CS422 Robotics and Automation Assignment 1

Maynooth University, Siyuan Zhan PhD

Due 30th Sept. Late submissions will not be accepted

Read Textbook Chapter 2 and answer the following questions:

1. Use Grübler's formula to verify that the Stewart mechanism (Fig. 1) indeed has six degrees of freedom (10pts)

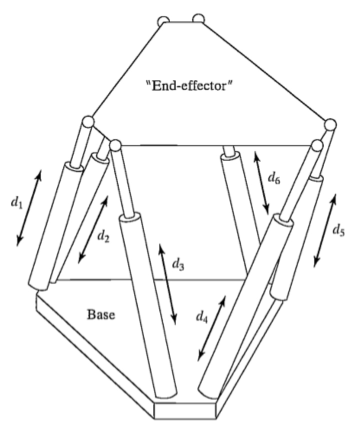

Figure 1: The Stewart mechanism is a six-degree-of-freedom fully parallel manipulator.

2. A vector P^A is rotated about $\hat{Z}_A$ by θ degrees and is subsequently rotated about $\hat{X}_A$ by ϕ degrees. Give the rotation matrix that accomplishes these rotations in the given order (5 pts).

3. A frame $\{B\}$ is located initialy coincident with a frame $\{A\}$. We rotate {B} about $\hat{Z}_B$ by θ degrees, and then we rotate the resulting frame about $\hat{X}_B$ by θ degrees. Give the rotation matrix that will change the descriptions of vectors from P^B to P^A (5pts)

4. Referring to Fig. 2

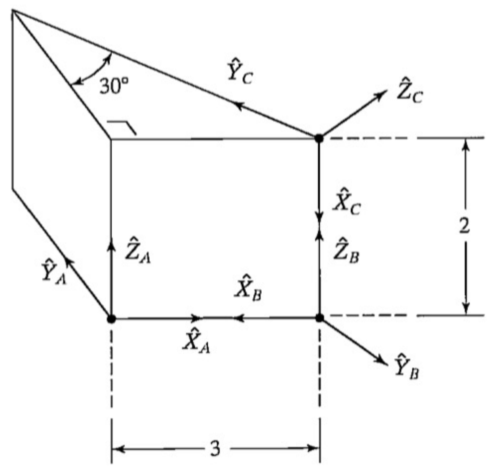

Figure 2: Frames at the corners of a wedge

(a) give the value of H_B^A
(b) give the value of H_C^A
(c) give the value of H_C^B
(d) give the value of H_A^C
(e) give the value of H_A^B (10pts)

5. Given
$$H_B^A = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
find H_A^B (5pts)

CS422 Robotics and Automation Assignment 1

Hanlin Cai (20122161)

Question 1 Sol:

As the Figure 1 shown, the number of joints is 18 (6 universal, 6 ball and socket and 6 prismatic in the actuators), while the number of links is 14 (2 parts for each actuator, the end-effector, and the base).

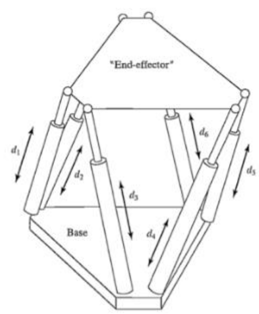

Figure 1 Six-degree-of-freedom Fully Parallel Manipulator

Finally, the sum of all the joint freedoms is 36. So, we can get

$$F=6*(14—18—1)+36=6. \quad (1.)$$

Question 2 Sol:

Step I, the vector P^A rotate about $\hat{Z}_A$ by θ degrees, the rotation matrix is

$$A_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2.)$$

Step II, it is subsequently rotated about $\hat{X}_A$ by φ degrees, then, the rotation matrix is

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}. \quad (3.)$$

So, the rotation matrix which will accomplish these rotations in the given order would be

$$A = A_2 \cdot A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos\phi\sin\theta & \cos\phi\cos\theta & -\sin\phi \\ \sin\phi\sin\theta & \sin\phi\cos\theta & \cos\phi \end{bmatrix} \quad (4.)$$

Question 3 Sol:

As the question illustrated, this is a problem about the coordinate transformation. The frame B is initially coincident with the frame A.

The frame B_1 is obtained by rotating frame B about $\hat{Z}_B$ by θ degrees, which can be represented by the 3 × 3 matrix

$$A_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5.)$$

The frame B_2 is obtained by rotating frame B_1 about $\hat{X}_B$ by θ degrees, which can also be represented by the 3 × 3 matrix

$$A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad (6.)$$

Then, the coordinates transformation equation is

$$P_A = A_1 \cdot A_2 \cdot P_B \quad (7.)$$

So, the rotation matrix which will change the description of vector P_B to P_A would be

$$R_A^B = A_1 \cdot A_2 = \begin{bmatrix} cos\ \theta & -sin\ \theta & 0 \\ sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} cos\ \theta & -cos\theta sin\theta & sin^2\theta \\ sin\ \theta & \cos^2\theta & -cos\theta sin\theta \\ 0 & sin\ \theta & cos\ \theta \end{bmatrix} \quad (8.)$$

Question 4 Sol:

As the Figure 1 shown, it is the frames at the corners of a wedge.

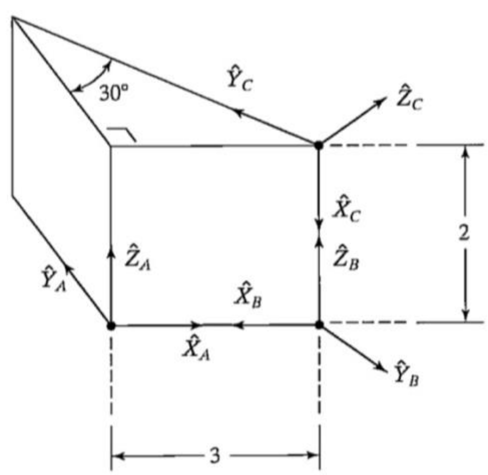

Figure 2 Frames at the corners of a wedge

So, we can get

$$H_B^A = \begin{bmatrix} \hat{x}_B \cdot \hat{x}_A & \hat{y}_B \cdot \hat{x}_A & \hat{z}_B \cdot \hat{x}_a & a \\ \hat{x}_B \cdot \hat{y}_A & \hat{y}_B \cdot \hat{y}_A & \hat{z}_B \cdot \hat{y}_A & b \\ \hat{x}_B \cdot \hat{z}_A & \hat{y}_B \cdot \hat{z}_A & \hat{z}_B \cdot \hat{z}_A & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9.}$$

Note that

$$P_{BORG}^A = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{10.}$$

$$\begin{cases} {}^A P = {}_B^A R \, {}^B P + {}^A P_{BORG} \\ \begin{bmatrix} {}^A P \\ 1 \end{bmatrix} = \begin{bmatrix} & {}_B^A R & & {}^A P_{BORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^B P \\ 1 \end{bmatrix} \end{cases} \tag{11.}$$

So, we can get

$$H_B^A = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12.}$$

$$H_C^A = \begin{bmatrix} 0 & -0.5 & 0.866 & 3 \\ 0 & 0.866 & 0.5 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13.}$$

$$H_C^B = \begin{bmatrix} 0 & 0.5 & -0.866 & 2 \\ 0 & -0.866 & -0.5 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14.}$$

$$H_A^C = \begin{bmatrix} 0 & 0 & -1 & -3 \\ -0.5 & 0.866 & 0 & -2 \\ 0.866 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15.}$$

$$H_A^B = \begin{bmatrix} -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16.}$$

Question 5 Sol:

We have

$$H_B^A = \begin{bmatrix} 0.25 & 0.43 & 0.86 & 5.0 \\ 0.87 & -0.50 & 0.00 & -4.0 \\ 0.43 & 0.75 & -0.50 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17.}$$

Easily get (by MATLAB)

$$H_A^B = inv(H_B^A) = \begin{bmatrix} 0.2511 & 0.8638 & 0.4319 & 0.9040 \\ 0.4369 & -0.4970 & 0.7515 & -6.4271 \\ 0.8713 & 0.0026 & -0.5013 & -2.8632 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \tag{18.}$$

Or we can calculate directly

$$H_A^B = \begin{bmatrix} {}^A R_B^H & -{}^A R_B^H \cdot {}^A P_B \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.87 & 0.43 & 0.90 \\ 0.43 & -0.5 & 0.75 & -6.4 \\ 0.86 & 0.0 & -0.50 & -2.8 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \tag{19.}$$

CS422 Robotics and Automation Assignment 2

Maynooth University, Siyuan Zhan PhD

Due 8th Nov. Late submissions will not be accepted

Read Textbook Chapter 3 and answer the following questions[1]:

The two problems are based on the textbook, Robot Modeling and Control by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions when provided.

1. **SHV 3-7 – Three-link Cartesian Robot (20pts):**
   Your solution should include a schematic of the manipulator with appropriately placed coordinate frames, a table of the DH parameters, and the final transformation matrix. Then answer the following question: What are the x, y, and z coordinates of the tip of the robot's end-effector in the base frame (as a function of the robot parameters and the joint coordinates)?

2. **SHV 3-6 – Three-link Articulated Robot (20 points)**

   Your solution should include a schematic of the manipulator with appropriately placed coordinate frames, a table of the DH parameters, and the final transformation matrix. Then answer the following question: What are the x, y, and z components in the base frame of a unit vector pointing along the robot's last link (from the third joint to the tip, as a function of the robot parameters and the joint coordinates)?

You can use either the standard DH convention following our textbook or the modified DH convention following "Introduction to Robotics: Mechanics and Control (3rd Edition)".

[1] Some of the assignment materials were adapted from the open materials used in MEAM520 at the University of Pennsylvania, created by Dr Katherine J. Kuchenbecker

CS323 Robotics & Automation Assignment 2

Hanlin CAI (20122161)   2022/10/31

Q1 SHV 3-7 – Three-link Cartesian Robot (20pts):

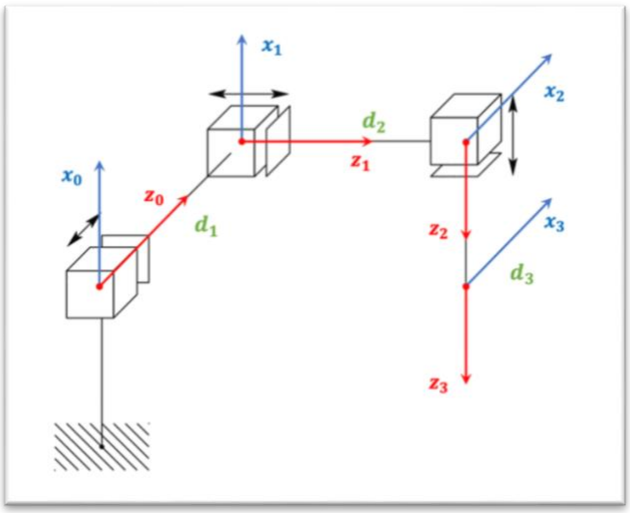

Figure 1 Three-link cartesian robot

Figure 1 shows the schematic of the manipulator with appropriately placed coordinate frames. And the Table 1 illustrates the summary of DH parameters of this manipulator.

Table 1 Summary of DH Parameters

#	θ	d	a	α
0-1	0	d_1	0	$-90°$
1-2	$90°$	d_2	0	$-90°$
2-3	0	d_3	0	0

The transformation matrix is:

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta 1 & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get:

$$
{}^{0}T_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

And we know,

$$
[{}^{0}T_3] = [{}^{0}T_1] \cdot [{}^{1}T_2] \cdot [{}^{2}T_3]
$$

So, we get the final transformation matrix:

$$
[{}^{0}T_3] = \begin{bmatrix} 0 & 0 & -1 & \boldsymbol{-d3} \\ 0 & -1 & 0 & \boldsymbol{d2} \\ -1 & 0 & 0 & \boldsymbol{d1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Ultimately, the x, y, and z components in the base frame of a unit vector pointing along the robot's last link is:

$$
[P] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} -d3 \\ d2 \\ d1 \\ 1 \end{bmatrix}
$$

Q2 SHV 3-6 – Three-link Articulated Robot (20 points)

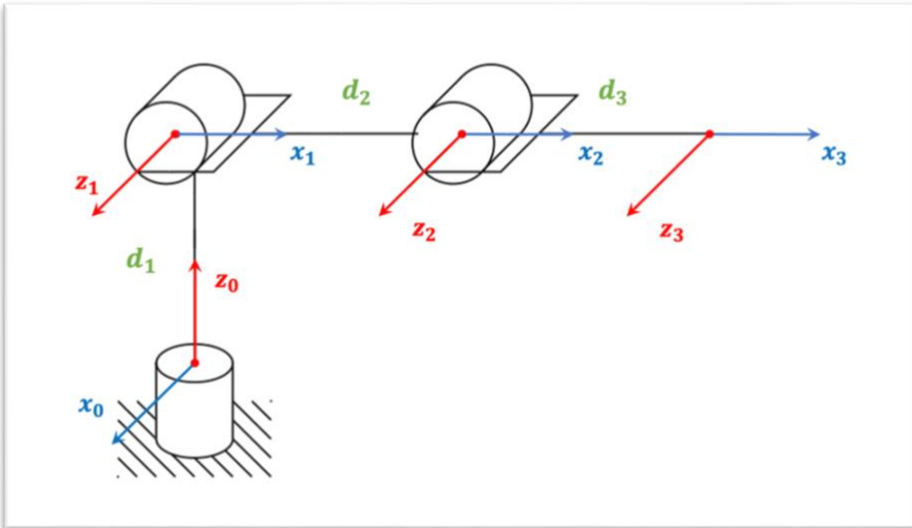

Figure 2 Three-link Articulated Robot

Figure 2 shows the schematic of the manipulator with appropriately placed coordinate frames. And the Table 2 illustrates the summary of DH parameters of this manipulator.

Table 2 Summary of DH Parameters

#	θ	d	a	α
0-1	$90°$	d_1	0	$90°$
1-2	0	0	d_2	0
2-3	0	0	d_3	0

The transformation matrix is:

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get:

$$^0T_1 = \begin{bmatrix} \cos\theta 1 & 0 & 1 & 0 \\ \sin\theta 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{1}T_2 = \begin{bmatrix} \cos\theta 2 & -sin\theta 2 & 0 & d2 \cdot cos\theta 2 \\ \sin\theta 2 & cos\theta 2 & 0 & d2 \cdot sin\theta 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix} \cos\theta 2 & -sin\theta 2 & 0 & d3 \cdot cos\theta 3 \\ \sin\theta 2 & cos\theta 2 & 0 & d3 \cdot sin\theta 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

And we know,

$$
[{}^{0}T_3] = [{}^{0}T_1] \cdot [{}^{1}T_2] \cdot [{}^{2}T_3]
$$

So, we get the final transformation matrix:

$$
[{}^{0}T_3] = \begin{bmatrix} 0 & 0 & 1 & \mathbf{0} \\ 1 & 0 & 0 & \boldsymbol{d3+d2} \\ 0 & 1 & 0 & \boldsymbol{d1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Ultimately, the x, y, and z components in the base frame of a unit vector pointing along the robot's last link is:

$$
[P] = \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1(d_2\cos\theta_2 + d_3\sin(\theta_2+\theta_3)) \\ \sin\theta_1(d_2\cos\theta_1 + d_3\sin(\theta_2+\theta_3)) \\ d_1 + d_2\sin\theta_2 - d_3\cos(\theta_2+\theta_3)) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ d3+d2 \\ d1 \\ 1 \end{bmatrix}
$$

CS422 Robotics and Automation Assignment 3

Maynooth University, Siyuan Zhan PhD

Due 25th Nov. Late submissions will not be accepted

Read Textbook Chapter 3 and answer the following questions[1]:

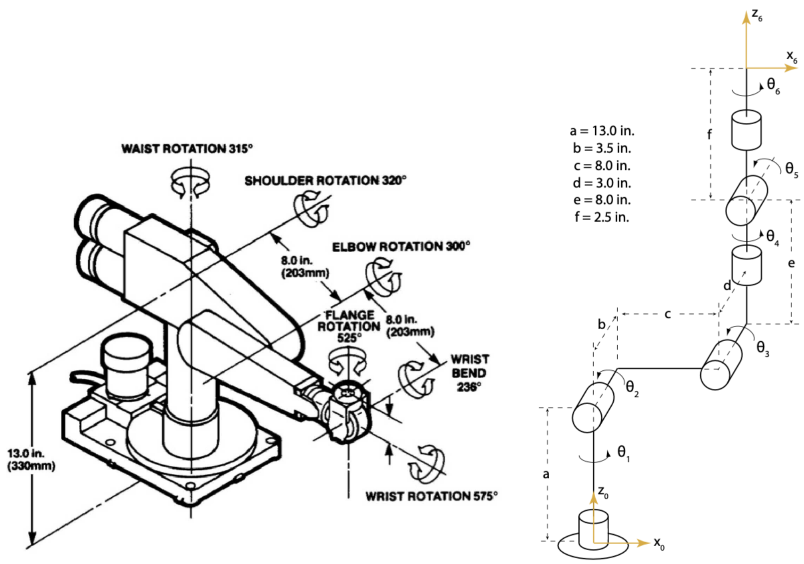

Figure 1: The schematic above on the right shows the zero configuration we have chosen for use in this class (a different pose from the drawing at the left). The joint angle arrows show the positive direction for each revolute joint (θ_1 to θ_6). All of the joints are shown at $\theta_i = 0$. The diagram also gives the measurements for the constant dimensions (a to f), all in inches.

The first two problems center on the forward kinematics of the PUMA 260. We will be using this robot for the manipulator virtual labs in this class. It is an articulated robot (RRR) with lateral offsets, plus a spherical wrist (RRR). The drawing in Fig 1. the left shows the robot and the arrangement of its joints.

[1] Some of the assignment materials were adapted from the open materials used in MEAM520 at the University of Pennsylvania, created by Dr Katherine J. Kuchenbecker

1. **DH Parameters for the PUMA 260 (20pts):**
   Annotate the full-page schematic of the PUMA (provided later in this document) with appropriately placed coordinate frames, and then write a table of the corresponding DH parameters; use degrees for the angles. Do this in pencil so that you can make corrections if needed. You may find it useful to follow the steps provided in SHV Section 3.4.

2. **Solve the Inverse Kinematics for the PUMA 260 (40 pts)**

CS323 Robotics & Automation Assignment 3

HANLIN CAI (20122161) *2022/11/22*

Q1 DH Parameters for the PUMA 260 (20pts).

Figure 1 shows the PUMA 260 robot and the arrangement of its joints and the schematic of the manipulator with appropriately placed coordinate frames.

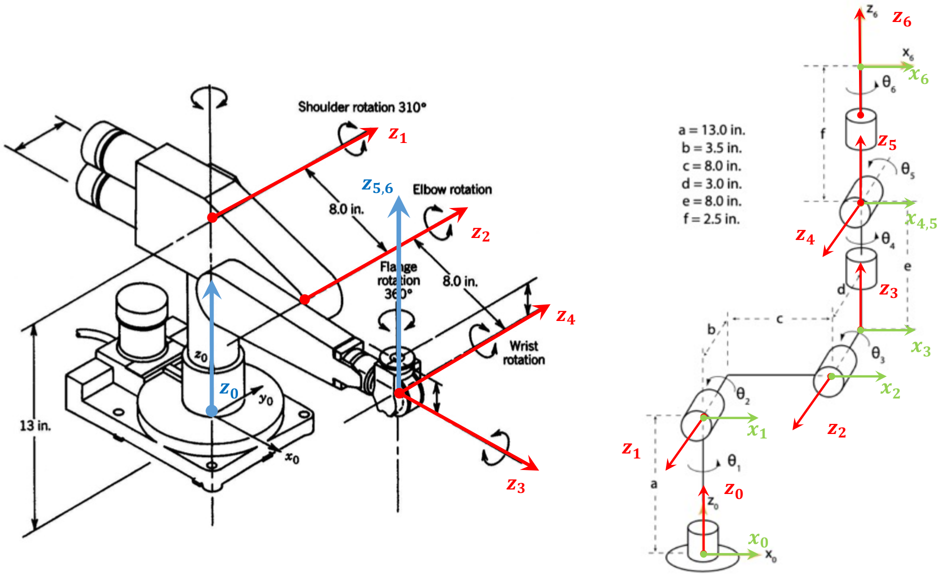

Figure 1 PUMA 260 manipulator

And the Table 1 illustrates the summary of DH parameters of this manipulator.

Table 1 DH Parameters (v1)

#	θ	d	a	α
0-1	θ_1	13.0	0	90°
1-2	θ_2	−3.5	8.0	0
2-3	θ_3	−3.0	0	−90°
3-4	θ_4	8.0	0	90°
4-5	θ_5	0	0	−90°
5-6	θ_6	2.5	0	0

The standard homogenous matrix is given by

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Then, we can get the matrix of each coordinate system, as follows

$${}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$${}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 8\cdot\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 8\cdot\sin\theta_2 \\ 0 & 0 & 1 & -3.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$${}^{2}T_3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$${}^{3}T_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$${}^{4}T_5 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$${}^{5}T_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Next, we can calculate the transformation matrix by

$$[{}^{0}T_6] = [{}^{0}T_1]\cdot[{}^{1}T_2]\cdot[{}^{2}T_3]\cdot[{}^{3}T_4]\cdot[{}^{4}T_5]\cdot[{}^{5}T_6] \tag{8}$$

To solve this equation, we need to compute some intermediate results

$$ {}^{4}T_6 = [{}^{4}T_5] \cdot [{}^{5}T_6] $$

$$ = \begin{bmatrix} c\theta_5 \cdot c\theta_6 & -c\theta_5 \cdot s\theta_6 & s\theta_5 & -2.5 \cdot s\theta_5 \\ s\theta_5 \cdot c\theta_6 & -s\theta_5 \cdot s\theta_6 & -c\theta_5 & 2.5 \cdot c\theta_5 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9} $$

And then,

$$ {}^{3}T_6 = [{}^{3}T_4] \cdot [{}^{4}T_6] $$

$$ = \begin{bmatrix} c\theta_4 c\theta_5 c\theta_6 - s\theta_4 s\theta_6 & -c\theta_4 c\theta_5 s\theta_6 - s\theta_4 c\theta_5 & c\theta_4 s\theta_5 & 2.5 \cdot c\theta_4 s\theta_5 \\ s\theta_4 c\theta_5 c\theta_6 + c\theta_4 s\theta_6 & -s\theta_4 c\theta_5 s\theta_6 + c\theta_4 c\theta_6 & s\theta_4 s\theta_5 & -2.5 \cdot s\theta_4 s\theta_5 \\ -s\theta_5 c\theta_6 & s\theta_5 s\theta_6 & c\theta_4 & 8 + 2.5 \cdot c\theta_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10} $$

Because, Joint 2 and joint 3 are parallel (using shorthand here),

$$ {}^{1}T_3 = [{}^{1}T_2] \cdot [{}^{2}T_3] $$

$$ = \begin{bmatrix} c_{23} - s_{23} & 0 & c_2 s_3 + s_2 c_3 & 8 \cdot c_2 \\ s_2 c_3 + c_2 s_3 & 0 & s_{23} - c_{23} & 8 \cdot s_2 \\ 0 & 1 & 0 & -6.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11} $$

And then,

$$ {}^{0}T_3 = [{}^{0}T_1] \cdot [{}^{1}T_3] $$

$$ = \begin{bmatrix} c_1 c_{23} - c_1 s_{23} & s_1 & c_1 c_2 s_3 - c_1 s_2 c_3 & 8 c_1 c_3 - 6.5 s_1 \\ s_1 c_{23} - s_1 s_{23} & -c_1 & s_1 c_2 s_3 - s_1 s_2 c_3 & s_1 (a_1 + a_2 c_2 + a_3 c_{23}) \\ s_2 c_3 + s_3 c_2 & 0 & s_{23} - c_{23} & 8 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

Therefore, the transformation matrix can be simplified to

$$ [{}^{0}T_6] = [{}^{0}T_3] \cdot [{}^{3}T_6] \tag{13} $$

Due to large number of different robot forms, it is unnecessary to expand the final calculations, so we will not provide final results here.

Supplement: *Moreover, the wrist center of the manipulator can be defined as point C. To make it easier to resolve the position of the wrist center. We can simplify the PUMA 260 as the following 6R manipulator.*

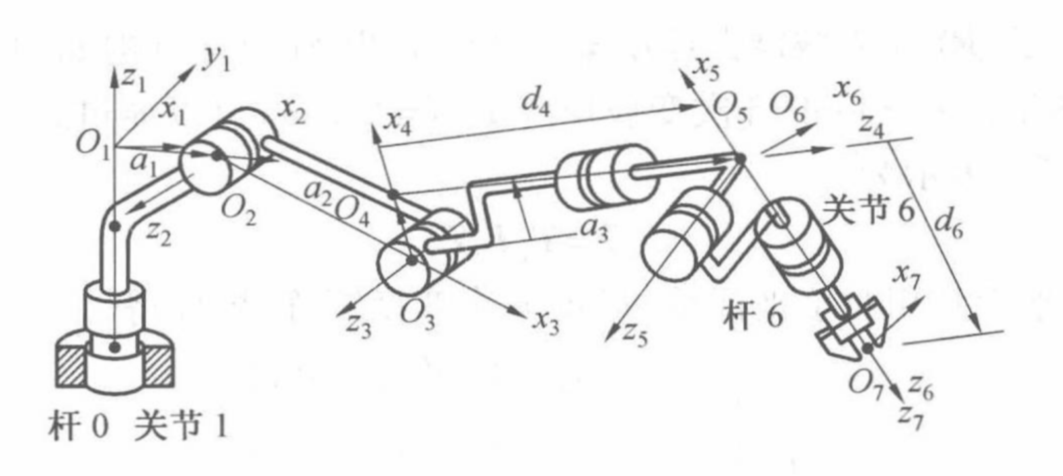

Figure 2 Simplified PUMA 260 manipulator (6R)

In this case, the DH parameters are shown in the Table 2.

Table 2 DH Parameters (v2)

#	θ	d	a	α
0-1	θ_1	0	α_1	$90°$
1-2	θ_2	0	α_2	0
2-3	θ_3	0	α_3	$90°$
3-4	θ_4	d_4	0	$-90°$
4-5	θ_5	0	0	$90°$
5-6	θ_6	d_6	0	0

Finally, utilizing Table 2 to calculate the location of the wrist center $\boldsymbol{p_C}$*,*

$$\boldsymbol{p_C} = \boldsymbol{p_{O_4}} + {}^{\mathbf{1}}\boldsymbol{Q_4 a_4}$$

$$= \begin{bmatrix} c_1(a_1 + a_2c_2 + a_3c_{23}) \\ s_1(a_1 + a_2c_2 + a_3c_{23}) \\ a_2s_2 + a_3s_{23} \end{bmatrix} + \begin{bmatrix} c_1c_{23} & s_1 & c_1s_{23} \\ s_1c_{23} & -c_1 & s_1s_{23} \\ s_{23} & 0 & -c_{23} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d_4 \end{bmatrix} \tag{14}$$

$$= \begin{bmatrix} c_1(a_1 + a_2c_2 + a_3c_{23} + d_4s_{23}) \\ s_1(a_1 + a_2c_2 + a_3c_{23} + d_4s_{23}) \\ a_2s_2 + a_3s_{23} - d_4c_{23} \end{bmatrix}$$

Q2 Solve the Inverse Kinematics for the PUMA 260 (40 pts).

Generally speaking, it is very difficult to resolve the inverse kinematics for the space-6R manipulator. However, in this case, the PUMA 260 decouples at the Wrist Center (point C).

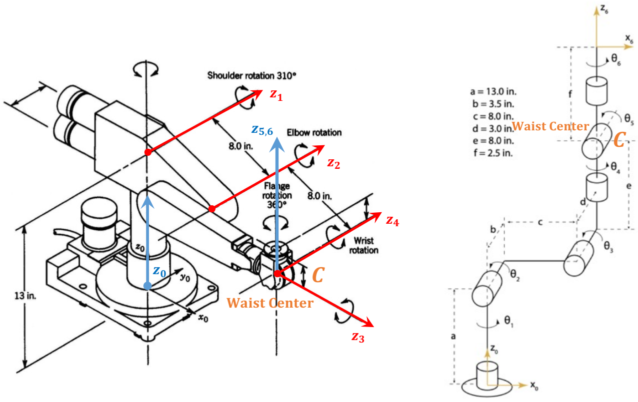

Figure 3 Waist center of PUMA 260 manipulator

Then, the inverse kinematics can be decomposed into two relatively simple problems: **(1)Position Inverse** and **(2) Orientation Inverse**. Given that:

$$\begin{cases} \boldsymbol{Q}_6^0(\theta_1, \cdots, \theta_6) = \boldsymbol{Q} \\ \boldsymbol{p}_6^0(\theta_1, \cdots, \theta_6) = \boldsymbol{p} \end{cases} \tag{15}$$

Illustrated in Equation (14) and (15), we know that

$$\boldsymbol{p} = \boldsymbol{p}_C + d_6 \boldsymbol{Q} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{16}$$

Now, we define $\boldsymbol{Q}$ and $\boldsymbol{p}$ as follows,

$$\boldsymbol{Q} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, \boldsymbol{p} = \begin{bmatrix} x_\mathrm{e} \\ y_\mathrm{e} \\ z_\mathrm{e} \end{bmatrix} \tag{17}$$

Then, $\boldsymbol{p}_C$ can be defined as

$$\boldsymbol{p}_C = \begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix} = \begin{bmatrix} x_\mathrm{e} - d_6 r_{13} \\ y_\mathrm{e} - d_6 r_{23} \\ z_\mathrm{e} - d_6 r_{33} \end{bmatrix} \tag{18}$$

(1) Position Inverse *(Geometric method)*

Noted that, to facilitate the marking of the manipulator, the variables adopt new symbols, as shown in the following figures. Thanks a lot!

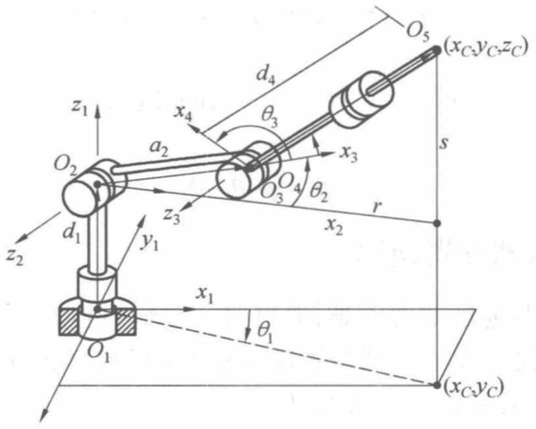

Figure 4 First three joint (as 3R manipulator)

First and foremost, we resolve the first three joints. Figure 4 illustrates the general organization. Then, we can easily define the θ_1 as:

$$\theta_1 = \begin{cases} \arctan 2(y_C, x_C) \\ \arctan 2(y_C, x_C) + \pi \end{cases} \tag{19}$$

So, there are two possible singularity posture in this case, as shown in Figure 5.

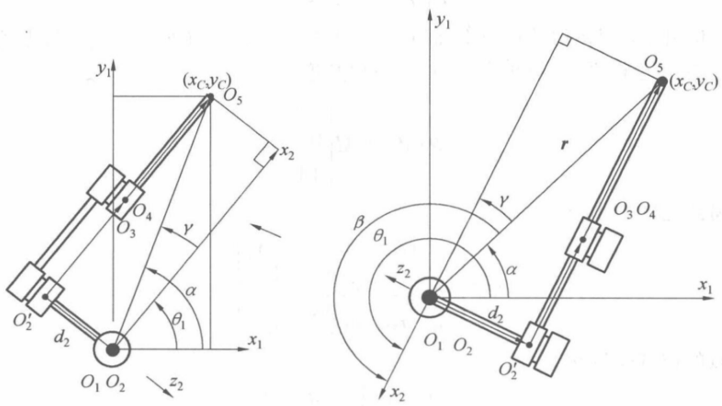

Figure 5 Shoulder Right & Shoulder Left

To simplify the calculation, as for the specific PUMA 260 shown in Figure 1, we will only consider the manipulator follows the condition of **Shoulder Left**. So, as illustrated in Figure 5

$$\theta_1 = \alpha - \beta \tag{20}$$

In the Equation (20),

$$\begin{cases} \alpha = \operatorname{atan}\ 2(y_C, x_C) \\ \beta = \pi + \gamma \\ \gamma = \operatorname{atan}\ 2\left(d_2, \sqrt{x_C^2 + y_C^2 - d_2^2}\right) \end{cases} \tag{21}$$

Then, we get

$$\theta_1 = \operatorname{atan} 2(y_C, x_C) - \operatorname{atan} 2\left(d_2, \sqrt{x_C^2 + y_C^2 - d_2^2}\right) + \pi \tag{22}$$

After calculating θ_1, now we consider θ_2 and θ_3, the following Figure 6 illustrates the projection of the Joint 2 and Joint 3.

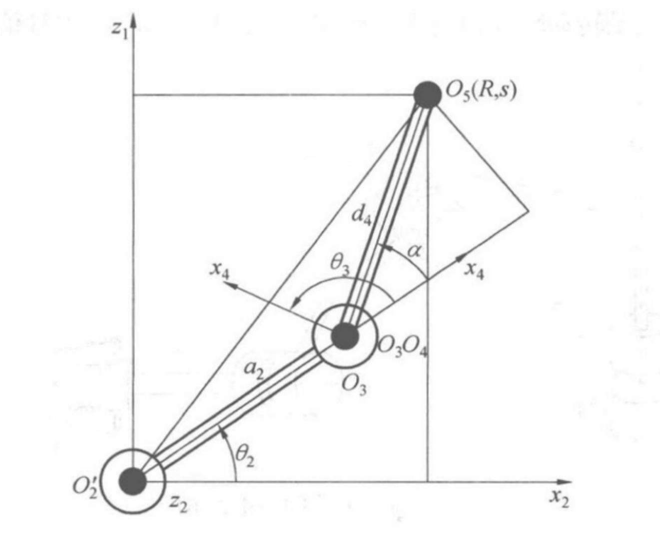

Figure 6 Projection of the Joint 2 and Joint 3

$$\begin{cases} \cos\ \alpha = \dfrac{R^2 + s^2 - a_2^2 - d_4^2}{2a_2 d_4} \equiv D \\ R = \sqrt{x_C^2 + y_C^2 - d_2^2} \\ s = z_C - d_1 \\ \alpha = \operatorname{atan}\ 2\left(\pm\sqrt{1 - D^2}, D\right) \end{cases} \tag{23}$$

So, we can get

$$\begin{cases} \theta_2 = \operatorname{atan} 2(s, R) - \operatorname{atan} 2(d_4 s\alpha, a_2 + d_4 c\alpha) \\ \theta_3 = \dfrac{\pi}{2} + \alpha \end{cases} \tag{24}$$

(2) Orientation Inverse *(Algebraic + Geometric)*

After resolving the $\boldsymbol{\theta}_{1,2,3}$, we can get the corresponding rotation matrix, defined as $\boldsymbol{Q}_{1,2,3}$. So, the rotation matrix from the coordinate system 4 to coordinate system 6 can be defined as

$$\boldsymbol{M} = (\boldsymbol{Q}_3)^{\mathrm{T}}(\boldsymbol{Q}_2)^{\mathrm{T}}(\boldsymbol{Q}_1)^{\mathrm{T}}\boldsymbol{Q} = ({}^1\boldsymbol{Q}_4)^{\mathrm{T}}\boldsymbol{Q} \tag{25}$$

Similar as Equation (17), we define $\boldsymbol{M}$ as follows

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{26}$$

Now, considering the last three Joint 4, 5 and 6. Figure 7 illustrates the general organization, where the three axes of rotation intersect at one point.

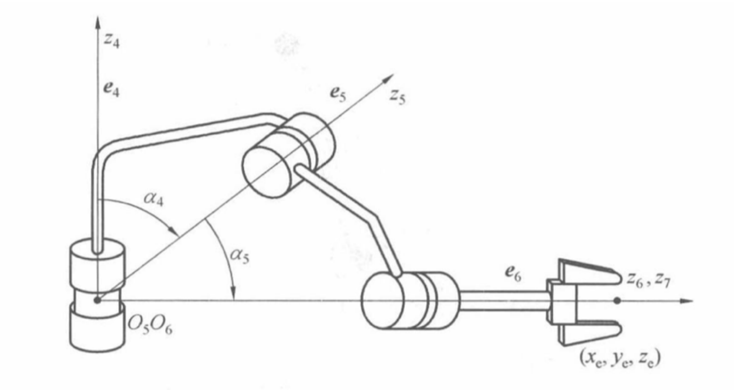

Figure 4 Last three joint

To begin with, we can define the Joint i possess the axis unit vector $\boldsymbol{e}_i$.

$${}^4\boldsymbol{e}_6 = \begin{bmatrix} m_{13} \\ m_{23} \\ m_{33} \end{bmatrix}, {}^4\boldsymbol{e}_5 = \begin{bmatrix} \mu_4 s_4 \\ -\mu_4 c_4 \\ \lambda_4 \end{bmatrix} \tag{27}$$

Then, we get

$$\begin{cases} {}^4\boldsymbol{e}_6 \cdot {}^4\boldsymbol{e}_5 = \lambda_5 \\ m_{13}\mu_4 s_4 - m_{23}\mu_4 c_4 + m_{33}\lambda_4 = \lambda_5 \end{cases} \tag{28}$$

Now, we assume that

$$\begin{cases} A = m_{13}\mu_4 \\ B = m_{23}\mu_4 \\ C = \lambda_5 - m_{33}\lambda_4 \end{cases} \tag{29}$$

Thus we have $As_4 - Bc_4 = C$, then utilizing the **Transformation of Triangle,**

$$\begin{cases} A = \rho \cos \varphi \\ B = \rho \sin \varphi \\ \rho = \sqrt{A^2 + B^2} \\ \varphi = \text{atan } 2(B, A) \end{cases} \tag{30}$$

We can get the $\theta_{4,5,6}$, as shown in following procedures:

a. For θ_4

$$\left.\begin{array}{l} \sin(\theta_4 - \varphi) = C/\rho, \cos(\theta_4 - \varphi) = \pm\sqrt{1 - (C/\rho)^2} \\ \theta_4 - \varphi = \text{atan } 2\left(C/\rho, \pm\sqrt{1 - (C/\rho)^2}\right) \\ \boldsymbol{\theta_4} = \textbf{atan } \mathbf{2}(\boldsymbol{B}, \boldsymbol{A}) + \textbf{atan } \mathbf{2}\left(\boldsymbol{C}/\boldsymbol{\rho}, \pm\sqrt{\mathbf{1} - (\boldsymbol{C}/\boldsymbol{\rho})^2}\right) \end{array}\right\} \tag{31}$$

b. For θ_5

$$\boldsymbol{Q}_5 = (\boldsymbol{Q}_4)^{\mathrm{T}} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} (\boldsymbol{Q}_6)^{\mathrm{T}} \tag{32}$$

So, we can resolve the $\boldsymbol{\theta_5}$ through the following equations (33),

$$\begin{cases} \mu_5 s_5 = (m_{12}\mu_6 + m_{13}\lambda_6)c_4 + (m_{22}\mu_6 + m_{23}\lambda_6)s_4 \\ \mu_5 c_5 = \lambda_4(m_{12}\mu_6 + m_{13}\lambda_6)s_4 - \lambda_4(m_{22}\mu_6 + m_{23}\lambda_6)c_4 - \mu_4(m_{32}\mu_6 + m_{33}\lambda_6) \end{cases} \tag{33}$$

c. For θ_6

$$\boldsymbol{Q}_6 = \boldsymbol{Q}_5^{\mathrm{T}} \boldsymbol{Q}_4^{\mathrm{T}} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{34}$$

Also, we can get the result of $\boldsymbol{\theta_6}$ through the Equation (34). As for the different $(\boldsymbol{\theta_4}, \boldsymbol{\theta_5})$ correspond to different $\boldsymbol{\theta_6}$, thus the Orientation Inverse possesses two sets of solution, which will be suggested in the following summary.

(3) Summary

Noted that, to facilitate the marking of the manipulator, the variables adopt new symbols, as shown in the above figures. Thanks a lot!

Refer to the above Equation (17, 26, 23, 30, 33), the summary of the Inverse Kinematics for the PUMA 260 is shown as follows,

$$\begin{cases} \theta_1 = \operatorname{atan} 2(y_C, x_C) - \operatorname{atan} 2\left(d_2, \sqrt{x_C^2 + y_C^2 - d_2^2}\right) + \pi \\ \theta_2 = \operatorname{atan} 2(s, R) - \operatorname{atan} 2(d_4 s\alpha, a_2 + d_4 c\alpha) \\ \theta_3 = \dfrac{\pi}{2} + \alpha \\ \theta_4 = \operatorname{atan}\ 2(B, A) + \operatorname{atan}\ 2\left(C/\rho, \pm\sqrt{1-(C/\rho)^2}\right) \\ \theta_5 = \operatorname{atan}\ 2(c_4 m_{13} + s_4 m_{23}, m_{33}) \\ \theta_6 = \operatorname{atan}\ 2(-s_4 m_{11} + c_4 m_{21}, -s_4 m_{12} + c_4 m_{22}) \end{cases} \tag{35}$$

Acknowledgements

I gratefully acknowledge Dr. Siyuan Zhan for his generous guidance and support during the EE422FZ course. Also, I would like to thank the tutor who carefully evaluate this report. Thank you!

Hanlin CAI

Nov 26th 2022

CS422 Robotics and Automation Assignment 4

Maynooth University, Siyuan Zhan PhD

Due 10th Dec. Late submissions will not be accepted*

This entire assignment is written and consists of two significantly adapted problems from the textbook, Robot Modeling and Control by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions on both questions.

1. **Three-link Cylindrical Manipulator (30 points)**
   The book works out the DH parameters and the transformation matrix T_3^0 for this robot in Example 3.2; you are welcome to use these results directly without rederiving them.

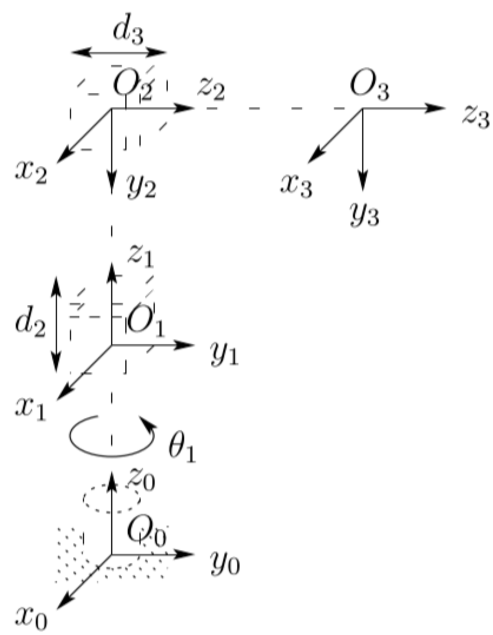

Figure 1: Three-link cylindrical manipulator

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

Table 1: Link DH parameters for 3-link cylindrical manipulator (* variables)

*Some of the assignment materials were adapted from the open materials created by Dr Katherine J. Kuchenbecker

T matrices are

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian J_v for the three-link cylindrical manipulator of Figure 1.

(b) Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

(c) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

(d) Find this robot's 6×3 Jacobian J.

(e) Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

(f) For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

(h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

2. **Three-link Spherical Manipulator (30 points)**
The book does not seem to work out the forward kinematics for this robot anywhere. Please use the diagram on the left side of the following Figure to define the positive joint directions and the zero configuration for the robot.

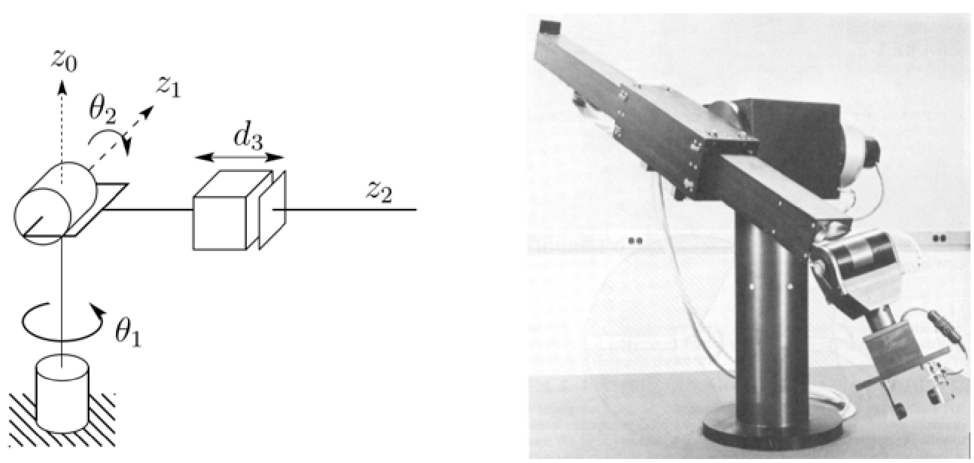

Figure 2: Left: the spherical manipulator; Right: an example of a particular design of spherical manipulator - the Stanford Arm

If we additionally choose the x_0 axis to point in the direction the robot arm points in the zero configuration, you can calculate that the tip of the spherical manipulator is at $\begin{bmatrix} x & y & z \end{bmatrix}^T = \begin{bmatrix} c_1c_2d_3 & s_1c_2d_3 & d_1 - s_2d_3 \end{bmatrix}^T$. In this expression θ_1, θ_2, and d_3 are the joint variables; s_i is $\sin\theta_i$ and c_i is $\cos\theta_i$; and d_1 is a constant.

(a) Calculate the 3×3 linear velocity Jacobian J_v for the spherical manipulator with no offsets shown in the left side of Figure 2. You may use any method you choose.

(b) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

(c) Find this robot's 6×3 Jacobian J.

(d) Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is ω_3^0 , the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

(e) For the same configuration described in the previous question, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

(f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = \begin{bmatrix} 0 \text{ m/s} & 0.5 \text{ m/s} & 0.1 \text{ m/s} \end{bmatrix}^T$? Make sure to provide units with your answer.

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

(h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

(i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

CS323 Robotics & Automation Assignment 4

Hanlin CAI (20122161)   2022/12/17

Q1 Adapted SHV 4-20, page 160– Three-link Cylindrical Manipulator

(a) Use the position of the end-effector in the base frame to calculate the 3×3 linear velocity Jacobian $\mathrm{J_v}$ for the three-link cylindrical manipulator of Figure 3-7 on page 85.

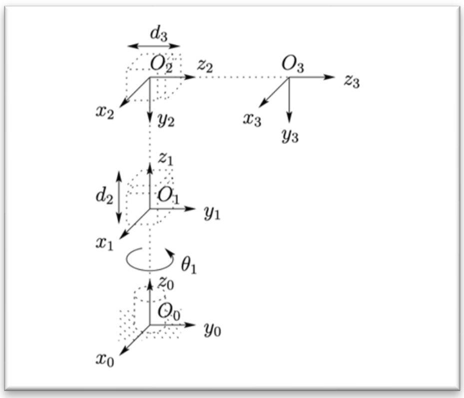

Figure 1 Three-link cylindrical manipulator

Figure 1 shows the schematic of the three-link cylindrical manipulator. We can easily get that

$$\begin{cases} x = -s_1 d_3 \\ y = c_1 d_3 \\ z = d_2 \end{cases}$$

By differential transformation, we can get the linear velocity Jacobian matrix as follows

$$J_v = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\partial q_3} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b) Use the positions of the origins o_i and the orientations of the z-axes z_i to calculate the 3×3 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

As for **Revolute Joints**, the linear velocity is $J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$.

As for **Prismatic Joints**, the linear velocity is $J_{v_i} = z_{i-1}$.

When $\mathrm{i}=1, \mathrm{n}=3,\ J_{v1}=Z_0(O_3-O_0)$, then

$$J_{v1}=\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}-s\theta_1 d_3\\c\theta_1 d_3\\d_1+d_2\end{pmatrix}=\begin{vmatrix}i & j & k\\0 & 0 & 1\\-s_1 d_3 & c_1 d_3 & d_1+d_2\end{vmatrix}$$

$$J_{v1}=-\mathrm{c}_1 d_3 i-s_1 d_3 j+0k$$

When $\mathrm{i}=2, \mathrm{n}=3,\ J_{v1}=Z_1$

$$J_{v2}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

When $\mathrm{i}=3, \mathrm{n}=3,\ J_{v1}=Z_2$

$$J_{v3}=\begin{pmatrix}-s_1\\c_1\\0\end{pmatrix}$$

Then, we can get the Jacobian matric as follows, which is the same as (a).

$$\therefore J_v=\begin{bmatrix}J_{v_1}\\J_{v_2}\\J_{v_3}\end{bmatrix}=\begin{bmatrix}-c_1 d_3 & 0 & -s_1\\-s_1 d_3 & 0 & c_1\\0 & 1 & 0\end{bmatrix}$$

(c) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

As for the angular velocity

$$J_{\omega_i}=\begin{cases}z_{i-1} & \text{revoluate}\\0 & \text{prismatic}\end{cases}$$

We can easily get that

$$J_\omega=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\\mathbf{1} & 0 & 0\end{bmatrix}$$

(d) Find this robot's 6×3 Jacobian J.

We easily combine the J_v and J_ω, hence

$$J_{6\times 3}=\begin{bmatrix}-c1d3 & 0 & -s1\\-s1d3 & 0 & c1\\0 & 1 & 0\\0 & 0 & 0\\0 & 0 & 0\\1 & 0 & 0\end{bmatrix}$$

(e) Imagine this robot is at $\theta_1 = \pi/2\text{rad}, d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1\text{rad/s}, \dot{\theta}_2 = 0.25$ m/s, and $\dot{d}_3 = -0.05$ m/s. What is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

$$v_3^0 = J_v\dot{q} = \begin{bmatrix} 0 & 0 & -1 \\ -0.3 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0.05 \\ -0.03 \\ 0.25 \end{bmatrix}\text{m/s}$$

(f) For the same situation, what is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

$$\omega_3^0 = J_\omega\dot{q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}\text{rad/s}$$

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

$$\det(J) = 0 = \det\begin{bmatrix} -c_1d_3 & 0 & -s_1 \\ -s_1d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix} = c_1^2d_3 + s_1^2d_3 = d_3$$

Hence, when $d_3 = 0$, the robot arm gets the singular configurations.

(h) Sketch the cylindrical manipulator in each singular configuration that you found and explain what effect the singularity has on the robot's motion in that configuration.

As the following Figure 2 shown, when the manipulator is in the singular configuration, no matter how the Joint 0 turn around, the end-effector would not take any movement.

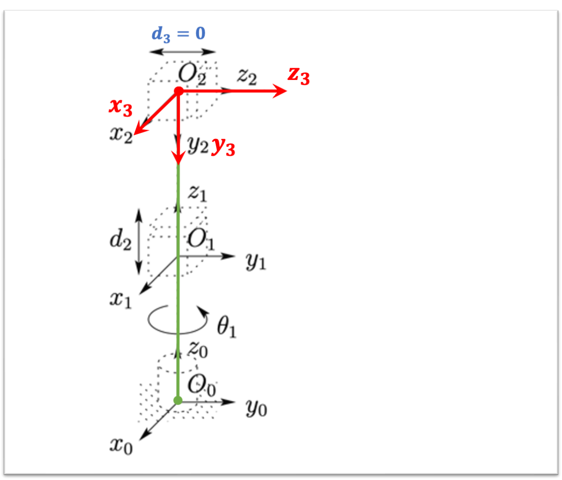

Figure 2 Manipulator in singular configuration

Q2 Adapted SHV 4-18, page 160–Three-link Spherical Manipulator

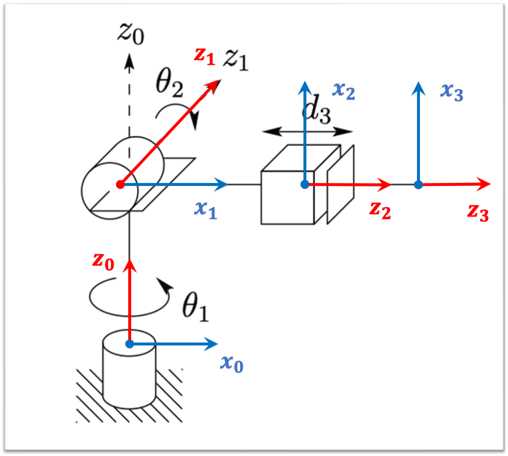

Figure 3 Three-link Spherical Robot

Figure 3 shows the schematic of the spherical manipulator with appropriately placed coordinate frames. And the Table 1 illustrates the summary of DH parameters of this manipulator.

Table 1 Summary of DH Parameters

#	θ	d	a	α
0-1	θ_1	d_1	0	90°
1-2	$\theta_2 - 90°$	0	0	90°
2-3	0°	d_3	0	0°

The transformation matrix is

$$T = RTTR = \begin{bmatrix} \cos\theta & -\sin\theta\cdot\cos\alpha & \sin\theta\cdot\sin\alpha & a\cdot\cos\theta \\ \sin\theta & \cos\theta\cdot\cos\alpha & -\cos\theta\cdot\sin\alpha & a\cdot\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can get that:

$${}^0T_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} s_2 & 0 & c_2 & 0 \\ -c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And,

$$
{}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Now, we know that

$$
{}^0T_2 = {}^0T_1\,{}^1T_2 = \begin{bmatrix} c_1 s_2 & -s_1 & c_1 c_2 & 0 \\ s_1 s_2 & c_1 & s_1 c_2 & 0 \\ c_2 & 0 & -s_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_3 = {}^0T_2\,{}^2T_3 = \begin{bmatrix} c_1 c_2 & -s_1 & -c_1 s_2 & c_1 c_2 d_3 \\ s_1 c_2 & c_1 & -s_1 s_2 & s_1 c_2 d_3 \\ c_2 & 0 & -s_2 & d_1 - s_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Based on the Equations above, we can easily get

$$
o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_3 = \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ d_1 - s_2 d_3 \end{bmatrix}
$$

$$
z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, z_2 = \begin{bmatrix} c_1 c_2 \\ s_1 c_2 \\ -s_2 \end{bmatrix}
$$

(a) Calculate the 3×3 linear velocity Jacobian J_v for the spherical manipulator with no offsets shown in the left side of Figure 1.12 on page 15 of SHV. You may use any method you choose.

The spherical robot consists of two revolute joints and one prismatic joint. So the linear velocity Jacobian J_v is

$$
J_v = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\partial q_3} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -s_1 c_2 d_3 & -c_1 s_2 d_3 & c_1 c_2 \\ c_1 c_2 d_3 & -s_1 s_2 d_3 & s_1 c_2 \\ 0 & -c_2 d_3 & -s_2 \end{bmatrix}
$$

(b) Find the 3×3 angular velocity Jacobian J_ω for the same robot.

As for the angular velocity

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{revoluate} \\ 0 & \text{prismatic} \end{cases}$$

We can easily get that

$$J_\omega = \begin{bmatrix} 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(c) Find this robot's 6×3 Jacobian J.

$$J = \begin{bmatrix} -s_1 c_2 d_3 & -c_1 s_2 d_3 & c_1 c_2 \\ c_1 c_2 d_3 & -s_1 s_2 d_3 & s_1 c_2 \\ 0 & -c_2 d_3 & -s_2 \\ 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(d) Imagine this robot is at $\theta_1 = \pi/4\text{rad}, \theta_2 = 0\text{rad}$, and $d_3 = 1$ m. What is ω_3^0, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1, \dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

$$\begin{cases} \theta_1 = \dfrac{\pi}{4} \\ \theta_2 = 0 \\ d_3 = 1 \end{cases}$$

$$\omega_3^0 = J_\omega \dot{q} = \begin{bmatrix} 0 & -\dfrac{\sqrt{2}}{2} & 0 \\ 0 & \dfrac{\sqrt{2}}{2} & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2}\dot{\theta}_2 \\ \dfrac{\sqrt{2}}{2}\dot{\theta}_2 \\ \dot{\theta}_1 \end{bmatrix} \text{rad/s}$$

(e) For the same configuration described in the previous question, what is v_3^0, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1, \dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

$$v_3^0 = J_v \dot{q} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \\ \dfrac{\sqrt{2}}{2} & 0 & \dfrac{\sqrt{2}}{2} \\ 0 & -\dfrac{\sqrt{2}}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{\sqrt{2}}{2}\dot{\theta}_1 + \dfrac{\sqrt{2}}{2}\dot{d}_3 \\ \dfrac{\sqrt{2}}{2}\dot{\theta}_1 + \dfrac{\sqrt{2}}{2}\dot{d}_3 \\ -\dfrac{\sqrt{2}}{2}\dot{\theta}_2 \end{bmatrix} \text{m/s}$$

(f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = [0\ \mathrm{m/s} \quad 0.5\ \mathrm{m/s}\ 0.1\ \mathrm{m/s}]^T$? Make sure to provide units with your answer.

We have already known that

$$v_3^0 = \begin{bmatrix} 0\mathrm{m/s} \\ 0.5\mathrm{m/s} \\ 0.1\mathrm{m/s} \end{bmatrix}$$

Hence, convert the linear velocity into angular velocity,

$$v_3^0 = \begin{bmatrix} 0\ \mathrm{m/s} \\ 0.5\ \mathrm{m/s} \\ 0.1\ \mathrm{m/s} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ \frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ -\frac{\sqrt{2}}{2}\dot{\theta}_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{4}\ \mathrm{rad/s} \\ \frac{\sqrt{2}}{10}\ \mathrm{rad/s} \\ \frac{\sqrt{2}}{4}\ \mathrm{m/s} \end{bmatrix} = \omega_3^0$$

(g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

Not easy to calculate that

$$\det(J) = 0 = \det \begin{bmatrix} -s_1c_2d_3 & -c_1s_2d_3 & c_1c_2 \\ c_1c_2d_3 & -s_1s_2d_3 & s_1c_2 \\ 0 & -c_2d_3 & -s_2 \end{bmatrix} = -c_2 \cdot (\mathrm{d}_3)^2$$

So, when $c_2 = 0$ or $d_3 = 0$, then $\det(J) = 0$. **That is when,**

$$\begin{cases} \theta_2 = \pm\frac{\pi}{2} \\ \text{or} \\ d_3 = 0 \end{cases}$$

The robot arm will get the singular configurations.

(h) Sketch the cylindrical manipulator in each singular configuration that you found and explain what effect the singularity has on the robot's motion in that configuration.

As we mention in the question above, when $c_2 = 0$ or $d_3 = 0$, then the robot arm gets the singular configurations. The following Figure 4,5,6 show the sketches of the configuration.

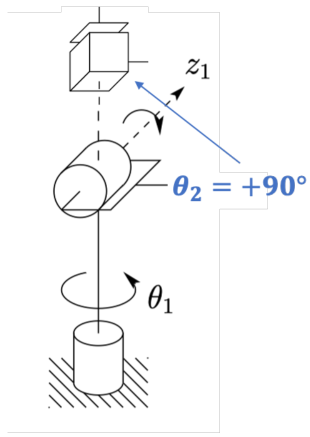

Figure 4 Three-link Spherical Robot

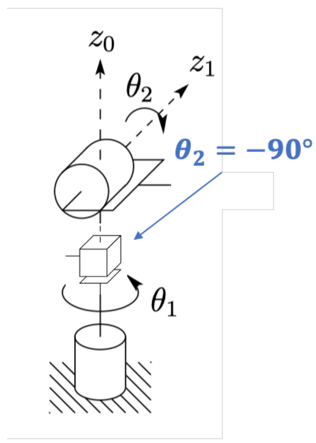

Figure 5 Three-link Spherical Robot

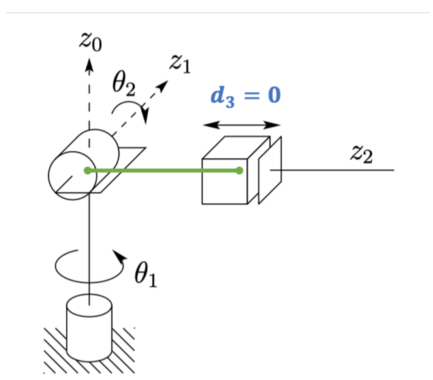

Figure 6 Three-link Spherical Robot

(i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

Noted that singularity of the robot exists objectively, we cannot avoid the singular configurations through changing the direction or zero configuration. So in practice, we need to carefully program to prevent robots from getting into ‘trouble’, as shown in Figure 7.

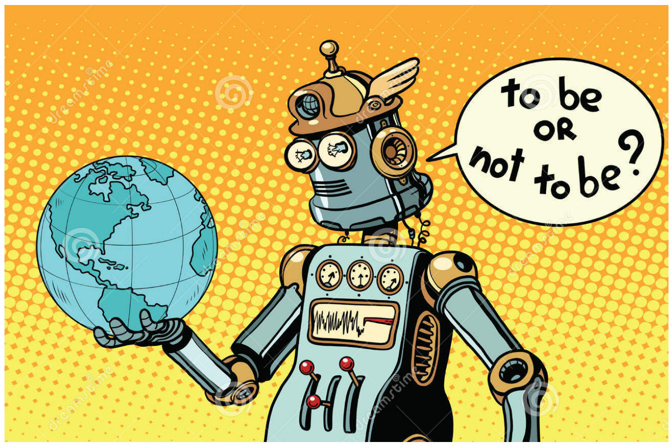

Figure 7 Robotics

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Thank you! Take care and keep warm!

Hanlin CAI

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