蔡汉霖 20122161 Hanlin Cai
83200211?

Q1.

$G(s) = \frac{1}{s(s+1)}$ $H(s) = \frac{1}{s+2}$ $K(s) = k$

→(a) open-loop : $G(s)H(s)K(s) = \frac{k}{s(s+1)(s+2)}$

poles : $p_1 = 0$ $p_2 = -1$ $p_3 = -2$ Hence $n=3$ $m=0$

$n > m$ the root loci has three parts.

angles : $\varphi_a = \frac{(2k+1)\pi}{n-m} = \frac{(2k+1)\pi}{3}$ $\therefore \varphi_a = \frac{\pi}{3}, \pi, \frac{5}{3}\pi$

~~asym~~ point : $\sigma_a = \frac{\sum_{i=1}^{3} P_i}{3} = \frac{-3}{3} = -1$

Hence $\varphi_a = \frac{1}{3}\pi, \pi, \frac{5}{3}\pi$ $\sigma_a = -1$ ①

→(b) breaking point : $\frac{1}{d} + \frac{1}{d+2} + \frac{1}{d+1} = 0$

$\therefore d_1 = (-3+\sqrt{3})/3 \approx -0.4226$ $d_2 = (-3-\sqrt{3})/3 = -1.577$

Because the loci lies in $(-\infty, -2]$ and $(-1, 0]$, So $d = d_1$

$\therefore s_b = d_1 = -0.4226$

$\therefore k = \left(s(s+1)(s+2) \right)\Big|_{s=-0.4226} = -0.3849$ ②

→(c) cross Im axis :

$G(s) = \frac{G(s)K(s)}{1+G(s)K(s)H(s)}$ $\therefore D(s) = s^3 + 3s^2 + 2s + k$

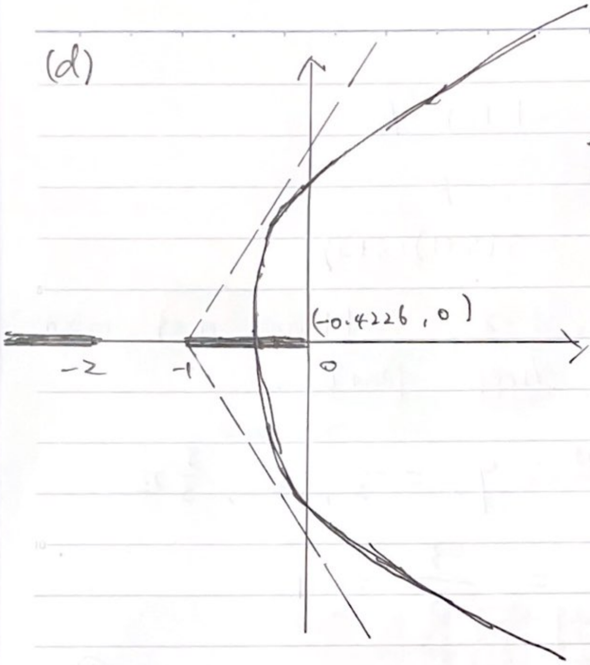
$\therefore D(j\omega) = -j\omega^3 - \omega^2 + 2j\omega + k = j(-\omega^3 + 2\omega) - \omega^2 + k = 0$

$$\begin{cases} -\omega^3 + 2\omega = 0 \\ \omega = \pm\sqrt{2} \rightarrow K = 2 \end{cases}$$ ③

No. Hanlin Cai

Date 20122161

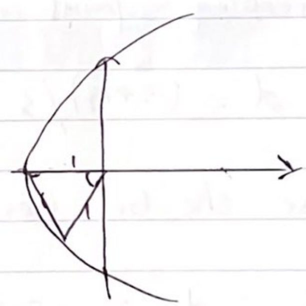
(d)



$\begin{cases} (-\infty, -2) \text{ and } (-1, 0) \\ \text{and the curve.} \end{cases}$

(e) Due to $\zeta = \cos\varphi = \frac{1}{2}$

$\therefore \varphi = \frac{\pi}{3}$



Hence the point could be $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Then $\left| G(s)H(s)K(s) \right| = 0$

$\therefore \left| -\frac{1}{2} - \frac{\sqrt{3}}{2}j \right| \cdot \left| \frac{1}{2} - \frac{\sqrt{3}}{2}j \right| \left| \frac{3}{2} - \frac{\sqrt{3}}{2}j \right| = K = \sqrt{3}$

we can get $K = \sqrt{3}$

Q2. $A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 0 \end{bmatrix}$

(a)

$$M = [B \quad A^0 B] = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

Because rank $(M) = 2 = \max$, Hence the system is controllable.

(b)

$$N = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Because rank $(N) = 2 = \max$, the system is observable.

(c)

$$G(s) = C(sI - A)^{-1} B = [2 \quad 0] \begin{pmatrix} s & -1 \\ 2 & s+2 \end{pmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore G(s) = [2, 0] \frac{\begin{bmatrix} s+2 & 1 \\ -2 & s \end{bmatrix}}{s^2+2s+2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{2}{s^2+2s+2}$$

Let $s^2 - 2s + 2 = 0 \Rightarrow s_1 = -1 + i \quad s_2 = -1 - i$

Since $s_1, s_2 < 0$, the system is stable.

(d) (i) $\lim_{s \to \infty} G(s) = \lim_{s \to 0} s G(s)$

Hence $\lim_{s \to 0} s \cdot G(s) = \lim_{s \to 0} \frac{2}{s^2+2s+2} = \frac{2}{2} = 1$

(ii) $T_s = \frac{4}{\xi \omega_n}$ $\xi = \frac{\sqrt{2}}{2}$ $\omega_n = \sqrt{2}$

$\therefore T_s = 4s$

(iii) $Po\% = e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)} = e^{\frac{-\frac{\sqrt{2}}{2}\pi}{\frac{\sqrt{2}}{2}}} = 4.32\%$

(iv) $\omega_d = \omega_n \sqrt{1 - \xi^2} = 1$

Q2.

(e) desired poles : $-2 \pm 2\sqrt{3}j$

Hence the expected system is $s^2 + 4s + 16$

$$|sI - A + BK| = \begin{vmatrix} s & -1 \\ 2+k_1 & s+2+k_2 \end{vmatrix} = s^2 + (k_2+2)s + (2+k_1)$$

$$\therefore \begin{cases} k_2 + 2 = 4 \\ 2 + k_1 = 16 \end{cases} \quad \therefore \begin{cases} k_1 = 14 \\ k_2 = 2 \end{cases}$$

$\therefore$ the state feedback controller is $[14 \quad 2] \cdot x$.

(f) desired poles = $-8 \pm 8j$

Hence, the expect equation is $s^2 + 16s + 128$.

$$|sI - A + KC| = \begin{vmatrix} s+2k_1 & -1 \\ 2+2k_2 & s+2 \end{vmatrix} = s^2 + (2+2k_1)s + (4k_1 + 2k_2 + 2)$$

$$\therefore \begin{cases} 2+2k_1 = 16 \\ 4k_1 + 2k_2 + 2 = 128 \end{cases} \quad \begin{cases} k_1 = 7 \\ k_2 = 49 \end{cases}$$

$$\therefore K_e = \begin{bmatrix} 7 \\ 49 \end{bmatrix}$$

$\therefore$ (e) $[14 \quad 2]$

(f) $\begin{bmatrix} 7 \\ 49 \end{bmatrix}$

20122161 $\frac{P5}{P8}$

$\angle G_{op} = 0 - \tan^{-1} w - \tan^{-1}\frac{w}{2} - \tan^{-1}\frac{w}{4}$

Q3. $G(s) = \frac{40}{(s+2)(s+4)}$ $H(s) = \frac{1}{s+1}$ $K(s) = 1$

(a)
$G_{op}(s) = \frac{40}{(s+1)(s+2)(s+4)}$ $G_{op}(jw) = \frac{40}{(jw+1)(jw+2)(jw+4)}$

$\therefore G_{op}(jw) = \frac{40[jw(w^2-14) + (8-7w^2)]}{(w^2+1)(w^2+4)(w^2+16)}$. and $|G_{op}| = \frac{40}{\sqrt{1+w^2}\sqrt{4+w^2}\sqrt{16+w^2}}$

① $Im(G(jw)) = 0$ $w^2 - 14 = 0 \rightarrow w \simeq \pm 3.7417$

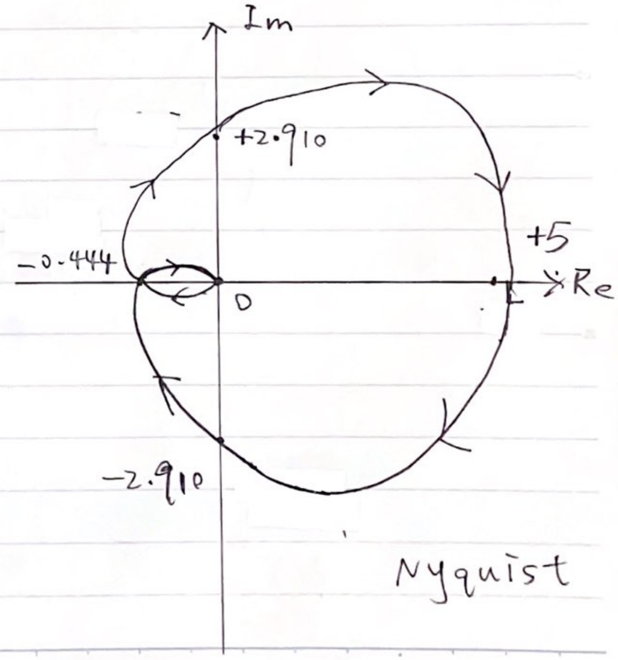
Then, $Re[G(jw)]$ $A_1 = |G_{op}| = \frac{4}{9}$ $\angle GH_1 = -180°$

② $Re(G(jw)) = 0$ $8 - 7w^2 = 0 \rightarrow w \simeq \pm 1.06904$

Then $Im[G(jw)]$ $A_2 = |G_{op}| = 2.910$ $\angle GH_2 = -90°$

Nyquist $\begin{cases} w = 0^+ \rightarrow A = 5 & \angle GH = 0° \\ w = \infty \rightarrow A = 0 & \angle GH = -270° \end{cases}$

w	A	P
0_+	5	0°
∞	0	−270°
3.7417	0.444	−180°
1.06904	2.910	−90°
−1.06904	−2.910	90°



Nyquist

(b) Gain Margin

As the equation above show, when $Im[G_{op}] = 0$

we know : $w = \pm 3.7417$

$$|G| = \frac{40}{\sqrt{1+w^2}\sqrt{4+w^2}\sqrt{16+w^2}} = \frac{4}{9}$$

Hence $GM = 20 \log \frac{1}{|G|} = 7.0437 \text{ dB}$

Q4

(a) $$\frac{Y(s)}{R(s)} = \frac{(k_p+\frac{k_i}{s})(\frac{k_m}{s\tau_m+1})}{1+(k_p+\frac{k_i}{s})(\frac{k_m}{s\tau_m+1})} \quad ①$$

$$\frac{E(s)}{R(s)} = \frac{R(s)-Y(s)}{R(s)} \quad ②$$

$$e_{ss} = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0}\left(s\cdot\frac{1}{s}\right)\frac{s(s\tau_m+1)}{s(s\tau_m+1)+(sk_p+k_i)k_m} = 0 \quad ③$$

Hence $e_{ss} = 0$

(b) only P-controller

$$\frac{Y(s)}{R(s)} = \frac{k_p k_m}{s\tau_m+1+k_p k_m} \quad ④$$

$$\frac{E(s)}{R(s)} = \frac{R(s)-Y(s)}{R(s)} = \frac{s\tau_m+1}{s\tau_m+1+k_p k_m} \quad ⑤$$

$$e_{ss} = \lim_{t\to\infty} E(s) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0}\left(s\cdot\frac{1}{s}\right)\frac{s\tau_m+1}{s\tau_m+1+k_p k_m} = \frac{1}{k_p k_m+1} \quad ⑥$$

Hence $e_{ss} = \dfrac{1}{k_p k_m+1}$

No. 20122161 $\frac{P8}{P8}$

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(C)

$$G(s) = \frac{100}{(s+1)(s+2)(s+3)} \quad K(s) = K_p + K_d s + \frac{k_i}{s} \quad H(s) = 1$$

① let $k_d = k_i = 0$

$$\therefore D(s) = (s+1)(s+2)(s+3) + 100 K_{pc} = 0$$

$$s^3 + 6s^2 + 11s + (6 + 100 K_p) = 0$$

$$\left.\begin{array}{lll} s^3 & 1 & 11 \\ s^2 & 6 & 6+100K_{pc} \\ s^1 & \frac{66-(6+100K_{pc})}{6} & \\ s^0 & 6+100K_p & \end{array}\right\} \quad \begin{array}{l} \therefore K_{pc} = 0.6 \\ \therefore s^2 : \quad 6s^2 + 66 = 0 \end{array}$$

$$\therefore \begin{cases} k_{pc} = 0.6 \\ w_c = \pm\sqrt{11} = \sqrt{11} \end{cases} \rightarrow t_c = \frac{2\pi}{w_c} = 1.8945 \text{ s}$$

$$\therefore k_p = 0.6 k_{pc} = 0.36 \quad ①$$

$$k_i = \frac{k_p}{t_i} = \frac{2k_p}{t_c} = 0.38 \quad ②$$

$$k_d = k_p t_d = \cancel{k_d}\, k_p \frac{t_c}{8} = 0.0853 \quad ③$$

$$\therefore K_p = 0.36 \quad K_i = 0.38 \quad K_D = 0.0853$$

$$\therefore PID = (k_d s^2 + k_p s + k_i)/s$$

$$= \frac{(0.0853 s^2 + 0.36 s + 0.38)}{s}$$

(OVER)

Campus