CS422 Robotics and Automation Assignment 1

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Question 1 Sol:

As the Figure 1 shown, the number of joints is 18 (6 universal, 6 ball and socket and 6 prismatic in the actuators), while the number of links is 14 (2 parts for each actuator, the end-effector, and the base).

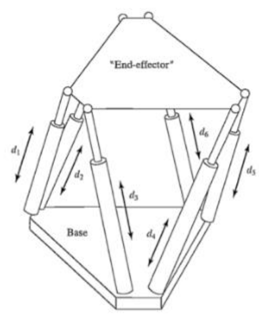

Figure 1 Six-degree-of-freedom Fully Parallel Manipulator

Finally, the sum of all the joint freedoms is 36. So, we can get

$$F=6*(14—18—1)+36=6. \tag{1.}$$

Question 2 Sol:

Step I, the vector P^A rotate about $\hat{Z}_A$ by θ degrees, the rotation matrix is

$$A_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.}$$

Step II, it is subsequently rotated about $\hat{X}_A$ by φ degrees, then, the rotation matrix is

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}. \tag{3.}$$

So, the rotation matrix which will accomplish these rotations in the given order would be

$$A = A_2 \cdot A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos\phi\sin\theta & \cos\phi\cos\theta & -\sin\phi \\ \sin\phi\sin\theta & \sin\phi\cos\theta & \cos\phi \end{bmatrix} \tag{4.}$$

Question 3 Sol:

As the question illustrated, this is a problem about the coordinate transformation. The frame B is initially coincident with the frame A.

The frame B_1 is obtained by rotating frame B about $\hat{Z}_B$ by θ degrees, which can be represented by the 3 × 3 matrix

$$A_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.}$$

The frame B_2 is obtained by rotating frame B_1 about $\hat{X}_B$ by θ degrees, which can also be represented by the 3 × 3 matrix

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{6.}$$

Then, the coordinates transformation equation is

$$P_A = A_1 \cdot A_2 \cdot P_B \tag{7.}$$

So, the rotation matrix which will change the description of vector P_B to P_A would be

$$R_A^B = A_1 \cdot A_2 = \begin{bmatrix} cos\ \theta & -sin\ \theta & 0 \\ sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} cos\ \theta & -cos\theta sin\theta & sin^2\theta \\ sin\ \theta & \cos^2\theta & -cos\theta sin\theta \\ 0 & sin\ \theta & cos\ \theta \end{bmatrix} \tag{8.}$$

Question 4 Sol:

As the Figure 1 shown, it is the frames at the corners of a wedge.

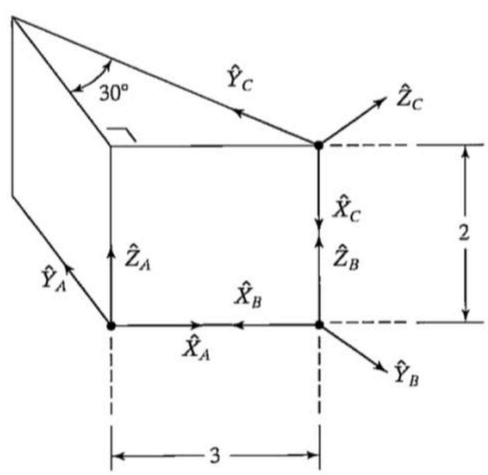

Figure 2 Frames at the corners of a wedge

So, we can get

$$H_B^A = \begin{bmatrix} \hat{x}_B \cdot \hat{x}_A & \hat{y}_B \cdot \hat{x}_A & \hat{z}_B \cdot \hat{x}_a & a \\ \hat{x}_B \cdot \hat{y}_A & \hat{y}_B \cdot \hat{y}_A & \hat{z}_B \cdot \hat{y}_A & b \\ \hat{x}_B \cdot \hat{z}_A & \hat{y}_B \cdot \hat{z}_A & \hat{z}_B \cdot \hat{z}_A & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9.}$$

Note that

$$P_{BORG}^A = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{10.}$$

$$\begin{cases} {}^{A}P = {}_{B}^{A}R\,{}^{B}P + {}^{A}P_{BORG} \\ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} & {}_{B}^{A}R & & {}^{A}P_{BORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} \end{cases} \tag{11.}$$

So, we can get

$$H_B^A = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12.}$$

$$H_C^A = \begin{bmatrix} 0 & -0.5 & 0.866 & 3 \\ 0 & 0.866 & 0.5 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13.}$$

$$H_C^B = \begin{bmatrix} 0 & 0.5 & -0.866 & 2 \\ 0 & -0.866 & -0.5 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14.}$$

$$H_A^C = \begin{bmatrix} 0 & 0 & -1 & -3 \\ -0.5 & 0.866 & 0 & -2 \\ 0.866 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15.}$$

$$H_A^B = \begin{bmatrix} -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16.}$$

Question 5 Sol:

We have

$$H_B^A = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17.}$$

Easily get (by MATLAB)

$$H_A^B = inv(H_B^A) = \begin{bmatrix} 0.2511 & 0.8638 & 0.4319 & 0.9040 \\ 0.4369 & -0.4970 & 0.7515 & -6.4271 \\ 0.8713 & 0.0026 & -0.5013 & -2.8632 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \tag{18.}$$

Or we can calculate directly

$$H_A^B = \begin{bmatrix} {}^A R_B^H & -{}^A R_B^H \cdot {}^A P_B \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.87 & 0.43 & 0.90 \\ 0.43 & -0.5 & 0.75 & -6.4 \\ 0.86 & 0.0 & -0.50 & -2.8 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{19.}$$