

Tutorial 4

Find the Laplace Transform of the following functions.

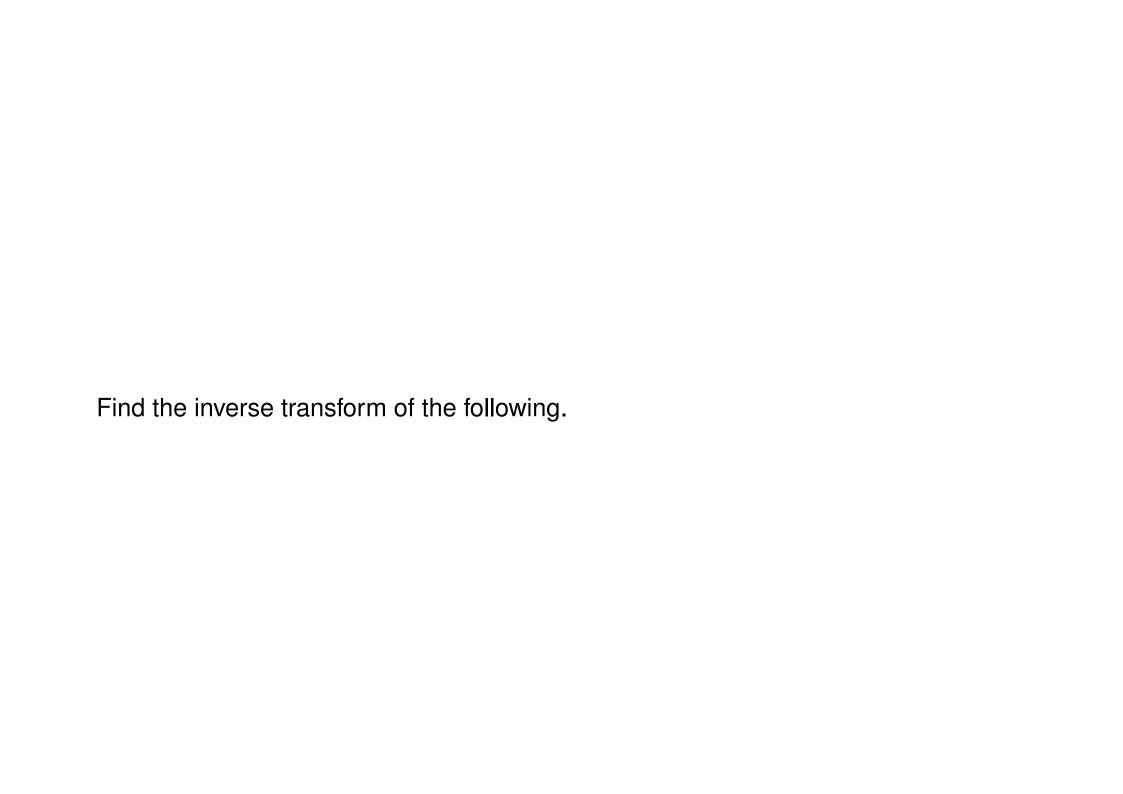
Problem 1b: $f(t) = (1 + e^{2t})^2$

$$\mathcal{L}{f(t)} = \mathcal{L}{(1 + e^{2t})^2}$$

$$= \mathcal{L}{1 + 2e^{2t} + e^{4t}}$$

$$= \mathcal{L}{1} + 2\mathcal{L}{e^{2t}} + \mathcal{L}{e^{4t}}$$

$$= \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$$



Problem 2b: $\mathcal{L}^{-1} \left\{ \frac{1}{s^3 + 5s} - \frac{48}{s^5} \right\}$

$$\frac{1}{s^3 + 5s} = \frac{A}{s} + \frac{Bs + C}{s^2 + 5}$$

$$1 = As^2 + 5A + Bs^2 + Cs$$

$$5A = 1 \implies A = \frac{1}{5}$$

$$C = 0$$

$$A + B = 0 \implies B = -A \implies B = -\frac{1}{5}$$

$$\frac{1}{s^3 + 5s} = \frac{\frac{1}{5}}{s} + \frac{-\frac{1}{5}s}{s^2 + 5}$$

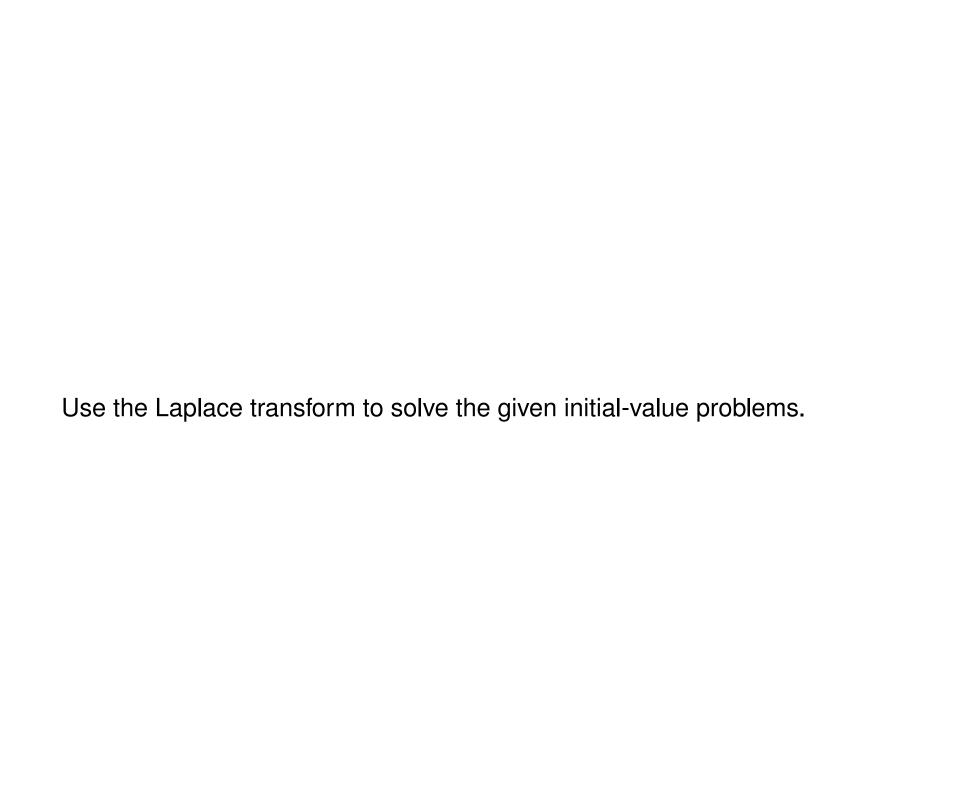
$$\mathcal{L}^{-1} \left\{ \frac{1}{s^3 + 5s} - \frac{48}{s^5} \right\} = \mathcal{L}^{-1} \left\{ \frac{1}{s^3 + 5s} \right\} - \mathcal{L}^{-1} \left\{ \frac{48}{s^5} \right\}$$

$$= \mathcal{L}^{-1} \left\{ \frac{\frac{1}{5}}{s} + \frac{-\frac{1}{5}s}{s^2 + 5} \right\} - \mathcal{L}^{-1} \left\{ \frac{48}{s^5} \right\}$$

Using
$$\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$$
 and $\mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos kt$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s} - \frac{48}{s^5}\right\} = \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+5}\right\} - 2\mathcal{L}^{-1}\left\{\frac{4!}{s^5}\right\}$$

$$= \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t - 2t^4$$



Problem 3a:
$$y'' + 5y' + 4y = 0$$
, $y(0) = 1$, $y'(0) = 0$

$$\mathcal{L}\{y'' + 5y' + 4y = 0\} = \mathcal{L}\{y''\} + 5\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 0$$

$$s^{2}Y(s) - sy(0) - y'(0) + 5sY(s) - 5y(0) + 4Y(s) = 0$$

$$(s^{2} + 5s + 4)Y(s) - s - 5 = 0$$

$$Y(s) = \frac{s + 5}{s^{2} + 5s + 4}$$

$$= \frac{s + 5}{(s + 4)(s + 1)}$$

$$\frac{A}{s + 4} + \frac{B}{s + 1} = \frac{A(s + 1) + B(s + 4)}{(s + 4)(s + 1)} = \frac{s + 5}{(s + 4)(s + 1)}$$

$$s = -1 \quad A(0) + B(3) = 4 \quad \Rightarrow B = \frac{4}{3}$$

$$s = -4 \quad A(-3) + B(0) = 1 \quad \Rightarrow A = -\frac{1}{3}$$

$$Y(s) = -\left(\frac{1}{3}\right)\frac{1}{s + 4} + \left(\frac{4}{3}\right)\frac{1}{s + 1}$$

$$y(t) = -\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\} + \frac{4}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$
$$= -\frac{1}{3}e^{-4t} + \frac{4}{3}e^{-t}$$