832002117 Hanlin CAI 20122161. No. P1/P5 Date EE304FZ

min 78 −12 −10

max 86

Q1. Sol.

(a) (i) 3 red 3 blue 3 green

$P_i = \left(\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}\right) \times 3 = \frac{1}{28}$ $P = \frac{1}{28}$ −2

(ii)

$P_{ii} = \frac{3}{9} \times \frac{3}{8} \times \frac{3}{7} = \frac{1}{56}$ $\frac{3}{56}$ $P = \cancel{\frac{1}{56}}$

(b) Three balls 10 cups −2

$N = (10 \times 9 \times 8) A_3^3 = 2160$ (ways)

(c) Undiscovered typos: U

$U = T - (A + B - C) = \frac{ABC}{C} - (A + B + C) = \frac{(A-C)(B-C)}{C}$

(d) $\mu = 58$ $\sigma = 12$

Let $Z = \frac{X - \mu}{\sigma}$ $P = \Phi(Z). = 0.95$

Referring to the normal distribution table, we know

$Z = 1.645 = \frac{X - 58}{12}$ Hence $X = 77.74 \approx 77.8$

Therefore, the minimum would be 77.8.

(e) As the question shown, $\lambda = \frac{1}{4}$ Hence $T \sim E(\frac{1}{4})$: $P = e^{-\frac{1}{4}x}$.

If the new customer can be served, then the total number of customer must be lower than 3. 4

Hence we obtain:

$P_{N(2)}(0) = e^{-\frac{1}{4} \times 2} = 0.606$ $1 - P = 0.393$

$P = (0.393)^3 \, 0.606 = \cancel{0.0367}$ −5

KOKUYO

P2/P5

(f) Sol. Scale $\delta = 12$ $\beta = 2$ $F(x) = 1 - e^{-(\frac{x}{\delta})^{\beta}}$

Hence the weibull distribution would be $F(x) = 1 - e^{-(\frac{x}{12})^2}$.

$1 - e^{-(\frac{x}{12})^2} = 0.7$, then $e^{-(\frac{x}{12})^2} = 0.3$

$\therefore -(\frac{x}{12})^2 = \ln(0.3) \longrightarrow (\frac{x}{12})^2 = 1.204$

$\therefore x = 13.17$ (months) ≈ 13 (months)

Therefore, the part need to be changed 13 month. (or 13.1)

(g) (i) As the question shown, the sample is large, and the mean lies within a range centred at the sample mean, $1 - 0.96 = 0.04$

Hence $Z_{\frac{\alpha}{2}} = Z_{=0.98} = 2.05$ ①

(ii) The sample is small (less than 30)

Hence $t_{n-1, \frac{\alpha}{2}} = t_{14, 0.025} = 2.145$ ②

(h) Sol. $y = \beta_1 x + \beta_0$ (y_i, x_i)

(i) ~~To obtain the estimates of β_0 and β_1, we know that~~

$$\begin{cases} \beta_0 = \bar{y} - \bar{x} \cdot \beta_1 & ① \\ \beta_1 = \frac{L_{xy}}{L_{xx}} = \frac{x_1 y_1 + x_2 y_2 + \dots + x_n y_n - n \cdot \bar{x} \bar{y}}{x_1^2 + x_2^2 + \dots + x_n^2 - n \cdot \bar{x}^2}. & ② \end{cases}$$

If we have got the best straight line, the β_0 is the value of y when $x = 0$, and β_1 can be obtained by $\beta_1 = \frac{x_i - \beta_0}{y_i}$.

(ii) $S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$ $S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$

$SSE = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_i x_i)^2$

$\therefore$ ③ $\hat{\sigma}^2 = \frac{SSE}{n-2}$ to estimate σ^2

5

3

Campus

Hanlin CAI. P3/P5

Q2. sol

(a) $P = 10^{-3}$ $\quad 1-P = \frac{999}{1000}$.

$$P_a = \left(\frac{999}{1000}\right)^{100} = 0.9048$$

(b)

$$P_b = C_{100}^{1}\left(\frac{999}{1000}\right)^{99}\left(\frac{1}{1000}\right) = 0.09057$$

(c) $P_c = 1 - P_a = 0.095208$

(d) Since single bit errors in a block can be corrected, hence $P_d = P_c - P_b = 4.6379 \times 10^{-3}$

Hanlin CAI. P4/P5

Q4.

(a) The sample is 100, which is large.

$\bar{x} = 99.5$ $S = 1.5$

$$\begin{cases} H_0: \text{The bottles contain } 100 \text{ mL.} \\ H_1: \text{The bottles do not contain } 100 \text{ mL.} \end{cases}$$

(b)

$$Z_0 = \frac{99.5 - 100}{1.5/\sqrt{100}} = -3.333$$

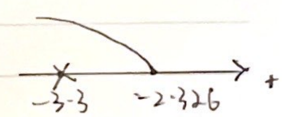
(c) Since, the problem is a two-side case

Hence, $Z_{\frac{\alpha}{2}} = 2.326$

-3

(d) The critical region is $x_1 = 98.5$ $x_2 = 101.5$

$\therefore$ $x > 101.5$ and $x < 98.5$.



(e) Since $Z_0 = -3.33 < -Z_{\frac{\alpha}{2}} = -2.326$,

Hence it is appropriate to reject H_0, so the bottles do not contain 100 ml generally.

-2

Q5.

No.	0	1	2	3	4	>4
O_i	18	35	26	15	6	0
E_i	18.34	33.90	28.20	13.90	5.66	
$\frac{(E_i-O_i)^2}{E_i}$	$\frac{0.74^2}{18.34}$	$\frac{1.21}{33.90}$	$\frac{4.84}{28.2}$	$\frac{1.21}{13.90}$	$\frac{0.1156}{5.66}$	

sol

(a) $N = 18 + 35 + 26 + 15 + 6 = 100$

$$X_0^2 = \sum_{i=1}^{4} \frac{(E_i - O_i)^2}{E_i}$$

(b) $N = 100 \times 10 = 1000$

$n = 1 \times 35 + 2 \times 26 + 3 \times 15 + 4 \times 6 = 156$

$P = \frac{n}{N} = 0.156$

(c) Since we estimate the frequencies of the number above

so $D.F = \nu = 5 - 1 - 1 = 3$

$$X^2_{3,0.01} = 11.345$$

(d) test statistic: $X_0^2 = \sum_{i=1}^{4} \frac{(E_i - O_i)^2}{E_i}$

$$\begin{cases} 1-p = 0.844 \\ p = 0.156 \end{cases}$$

$$\begin{cases} P(X=0) = (0.844)^{10} = 0.1834 \\ P(X=1) = \binom{10}{1}(0.156)(0.844)^9 = 0.3390 \\ P(X=2) = 0.28197 = 0.2820 \\ P(X=3) = 0.13898 \\ P(X=4) = 0.05662 \end{cases}$$

As the table (above) shonw,

$\longrightarrow X_0^2 = 0.3211$

(e) Since $X_0^2 = 0.3211 < X^2_{3,0.01} = 11.345$

So we cannot reject the null hypothesis, which means that obesity is a trait of people independent of community they live in.