

Introduction to Computer Vision

Feature points

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Applications

- Local features (feature points) are useful for
 - matching
 - tracking of objects
 - robot navigation
 - object recognition
 - pose estimation & camera calibration
 - scene classification
 - texture analysis
 - indexing and image retrieval
 - video mining
 - augmented reality
 - ...



Pair and discuss: how can we estimate the translation?

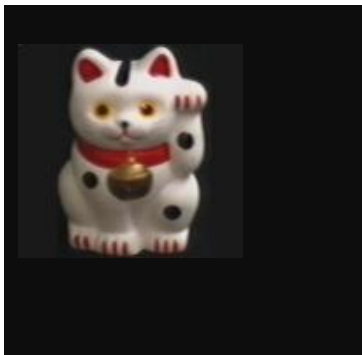


Image A

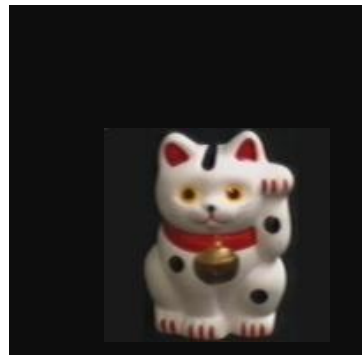
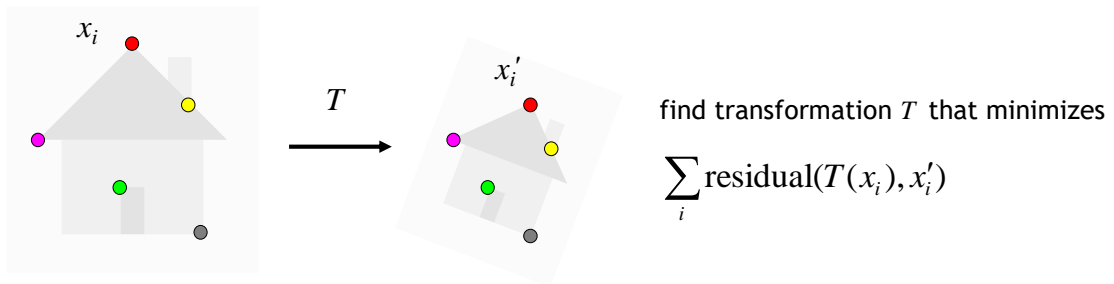


Image B

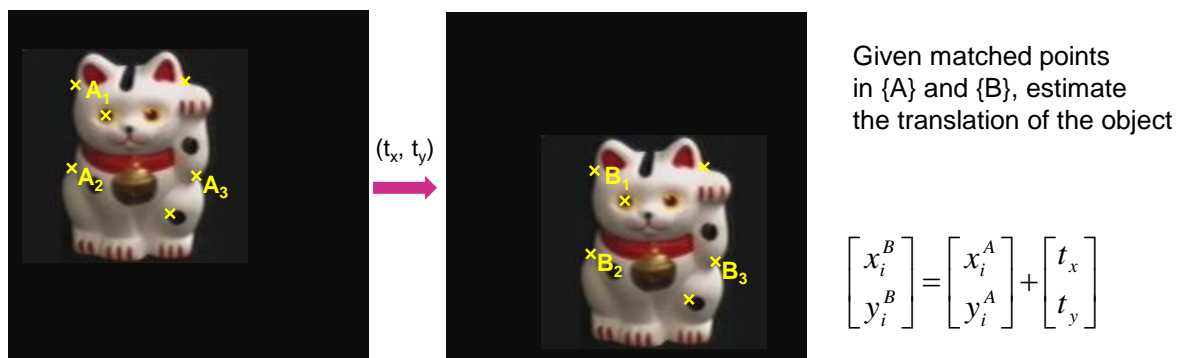
recall: geometric transformations

Alignment

- Find the parameters of the **transformation** that best align matched points [or](#)
- Fit a model to a transformation between pairs of features (*matches*) in 2 images



Example: solving for translation



Least squares solution

Write down objective function

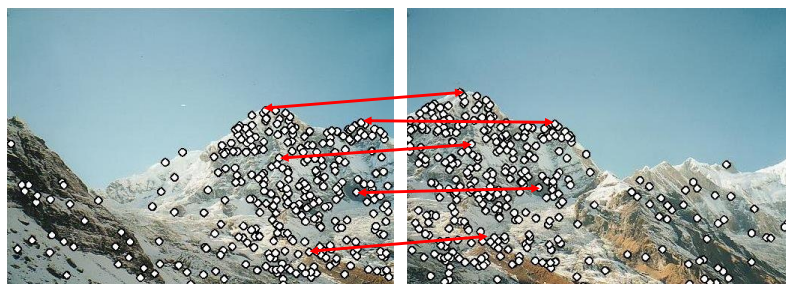
Derived solution: compute derivative \rightarrow compute solution

Computational solution: write in form $Ax=b \rightarrow$ solve using pseudo-inverse $x = A^{-1}b$ or eigenvalue decomposition

Application: panorama stitching



Feature matching



“What stuff in the left image matches with stuff on the right?”

Source: Steve Seitz and Rick Szeliski

Building a panorama

1. Detect feature points in both images
2. Find corresponding pairs
3. Find a parametric transformation (homography)
4. Warp (right image to left image)



$$\forall \text{ matching pair } \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\text{left}} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{\text{right}}$$

Feature matching for panorama stitching

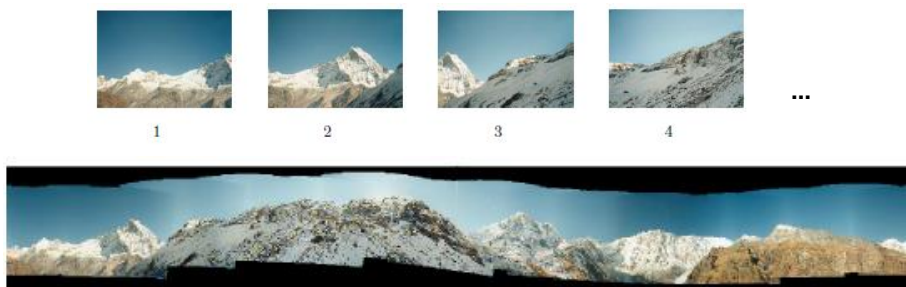


Image matching



“What stuff in the left image matches with stuff on the right?”

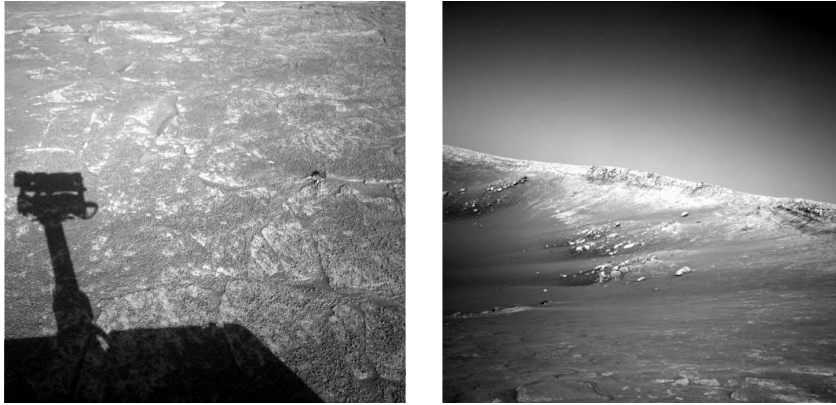
Image matching: harder case



“What stuff in the left image matches with stuff on the right?”

Image matching: even harder case

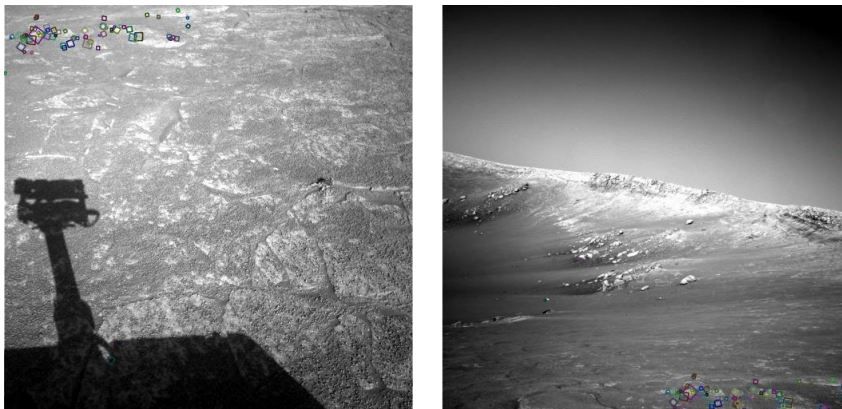
NASA Mars Rover images



“What stuff in the left image matches with stuff on the right?”

Interest point matching

NASA Mars Rover images with SIFT feature matches



Figures by Noah Snavely

Deformations

- Let us compare two images I_1 and I_2
 - I_2 may be a transformed version of I_1
 - What kind of transformations are we likely to encounter?

zoom + rotation

blur

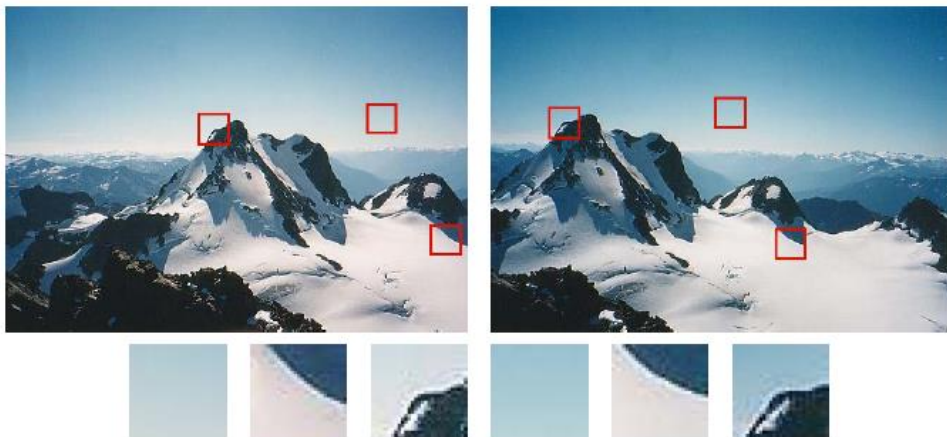
change in view point

change in light



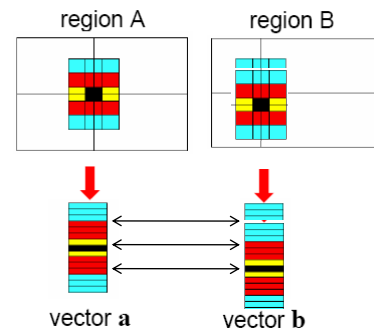
What features to use?

- Two images of the same scene to be aligned by matching features
 - what would be **good features** considering the transformations we are likely to encounter?

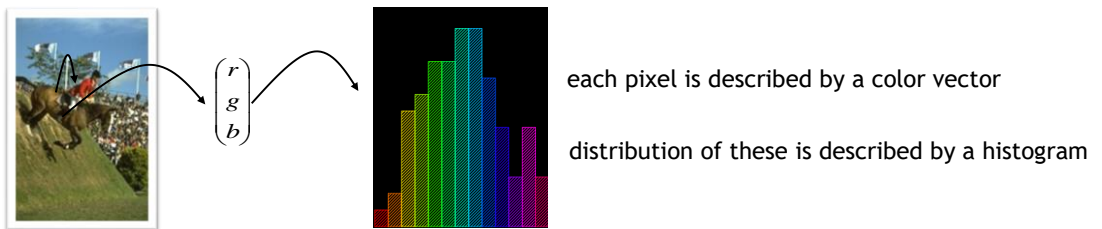


Simplest descriptions of neighbourhood around interest point

- **List of intensities** to form a feature vector
 - very sensitive to even small shifts, rotations



- **Colour histogram**
 - may be not distinctive enough (no spatial information)



Pair and discuss: what makes a feature unique?

- Look for image regions that are unique,
i.e. that lead to unambiguous matches in other images (uniqueness)

Local features

- Image pattern that **differs** from its neighbours in terms of
 - intensity
 - colour
 - texture
- Local features can be
 - points/corners
 - small image patches/regions
- Typically, measurements are taken from a region centred on a local feature and converted into **descriptors**

Local features

- Importance of local features for object recognition by the human visual system
 - removing the **corners** from images impedes human recognition, while
 - removing most of the **straight edge** information does not



Source: T. Tuytelaars & K. Mikolajczyk

Local features: main components

- **Detection**

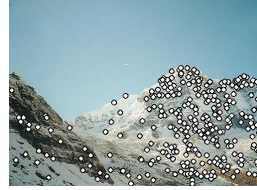
- identify the interest points

- **Description**

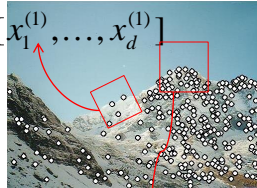
- extract vector feature descriptor surrounding each interest point

- **Matching**

- Determine correspondence between descriptors in two views



$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$



Source: Kristen Grauman

Properties of good features

- **Repeatability**

- given two images of the same object or scene, **taken under different viewing conditions**, a high percentage of the features detected on the scene part visible in both images should be found in both images

- **Distinctiveness**

- the intensity patterns underlying the detected features should show **sufficient variation** so that features can be distinguished and matched

- **Locality**

- the features should be local, so as to reduce the probability of **occlusion** and to allow simple **model approximations** of the geometric and photometric deformations between two images taken under different viewing conditions

Properties of good features

- **Quantity**

- the number of detected features should be sufficiently large such that a reasonable number of features are detected even on small objects

- **Accuracy**

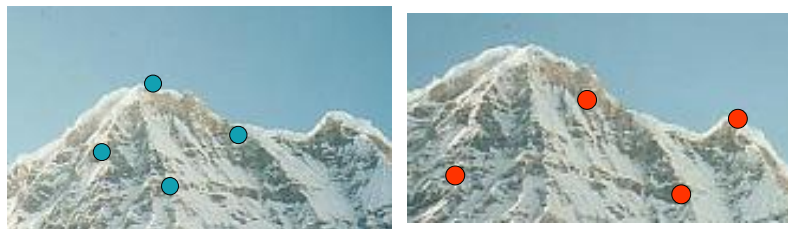
- the detected features should be accurately localized, in **image location** and with respect to **scale**

- **Efficiency**

- the detection of features in a new image should allow for time-critical applications

Interest operator repeatability

- Ability to detect (at least some of) the same points in both images

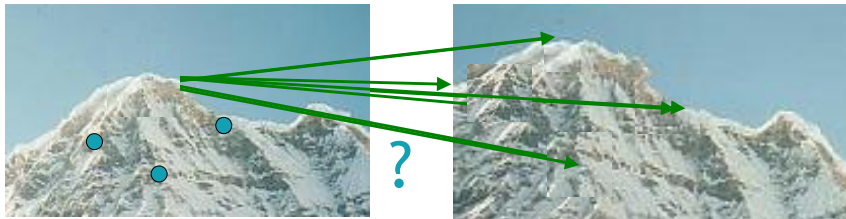


No chance to find true matches!

- Yet ability to run the detection procedure **independently** per image

Descriptor distinctiveness

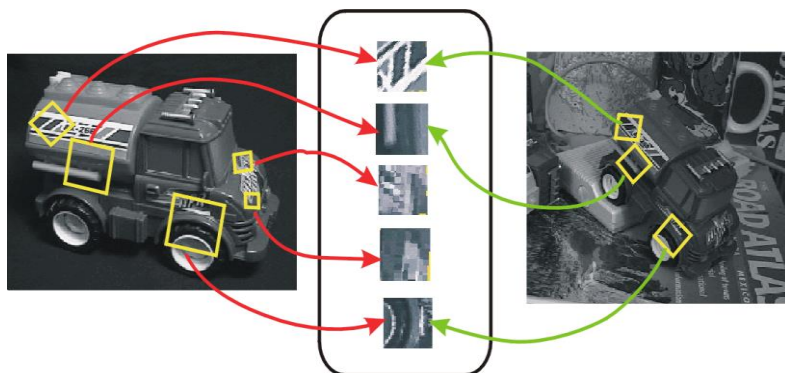
- Ability to reliably determine which point goes with which



- Must provide some **invariance** to geometric and photometric differences between the two views

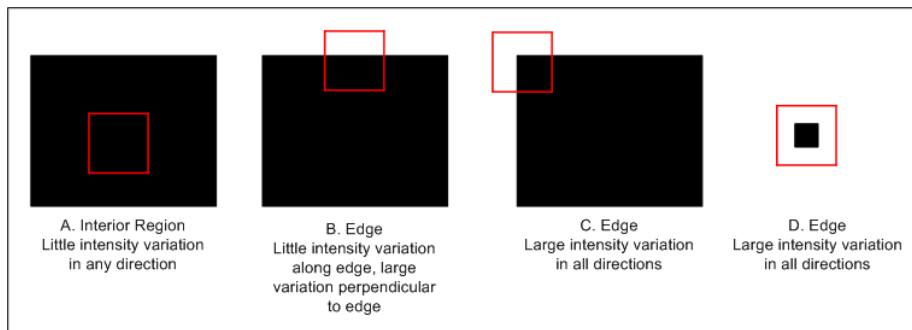
Local invariant features

- Finding points and representing their patches should produce similar results even when conditions vary
 - **photometric** invariance: brightness, exposure, ...
 - **geometric** invariance: translation, rotation, scale, ...



Local measures of uniqueness

- Suppose we only consider a small window of pixels
 - what defines whether a feature is a good or bad candidate?



Corners

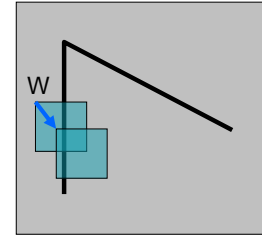
- Corner
 - where **two edges** come together
 - where the image gradient has significant components in the x and y direction
 - can establish corners from the gradient rather than the edge images
 - corresponds to point in both the world and image spaces
- Point features / interest points / corners / feature points
- **Variance**-based interest operators
 - to select image points
 - interest (corner) points
 - measure: usually the variance of the intensity values on a small window
 - the simplest is **Moravec**

Variance-based interest point detection

- When shifting the window W by (u,v)
 - how do the pixels in W change?
 - compare each pixel **before and after the shift** by **summing the squared differences (SSD)**

$$E(u, v) = \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2$$

↑ shifted intensity ↑ intensity



Moravec interest point detector

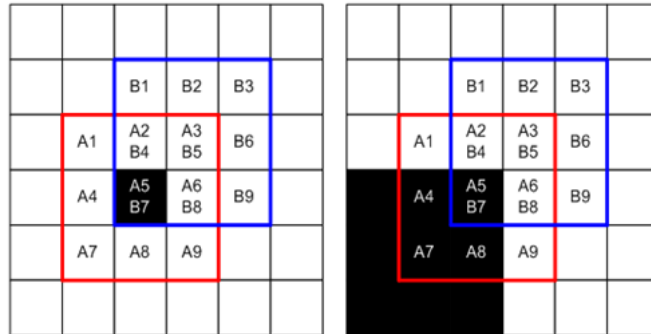
- Defines interest points as
 - points where there is a **large intensity variation in every direction**
- Measures the intensity variation by
 - placing a small square window (e.g. 3x3, 5x5, 7x7) centred at a given point
 - shifting this window by one pixel in each of the eight principle directions
 - horizontally, vertically, and four diagonals
 - intensity variation for a given shift is calculated by taking the **sum of squares of intensity differences** of corresponding pixels in these two windows

Moravec detector: example

- Calculate intensity variation for 3x3 window in the **upper right diagonal direction**
 - for an isolated black pixel on a white background
 - and on an ideal corner

$$E = \sum_{i=1}^9 (A_i - B_i)^2 = 2 \times 255^2$$

$$E = \sum_{i=1}^9 (A_i - B_i)^2 = 3 \times 255^2$$



Moravec interest point detector

- Moravec operator
 - gives a measure of **cornerness** to each pixel in the image
 - this measure is the **minimum intensity value** found over the eight shift directions
- Moravec operator applied to each pixel in an image creates a **cornerness map**
 - corners are the **local maxima** in the cornerness map

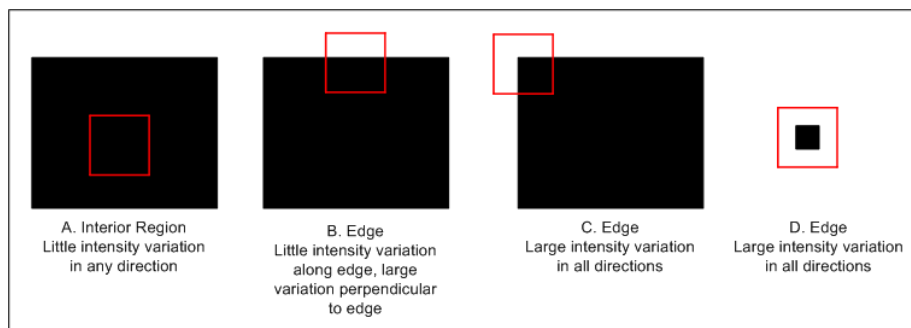
| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| x | x | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | x | x | x |
| x | x | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 2 | 1 | x | x | x |
| x | x | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 1 | 1 | x | x | x |
| x | x | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | x | x | x |
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |
| x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |

cornerness map for an image when a 3x3 window is used

Moravec interest point detector

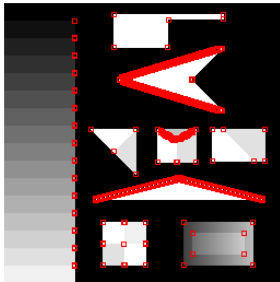
- Computationally efficient
- Sensitive to noise
 - many local maxima do not correspond to corners
 - isolated pixels can be detected as corners
- Using a larger window size
 - makes the detector more robust
 - true corner: larger intensity variation than the isolated pixel
 - setting all cornerness values below a threshold to zero helps
 - choosing this threshold is difficult
 - must be set high 'enough' to avoid these false corners
 - but low 'enough' to retain as many true corners as possible

Moravec interest point detector

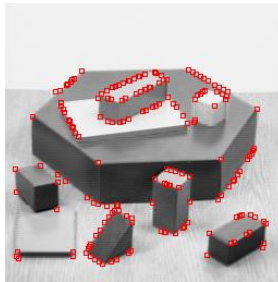


Four types of window positions that were considered by the Moravec operator

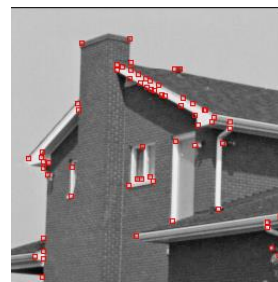
Moravec operator with a 3x3 window: examples



threshold near zero



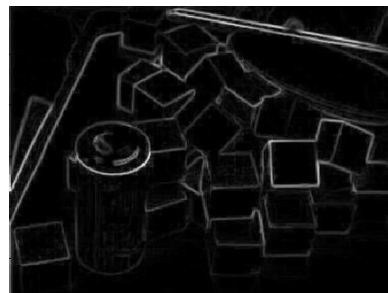
threshold chosen to detect most of the corners while minimizing the number of false corners detected



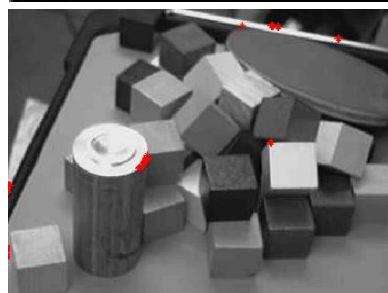
threshold chosen high to avoid detecting corners due to the texture of the house

Examples: detected interest points with different thresholds

original image



Moravec operator applied



Example: high & low threshold



original



Moravec interest points
for a low threshold

Example: high & low threshold



original



Moravec interest points
for a high threshold

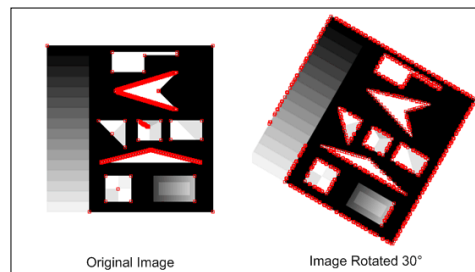
Moravec interest point detector: limitations

- **Noisy** response

- estimation of the local intensity variation can be improved with a **Gaussian window**, i.e. by placing **more weight** on the intensity differences of pixel pairs near the **centre** of the window (the difference of pixel pairs near the centre is a better indication of the local variance)

- **Poor repeatability** due to the anisotropic response of operator

- intensity variation calculated only at a discrete set of shifts in the eight principle directions
- the operator is **not rotationally invariant**



Variance-based interest point detection

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Diagram illustrating the components of the variance-based interest point detection formula:

- $w(x, y)$: window function
- $I(x + u, y + v)$: shifted intensity
- $I(x, y)$: intensity

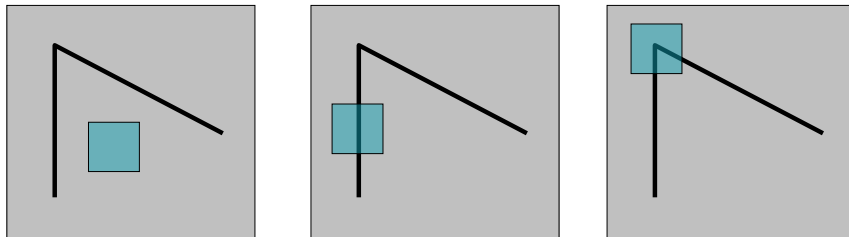
Below the formula, two possible window functions are shown:

- $w(x, y) =$ 1 in window, 0 outside (represented by a red rectangle)
- or Gaussian (represented by a red bell curve)

Source: R. Szeliski

What defines whether a feature is a good or bad candidate?

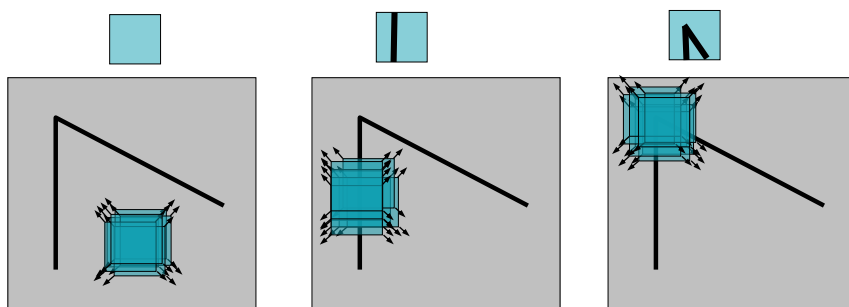
- Let us only consider a small window



Source: Darya Frolova, Denis Simakov, Weizmann Institute.

Local measures of uniqueness

- How does the window change when it is shifted?
- Shifting the window in **any direction** causes a **big change**



flat region:
no change in any directions

edge:
no change along the
edge direction

corner:
significant change in all directions

Source: Darya Frolova, Denis Simakov, Weizmann Institute.

Interest point detection using derivatives

- Taking the sum of differences between corresponding pixels in the two windows is an approximation of the **image gradient**
 - For an arbitrary shift (u,v) we can state the intensity variation as:

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

- Taylor series expansion of I :

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

- If the motion (u,v) is small, then **first-order approximation** is good

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

Interest point detection using derivatives

- Plugging this into the original formula:

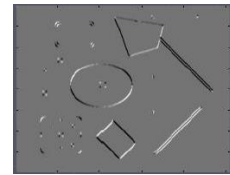
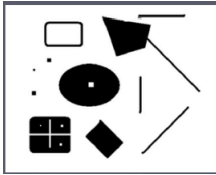
$$\begin{aligned} E(u, v) &= \sum_{(x,y) \in W} [I(x+u, y+v) - I(x, y)]^2 \\ &\approx \sum_{(x,y) \in W} [I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y)]^2 \\ &\approx \sum_{(x,y) \in W} \left[[I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2 \end{aligned}$$

- This can be re-written as:

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

Interest point detection using derivatives

- M is 2×2 matrix of **image derivatives**
 - averaged in neighborhood of a point
 - depends on image properties



$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$

$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$

$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$

Interest point detection using derivatives

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_M \begin{bmatrix} u \\ v \end{bmatrix}$$

- M contains all the differential operators describing the geometry of the image surface at a given point (x, y)
- We want E to be large for small shifts in all directions
- **Eigenvalues** and **eigenvectors** of M
 - define shifts with the smallest and largest change (E value)
 - will be proportional to the principle curvatures of the image surface and form a rotationally invariant description of M

Quick eigenvalue/eigenvector review

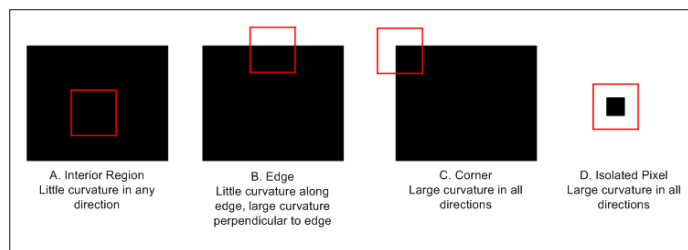
The **eigenvectors** of a matrix A are the vectors x that satisfy: $Ax = \lambda x$

The scalar λ is the **eigenvalue** corresponding to x

- the eigenvalues are found by solving: $\det(A - \lambda I) = 0$
- in our case, $A = M$ is a 2x2 matrix, so $\det \begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix} = 0$
- the solution: $\lambda_{\pm} = \frac{1}{2} \left[(m_{11} + m_{22}) \pm \sqrt{4m_{12}m_{21} + (m_{11} - m_{22})^2} \right]$

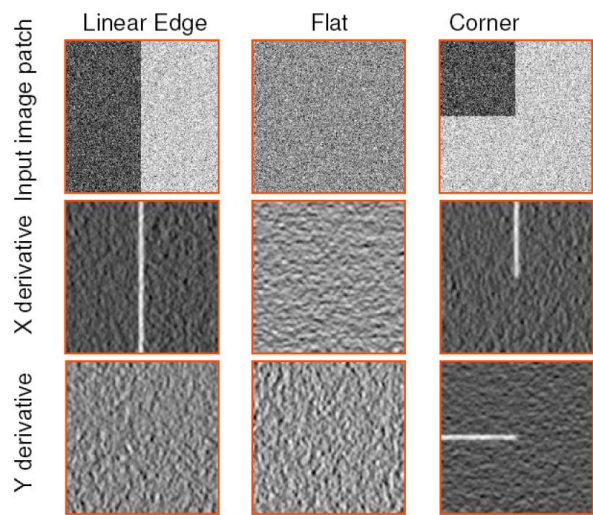
Once you know λ , find x by solving $\det \begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Interpreting the eigenvalues

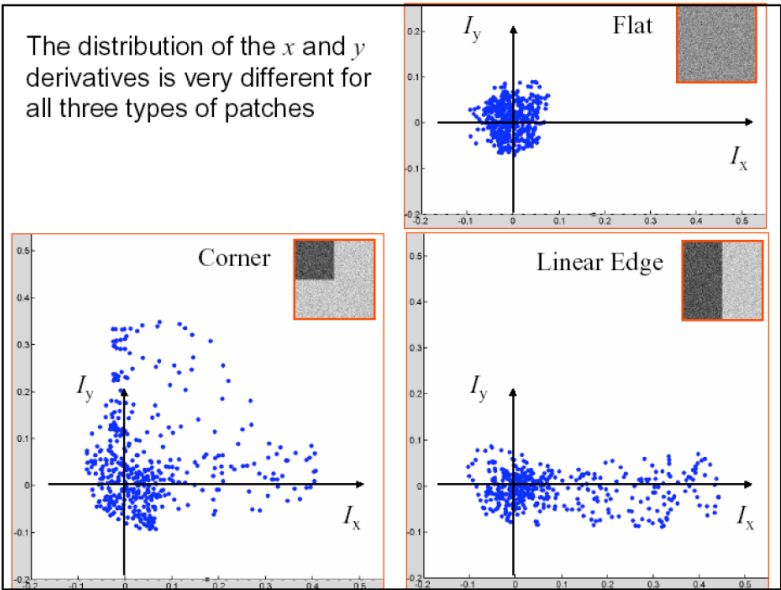


- In position A
 - image intensity will be relatively constant within the window
 - little curvature in the surface within this window
 - **both eigenvalues will be relatively small**
- In position B
 - significant curvature perpendicular to the edge and little curvature along the edge
 - **one of the eigenvalues will be large and the other small**
- Both positions C & D have significant curvature in both directions
 - **both eigenvalues will be large**

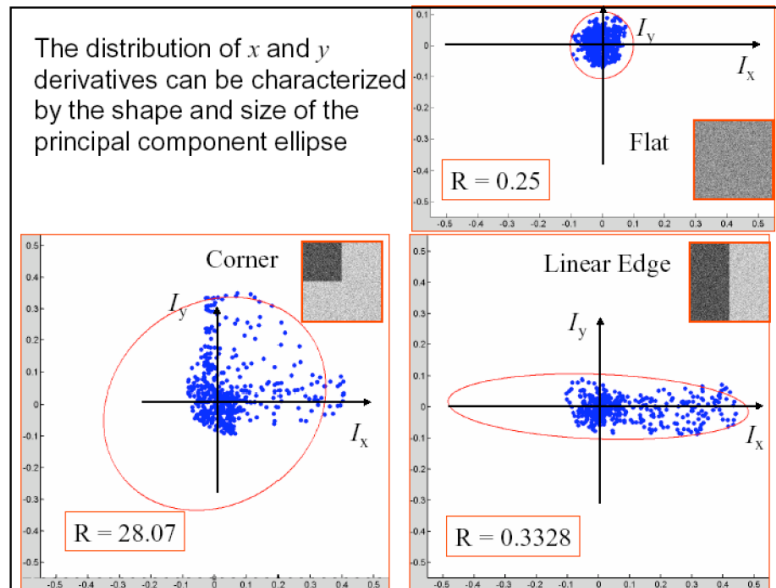
Interpreting the image derivatives



Interpreting the image derivatives

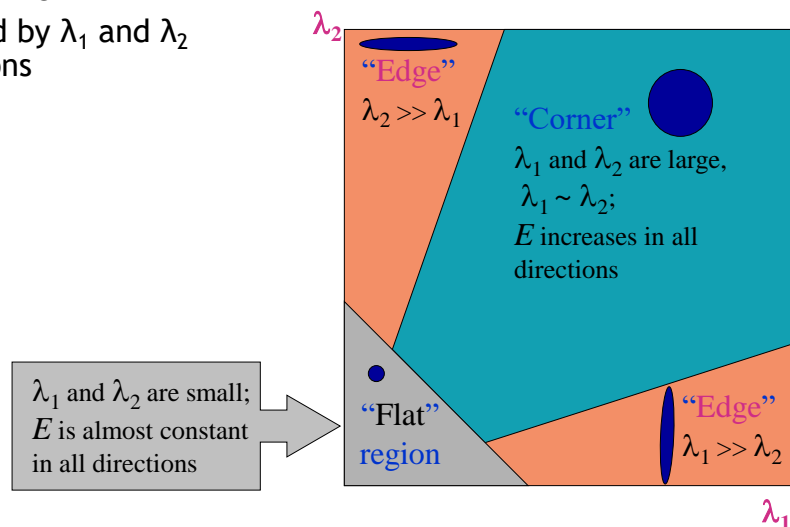


Interpreting the image derivatives



Interpreting the eigenvalues

- Let λ_1 and λ_2 be the eigenvalues of M
- The plane described by λ_1 and λ_2 is divided into regions



Harris corner detector

- Cornerness of a point is calculated as

$$R(x_0, y_0) = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

α scale constant (0.04 to 0.06)

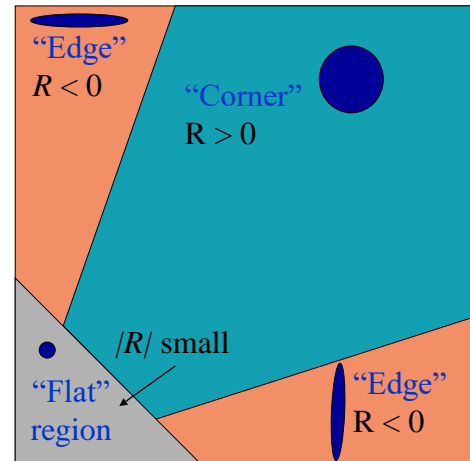
- Computed using two approximations

$$R(x_0, y_0) = \det(M) - \alpha \text{trace}(M)^2$$

M : 2x2 matrix computed from image derivatives

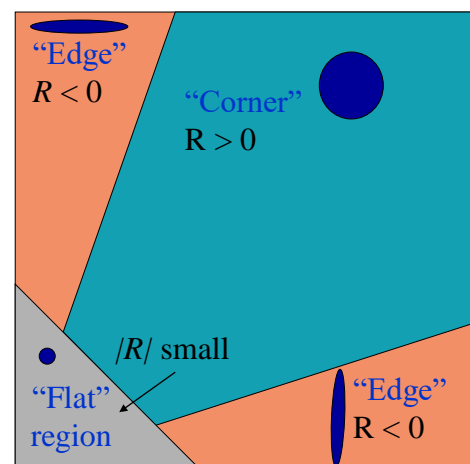
$$M(x_0, y_0) = \sum_{x, y \in \text{neighborhood}(x_0, y_0)} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

weight function



Harris corner detector

- R depends only on eigenvalues of M
- **Corner**: R is large
- **Edge**: R is negative, with large magnitude
- **Flat** region: $|R|$ is small



Harris corner detector

- Based on the **second moment matrix**
 - used for describing local image structures
 - the matrix describes the **gradient distribution** in a local neighbourhood of a point
 - the **eigenvalues** of this matrix represent the principal signal changes in two orthogonal directions in a neighbourhood around the point
- Corners
 - locations in the image for which the image signal **varies significantly in both directions** (i.e. for which **both eigenvalues are large**)
- **Cornerness measure**
 - combines the two eigenvalues in a single measure
 - is computationally less expensive

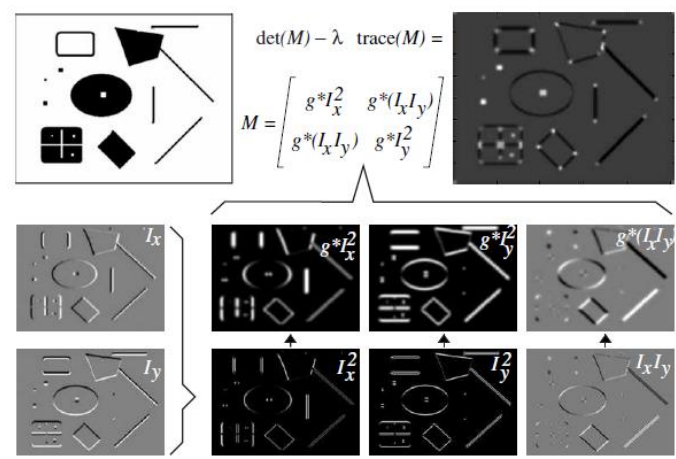
Harris corner detector

$$R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

α : (0.04 to 0.06)

- Since the **determinant** of a matrix is equal to the **product** of its eigenvalues and the **trace** corresponds to the **sum**
 - high values of the cornerness measure correspond to both eigenvalues being large
 - adding the second term with the trace reduces the response of the operator on strong straight contours
 - computing this measure based on the determinant and the trace is computationally less demanding than computing the eigenvalues
- **Thresholding** is still required to distinguish between corners and other points

Harris detector: example

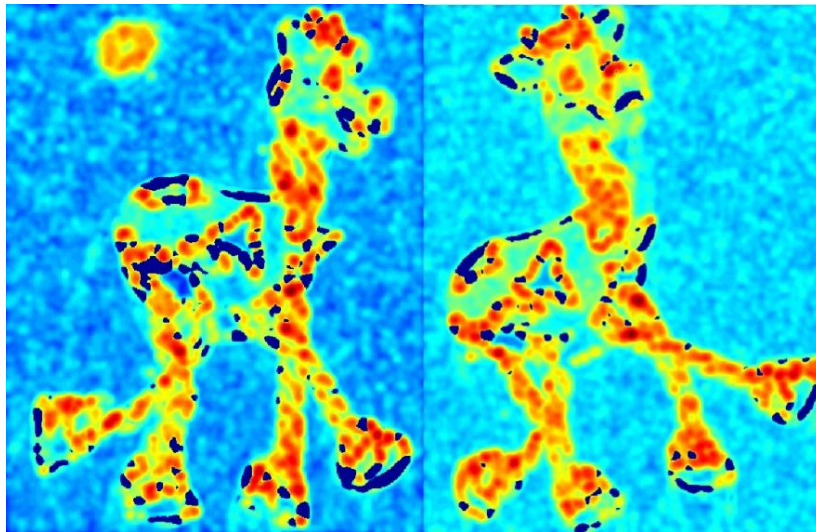


Components of the second moment matrix and Harris cornerness measure

Harris corner detector: example

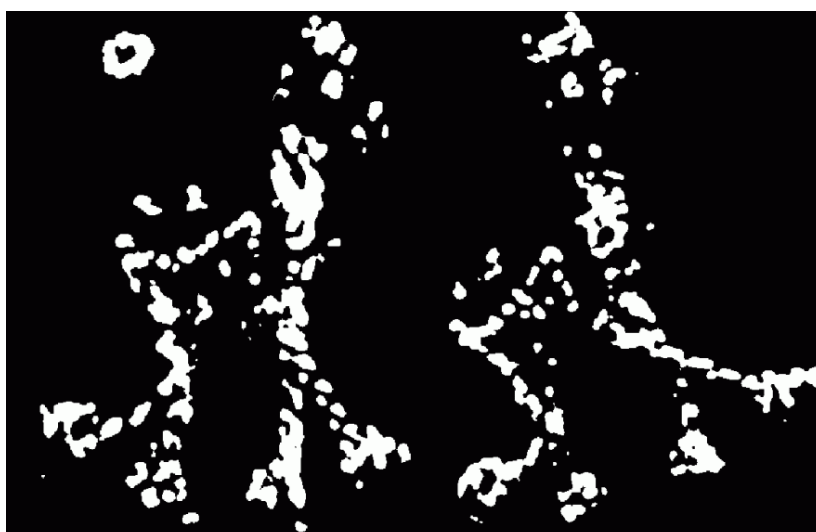


Harris corner detector: example



computed corner response R

Harris corner detector: example



points with large corner response: $R > \text{threshold}$

Harris corner detector: example



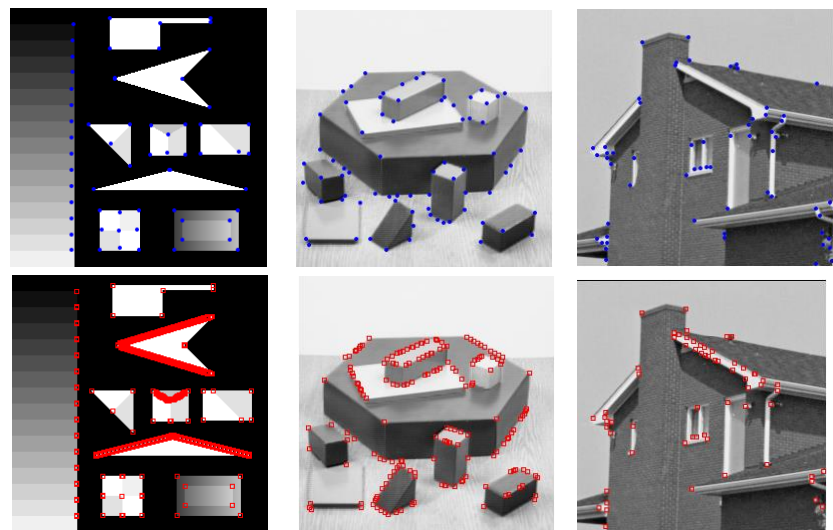
only the points of local maxima of R

Harris corner detector: example

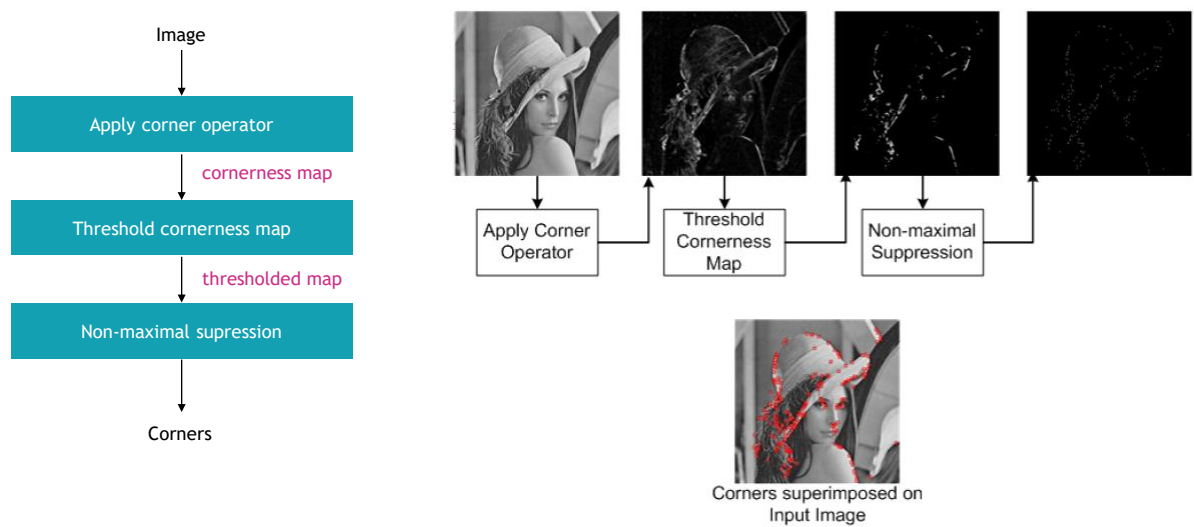


detected corners

Harris vs. Moravec



Corner detectors: summary

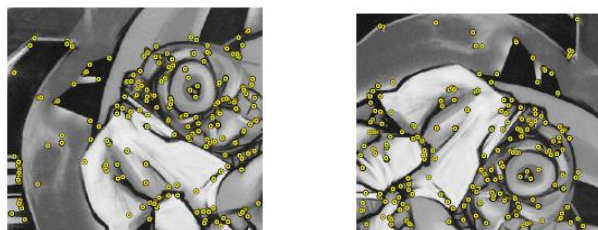


Corner detection: invariance

- Will we still pick up the same features if we **change the brightness** of the image / **rotate** the image by some angle / **scale** the image?
- We want corner locations to be **invariant** to photometric transformations and **covariant** to geometric transformations
 - **invariance**: image is transformed and corner locations do not change
 - **covariance**: if we have two transformed versions of the same image, features should be detected in corresponding locations



Harris corner detection: invariance

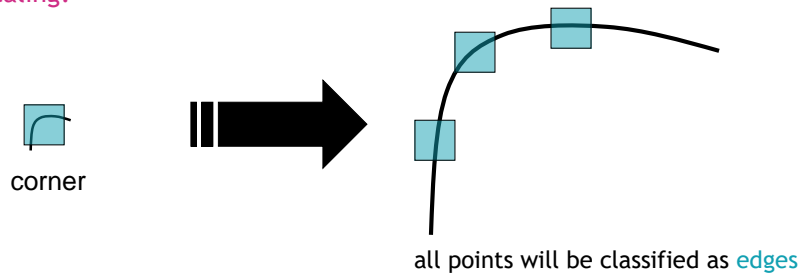


Many of the features detected in the original image (left) have also been found in the rotated version (right)

- The **repeatability** of the Harris detector under **rotations** is high
- Features are typically found at locations which are informative (i.e. with a high variability in the intensity pattern)

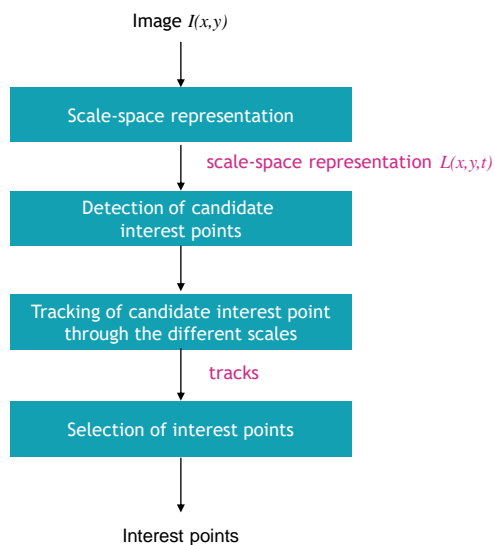
Harris corner detection: invariance

- Corner locations are
 - partially invariant to intensity change
 - covariant w.r.t. translation
 - covariant w.r.t. rotation
 - **not covariant to scaling!**



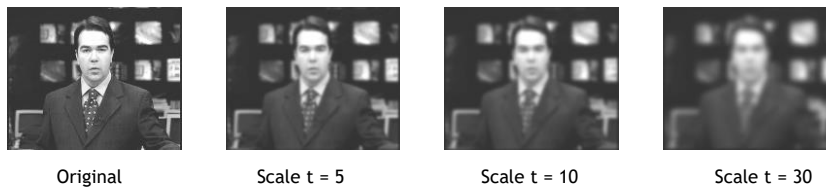
Solution: scale-space approach

Scale-space approach



Scale-space approach

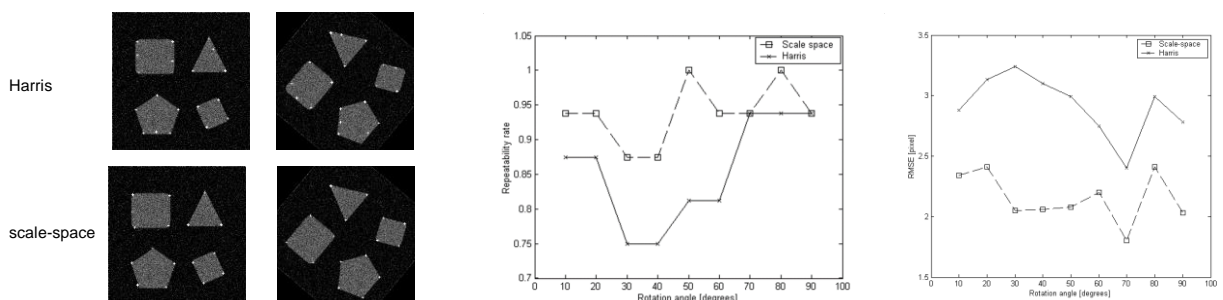
- Scale-space representation
 - parametric set of images **gradually smoothed**
 - the most appropriate scale for extracting the interest points is not known



- Criterion for the selection of interest points: **life-span of a track**
 - the life-span of points due to noise is **short**
 - the life-span of interest points is **long**

Evaluation: Harris vs. scale-space

- Repeatability
 - **Stability** of interest points with viewpoint changes and/or geometric transformations
 - $\text{Repeatability} = (\# \text{ of interest points repeated}) / (\text{total } \# \text{ of interest points})$
- Accuracy
 - High accuracy in **localisation**
 - Measure of the precision of estimated interest point compared to the 'real' interest point
 - **Accuracy** \rightarrow **Root Mean Square Error**



Scale Invariant Feature Transform (SIFT)

- Extracts **highly distinctive** features that are
 - **invariant** to rotation and scaling
 - **partially invariant** to changes in illumination
 - **partially invariant** to affine transformations
- SIFT: main steps
 - scale-space **extrema** detection
 - extract scale and rotation invariant interest points (i.e. keypoints)
 - keypoint **localization**
 - determine **location and scale** for each interest point
 - **orientation** assignment
 - assign **one or more** orientations to each keypoint
 - keypoint **descriptor**
 - use local image gradients **at the selected scale**

Scale invariant detection

- Functions (kernels) for determining scale

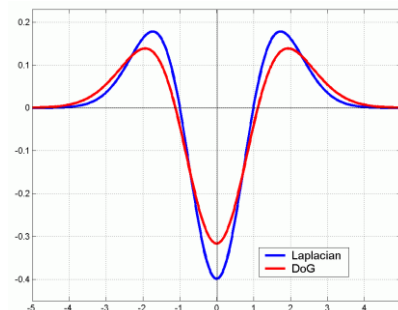
$$L = \sigma^2 (G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma))$$

Laplacian

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

Difference of Gaussians

$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



L or **DoG** kernel is a matching filter. It finds **blob-like structure**. It is also successful in getting characteristic scale of other structures, e.g. corners.

$$f = \text{Kernel} * \text{Image}$$

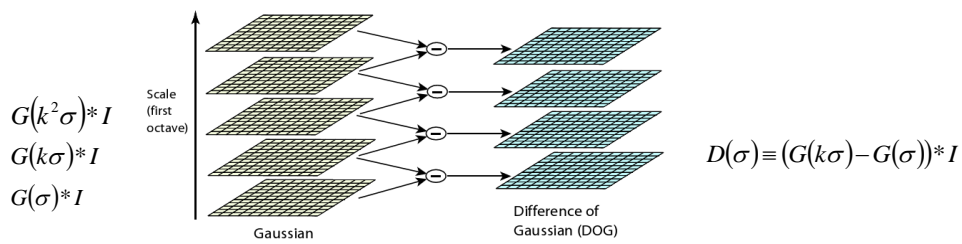
Keypoint detection

- Image: convolved with Gaussian filters at different scales

$$L(x, y, k\sigma) = G(x, y, k\sigma) * I(x, y) \quad k\sigma: \text{scale (blur)}$$

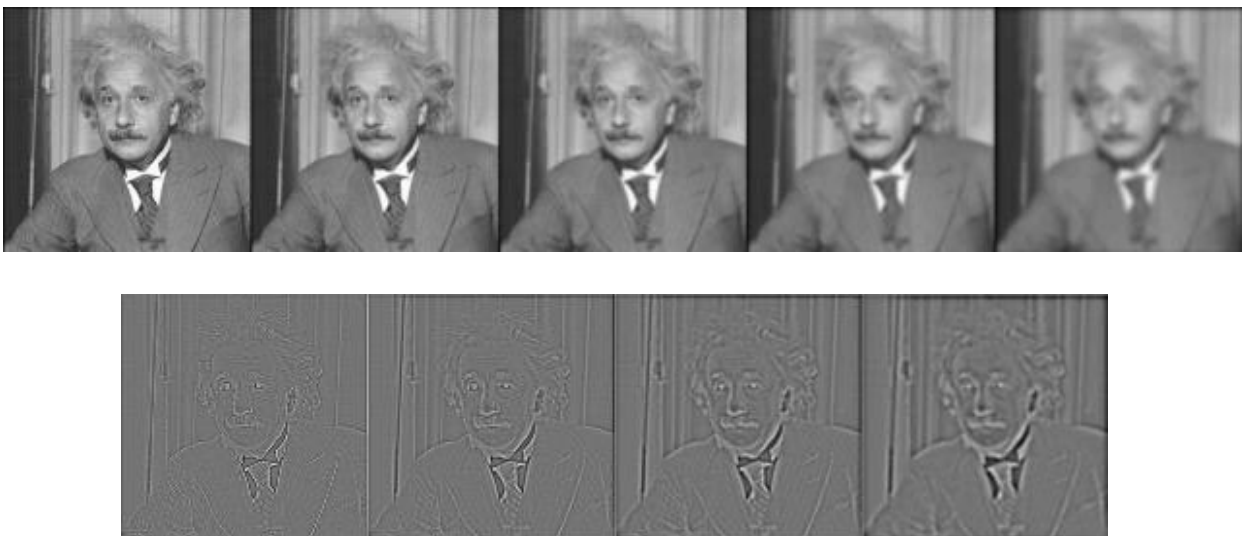
- Difference of successive Gaussian-blurred images are taken (DoG)

- DoG image: $D(x, y, \sigma) = L(x, y, k_i\sigma) - L(x, y, k_j\sigma)$



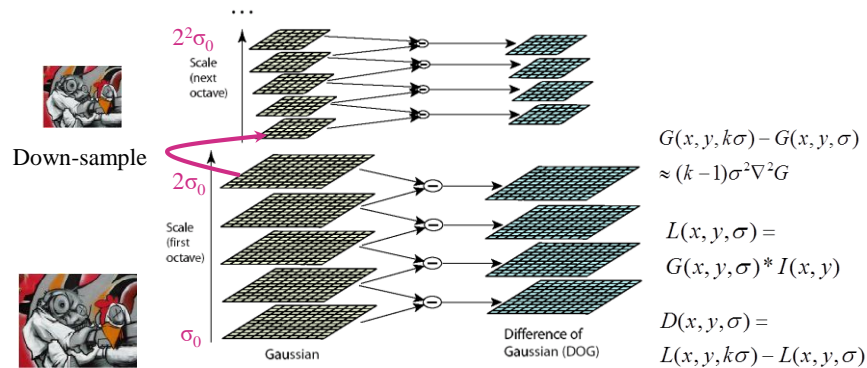
- Keypoints are **maxima/minima** of the DoG that occur at multiple scales

Difference of Gaussians: example



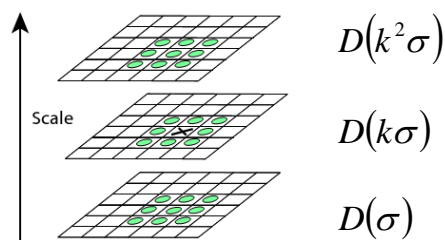
Scale-space extrema detection

- DoG images are **grouped by octaves**
 - octave: corresponds to **doubling the value of σ**
 - fixed number of scales (i.e. levels) per octave



Keypoint detection

- Keypoints: **local minima/maxima** of the DoG images across scales
 - choose all extrema in **DOG pyramid** within $3 \times 3 \times 3$ neighborhood
 - compare each pixel in the DoG images to its **eight** neighbors at the same scale and **nine** corresponding neighboring pixels in each of the neighboring scales
 - comparisons with the nearest **26 neighbours** in a discretized **scale-space volume** ($x, y, scale$)
 - if pixel value is the max/min among all compared pixels \rightarrow selected as candidate keypoint



Keypoint localisation

- Scale-space extrema detection
 - produces **too many** keypoint candidates & some are **unstable**
→ discard **low contrast** keypoints & those located **on edges**
- Next step: accurate location, scale, and ratio of principal curvatures
 - determine the location & scale of keypoints to **sub-pixel & sub-scale accuracy** by fitting a 3D quadratic function at each keypoint

$$X_i = (x_i, y_i, \sigma_i) \rightarrow X_i + \Delta X$$

- compute its maxima → yields a non integer position (in x,y)

Recall

- Harris interest point filtering uses the 2nd order moment matrix to reject points lying on edges

$$M(x_0, y_0) = \sum_{x, y \in \text{neighborhood}(x_0, y_0)} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$R(M) = \det(M) - \alpha \text{trace}^2(M)$$

or

$$R(M) = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

Keypoint filtering

- **Reject points with bad contrast**

- reject keypoint if $|D(X_i + \Delta X)| < 0.03$
- assumes that image values have been normalized in $[0, 1]$

- Laplacian gives strong response on edges \rightarrow additional filtering is needed

- eigenvalues of the Hessian matrix are computed & their strengths evaluated

- **Reject edges**

- SIFT uses the Hessian matrix for efficiency $\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$

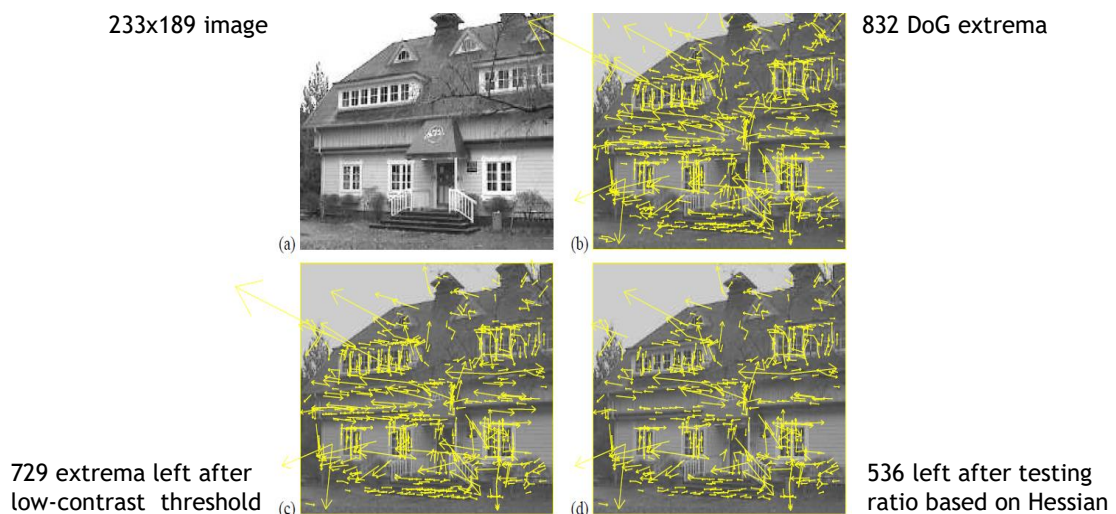
α : largest eigenvalue (λ_{\max}); β : smallest eigenvalue (λ_{\min}) \rightarrow proportional to principal curvatures

$$\begin{aligned} \text{Tr}(\mathbf{H}) &= D_{xx} + D_{yy} = \alpha + \beta, \\ \text{Det}(\mathbf{H}) &= D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta. \end{aligned} \quad \rightarrow \quad \frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(r\beta + \beta)^2}{r\beta^2} = \frac{(r + 1)^2}{r},$$

$$\frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} < \frac{(r + 1)^2}{r}$$

$r = \alpha/\beta$
SIFT uses $r = 10$

Keypoint localisation: example



Orientation assignment

- By assigning a consistent orientation, the descriptor can be orientation invariant
- Create **histogram of gradient directions**, within a region around the keypoint, **at selected scale**
- Let, for a keypoint, L be the Gaussian image with the closest scale
 - compute **gradient magnitude** and **orientation** using finite differences:

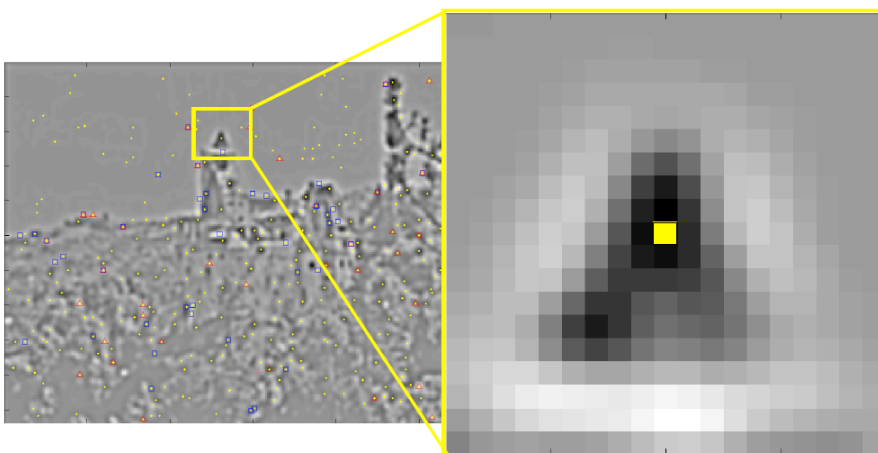
$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y) \quad \text{GradientVector} = \begin{bmatrix} L(x+1, y) - L(x-1, y) \\ L(x, y+1) - L(x, y-1) \end{bmatrix}$$

$$m(x, y) = \sqrt{(L(x+1, y) - L(x-1, y))^2 + (L(x, y+1) - L(x, y-1))^2}$$

$$\theta(x, y) = a \tan 2((L(x, y+1) - L(x, y-1)) / (L(x+1, y) - L(x-1, y)))$$

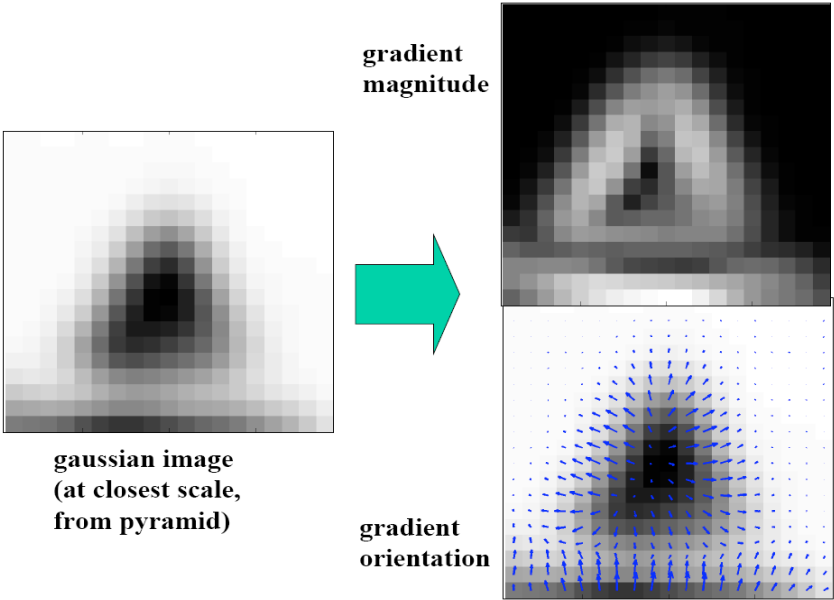
- Histogram entries are **weighted** by
 - gradient magnitude and
 - a Gaussian function with $\sigma = 1.5 \times (\text{scale of the keypoint})$

Orientation assignment

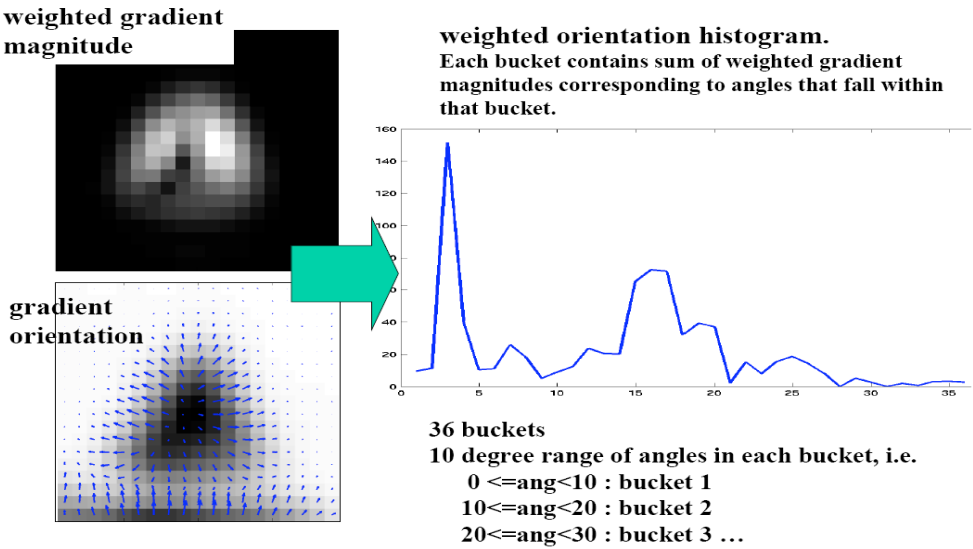


- Keypoint location = extrema location
- Keypoint scale is scale of the DOG image

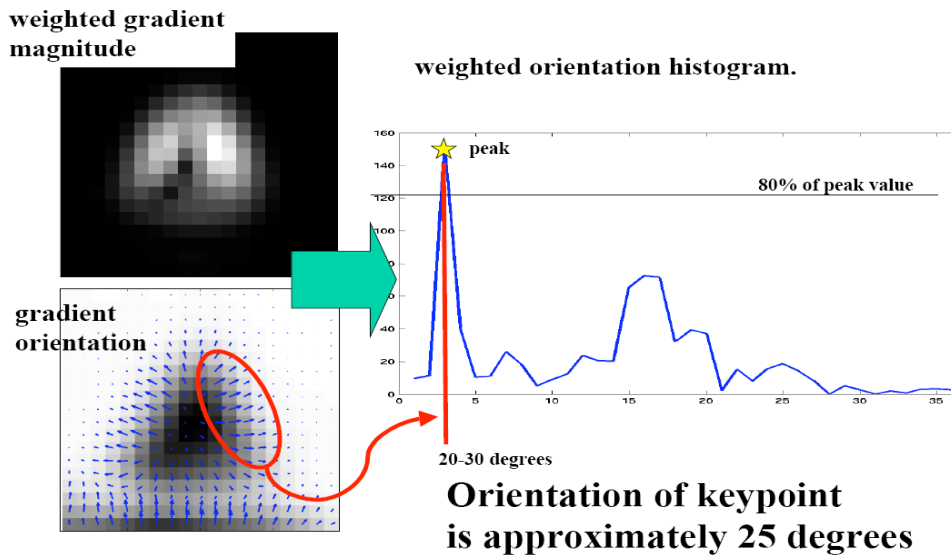
Orientation assignment



Orientation assignment

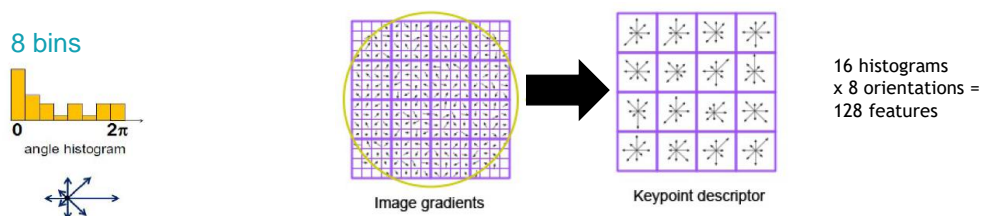


Orientation assignment



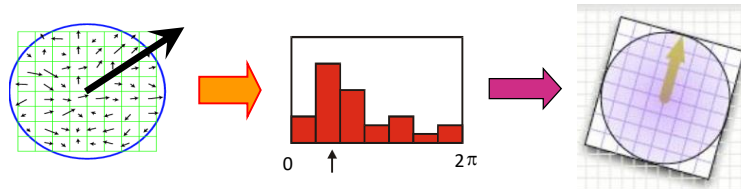
SIFT descriptor

- Take a 16 x 16 window around the detected interest point
- Divide into a 4x4 grid of cells
- Compute histogram in each cell (4x4 samples) in 8 directions
 - Gaussian weighting around center, with $\sigma = 0.5 \times$ (that of the scale of a keypoint)
 - $4 \times 4 \times 8 = 128$ dimensional feature vector



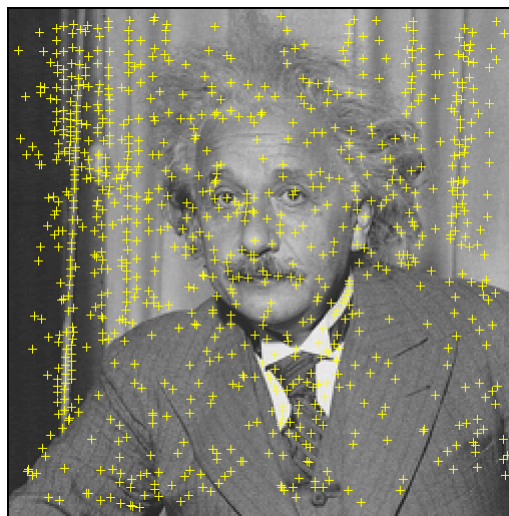
SIFT descriptor

- Assign canonical orientation at **peak of smoothed histogram**
 - fit parabola to better localize peak



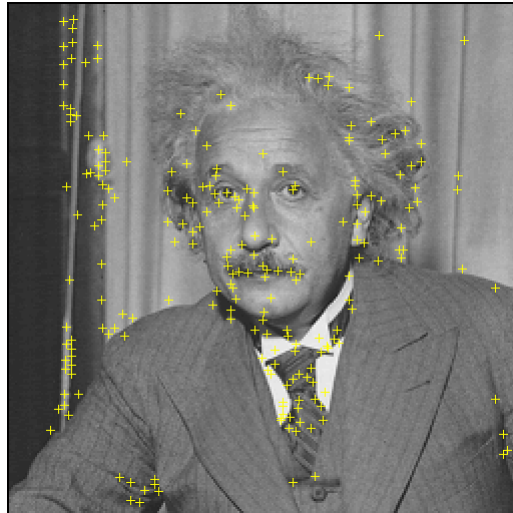
- If other peaks are within 80% of highest peak → assign **multiple orientations**
 - significantly improves stability of matching
 - about 15% of keypoints has multiple orientations assigned

SIFT keypoint localisation: example



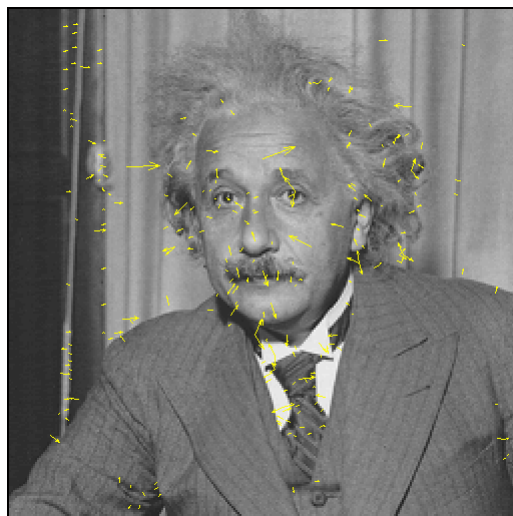
Maxima

SIFT keypoint localisation: example



Remove low contrast and edges

SIFT keypoint localisation: example



Extracted keypoints
(arrows indicate scale and orientation)

Matching SIFT features

- Given a feature in I_1 , how to find the best match in I_2 ?
 - Define **distance function** that compares two descriptors
 - Test all the features in I_2 , find the one with **min distance**

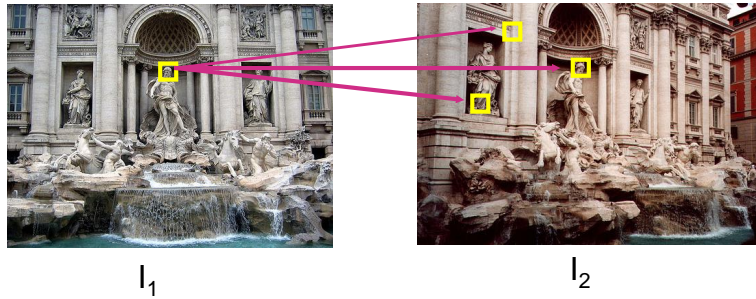
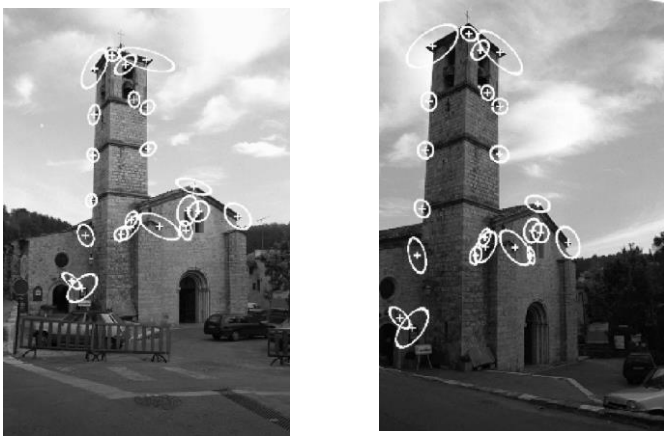


Image retrieval: example

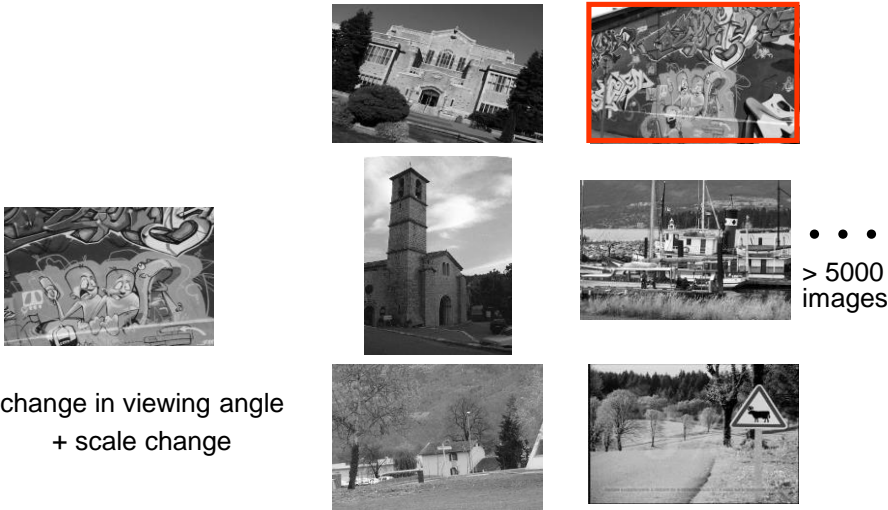


Example: matches



22 correct matches

Image retrieval: example 2



Example: matches



33 correct matches

Properties of SIFT

- Very robust
 - handles **changes in viewpoint**
 - up to about 60 degree out of plane rotation
 - handles significant **changes in illumination**
 - day vs. night
- Fast and efficient
 - can run in real time
- **SURF**
 - Speeded Up Robust Features
 - faster than SIFT

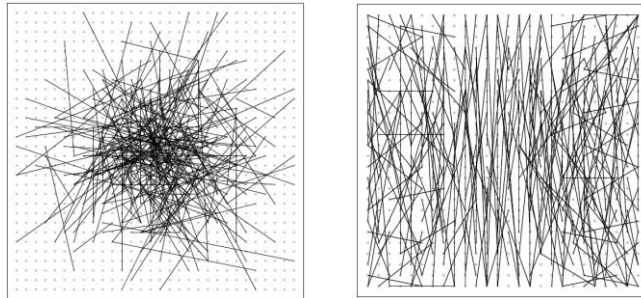
[SURF: Speeded Up Robust Features,](#)
[Computer Vision and Image Understanding, 2008](#)



Binary descriptors

- Principles

- computed from local image patches from a set of **pairwise intensity comparisons**
- each bit: the result of one comparison
- similarity measure: **Hamming distance**

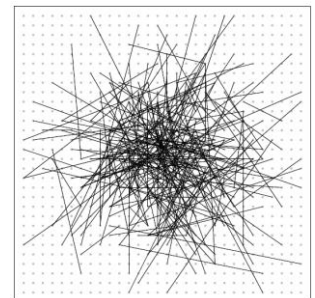


http://cs.unc.edu/~jheinly/binary_descriptors.html

BRIEF

- Binary Robust Independent Elementary Features

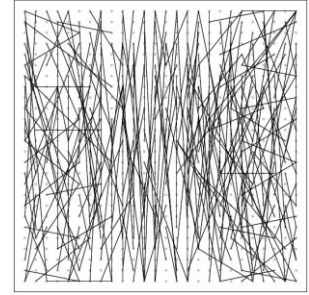
- **sampling pattern**
 - 128, 256, or 512 comparisons → 128, 256, or 512 bits
- sample points
 - selected randomly from an isotropic Gaussian distribution centred at the feature location
- low computational cost
- low storage requirements



BRIEF: Binary Robust Independent Elementary Features. ECCV 2010

ORB

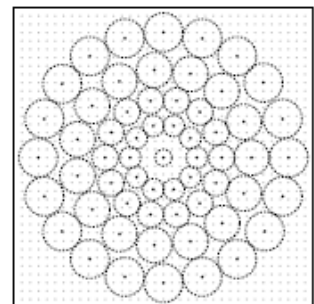
- Oriented FAST and Rotated BRIEF
 - overcomes the lack of rotation invariance of BRIEF
 - local orientation
 - vector between the feature location and the **intensity centroid** (i.e. weighted average of intensities in the patch)
 - **256** pairwise intensity comparisons
 - constructed via machine learning
 - maximizing the descriptor's variance
 - minimizing the correlation under various orientations



ORB: An Efficient Alternative to SIFT or SURF, ICCV 2011

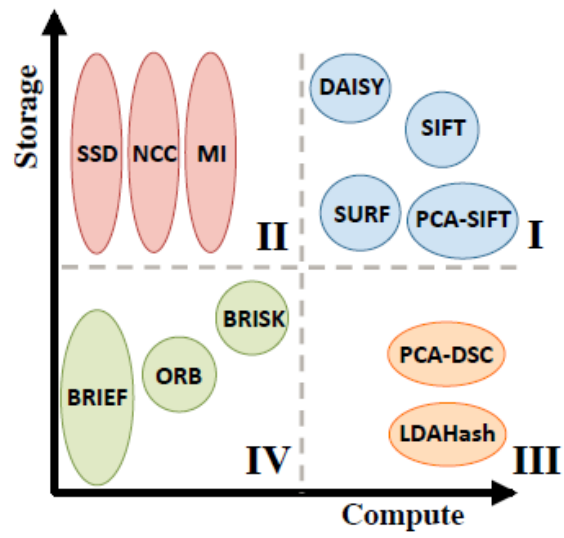
BRISK

- Binary Robust Invariant Scalable Keypoints
 - scale invariant
 - detects keypoints in a **scale-space pyramid**
 - non-maxima suppression and interpolation across all scales
 - rotation invariant
 - symmetric pattern
 - sample points in concentric circles surrounding the feature
 - each sample point represents a Gaussian blurring of its surrounding
 - standard deviation of blurring increases with distance from centre
 - high computational cost
 - high storage requirements



BRISK: Binary Robust Invariant Scalable Keypoints, ICCV 2011

Comparison



Comparative Evaluation of Binary Features. ECCV 2012