HW3: Inverse Kinematics

EECS C106A/206A Fall 2018

Due: Thursday, September 20, 2018 at 11:59 PM on Gradescope

Feel free to use a computer to help you with this problem set. If you do write any code to help you solve a problem, attach the code at the end of your problem set.

1 Paden-Kahan Subproblems

Consider the manipulator shown in Figure 1.

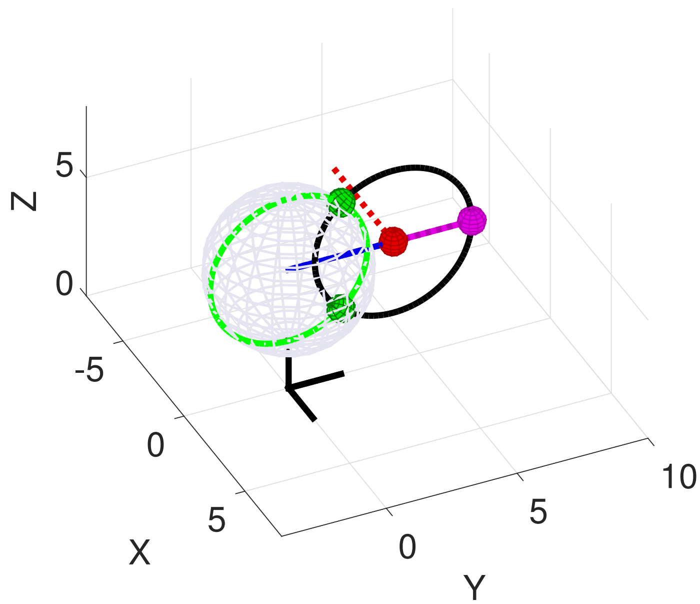

Figure 1: Pink end effector $\boldsymbol{p}$, rotating about the red axis $\boldsymbol{\omega}$ which passes through the red point $\boldsymbol{r}$. A ball of radius δ is shown about point $\boldsymbol{q}$. The two solutions are shown in green.

1. Using Paden Kahan subproblems, show the inverse kinematic solutions to this system are $\theta_3 = -2.301$ and $\theta_3 = 2.301$ when:

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 7 \\ 5 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix} \quad \boldsymbol{\omega} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \delta = 3$$

(2)

Consider the manipulator shown in Figure 2.

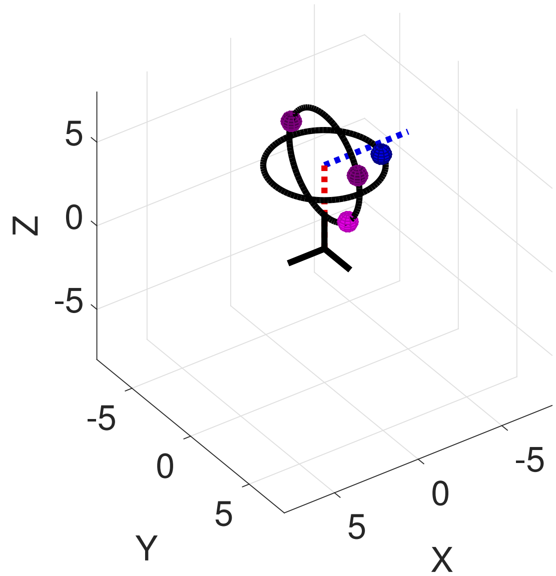

Figure 2: The pink end effector $\boldsymbol{p}$, rotates about the red axis $\boldsymbol{\omega}_1$ and blue $\boldsymbol{\omega}_2$ axes.. The desired end effector point is shown in blue. The two intersection points are shown in purple.

2. Using Paden Kahan subproblems, show the inverse kinematic solutions to this system are $\{\theta_1, \theta_2\} = \{-2.356, 2.640\}$ and $\{\theta_1, \theta_2\} = \{0.785, -1.181\}$ when:

$$\boldsymbol{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \boldsymbol{\omega}_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad \boldsymbol{p} = \begin{bmatrix} 0 \\ 1.999 \\ 2.765 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix} \tag{4}$$

2 Forward Kinematics

We wish to model the forward kinematics of the robot shown in Figure 3. This robot is annotated with coordinate frames as shown in Figure 4.

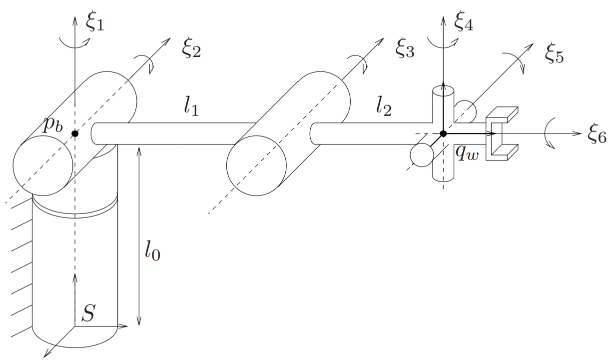

Figure 3: Sketch of the Elbow manipulator. In this problem $l_0 = 5$, $l_1 = 4$, $l_2 = 3$ and gripper G is located at a coordinates $[0, 2, 0]$ as given in the F frame.

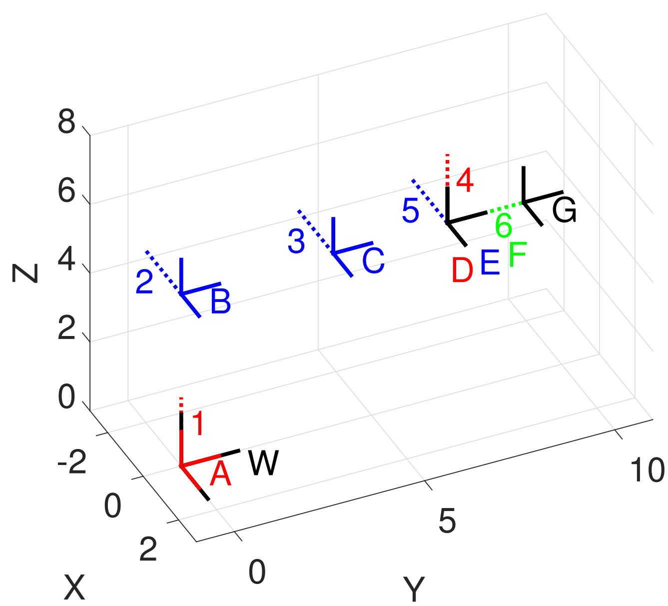

Figure 4: Frame and axis convention for the manipulator. The red frame A rotates about the red Z axis parametrised by variable θ_1 as measured in the black world (W) frame. The frames for the 4^{th}, 5^{th}, and 6^{th} joint (D, E, and F) initially coincide.

1. Show that the twists of this manipulator can be written:

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} 0 \\ -5 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_3 = \begin{bmatrix} 0 \\ -5 \\ 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_4 = \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_5 = \begin{bmatrix} 0 \\ -5 \\ 7 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_6 = \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

2. Show that configuration $g_{WB}\left(\theta_1, \theta_2\right)$ has the form:

$$g_{WB}\begin{pmatrix} 0.785 \\ -1.181 \end{pmatrix} = \begin{bmatrix} 0.7071 & -0.2688 & 0.6540 & 0 \\ 0.7071 & 0.2688 & -0.6540 & 0 \\ 0 & 0.9250 & 0.3801 & 5.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

3. Show that configuration $g_{WD}\left(\theta_1, \theta_2, \theta_3, \theta_4\right)$ has the form:

$$g_{WD}\begin{pmatrix} 0.785 \\ -1.181 \\ 2.301 \\ -1.160 \end{pmatrix} = \begin{bmatrix} 0.5652 & 0.5249 & -0.6363 & -2.0000 \\ 0.0000 & 0.7714 & 0.6363 & 2.0000 \\ 0.8249 & -0.3597 & 0.4360 & 6.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

4. Show that configuration $g_{WG}\left(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\right)$ has the form:

$$g_{WG}\begin{pmatrix} 0.785 \\ -1.181 \\ 2.301 \\ -1.160 \\ -0.690 \\ 0.970 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 Inverse Kinematics

We now wish to compute the full set of inverse Kinematic solutions for the Manipulator shown in Figures 3, 4.

1. Given the desired end effector configuration g_{WG} show that the required configuration of g_{WF} is given by:

$$g_{WG} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad g_{WF} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Show that θ_3 can be formulated as a Paden Kahan Subproblem 3:

$$\left\| e^{\hat{\xi}\theta} p - q \right\| = \delta \tag{1}$$

where:

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 7 \\ 5 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} 0 \\ -5 \\ 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \delta = 3$$

3. Show that the two solutions of this PK3 problem are: $\theta_3 = \{-2.301, 2.301\}$.

4. Show that when $\theta_3 = 2.301$ the angles θ_1, θ_2 be formulated the Paden Kahan subproblem 2:

$$e^{-\hat{\xi}_i\theta_i}\boldsymbol{q} = \boldsymbol{c} = e^{\hat{\xi}_j\theta_j}\boldsymbol{p} \tag{2}$$

where:

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 2 \\ 2.764 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad \boldsymbol{\xi}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{\xi}_j = \begin{bmatrix} 0 \\ -5 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

with the two intermediary points

$$\boldsymbol{c}_2 = \begin{bmatrix} 0 \\ -2.828 \\ 6 \end{bmatrix} \quad \boldsymbol{c}_2 = \begin{bmatrix} 0 \\ 2.828 \\ 6 \end{bmatrix}$$

5. Show that the two solutions of this PK2 problem are: $\{\theta_1, \theta_2\} = \{-2.356, 2.640\}$ and $\{\theta_1, \theta_2\} = \{0.785, -1.181\}$.

6. Show that when $\{\theta_1, \theta_2, \theta_3\} = \{0.785, -1.181, 2.301\}$ the angles θ_4, θ_5 be formulated the Paden Kahan subproblem 2, with:

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 9 \\ 5 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} 1.414 \\ 7.617 \\ 6.273 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 7 \\ 5 \end{bmatrix} \quad \boldsymbol{\xi}_i = \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \boldsymbol{\xi}_j = \begin{bmatrix} 0 \\ -5 \\ 7 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

with the two intermediary points

$$\boldsymbol{c}_1 = \begin{bmatrix} 0 \\ 5.457 \\ 6.273 \end{bmatrix} \quad \boldsymbol{c}_2 = \begin{bmatrix} 0 \\ 8.543 \\ 6.273 \end{bmatrix}$$

7. Show that the two solutions of this PK2 problem are: $\{\theta_5, \theta_6\} = \{1.982, -2.452\}$ and $\{\theta_5, \theta_6\} = \{-1.160, -0.690\}$.

8. Show that when $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\} = \{0.785, -1.181, 2.301, -1.160, -0.690\}$ the angle θ_6 be formulated the Paden Kahan subproblem 1:

$$e^{\hat{\xi}\theta}\boldsymbol{p} = \boldsymbol{q} \tag{3}$$

with:

$$\boldsymbol{p} = \begin{bmatrix} 0 \\ 9 \\ 6 \end{bmatrix} \quad \boldsymbol{q} = \begin{bmatrix} 0.825 \\ 9 \\ 5.565 \end{bmatrix} \quad \boldsymbol{r} = \begin{bmatrix} 0 \\ 7 \\ 5 \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

and show that the solution of this PK1 problem is: $\theta_6 = 0.970$.

9. How many unique solutions exist for this desired configuration?

10. Describe how your solve this problem if joint 3 was a prismatic joint along the y direction instead of a revolute joint. Which thetas would change? Which thetas would stay the same?