EE206 Assignment 6 *

Due 20th Nov.

1. Use the relation between multiplication of $f(t)$ (by t^n) and differentiation of $F(s)$ to find the Laplace transforms of the following
   (a) $f(t) = te^{-t}\cos(2t)$
2. Use the Laplace transform to solve the given initial-value problems
   (a) $y'' + y' = e^{-t}\cos t, \quad y(0) = 0, \quad y'(0) = 0.$
   (b) $y' + 2y = f(t), \quad y(0) = 0,$ where
   $$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ -1, & 1 \le t. \end{cases}$$
   (c) $y' + 3y = f(t), \quad y(0) = 0,$ where
   $$f(t) = \begin{cases} 1, & 0 \le t < 1, \\ 0, & 1 \le t. \end{cases}$$
3. Use the Convolution Theorem to find the Laplace Transform of the following functions (* stands for convolution)
   (a) $f(t) = t^3 * te^{-t}$
   (b) $f(t) = e^{2t} * \sin 3t$
4. Evaluate the given Laplace transforms without evaluating the integrals (Convolution theorem)
   (a) $\mathcal{L}\left\{\int_0^t \tau \sin\tau d\tau\right\}$
   (b) $\mathcal{L}\left\{\int_0^t 2\sin\tau\cos(t-\tau)d\tau\right\}$
5. Use the Laplace transform to solve the following problems
   (a) $f(t) + \int_0^t f(\tau)d\tau = 1$
   (b) $y'' + 9y = \cos 3t, \quad y(0) = 1, \quad y'(0) = 4$
   (c) $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi), \quad y(0) = 1, \quad y'(0) = 0$
   (d) $y'' + 2y' = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 1$

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