CS422 Robotics and Automation Assignment 4

Maynooth University, Siyuan Zhan PhD

Due 10th Dec. Late submissions will not be accepted*

This entire assignment is written and consists of two significantly adapted problems from the textbook, Robot Modeling and Control by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions on both questions.

1. **Three-link Cylindrical Manipulator (30 points)**
   The book works out the DH parameters and the transformation matrix T_3^0 for this robot in Example 3.2; you are welcome to use these results directly without rederiving them.

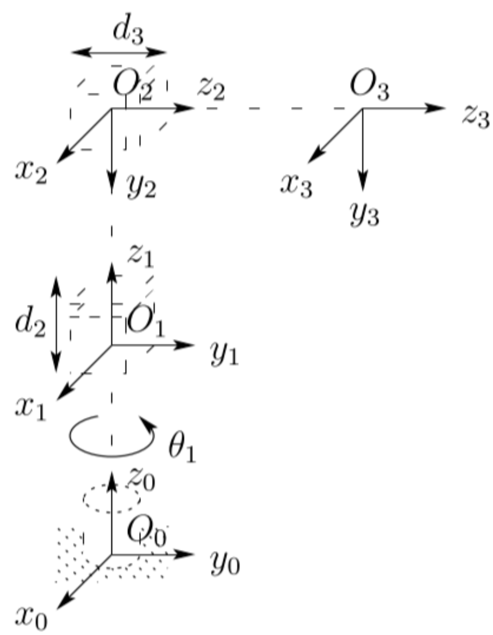

Figure 1: Three-link cylindrical manipulator

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

Table 1: Link DH parameters for 3-link cylindrical manipulator (* variables)

*Some of the assignment materials were adapted from the open materials created by Dr Katherine J. Kuchenbecker

T matrices are

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for the three-link cylindrical manipulator of Figure 1.

(b) Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

(c) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

(d) Find this robot's 6×3 Jacobian J.

(e) Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

(f) For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

(h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

2. **Three-link Spherical Manipulator (30 points)**
The book does not seem to work out the forward kinematics for this robot anywhere. Please use the diagram on the left side of the following Figure to define the positive joint directions and the zero configuration for the robot.

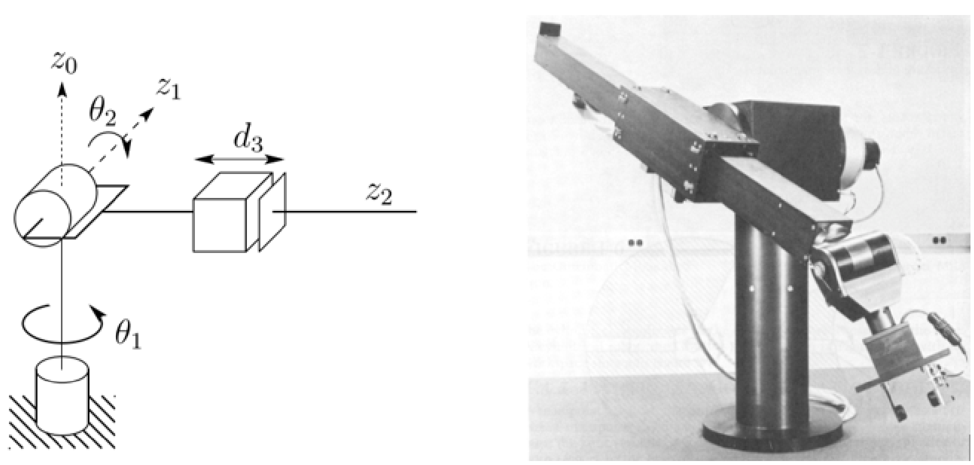

Figure 2: Left: the spherical manipulator; Right: an example of a particular design of spherical manipulator - the Stanford Arm

If we additionally choose the x_0 axis to point in the direction the robot arm points in the zero configuration, you can calculate that the tip of the spherical manipulator is at $\begin{bmatrix} x & y & z \end{bmatrix}^T = \begin{bmatrix} c_1 c_2 d_3 & s_1 c_2 d_3 & d_1 - s_2 d_3 \end{bmatrix}^T$. In this expression θ_1, θ_2, and d_3 are the joint variables; s_i is $\sin\theta_i$ and c_i is $\cos\theta_i$; and d_1 is a constant.

(a) Calculate the 3×3 linear velocity Jacobian J_v for the spherical manipulator with no offsets shown in the left side of Figure 2. You may use any method you choose.

(b) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

(c) Find this robot's 6×3 Jacobian J.

(d) Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0 , the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

(e) For the same configuration described in the previous question, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

(f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = \begin{bmatrix} 0 \text{ m/s} & 0.5 \text{ m/s} & 0.1 \text{ m/s} \end{bmatrix}^T$? Make sure to provide units with your answer.

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

(h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

(i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.