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EE211 MATLAB Assignment 3:

Personal Statement

Alternatives： A MATLAB/Simulink environment

The data set given to me by the TA is shown below:

A = 4; B_1 = 10; B_2 =25

All the Code & Picture were designed and created by myself, and I never share with others.

Procedure 1

The figure of the dynamical system in this assign3 is shown below:

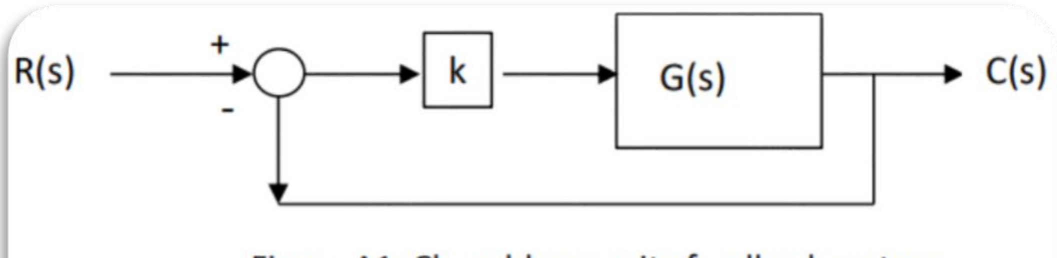


Figure A1: Closed-loop unity feedback system

Fig-1

And its transfer function G(s) is given:

$$G(s) = A\frac{s-1}{s^2 + B_1 * s + B_2}$$

Where A = 4; B_1 = 10; B_2 =25

So, the transfer function is:

$$G(s) = \frac{4s - 4}{s^2 + 10s + 25}$$

Firstly, we split the transfer function into 2 parts by defining an intermediate variable Z(s) as follows:

$$\frac{Z(s)}{U(s)} = \frac{1}{s^2 + 10s + 25}$$

$$\frac{Y(s)}{Z(s)} = 4s - 4$$

Then, converting to the time domain gives:

$$\ddot{z} + 10\dot{z} + 25z = u$$

$$4\dot{z} - 4z \ = y$$

Setting the states as

$$\begin{aligned} x_1 &= z \\ x_2 &= \dot{z} \end{aligned}$$

Then we get the state equation as follows:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -25x_1 - 10x_2 + u$$

Giving:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$[y] = [-4 \ \ 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

So, we know that:

$$A = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -4 & 4 \end{bmatrix}$$

$$D = 0$$

Procedure 2

Through Producer1, we have obtained that:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} -4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

So, we can calculate the eigenvalues of the state space system:

$$\begin{vmatrix} \lambda & -1 \\ 25 & \lambda + 10 \end{vmatrix} = 0$$

That is：

$$\lambda^2 + 10\lambda + 25 = 0$$

So, the eigenvalues are:

$$\lambda_1 = \lambda_2 = -5$$

Procedure 3

Through the producer1, we have got the transfer function:

$$G(s) = \frac{4s - 4}{s^2 + 10s + 25}$$

Then, we can calculate the poles of the transfer function:

$$s^2 + 10s + 25 = 0$$

So, we can easily get the poles:

$$s_1 = s_2 = -5$$

Just the same as the eigenvalues calculated in Procedure 2:

$$s_1 = s_2 = -5 = \lambda_1 = \lambda_2$$

Procedure 4

Due to the fact that all the eigenvalues are:

$$Re(\lambda) < 0$$

And the poles of the dynamical system are:

$$s_1 = s_2 = -5 < 0$$

Hence, the dynamical system is asymptotically stable and critically damped.

Procedure 5

Through Procedure1, we have obtained that:

$$A = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -4 & 4 \end{bmatrix} \qquad D = 0$$

So, we use the SIMULINK for simulation:

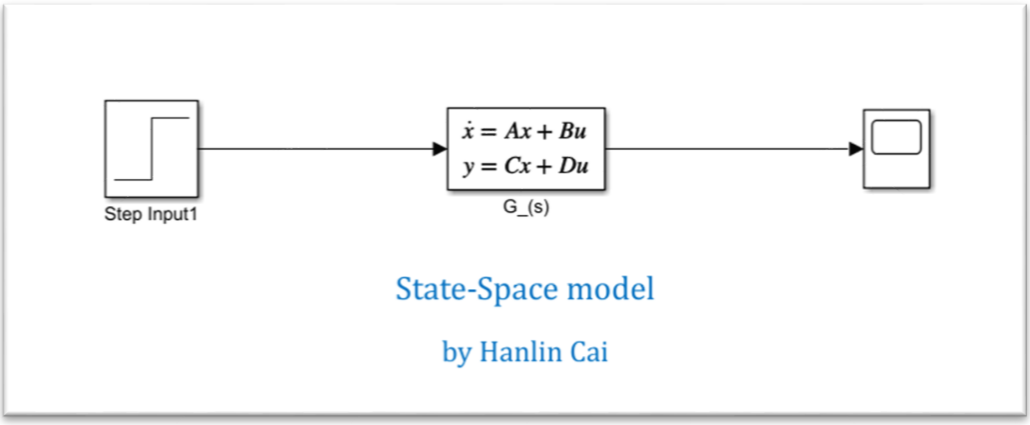


Fig-2

Then we can get the Scope of the transfer function:

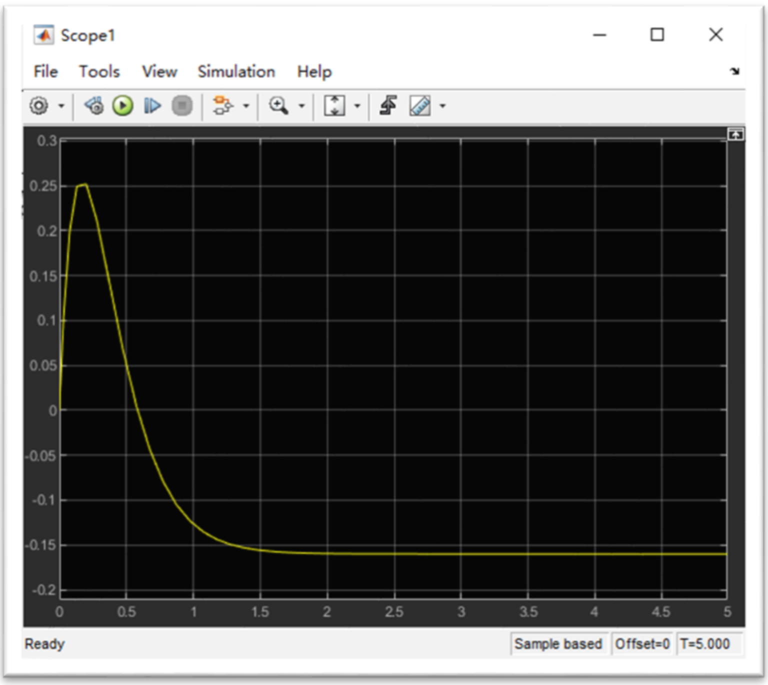


Fig-3

When we change the system input:

$$when\ system\ input = 0$$

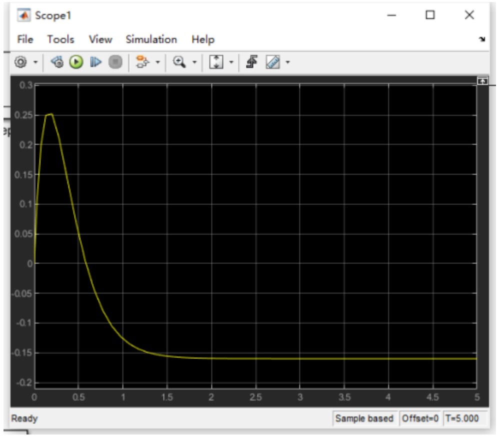


Fig-4

$$when\ system\ input = -2$$

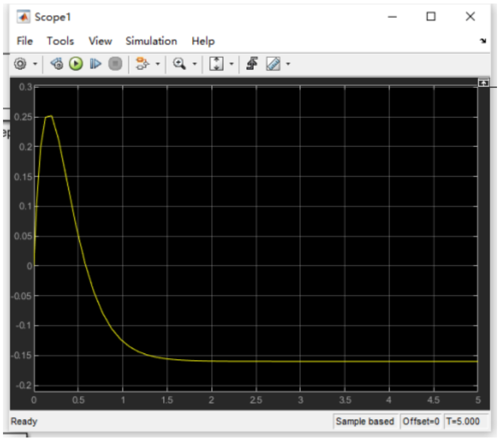


Fig-5

$$when\ system\ input = +100$$

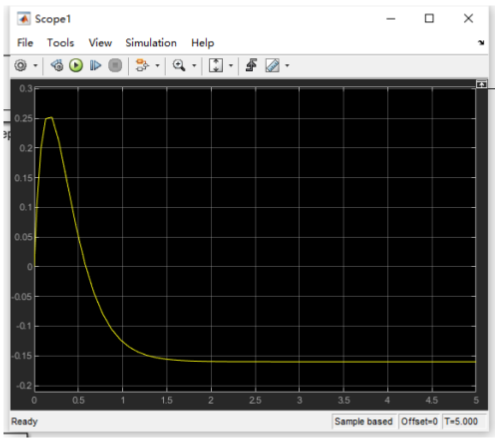


Fig-6

So, we can conclude that stability is independent of the system input.

We have known that the figure of the dynamical system:

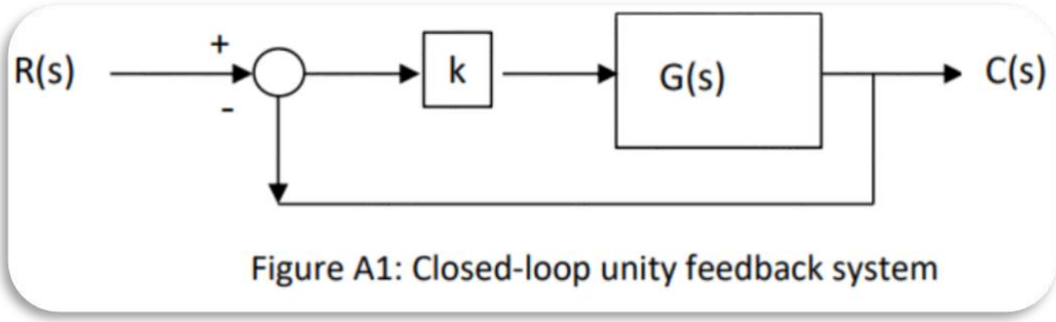


Fig-7

Then, we can use SIMULINK for system simulation:

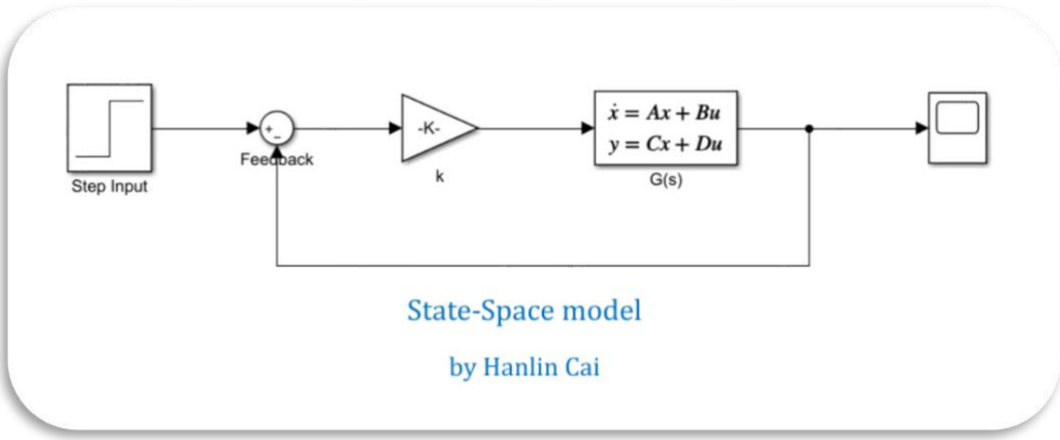


Fig-8

When the $k = 2$:

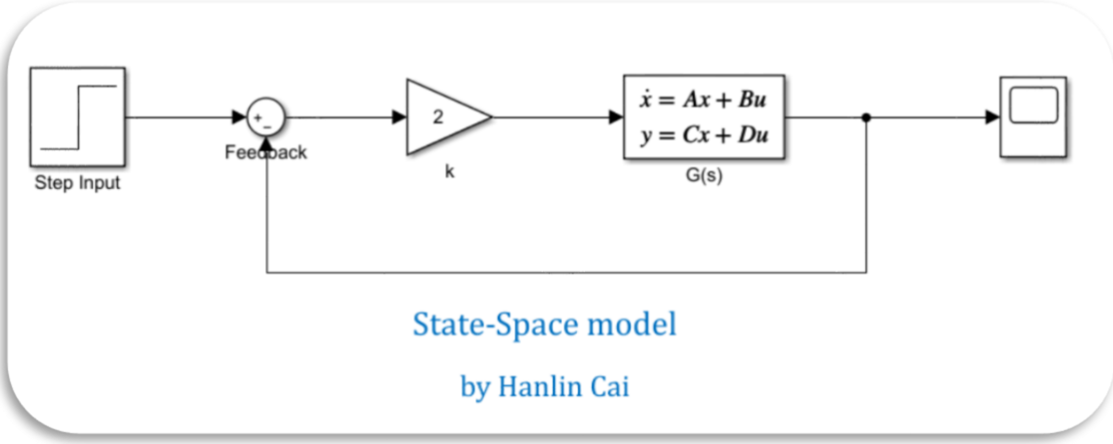


Fig-9

We can get the Scope of the system, as Fig-10 shown, the system is asymptotically stable:

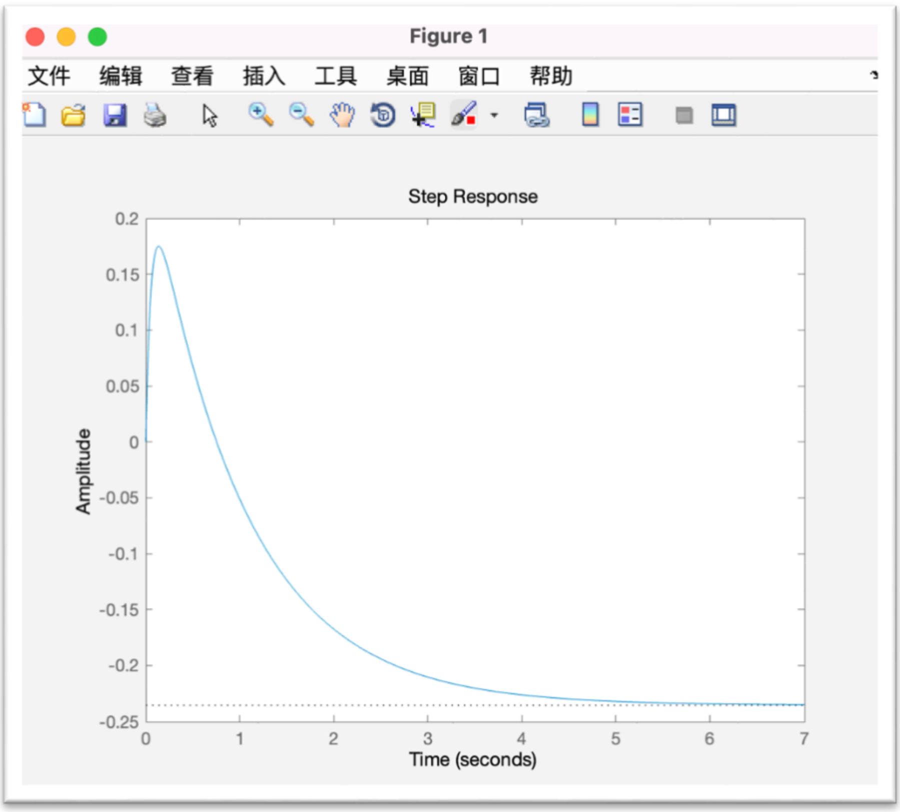


Fig-10

As Fig-10 shown, the dynamical system is asymptotically stable.

Procedure 7

We have obtained the SIMULINK simulation for the system:

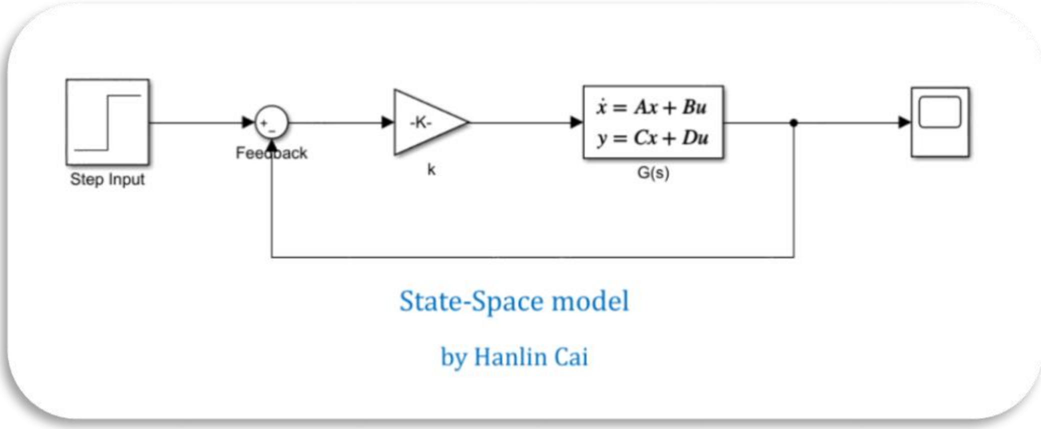


Fig-11

When we change the value of k, we can get different Scope:

$$when\ k = +25/4$$

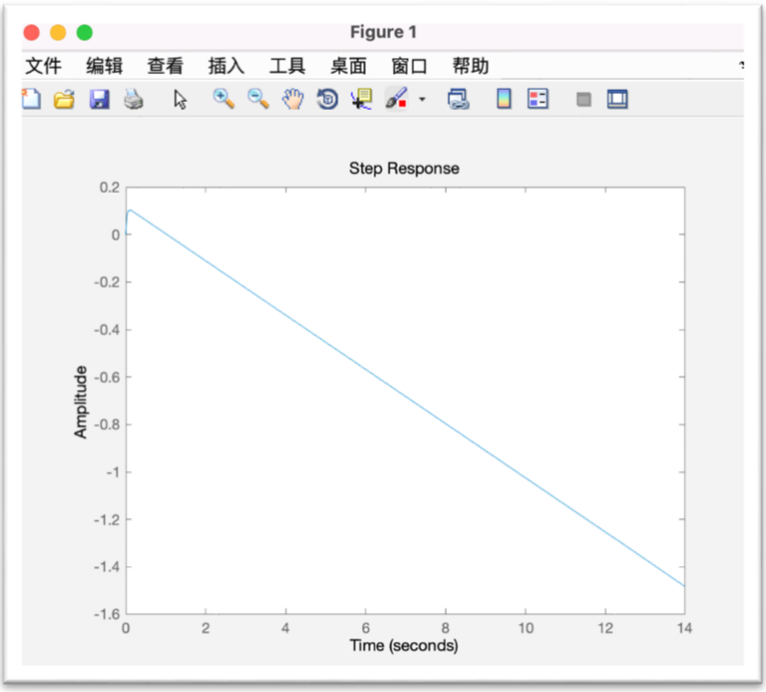


Fig-12 Already Overdamped

$when\ k = -5$

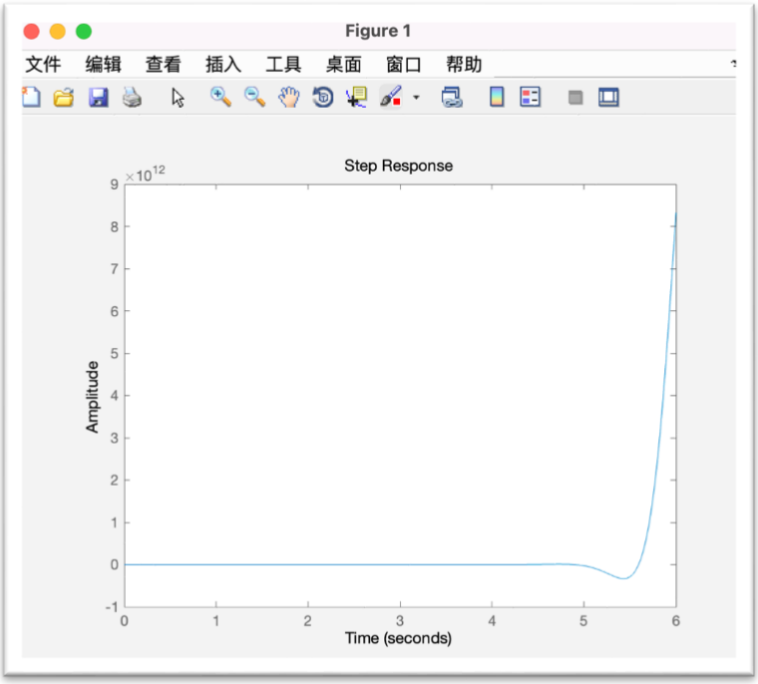


Fig-13 Overdamped

$when\ k = -5/2$

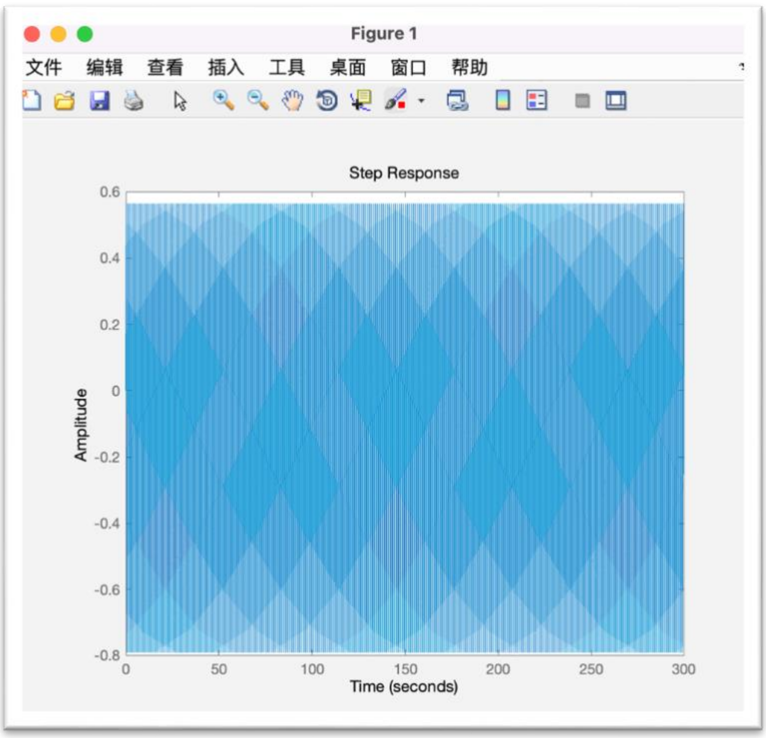


Fig-14 Marginally Stable

$$when\ k = +10$$

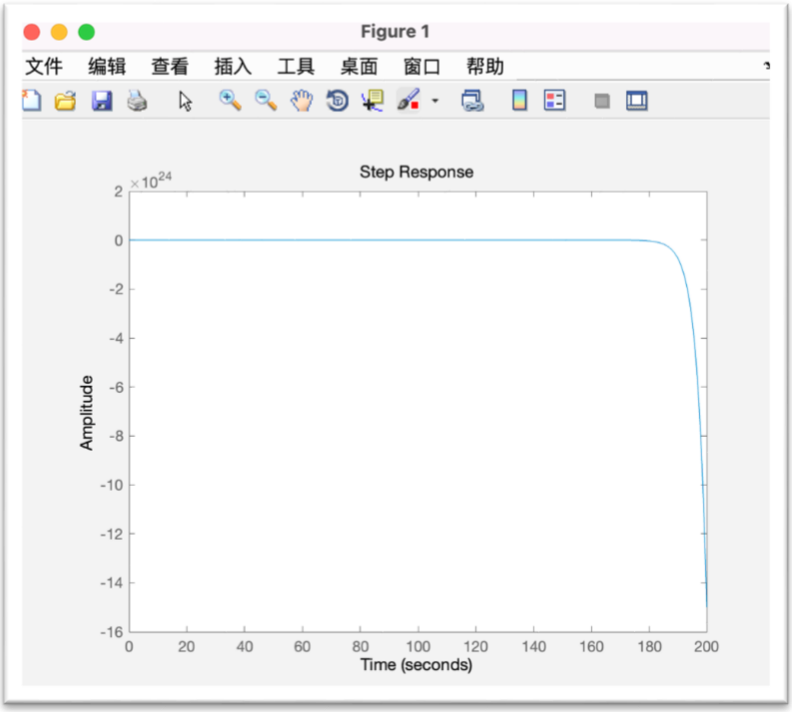


Fig-15 Overdamped

Comment:

As the Fig10-15 shown, when we change different value of k, the transient response will change. When k $is\ in\ the\ scope$ $(-\frac{5}{2}, +\frac{25}{4})$, the system is asymptotically stable.

In this scope, when k become larger, the transient response will become weaker, and the system tends to be overdamped. (When $k = \frac{25}{4}$, the system will already become overdamped.)

Meanwhile, when k become smaller, the transient response will become stronger, and the system tends to be overdamped, too. (When $k = -\frac{5}{2}$, the system is marginally stable.)

Overall, when $k\ is\ in$ $(-\frac{5}{2}, +\frac{25}{4})$, the system is asymptotically stable. And when $k\ is\ out\ of$ $(-\frac{5}{2}, +\frac{25}{4})$, the system is unstable.

Procedure 8 & Procedure 9

We have let $k = 2$, and the figure of the closed-loop transfer function (CLTF) for the system is given as Fig-16. So, we can calculate the poles of the system by MATLAB.

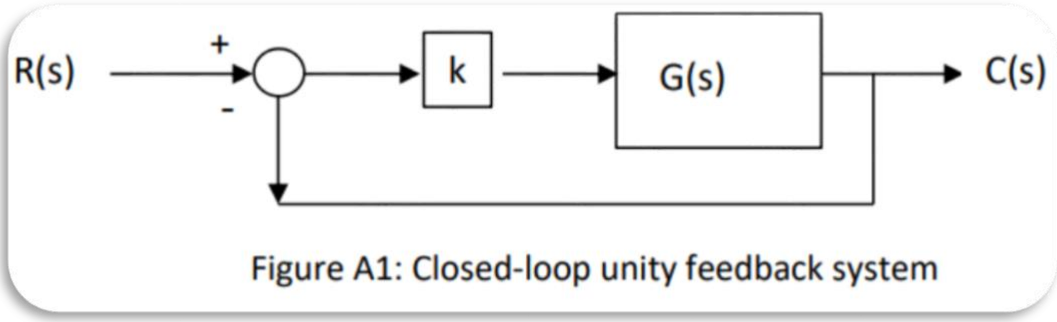


Figure A1: Closed-loop unity feedback system

Fig-16

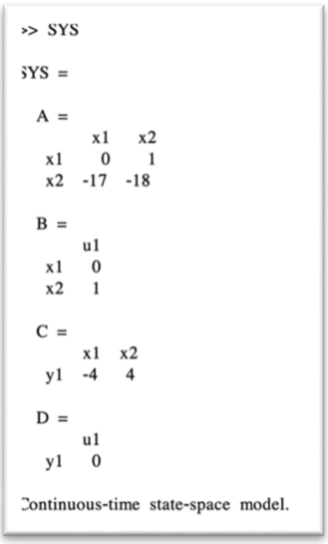
Through MATLAB Code shown below, through 'root' function, we can get the poles:

$$pole_1 = -5$$

$$pole_2 = -5$$

Because $poles < 0$, so the system is asymptotically stable.

And through the 'feedback' function, we can get the state-space representation for the CLTF for the system, as Fig-17 shown.



```
>> SYS

SYS =

  A =
          x1    x2
    x1     0     1
    x2   -17   -18

  B =
         u1
    x1    0
    x2    1

  C =
         x1   x2
    y1   -4    4

  D =
         u1
    y1    0

Continuous-time state-space model.
```

Fig-17

So, the state-space representation for the CLTF for the system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -17 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$[y] = [-4 \;\; 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

And through the ‘eig’ function, we can get the eigenvalues of this system:

$$eigvalue_1 = -1$$

$$eigvalue_2 = -17$$

Due to the fact that:

$$All \ Re(\lambda) < 0$$

So, we can draw the conclusion that:

The dynamical system is asymptotically stable.

MATLAB Code:

```
%% Assign3 Hanlin Cai 832002117
% The dataset given to me by the TA is: A = 4; B1 = 10; B2 =25
% Statement: All the Code and Pic were created by myself, and
I never share with others.

%% Q4-Q5
% 建立系统状态空间表达式模型 State-Space Modelling
A = [0 1 ; -25 -10];
B = [0;1];
C = [-4 4];
D = 0;
G = ss(A,B,C,D);  % Continuous-time state-space model.
% step(G,5)   % using a unit step input
```

```
% 系统多项式形式的传递函数模型 sys1
sys1 = tf(G);  % Continuous-time transfer function.

% 系统零-极点形式的传递函数模型 sys2
sys2 = zpk(sys1);  % Continuous-time zero/pole/gain model.

% 系统传递函数多项式形式的分子、分母多项式系数
[num ,den] = ss2tf(A,B,C,D);  % Numerator and denominator polynomial coefficients

% 系统零-极点: z 为零点, p 为极点
[z,p] = ss2zp(A,B,C,D);  % z is the zero point, and p is the pole.


%% Q8
% calculate the poles of the closed loop system.

syms s;
sys11 = s^2 + 10*s + 25;
% root(sys11,s);
poles = solve(sys11,s);  % poles = -5 & -5
%% Q9
% determine a state-space representation for the closed-loop transfer function (CLTF) for the system in Fig.A1 \
% and calculate the eigenvalues of this system.

k = 2;  % when k = 2, the system is asymptotically stable
sign = -1;
SYS = feedback(G,k,sign);
SYS1 = tf(SYS);  % Continuous-time transfer function.
eigvalues = eig(SYS);  % when k = 2, eigvalues = -1 & -17

%% EE211 Assignment3 MATLAB code by Hanlin Cai
```

A summary of what I gained in this Assignment3

Through this assignment, I get familiar with how to simulate a complex model by **MATLAB/Simulink**. Although this assignment was very difficult for me at the beginning, I finally finished it well through independent study and continuous trying.

Acknowledgement

I would like to thank my professor, **Zhicong Chen**, and my TA, **Honghui Chen.** I'm really appreciate for your grateful patience, hard-work, and wisdom! Without your help, I could not finish this difficult assignment3 well. Thank you so much!