CS323 Robotics & Automation Assignment 4

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Q1 Adapted SHV 4-20, page 160– Three-link Cylindrical Manipulator

(a) Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian $\mathrm{J_v}$ for the three-link cylindrical manipulator of Figure 3-7 on page 85.

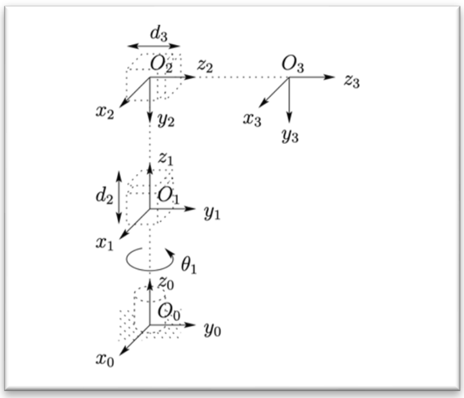

Figure 1 Three-link cylindrical manipulator

Figure 1 shows the schematic of the three-link cylindrical manipulator. We can easily get that

$$\begin{cases} x = -s_1 d_3 \\ y = c_1 d_3 \\ z = d_2 \end{cases}$$

By differential transformation, we can get the linear velocity Jacobian matrix as follows

$$J_v = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\partial q_3} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b) Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

As for **Revolute Joints**, the linear velocity is $J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$.

As for **Prismatic Joints**, the linear velocity is $J_{v_i} = z_{i-1}$.

When $\mathrm{i} = 1, \mathrm{n} = 3,\ J_{v1} = Z_0(O_3 - O_0)$, then

$$J_{v1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} -s\theta_1 d_3 \\ c\theta_1 d_3 \\ d_1 + d_2 \end{pmatrix} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ -s_1 d_3 & c_1 d_3 & d_1 + d_2 \end{vmatrix}$$

$$J_{v1} = -\mathrm{c}_1 d_3 i - s_1 d_3 j + 0k$$

When $\mathrm{i} = 2, \mathrm{n} = 3,\ J_{v1} = Z_1$

$$J_{v2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

When $\mathrm{i} = 3, \mathrm{n} = 3,\ J_{v1} = Z_2$

$$J_{v3} = \begin{pmatrix} -s_1 \\ c_1 \\ 0 \end{pmatrix}$$

Then, we can get the Jacobian matric as follows, which is the same as (a).

$$\therefore J_v = \begin{bmatrix} J_{v_1} \\ J_{v_2} \\ J_{v_3} \end{bmatrix} = \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

As for the angular velocity

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{revoluate} \\ 0 & \text{prismatic} \end{cases}$$

We can easily get that

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathbf{1} & 0 & 0 \end{bmatrix}$$

(d) Find this robot's 6×3 Jacobian J.

We easily combine the J_v and J_ω, hence

$$J_{6\times 3} = \begin{bmatrix} -c1d3 & 0 & -s1 \\ -s1d3 & 0 & c1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(e) Imagine this robot is at $\theta_1 = \pi/2\text{rad}, d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1\text{rad/s}, \dot{\theta}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

$$v_3^0 = J_v\dot{q} = \begin{bmatrix} 0 & 0 & -1 \\ -0.3 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0.05 \\ -0.03 \\ 0.25 \end{bmatrix}\text{m/s}$$

(f) For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

$$\omega_3^0 = J_\omega\dot{q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}\text{rad/s}$$

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

$$\det(J) = 0 = \det\begin{bmatrix} -c_1d_3 & 0 & -s_1 \\ -s_1d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix} = c_1^2d_3 + s_1^2d_3 = d_3$$

Hence, when $d_3 = 0$, the robot arm gets the singular configurations.

(h) Sketch the cylindrical manipulator in each singular configuration that you found and explain what effect the singularity has on the robot's motion in that configuration.

As the following Figure 2 shown, when the manipulator is in the singular configuration, no matter how the Joint 0 turn around, the end-effector would not take any movement.

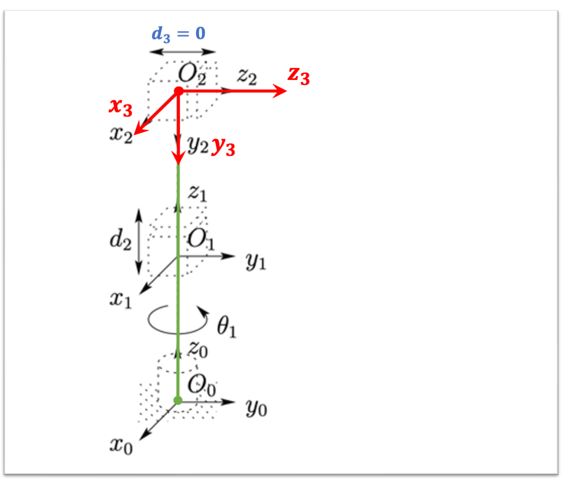

Figure 2 Manipulator in singular configuration

Q2 Adapted SHV 4-18, page 160–Three-link Spherical Manipulator

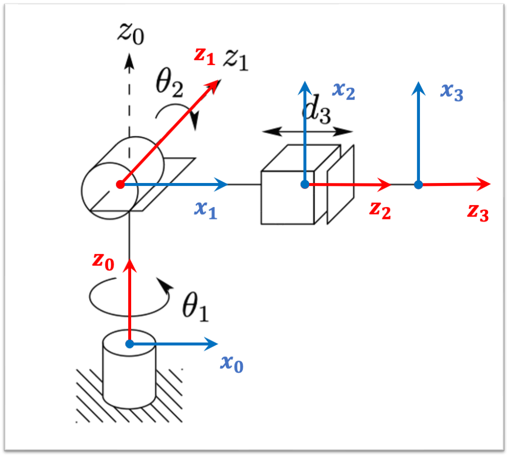

Figure 3 Three-link Spherical Robot

Figure 3 shows the schematic of the spherical manipulator with appropriately placed coordinate frames. And the Table 1 illustrates the summary of DH parameters of this manipulator.

Table 1 Summary of DH Parameters

#	θ	d	a	α
0-1	θ_1	d_1	0	90°
1-2	$\theta_2 - 90°$	0	0	90°
2-3	0°	d_3	0	0°

The transformation matrix is

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta \cdot \cos\alpha & \sin\theta \cdot \sin\alpha & a \cdot \cos\theta \\ \sin\theta & \cos\theta \cdot \cos\alpha & -\cos\theta \cdot \sin\alpha & a \cdot \sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get that:

$$^0T_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} s_2 & 0 & c_2 & 0 \\ -c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And,

$$
{}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Now, we know that

$$
{}^{0}T_{2} = {}^{0}T_{1}\,{}^{1}T_{2} = \begin{bmatrix} c_1 s_2 & -s_1 & c_1 c_2 & 0 \\ s_1 s_2 & c_1 & s_1 c_2 & 0 \\ c_2 & 0 & -s_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_{3} = {}^{0}T_{2}\,{}^{2}T_{3} = \begin{bmatrix} c_1 c_2 & -s_1 & -c_1 s_2 & c_1 c_2 d_3 \\ s_1 c_2 & c_1 & -s_1 s_2 & s_1 c_2 d_3 \\ c_2 & 0 & -s_2 & d_1 - s_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Based on the Equations above, we can easily get

$$
o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_3 = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ d_1 - s_2 d_3 \end{bmatrix}
$$

$$
z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, z_2 = \begin{bmatrix} c_1 c_2 \\ s_1 c_2 \\ -s_2 \end{bmatrix}
$$

(a) Calculate the 3×3 linear velocity Jacobian J_v for the spherical manipulator with no offsets shown in the left side of Figure 1.12 on page 15 of SHV. You may use any method you choose.

The spherical robot consists of two revolute joints and one prismatic joint. So the linear velocity Jacobian J_v is

$$
J_v = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\partial q_3} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -s_1 c_2 d_3 & -c_1 s_2 d_3 & c_1 c_2 \\ c_1 c_2 d_3 & -s_1 s_2 d_3 & s_1 c_2 \\ 0 & -c_2 d_3 & -s_2 \end{bmatrix}
$$

(b) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

As for the angular velocity

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{revoluate} \\ 0 & \text{prismatic} \end{cases}$$

We can easily get that

$$J_\omega = \begin{bmatrix} 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(c) Find this robot's 6×3 Jacobian J.

$$J = \begin{bmatrix} -s_1 c_2 d_3 & -c_1 s_2 d_3 & c_1 c_2 \\ c_1 c_2 d_3 & -s_1 s_2 d_3 & s_1 c_2 \\ 0 & -c_2 d_3 & -s_2 \\ 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(d) Imagine this robot is at $\theta_1 = \pi/4 \text{rad}, \theta_2 = 0 \text{rad}$, and $d_3 = 1$ m. What is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1, \dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

$$\begin{cases} \theta_1 = \dfrac{\pi}{4} \\ \theta_2 = 0 \\ d_3 = 1 \end{cases}$$

$$\omega_3^0 = J_\omega \dot{q} = \begin{bmatrix} 0 & -\dfrac{\sqrt{2}}{2} & 0 \\ 0 & \dfrac{\sqrt{2}}{2} & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2} \dot{\theta}_2 \\ \dfrac{\sqrt{2}}{2} \dot{\theta}_2 \\ \dot{\theta}_1 \end{bmatrix} \text{rad/s}$$

(e) For the same configuration described in the previous question, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1, \dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

$$v_3^0 = J_v \dot{q} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \\ \dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \\ 0 & -\dfrac{\sqrt{2}}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2} \dot{\theta}_1 + \dfrac{\sqrt{2}}{2} \dot{d}_3 \\ \dfrac{\sqrt{2}}{2} \dot{\theta}_1 + \dfrac{\sqrt{2}}{2} \dot{d}_3 \\ -\dfrac{\sqrt{2}}{2} \dot{\theta}_2 \end{bmatrix} \text{m/s}$$

(f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = [0 \text{ m/s} \quad 0.5 \text{ m/s}\ 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.

We have already known that

$$v_3^0 = \begin{bmatrix} 0\text{m/s} \\ 0.5\text{m/s} \\ 0.1\text{m/s} \end{bmatrix}$$

Hence, convert the linear velocity into angular velocity,

$$v_3^0 = \begin{bmatrix} 0 \text{ m/s} \\ 0.5 \text{ m/s} \\ 0.1 \text{ m/s} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ \frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ -\frac{\sqrt{2}}{2}\dot{\theta}_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{4} \text{ rad/s} \\ \frac{\sqrt{2}}{10} \text{ rad/s} \\ \frac{\sqrt{2}}{4} \text{ m/s} \end{bmatrix} = \omega_3^0$$

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

Not easy to calculate that

$$\det(J) = 0 = \det \begin{bmatrix} -s_1c_2d_3 & -c_1s_2d_3 & c_1c_2 \\ c_1c_2d_3 & -s_1s_2d_3 & s_1c_2 \\ 0 & -c_2d_3 & -s_2 \end{bmatrix} = -c_2 \cdot (\mathrm{d}_3)^2$$

So, when $c_2 = 0$ or $d_3 = 0$, then $\det(J) = 0$. **That is when,**

$$\begin{cases} \theta_2 = \pm\frac{\pi}{2} \\ \text{or} \\ d_3 = 0 \end{cases}$$

The robot arm will get the singular configurations.

(h) Sketch the cylindrical manipulator in each singular configuration that you found and explain what effect the singularity has on the robot's motion in that configuration.

As we mention in the question above, when $c_2 = 0$ or $d_3 = 0$, then the robot arm gets the singular configurations. The following Figure 4,5,6 show the sketches of the configuration.

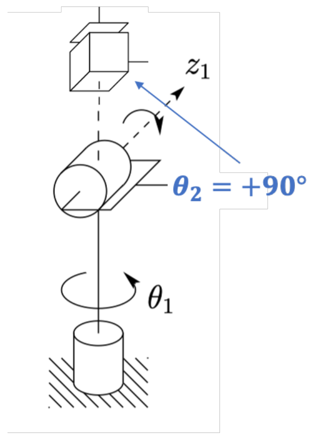

Figure 4 Three-link Spherical Robot

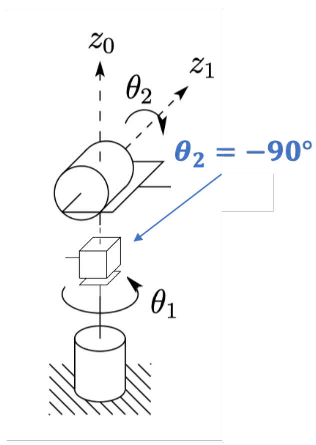

Figure 5 Three-link Spherical Robot

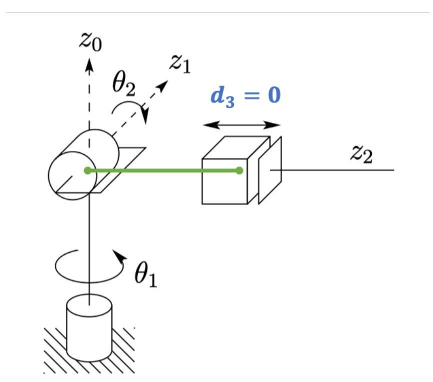

Figure 6 Three-link Spherical Robot

(i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

Noted that singularity of the robot exists objectively, we cannot avoid the singular configurations through changing the direction or zero configuration. So in practice, we need to carefully program to prevent robots from getting into 'trouble', as shown in Figure 7.

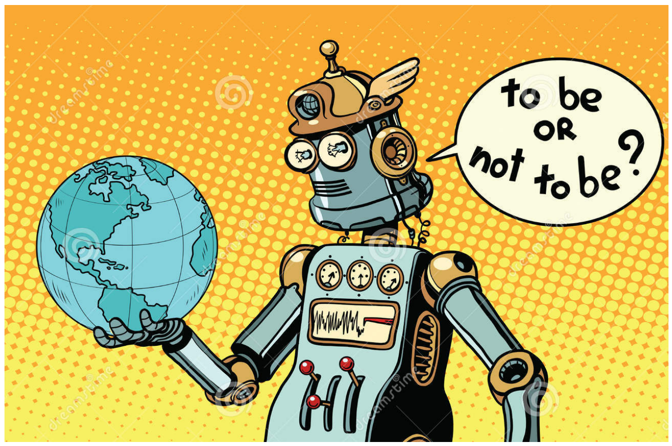

Figure 7 Robotics

Acknowledgements

This is the last assignment for this course. I gratefully acknowledge Dr. Siyuan Zhan for his generous guidance and encouragement during the EE323FZ course. Also, I would like to thank the tutor who carefully evaluate this report.

Thank you! Take care and keep warm!

Hanlin CAI

Dec 17th, 2022