HW3

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Note: look at return of the ind, not ind themselves

$x_t = F x_{t-1} + w_t$, $w_t \sim N(0, Q)$ ← State

$z_t = H x_t + V_t$, $V_t \sim N(0, R)$ ← Measurement

$= H(F x_{t-1} + w_t) + V_t$

$= H F x_{t-1} + H w_t + V_t$

opt state var $= N(x_{t|t}, P_{t|t})$, $K = P_{t|t-1} H^T (H P_{t|t-1} H^T + R)^{-1}$

$x_{t|t} = x_{t|t-1} + K_t (z_t - H x_{t|t-1})$

$P_{t|t} = P_{t|t-1} + K H P_{t|t-1}$

derivation of K: $\begin{pmatrix} x \\ z \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_x \\ \mu_z \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \right)$

$x \mid (z = \bar{z}) \sim N(\hat{\mu}, \hat{\Sigma})$

$\hat{\mu} = \mu_x + \Sigma_{12} \Sigma_{22}^{-1} (z - \mu_z)$

$\hat{\Sigma} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$

Fundamental knowledge ↑

$S_{t,AU} = \beta_t S_{t,CA} + v_t$, $v_t \sim N(0, \sigma_v^2)$ ← Measurement

$\beta_t = F_t \beta_{t-1} + w_t$, $w_t \sim N(0, Q_t)$ ← State

∴ opt. state cov. $= N(\hat{\beta}_{t|t}, \hat{P}_{t|t})$,

where $\hat{\beta}_{t|t} = \hat{\beta}_{t|t-1} + K_t (S_{t,AU} - S_{t,CA} \hat{\beta}_{t|t-1})$

$\hat{P}_{t|t} = \hat{P}_{t|t-1} - S_{t,CA} K \hat{P}_{t|t-1}$

$K_t = \hat{P}_{t|t-1} S_{t,CA}^T (S_{t,CA} \hat{P}_{t|t-1} S_{t,CA}^T + \sigma_v^2)^{-1}$

$$= \frac{\hat{P}_{t|t-1} S_{t,CA}^T}{S_{t,CA}^2 \cdot \hat{P}_{t|t-1} + \sigma_v^2} \quad \begin{matrix} / \hat{P}_{t|t-1} \\ / \hat{P}_{t|t-1} \end{matrix}$$

$$= \frac{S_{t,CA}}{S_{t,CA}^2 + \frac{\sigma_v^2}{\hat{P}_{t|t-1}}} \quad \left| \ \frac{\sigma_v^2}{\hat{P}_{t|t-1}} = \gamma^{-1} \right.$$

$$\therefore K_t = \frac{S_{t,CA}}{S_{t,CA}^2 + \gamma^{-1}}$$

$$\therefore \boxed{\hat{\beta}_{t|t} = \hat{\beta}_{t|t-1} + \frac{S_{t,CA}}{S_{t,CA}^2 + \gamma^{-1}} \cdot (S_{t,AU} - S_{t,CA} \hat{\beta}_{t|t-1})}$$

last formula on Pg 72. (QED.)

$$= \hat{\beta}_{t|t-1} \times 1 - \frac{S_{t,CA}^2 \hat{\beta}_{t|t-1}}{S_{t,CA}^2 + \gamma^{-1}} + \frac{S_{t,CA} S_{t,AU}}{S_{t,CA}^2 + \gamma^{-1}}$$

$$= \left(1 - \frac{S_{t,CA}^2}{S_{t,CA}^2 + \gamma^{-1}}\right) \hat{\beta}_{t|t-1} + \frac{S_{t,CA} S_{t,AU}}{S_{t,CA}^2 + \gamma^{-1}}$$

if large γ (SNR), γ^{-1} small, $\frac{a}{b + \gamma^{-1}} \to \frac{a}{b}$

$\therefore \hat{\beta}_{t|t} \doteq \frac{S_{t,AU}}{S_{t,CA}}$

if small γ (SNR), γ^{-1} large, $\frac{a}{b + \gamma^{-1}} \to 0$.

$\therefore \hat{\beta}_{t|t} \doteq \hat{\beta}_{t|t-1}$

? Kalman filter $\hat{\beta}_{t|t}$

Or use OLS: $S_{t,AU} = \beta S_{t,CA} + V_t$, $V_t \sim N(0, \sigma_v^2)$

↳ Rolling LR, lookback window of 6mo.

At EOD, update β w/ close, then find residual

∴ $V_t = S_{t,AU} - \beta S_{t,CA}$ & record σ_v^2.

$V_t > k \sigma_v^{\frac{1}{2}}$ → +CA −AU

$<$ → −CA +AU

} $k = 0.5, 1.2$ → arbitrary?

↳ do for OLS & Kalman filter, $k = 0.5$.