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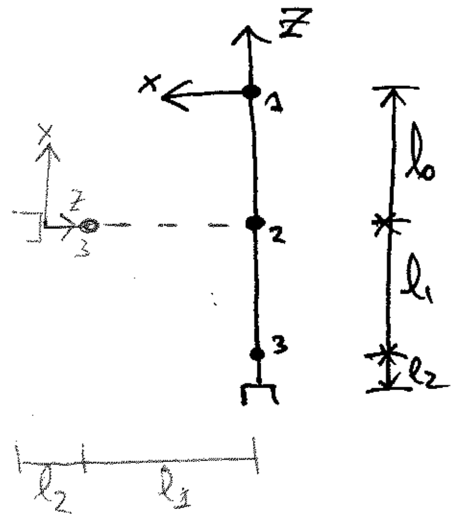
HW7

1) Initial Config

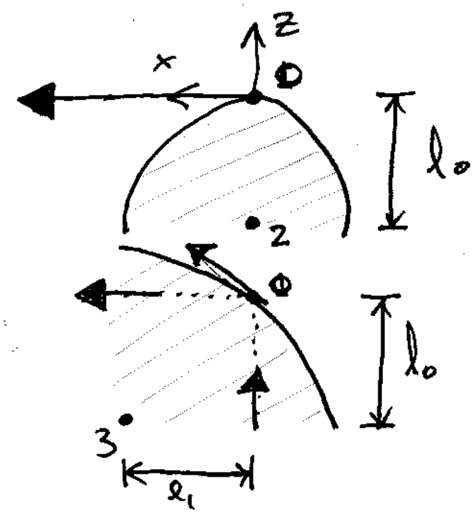
a) Config 1



b) $J_1^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ - circle centered at $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ radius 0 rotating around y axis

$J_2^s = \begin{bmatrix} l_0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

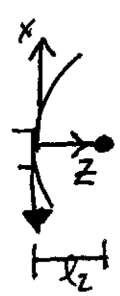
$J_3^s = \begin{bmatrix} l_0 \\ 0 \\ l_1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$



circle @ $\begin{bmatrix} 0 \\ 0 \\ -l_0 \end{bmatrix}$ radius $[l_0]$
rotating about y-axis

circle @ $\begin{bmatrix} l_1 \\ 0 \\ -l_0 \end{bmatrix}$ radius $\sqrt{l_0^2+l_1^2}$
rotating about y-axis

$J_3^b = \begin{bmatrix} -l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$



circle @ $\begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}$ radius l_2
rotating about y-axis

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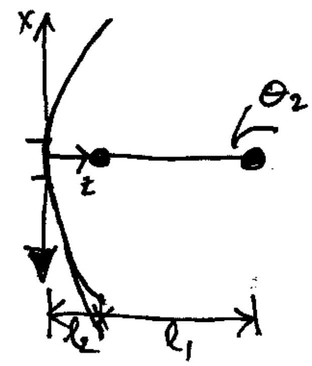
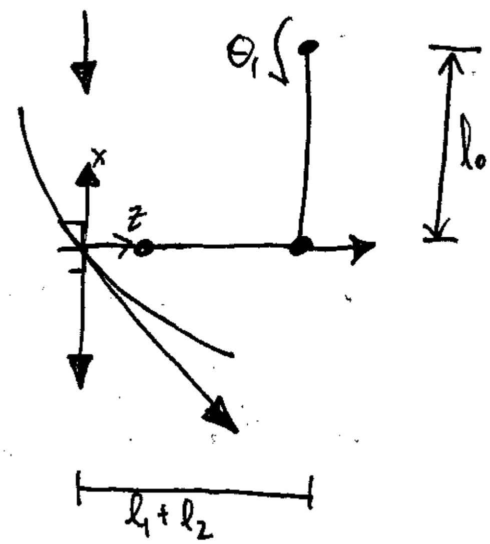
1b$_{II}$) $J^b_2 = \begin{bmatrix} -l_1-l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$



Circle @ $\begin{bmatrix} 0 \\ 0 \\ l_1+l_2 \end{bmatrix}$ radius l_1+l_2 rotating about y-axis

$J^b_1 = \begin{bmatrix} -l_1-l_2 \\ l_0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$



Circle @ $\begin{bmatrix} l_0 \\ 0 \\ l_1+l_2 \end{bmatrix}$ radius $\sqrt{(l_0)^2+(l_1+l_2)^2}$ rotating about y-axis

c) Nonsingular: - independent control of $(v_x^s, v_z^s, \omega_y^s)$ or $(v_x^b, v_z^b, \omega_y^b)$

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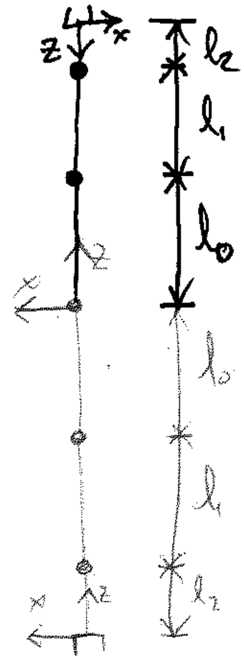
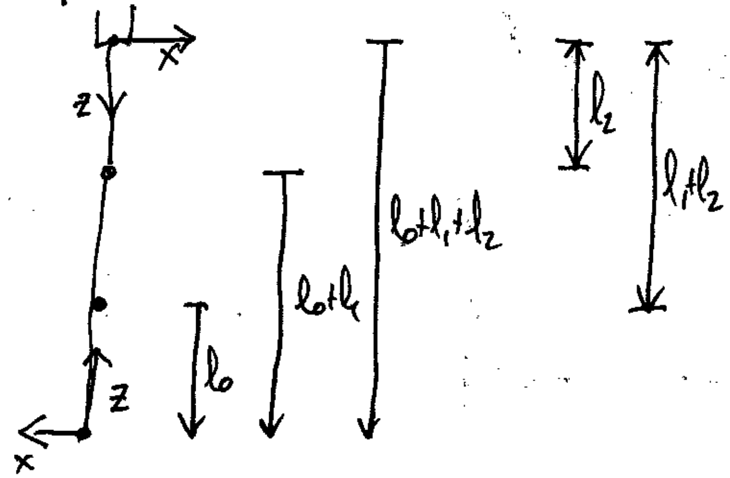
1d) Initial Config



Config 2

e)



$J_1^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle@ origin, radius 0 along y-axis

$J_2^s = \begin{bmatrix} -l_0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle@ $\begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$, radius l_0 along y-axis

$J_3^s = \begin{bmatrix} -l_0-l_1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle@ $\begin{bmatrix} 0 \\ 0 \\ l_0+l_1 \end{bmatrix}$, radius l_0+l_1 along y-axis

$J_3^b = \begin{bmatrix} -l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle @ $\begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}$, radius l_2 along y-axis

$J_2^b = \begin{bmatrix} -l_1-l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle @ $\begin{bmatrix} 0 \\ 0 \\ l_1+l_2 \end{bmatrix}$ radius l_1+l_2 along y-axis

$J_1^b = \begin{bmatrix} -l_0-l_1-l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ circle @ $\begin{bmatrix} 0 \\ 0 \\ l_0+l_1+l_2 \end{bmatrix}$ radius $l_0+l_1+l_2$ along y-axis

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1f) This is a singular configuration: rank = 2.

Consider J^s: $\begin{bmatrix} 0 & -l_0 & -l_0-l_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} | & | & | \\ c_1 & c_2 & c_3 \\ | & | & | \end{bmatrix}$

$$c_3 - c_1 = \begin{bmatrix} -l_0-l_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad c_2 - c_1 = \begin{bmatrix} -l_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{c_3 - c_1}{-(l_0+l_1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \frac{c_2 - c_1}{-l_0} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow c_3 - c_1 = \frac{(l_0+l_1)}{l_0}(c_2 - c_1)$$

$$c_3 = \left(\frac{l_0+l_1}{l_0}\right)(c_2 - c_1) + c_1$$

$\Rightarrow$ rank 2.

instantaneous forces/velocities in linear x, rotational y only.

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1g)

$$\xi_1 = \begin{bmatrix} -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -l_0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} l_0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\xi_3 = \begin{bmatrix} -\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -l_0-l_1 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} l_0+l_1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_1^s = \xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_2^s = Ad_g \xi_2 = Ad_{e^{\hat{\xi}_1\theta_1}} \xi_2$$

$$= \begin{bmatrix} c_1 & 0 & s_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -s_1 & 0 & c_1 & 0 & 0 & 0 \\ & & & c_1 & 0 & s_1 \\ & 0 & & 0 & 1 & 0 \\ & & & -s_1 & 0 & c_1 \end{bmatrix}\begin{bmatrix} l_0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_0 c_1 \\ 0 \\ -l_0 s_1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_3^s = Ad_{e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}} \xi_3 = \begin{bmatrix} c_{1+2} & 0 & s_{1+2} & 0 & -l_0(c_{1+2}-c_1) & 0 \\ 0 & 1 & 0 & -l_0(c_2-1) & 0 & -l_0 s_2 \\ -s_{1+2} & 0 & c_{1+2} & 0 & l_0(s_{1+2}-s_1) & 0 \\ & & & c_{1+2} & 0 & s_{1+2} \\ & 0 & & 0 & 1 & 0 \\ & & & -s_{1+2} & 0 & c_{1+2} \end{bmatrix}\begin{bmatrix} l_0+l_1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (l_0+l_1)c_{1+2} - l_0(c_{1+2}-c_1) \\ 0 \\ -(l_0+l_1)s_{1+2} + l_0(s_{1+2}-s_1) \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_0 c_1 + l_1 c_{1+2} \\ 0 \\ l_0 s_1 + l_1 s_{1+2} \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

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1 g_{II})

$$J_3^b = Ad_{e^{\hat{\xi}_3\theta_3} g_{wT}(0)} \xi_3 = \begin{bmatrix} c_3 & 0 & -s_3 & 0 & -l_0 c_3 - l_1 c_3 - l_2 & 0 \\ 0 & 1 & 0 & l_0+l_1+l_2 s_3 & 0 & -l_2 s_3 \\ s_3 & 0 & c_3 & 0 & -(l_0+l_1) s_3 & 0 \\ & & & c_3 & 0 & -s_3 \\ & 0 & & 0 & 1 & 0 \\ & & & s_3 & 0 & c_3 \end{bmatrix} \begin{bmatrix} l_0+l_1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (l_0+l_1)c_3 - (l_0+l_1)c_3 - l_2 \\ 0 \\ (l_0+l_1)s_3 - (l_0+l_1)s_3 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_2^b = Ad_{e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3}} \xi_2 = \begin{bmatrix} c_{2+3} & 0 & -s_{2+3} & 0 & l_0 c_{2+3} - l_1 c_3 - l_2 & 0 \\ 0 & 1 & 0 & l_0 + l_1 c_2 + l_2 c_{2+3} & 0 & -l_1 s_2 \; l_2 s_{2+3} \\ s_{2+3} & 0 & c_{2+3} & 0 & -l_0 s_{2+3} - l_1 s_3 & 0 \\ & & & c_{2+3} & 0 & -s_{2+3} \\ & 0 & & 0 & 1 & 0 \\ & & & s_{2+3} & 0 & c_{2+3} \end{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_0 c_{2+3} - l_0 c_{2+3} - l_1 c_3 - l_2 \\ 0 \\ l_0 s_{2+3} - l_0 s_{2+3} - l_1 s_3 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_1 c_3 - l_2 \\ 0 \\ -l_1 s_3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

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1g III) $J_1^b = Ad_{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3}} \xi_3 = \begin{bmatrix} c_{2+3} & 0 & -s_{2+3} & 0 & -l_0 c_{2+3} - l_1 c_3 - l_2 & 0 \\ 0 & 1 & 0 & l_0 + l_1 c_2 + l_2 c_{2+3} & 0 & -l_1 s_2 - l_2 s_{2+3} \\ s_{2+3} & 0 & c_{2+3} & 0 & -l_0 s_{2+3} - l_1 s_3 & 0 \\ & & & c_{2+3} & 0 & -s_{2+3} \\ & & & 0 & 1 & 0 \\ & & & s_{2+3} & 0 & c_{2+3} \end{bmatrix} \xi_3$

$$= \begin{bmatrix} (l_0+l_1)c_{2+3} - l_0 c_{2+3} - l_1 c_3 - l_2 \\ (l_0+l_1) s_{2+3} - l_0 s_{2+3} - l_1 s_3 \\ 0 \\ 1 \\ 0 \end{bmatrix} =$$

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1 g III)

$$J_1^b = Ad_{e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} g_{ST}(0)}^{-1} \xi_1$$

$$= \begin{bmatrix} c_{1+2+3} & 0 & -s_{1+2+3} & 0 & -l_0 c_{2+3} - l_1 c_3 - l_2 & 0 \\ 0 & 1 & 0 & l_0 c_1 + l_1 c_{1+2} + l_2 c_{1+2+3} & 0 & -l_0 s_1 - l_1 s_{1+2} - l_2 s_{1+2+3} \\ s_{1+2+3} & 0 & c_{1+2+3} & 0 & -l_0 s_{2+3} - l_1 s_3 & 0 \\ & & & c_{1+2+3} & 0 & -s_{1+2+3} \\ & 0 & & 0 & 1 & 0 \\ & & & s_{1+2+3} & 0 & c_{1+2+3} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_0 c_{2+3} - l_1 c_3 - l_2 \\ 0 \\ -l_0 s_{2+3} - l_1 s_3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

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1h) ξ_i' and $\xi_i^{\dagger}$ show the new representations of the twist ξ_i after some motion produced by Θ. ξ_i' gives the representation of ξ_i as viewed in the world frame, while $\xi_i^{\dagger}$ gives the representation of ξ_i as viewed in the tool frame.

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1Ki) Singular when rank drops:

Case 1: $\theta_2 = 0$: $J_s = \begin{bmatrix} 0 & l_0 c_1 & l_0 c_1 + l_1 c_1 \\ 0 & 0 & 0 \\ 0 & -l_0 s_1 & -l_0 s_1 - l_1 s_1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad c_3 = c_1 + \frac{l_0 + l_1}{l_0}(c_2 - c_1)$

Case 2: $\theta_2 = \pi$: $J_s = \begin{bmatrix} 0 & l_0 c_1 & (l_0 - l_1) c_1 \\ 0 & 0 & 0 \\ 0 & -l_0 s_1 & -(l_0 - l_1) s_1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad c_3 = c_1 + \frac{(l_0 + l_1)}{l_0}(c_2 - c_1)$

Case 3: $\theta_2 = \pi$ & $l_0 = l_1$: $J_s = \begin{bmatrix} 0 & l_0 c_1 & 0 \\ 0 & 0 & 0 \\ 0 & -l_0 s_1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad c_3 = c_1$

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1 j) Config 1: $\Theta = \begin{bmatrix} 0 \\ -\pi/2 \\ 0 \end{bmatrix} \Rightarrow J_b = \begin{bmatrix} -l_1-l_2 & -l_1-l_2 & -l_2 \\ 0 & 0 & 0 \\ l_0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} -l_1-l_2 & 0 & l_0 & 0 & 1 & 0 \\ -l_1-l_2 & 0 & 0 & 0 & 1 & 0 \\ -l_2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -mg \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (l_1+l_2)mg \\ (l_1+l_2)mg \\ l_2\, mg \end{bmatrix}$$ ~~Nm~~

Config 2: $\Theta = \begin{bmatrix} -\pi \\ 0 \\ 0 \end{bmatrix} \Rightarrow J_b = \begin{bmatrix} -l_0-l_1-l_2 & -l_1-l_2 & -l_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} -l_0-l_1-l_2 & 0 & 0 & 0 & 1 & 0 \\ -l_1-l_2 & 0 & 0 & 0 & 1 & 0 \\ -l_2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ mg \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ ~~Nm~~

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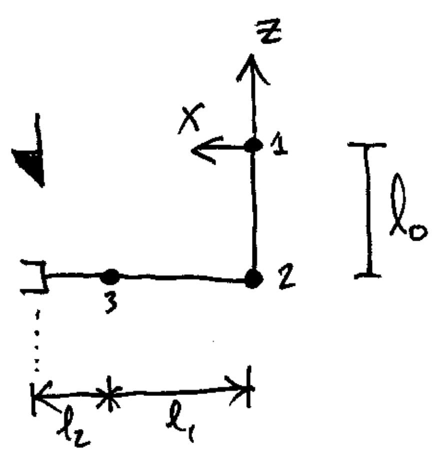
1k)
$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ l_0 & 0 & 0 & 0 & 1 & 0 \\ l_0 & 0 & l_1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \\ (l_1+l_2)mg \\ 0 \end{bmatrix} = \begin{bmatrix} (l_1+l_2)mg \\ (l_1+l_2)mg \\ l_2\ mg \end{bmatrix}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ -l_0 & 0 & 0 & 0 & 1 & 0 \\ -l_0-l_1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Config 1:

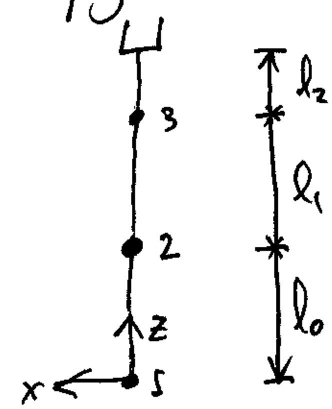


Circle @ ~~origin~~ origin extending to manipulator contact:

$\tau_y = (l_1+l_2)mg$

$F_z = -mg$

Config 2:



Circle @ origin extending to manipulator contact

$F_z = -mg$.