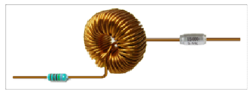
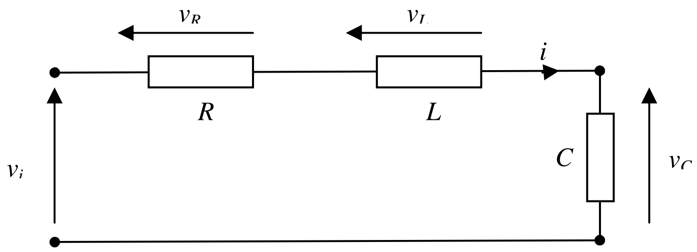
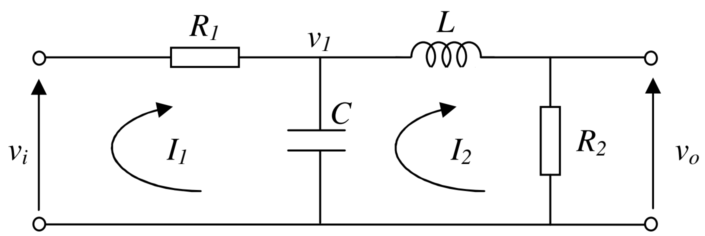
Tutorial Sheet 2 – Modelling Dynamical Systems

Q1 Determine a mathematical model for the following radio tuning circuit (effectively a series RLC circuit), relating the input voltage v_i to the voltage across the capacitor v_C.

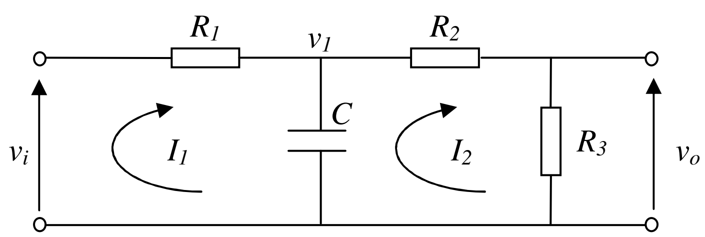




Q2 Determine the transfer function model relating the input and output voltages in each of the following circuit:

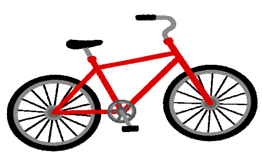


Circuit A

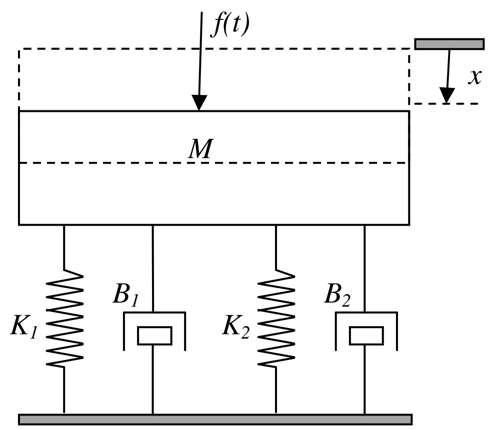


Circuit B

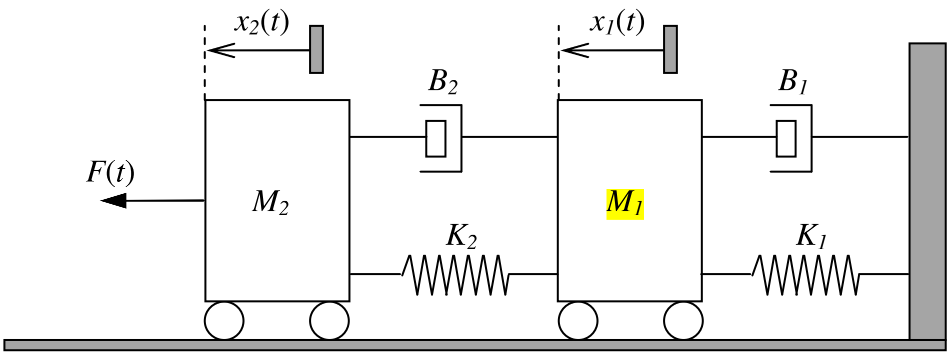
Q3 Determine a mathematical model for the bicycle suspension (spring-mass-damper) system, whose physical model is as follows:



In this bicycle suspension model, K_1 and B_1 represent the spring and damping coefficients for the back wheel while K_2 and B_2 represent the spring and damping coefficients for the front wheel.



Q4 Consider the mechanical system shown below:



Determine the transfer function model relating the input force $F(t)$ to the position of mass M_1.

Q5 The data in the table below are samples of the input and output of an unknown dynamical system.

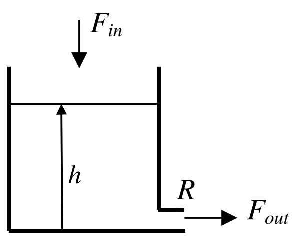
Sample instant (k)	**1**	**2**	**3**	**4**	**5**	**6**
input $u(k)$	0.065	0.065	–0.15	–0.15	–0.1	0.11
output $y(k)$	0.098	0.109	0.117	–0.019	–0.128	–0.1

Experimental data from an unknown process

(i) Determine the 1st order and 2nd order difference equation models that best fit the data. *Use Matlab to perform the required matrix calculations.*

(ii) Express each model as a transfer function.

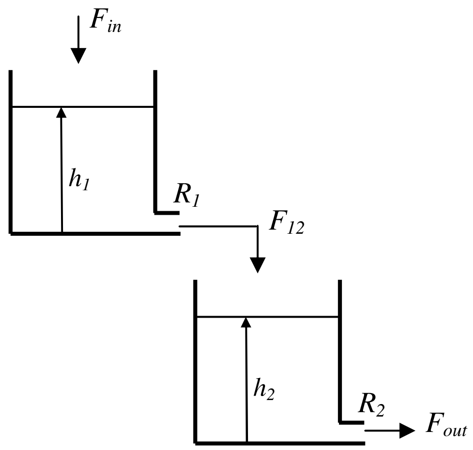
Q6 (i) Derive a mathematical model for the following single tank system, relating input flow rate, F_{in}, to the height of the liquid, h, given that the cross sectional area of the tank is A. Assume that water flow is laminar and hence $F_{out} = \dfrac{h}{R}$, where parameter R, in this case, is the resistance to flow.



Single tank system

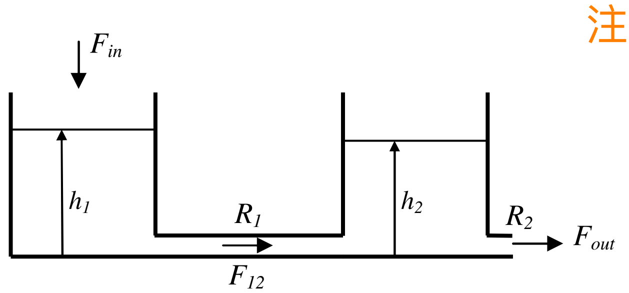
(ii) How would a turbulent flow have affect F_{out} and hence your model?

Q7 Using your answer from Q6, derive a mathematical model relating input flow rate, F_{in}, to the height of the liquid in tank 2, h_2, for the dual **non-interacting** tank system given below. Assume that the cross sectional area of tanks 1 and 2 is A_1 and A_2 respectively. Also assume laminar flow with flow resistances R_1 and R_2 as shown.



Q8 Derive a model for the dual **interacting** tank system given below, relating the input flow rate, F_{in}, to the height of the liquid in tank 2, h_2. Assume that the cross sectional area of tanks 1 and 2 is A_1 and A_2 respectively. Also assume laminar flow with flow resistances R_1 and R_2 as shown.

注意题目问法!



Note for Q6 – Q8, we state that the output flowrate is inversely proportional to the height of water in the tank, given as $F_{out} = \dfrac{h}{R}$.

In the lecture notes, we expressed this equation as $F_{out} = kh$, *where k is a constant of proportionality.*

In essence, therefore, $k \equiv \dfrac{1}{R}$