Tutorial Sheet 4 – System analysis & Stability

(Covering sections 6 & 7 of the notes)

Q1 State the order of each of the following systems, justifying your answer in each case:

(i) $$\frac{s}{(s+2)(s+5)}$$

(ii) $$\frac{s^2+8}{s(s^2+2s-8)}$$

(iii) $$\frac{dx(t)}{dt}+3x(t)-4=0$$

(iv) $$\frac{d^2x(t)}{dt}-4x(t)=4$$

Q2 (i) The series RL circuit (see Q5, tutorial 1) can be modelled by the following first order differential equation:

$$L\frac{di}{dt}+iR=v_i$$

Solve this equation directly, obtaining an expression for $i(t)$, given that $i(0) = 0$.

(ii) The transfer function model for the same circuit is given as:

$$\frac{I(s)}{V_i(s)}=\frac{1}{sL+R}$$

Use Inverse Laplace Transforms to solve this model and verify your answer in part (i) above. Take $v_i(t)$ to be a constant (dc) voltage source.

Q3 Obtain a solution for $y(t)$ from the following model, given that the input $u(t) = 1$:

$$\frac{Y(s)}{U(s)}=\frac{1}{s^2+6s+8}$$

Q4 Solve the following differential equation by first converting it into transfer function form (assume zero initial conditions) and subsequently obtaining the inverse Laplace transform.

$$\frac{d^2x(t)}{dt}-4x(t)=4$$

Q5 List the three possible stability states for an arbitrary system and clearly relate each one to:

(i) the system's natural response

(ii) the location of the system's poles

Q6 Draw the pole-zero diagram for each of the following systems and, hence, comment on the system's stability:

(i) $\dfrac{s}{(s+2)(s+5)}$ (ii) $\dfrac{s+3}{s(s^2+2s-8)}$

(iii) $\dfrac{1}{s(s+2)(s-2)}$ (iv) $\dfrac{s^2-3s+2}{(s^2+2s)(s+3)}$

Q7 Determine the conditions for α such that each of the following systems is stable:

(i) $\dfrac{1}{(s+2)(s+\alpha)}$ (ii) $\dfrac{s+\alpha}{(s^2+4s+4)}$ (iii) $\dfrac{s}{(s-2)(s+\alpha)}$

Q8 Consider the system represented by block diagram below and carry out the following:

(i) obtain a single transfer function block for this system

(ii) determine the order of the system

(iii) plot the pole-zero diagram for the system

(iv) comment on the stability of the system

(v) solve for the output $y(t)$ given that $u(t) = 1$