Engineering Mathematics 1 (Fall 2021)

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EE106FZ-Introduciton

- Module name and code: Engineering Mathematics 1, EE106FZ
- Credit rating: 5 ECTS Credits
- Pre-requisites: None
- Aims: To teach basic mathematics, especially calculus
- Module exam and assignments:
  - Final Exam (2h) 60%
  - Continuous assessments: Test 1 (2h) and Test 2 (2h) 30%
  - Maths assignments: 16 (1h each) 10%
- Pass standard: 40%
- Penalties: Late submission, 10% penalty each assessment

Course text and references

- Course text: Booth D. J. and K. A. Stroud, Engineering mathematics, Palgrave MacMillan, (2007).

- References:
  - Kreyszig E., Advanced Engineering Mathematics, Wiley (2010).
  - Hobson M. P. and Riley K. F., Essential mathematical methods for the physical sciences, Cambridge University Press, (2011).
  - Spivak M., Calculus, Cambridge University Press, (2006)
- Time allowance: Lectures (36h), Tutorials (12h), Assignments (12h), Independent study (63h), Semester examination (2h)

Syllabus by bullets

- Introduction, motivation and scope of course
- Number systems
  - Complex numbers
  - Complex conjugate
  - Exponential and polar form
  - Inequalities
- Sequences and series
  - Arithmetic and geometric series
  - Finite and infinite sums
  - Convergence
  - Limits and continuity
  - Maclaurin and Taylor series
- Differential calculus
  - Tangents to curves
  - L'Hopital's rule
  - Sum, product, quotient and chain rules for derivatives
  - Critical points, maxima, minima and points of inflexion

- Integral calculus
  - Integrals as areas and limits of sums
  - Definite and indefinite integrals, integration techniques
- Applications of integration
  - Calculation of lengths, areas and volumes
  - Mean and RMS values
- Introduction to numerical integration
- Introduction to ordinary differential equations (ODEs)

Students should be able to (after learning)

- Add, subtract and multiply complex numbers
- Convert complex numbers between Cartesian and polar forms
- Differentiate all commonly occurring functions including polynomials, exponential functions, trigonometric functions, hyperbolic functions and combinations thereof
- Distinguish between the two main meanings of a derivative, namely the derivative as a tangent and the derivative as a rate of change
- Integrate certain standard functions, constructed from polynomials, exponential functions, trigonometric functions, hyperbolic functions and combinations thereof
- Distinguish between the two main meanings of integration, namely the integral as the inverse of the derivative and the integral as the area under a curve
- Apply Taylor series to numerically approximate functions
- Apply Simpson's rule to numerically evaluate integrals
- Solve simple first and second order ordinary differential equations
- Apply and select the appropriate mathematical techniques to solve a variety of associated engineering problems

Lecture 1: Complex number (part 1)

1. Symbol i j imaginary unit

Quadratic equation $x^2 + 1 = 0$ $\Rightarrow$ $i^2 = -1$

$x^2 - 1 = 0$ $x_1 = 1$ & $x_2 = -1$

$x^2 = -1$ $i^2 = -1$ $\therefore$ $x = i$

2. Power of i

Positive integer powers

$i^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = (-1) \times (-1) = 1$

....

Negative integer powers

$i^2 = -1$, $i \cdot i = -1$, $i = \frac{-1}{i}$, $-i = \frac{1}{i} = i^{-1}$

$i^{-2} = (i^{-1})^2 = (-i)^2 = i^2 = -1$

$i^{-3} = (i^{-1})^3 = (-i)^3 = -i^3 = i$

$i^{-4} = (i^{-1})^4 = (-i)^4 = i^4 = 1$

....

3. Complex numbers, Cartesian form $z = a + ib$ $\rightarrow A(a, b) \longrightarrow \overrightarrow{OA}$

number point vector

a: Real part, b: Imaginary part

Addition and substraction

Let $Z_1 = a_1 + ib_1$, $Z_2 = a_2 + ib_2$ ✓

$Z_1 + Z_2 = a_1 + a_2 + i(b_1 + b_2)$

$Z_1 - Z_2 = a_1 - a_2 + i(b_1 - b_2)$

Multiplication

$Z_1 \cdot Z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$= (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$

$$Z_1 = a_1 + ib_1. \qquad \bar{Z}_1 = a_1 - ib_1$$

Conjugate of Z_1

$$Z_1 \cdot \bar{Z}_1 = (a_1+ib_1)(a_1-ib_1) = a_1^2 + b_1^2 + \boxed{i(a_1 b_1 - a_1 b_1)}$$

$$0$$

$$= a_1^2 + b_1^2 = |Z_1|^2$$

norm of Z_1

Division (Conjugate)

$$\frac{Z_1}{Z_2} = \frac{a_1+ib_1}{a_2+ib_2} = \frac{(a_1+ib_1)(a_2-ib_2)}{(a_2+ib_2)(a_2-ib_2)}$$

$$= \frac{a_1a_2 + b_1b_2 + i(a_2b_1 - a_1b_2)}{a_2^2+b_2^2}$$

$$= \frac{a_1a_2+b_1b_2}{a_2^2+b_2^2} + i\,\frac{a_2b_1 - a_1b_2}{a_2^2+b_2^2}$$

Equal complex numbers

$$Z_1 = Z_2 \qquad (a, b) \rightleftarrows Z = a+ib$$

1-1 mapping

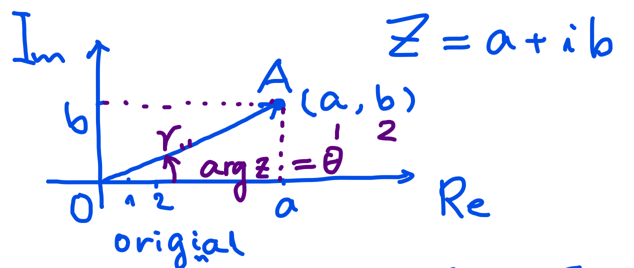
$$a_1 + ib_1 = a_2 + ib_2$$

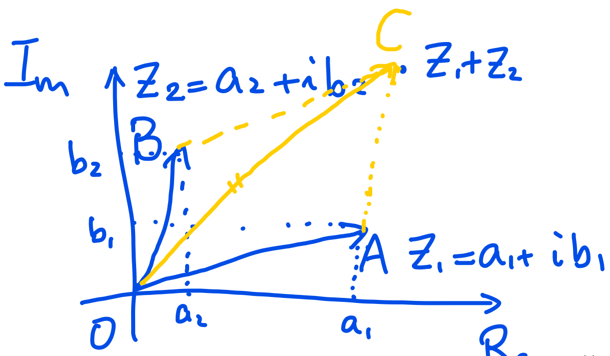
$$\therefore a_1 = a_2 \;\&\; b_1 = b_2$$

4. Graphical addition of complex numbers

Argand diagram: In 1806, the French mathematician Jean-Robert Argand devised a means of representing a complex number using the same Cartesian coordinate system.

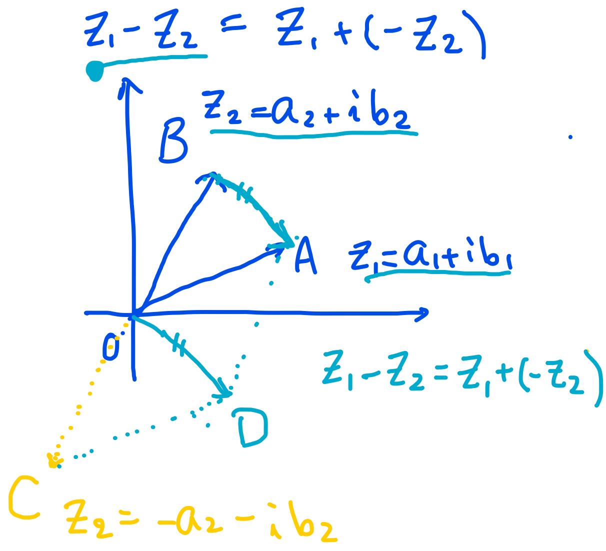
Complex plane





$$Z_1 + Z_2 = a_1 + a_2 + i(b_1+b_2)$$

sum



5. Polar form of a complex number $z = r(\cos\theta + i\sin\theta)$

r is called the modulus (length) of the complex number z, $|z| = r = \sqrt{a^2+b^2}$

θ is called the argument (angle) of the complex number z, $\arg z = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

Let $z = a+ib$. $|z|^2 = a^2+b^2 = z\cdot\bar{z}$. $r = |z| = \sqrt{a^2+b^2}$

$z = r(\cos\theta + i\sin\theta) = r\cos\theta + ir\sin\theta$

$a = r\cos\theta$, $b = r\sin\theta$ $\because \cos^2\theta + \sin^2\theta = 1$ $\therefore \left(\frac{a}{r}\right)^2 + \left(\frac{b}{r}\right)^2 = \frac{a^2+b^2}{r^2} = 1$

$\therefore r^2 = a^2+b^2$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{b}{r}\cdot\frac{r}{a} = \frac{b}{a}$ $\therefore \theta = \arg z = \tan^{-1}\left(\frac{b}{a}\right)$

Shorthand version of polar form, decimal places (dp)

$z = a+ib$

anti-clockwise

$z = r(\cos\theta + i\sin\theta) = r\angle\theta$

$\theta = \tan^{-1}\left(\frac{b}{a}\right) = 35^\circ.176$ 3dp

$= 35.17$ 2dp

$= 35.2$ 1dp

6. Exponential form of a complex number $z = re^{i\theta}$

$e^{i\theta} = \cos\theta + i\sin\theta$ ✓

$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

nonlinear linear

7. Logarithm of a complex number $\ln z = \ln r + i\theta$

$z = re^{i\theta}$

$\ln z = \ln(re^{i\theta}) = \ln r + \ln e^{i\theta} = \ln r + i\theta$

Conclusions:

$z = a + ib \qquad r = \sqrt{a^2+b^2}, \ \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$z = r(\cos\theta + i\sin\theta), \quad r\underline{|\theta}$

$z = re^{i\theta} \quad \rightarrow \quad r\underline{|\theta}$

$\ln z = \ln r + i\theta$