EE406 Assignment-1

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(Q1) (20%)

Find $y(k)$ for $Y(z) = \frac{bz}{(z-p_1)^2(z-p_2)}$

Give your answer in fractional form, **not** in floating point form e.g. $y(k) = \frac{8}{3}\left(\frac{2}{27}\right)^k$ and not $y(k) = 2.67(0.074)^k$

- $p_1 = -1.1, \;\; p_2 = -2, \;\; b = 0.8$
- The answer is given at Page 3.

(Q2) (20%)

$y(k+2) + a_1 y(k+1) + a_2 y(k) = b_0$

(a) Find $Y(z)$. Give your final answer in standard polynomial form i.e. **not** in factorised form e.g. $Y(z) = \frac{4z+2}{2z^2-z-6}$ and not $Y(z) = \frac{2(2z+1)}{(z-2)(2z+3)}$

(b) Find $y(k)$ using the method of partial fractions.

Give your final answer in fractional form, **not** in floating point form e.g. $y(k) = \frac{8}{3}\left(\frac{2}{27}\right)^k$ and not $y(k) = 2.67(0.074)^k$

(c) Evaluate $y(0), \ldots, y(8)$. Tabulate your answer.

(d) Comment on the stability of $Y(z)$ and $y(k)$

- $a_1 = -0.7, \;\; a_2 = -0.44, \;\; b_0 = 0, \;\; y(0) = -8, \;\; y(1) = -1$
- The answer is given at Page 4.

(Q3) (20%)

Use Jury's test to establish the stability of the system with characteristic equation:

$A(z) = a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0 = 0$

State if the system is stable or not and why.

- $a_4 = 3, \;\; a_3 = -1, \;\; a_2 = -2.9, \;\; a_1 = 0.5, \;\; a_0 = 0.5$
- The answer is given at Page 5.

Q4 & Q5

- $b_2 = 11, \ \ b_3 = 16$
- $a_1 = 6.3, \ \ a_2 = 13, \ \ a_3 = 8.4$
- $n = 49, \ \ T(secs.) = 0.09$
- The answer is given at Page 6-10.

If you have any question or problem,

please feel free to contact me at: hanlin.cai@ieee.org

EE406 Assignment 1.

Q1. $P_1 = -1.1 \quad P_2 = -2 \quad b = 0.8$

Sol.

$$Y(z) = \frac{0.8z}{(z+1.1)^2(z+2)} \longrightarrow \frac{Y(z)}{z} = \frac{0.8}{(z+1.1)^2(z+2)}.$$

$$\therefore Y(z) = \frac{A_{11}}{(z+1.1)^2} + \frac{A_{12}}{(z+1.1)} + \frac{A_2}{(z+2)}.$$

$$\begin{cases} A_{11} = (z+1.1)^2 \dfrac{Y(z)}{z}\Big|_{z=-1.1} = \dfrac{0.8}{z+2}\Big|_{z=-1.1} = \dfrac{8}{9} & ① \\ A_{12} = \dfrac{d}{dz}(z+1.1)^2 \dfrac{Y(z)}{z}\Big|_{z=-1.1} = \dfrac{d}{dz}\dfrac{0.8}{z+2}\Big|_{z=-1.1} = -\dfrac{80}{81} & ② \\ A_2 = (z+2)\dfrac{Y(z)}{z}\Big|_{z=-2} = \dfrac{80}{81} & ③ \end{cases}$$

$$\therefore \frac{Y(z)}{z} = \frac{8}{9}\frac{1}{(z+1.1)^2} - \frac{80}{81}\cdot\frac{1}{z+1.1} + \frac{80}{81}\cdot\frac{1}{z+2}$$

$$\therefore Y(z) = \frac{8}{9} z \frac{1}{(z+1.1)^2} - \frac{80}{81} z \frac{1}{z+1.1} + \frac{80}{81} z \cdot \frac{1}{z+2}$$

$$\therefore y(k) = \frac{8}{9} k \left(-\frac{11}{10}\right)^{k-1} - \frac{80}{81}\left(-\frac{11}{10}\right)^{k} + \frac{80}{81}(-2)^{k}.$$

Q2. Given $y(k+2) - 0.7y(k+1) - 0.44y(k) = 0 \qquad y(0) = -8 \quad y(1) = -1$

(a)

sol. $z^2Y(z) - z^2y(0) - zy(1) - 0.7\left(zY(z) - zy(0)\right) - 0.44Y(z) = 0$

$$\rightarrow Y(z) = \frac{-8z^2 + 4.6z}{z^2 - 0.7z - 0.44} = \frac{-400z^2 + 230z}{50z^2 - 35z - 22}$$

(b) $$Y(z) = \frac{-400z^2 + 230z}{50z^2 - 35z - 22} = \frac{-400z^2 + 230z}{(5z+2)(10z-11)}.$$

$$\therefore \frac{Y(z)}{z} = \frac{A_1}{5z+2} + \frac{A_2}{10z-11}$$

$$\begin{cases} A_1 = (5z+2)\frac{Y(z)}{z}\Big|_{-0.4} = -26 & ① \\ A_2 = -28. & ②. \end{cases}$$

$$\therefore Y(z) = \frac{-26z}{5z+2} - \frac{28z}{10z-11} \Rightarrow y(k) = \frac{-26}{5}\left(\frac{2}{5}\right)^k - \frac{14}{5}\left(\frac{11}{10}\right)^k$$

(c) $y(0) = -8$

$y(1) = -5.16$

$y(2) = -4.22$

$y(3) = -4.0596$

$y(4) = -4.2326$

$y(5) = -4.562676$

$y(6) = -4.98167$

$y(7) = -5.46492756$

$y(8) = -6.00545654.$

(d) Pole : $50z^2 - 35z - 22 = 0$

$$\begin{cases} z_1 = 1.1 \quad > 1 \\ z_2 = -0.4 \end{cases}$$

$\therefore$ the system is unstable.

For $y(k) = -\frac{26}{5}\left(\frac{2}{5}\right)^k - \frac{14}{5}\left(\frac{11}{10}\right)^k.$

$$y(\infty) = \lim_{k \to \infty}\left[-\frac{26}{5}\left(\frac{2}{5}\right)^k - \frac{14}{5}\left(\frac{11}{10}\right)^k\right]$$

$$= -\infty$$

$\therefore$ the system is unstable as expected!

Q3. Given $A(z) = 3z^4 - z^3 - 2.9z^2 + 0.5z + 0.5 = 0$

① $a_4 = 3 \rightarrow$ can apply Jury Test.

② $A(1) = 3 - 1 - 2.9 + 0.5 + 0.5 = 0.1 > 0$ Test 1 ✓

③ $(-1)^4 A(-1) = A(-1) = 3 + 1 - 2.9 + 0.5 - 0.5 = 1.1$

Test 2 ✓

④ Jury Table.

	z^0	z^1	z^2	z^3	z^4
1	$\frac{1}{2}$	$\frac{1}{2}$	-2.9	-1	3
2	3	-1	-2.9	$\frac{1}{2}$	$\frac{1}{2}$
3	-8.75	3.25	7.25	-2	
4	-2	7.25	3.25	-8.75	
5	72.5625	-13.9375	-56.9375		

⑤ We get $\begin{cases} |a_0| = \frac{1}{2} < |a_4| = 3 \\ |b_0| = 8.75 > |b_3| = 2 \\ |c_0| > |c_2| \end{cases}$

Thus, this system is stable as all test passed.

(OVER).

(Q4) (20%)

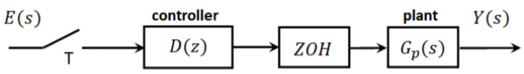


Figure 1 Open-loop sampled-data control system

The plant is $G_p(s) = \frac{b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$, the controller is $D(z) = 1$ and the sample period is T.

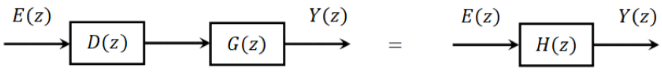


Figure 2 Open-loop discrete-time control system

Use MATLAB to:

1. find the system discrete-time transfer function $H(z)$.
2. simulate and plot the unit step response for n samples. Your plot should be formatted like the example on slide 27 of notes 3.
3. Find the zeros, poles and DC gain of your open-loop system.

Discuss your result.

Given $G_p(s) = \frac{b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3} = \frac{11s + 16}{s^3 + 6.3s^2 + 13s + 8.4}$

where $n = 49$, $T = 0.09$

The MATLAB code was programmed like below:

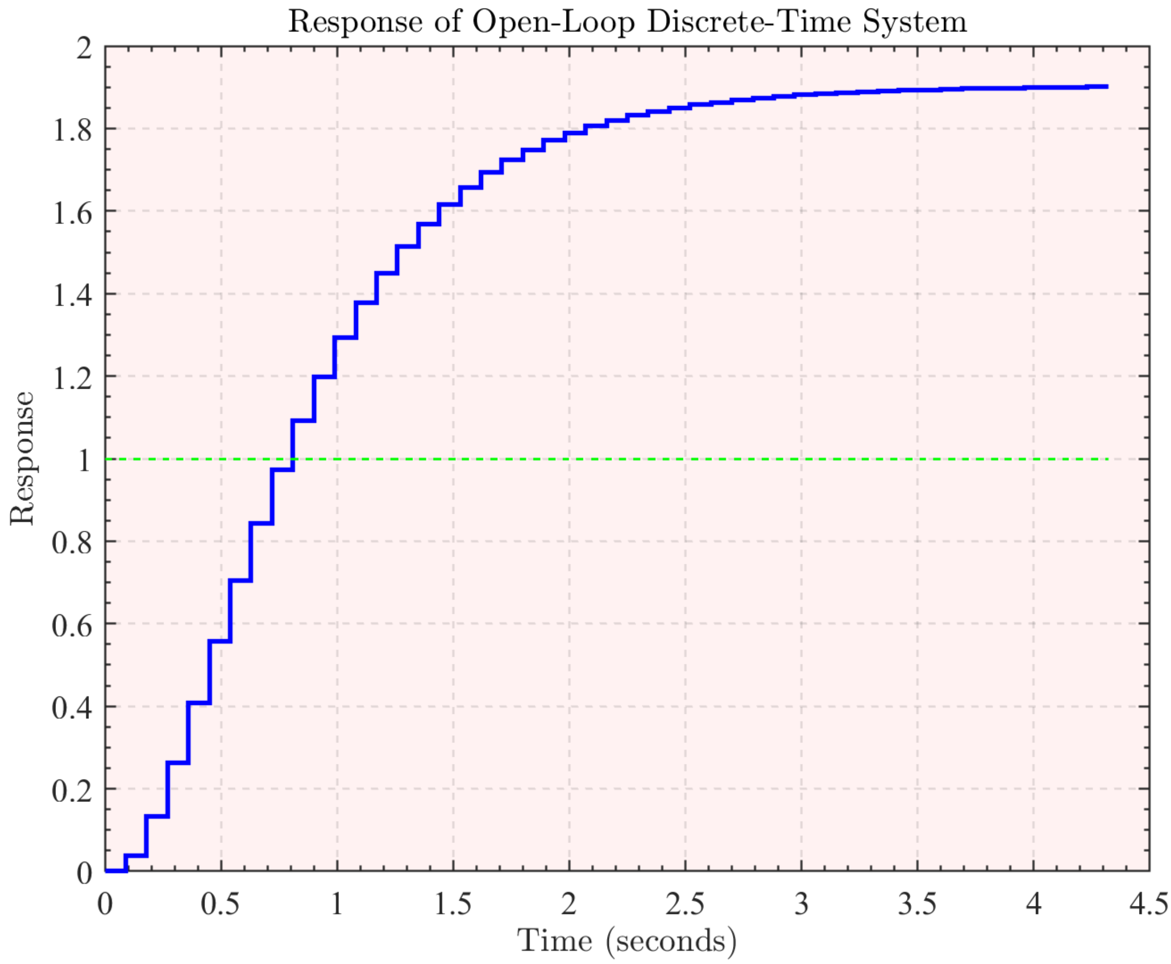
```
T = 0.09;
Gs = tf([11 16], [1 6.3 13 8.4]); % Continuous-time transfer function
Gz = c2d(Gs, T, 'zoh'); % Discrete-time system using Zero-Order Hold
Dz = tf(1, 1, T); % Discrete-time controller (unity feedback)
DGz = Dz * Gz; % Combined transfer function
n = 49; % Number of samples for the unit step response
t = 0:T:(n-1)*T; % Time vector for n samples
e = ones(size(t)); % Unit step input vector
y = lsim(DGz, e, t); % Simulation of the system response

% Plotting the unit step input and the output response
figure;
stairs(t, y, 'b', 'LineWidth', 2); % Stairs used for discrete-time response
hold on;
stairs(t, e, '--g', 'LineWidth', 1); % Input as a green dashed line
hold off;
legend('y(k): output', 'e(k): input');
xlabel('Time (seconds)');
ylabel('Response');
title('Response of Open-Loop Discrete-Time System');
grid on;

% Displaying the transfer function, poles, zeros, and DC gain
DGz
pole(DGz)
zero(DGz)
dcgain(DGz)
```

The output gives that:

- The system discrete-time transfer function is

$H(z) = \frac{0.03857z^2 - 0.0004868z - 0.02926}{z^3 - 2.485z^2 + 2.057z - 0.5672}$

- The unit step response for 49 samples is



- Zeros of the open-loop system are

$zero_1 = 0.8773$
$zero_2 = -0.8647$

- Poles of the open-loop system are

$pole_1 = 0.8919 + 0.0000i$
$pole_2 = 0.7966 + 0.0383i$
$pole_3 = 0.7966 - 0.0383i$

- DC Gain of the open-loop system is

1.9048

(Q5) (20%)

Use your Q4 plant, and controller $D(z) = \frac{2z-1}{z-1}$. Use the Q4 sample period.

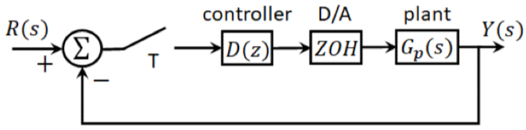


Figure 3 Closed-loop sampled-data control system

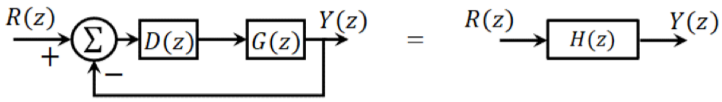


Figure 4 Closed-loop discrete-time control system

Use MATLAB to:

1. find the system discrete-time transfer function $H(z)$.
2. simulate and plot the unit step response for n samples. Your plot should be formatted like the examples on slide 33 of notes 3.
3. Find the zeros, poles and DC gain of your closed-loop system.

Discuss your result.

The MATLAB code was programmed like below:

```
T = 0.09;
n = 49; % Number of samples for step response

% Plant definition
Gs = tf([11 16], [1 6.3 13 8.4]); % (11s+16)/(s^3+6.3s^2+13s+8.4)

% Discretize the plant with zero-order hold
Gz = c2d(Gs, T, 'zoh');

% Controller definition as per Q5
numDz = [2 -1]; % Numerator coefficients
denDz = [1 -1]; % Denominator coefficients
Dz = tf(numDz, denDz, T);

% The closed-loop transfer function H(z)
SYSz = feedback(Dz*Gz, 1);

% Time vector for step response
t = 0:T:T*(n-1);

% Simulate the unit step response
[y, t_out] = step(SYSz, t);

% Plot the step response
figure;
stairs(t_out, y, 'b', 'LineWidth', 2);
xlabel('Time (seconds)');
ylabel('Output');
title('Closed-Loop Step Response');
```

```
grid on;

% Print system characteristics
disp(SYSz);
disp(pole(SYSz));
disp(zero(SYSz));
disp(dcgain(SYSz));
```

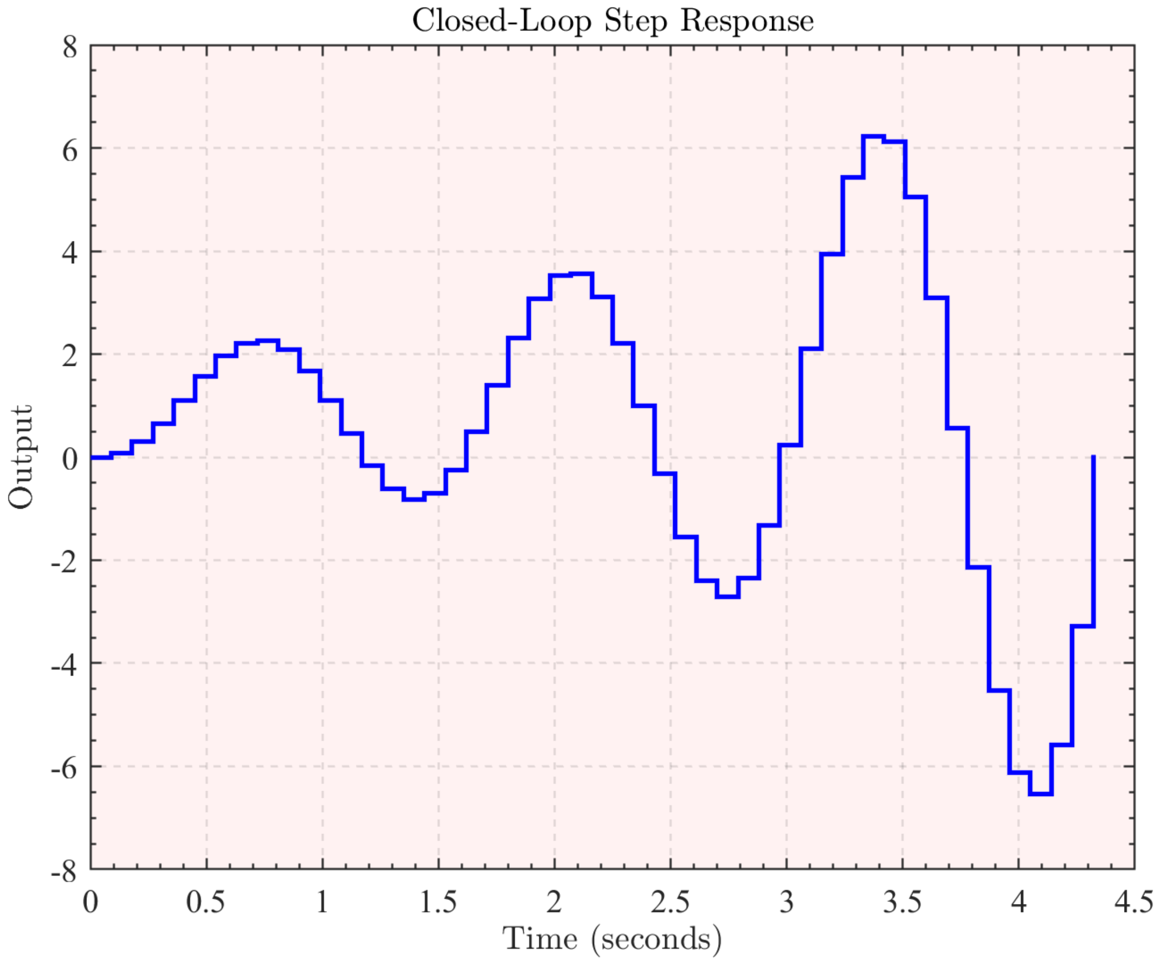
- The system discrete-time transfer function is

$H(z) = \frac{0.0771z^3 - 0.0395z^2 - 0.0580z - 0.0293}{z^4 - 3.4079z^3 + 4.5023z^2 - 2.6821z - 0.5965}$

Where the parameter list is as follows:

```
    Numerator: {[0 0.0771 -0.0395 -0.0580 0.0293]}
  Denominator: {[1 -3.4079 4.5023 -2.6821 0.5965]}
     Variable: 'z'
      IODelay: 0
   InputDelay: 0
  OutputDelay: 0
           Ts: 0.0900
     TimeUnit: 'seconds'
    InputName: {''}
    InputUnit: {''}
   InputGroup: [1×1 struct]
   OutputName: {''}
   OutputUnit: {''}
  OutputGroup: [1×1 struct]
        Notes: [0×1 string]
     UserData: []
         Name: ''
 SamplingGrid: [1×1 struct]
```

- The unit step response for 49 samples is



- Poles of the open-loop system are

$pole_1 = 0.9565 + 0.4311i$
$pole_2 = 0.9565 - 0.4311i$
$pole_3 = 0.8770 + 0.0000i$
$pole_4 = 0.6179 + 0.0000i$

- Zeros of the open-loop system are

$zero_1 = 0.8773$
$zero_2 = -0.8647$
$zero_2 = 0.5000$

- DC Gain of the open-loop system is

1.0000