Tutorial Sheet 2 – Control & Dynamical Systems

Q1 (i) Given that viscous friction is proportional to velocity squared show that the dynamic model for a skydiver free falling is given by:

$$M\frac{dv}{dt} + Bv^2 = Mg$$

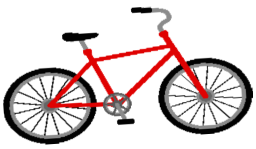
where M is the mass of the skydiver, g is the acceleration due to gravity, v is the velocity of the skydiver and B is the coefficient of friction. Hence calculate the formula for terminal velocity, v_T (*hint: terminal velocity occurs when the rate of change of velocity, i.e. acceleration, is 0*).

(ii) Given that a 90 kg skydiver free falls at a terminal velocity of 55 m/s (approx. 120 mph), determine the value of the coefficient of friction.

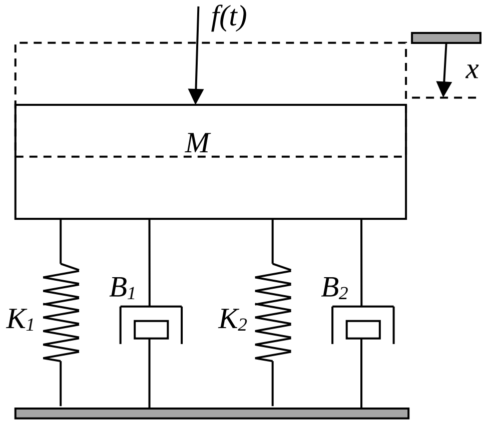
(iii) Assuming B is proportional to the surface area perpendicular to the direction of motion, estimate the velocity the skydiver can achieve by going into a vertical dive which results in a reduction in surface area from 0.8 m^2 to 0.2 m^2.

(iv) When the skydiver's parachute opens an additional drag force, $B_p v^2$, is introduced. If the coefficient of friction of the parachute is given by 2.5A where A is the cross-sectional area of the parachute, determine the size of parachute needed to obtain a landing speed of 4 m/s.

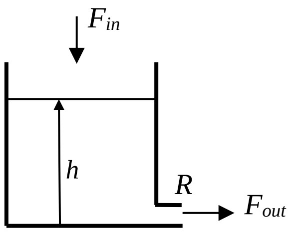
Q2 Determine a mathematical model for the bicycle suspension (spring-mass-damper) system, whose physical model is as follows:



In this bicycle suspension model, K_1 and B_1 represent the spring and damping coefficients for the back wheel while K_2 and B_2 represent the spring and damping coefficients for the front wheel.



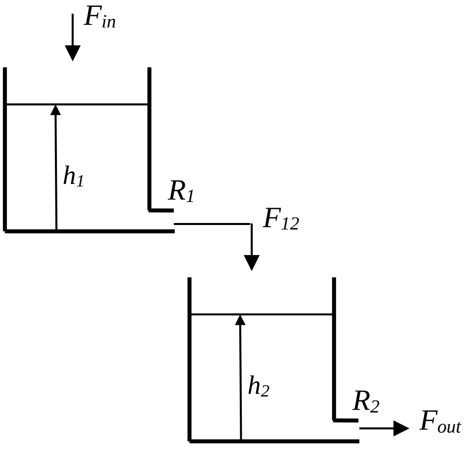
Q3 (i) Derive a mathematical model for the following single tank system, relating input flow rate, F_{in}, to the height of the liquid, h, given that the cross sectional area of the tank is A. Assume that water flow is laminar and hence $F_{out} = \frac{h}{R}$, where parameter R, in this case, is the resistance to flow.



Single tank system

(ii) How would a turbulent flow have affect F_{out} and hence your model?

Q4 Using your answer from Q9, derive a mathematical model relating input flow rate, F_{in}, to the height of the liquid in tank 2, h_2, for the dual **non-interacting** tank system given below. Assume that the cross sectional area of tanks 1 and 2 is A_1 and A_2 respectively. Also assume laminar flow with flow resistances R_1 and R_2 as shown.



Q5 Derive a model for the dual **interacting** tank system given below. Assume that the cross sectional area of tanks 1 and 2 is A_1 and A_2 respectively. Also assume laminar flow with flow resistances R_1 and R_2 as shown. Present your model as two differential equations.