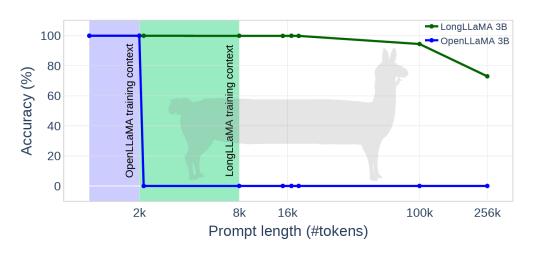
# On Long Context Language Modeling

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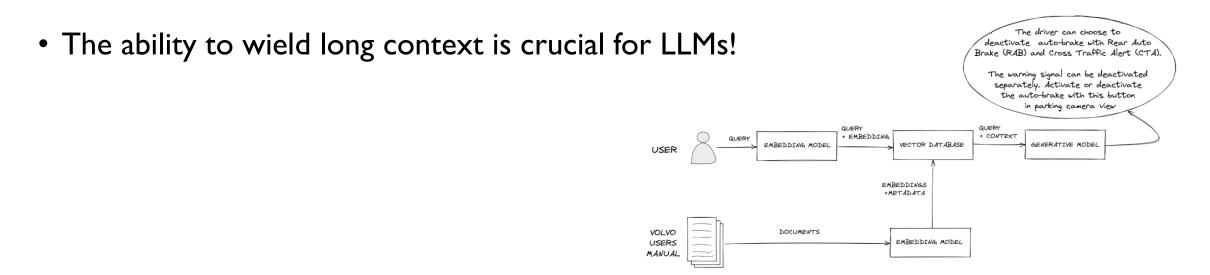
#### Overview

- Long context modeling is a key ability for LM's application (e.g., processing long documents, coherent multi-round dialogs...).
- Current LMs are pre-trained with context windows less than 10k tokens (LLaMA: 2048; LLaMA-2, ChatGPT: 4096)
- Pre-trained LMs need to be effectively and efficiently adapted for long context language modeling.



#### Overview

- Currently, a decent way to deploy LLMs for practical use is Retrieval Augmented Generation (RAG).
  - Instead of tackling the 'internal' problems of LLMs, such as static, lack of domain-specific knowledge, hallucinations, and costly adaptations, we could retrieval grounding external context to LLMs for generation purpose.



#### Outline

- Model Refinement
  - Positional encoding
  - Attention scheme
- Data Mix
- Training
- Application-oriented Evaluation

### Sinusoidal Positional Encoding

- The attention module has no intrinsic ability of order modeling, compared with convolution or recurrence.
- We need to inject some information about the relative and absolute position of the tokens in the sequence, namely positional encoding (PE).
- Formally, denote the dim of embeddings as  $d_{\rm model}$ , the sinusoidal PE is defined as sine and cosine functions of different frequencies:

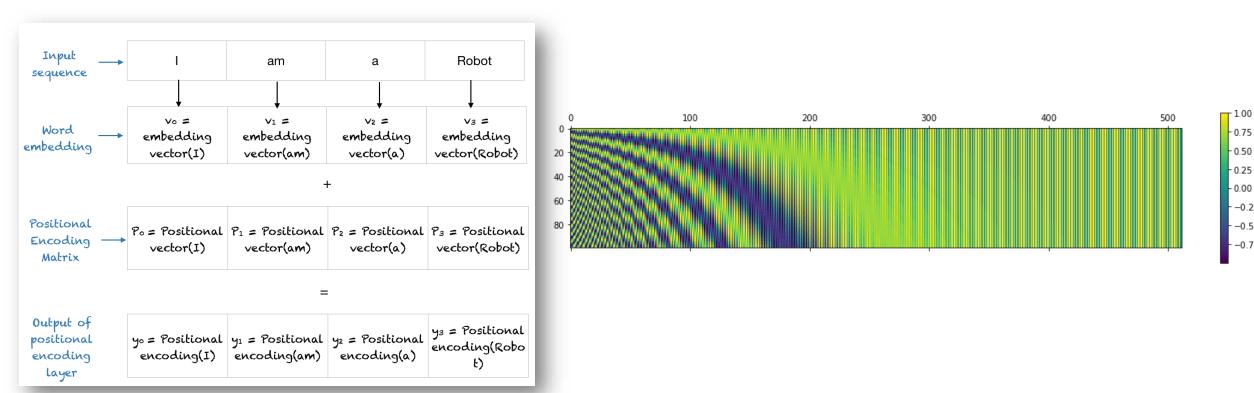
$$PE_{(pos,2i)} = \sin(\frac{pos}{10000^{2i/d_{\text{model}}}})$$

$$PE_{(pos,2i+1)} = \cos(\frac{pos}{10000^{2i/d_{\text{model}}}})$$

where pos is the position and i is the dimension.

#### Sinusoidal Positional Encoding

Input embedding = word embedding + positional encoding



### Sinusoidal Positional Encoding

#### Some intuition about how it works

- Denote the model as a scalar function  $f(\cdots, x_m, \cdots, x_n, \cdots)$ , where  $x_m$  and  $x_n$  are m-th and n-th elements.
- Attention-based models will yield  $f(\cdots, \boldsymbol{x}_m, \cdots, \boldsymbol{x}_n, \cdots) = f(\cdots, \boldsymbol{x}_n, \cdots, \boldsymbol{x}_m, \cdots)$
- Positional encoding could break such symmetry by:

$$\widetilde{f}(\cdots,oldsymbol{x}_m,\cdots,oldsymbol{x}_n,\cdots)=f(\cdots,oldsymbol{x}_m+oldsymbol{p}_m,\cdots,oldsymbol{x}_n+oldsymbol{p}_n,\cdots)$$

- Question: How can we come up with a reasonable design for p ?
- Taylor expansion at  $(\cdots, \boldsymbol{x}_m, \cdots, \boldsymbol{x}_n, \cdots)$ :

$$\tilde{f} \approx f + \boldsymbol{p}_{m}^{\top} \frac{\partial f}{\partial \boldsymbol{x}_{m}} + \boldsymbol{p}_{n}^{\top} \frac{\partial f}{\partial \boldsymbol{x}_{n}} + \frac{1}{2} \boldsymbol{p}_{m}^{\top} \frac{\partial^{2} f}{\partial \boldsymbol{x}_{m}^{2}} \boldsymbol{p}_{m} + \frac{1}{2} \boldsymbol{p}_{n}^{\top} \frac{\partial^{2} f}{\partial \boldsymbol{x}_{n}^{2}} \boldsymbol{p}_{n} + \underbrace{\boldsymbol{p}_{m}^{\top} \frac{\partial^{2} f}{\partial \boldsymbol{x}_{m}^{2} \partial \boldsymbol{x}_{n}} \boldsymbol{p}_{n}}_{\boldsymbol{p}_{m}^{\top} \boldsymbol{\mathcal{H}} \boldsymbol{p}_{n}}$$

#### Sinusoidal Positional Encoding $\tilde{f} \approx f + p_m^{\top} \frac{\partial f}{\partial x_m} + p_n^{\top} \frac{\partial f}{\partial x_n} + \frac{1}{2} p_m^{\top} \frac{\partial^2 f}{\partial x_n^2} p_m + \frac{1}{2} p_n^{\top} \frac{\partial^2 f}{\partial x_n^2} p_n + p_m^{\top} \frac{\partial^2 f}{\partial x_n \partial x_n} p_n$ Some intuition about how it works

$$\widetilde{f}pprox f+oldsymbol{p}_m^{ op}rac{\partial f}{\partial oldsymbol{x}_m}+oldsymbol{p}_n^{ op}rac{\partial f}{\partial oldsymbol{x}_n}+rac{1}{2}oldsymbol{p}_m^{ op}rac{\partial^2 f}{\partial oldsymbol{x}_m^2}oldsymbol{p}_m+rac{1}{2}oldsymbol{p}_n^{ op}rac{\partial^2 f}{\partial oldsymbol{x}_n^2}oldsymbol{p}_n+oldsymbol{p}_m^{ op}rac{\partial^2 f}{\partial oldsymbol{x}_m\partial oldsymbol{x}_n}oldsymbol{p}_n$$

- In principle, an ideal positional encoding should contain the information of relative position between tokens.
- A promising term is  $m{p}_m^ op m{\mathcal{H}} m{p}_n$ . For simplicity, consider  $d_{\mathrm{model}} = 2, m{\mathcal{H}} = m{I}$ , we want  $p_m^{\top} \tilde{\mathcal{H}} p_n$  could encode the relative position, i.e.,  $\langle p_m, p_n \rangle = g(m-n)$ .
- Derive with complex numbers:

$$\langle \boldsymbol{p}_{m}, \boldsymbol{p}_{n} \rangle = \operatorname{Re}[\boldsymbol{p}_{m}\boldsymbol{p}_{n}^{*}] \quad \Leftarrow \quad \boldsymbol{p}_{m}\boldsymbol{p}_{n}^{*} = \boldsymbol{q}_{m-n}$$
 $\boldsymbol{p}_{m} = r_{m}e^{\mathrm{i}\phi_{m}}, \boldsymbol{p}_{n}^{*} = r_{n}e^{-\mathrm{i}\phi_{n}}, \boldsymbol{q}_{m-n} = R_{m-n}e^{\mathrm{i}\Phi_{m-n}}$ 
 $r_{m}r_{n}e^{\mathrm{i}(\phi_{m}-\phi_{n})} = R_{m-n}e^{\mathrm{i}\Phi_{m-n}} \quad \Rightarrow \quad \begin{cases} r_{m}r_{n} = R_{m-n}\\ \phi_{m} - \phi_{n} = \Phi_{m-n} \end{cases}$ 

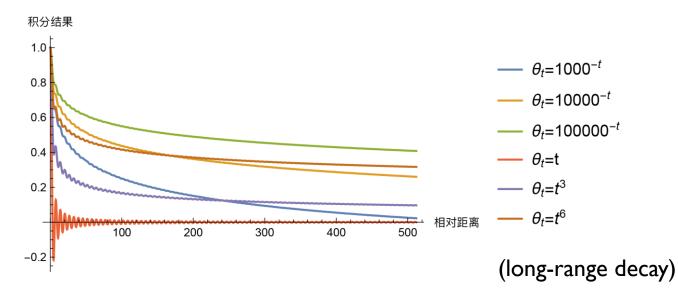
One solution

$$\boldsymbol{p}_m = e^{\mathrm{i}m\theta} \quad \Leftrightarrow \quad \boldsymbol{p}_m = \begin{pmatrix} \cos m\theta \\ \sin m\theta \end{pmatrix}$$

#### Sinusoidal Positional Encoding Some intuition about how it works

on: 
$$m{p}_m = egin{pmatrix} e^{\mathrm{i}m heta_0} \ dots \ e^{\mathrm{i}m heta_{d/2-1}} \end{pmatrix}$$

• Extend to high dimension:  $p_m = \begin{pmatrix} e^{\mathrm{i}m\theta_0} \\ \vdots \\ e^{\mathrm{i}m\theta_{d/2-1}} \end{pmatrix} \quad \Leftrightarrow \quad p_m = \begin{pmatrix} \cos m\theta_0 \\ \sin m\theta_0 \\ \vdots \\ \cos m\theta_{d/2-1} \\ \sin m\theta_{d/2-1} \end{pmatrix}$  • Choose  $\theta^i$ :



### Relative Positional Encodings

- Consider the attention module with absolute positional encoding:
- Expand the QK term:

$$\boldsymbol{q}_{i}\boldsymbol{k}_{j}^{\top}=\left(\boldsymbol{x}_{i}+\boldsymbol{p}_{i}\right)\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\left(\boldsymbol{x}_{j}+\boldsymbol{p}_{j}\right)^{\top}=\boldsymbol{x}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\boldsymbol{x}_{j}^{\top}+\boldsymbol{x}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\boldsymbol{p}_{j}^{\top}+\boldsymbol{p}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\boldsymbol{x}_{j}^{\top}+\boldsymbol{p}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\boldsymbol{p}_{j}^{\top}$$

- Shaw et al., 2018:
- XLNET:
- T5:
- DeBERTa:

$$m{x}_im{W}_Qm{W}_K^ opm{x}_j^ op+m{x}_im{W}_Qm{R}_{i,j}^K^ op$$
 , where  $m{R}_{i,j}^K$  is a function of  $i-j$ 

 $\left\{egin{aligned} oldsymbol{q}_i &= (oldsymbol{x}_i + oldsymbol{p}_i) oldsymbol{W}_Q \ oldsymbol{k}_j &= (oldsymbol{x}_j + oldsymbol{p}_j) oldsymbol{W}_K \ oldsymbol{v}_j &= oldsymbol{x}_j oldsymbol{W}_V \end{aligned}
ight.$ 

 $egin{aligned} a_{i,j} = softmax\left(oldsymbol{q}_i oldsymbol{k}_j^ op 
ight) \ oldsymbol{o}_i = \sum_{i} a_{i,j} oldsymbol{v}_j \end{aligned}$ 

$$\boldsymbol{x}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K}^{\top}\boldsymbol{x}_{j}^{\top}+\boldsymbol{x}_{i}\boldsymbol{W}_{Q}\boldsymbol{W}_{K,R}^{\top}\boldsymbol{R}_{i-j}^{\top}+\boldsymbol{u}\boldsymbol{W}_{K}^{\top}\boldsymbol{x}_{j}^{\top}+\boldsymbol{v}\boldsymbol{W}_{K,R}^{\top}\boldsymbol{R}_{i-j}^{\top}$$

$$oldsymbol{x}_i oldsymbol{W}_Q oldsymbol{W}_K^ op oldsymbol{x}_j^ op + oldsymbol{eta}_{i,j}$$

$$oldsymbol{x}_i oldsymbol{W}_Q oldsymbol{W}_K^ op oldsymbol{x}_j^ op + oldsymbol{x}_i oldsymbol{W}_Q oldsymbol{W}_K^ op oldsymbol{X}_i^ op + oldsymbol{R}_{j,j} + oldsymbol{R}_{j,i} oldsymbol{W}_Q oldsymbol{W}_K^ op oldsymbol{x}_j^ op$$

Shaw, Peter, Jakob Uszkoreit, and Ashish Vaswani. "Self-attention with relative position representations." (NAACL 2018).

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Raffel, Colin, et al. "Exploring the limits of transfer learning with a unified text-to-text transformer." (JMLR 2020).

He, Pengcheng, et al. "Deberta: Decoding-enhanced bert with disentangled attention." (ICLR 2021).

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### RoPE: Rotary Position Embedding

- Implement relative PE with absolute PE.
- Attention with absolute PE:  $\tilde{m{q}}_m = m{f}(m{q},m), \quad \tilde{m{k}}_n = m{f}(m{k},n)$
- Find a tractable solution of:  $\langle \boldsymbol{f}(\boldsymbol{q},m), \boldsymbol{f}(\boldsymbol{k},n) \rangle = g(\boldsymbol{q},\boldsymbol{k},m-n)$ 
  - Initial condition: f(q, 0) = q, f(k, 0) = k
  - Derive with complex number:

$$\operatorname{Re}[\boldsymbol{f}(\boldsymbol{q},m)\boldsymbol{f}^*(\boldsymbol{k},n)] = g(\boldsymbol{q},\boldsymbol{k},m-n) \Leftarrow \boldsymbol{f}(\boldsymbol{q},m)\boldsymbol{f}^*(\boldsymbol{k},n) = \boldsymbol{g}(\boldsymbol{q},\boldsymbol{k},m-n)$$

• Let:

 $m{f}(m{q},m) = R_f(m{q},m)e^{\mathrm{i}\Theta_f(m{q},m)}$  Equation:

$$f(q,m) = R_f(q,m)e^{i\Theta_f(q,m)}$$
 $f(k,n) = R_f(k,n)e^{i\Theta_f(k,n)}$ 
 $\Theta_f(q,k,m-n) = R_g(q,k,m-n)e^{i\Theta_g(q,k,m-n)}$ 

$$R_f(\mathbf{q}, m)R_f(\mathbf{k}, n) = R_g(\mathbf{q}, \mathbf{k}, m - n)$$
  
$$\Theta_f(\mathbf{q}, m) - \Theta_f(\mathbf{k}, n) = \Theta_g(\mathbf{q}, \mathbf{k}, m - n)$$

• One solution:  $m{f}(m{q},m) = m{q}e^{\mathrm{i}m heta}$  , i.e., rotation operation  $m{f}(m{q},m) = \begin{pmatrix} \cos m heta & -\sin m heta \\ \sin m heta & \cos m heta \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix}$ 

### RoPE: Rotary Position Embedding

• High dimension:  $\theta_i = 10000^{-2i/d}$ 

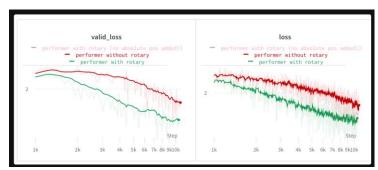
$$\underbrace{\begin{pmatrix} \cos m\theta_0 & -\sin m\theta_0 & 0 & 0 & \cdots & 0 & 0 \\ \sin m\theta_0 & \cos m\theta_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cos m\theta_1 & -\sin m\theta_1 & \cdots & 0 & 0 \\ 0 & 0 & \sin m\theta_1 & \cos m\theta_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \sin m\theta_{d/2-1} & -\sin m\theta_{d/2-1} \\ 0 & 0 & 0 & 0 & \cdots & \sin m\theta_{d/2-1} & \cos m\theta_{d/2-1} \end{pmatrix}} \underbrace{\begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_{d-2} \\ q_{d-1} \end{pmatrix}}_{\mathcal{R}_m} \otimes \begin{pmatrix} \cos m\theta_0 \\ \cos m\theta_0 \\ \cos m\theta_1 \\ \vdots \\ q_{d-2} \\ q_{d-1} \end{pmatrix}} + \begin{pmatrix} -q_1 \\ q_0 \\ -q_3 \\ q_2 \\ \vdots \\ q_{d-2} \\ q_{d-1} \end{pmatrix} \otimes \begin{pmatrix} \sin m\theta_0 \\ \sin m\theta_0 \\ \sin m\theta_0 \\ \sin m\theta_1 \\ \vdots \\ q_{d-2} \\ q_{d-1} \end{pmatrix}}$$

which satisfies:  $(\mathcal{R}_m q)^{ op} (\mathcal{R}_n k) = q^{ op} \mathcal{R}_m^{ op} \mathcal{R}_n k = q^{ op} \mathcal{R}_{n-m} k$ 

• Superior performance in terms of ppl. on a wide-range of model architectures.

	Туре	OWT2 Loss	OWT2 Ppl.	
	Learned Absolute	2.809	16.59	
	T5 RPE	2.801	16.46	
	Rotary	2.759	15.78	
Final validation	loss / ppl scores on OV	VT2 validation	set at 55k step	s (~30B tokens

	Туре	Pile Loss	Pile Ppl.	
	Learned Absolute	2.240	9.393	
	T5 RPE	2.223	9.234	
	Rotary	2.173	8.784	
Final validation	n loss / ppl scores on P	ile validation	set at 8k step	s (~8B tokens)

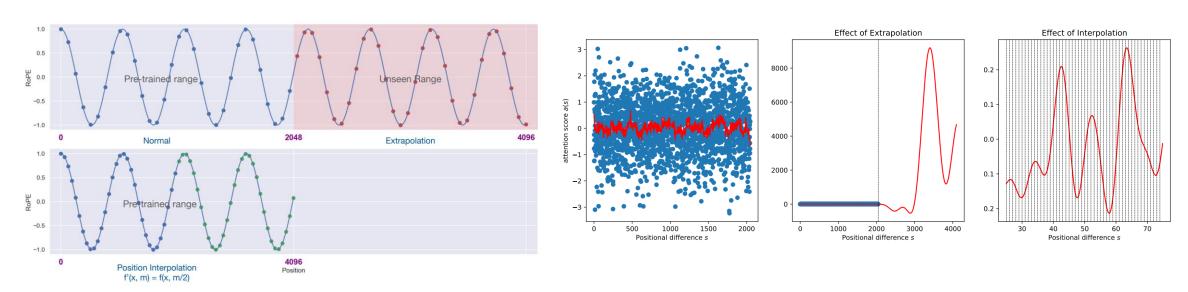


### Length Extrapolation / Generalization

- Length extrapolation: overcoming inconsistencies between training and inference
  - Unseen positional encoding during inference.
  - More attention participants during inference.
- For attention: confine the context window with attention mask (window attention), e.g., change  $q_m^{\top} k_n$  to  $q_m^{\top} k_n \lambda |m-n|$ .
- For PE: refine existing RoPE to support longer context while preserving performance within original context length.

## Position Interpolation for RoPE Length extrapolation w/ minimal fine-tuning

- Extrapolating outside the training positions may produce catastrophic values.
- Since PE can be applied on non-integer positions, one could interpolate the position encodings at neighboring integer positions.



Extrapolation v.s. Interpolation

Catastrophic behavior of extrapolation

## Position Interpolation for RoPE Length extrapolation w/ minimal fine-tuning

• Method: Given a pre-trained context length L and a new context length L'>L, position interpolation replaces RoPE by:

$$m{f}'(m{x},m) = m{f}(m{x},rac{mL}{L'})$$

• Numerical bound: for unbounded  $h_j := (q_{2j} + iq_{2j+1})(k_{2j} - ik_{2j+1})$ , the upper bound for difference between the result attention score and the typical attention score for interpolation is ~600x smaller than extrapolation.

## Position Interpolation for RoPE Length extrapolation w/ minimal fine-tuning

#### • Experiments:

• Extend LLaMA (7B, 13B, 33B, 65B) to context window sizes up to 32768 using direct fine-tuning (10k steps) or Position Interpolation(1k steps).

	Model		E	valuation	Context V	Window S	Size
Size	Context Window	Method	2048	4096	8192	16384	32768
7B	2048	None	7.20	$> 10^{3}$	$> 10^{3}$	$> 10^{3}$	$> 10^{3}$
7B	8192	FT	7.21	7.34	7.69	-	-
7B	8192	PI	7.13	6.96	6.95	-	-
7B	16384	PΙ	7.11	6.93	6.82	6.83	-
7B	32768	PI	7.23	7.04	6.91	6.80	6.77
13B	2048	None	6.59	-	-	-	-
13B	8192	FT	6.56	6.57	6.69	-	-
13B	8192	PI	6.55	6.42	6.42	-	-
13B	16384	PΙ	6.56	6.42	6.31	6.32	-
13B	32768	PI	6.54	6.40	6.28	6.18	6.09
33B	2048	None	5.82	-	-	-	-
33B	8192	FT	5.88	5.99	6.21	-	-
33B	8192	PI	5.82	5.69	5.71	-	-
33B	16384	PI	5.87	5.74	5.67	5.68	-
65B	2048	None	5.49	-	-	-	-
65B	8192	PI	5.42	5.32	5.37	-	-

Table 1: Evaluation perplexity on PG19 dataset (Rae et al., 2020). FT: Direct Fine-tuning. PI: Position Interpolation. Model fine-tuned with PI shows progressively lower perplexity with longer context window, showing that PI can leverage long context well, while the perplexity of FT increases over longer window. Note that overall the perplexity is higher compared to Table 2 since PG19 has very different writing styles.

	Model				Fine-tun	ing steps		
Size	Context Window	Method	200	400	600	800	1000	10000
7B	8192	FT	1792	2048	2048	2048	2304	2560
33B	8192	FT	1792	2048	1792	2048	2304	-
7B	8192	PI	8192	8192	8192	8192	8192	-
7B	16384	PI	16384	16384	16384	16384	16384	-
7B	32768	PI	32768	32768	18432	32768	32768	-
33B	8192	PI	8192	8192	8192	8192	8192	-
33B	16384	PI	16384	16384	16384	16384	16384	-

Table 4: Effective context window sizes after fine-tuning. FT: Direct fine-tuning. PI: Position Interpolation.

Model Size	Context Window	Fine-tune on	BoolQ	<b>PIQA</b>	Race-M	Race-H	WinoGrande
7B	2048	None	76.1	78.9	55.7	42.2	69.6
7B	8192	Pile	73.2	78.2	53.8	41.7	69.0
7B	16384	Pile	69.8	77.6	53.3	40.9	67.8
7B	32768	Pile	64.7	77.2	50.1	39.6	66.9
<b>7B</b>	8192	RedPajama	75.5	77.4	54.5	41.5	68.1
33B	2048	None	81.6	80.2	61.1	45.9	76.2
33B	8192	Pile	80.2	80.7	60.2	45.7	75.9

Table 5: Zero-shot performance on a subset of LLaMA Benchmarks. Models extended by Position Interpolation comparable performance as the original models, except for BoolQ dataset that may require models to pay close attention to word ordering in a short reference paragraph.

• Linear position interpolation (  $f'(x,m)=f(x,\frac{mL}{L'})$  ) may overly narrow the spacing on the unit-digit of the position:



• Amortize such 'narrowing' pressure to all digits via numeral system conversion:



• Recall that for a  $\beta$ -numerical system, the m-th digit of a number n is:

$$\left| \frac{n}{\beta^{m-1}} \right| \mod \beta$$

• RoPE is  $\beta$  -system for position n :

$$\left[\cos\left(\frac{n}{\beta^0}\right), \sin\left(\frac{n}{\beta^0}\right), \cos\left(\frac{n}{\beta^1}\right), \sin\left(\frac{n}{\beta^1}\right), \cdots, \cos\left(\frac{n}{\beta^{d/2-1}}\right), \sin\left(\frac{n}{\beta^{d/2-1}}\right)\right]$$

where  $\beta = 10000^{2/d}$ .

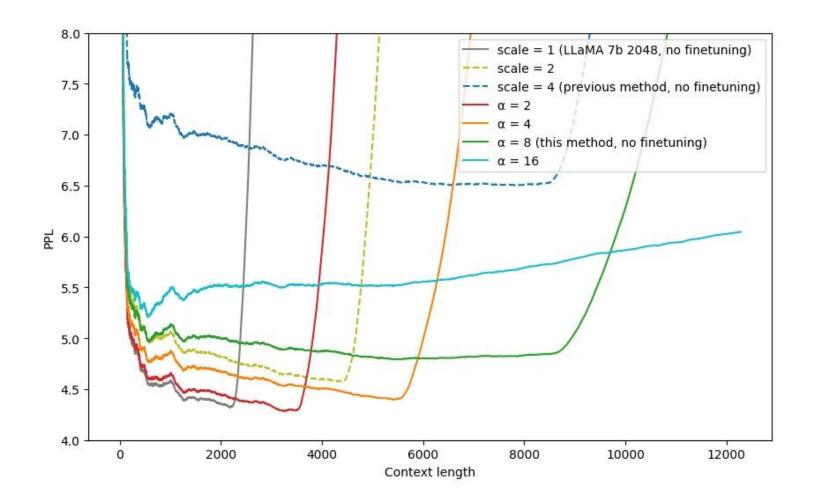
- To expand the length to kL within the same number of digits d, one could adopt  $\beta \sqrt[d]{k}$  -system.
- For RoPE, we could adopt  $\beta_k = 10000k^{2/d}$ .

- Another implementation: extrapolation for high-freq, interpolation for low-freq.
- Consider RoPE:

$$\left[\cos\left(\frac{n}{\beta^0}\right), \sin\left(\frac{n}{\beta^0}\right), \cos\left(\frac{n}{\beta^1}\right), \sin\left(\frac{n}{\beta^1}\right), \cdots, \cos\left(\frac{n}{\beta^{d/2-1}}\right), \sin\left(\frac{n}{\beta^{d/2-1}}\right)\right]$$

- The lowest-freq term is  $\frac{n}{\beta^{d/2-1}}$  , let  $\frac{n}{(\beta\lambda)^{d/2-1}}=\frac{n/k}{\beta^{d/2-1}}$  , then  $\lambda=k^{2/(d-2)}$
- For the highest-freq term  $\frac{n}{\beta}$  , since  $\lambda\approx 1$  when d is large. Hence, it's equivalent to direct position extrapolation.

• Experiments:



### Extensive Study on RoPE for Long Context Modeling

- Vanilla RoPE imposes a large decay on attention scores for distant tokens.
- RoPE introduces large "oscillation" in the long-range regions.
- Experiment with several RoPE variants:
  - Vanilla:  $oldsymbol{x}_j e^{\mathrm{i} m b^{rac{-2j}{d}}}$
  - Position Interpolation (PI):  $oldsymbol{x}_j e^{\mathrm{i} lpha m b^{rac{-2j}{d}}}$
  - Adjusted Base Frequency (ABF):  $oldsymbol{x}_j e^{\mathrm{i} m(eta b)^{rac{-2j}{d}}}$
  - Xpos + ABF:  $x_j e^{\xi_j m + \mathrm{i} m(\beta b)^{\frac{-2j}{d}}}$

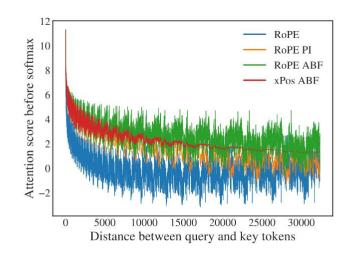
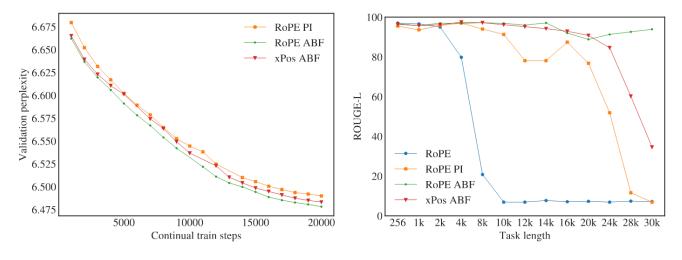


Figure 4: Decaying raw attention scores for distant tokens of explored positional encoding variants (assuming keys and queries are all-ones vectors).

### Extensive Study on RoPE for Long Context Modeling

#### • Experiments:

Continual pre-training + instruction fine-tuning on pre-trained LLaMA-2 checkpoints



<sup>(</sup>a) Validation PPL (16k-token sequences) on a held-out long-context dataset.

Figure 5: Comparison of positional encoding variants on synthetic sentence retrieval task and validation perplexity evolution during continual pretraining.

	HumanEval	Math	MMLU	HellaSwag	TQA
RoPE	14.63	3.62	45.69	76.31	65.23
ROPE PI ROPE-ABF xPos-ABF	15.24 <b>17.07</b> 16.46	3.08 3.52 3.54	45.84 <b>46.24</b> 45.72	76.65 <b>76.73</b> 76.68	65.96 66.04 <b>66.14</b>

Table 6: The performance of models with different positional encoding variants on standard short-context benchmarks.

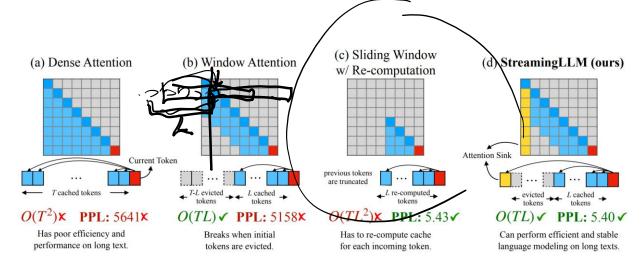
PE	Books	CC	Wikipedia
RoPE	6.548	6.816	3.802
RoPE PI RoPE ABF xPos ABF	6.341 <b>6.323</b> 6.331	6.786 <b>6.780</b> <b>6.780</b>	3.775 3.771 3.771

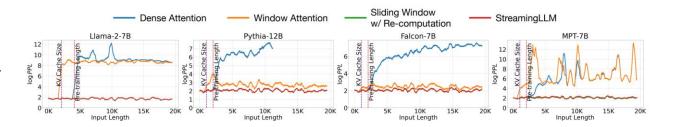
Table 5: Validation perplexity of models with different positional encoding variants. All samples are 32,768-token sequences (CC: CommonCrawl).

<sup>(</sup>b) Performance on FIRST-SENTENCE-RETRIEVAL task.

Improved Attention Scheme for Long Context Modeling Refine the window attention

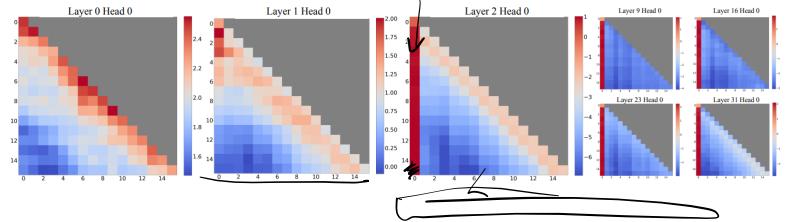
- For long context language modeling, there are two main challenges:
  - Pre-trained LMs have limited length extrapolation capabilities.
  - The KV cache on decoding stage can lead to excessive memory or increasing decoding latency (if we recompute some of them).
- A direct solution is window attention.
  - However, it is sub-optimal in quality or efficiency.
  - Window attention breaks when the first token is excluded.





### Improved Attention Scheme for Long Context Modeling How window attention breaks?

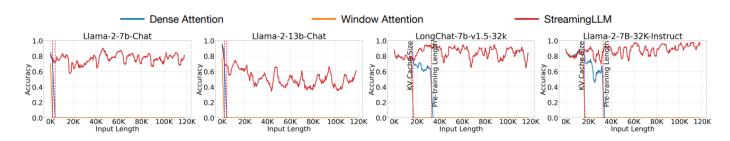
- Empirically, LMs consistently focuses on the initial tokens across all layers and heads except the bottom two layers.
  - Even when substituting the first token with a line break "\n".



• Intuitively, the model learned a biased behavior that tends to dump unnecessary attention values to specific tokens.

### Improved Attention Scheme for Long Context Modeling Window attention with attention sinks

- Method: preserve several start tokens when performing window attention.
  - Use attention sinks (i.e., four initial tokens) to stabilize the computation of attention score.
  - Use rolling KV cache around the most recent tokens to save memory.



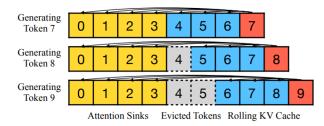


Figure 4: The KV cache of StreamingLLM.

Figure 9: Performance on the StreamEval benchmark. Accuracies are averaged over 100 samples.

Llama-2-13B	PPL (↓)
0 + 1024 (Window)	5158.07
4 + 1020	5.40
4"\n"+1020	5.60

Eval on 65k tokens

Cache Config	0+2048	1+2047	2+2046	4+2044	8+2040
Falcon-7B MPT-7B Pythia-12B	17.90 460.29 21.62	12.12 14.99 11.95	12.12 15.00 12.09	12.12 14.99 12.09	12.12 14.98 12.02
Cache Config	0+4096	1+4095	2+4094	4+4092	8+4088
Llama-2-7B	3359.95	11.88	10.51	9.59	9.54

Eval on 400k tokens

## Improved Attention Scheme for Long Context Modeling Explicitly build attention sink during pre-training

- Pre-training with attention sink:
  - Zero Sink: replace the regular Softmax in attention with Softmax-I:  $\operatorname{SoftMax}_1(x)_i = \frac{e^{x_i}}{1 + \sum_{i=1}^N e^{x_i}}$
  - Learnable Sink: Add a common token at the beginning of all samples.

Cache Config	0+1024	1+1023	2+1022	4+1020
Vanilla	27.87	18.49	18.05	18.05
Zero Sink	29214	19.90	18.27	18.01
Learnable Sink	1235	<b>18.01</b>	18.01	18.02

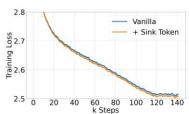
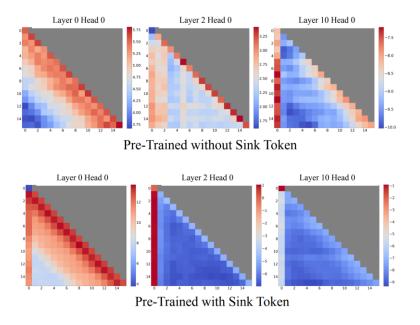


Figure 6: Pre-training loss curves of models w/ and w/o sink tokens. Two models have a similar convergence trend.

Table 4: Zero-shot accuracy (in %) across 7 NLP benchmarks, including ARC-[Challenge, Easy], HellaSwag, LAMBADA, OpenbookQA, PIQA, and Winogrande. The inclusion of a sink token during pre-training doesn't harm the model performance.

Methods	ARC-c	ARC-e	HS	LBD	OBQA	PIQA	WG
Vanilla	18.6	45.2	29.4	39.6	16.0	62.2	50.1
+Sink Token	19.6	45.6	29.8	39.9	16.6	62.6	50.8



### Data Mix for Continual Pre-training

- The data for continual pre-training Long LLaMA-2 consists of two parts:
  - Existing pre-training data of LLaMA-2
  - New long text data
- Adjusting the length distribution of the pretrain data does not provide major benefits for long context modeling.
- Long LLMs can be trained with limited long text data. And the performance gain mainly comes from the data quality itself instead of length distribution.

Continual Pretrain Data	NarrativeQA $\Delta$ F1	Qasper Δ F1	Quality $\Delta$ EM	$\begin{array}{c} {\rm QMSum} \\ {\rm \Delta \; ROUGE\text{-}geo} \end{array}$
LLAMA 2 LONG data mix	23.70%	43.64%	75.5%	45.70%
LLAMA 2 data mix	18.23%	38.12%	60.3%	44.87%
- remove long text data	19.48%	39.14%	67.1%	36.60%
- upsample existing long text data	22.15%	36.82%	65.0%	42.83%

Table 7: Comparison of different pretraining data mix on long-context tasks. Instead of showing the absolute performance, we report relative improvements over the 7B LLAMA 2 which has a 4,096-token context window. All models are evaluated with prompts truncated at 16,384 tokens.

Continual Pretrain Data	HumanEval	Math	MMLU	HellaSwag	TQA
LLAMA 2 LONG data mix LLAMA 2 data mix	<b>17.08</b> 15.24	<b>4.09</b> 3.61	<b>48.62</b> 46.30	76.74 76.63	66.24 <b>66.71</b>
- remove long text data	17.07	3.57	46.25	76.03 <b>76.76</b>	65.90
- upsample existing long text data	17.07	3.53	46.25	76.74	66.04

Table 8: Standard short task performance of long-context models with different pretrain data mix.

### Instruction Tuning

- Fine-tuning only on the short instruction data is enough to produce a decent long chat LM.
- Mixing long context pre-training data with instruction data could further boost LM's performance.
- Different from common instruction tuning that only calculates LM loss on the response part, calculating the LM loss on the whole sequence (i.e., instruction-response pair) is better for long context LMs.

Settings	Qasper	NarrativeQA	QuALITY	SummScreenFD	QMSum
LLAMA 2 CHAT baseline	12.2	9.13	56.7	10.5	14.4
LLAMA 2 LONG finetuned with: "RLHF V5" "RLHF V5" mix pretrain "RLHF V5" mix self-inst w/o LM loss "RLHF V5" mix self-inst with LM loss	22.3 23.7 35.7 38.9	13.2 16.6 22.3 <b>23.3</b>	71.4 76.2 59.3 <b>77.3</b>	14.8 <b>15.7</b> 12.2 14.5	16.9 17.8 13.4 <b>18.5</b>

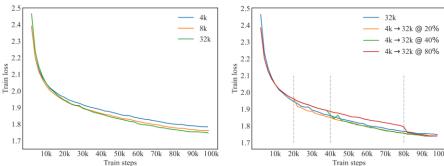
Table 9: Comparison of different instruction finetuning data mixes.

### Training Curriculum

- Question: does pre-training from scratch with long sequences yield better performance than continual pre-training?
- Experiment: two-stage training that begins with 4096 context length and switch to 32768 when the model completes 20%, 40%, 80% of whole training process.
  - Same number of training tokens.
  - Same number of tokens per each gradient update.
- Pre-trained LM could efficiently obtain long context modeling ability with continual pre-training (save ~40% FLOPs with no performance loss).

Pretrain Curriculum	FLOPs	NarrativeQA F1	Qasper F1	Quality EM	QMSum ROUGE-geo
32k from scratch	$3.783 \times 10^{22}$	18.5	28.6	37.9	11.46
$4k \rightarrow 32k @ 20\%$	$3.405 \times 10^{22}$	20.0	28.1	38.8	12.09
$4k \rightarrow 32k @ 40\%$	$3.026 \times 10^{22}$	20.1	27.0	37.4	12.44
$4k\rightarrow 32k @ 80\%$	$2.270\times10^{22}$	18.5	25.0	38.3	11.00

Table 10: Comparison of models with different training curricula on long context QA tasks.



## How LMs use Long Contexts Have they already met our needs?

 How well recent long LLMs utilize long texts.

#### Models:

- Open: MPT-30B-Instruct (uses AliBi, 8k), LongChat-13B (16k, RoPE)
- Close: GPT-3.5-Turbo (4k & 16k), Claude-1.3 (100k)

#### • Task I: Multi-document QA task

- Input: a user query and k documents (2.7k tokens each), where exactly **one** document contains the answer.
- Variable: Total input context length and the position of relevant information.
- The query is put after the given context.

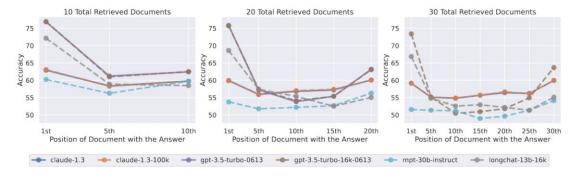


Figure 5: The effect of changing the position of relevant information (document containing the answer) on multidocument question answering performance. Lower positions are closer to the start of the input context. Performance is generally highest when relevant information is positioned at the very start or very end of the context, and rapidly degrades when models must reason over information in the middle of their input context.

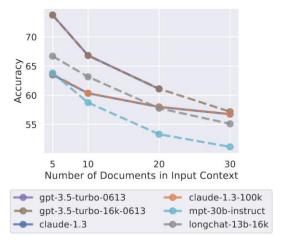


Figure 6: Language model performance (averaged across position of relevant information) on the multi-document question answering task decreases as the input context grows longer.

### How LMs use Long Contexts

• Task2: Synthetic Key-Value retrieval task

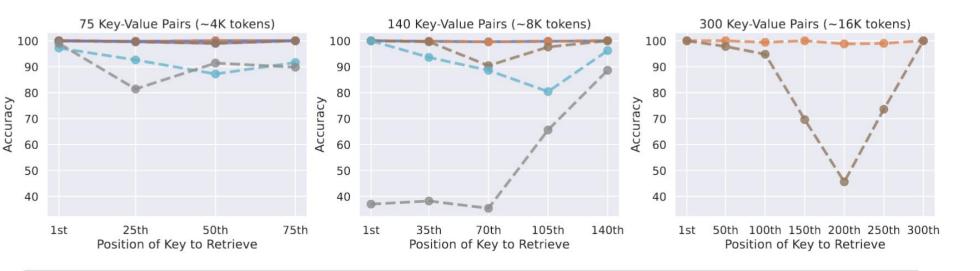
claude-1.3-100k

claude-1.3

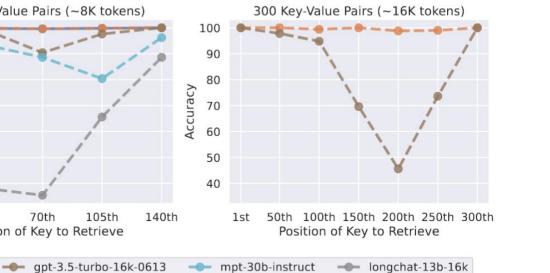
• A string-serialized JSON object with k key-value pairs.

--- gpt-3.5-turbo-0613

- Each key and value are randomly-generated UUIDs.
- A particular key to be extracted.

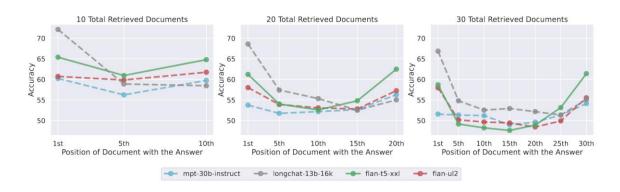






### Why LLMs "Lost in the Middle"?

- Model architecture:
  - Compare with the FLAN-T5 (Encoder-Decoder) model.



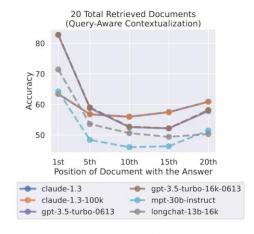
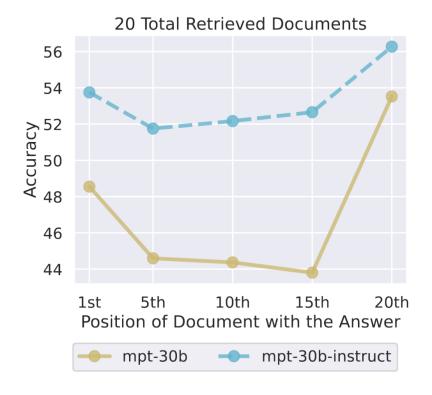


Figure 12: Query-aware contextualization (i.e., placing the question before *and* after the documents in the input context) improves multi-document QA performance when relevant information occurs at the very beginning, but slightly decreases performance otherwise.

- FLANs suffer less from the "lost in the middle" effect.
- Intuitively, since the query is put after the context, decoder-only LMs could not attend to the query when read through the context.
- Put the query at both the beginning and ending of the context could achieve nearly prefect performance.

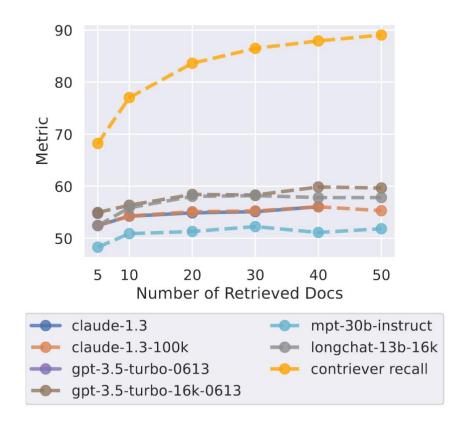
### Why LLMs "Lost in the Middle"?

- Effect of instruction tuning:
  - Pre-trained and instruction-tuned LMs exhibit similar U-shape patterns.
  - Contradict to the traditional wisdom about pretrained LMs that they suffer from recency bias, the result demonstrates that pre-trained LMs could effectively utilize long-range information.
  - Such ability may come from specific pre-training data, e.g., StackOverflow.



#### The more, the better?

- Experiment with standard retriever-reader setup.
  - A retrieval system takes an input query and return k documents from Wikipedia.
  - Evaluate reader accuracy and retriever recall.
- Using more than 20 documents only marginally improves the accuracy.
- It is a promising research direction for understanding and improving how LMs utilize a long information.



### Summary

- With the combination of existing techniques, we can effectively and efficiently expand the context length of pre-trained models.
- However, there are still several remained questions:
  - Whether there are more principled ways to design position encoding.
  - Window attention seems to be an expediency for long context modeling.
  - LLMs do not treat a long context equally, they "lost in the middle".

### Thanks!