Maynooth University
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SEMESTER 1

2020-21

EE206FZ
Differential Equations and Transform Methods

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Time allowed: 2 hours

Question 1 is compulsory

Answer Question 1 and any two other questions

Question 1 is worth 50 marks

All other questions are worth 25 marks

Instructions

	Yes	**No**
Log Books Allowed	☐	☐
Formula Tables Allowed	☐	☐
Other Allowed (*enter details)*	☐	☐

General (*enter details)*

Question 1 (Compulsory)

(a) Solve using separation of variables **[5 marks]**

$$\frac{dy}{dx} = \frac{xy}{\ln y}, \qquad y(1) = 1$$

(b) Find the general solution of the linear first-order equation **[5 marks]**

$$\frac{dy}{dx} + 3x^2 y = e^{-x^3}$$

(c) Solve, without using the Laplace transform, the second order differential equation

$$y'' + 2y' = -y$$ **[5 marks]**

(d) Find the inverse Laplace transforms of **[5 marks]**

$$(i)\mathbf{L}^{-1}\left[\frac{1}{s-1} + \frac{1}{s-2} + \frac{2}{s-3}\right]$$

$$(ii)\mathbf{L}^{-1}\left[\frac{s+2}{s^2+4}\right]$$

$$(iii)\mathbf{L}^{-1}\left[\frac{1}{s^2(s^2+1)}\right]$$

$$(iv)\mathbf{L}^{-1}\left[\frac{1}{(s-2)^2+1}\right]$$

$$(v)\mathbf{L}^{-1}\left[\frac{2e^{-s}}{s^2} + \frac{3e^{-s}}{s}\right]$$

(e) Perform partial fractions decomposition and then inverse Laplace transforms of

[5 marks]

$$Y(s) = \frac{s+2}{(s+3)^2}$$

(f) Solve the initial value problem **[5 marks]**

$$y' - 3y = e^{3t}t^3, \qquad y(0) = 0$$

using the Laplace transform.

(g) Find the solution of the initial value problem defined as **[5 marks]**

$$y' + y = f(t), \qquad y(0) = 0$$

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ 2, & t \ge 2 \end{cases}$$

using the Laplace transform.

(h) Expand the given function in an appropriate cosine or sine series **[5 marks]**

$$f(x)=\begin{cases}-1, & -\pi \le t < 0\\ 1, & 0 \le x < \pi\end{cases}$$

(i) Find the Fourier transform of the function **[5 marks]**

$$f(x)=\begin{cases}2, & -1 \le t < 1\\ 0, & Otherwise\end{cases}$$

(j) Perform the Z-Transform of the following sequence **[5 marks]**

$$x_k = 5e^{-2k}$$

Question 2

(a) Solve the differential equation by the method of undetermined coefficients

$$x''(t)-10x'(t)+25x(t)=30t+3, \qquad x(0)=0, x'(0)=0$$ **[10 marks]**

(b) Find the solution of the initial value problem defined as

$$y''-y=tU\left(t-1\right), \qquad y(0)=0, y'(0)=0$$ **[15 marks]**

Question 3

(a) Obtain the trigonometric Fourier series of **[10 marks]**

$$f(x)=\begin{cases}-2-3x, & -2 \le x < 0\\ 2-3x, & 0 < x \le 2\end{cases}$$

(b) Solve the integro-differential equation **[15 marks]**

$$f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} d\tau$$

using the Laplace transform.

Question 4

(a) Solve the following recurrence relation

$$x_{k+2} - 3x_{k+1} + 2x_k = 1$$

where $x_0 = 0$ and $x_1 = 1$.

[10 marks]

(b) Find the Fourier transform of the function

$$f(x) = \begin{cases} 0, & x < -1 \\ e^{-|x|}, & -1 < x < 1 \\ 0, & x > 1 \end{cases}$$

[15 marks]