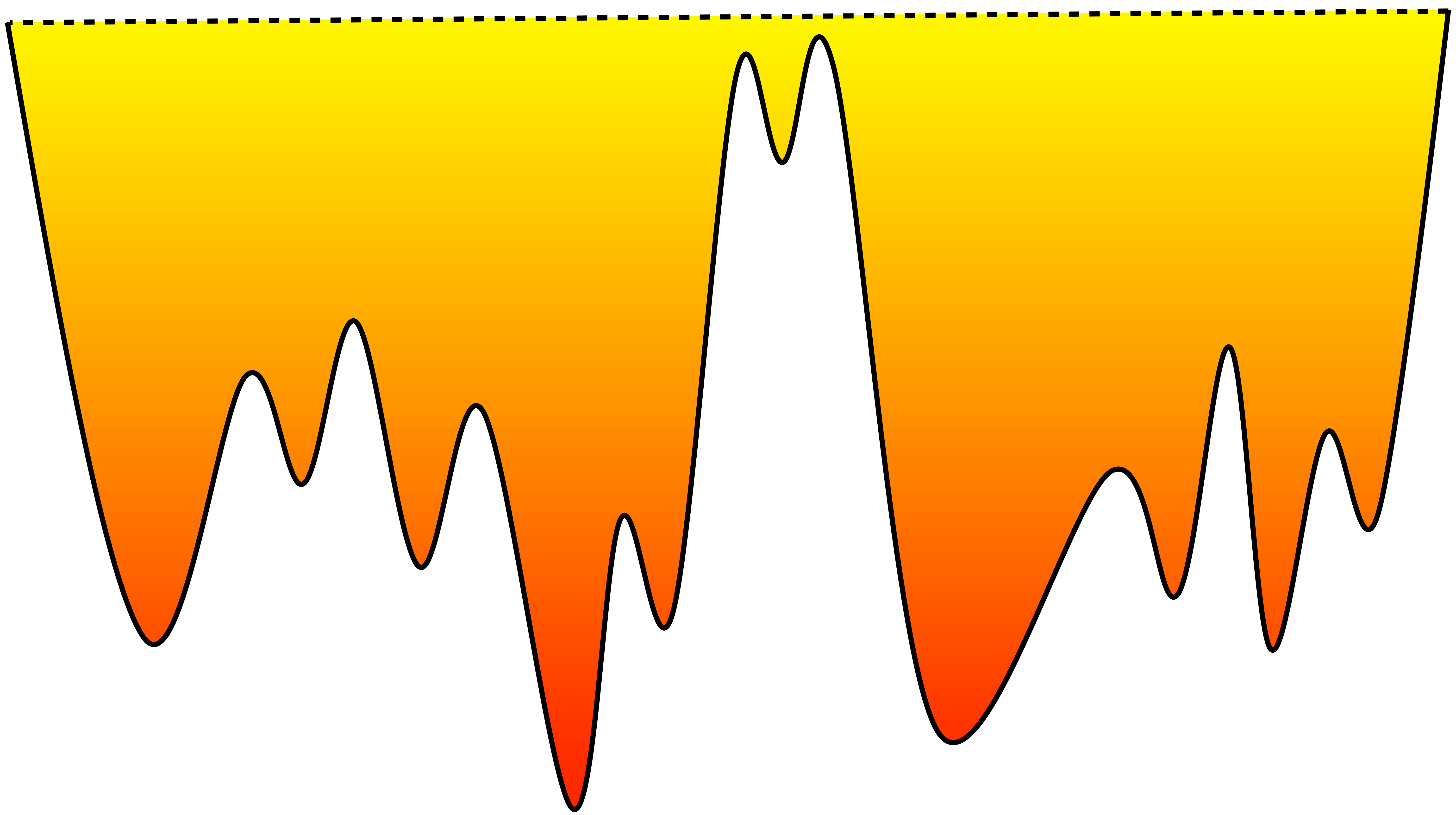
Discrete Optimization

Local Search: Part VIII

Goal of the Lecture

- Local search
  - meta-heuristics
  - multi-start search
  - simulated annealing
  - tabu search

Escaping Local Minima



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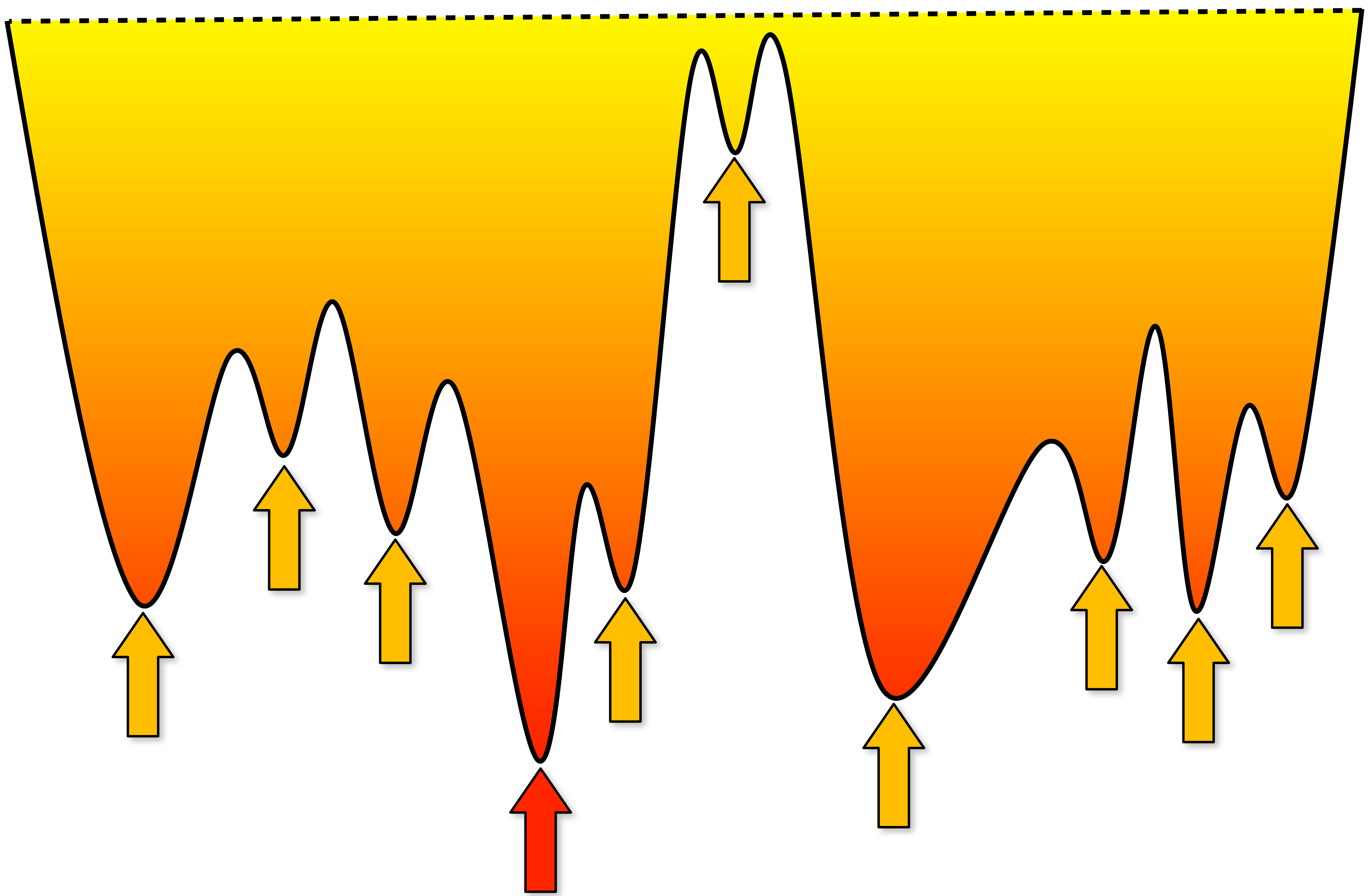
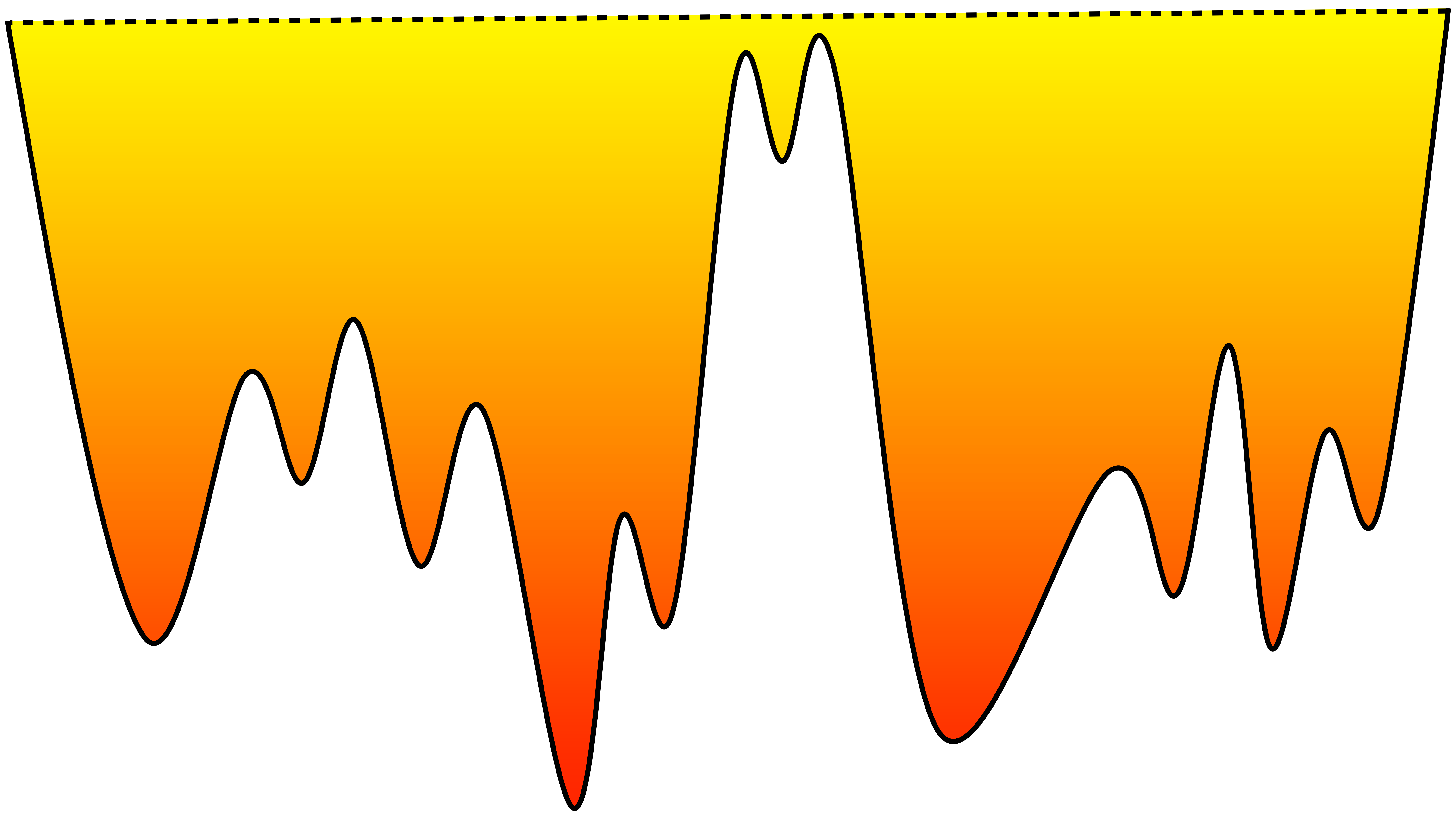


Iterated Local Search



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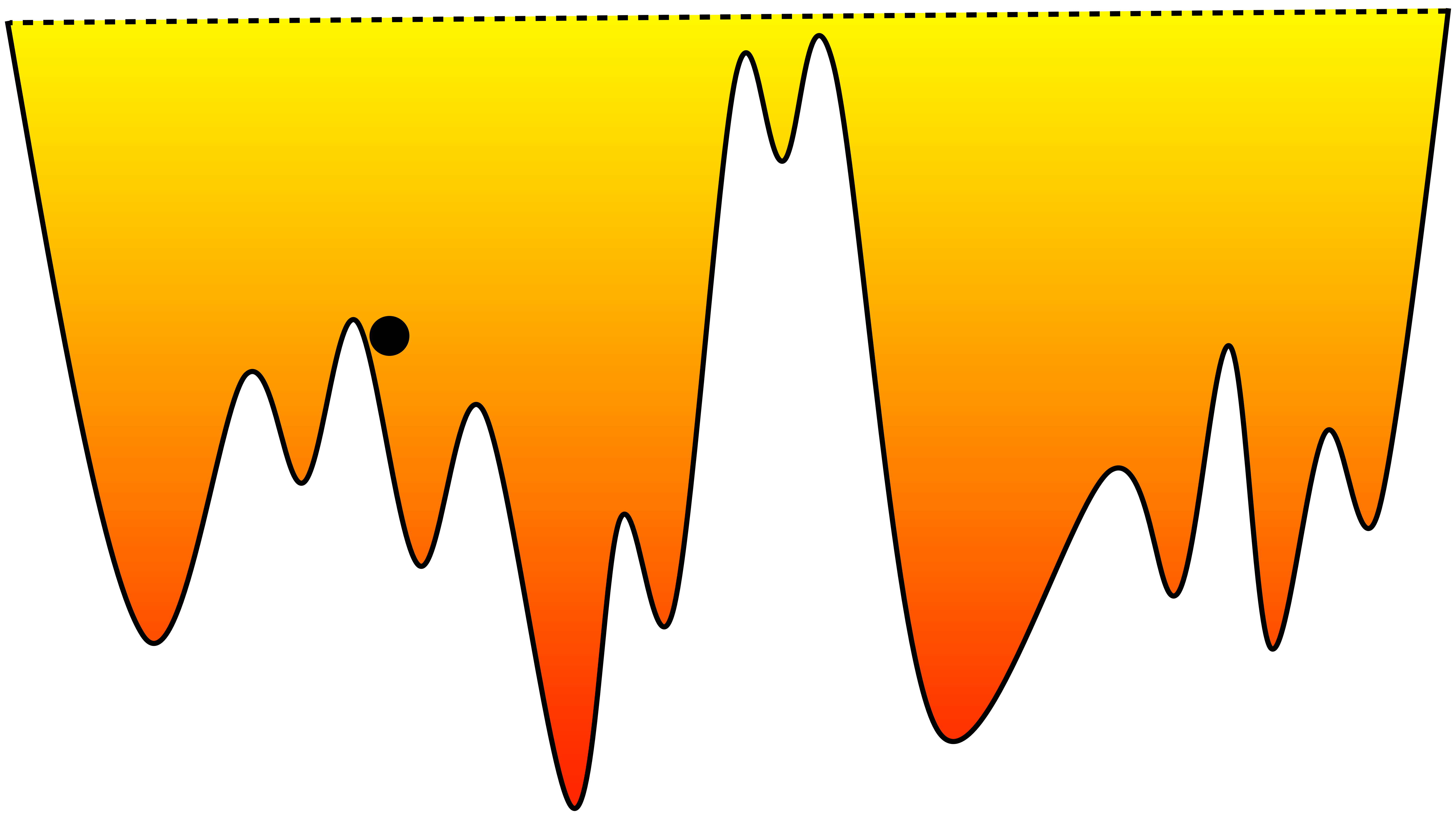

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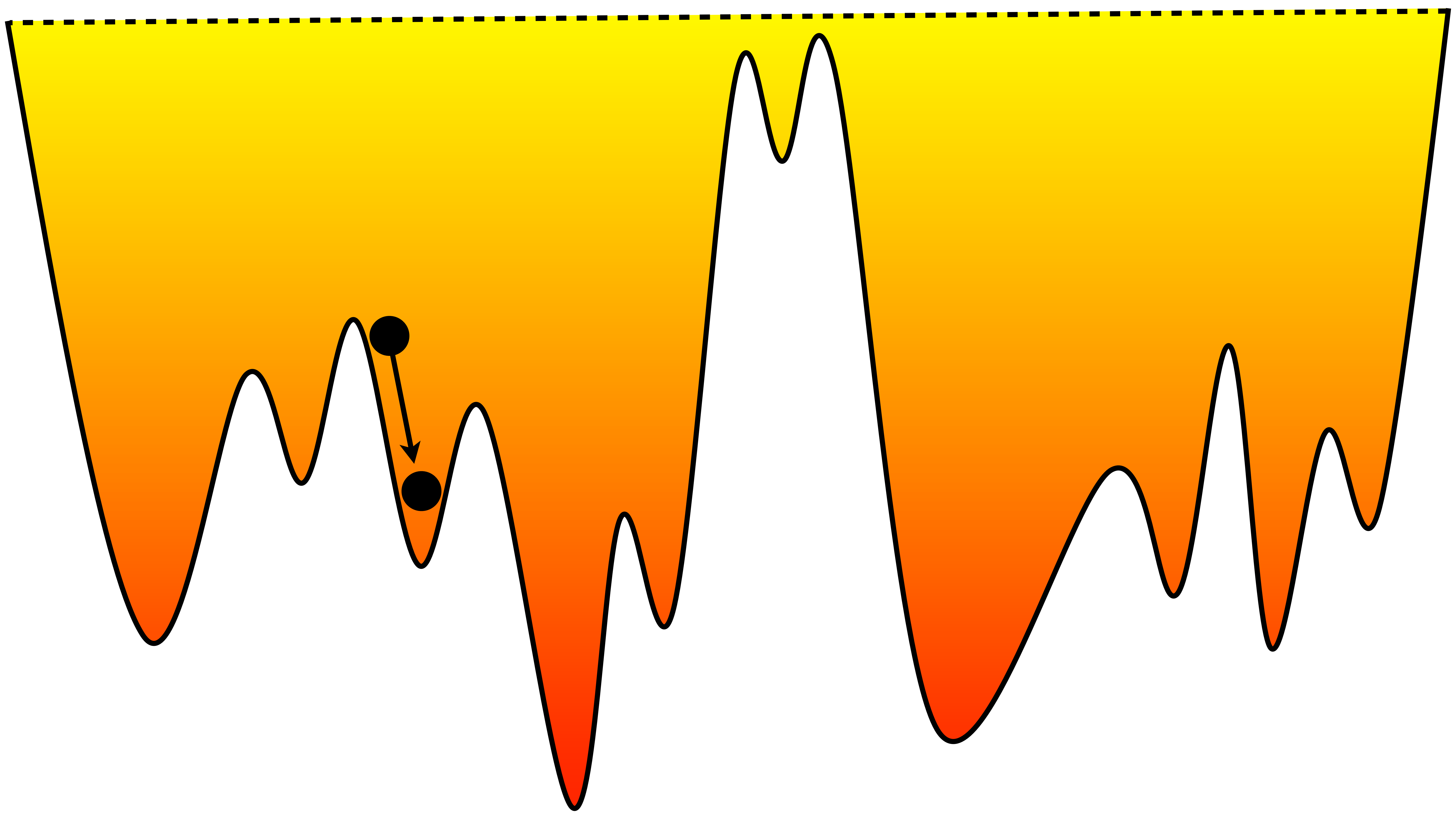

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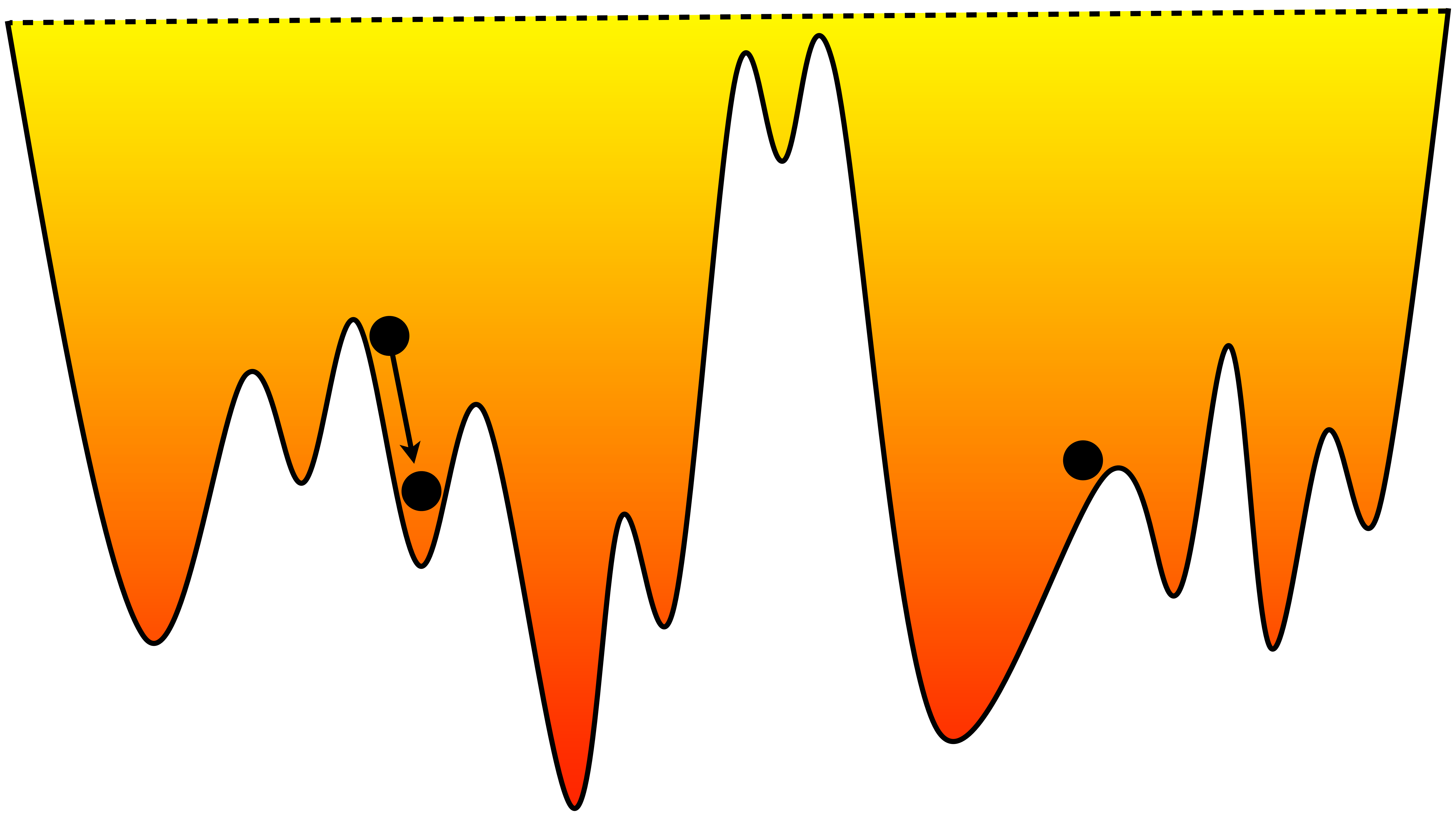

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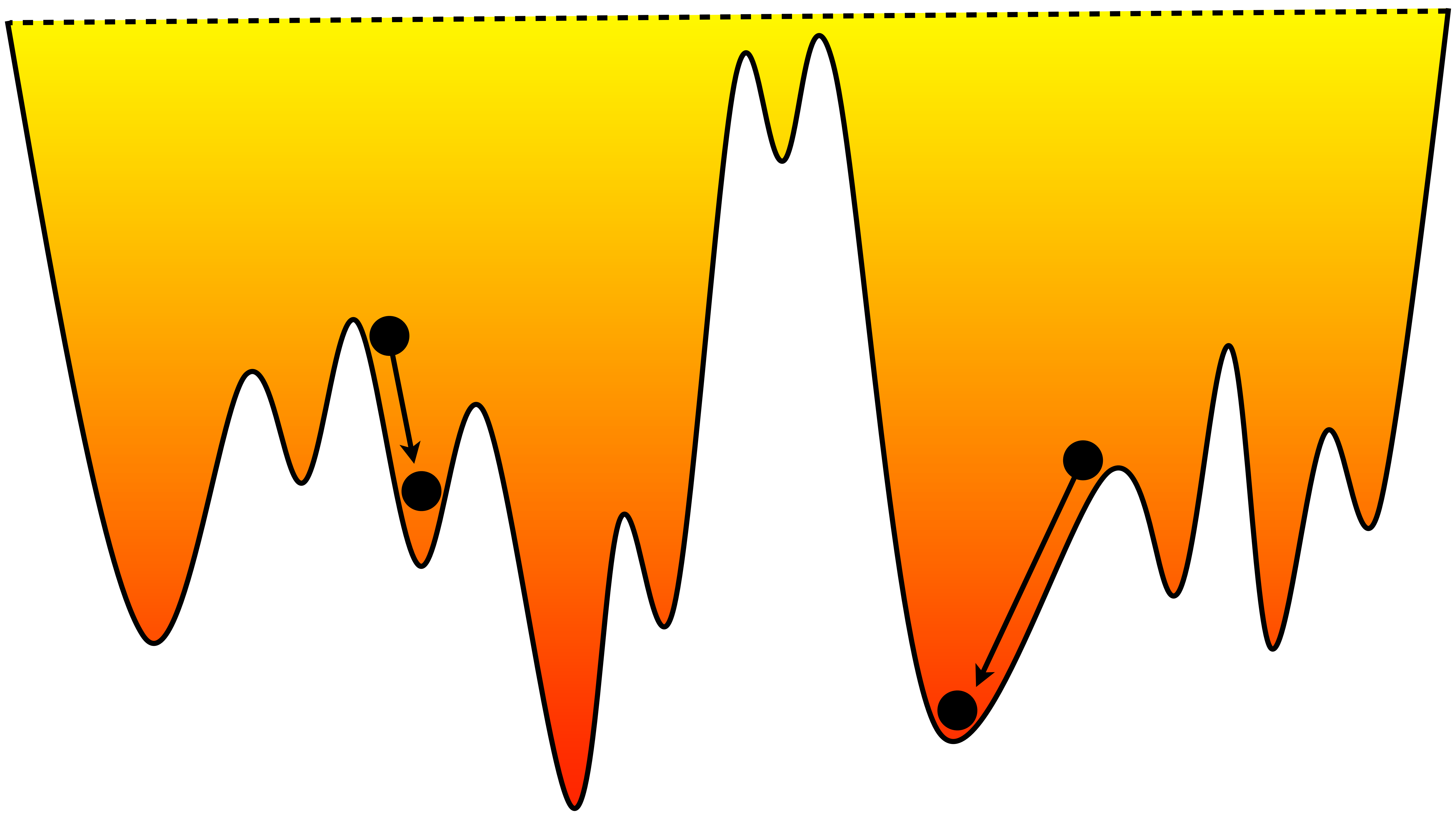

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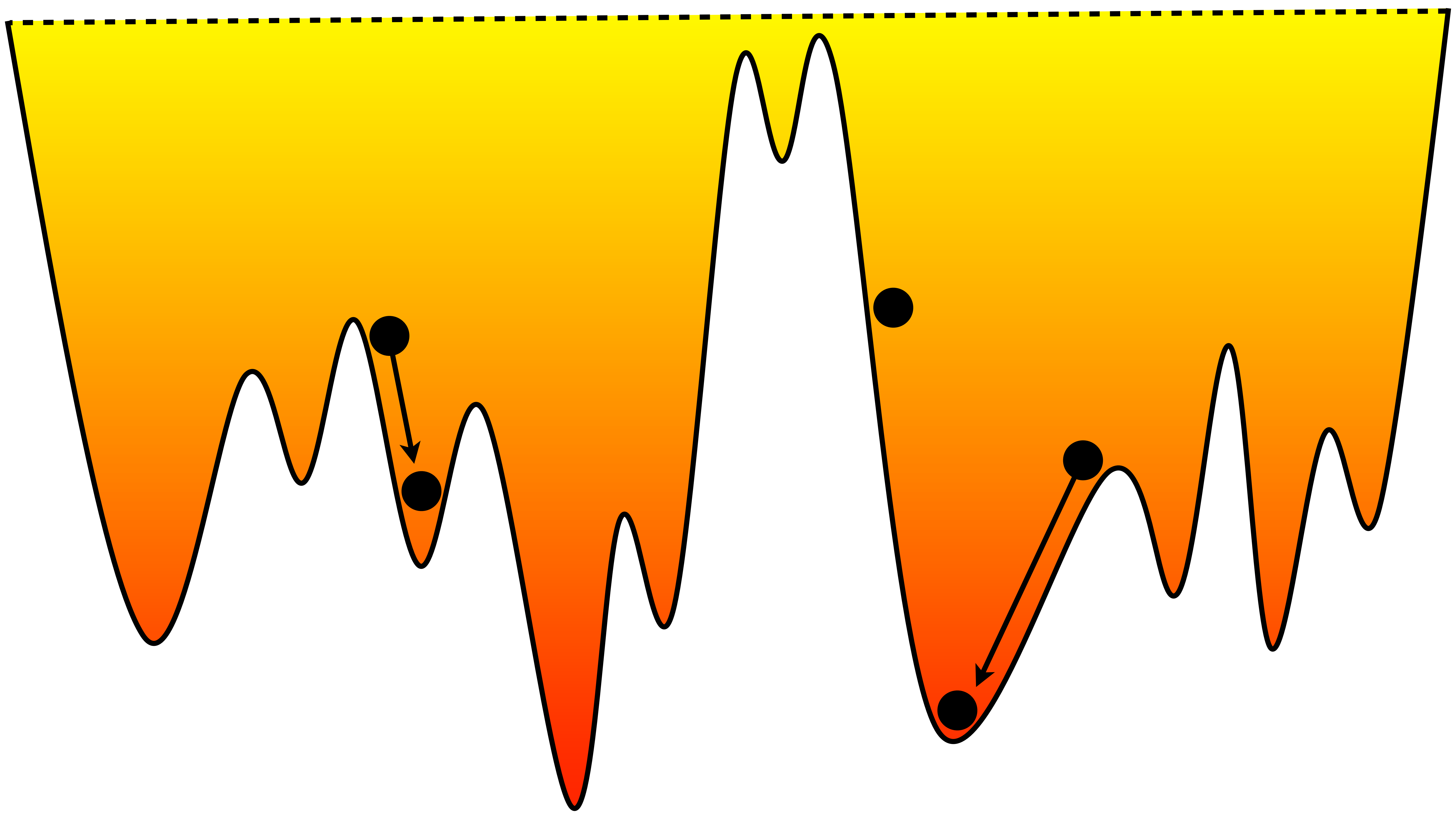

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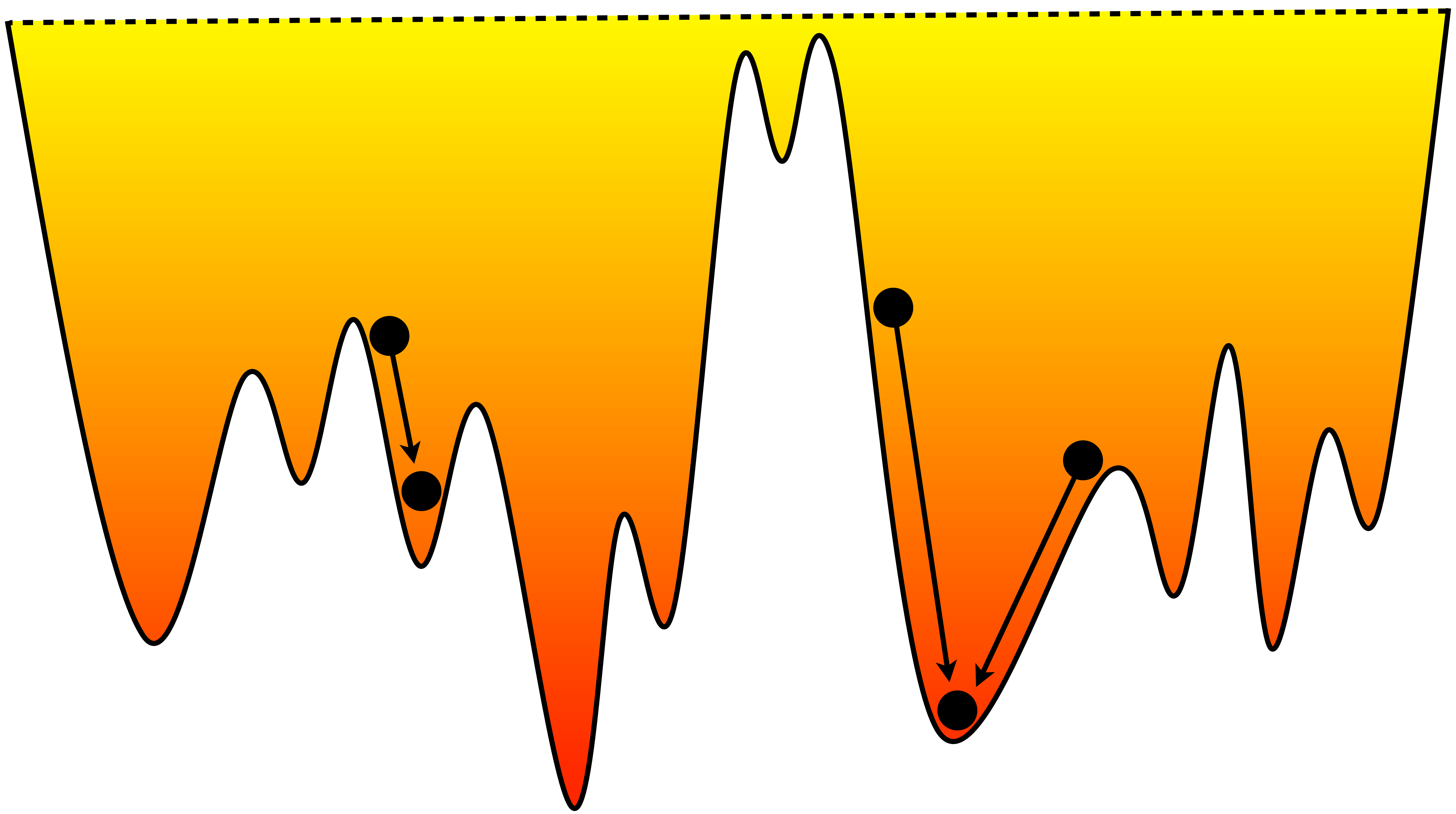

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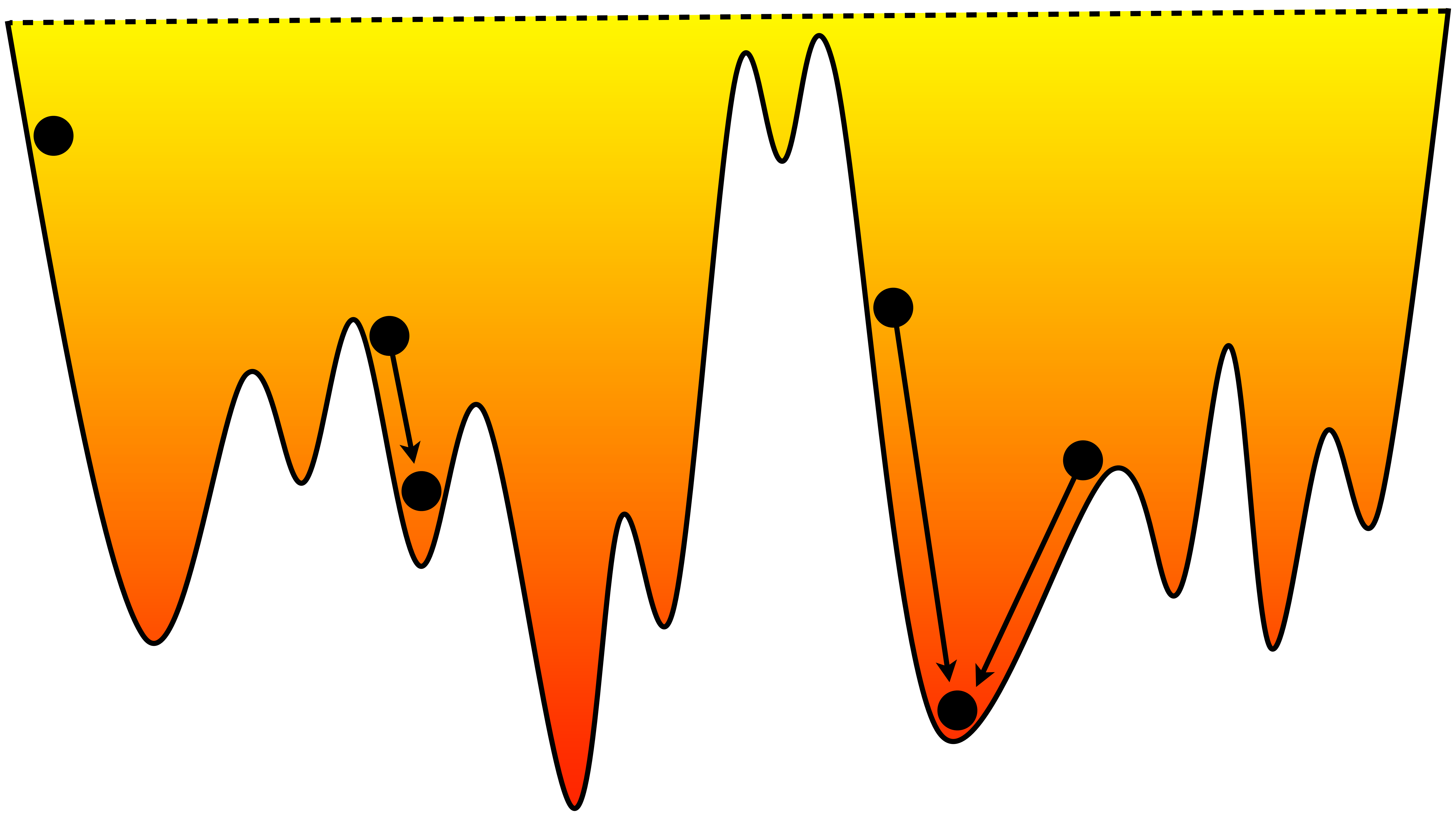

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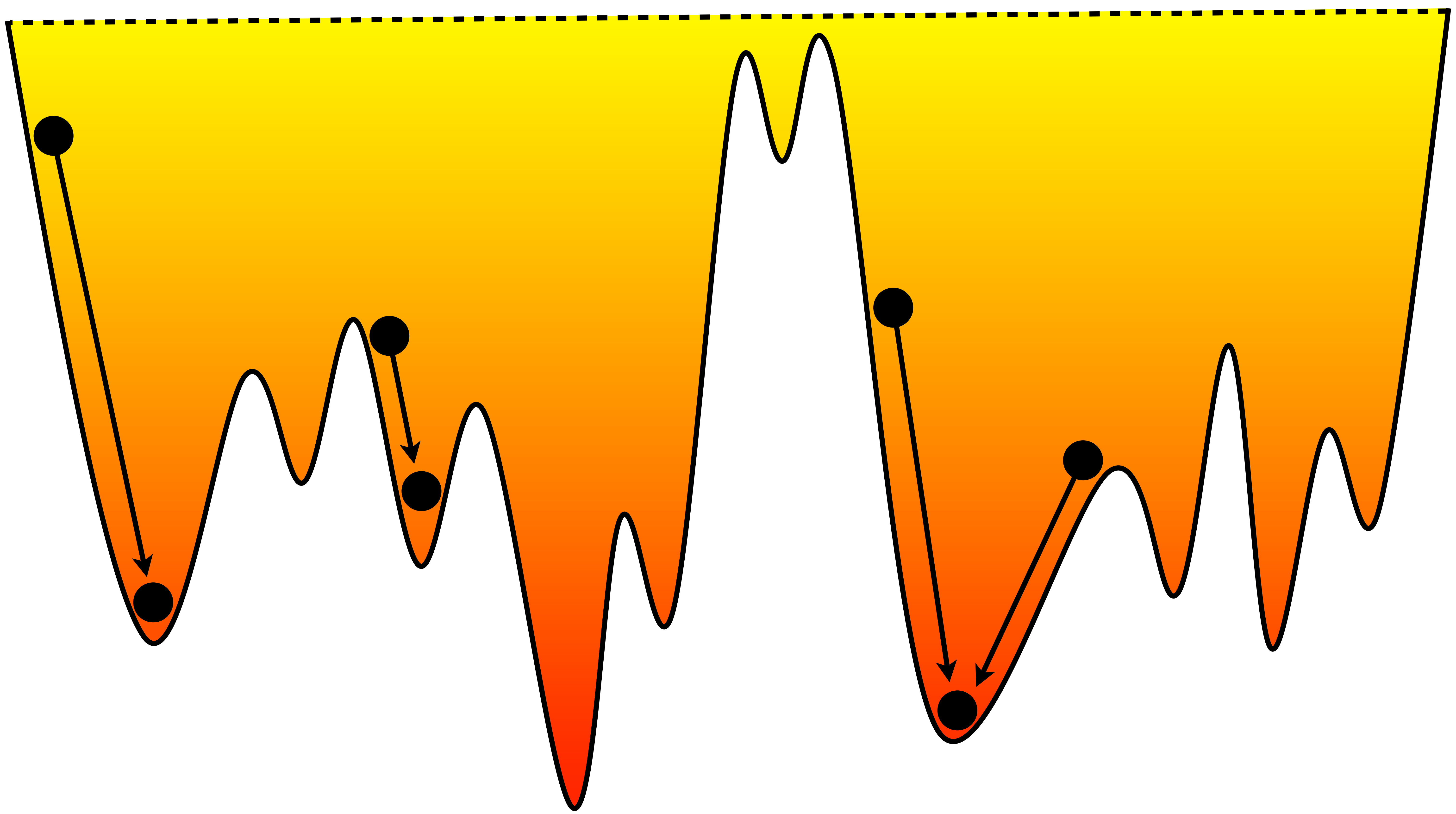

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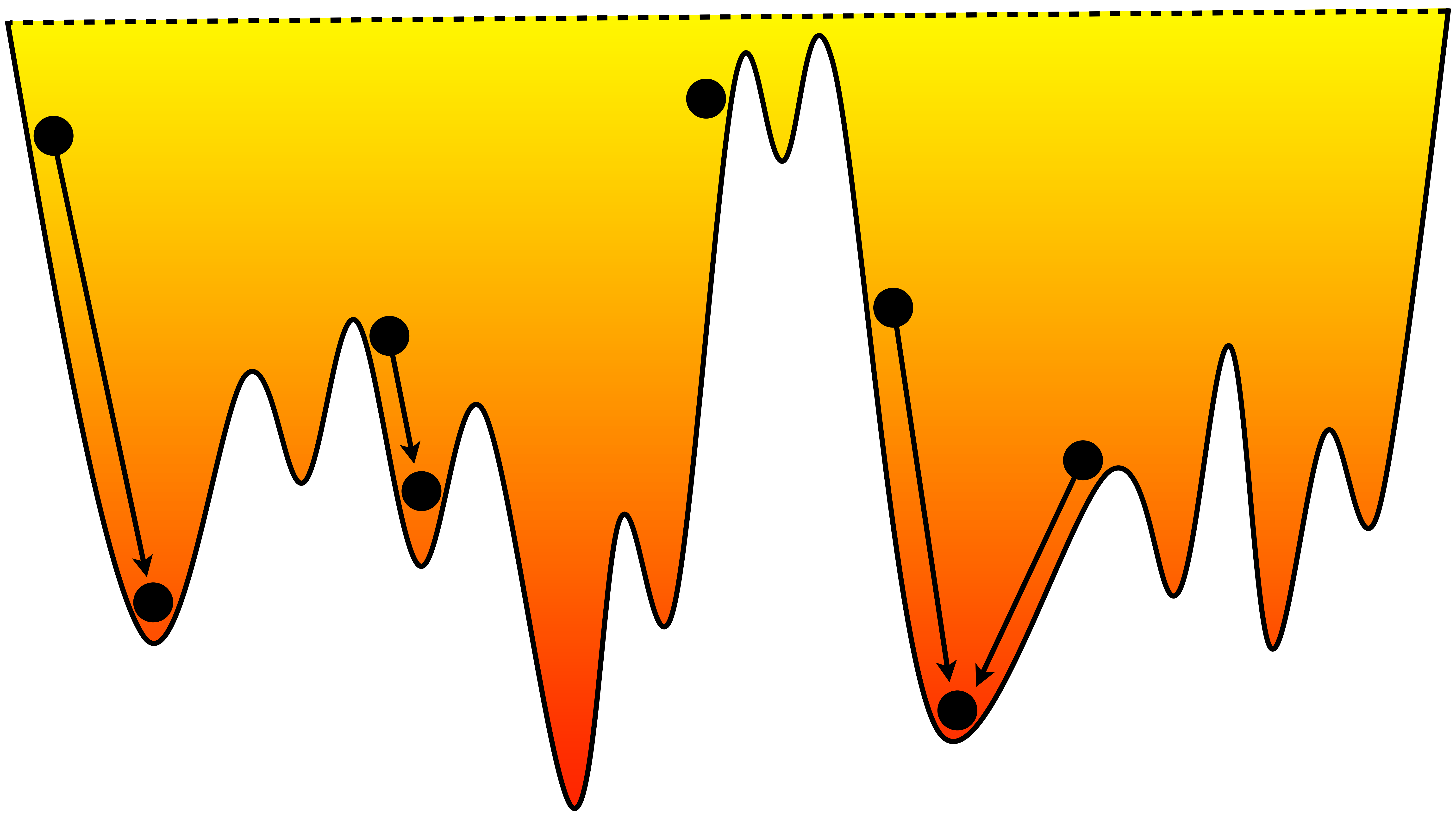

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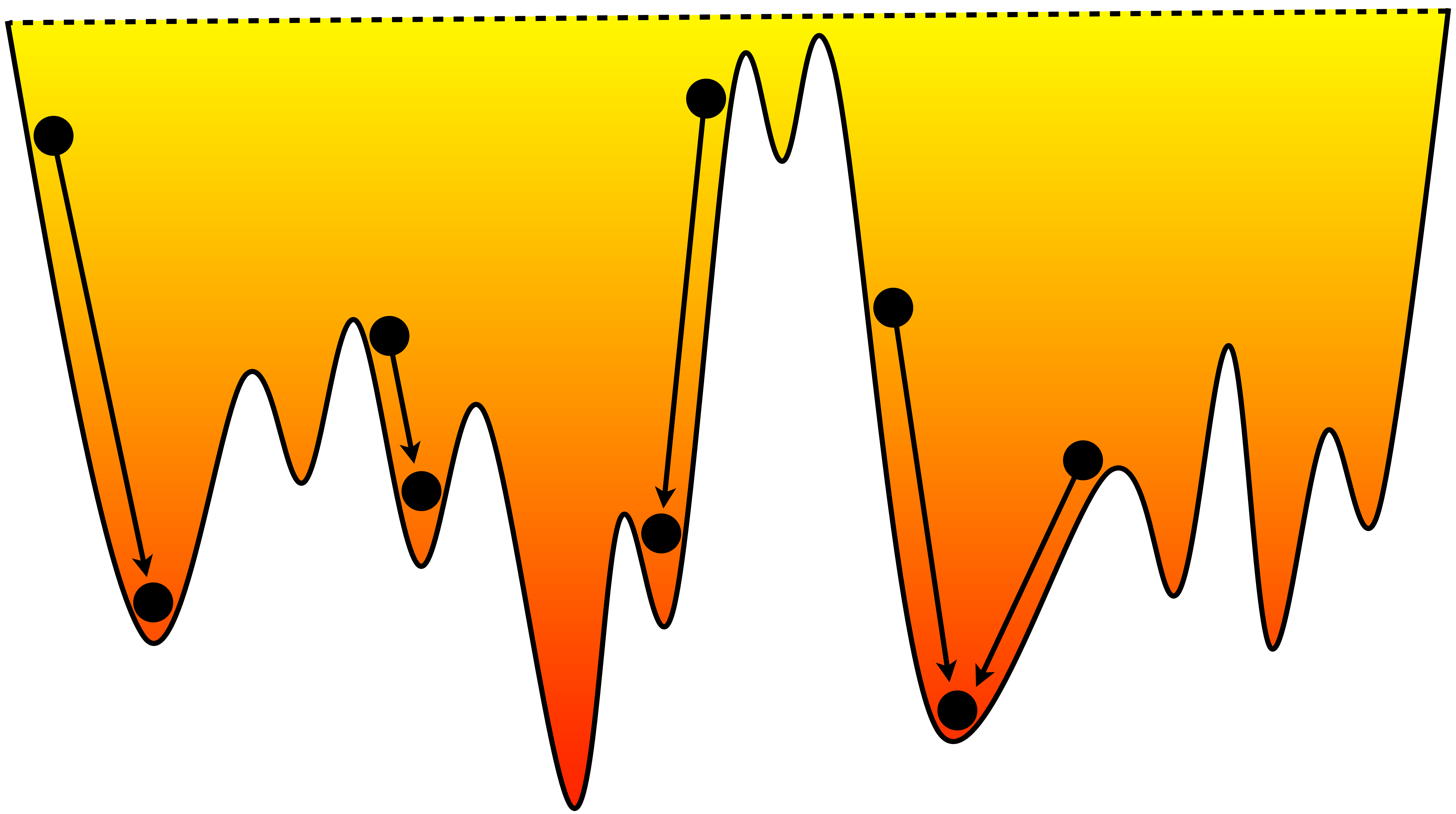

Iterated Local Search

- Execute multiple local search from different starting configuration
  - generic
    - can be combined with other metaheuristics
    - multistarts or restarts

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```
function ITERATEDLOCALSEARCH(f, N, L, S) {
    s := GENERATEINITIALSOLUTION();
    s* := s;
    for k := 1 to MaxSearches do
        s := LOCALSEARCH(f, N, L, S, s);
        if f(s) < f(s*) then
            s* := s;
        s := GENERATENEWSOLUTION(s);
    return s*;
}
```

Metropolis Heuristic

- Basic idea
  - accept a move if it improves the objective value
    or, in case it does not, with some probability
  - the probability depends on how “bad” the move is
  - inspired by statistical physics

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- Basic idea
  - accept a move if it improves the objective value or, in case it does not, with some probability
  - the probability depends on how "bad" the move is
  - inspired by statistical physics
- How is the probability chosen?
  - t is a temperature
  - Δ is the difference f(n) - f(s)
  - a degrading move is accepted with probability,

$$\exp(\frac{-\Delta}{t})$$

Metropolis Heuristic

```
function S-Metropolis[t](N,s)
    select n ∈ N with probability 1/#N;
    if f(n) ≤ f(s) then
        return n;
    else with probability exp(−(f(n)−f(s))/t)
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- What happens for a large $\Delta = f(n) - f(s)$?

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- What happens for a large $\Delta = f(n) - f(s)$?
  - the probability of accepting the move becomes very small

Metropolis Heuristic

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  - the probability of accepting a degrading move is large

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7. **else**
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- What happens for a small t ?
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  - heating and cooling schedules to produce crystals with few defects

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  - heating and cooling schedules to produce crystals with few defects
- Key idea
  - Use Metropolis algorithm but adjust the temperature dynamically
  - start with a high temperature
    - essentially a random walk
  - decrease the temperature progressively
  - when the temperature is low
    - essentially a local improvement search

Simulated Annealing

```
function SimulatedAnnealing(f, N) {
    s := generateInitialSolution();
    t_1 := initTemperature(s);
    s* := s;
    for k := 1 to MaxSearches do
        s := LocalSearch(f,N,L-All,S-Metropolis[t_k],s);
        if f(s) < f(s*) then
            s* := s;
        t_{k+1} := updateTemperature(s,t_k);
    return s*;
}
```

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  - connected neighborhood
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    - slower than exhaustive search

Simulated Annealing

- guaranteed to converge to a global optimum
  - connected neighborhood
  - slow schedule
    - slower than exhaustive search
- In practice
  - some excellent results on some hard benchmarks
    - e.g., TTP, minimizing tardiness in scheduling
  - reasonably fast schedule

$$t_{k+1} = \alpha \; t_k$$

Simulated Annealing

- Various additional techniques
  - restarts
  - reheats
  - see also tabu search later

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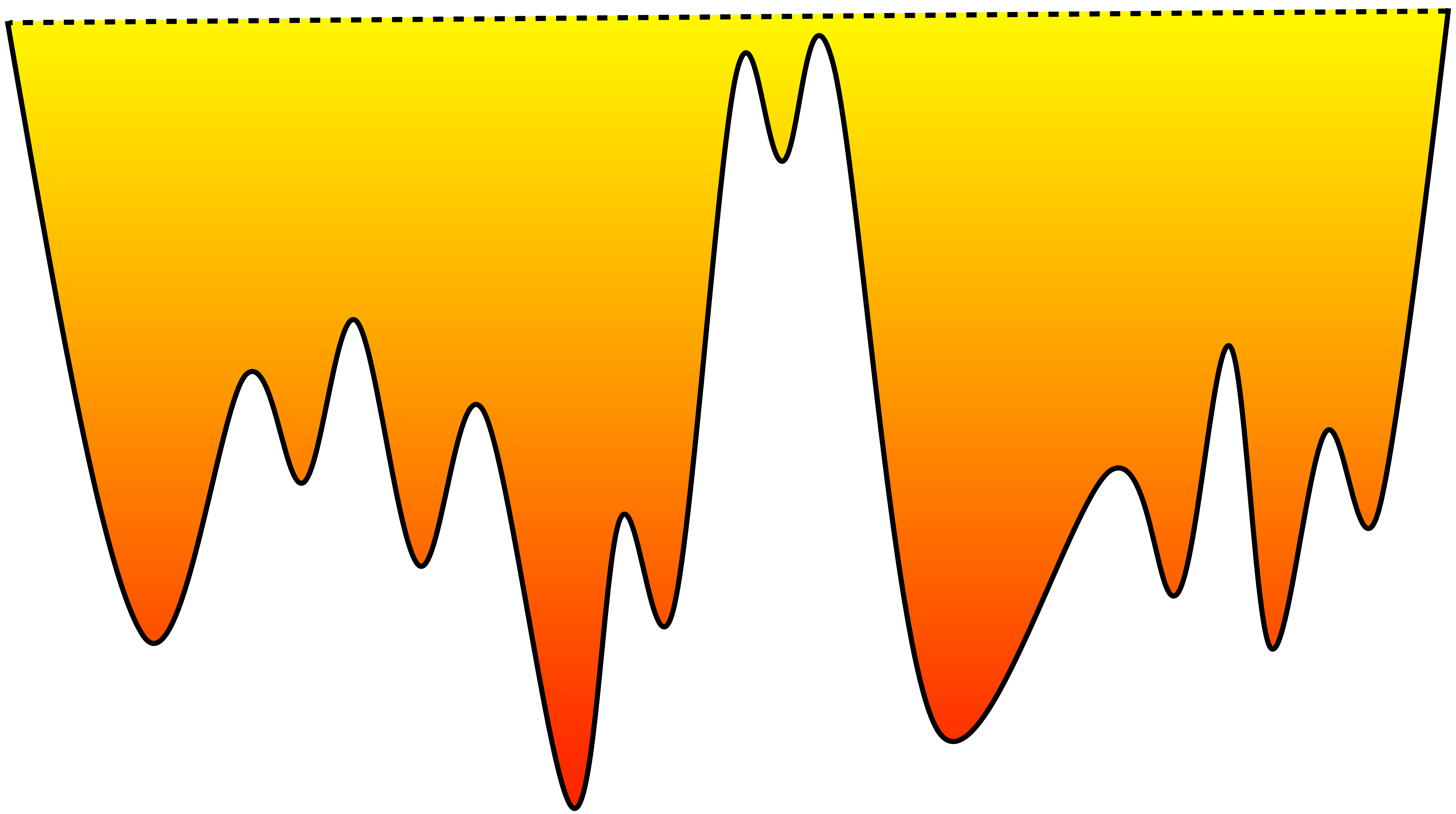
- Restarts
  - like in multi-start procedure
- Reheat
  - increase the temperature

Tabu Search



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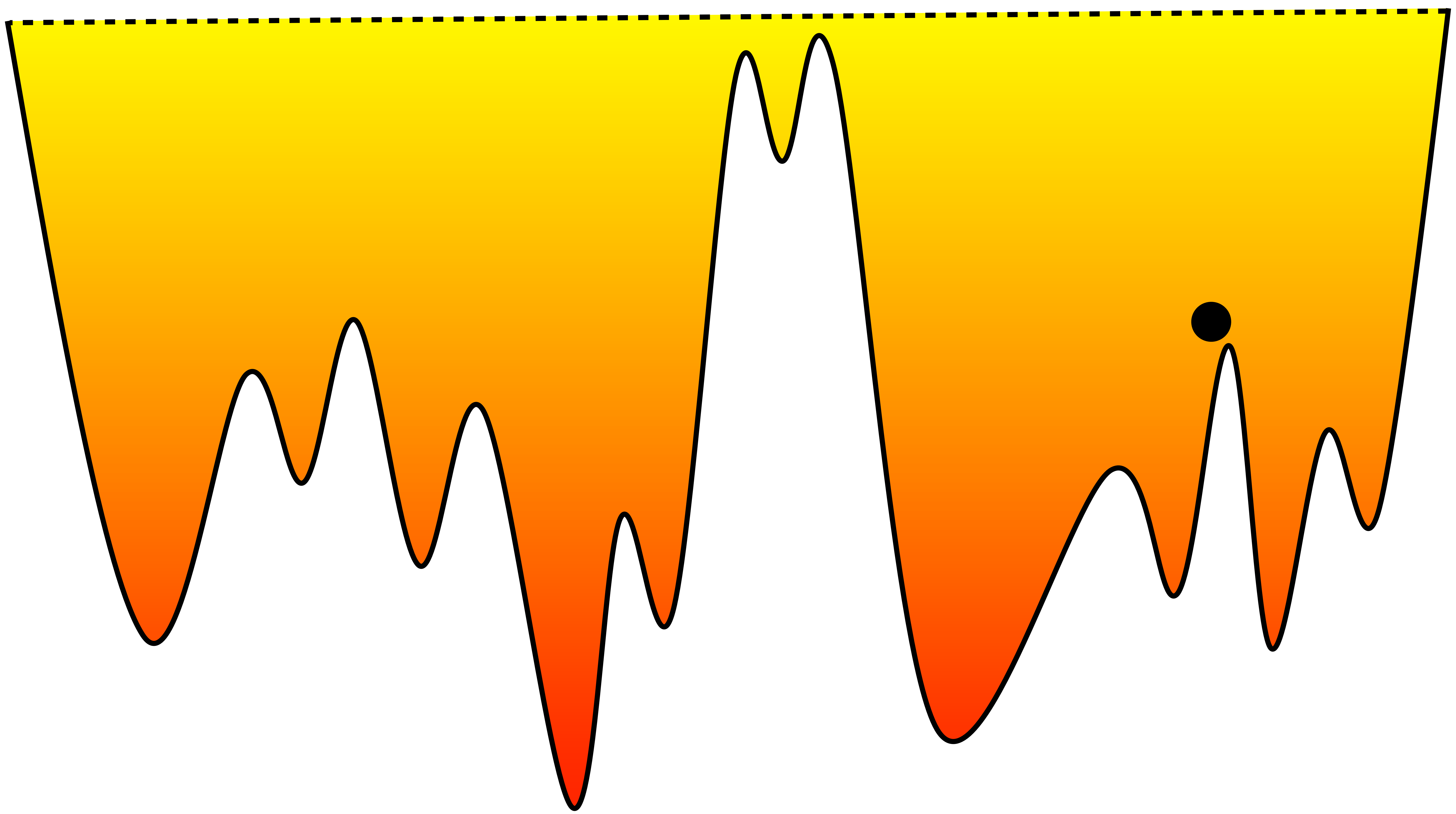

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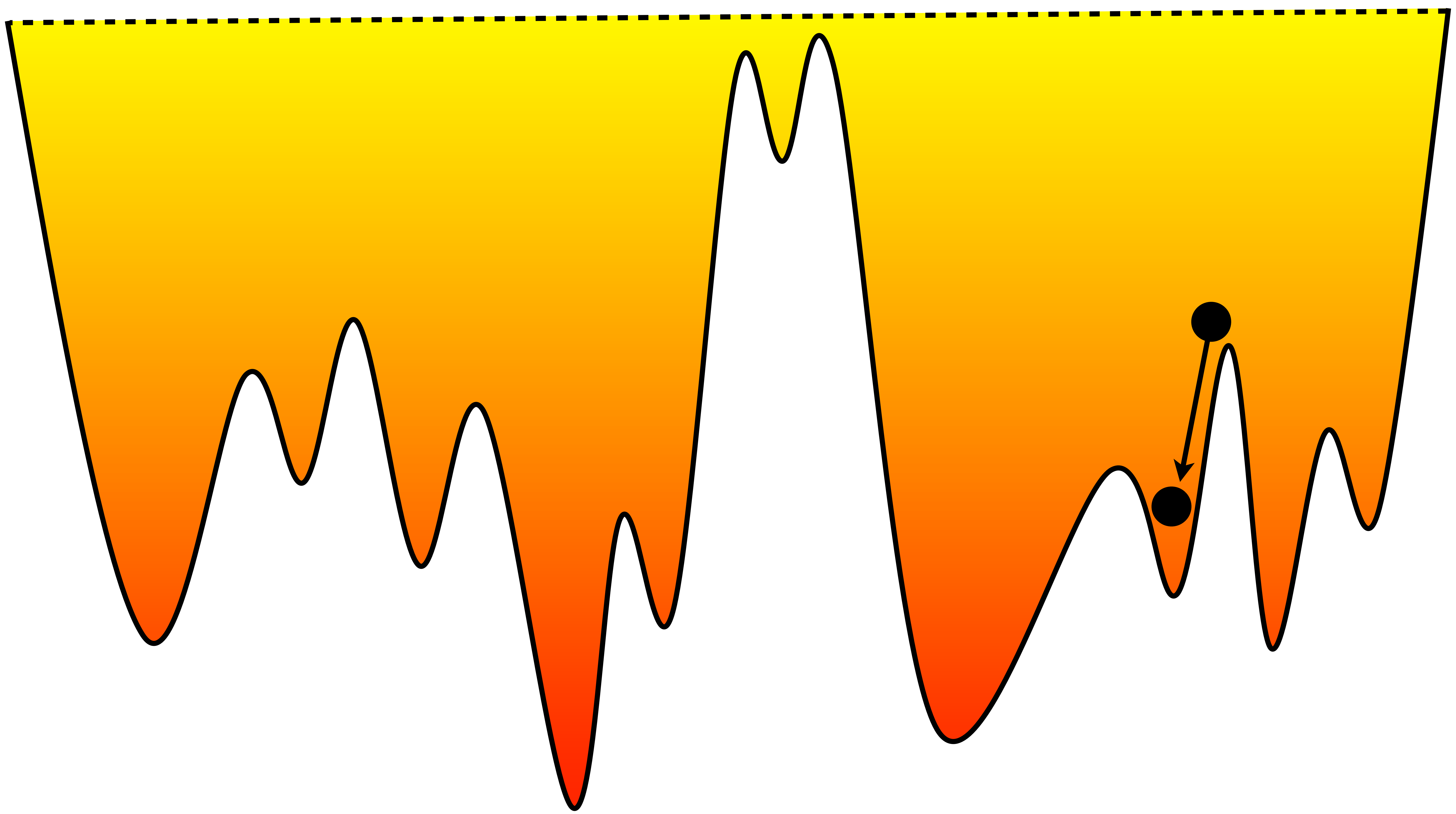

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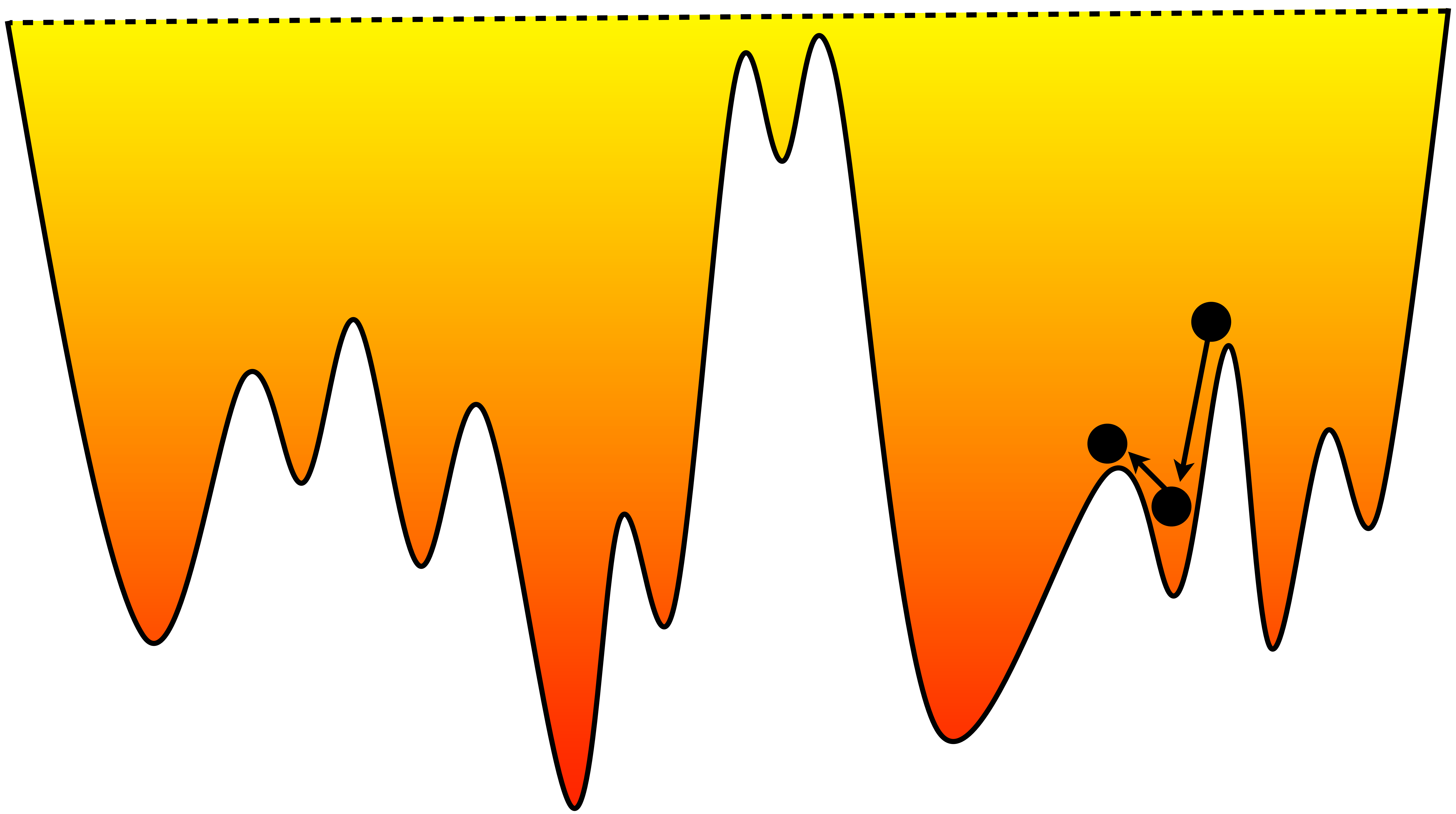

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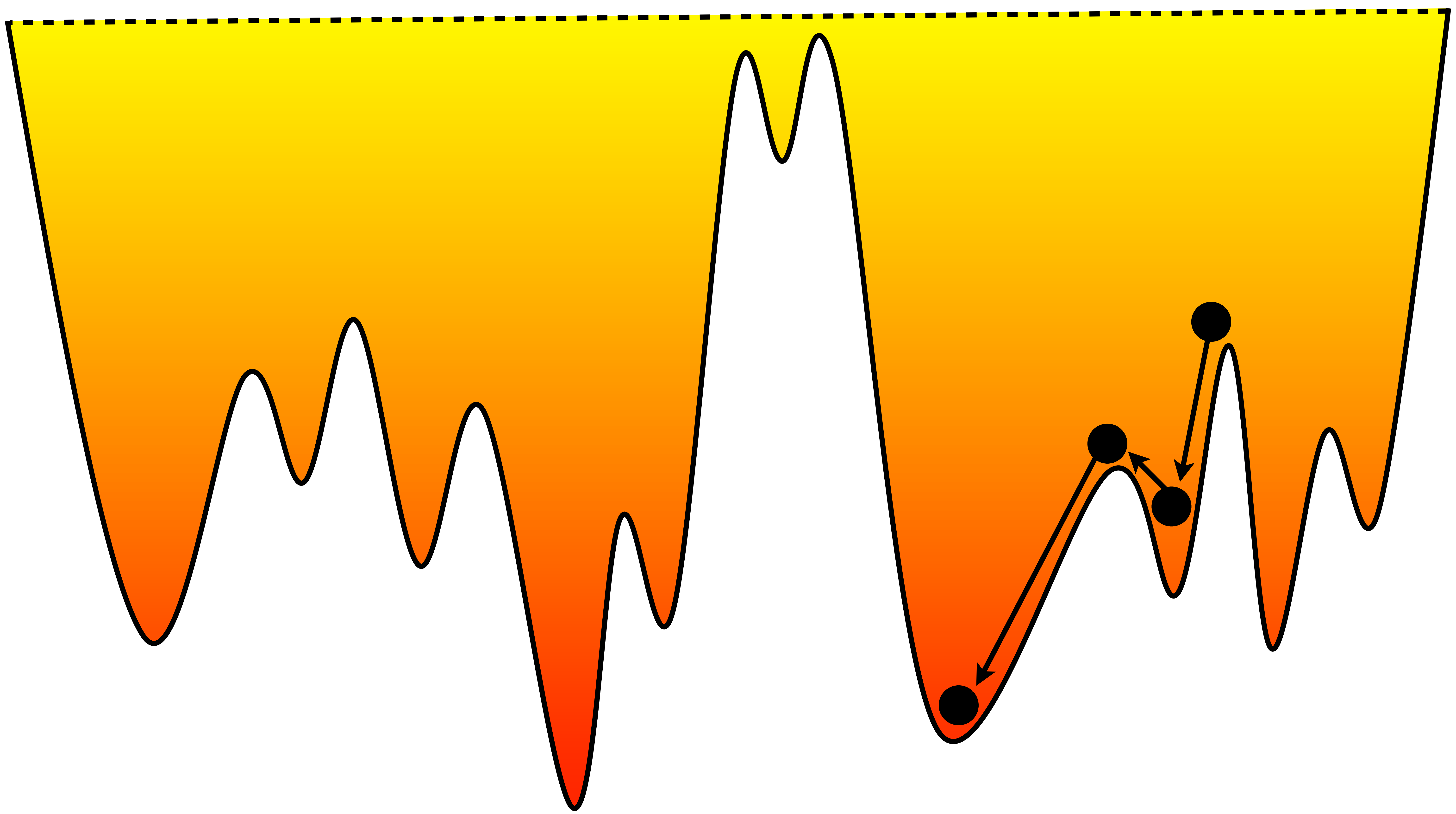

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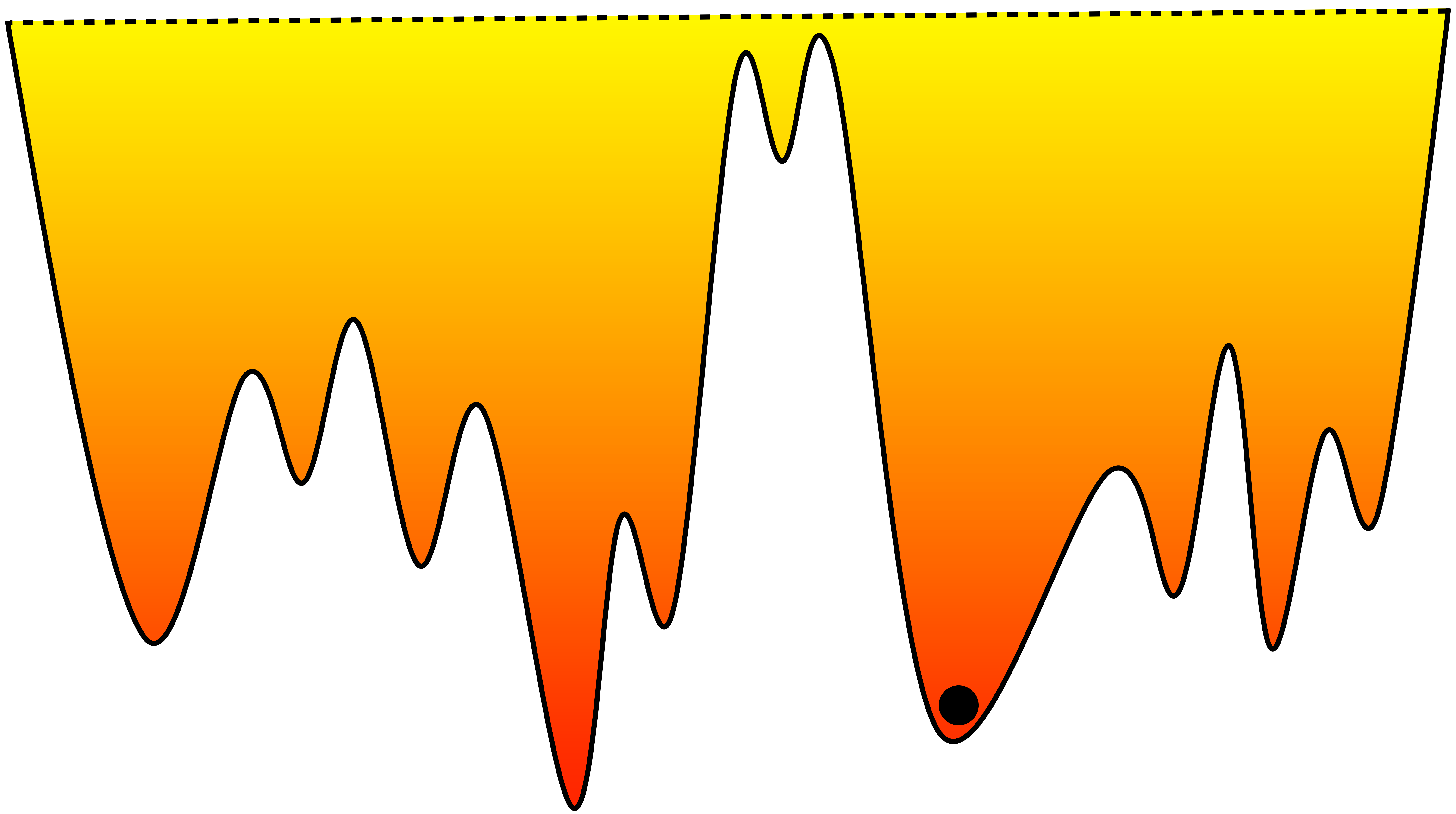

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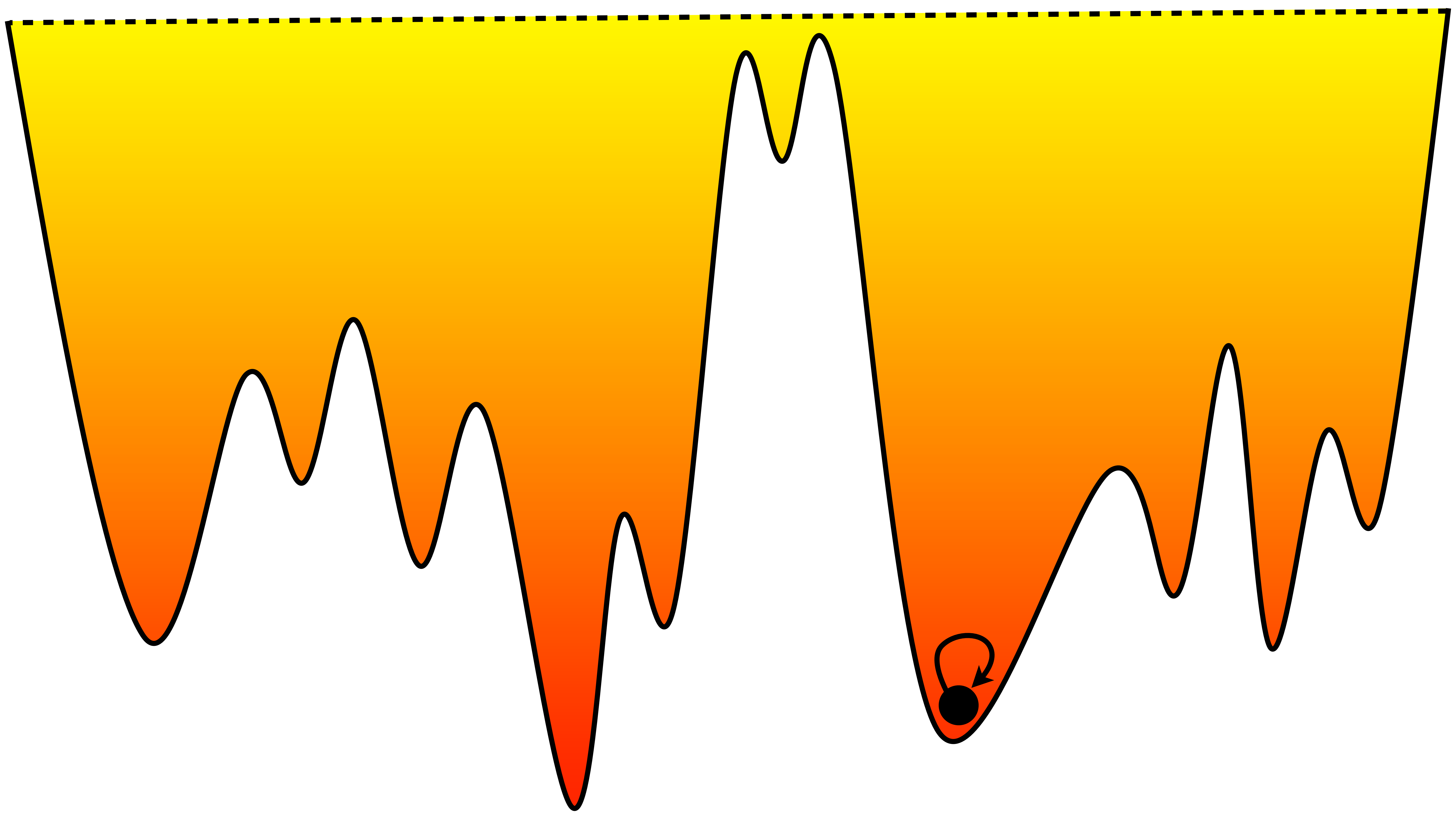

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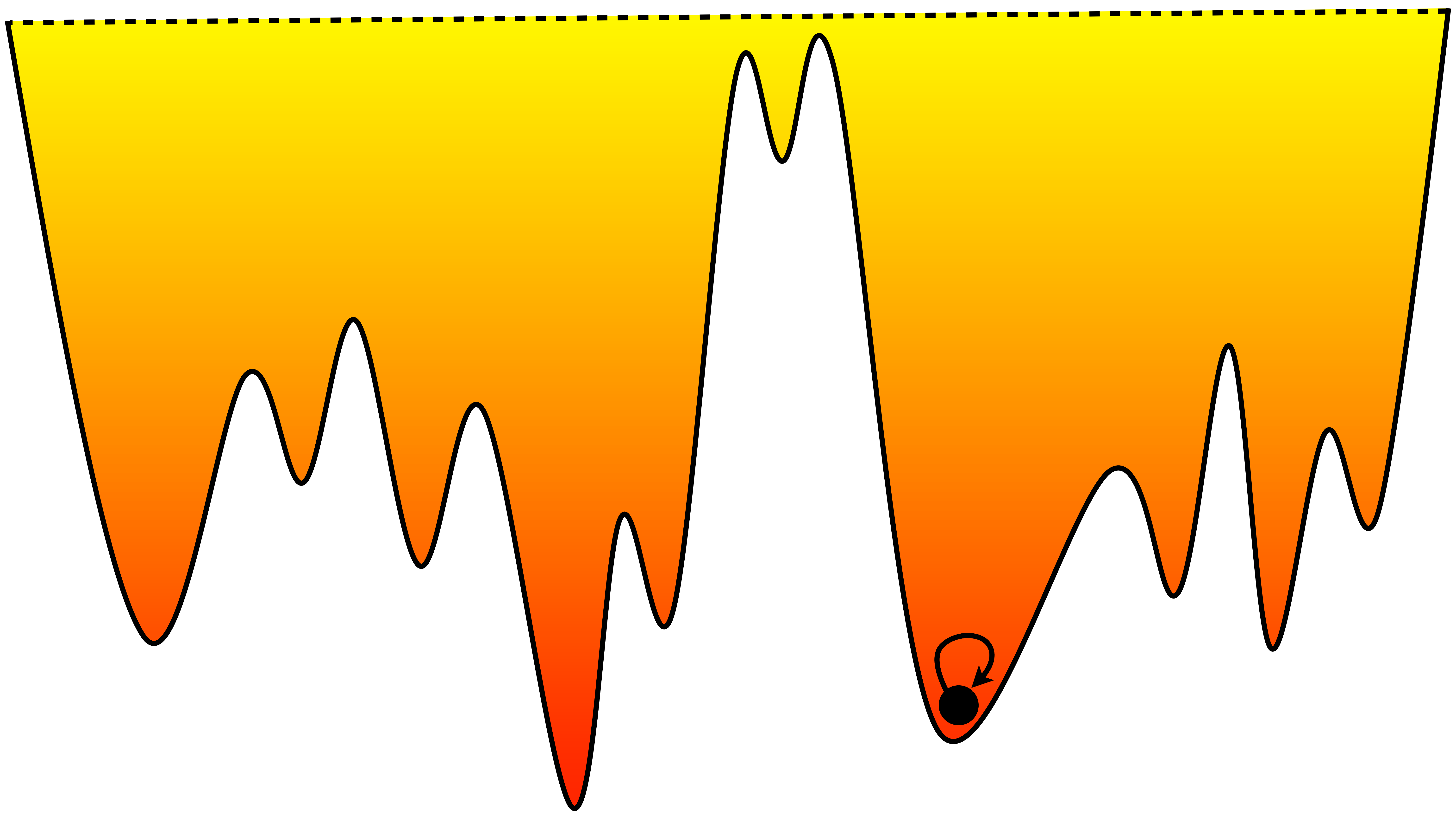
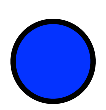


Tabu node  : node I have already visited

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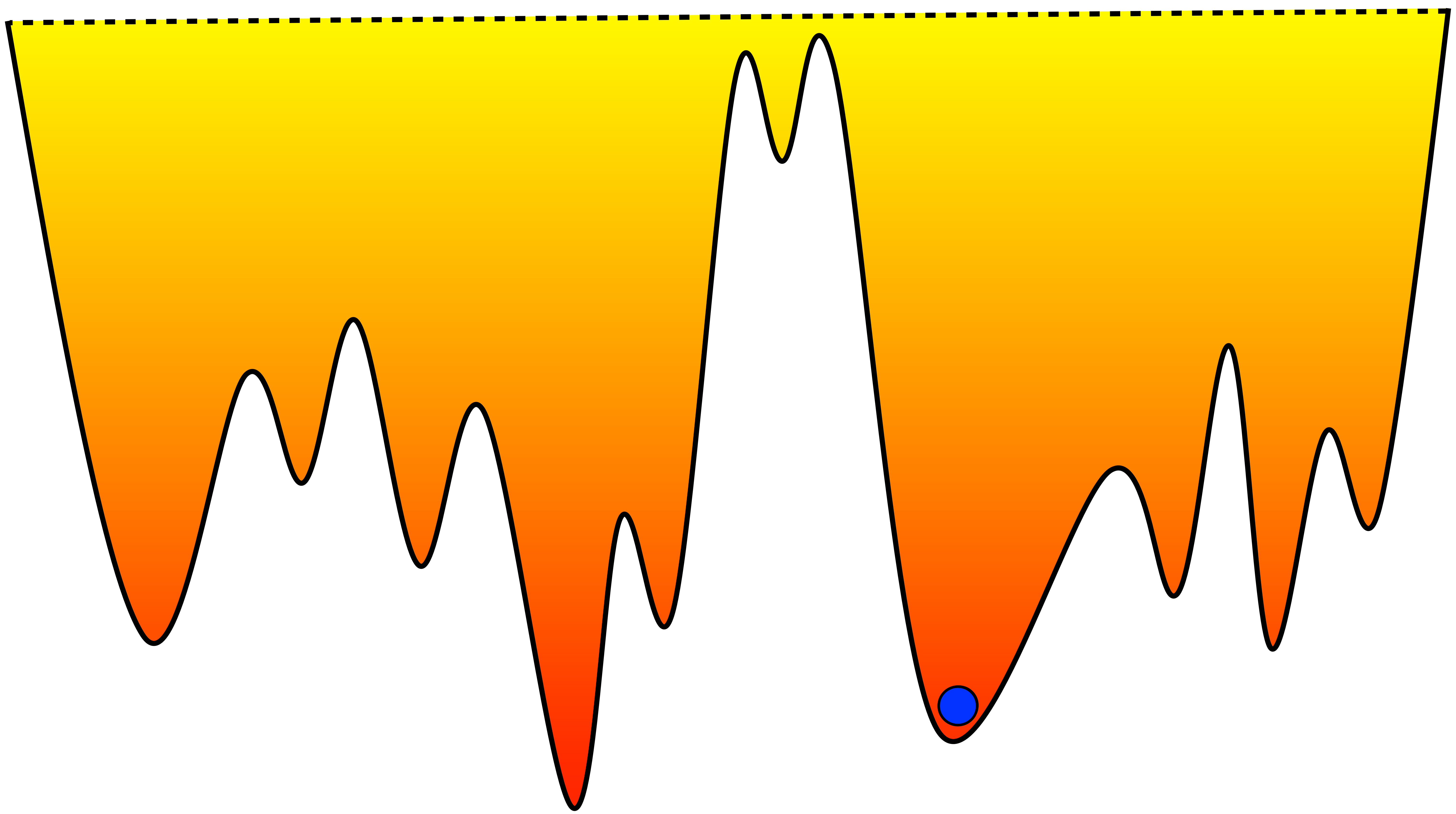

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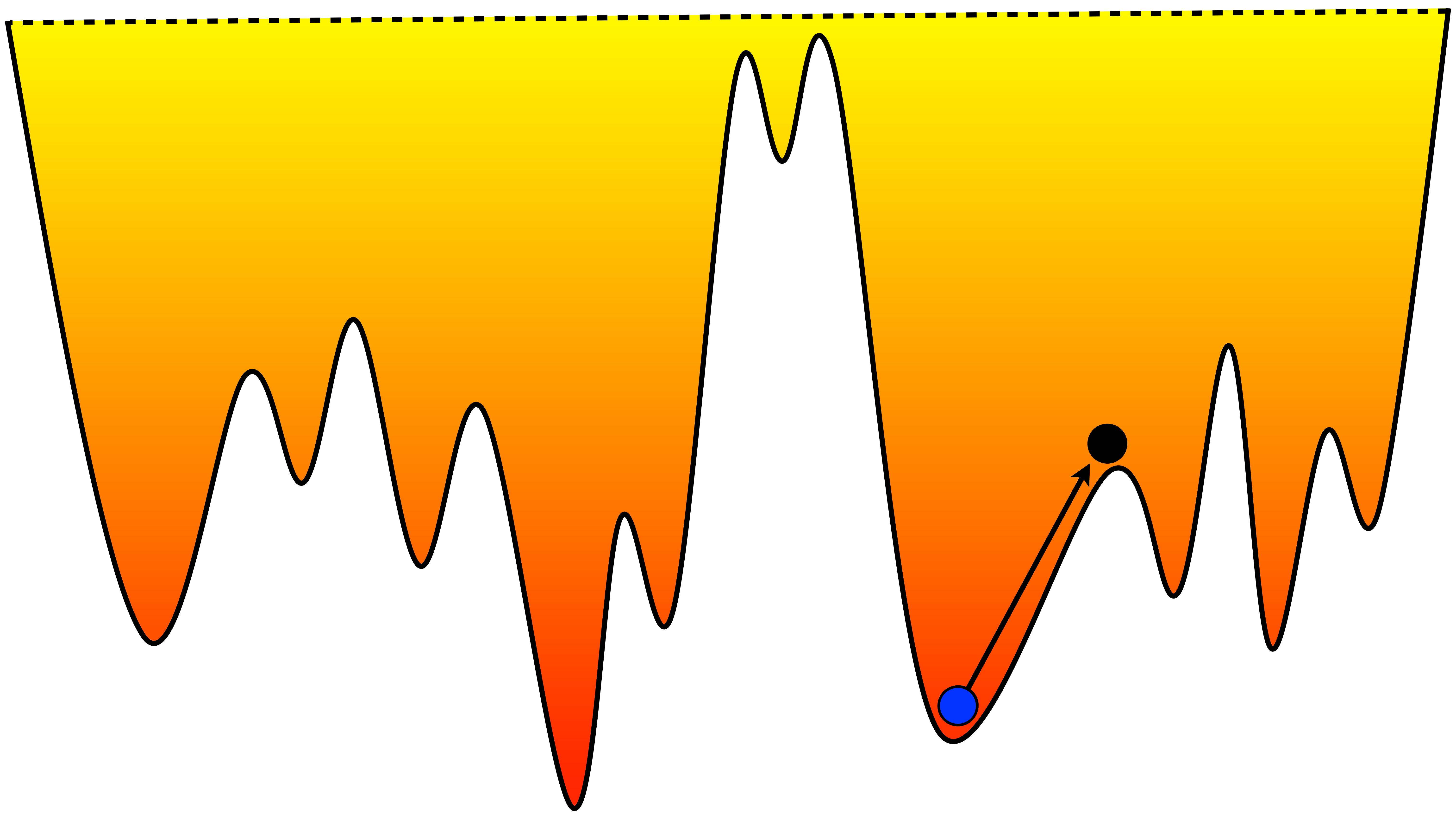

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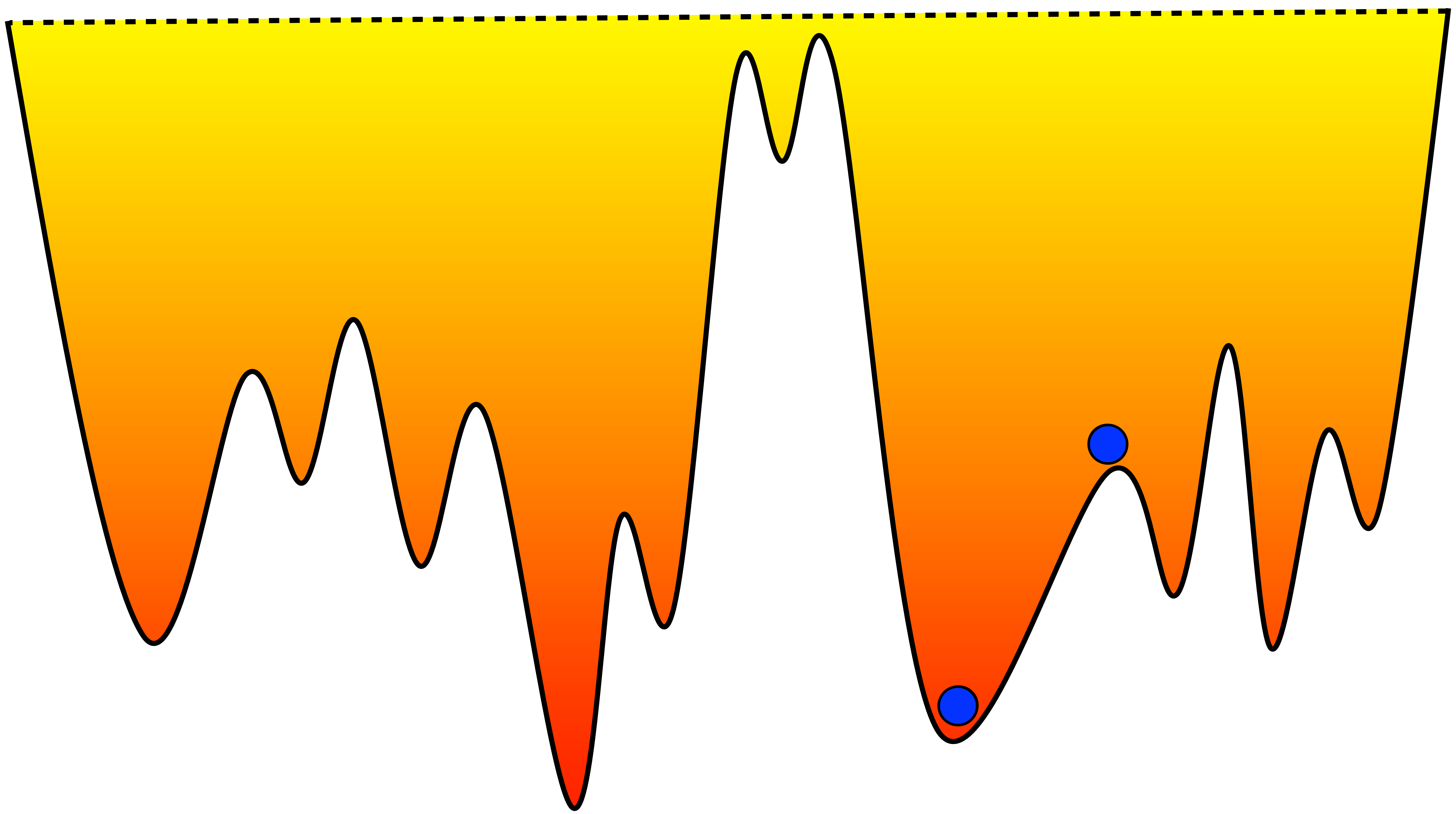

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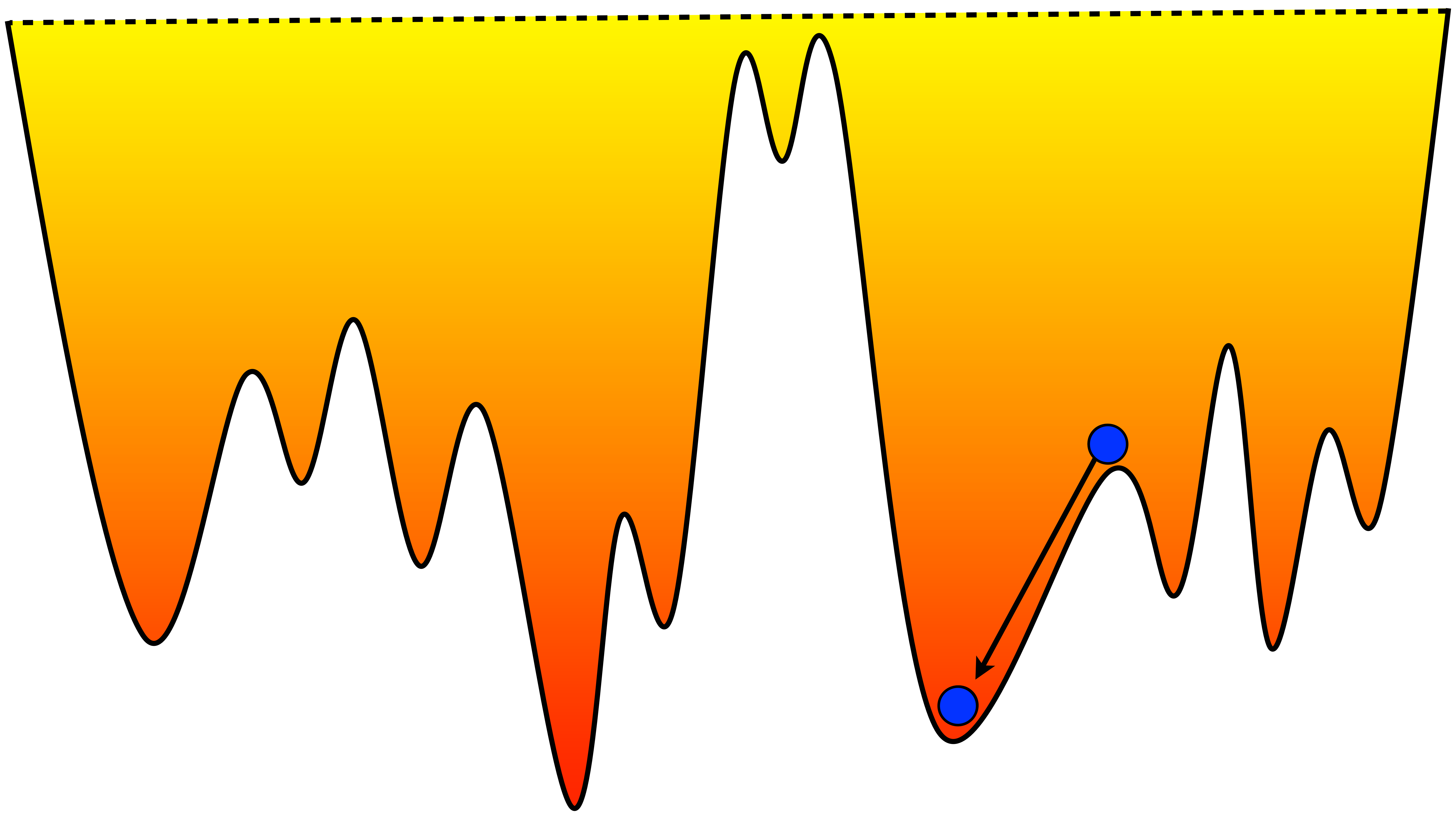
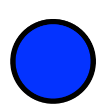


Tabu node  : node I have already visited

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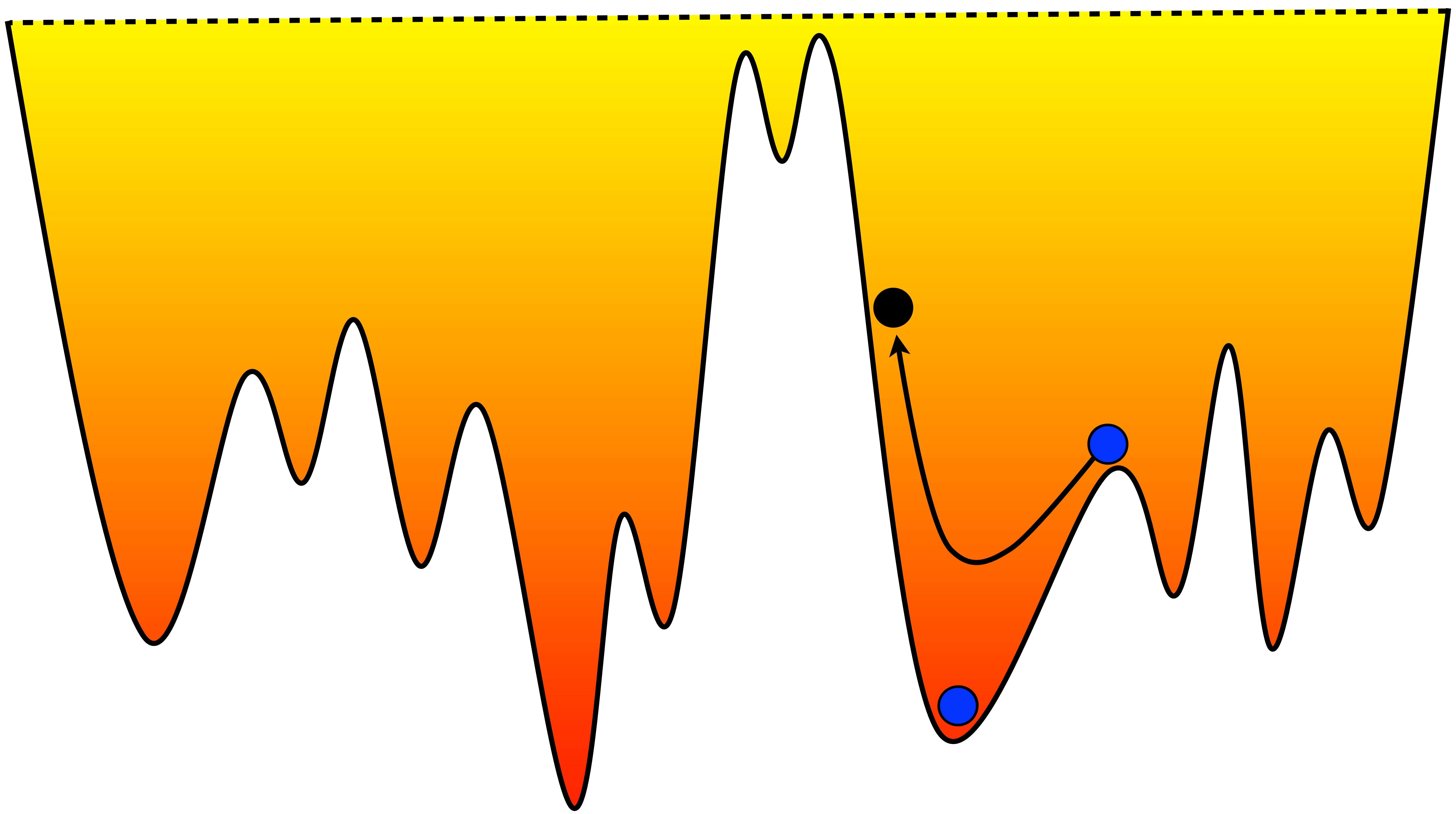

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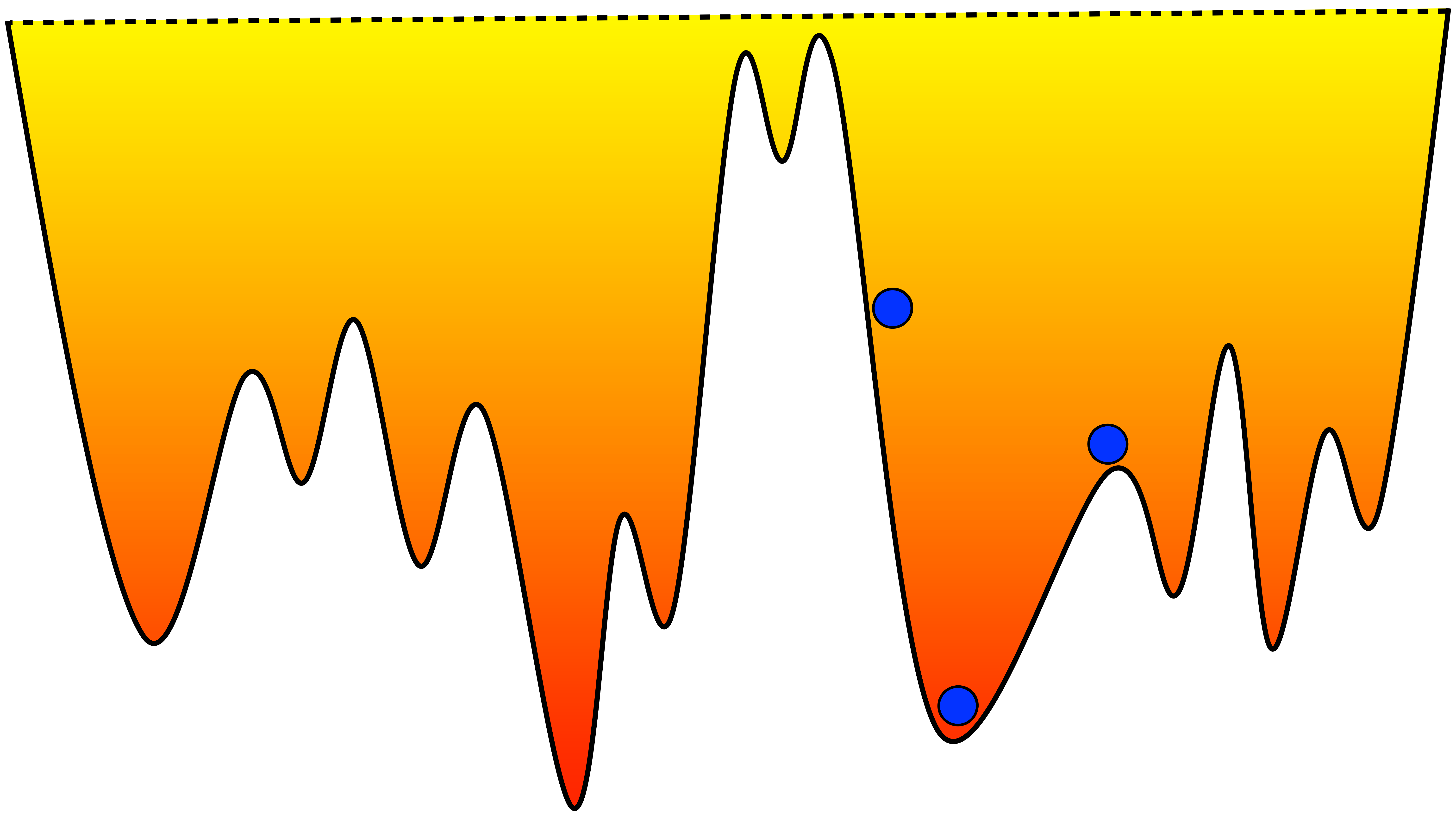

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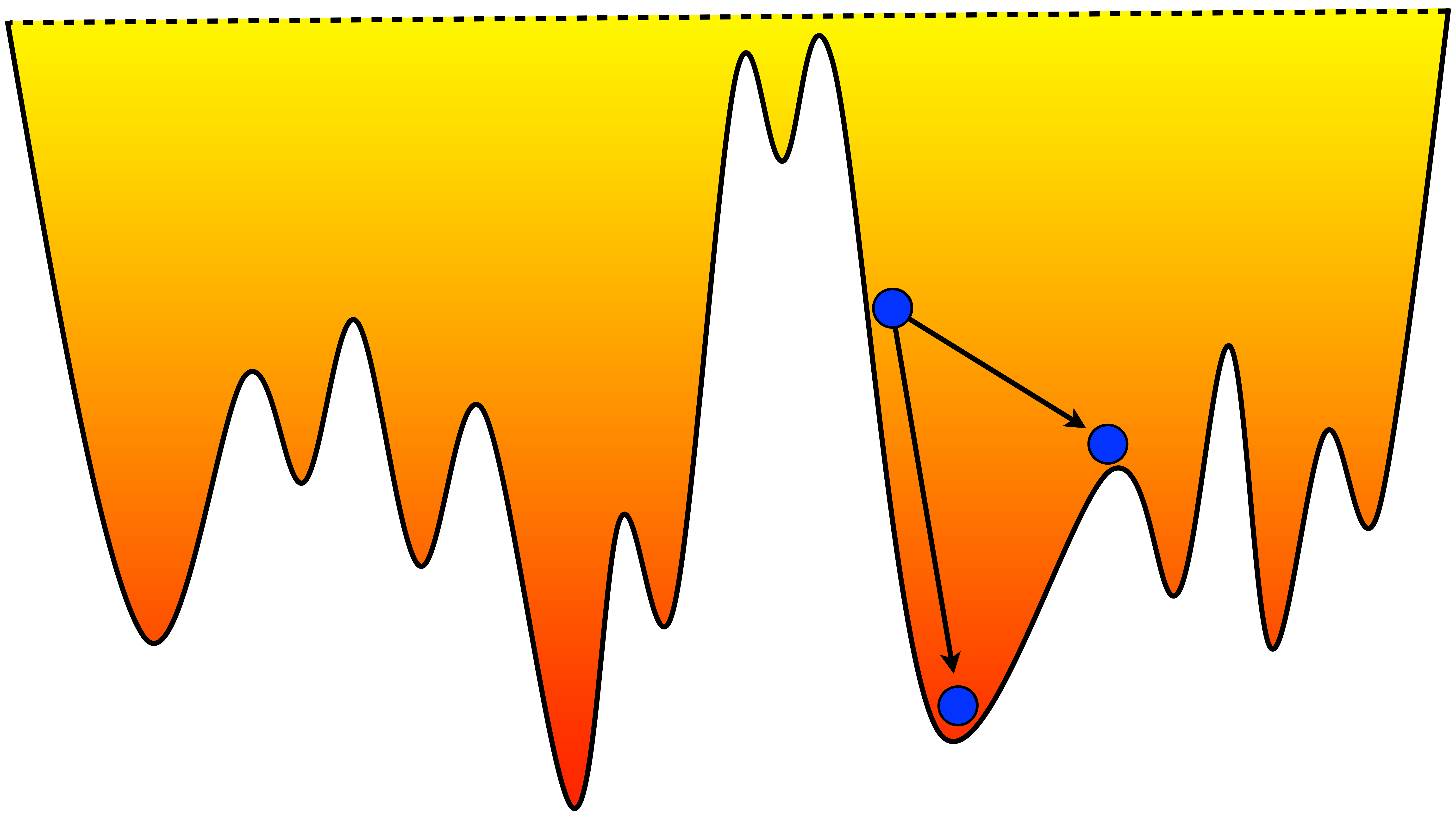

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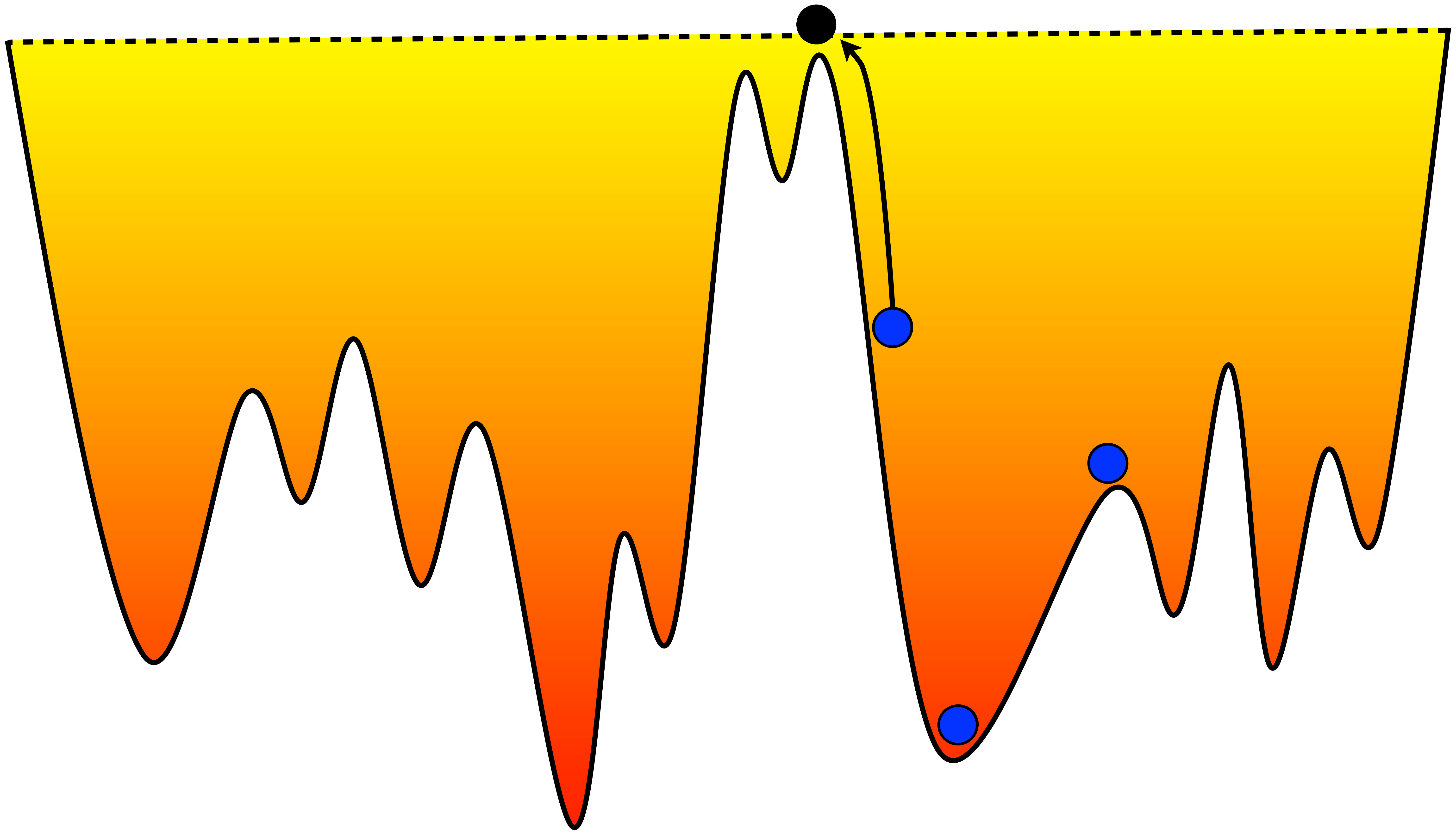

Tabu node ● : node I have already visited

Tabu Search

- Key abstract idea
  - maintain the sequence of nodes already visited
    - tabu list and tabu nodes

function $\textsc{LocalSearch}(f, N, L, S, s_1)$ $\{$

  $s^* := s_1;$

  $\tau := \langle s_1 \rangle;$

  for $k := 1$ **to** $MaxTrials$ **do**

    if $\mathit{satisfiable}(s) \wedge f(s_k) < f(s^*)$ **then**

      $s^* := s_k;$

    $s_{k+1} := S(L(N(s_k), \tau), \tau);$

    $\tau := \tau :: s_{k+1};$

  return $s^*;$

$\}$

Tabu Search

- Basic abstract tabu-search
  - select the best configurations that is not tabu, i.e., has not been visited before

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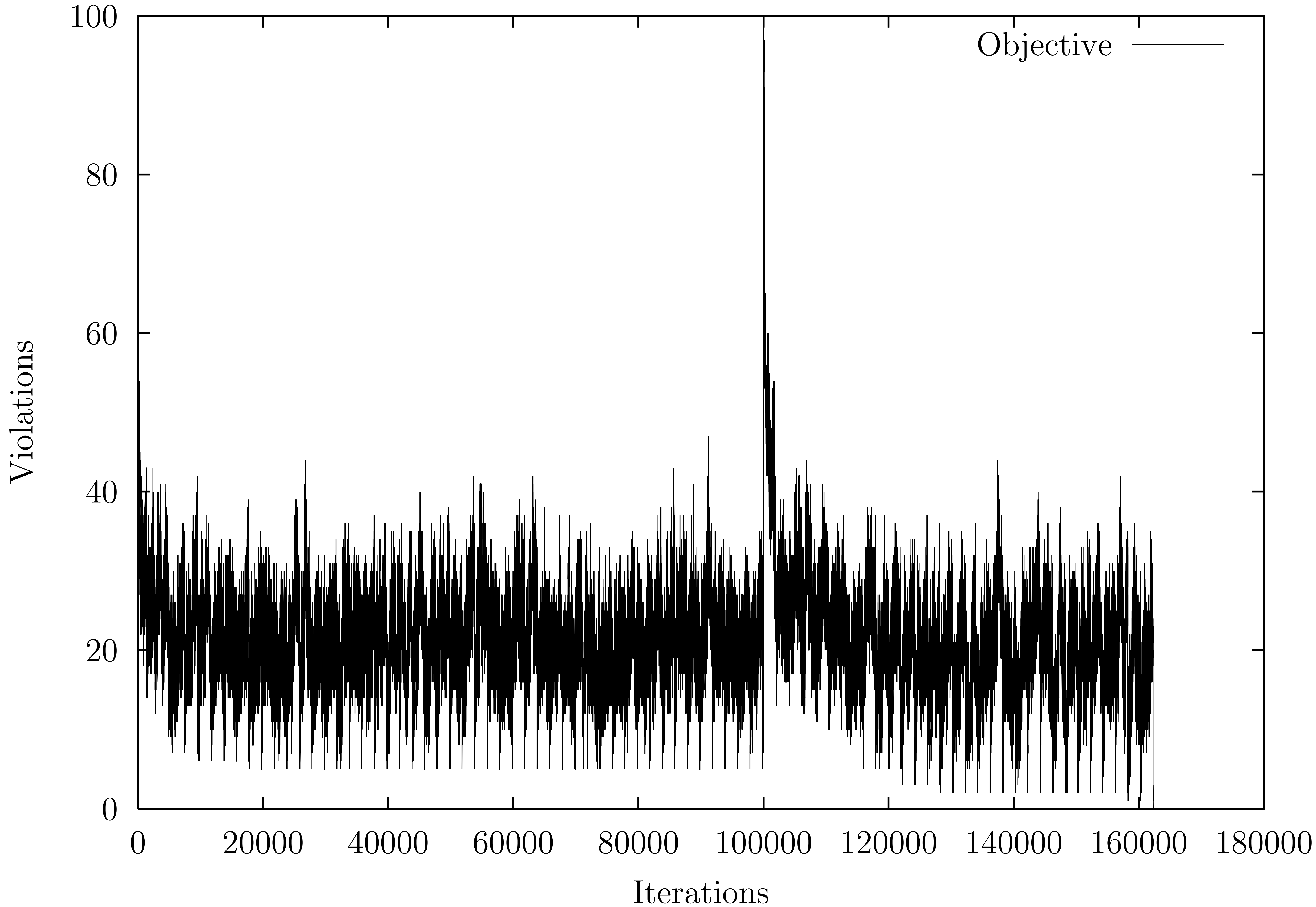
  - select the best configurations that is not tabu, i.e., has not been visited before

1. **function** TabuSearch(f,N,s)
2. **return** LocalSearch(f,N,L-NotTabu,S-Best);

where

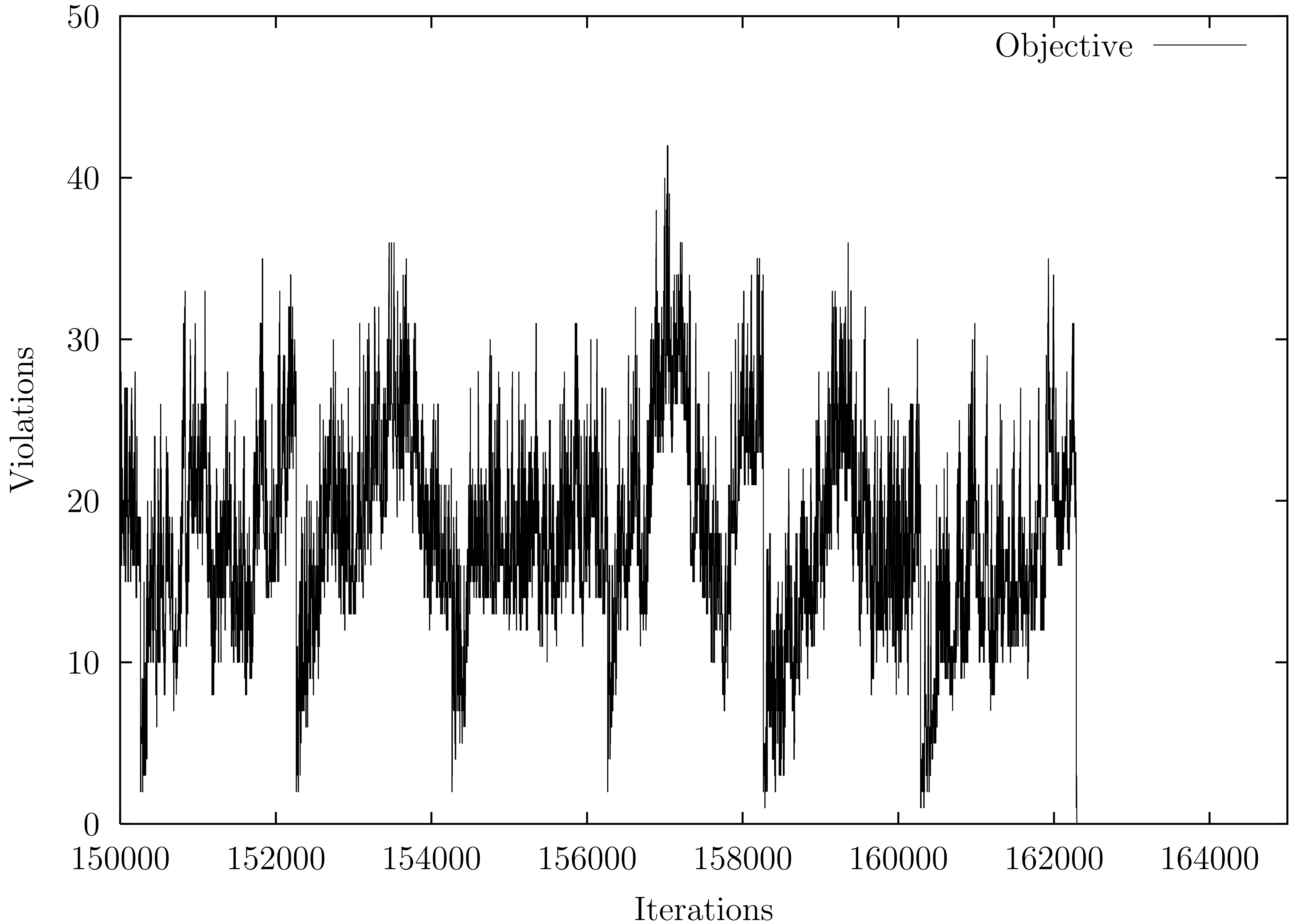
1. **function** L-NotTabu(N,τ)
2. **return** $\{ n \in N \mid n \notin \tau \}$;

Tabu Search

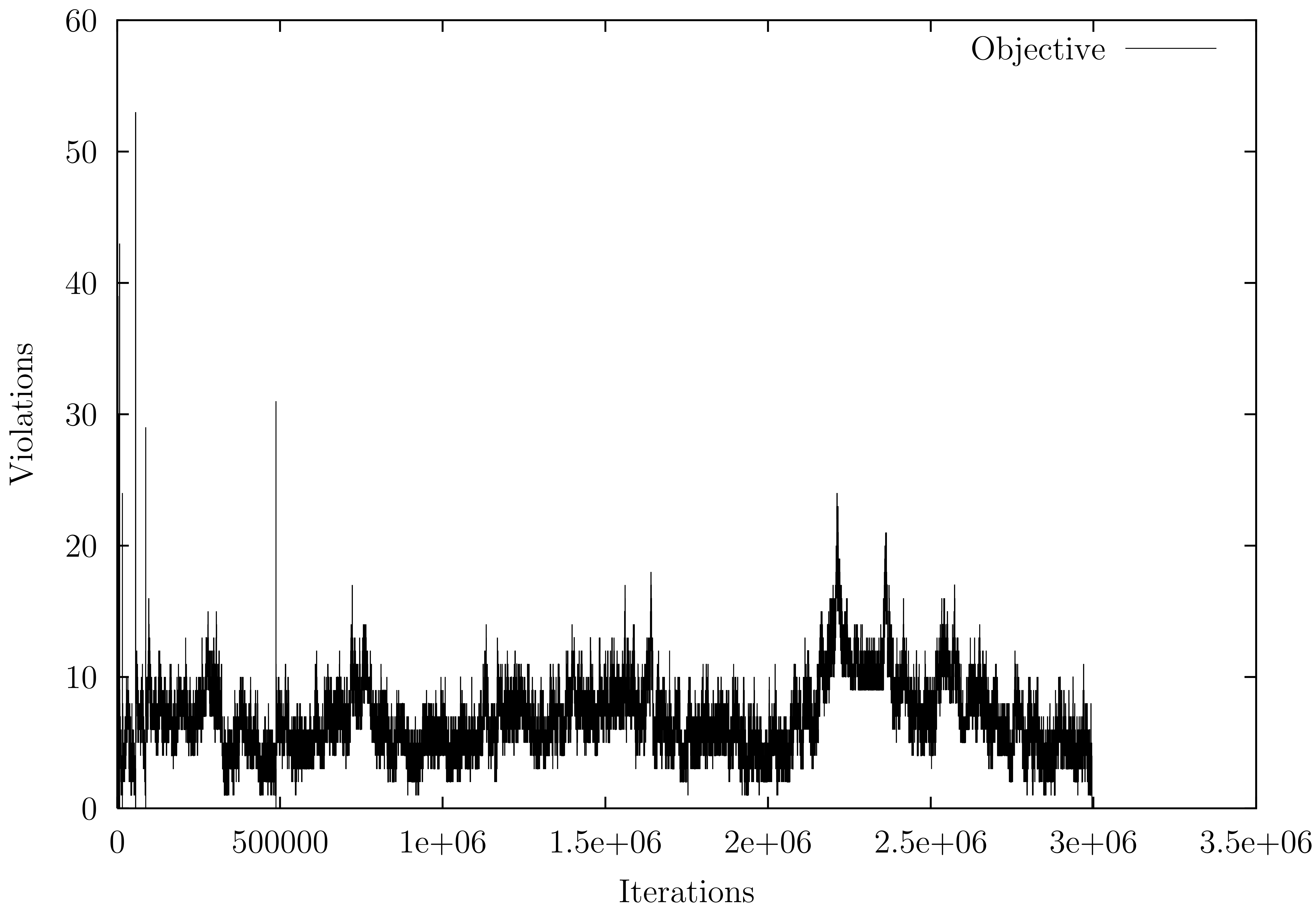


Objective
Violations
Iterations
0
20
40
60
80
100
0
20000
40000
60000
80000
100000
120000
140000
160000
180000

Tabu Search


Objective
Violations
Iterations
0
10
20
30
40
50
150000
152000
154000
156000
158000
160000
162000
164000

Tabu Search



Objective
Violations
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0
10
20
30
40
50
60
0
500000
1e+06
1.5e+06
2e+06
2.5e+06
3e+06
3.5e+06

Metaheuristics

- Many others
  - variable neighborhood search
  - guided local search
  - ant-colony optimization
  - hybrid evolutionary algorithms
  - scatter search
  - ...

Until Next Time