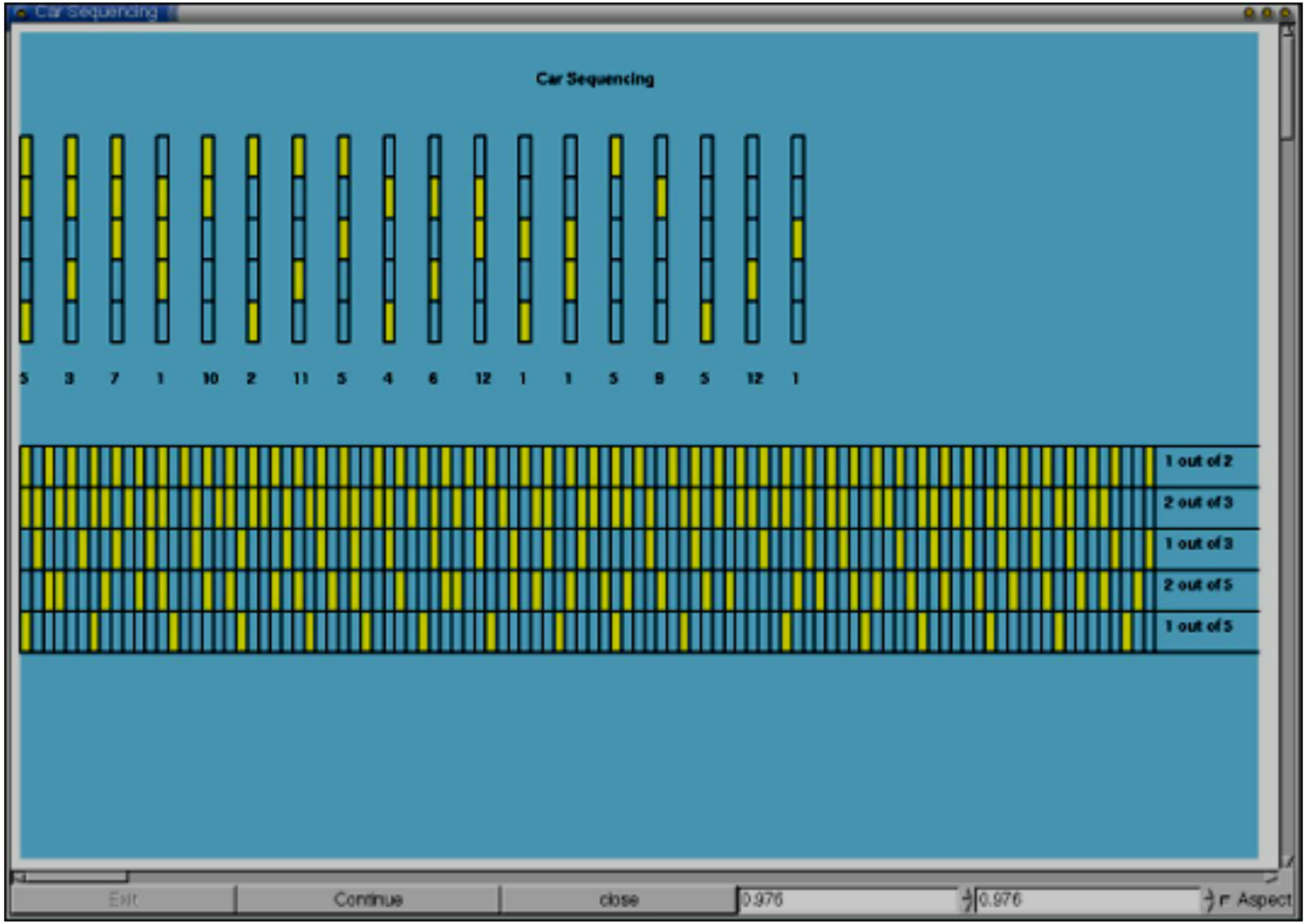
Discrete Optimization

Constraint Programming: Part VII

Car Sequencing

- Cars on an assembly line
- Cars require specific options
  - leather seats, moonroof
- Capacity constraints on the production units
  - at most 2 out of 5 successive cars can require a moonroof
- Sequence all the cars such that the capacity constraints are satisfied

Car Sequencing



上面的是每种type 的生产所需的option
和所需要生产的数量，下面是一个满足capacity
constraint要求的调度顺序

Car Sequencing

Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
Class 2				yes		1
Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
Capacity	1/2	2/3	1/3	2/5	1/5	

Car Sequencing

```
range Slots = ...;
range Configs = ...;
range Options = ...;
int demand[Configs] = ...;
int nbCars = sum(c in Configs) demand[c];
int lb[Options] = ...;
int ub[Options] = ...;
int requires[Options,Config] = ...;
var{int} line[Slots] in Configs;
var{int} setup[Options,Slots] in 0..1;

solve {
   forall(c in Configs)
      sum(s in Slots) (line[s] = c) = demand[c];

   forall(s in Slots,o in Options)
      setup[o,s] = requires[o,line[s]];

   forall(o in Options, s in 1..nbCars-ub[o]+1)
      sum(j in s..s+ub[o]-1) setup[o,s] <= lb[o];
}
```

上面的

line[s] denotes the type of car sequenced on slot s

setup[o,s]=1
if slot[s] has a car requiring option o

demand constraints

defines the setup variables

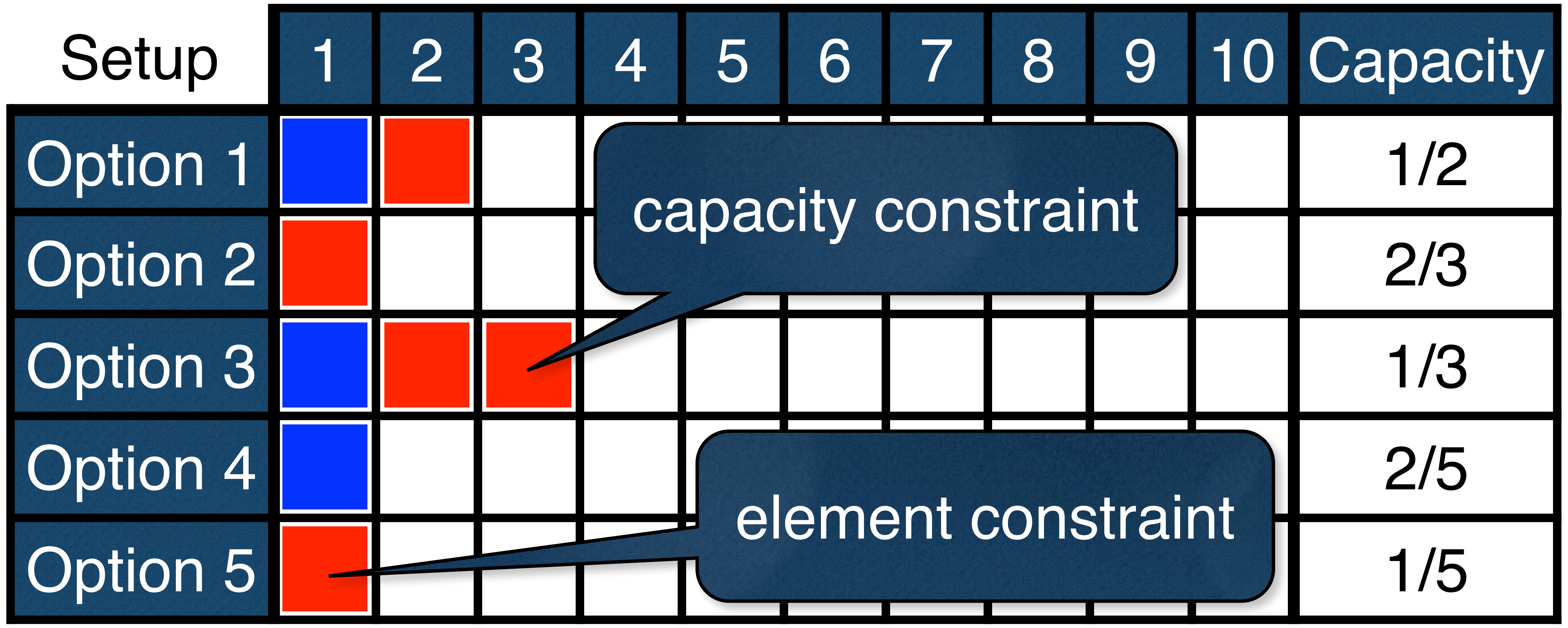
capacity constraints

Car Sequencing

Slots	1	2	3	4	5	6	7	8	9	10	Demand
Class 1											1
Class 2											1
Class 3											2
Class 4											2
Class 5											2
Class 6											2

Car Sequencing

Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
Class 2				yes		1
Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
Capacity	1/2	2/3	1/3	2/5	1/5	



Car Sequencing

```
range Slots = ...;
range Configs = ...;
range Options = ...;
int demand[Configs] = ...;
int lb[Options] = ...;
int ub[Options] = ...;
int requires[Options,Config] = ...;
var{int} line[Slots] in Configs;
var{int} setup[Options,Slots] in 0..1;

solve {
   forall(c in Configs)
      sum(s in Slots) (line[s] = c) = demand[c];

   forall(s in Slots,o in Options)
      setup[o,s] = requires[o,line[s]];

   forall(o in Options, s in 1..nbCars-ub[o]+1)
      sum(j in s..s+ub[o]-1) setup[o,s] <= lb[o];
}
```

Car Sequencing

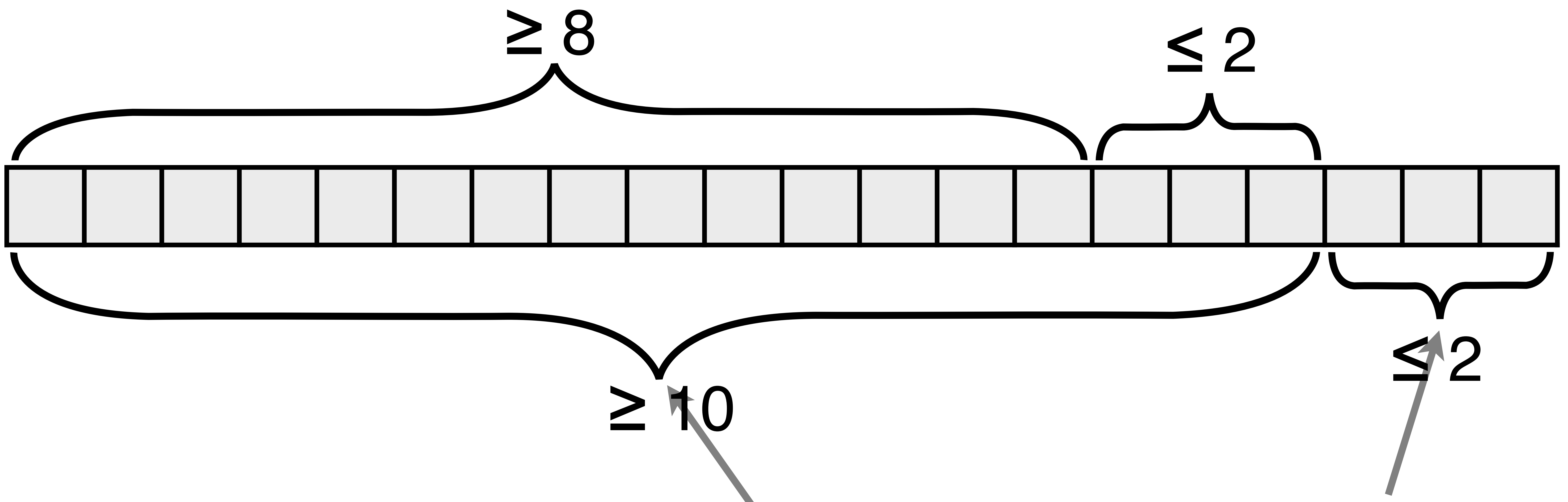
Setup	1	2	3	4	5	6	7	8	9	10	Capacity
Option 1											1/2
Option 2											2/3
Option 3											1/3
Option 4											2/5
Option 5											1/5

Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
Class 2				yes		1
Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
Capacity	1/2	2/3	1/3	2/5	1/5	

Slots	1	2	3	4	5	6	7	8	9	10	Demand
Class 1											1
Class 2											1
Class 3											2
Class 4											2
Class 5											2
Class 6											2

Car Sequencing: Redundant Constraints

- Consider an option *o* with
  - Capacity: 2 out of 3
  - Demand: 12 cars



- How many cars with option *o* can be in the last 3 slots?
- How many cars with option *o* must be in these slots?

Car Sequencing: Redundant Constraints

```
range Slots = ...;
range Configs = ...;
range Options = ...;
int demand[Configs] = ...;
int lb[Options] = ...;
int ub[Options] = ...;
int requires[Options,Config] = ...;
var{int} line[Cars] in Configs;
var{int} setup[Options,Slots] in 0..1;

solve {
   forall(c in Configs)
      sum(s in Slots) (line[s] = c) = demand[c];
   forall(s in Slots,o in Options)
      setup[o,s] = requires[o,line[s]];
   forall(o in Options, s in 1..nbCars-ub[o]+1)
      sum(j in s..s+ub[o]-1) setup[o,s] <= lb[o];

   forall(o in Options, i in 1..demand[o])
      sum(s in 1..nbCars-i*ub[o]) setup[o,s] >= demand[o] - i*lb[o];

}
```

Car Sequencing

Setup	1	2	3	4	5	6	7	8	9	10	Capacity
Option 1											1/2
Option 2											2/3
Option 3											1/3
Option 4											2/5
Option 5											1/5

Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
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Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
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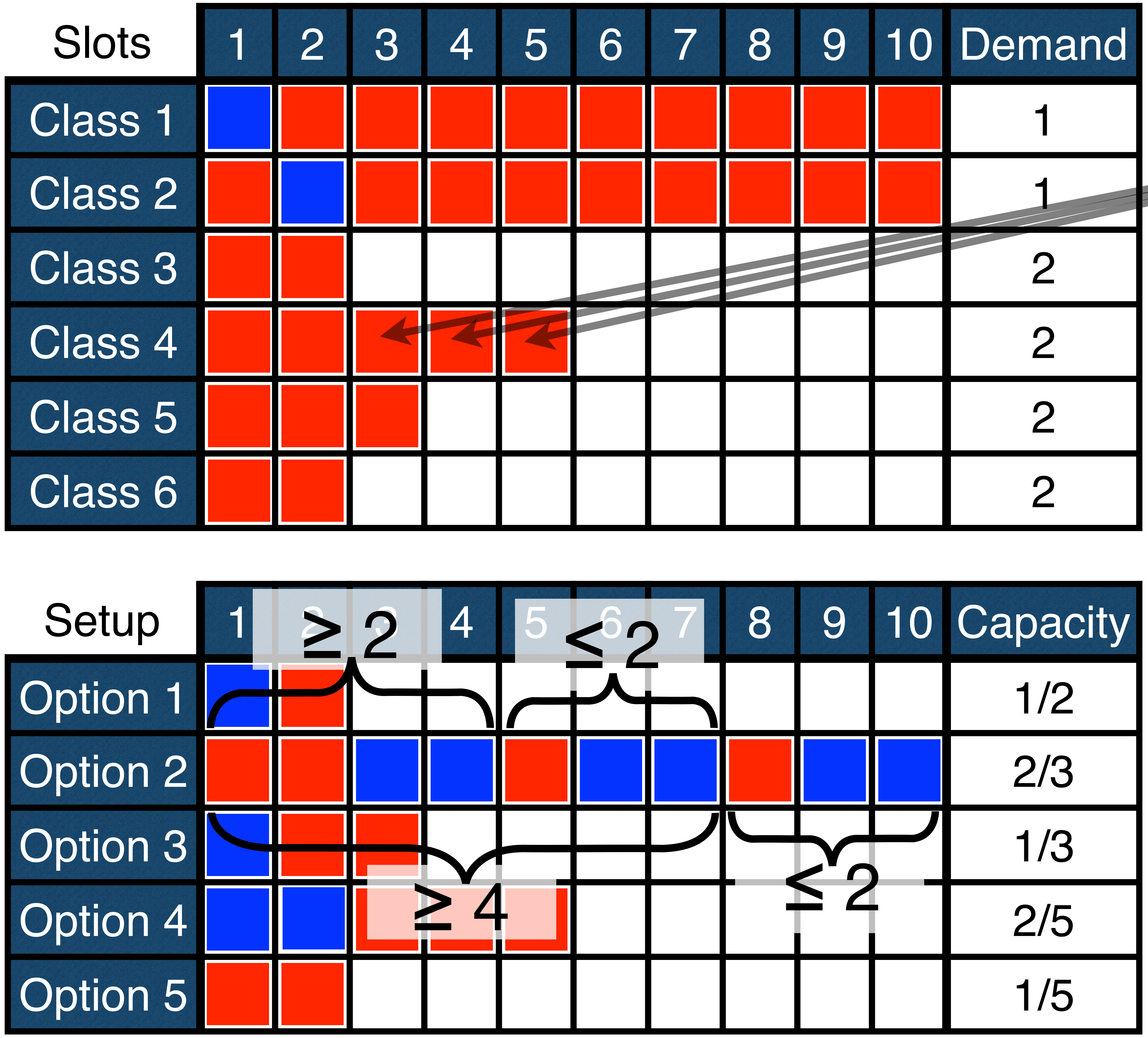
Slots	1	2	3	4	5	6	7	8	9	10	Demand
Class 1											1
Class 2											1
Class 3											2
Class 4											2
Class 5											2
Class 6											2

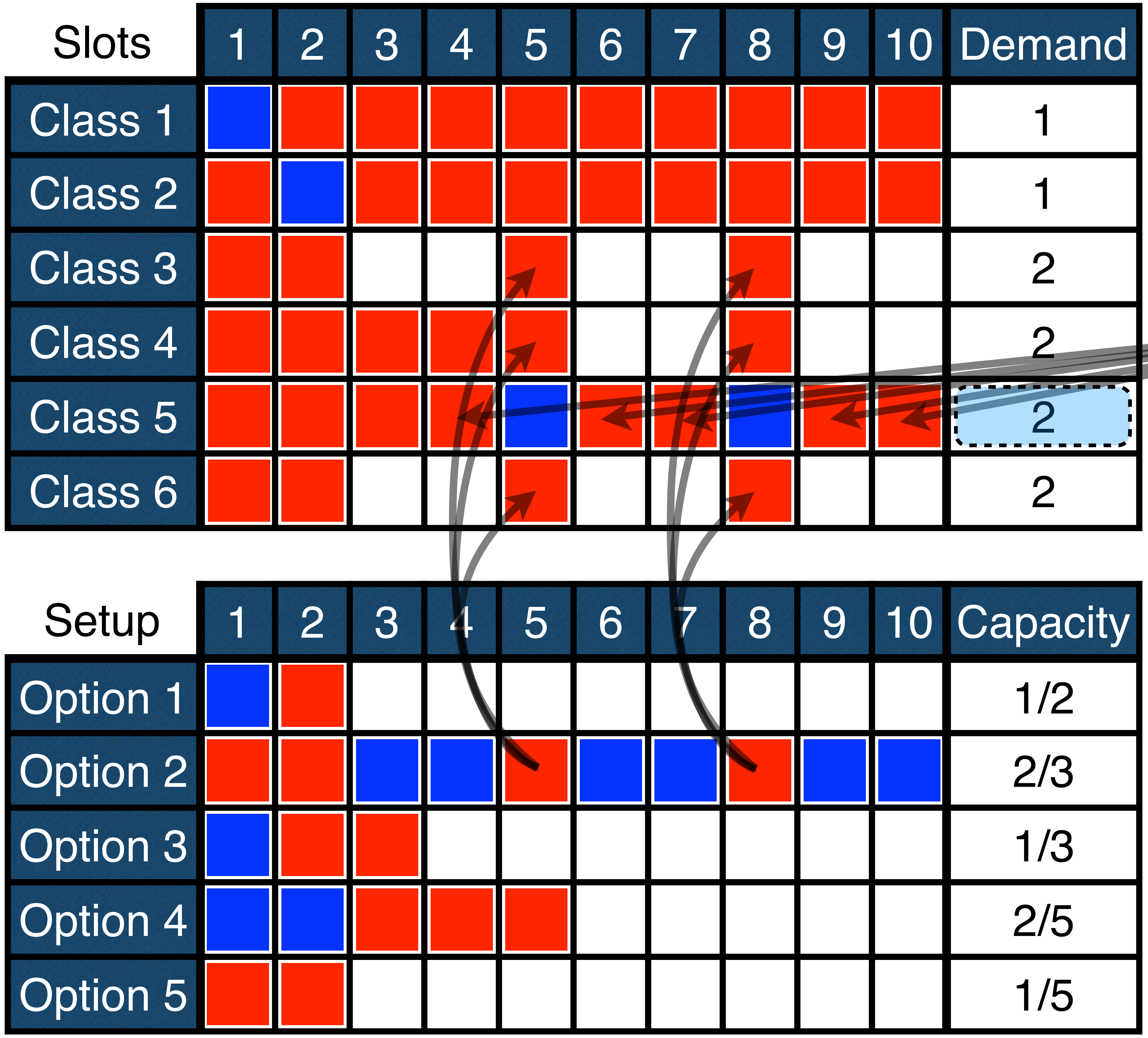
Car Sequencing



Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
Class 2				yes		1
Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
Capacity	1/2	2/3	1/3	2/5	1/5	

option 2： 6 cars

Car Sequencing



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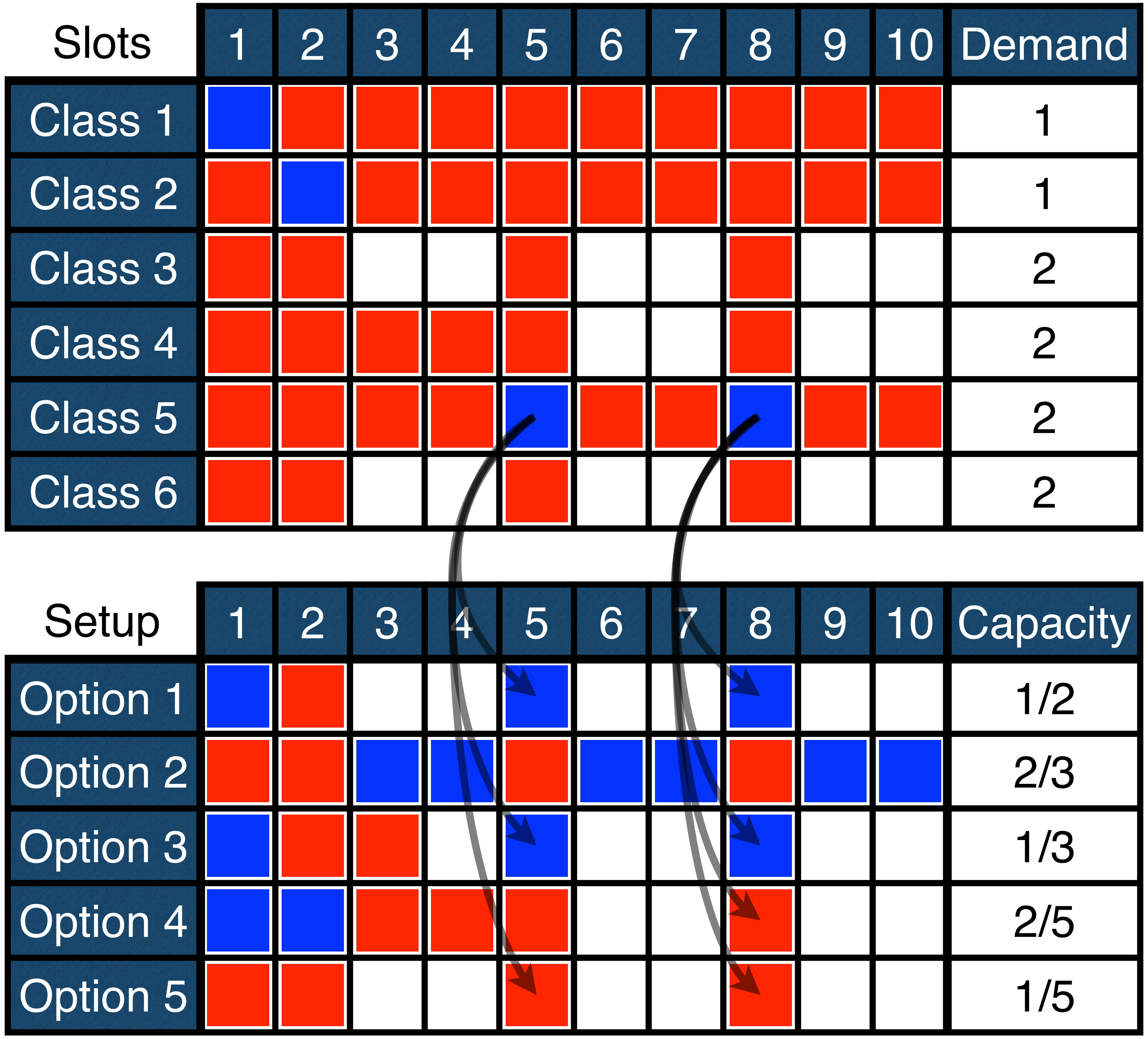
Car Sequencing



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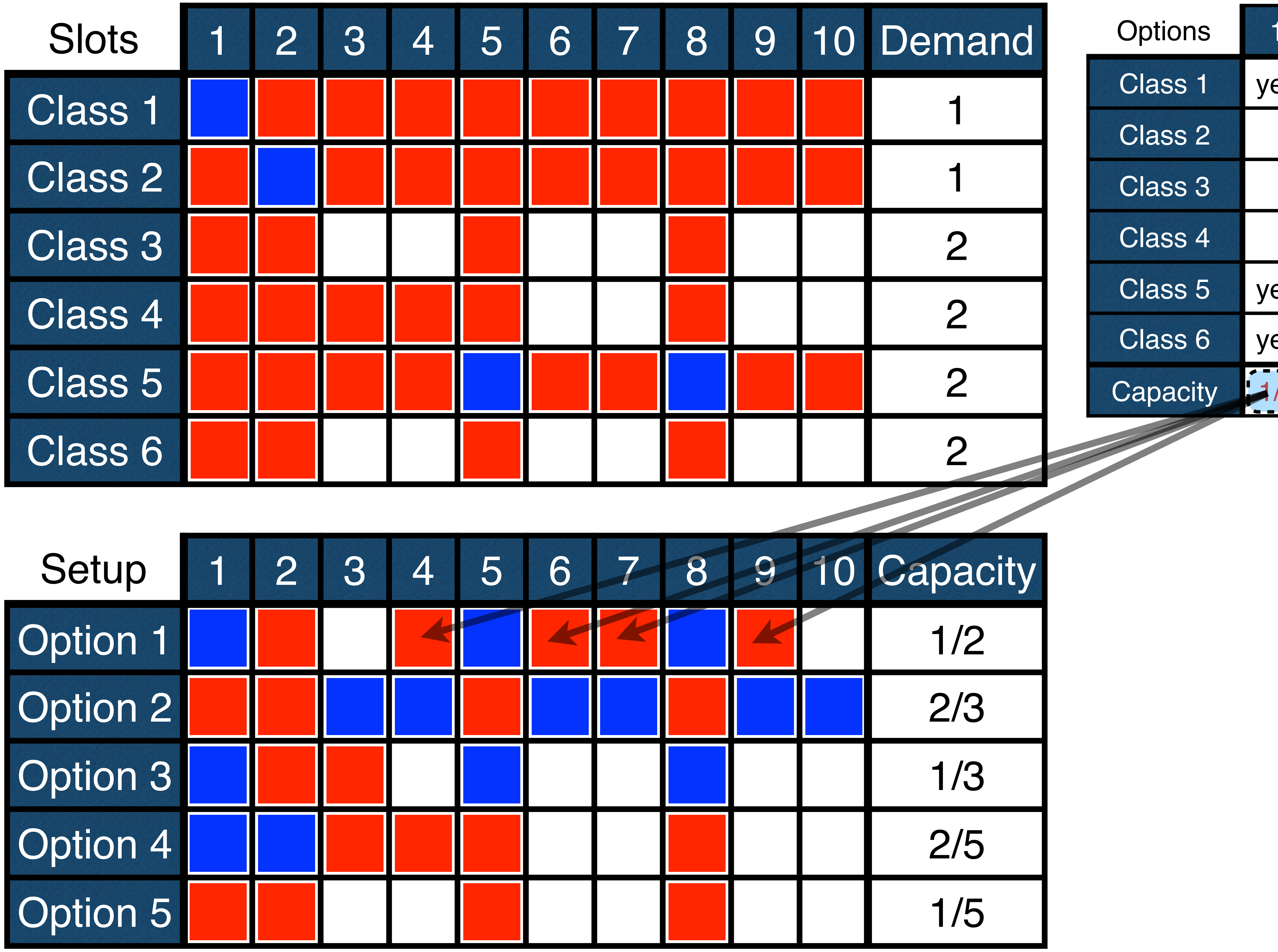
Car Sequencing



Options	1	2	3	4	5	Demand
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Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
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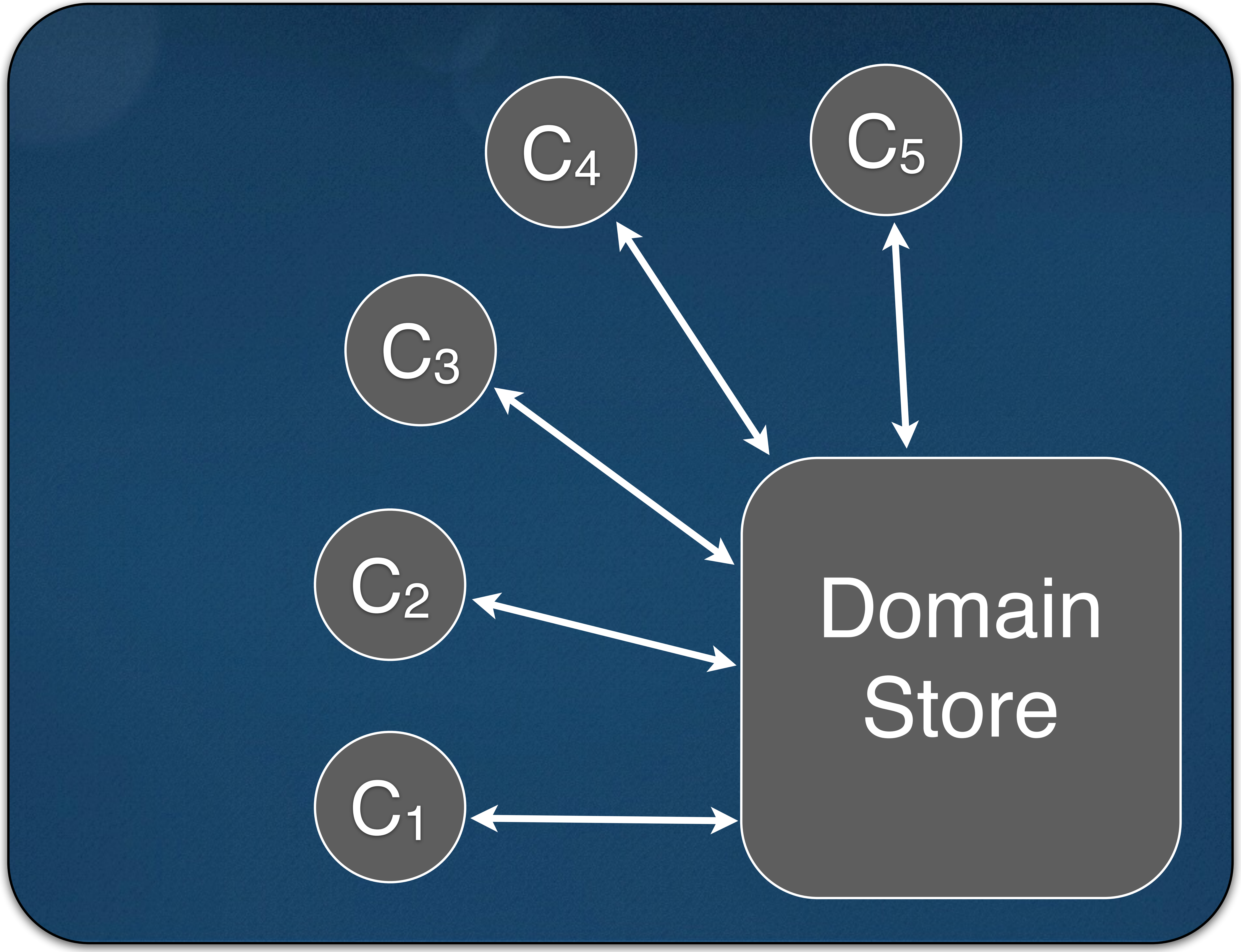
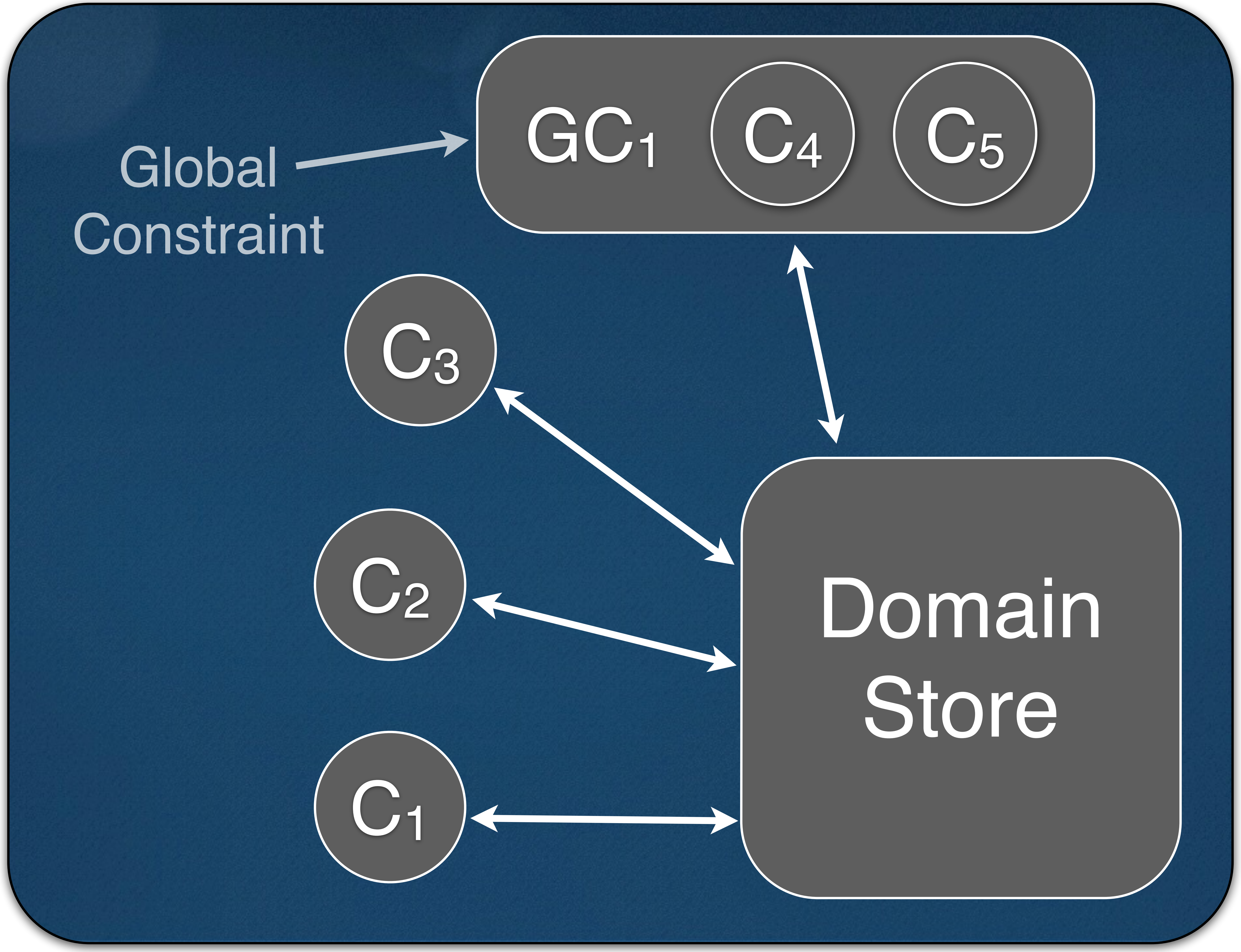
Improving Communication



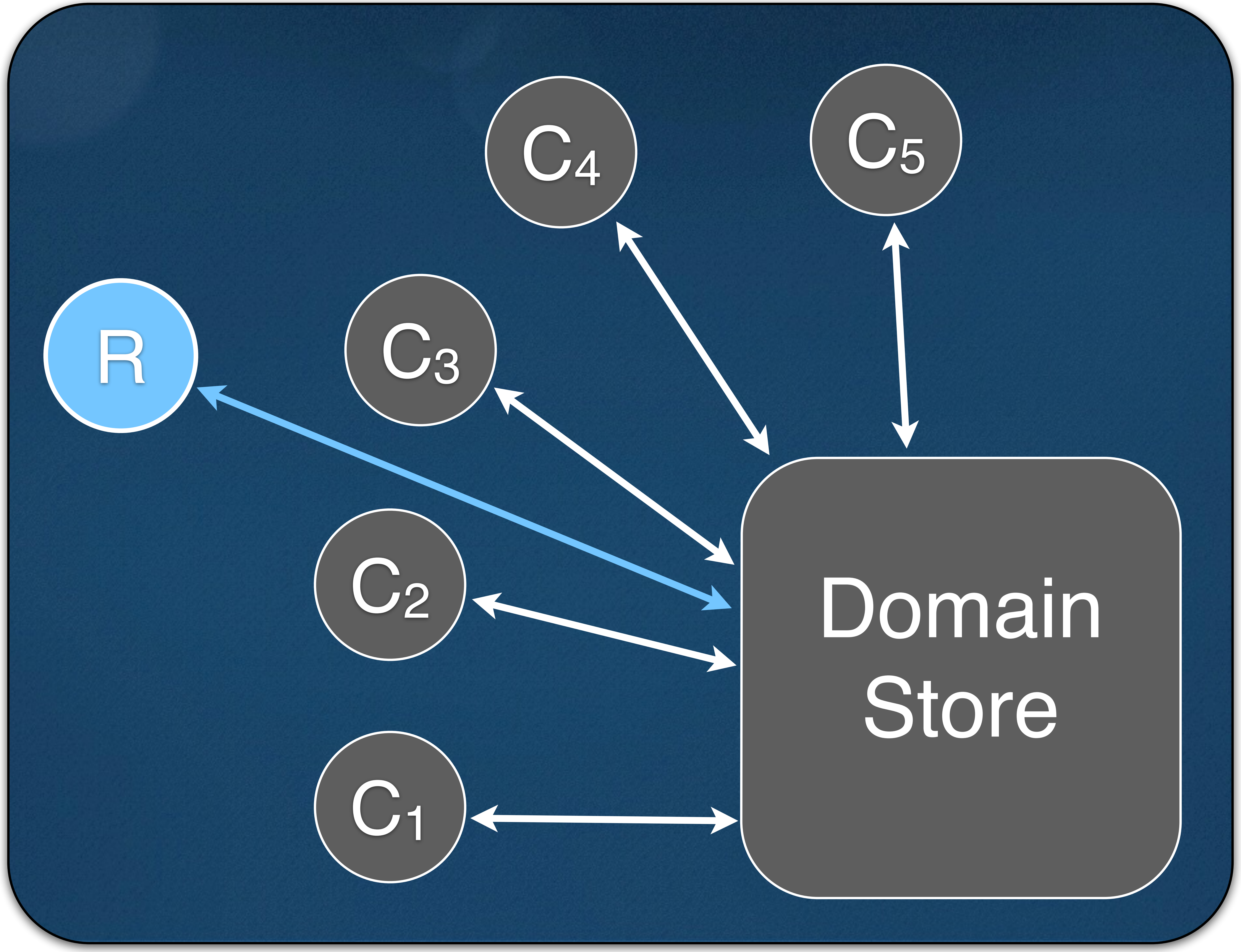
Constraint Store

Improving Communication



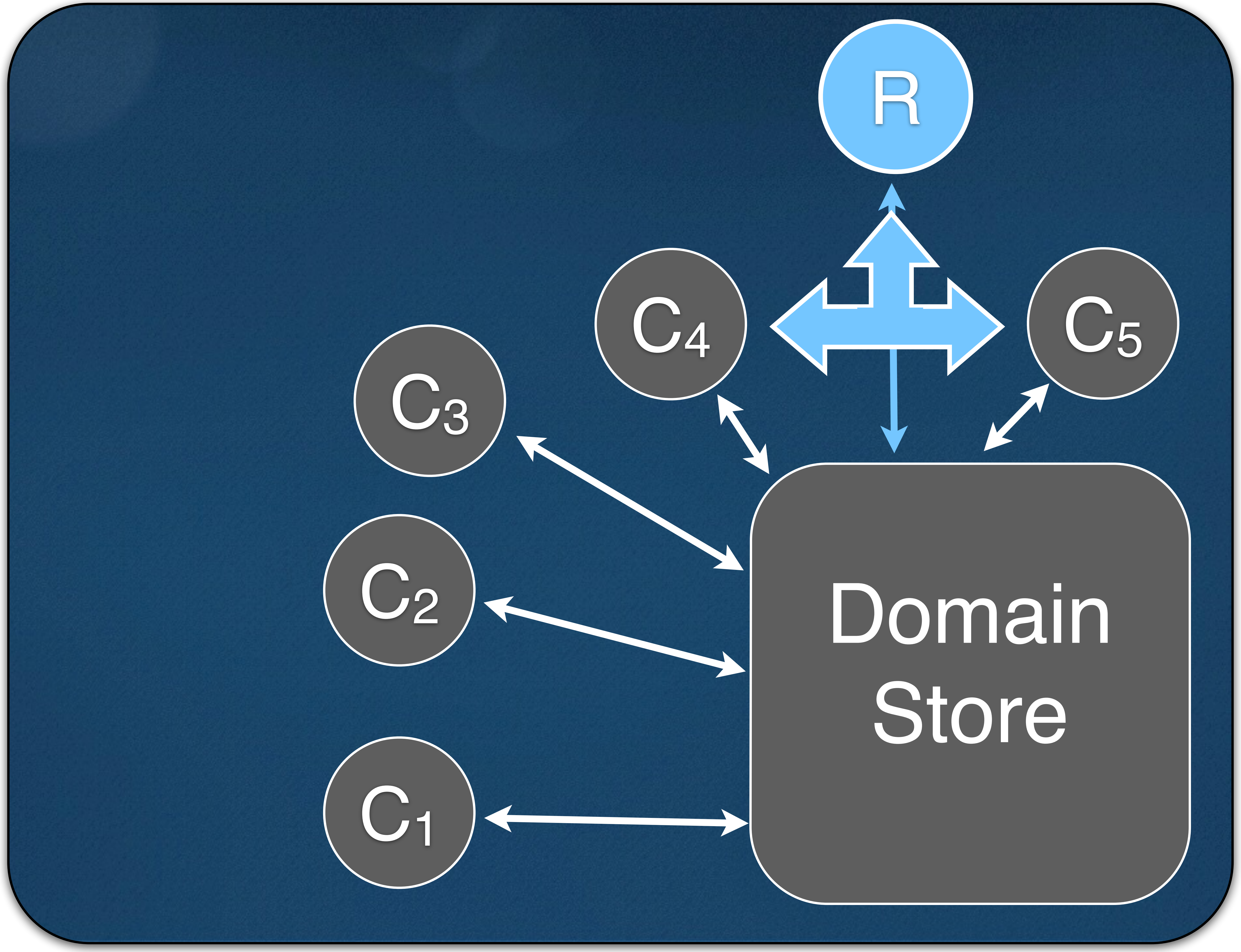
Constraint Store

Improving Communication



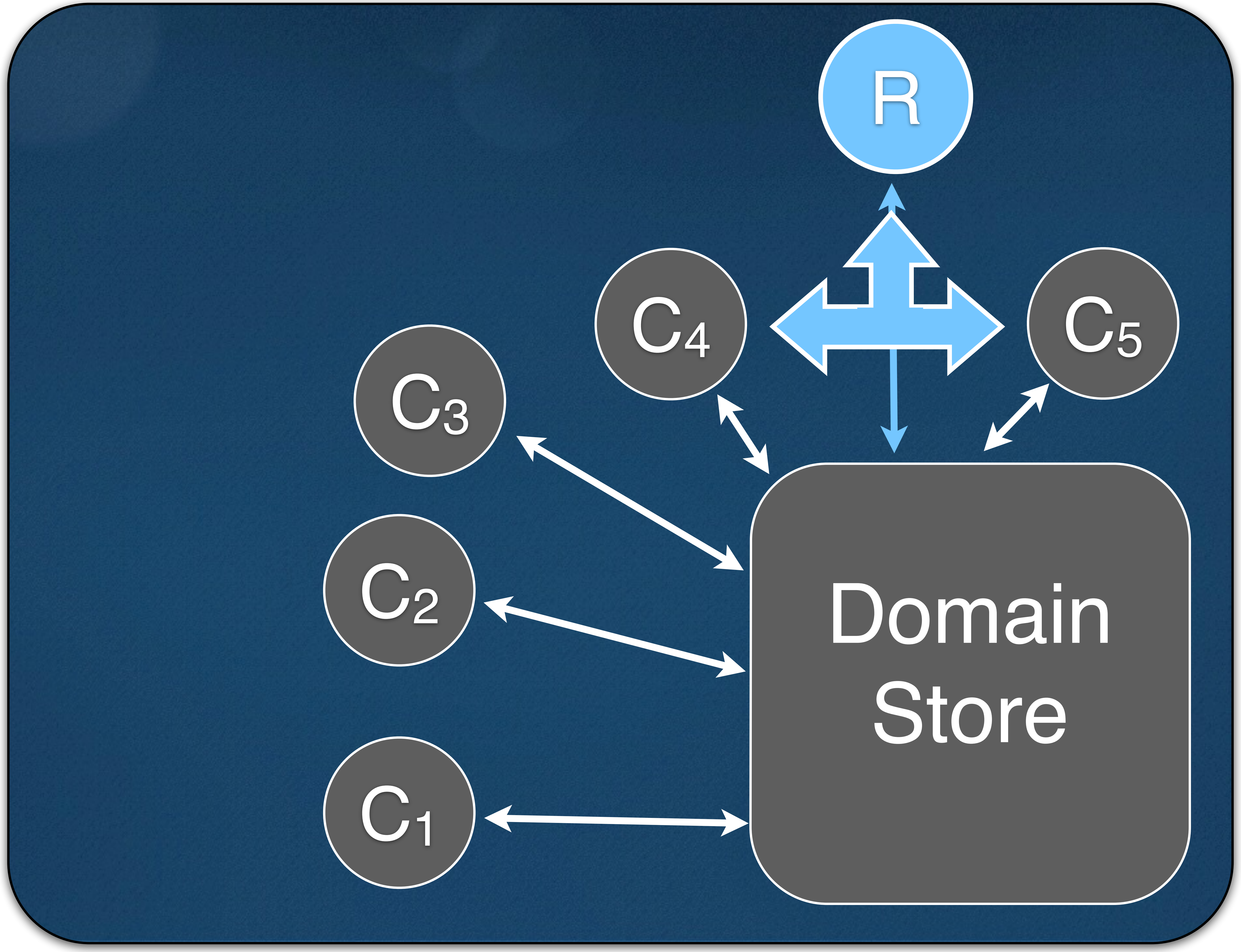
Constraint Store

Improving Communication: Surrogate Constraint



Constraint Store

Improving Communication: Implied Constraint



Constraint Store

Dual Modeling

- Sometimes there are multiple ways of modeling a problem
  - not the same decision variables
- The two models may have complementary strengths
  - hard to choose between them
  - some constraints are easier to express in one model and others in the other one
- Dual modeling
  - the idea of stating multiple models of a problem and linking them with constraints

Back to the 8-Queens Problem

- What are the decision variables?
  - many possible modelings
    - this is what makes optimization problems interesting :-)
- Here is one modeling
  - associate a decision variable with each column
  - the variable denotes the row of the queens placed in this column
- What are the constraints?
  - the queens cannot be placed on the same row
  - the queens cannot be placed on the same upward diagonal
  - the queens cannot be placed on the same downward diagonal

Back to the 8-Queens Problem

- What are the decision variables?
  - many possible modelings
    - this is what makes optimization problems interesting :-)
- Here is another modeling
  - associate a decision variable with each row
  - the variable denotes the column of the queens placed in this row
- What are the constraints?
  - the queens cannot be placed on the same row
  - the queens cannot be placed on the same upward diagonal
  - the queens cannot be placed on the same downward diagonal

The 8-Queens Problem with Dual Modeling

```
range R = 1..8;
range C = 1..8;
var{int} row[C] in R;
var{int} col[R] in C
solve {
   forall(i in R,j in R: i < j) {
      row[i] ≠  row[j];
      row[i] ≠  row[j] + (j - i);
      row[i] ≠  row[j] - (j - i);
   }
   forall(i in C,j in C: i < j) {
      col[i] ≠  col[j];
      col[i] ≠  col[j] + (j - i);
      col[i] ≠  col[j] - (j - i);
   }
   forall(r in R,c in C)
      (row[c] = r) <=> (col[r] = c);
}
```

connecting the two constraints

Citations

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