Channel Capacity

Signal-to-Noise Ratio (SNR) - *Recap*

SNR is a very important measure of **receive signal quality** since noise is the major cause of unwanted effects. It can be expressed in two ways:

$$SNR = \frac{P_{signal}}{P_{noise}} \qquad \textbf{\textit{or}} \qquad SNR = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \quad dB$$

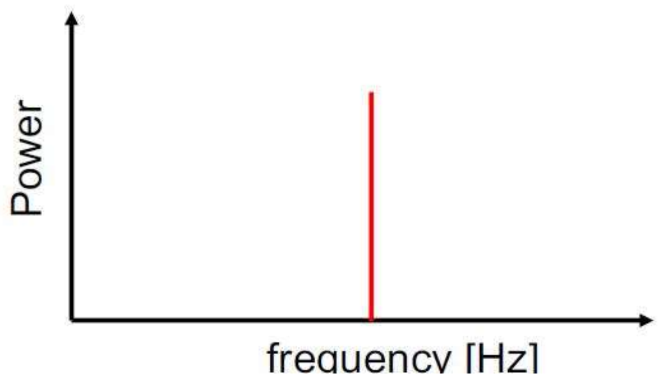
The stronger the signal and the weaker the noise, *the higher the SNR ratio*.

If the signal is weak and the noise is strong, the SNR ratio will be low and the reception will be unreliable.

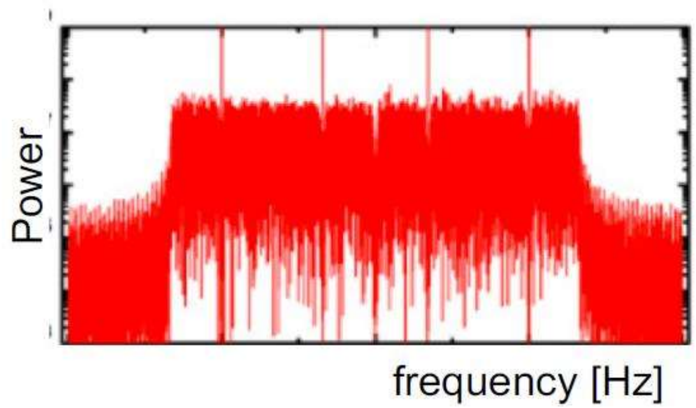
Signal's Spectrum

More complex signals consist of large number of frequencies => we talk about a signal's bandwidth (BW). In general, the larger a signal's BW, the more information it carries.

Spectrum is the plot of the power as a function of frequency.



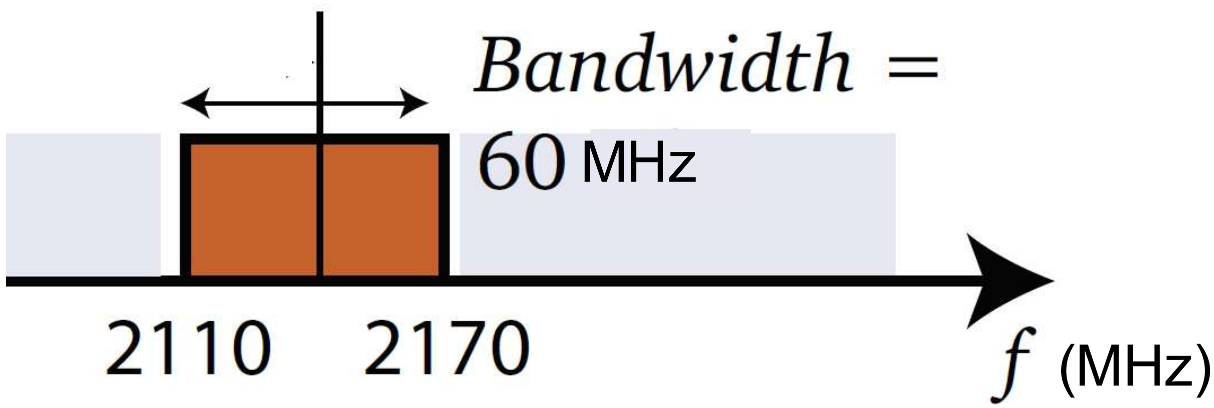
Single-frequency signal



Spectrum of a complex signal

Signal's Bandwidth

Bandwidth can be imagined as the span of a signal's frequency content, sort of as the 'fatness' of a signal.



$$BW = f_{high} - f_{low} = 2170\ \ MHz - 2110\ MHz = 60\ MHz$$

Introduction to Channel Capacity

There is a question we have not considered yet – ***what is the maximum amount of data that can be passed through a communication channel?***

The **Shannon–Hartley theorem** gives the channel capacity *C*, meaning the theoretical maximum information rate that can be sent in the presence of noise.

The theoretical maximum amount of information that can be transmitted is given by the equation:

$$Channel\ Capacity = (BW)\log_2(1 + SNR_{linear})$$

Capacity= maximum data rate (bits/sec)

BW = bandwidth in Hz

SNR_{linear} = ***linear*** *ratio of signal power to noise power*

$$SNR_{linear} = \frac{P_{signal}\ (W)}{P_{noise}\ (W)}$$

Shannon-Hartley Theorem

Shannon's theorem gives the capacity of a system in the presence of noise. The theoretical maximum amount of information that can be transmitted is given by the equation:

$$Channel\ Capacity = (BW)\log_2(1 + SNR_{linear}) \quad \textit{in bits/sec}$$

Channel capacity is the measure of how much data can be sent over a channel.

This sets a fundamental limit to the maximum data rate you can get from a given chunk of spectrum. The **data rate** can then only be increased if

- Increase the power level
- Increase the bandwidth
- Reduce the noise

Example 1

We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. Calculate the capacity of this channel.

$$Channel\ Capacity = (BW)\log_2(1 + SNR_{linear}) = 10^6 \log_2(1 + 63) = 6\ Mbps$$

Example 2

Consider an extremely noisy channel in which the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel, the capacity C is calculated as:

$$\text{Channel Capacity} = (\text{BW})\log_2(1 + \text{SNR}_{\text{linear}}) = (\text{BW})\log_2(1 + 0) = 0$$

Example 3

A WiFi-type signal, bandwidth of 20 MHz and power of 100 mW, is transmitted at 2.4 GHz. The background noise at the receiver in the bandwidth of interest is -140 dBm/Hz. Calculate the **theoretical maximum data throughput** at a distance of 100 meters.

Example 3

A WiFi-type signal, bandwidth of 20 MHz and power of 100 mW, is transmitted at 2.4 GHz. The background noise at the receiver in the bandwidth of interest is -140 dBm/Hz. Calculate the **theoretical maximum data throughput** at a distance of 100 meters.

The maximum data throughput (channel capacity) is given by:

$$Channel\ Capacity = (BW)\log_2(1 + \boldsymbol{SNR_{linear}})$$

$BW = 20 * 10^6\ Hz$

We must calculate the SNR_{linear} at 100 meters

Example 3

A WiFi-type signal, bandwidth of 20 MHz and power of 100 mW, is transmitted at 2.4 GHz. The background noise at the receiver in the bandwidth of interest is -140 dBm/Hz. Calculate the **theoretical maximum data throughput** at a distance of 100 meters.

$$Path\ Loss \approx 20\log_{10}\left(\frac{4\pi D}{\lambda}\right) dB$$

$$= 20\log_{10}\left(\frac{4\pi(100)}{0.125}\right)$$

$$= 20\log_{10}(10053)$$

$$= 80dB$$

We need to know the wavelength (λ) of the signal:

c is speed of light

$$c = f\lambda$$

$$3 * 10^8 = (2.4\ \times 10^9)\lambda$$

$$\lambda = 0.125m$$

So the signal starts at 20 dBm, it loses 80 dB travelling over the air.

That means that it hits the receiving antenna at 20 dBm – 80 dB = - 60 dBm

Example 3

The noise at the antenna is -140 dBm/Hz and we are told that the bandwidth of our signal is 20 MHz.

So total noise power is (power per Hertz) * (the bandwidth in Hertz)

-140 dBm * 20 MHz

Now this is log and linear, so that doesn't work.

$$Noise = -140dBm/Hz = 10^{-17}\ W/Hz$$

$$Total\ Noise = 10^{-17} * (20 * 10^{6})\text{W}$$
$$= -67dBm$$

Example 3

So the signal to noise ratio is given as

$$
\begin{aligned}
SNR_{dB} &= Signal(dBm) - Noise(dBm) \\
&= (-60) - (-67) \\
&= 7\ dB
\end{aligned}
$$

$$
\begin{aligned}
SNR_{linear} &= 10^{7/10} \\
&= 5.01
\end{aligned}
$$

Example 3

So the channel capacity is given by

$$
\begin{aligned}
Channel\ Capacity &= (BW)\log_2(1 + SNR_{linear}) \\
&= (20 * 10^6)\log_2(\,1 + 5.01) \\
&= (20 * 10^6)(2.58) \\
&= 51.7\ Mbits\ per\ second(Mbps)
\end{aligned}
$$