end
end
gDE

gAE = simplify(gAC*gCD*gDE);
subs(gAE,{'t1','t2','t3'},{-pi/2,2,2})
```

```
gBC =

    0.0000   -1.0000         0         0
    1.0000    0.0000         0    2.0000
         0         0    1.0000         0
         0         0         0    1.0000


gAB =

[ cos(t1), -sin(t1), 0, -2]
[ sin(t1),  cos(t1), 0,  0]
[       0,        0, 1,  0]
[       0,        0, 0,  1]


gAC =

[ -sin(t1), -cos(t1), 0, - 2*sin(t1) - 2]
[  cos(t1), -sin(t1), 0,       2*cos(t1)]
[        0,        0, 1,               0]
[        0,        0, 0,               1]


gCD =

[ 0, 0, -1, t2]
[ 0, 1,  0,  0]
[ 1, 0,  0,  0]
[ 0, 0,  0,  1]


gDE =

[ 1, 0, 0,  0]
[ 0, 1, 0,  0]
[ 0, 0, 1, t3]
[ 0, 0, 0,  1]
```

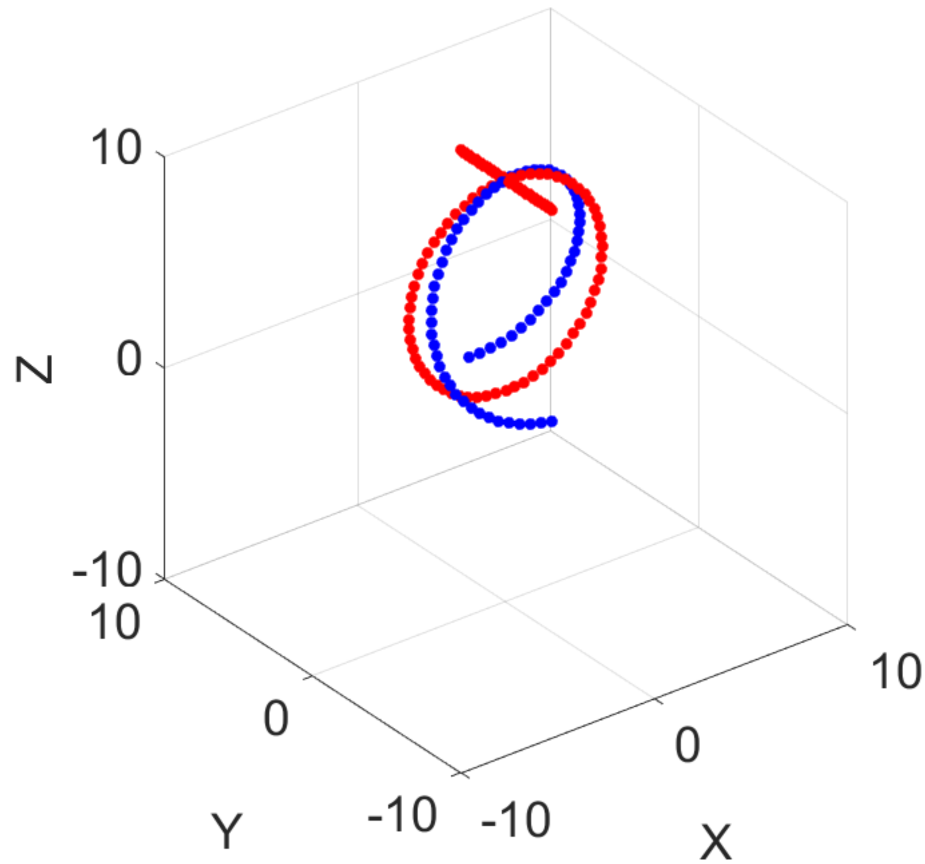
```
ans =

[ 0, 0, -1, 0]
[ 0, 1,  0, 0]
[ 1, 0,  0, 0]
[ 0, 0,  0, 1]
```

Exp Revolute

```
clear all
clc
syms t1
assume(t1,'real')

om1     = [0;1;0];
q1      = [0;0;5];
gAB0    = [eye(3),[0;0;10];[0,0,0,1]]

twist1  = [-f_hat(om1)*q1;om1]
gAB     = simplify(expm(f_wedge(twist1)*t1)*gAB0,'criterion','preferreal')

figure(1)
clf
for angle = -pi:0.1:pi
    hold on
    temp    = subs(gAB,'t1',angle);
    plot3(temp(1,4),temp(2,4),temp(3,4),'ro','markerfacecolor','r')
    axis equal
    axis([-1 1 -1 1 -1 1].*10)
    view(3)
    drawnow
end

% Exp Prismatic
% clear all
% clc
syms t2
assume(t2,'real')

v2     = [0;1;0];
gAB0    = [eye(3),[0;0;10];[0,0,0,1]]

twist2  = [v2;[0;0;0]]
gAB     = simplify(expm(f_wedge(twist2)*t2)*gAB0,'criterion','preferreal')

figure(1)
% clf
for angle = -pi:0.1:pi
    hold on
    temp    = subs(gAB,'t2',angle);
    plot3(temp(1,4),temp(2,4),temp(3,4),'ro','markerfacecolor','r')
    axis equal
    axis([-1 1 -1 1 -1 1].*10)
    view(3)
    drawnow
end

% Exp Prismatic
% clear all
% clc
% syms t2
% assume(t2,'real')
%
% v2     = [0;1;0];
% gAB0    = [eye(3),[0;0;10];[0,0,0,1]]
%
% twist2  = [v2;[0;0;0]]
% gAB     = simplify(expm(f_wedge(twist1)*t1)*expm(f_wedge(twist2)*t2)*gAB0,'criterion','preferreal')
%
% figure(1)
% % clf
% for angle = -pi:0.1:pi
%     hold on
%     temp    = subs(gAB,'t2',angle);
%     plot3(temp(1,4),temp(2,4),temp(3,4),'ro','markerfacecolor','r')
%     axis equal
%     axis([-1 1 -1 1 -1 1].*10)
%     view(3)
%     drawnow
% end


syms t3
assume(t3,'real')

twist3  = twist1+twist2;
gAB     = simplify(expm(f_wedge(twist3)*t3)*gAB0,'criterion','preferreal')

figure(1)
```

```
% clf
for angle = -pi:0.1:pi
    hold on
    temp    = subs(gAB,'t3',angle);
    plot3(temp(1,4),temp(2,4),temp(3,4),'bo','markerfacecolor','b')
    axis equal
    axis([-1 1 -1 1 -1 1].*10)
    view(3)
    drawnow
end

set(gca,'fontsize',28)
xlabel('X','fontsize',28)
ylabel('Y','fontsize',28)
zlabel('Z','fontsize',28)
grid on
```

```
gAB0 =

     1     0     0     0
     0     1     0     0
     0     0     1    10
     0     0     0     1


twist1 =

    -5
     0
     0
     0
     1
     0


gAB =

[  cos(t1), 0, sin(t1),     5*sin(t1)]
[        0, 1,       0,             0]
[ -sin(t1), 0, cos(t1), 5*cos(t1) + 5]
[        0, 0,       0,             1]


gAB0 =

     1     0     0     0
     0     1     0     0
     0     0     1    10
     0     0     0     1


twist2 =

     0
     1
     0
     0
     0
     0


gAB =

[ 1, 0, 0,  0]
[ 0, 1, 0, t2]
[ 0, 0, 1, 10]
[ 0, 0, 0,  1]


gAB =

[  cos(t3), 0, sin(t3),     5*sin(t3)]
[        0, 1,       0,            t3]
[ -sin(t3), 0, cos(t3), 5*cos(t3) + 5]
[        0, 0,       0,             1]
```


10
0
-10
Z
10
0
-10
Y
-10
0
10
X