Engineering Mathematics 1 (Fall 2021)

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September 28, 2021

Students should be able to (after learning)

- Add, subtract and multiply complex numbers
- Convert complex numbers between Cartesian and polar forms
- Differentiate all commonly occurring functions including polynomials, exponential functions, trigonometric functions, hyperbolic functions and combinations thereof
- Distinguish between the two main meanings of a derivative, namely the derivative as a tangent and the derivative as a rate of change
- Integrate certain standard functions, constructed from polynomials, exponential functions, trigonometric functions, hyperbolic functions and combinations thereof
- Distinguish between the two main meanings of integration, namely the integral as the inverse of the derivative and the integral as the area under a curve
- Apply Taylor series to numerically approximate functions
- Apply Simpson's rule to numerically evaluate integrals
- Solve simple first and second order ordinary differential equations
- Apply and select the appropriate mathematical techniques to solve a variety of associated engineering problems

Lecture 4: Sequences

1. Definitions of sequences and graphs

Sequence: any function f whose input is restricted to positive or negative integer values n has an output in the form of a sequence of numbers.

(output) $f \leftrightarrow n$ (input)

functions → 1–1 mapping, NOT multi-mapping

$f(n) = n^2$: $f(n)$ ↓ Range, n^2 ↓ Domain

Arithmetic sequence: general form: $f(n) = a + nd, n = 0, 1, 2, 3, \cdots$, where a is the first term and d is common difference.

Given $f(n) = a + nd$, $n = 0, 1, 2, 3, \cdots$

$n=0$, $f(0) = a$

$n=1$, $f(1) = a + d$

$n=2$, $f(2) = a + 2d \cdots$

. Write $a, a+d, a+2d, \cdots$ as the sequence.

Ex: $3, 5, 7, 9, \cdots$ (Given)

Sol: $a = 3$, $d = 5-3 = 7-5 = 2$,

$f(n) = a + nd$, $n = 0, 1, 2, 3, \cdots$

(formular is the solution)

Geometric sequence: general form: $f(n) = Ar^n, n = 0, 1, 2, 3, \cdots$, where A is the first term and r is common ratio.

Given formular $f(n) = Ar^n$, $n = 0, 1, 2, 3, \cdots$

$n=0$, $f(0) = A$,

$n=1$, $f(1) = Ar$

$n=2$, $f(2) = Ar^2, \cdots$

$A, Ar, Ar^2, Ar^3, \cdots$ the required solution.

Ex: $4, 8, 16, 32, \cdots$ (Given)

Sol: $A = 4$, $r = 2$

$f(n) = 4 \cdot 2^n$, $n = 0, 1, 2, 3, \cdots$

(formular, solution)

Harmonic sequence: if the reciprocals of its terms form an arithmetic sequence. General form: $f(n) = \frac{1}{n}, n = 1, 2, 3, \cdots$.

Given $f(n) = 1/n$, $n = 1, 2, 3, \cdots$

$n=1$, $f(1) = 1$,

$n=2$, $f(2) = \frac{1}{2}$,

$n=3$, $f(3) = \frac{1}{3}$,

$n=4$, $f(4) = \frac{1}{4}, \cdots$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$ the required sequence.

common factor $\frac{1}{2}$

Ex: $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \cdots$ (Given)

Sol: $\frac{1}{2}\times 1, \frac{1}{2}\times\frac{1}{2}, \frac{1}{2}\times\frac{1}{3}, \frac{1}{2}\times\frac{1}{4}, \frac{1}{2}\times\frac{1}{5}, \cdots$

$\frac{1}{2}\frac{1}{n} = \frac{1}{2n} = f(n)$

(formular, solution)

Recursive sequence: each term of the sequence is seen to depend upon another term of the same sequence. $f(n+1) = f(n) + 5$ and $f(1) = 3$

$n=1$, $f(2)=f(1)+5 = 3+5 = 8$ initial value

$n=2$, $f(3)=f(2)+5 = 8+5 = 13$ 1-step, OR, next term

$n=3$, $f(4)=f(3)+5 = 13+5 = 18, \dots$

1-order difference equation

$8, 13, 18, \dots$ the required sequence.

Ex: $\underbrace{3, 9}_{6}, \underbrace{9, 15}_{6}, \underbrace{15, 21}_{6}, \underbrace{21, 27}_{6}, \dots$

Sol: $a=3$, $d=6$, $f(n) = 3+6n$, $n=0,1,2,3,\dots$

$f(n+1) = 3+6(n+1) = 6n+6+3 = f(n)+6$, $n=0,1,2,3,\dots$

$f(n-1) = 3+6(n-1) = 3+6n-6 = f(n)-6$,

$f(n) = f(n-1)+6$, $n=1,2,3,\dots$

Fibonacci sequence: $0, 1, 1, 2, 3, 5, 8, 13, 21, \cdots$ General form: $f(n) = f(n-1) + f(n-2)$ and $f(0) = 0, f(1) = 1$. two initial values

$f(n-2)$ is two terms away from $f(n)$: 2-order difference equation

$f(0) = f(-1) + f(-2) = 0$

$f(1) = f(0) + f(-1) = 1$

$0 + f(-1) = 1$

$f(-1) = 1$

$f(-2) = -1$

$f(n) = 2f(n-1) + 3f(n-2) - 4f(n-3)$, $n \geq 0$

$f(0)=0$, $f(1)=2$, $f(2)=6$

3-order DE

2. Difference equations

$f(n+1) = f(n)$

$\Updownarrow$

$\boxed{\text{Solving 1-order difference equations:}}$ $f(n+1) - f(n) = 0, f(1) = 3$ $(*)$

Assume that $f(n) = kw^n$ is a solution with $k > 0, w > 0$

$f(n+1) - 12f(n) = 0, n \geq 0, f(1) = -3$

Sol: $\boxed{f(n) = kw^n}$ is the solution of $(*)$, $f(n+1) = kw^{n+1}$, $f(n) = ab^n$

$kw^{n+1} - kw^n = 0$, $f(1) = kw^1 = 3$

$\therefore kw^n(w-1) = 0$, $w-1=0$ $\therefore w = 1$ $\quad k = 3$

So, $f(n) = 3 \cdot 1^n = 3$, $n = 1, 2, 3, \ldots$ is the solution of $(*)$. #

Ex: $f(n+1) + 9f(n) = 0$, $n \geq 0$, $f(0) = 6$, find the solution. $f(n) = ?$

Sol: Assume that $\boxed{f(n) = kw^n}$ is the solution of $f(n+1) + 9f(n) = 0$.

$f(n+1) = kw^{n+1}$ $\therefore kw^{n+1} + 9kw^n = 0$ $\therefore kw^n[w+9] = 0$

$kw^n \neq 0$ $\therefore w+9 = 0$, $w = -9$

$f(0) = 6 = kw^0 = k$

$f(n) = 6(-9)^n$, $n = 0, 1, 2, \ldots$

is the solution for this topic. #

Ex: $f(n+1) - 12f(n) = 0$, $n \geq 0$, $f(1) = -3$, find $f(n)$.

Sol: Assume that $\boxed{f(n) = kw^n}$ is the solution, $f(n+1) = kw^{n+1}$

$kw^{n+1} - 12kw^n = 0$, $\therefore kw^n(w-12) = 0$ $\therefore w = 12$

$f(1) = -3 = kw^1 = kw$

$\therefore k = -\frac{1}{4}$ $\quad \therefore f(n) = -\frac{1}{4} \cdot 12^n$, $n = 0, 1, 2, 3, \ldots$ #

Solving 2-order difference equations with distinct roots:

$f(n+2) - 7f(n+1) + 12f(n) = 0, n \geq 0, f(0) = 0, f(1) = 1.$

Assume that $f(n) = kw^n$ is a solution with $k > 0, w > 0$.

Solving 2-order difference equations with equal roots:

$f(n+2) - 4f(n+1) + 4f(n) = 0, n \geq 0, f(0) = 0, f(1) = 1.$

Assume that $f(n) = kw^n$ is a solution with $k > 0, w > 0$.