CS323 Robotics & Automation Assignment 2

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Q1 SHV 3-7 – Three-link Cartesian Robot (20pts):

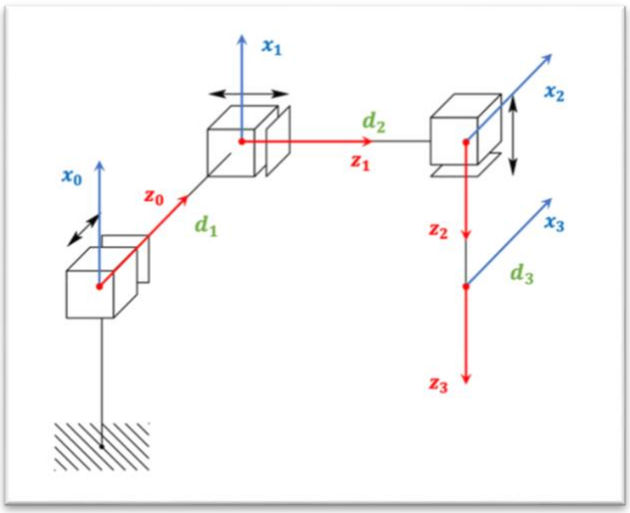

Figure 1 Three-link cartesian robot

Figure 1 shows the schematic of the manipulator with appropriately placed coordinate frames. And the Table 1 illustrates the summary of DH parameters of this manipulator.

Table 1 Summary of DH Parameters

#	θ	d	a	α
0-1	0	d_1	0	$-90°$
1-2	$90°$	d_2	0	$-90°$
2-3	0	d_3	0	0

The transformation matrix is:

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta 1 & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get:

$$ {}^{0}T_{1} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_{2} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

And we know,

$$ [{}^{0}T_{3}] = [{}^{0}T_{1}] \cdot [{}^{1}T_{2}] \cdot [{}^{2}T_{3}] $$

So, we get the final transformation matrix:

$$ [{}^{0}T_{3}] = \begin{bmatrix} 0 & 0 & -1 & \boldsymbol{-d3} \\ 0 & -1 & 0 & \boldsymbol{d2} \\ -1 & 0 & 0 & \boldsymbol{d1} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Ultimately, the x, y, and z components in the base frame of a unit vector pointing along the robot's last link is:

$$ [P] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} -d3 \\ d2 \\ d1 \\ 1 \end{bmatrix} $$

Q2 SHV 3-6 – Three-link Articulated Robot (20 points)

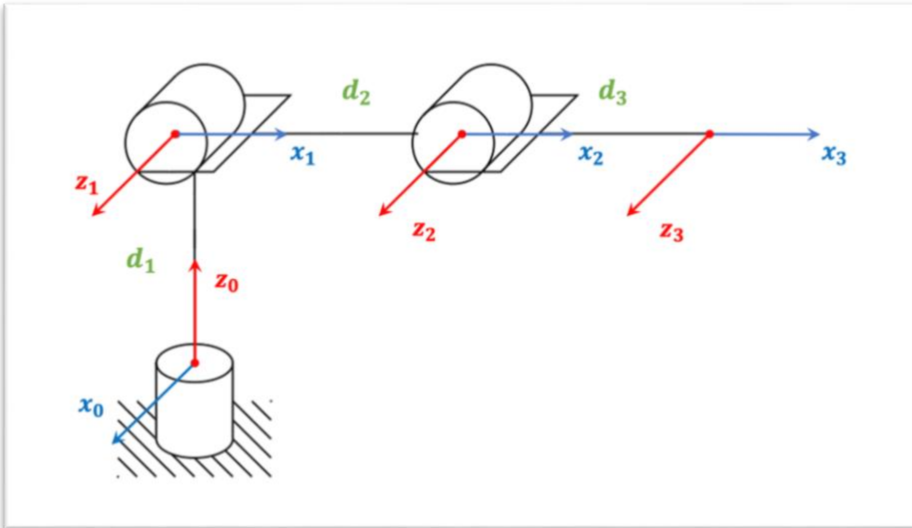

Figure 2 Three-link Articulated Robot

Figure 2 shows the schematic of the manipulator with appropriately placed coordinate frames. And the Table 2 illustrates the summary of DH parameters of this manipulator.

Table 2 Summary of DH Parameters

#	θ	d	a	α
0-1	$90°$	d_1	0	$90°$
1-2	0	0	d_2	0
2-3	0	0	d_3	0

The transformation matrix is:

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get:

$${}^{0}T_1 = \begin{bmatrix} \cos\theta 1 & 0 & 1 & 0 \\ \sin\theta 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{1}T_{2} = \begin{bmatrix} \cos\theta 2 & -sin\theta 2 & 0 & d2\cdot cos\theta 2 \\ \sin\theta 2 & cos\theta 2 & 0 & d2\cdot sin\theta 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_{3} = \begin{bmatrix} \cos\theta 2 & -sin\theta 2 & 0 & d3\cdot cos\theta 3 \\ \sin\theta 2 & cos\theta 2 & 0 & d3\cdot sin\theta 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

And we know,

$$
[{}^{0}T_{3}] = [{}^{0}T_{1}]\cdot[{}^{1}T_{2}]\cdot[{}^{2}T_{3}]
$$

So, we get the final transformation matrix:

$$
[{}^{0}T_{3}] = \begin{bmatrix} 0 & 0 & 1 & \mathbf{0} \\ 1 & 0 & 0 & \boldsymbol{d3+d2} \\ 0 & 1 & 0 & \boldsymbol{d1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Ultimately, the x, y, and z components in the base frame of a unit vector pointing along the robot's last link is:

$$
[P] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1(d_2\cos\theta_2 + d_3\sin(\theta_2+\theta_3)) \\ \sin\theta_1(d_2\cos\theta_1 + d_3\sin(\theta_2+\theta_3)) \\ d_1 + d_2\sin\theta_2 - d_3\cos(\theta_2+\theta_3)) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ d3+d2 \\ d1 \\ 1 \end{bmatrix}
$$