The basic circuit analysis theorems

Introduction

Two important definitions:

Loop is any simple closed path in a circuit.

Mesh is a loop that doesn't have a closed path in its interior.

And a reminder that the direction of current through a resistor will define the polarity of the voltage drop across the resistor:

MULTI-LOOP CIRCUIT ANALYSIS

Mesh/Loop Analysis

This is the systematic application of Kirchoff's Voltage Law

Step 1 **Identify all loops (don't forget the outer loop)**

Step 2 Identify what you want to learn

Step 3 Apply circuit simplification techniques to those parts of the circuits you do not care about

Step 4 Transform all current sources into their **equivalent voltage source form**

Step 5 **Draw directions for the current in each loop. Label these currents** (this is arbitrary as long as you stick with it).

Step 6 For each resistor in the loop, draw a +/- to indicate voltage drops. **Resistors always go + to- with current flow**, while batteries have their own signs.

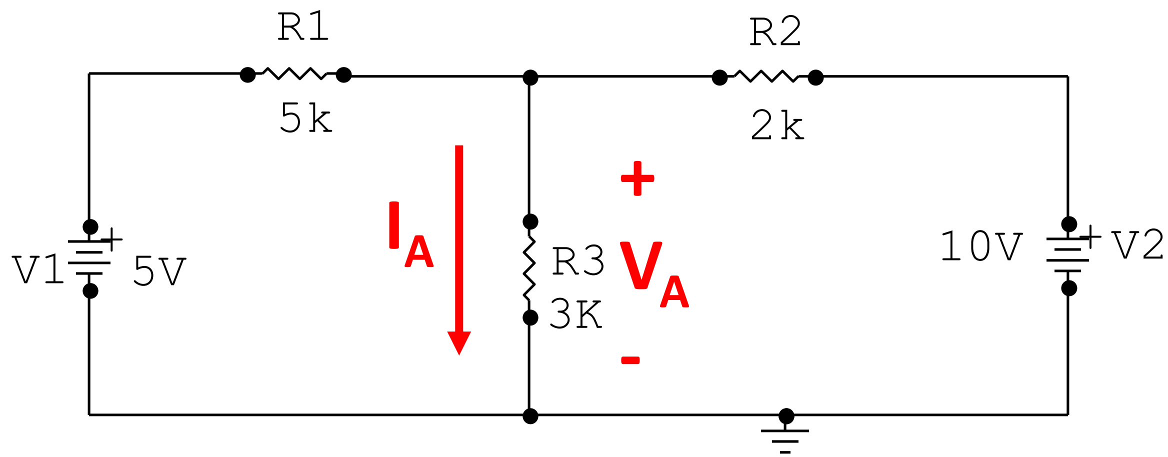
Step 7 Express the voltage drops in terms of the loop current(s)

Step 8 Declare whether drops are positive or negative

Note that for ANY and ALL loops, **the algebraic sum of voltages must equal zero.** This produces a set of simultaneous equations for each node. Solve the simultaneous equations (in any manner).

Multi-Loop Circuit Analysis

Calculate the current I_A and the voltage (across R_3) V_A in the circuit below:

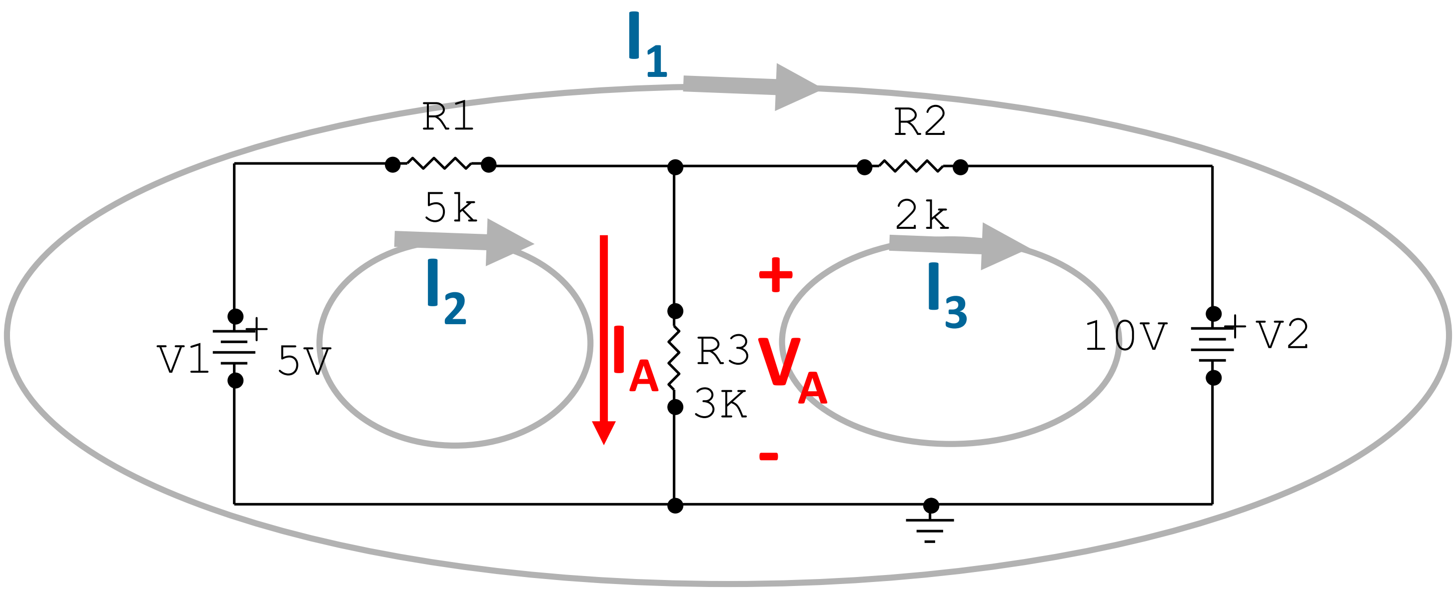


Ohms Law: V=IR

Kirchoff's Current Law: current is preserved at a node

Kirchoff's Voltage Law: voltage in a loop sums to zero

Mesh Analysis - Example



Step 1 **Identify all loops:** two inner loops, and one outer loop.

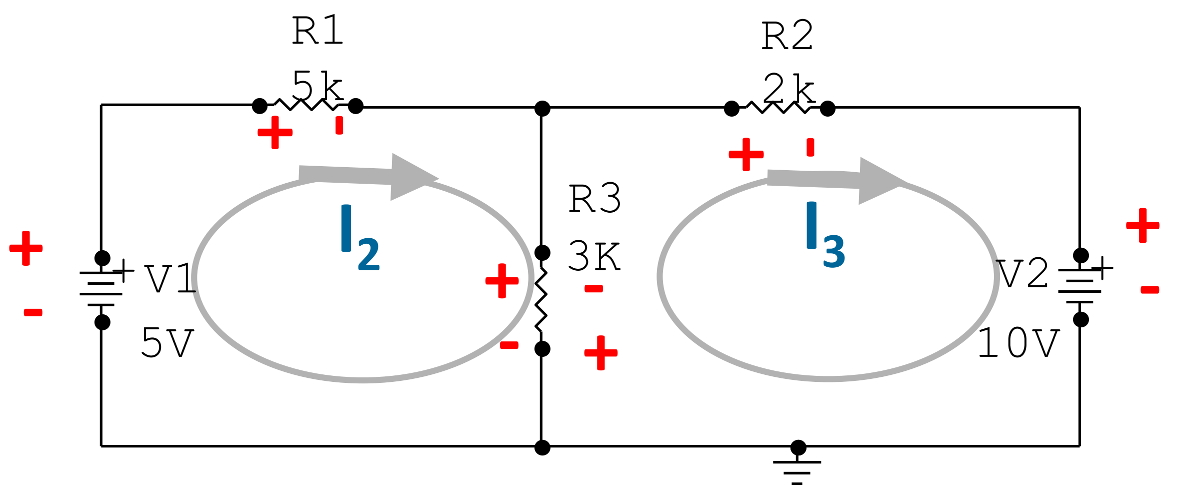
Step 2 Identify what you want to learn (I_A, V_A)

Step 3 Apply circuit simplification techniques to those parts of the circuits you do not care about

Step 4 **Transform current sources into voltage sources:** none

Step 5 **Draw current directions**

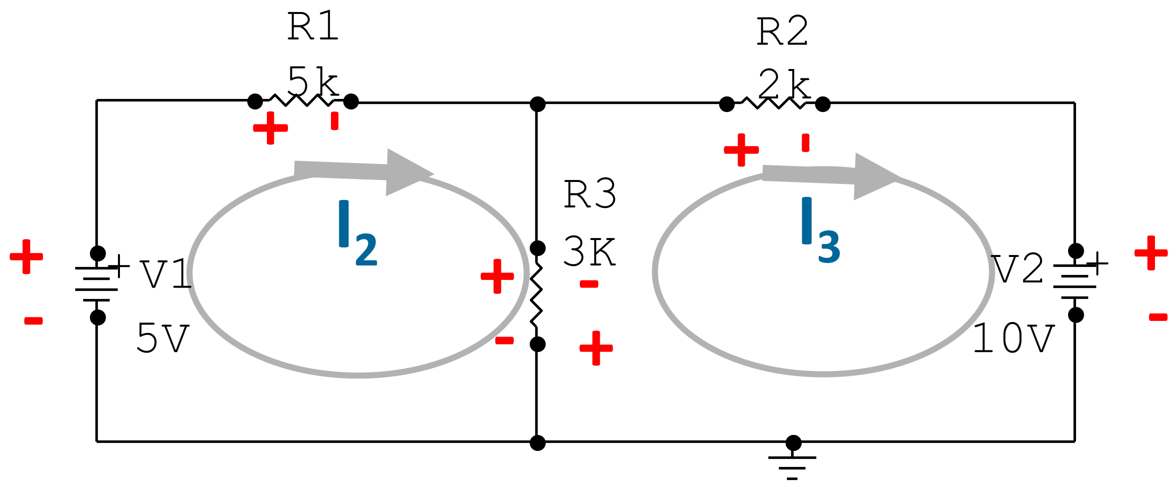
Mesh Analysis – Example



Step 6 Indicate Voltage Drops

- you only need to do the minimum set of loops provided that all elements are included in the loops you do select (I'll pick the two inner ones).
- nothing goes wrong if you do them all.. Just a little more work.

Mesh Analysis – Example



Step 7 Express the voltage drops in terms of the loop current(s). Declare whether a drop is positive or negative

If I pick voltage drops as positive:

Loop2: $-V_1 + I_2R_1 + (I_2 - I_3)R_3 = 0$

Loop3: $+V_2 + I_3R_2 + (I_3 - I_2)R_3 = 0$

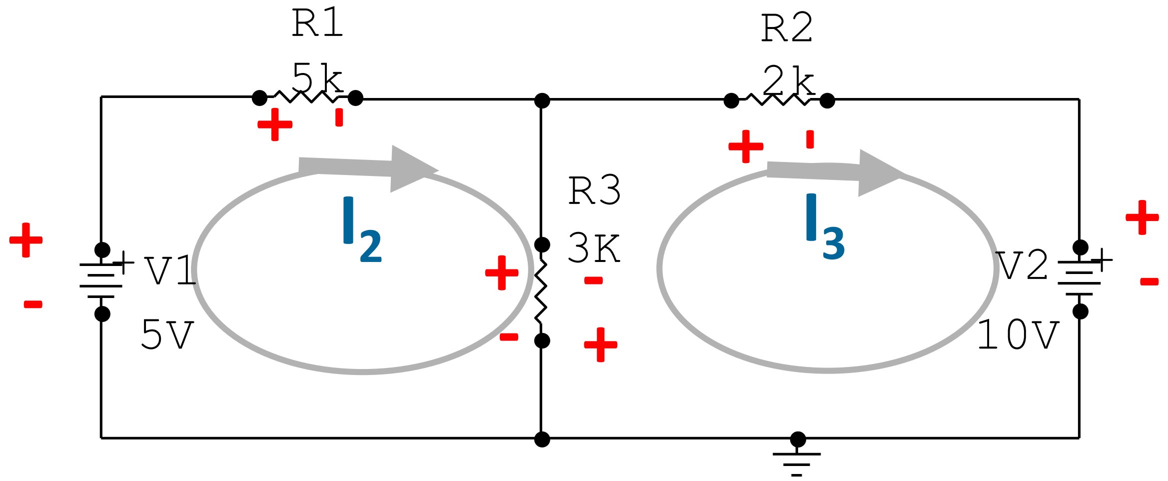
If I pick voltage drops as negative:

Loop2: $V_1 - I_2R_1 - (I_2 - I_3)R_3 = 0$

Loop3: $-V_2 - I_3R_2 - (I_3 - I_2)R_3 = 0$

Currents rotating in opposite directions in the same device… then we use the difference of the two currents for use in the device.

Mesh Analysis - Example



$$I_2R_1 - (I_3 - I_2)R_3 = +V_1$$

$$+I_3R_2 + (I_3 - I_2)R_3 = -V_2$$

Mesh Analysis – Example

Lets just use simultaneous equations (plug in numbers)

$$I_2 5000 - (I_3 - I_2)3000 = +5$$
$$+I_3 2000 + (I_3 - I_2)3000 = -10$$
$$8I_2 - 3I_3 = +0.005$$
$$-3I_2 + 5I_3 = -0.010$$

Scale and cancel one variable, then get the other

$$8I_2 - 3I_3 = +0.005$$
$$-3\left(\frac{3}{5}\right)I_2 + 5\left(\frac{3}{5}\right)I_3 = -0.010\left(\frac{3}{5}\right)$$
$$adding$$
$$\left(8 - \frac{9}{5}\right)I_2 - 00 = +0.005 - 0.010\left(\frac{3}{5}\right)$$
$$I_2 = -0.001\left(\frac{5}{31}\right)$$
$$I_3 = -(.0005 - 8I_2)/3$$

watch your signs

Mesh Analysis – Example

Now calculate the voltage and current through R3

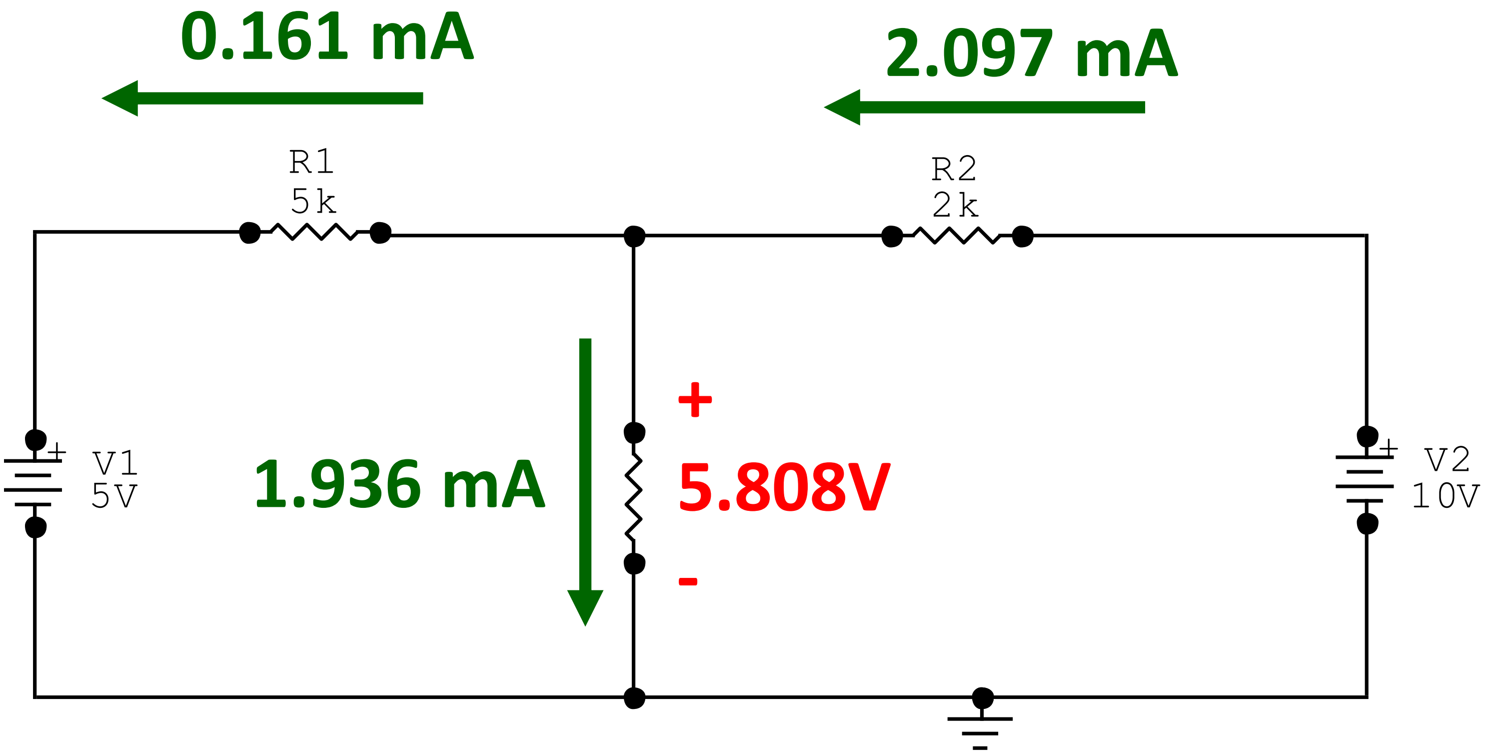
Current through R3 from top-to-bottom is (I_2-I_3) as drawn on chart

$$I_2 - I_3 = (-0.161m) - (-2.097m) = +1.936mA$$

Voltage is current by resistance, direction is + at top, due to current direction

$$V = IR = 3000 * 1.936mA$$
$$= 5.808V$$

Mesh Analysis – Example



NODE ANALYSIS

Node Analysis

This is the systematic application of Kirchoff's Current Law

Step 1 **Identify all nodes (greater than 2 paths).** Note that the ground node is a valid node but you'll see that you only need to look at n-1 nodes to solve the circuit… so note but ignore ground.

Step 2 Identify what you want to learn.

Step 3 Apply circuit simplification techniques to those parts of the circuits you do not care about.

Step 4 Transform all voltage sources into their **equivalent Current Source form.**

Step 5 Draw directions for the currents in each branch (direction doesn't matter at this stage, but current sources have their own directions).

Step 6 For each branch, identify the magnitude of the current in terms of the voltage drop across that branch.

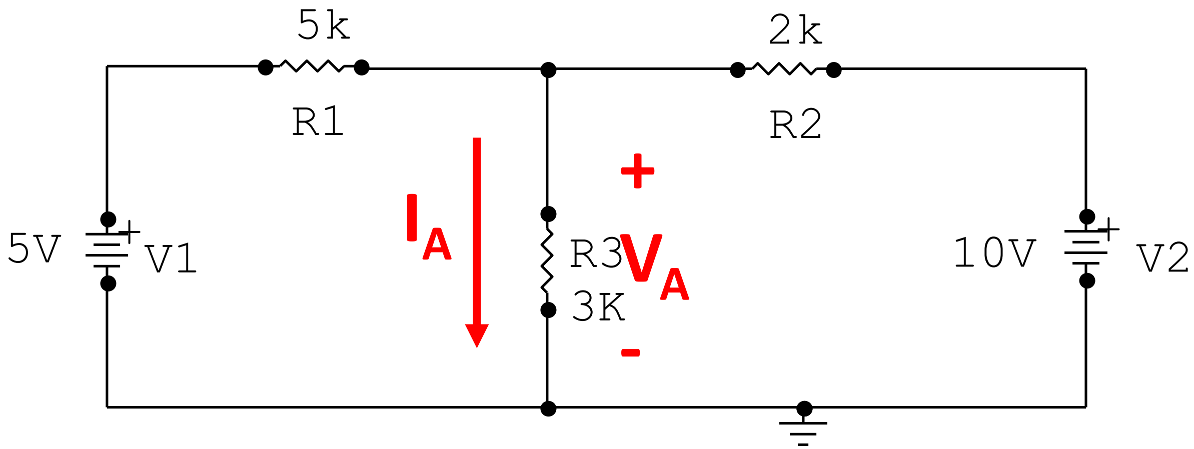
Step 7 **Pick a direction (in or out) as positive** (it doesn't matter which but be consistent).

Step 8 Note that at ANY and ALL nodes**, the algebraic sum of currents must equal zero.** This produces a set of simultaneous equations for each node.

Step 9 Solve the simultaneous equations (in any manner).

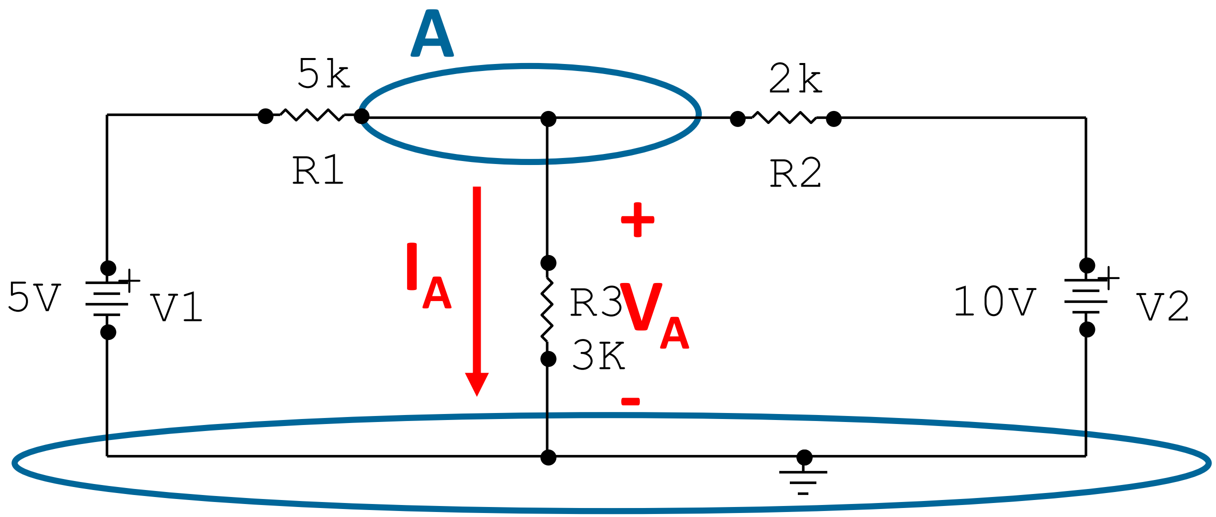
Node Analysis – Example

Calculate the current I_A and the voltage (across R_3) V_A in the circuit below



Node Analysis – Example

Calculate the current I_A and the voltage (across R_3) V_A in the circuit below



Step 1 Identify all nodes:

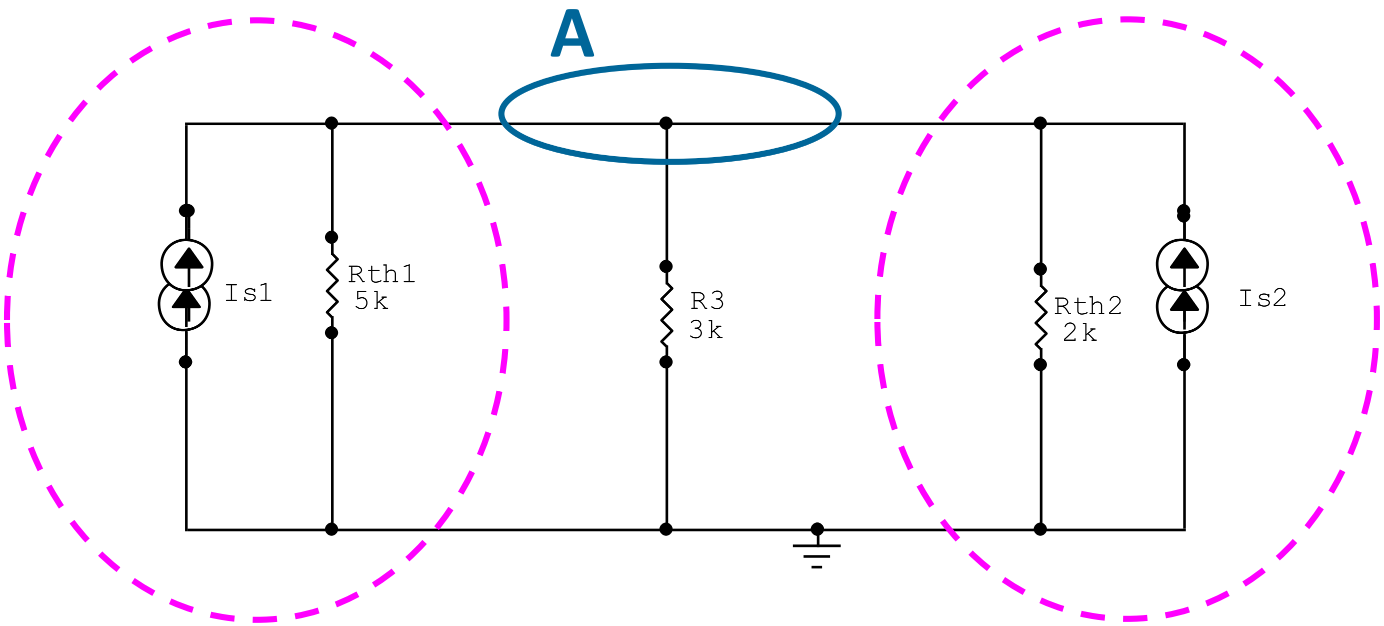
- The 2-path nodes are trivial cases and can be ignored.
- The ground node is relevant but we don’t tend to need to use it.
- That just leaves node A.

Node Analysis – Example (E3)

Step 2 Identify what you want to learn: I_A and V_A

Step 3 & Step 4 Apply circuit simplification techniques:

- we need to convert the voltage sources to current sources.
- keep the resistors the same, and $I_S=V_S/R_S$.



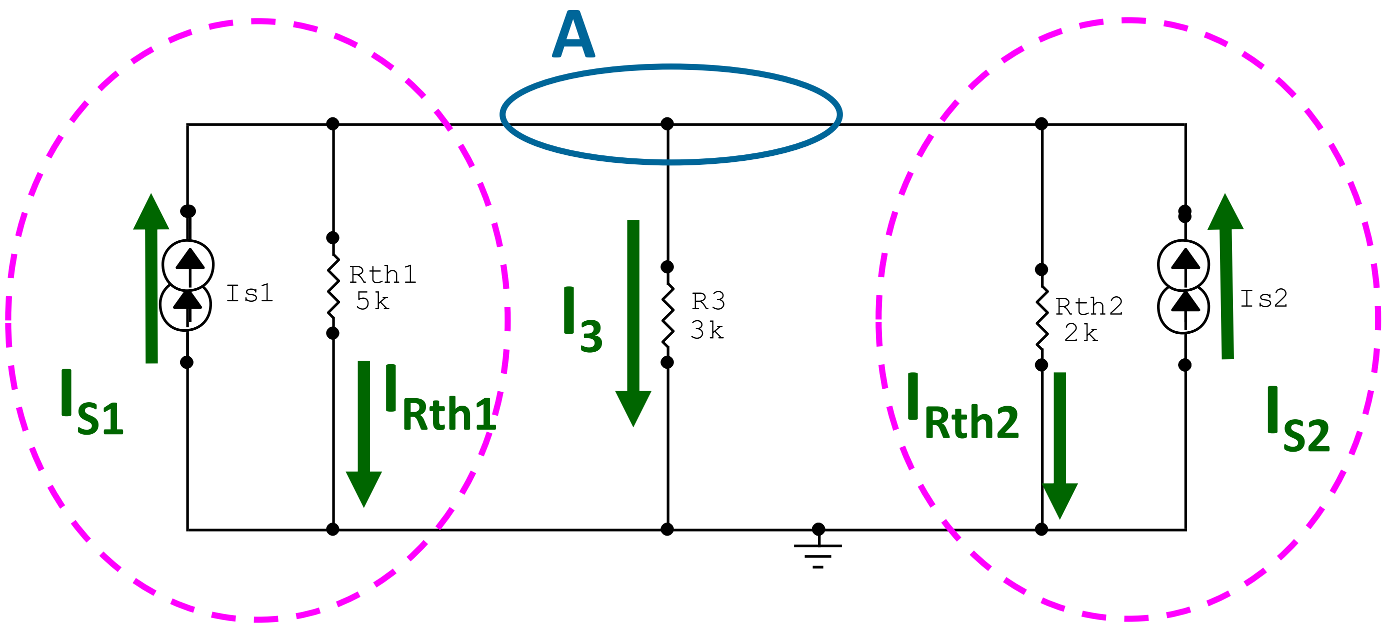
I_{s1}=1mA

I_{s2}=5mA

Node Analysis – Example (E5)

Step 5 Draw current directions in each branch

- the direction does not matter as long as you stay consistent
- for me, I choose either **all** currents in or out of the node (this time out)
- it helps if you do try to use common sense ☺
- it is important to be careful with your labels!!



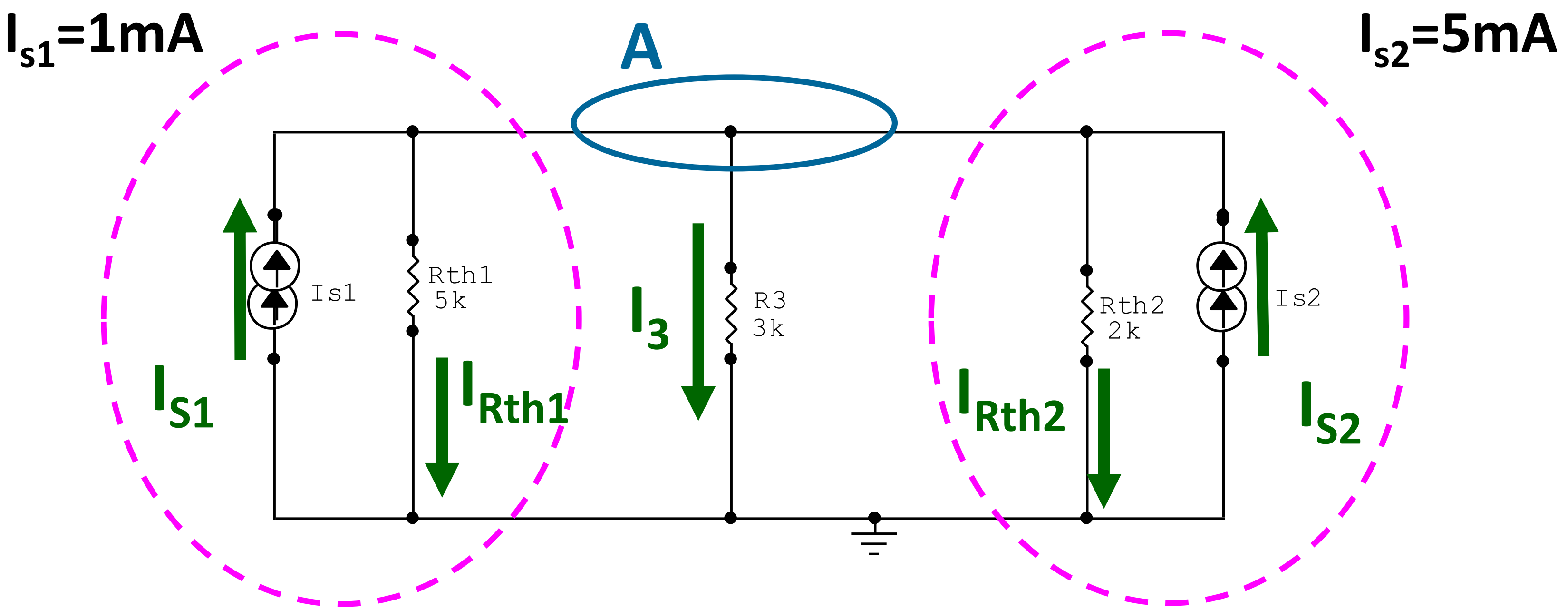
I_{s1}=1mA

I_{s2}=5mA

Node Analysis – Example (E6)

Step 6 **Express all currents in terms of voltages or constants (worry about signs later)**

$$I_{S1} = 1mA$$
$$I_{S2} = 5mA$$
$$I_3 = V_A/R3 = V_A/3000$$
$$I_{RTH1} = V_A/R_{th1} = V_A/5000$$
$$I_{RTH1} = V_A/R_{th2} = V_A/2000$$



Node Analysis – Example (E7)

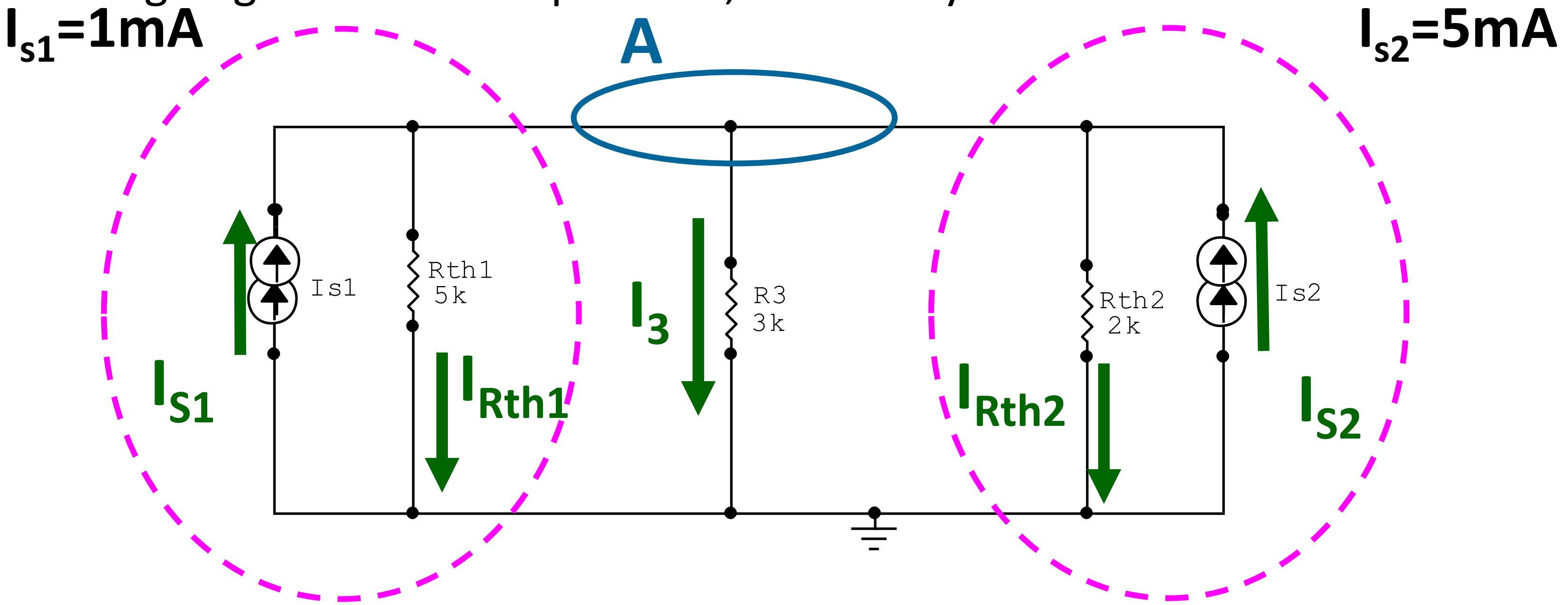
Step 7 Pick a sign for a direction (it doesn't matter which)

I picked **OUT** as being positive

Step 8 State Kirchoff's Current Law for all Nodes (there's still only 1 + ground)

$$-I_{S1} + I_{RTH1} - I_{S2} + I_{RTH2} + I_{R3} = 0$$
$$(-1m) + (V_A/5000) + (-5m) + (V_A/2000) + (V_A/3000) = 0$$

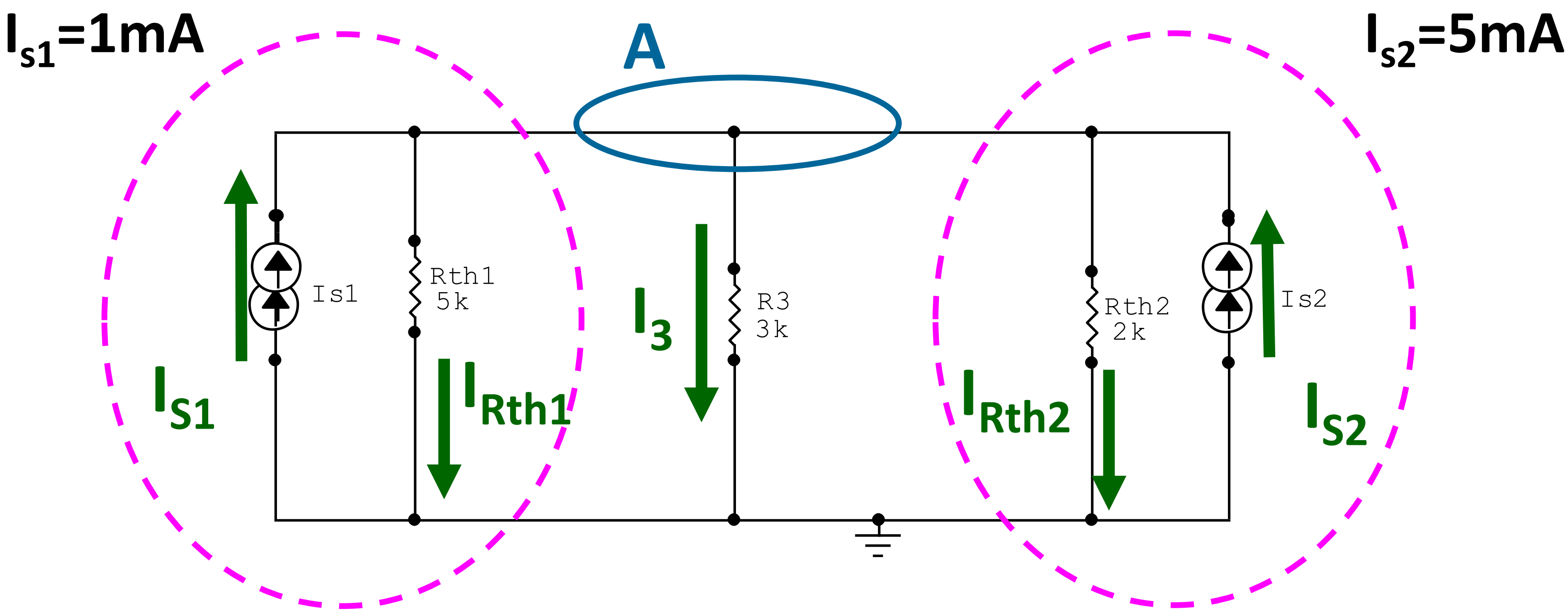
This going to be a trivial problem, as it's only one node.



Node Analysis – Example (E8)

Step 9 **Solve for V_A and then I_3**

$$(V_A/5)+(V_A/2)+(V_A/3)=6$$
$$V_A(0.2+0.5+0.333)=6$$
$$V_A=5.808\ Volts$$
$$I_A=1.936\ Amperes$$



Observations

Mesh or Loop Analysis: you end up solving for currents, even though we start with voltages.

Node Analysis: you end up solving for voltages, even though we start with currents.

Which one to use?

- **Complexity** is based on the number of nodes/loops you have in the final circuit. The one with the least is the easiest.
- You may have no choice depending on your current and voltage sources.