EE114
Intro to Systems & Control

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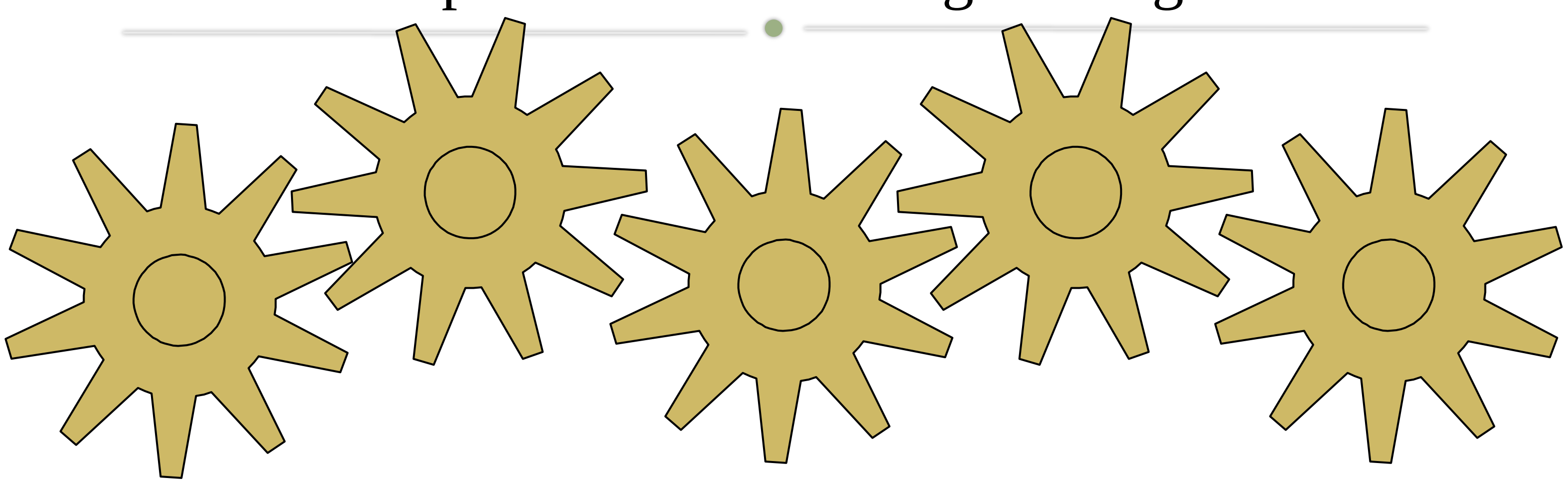
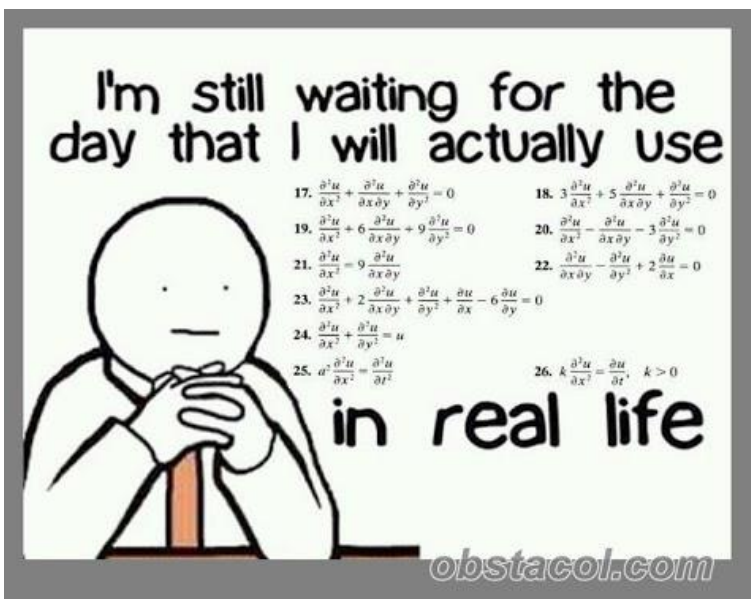
Dept. of Electronic Engineering



So far …

- We've modelled various systems as ordinary differential equations and as transfer function models, the latter using Laplace Transforms.



So far …

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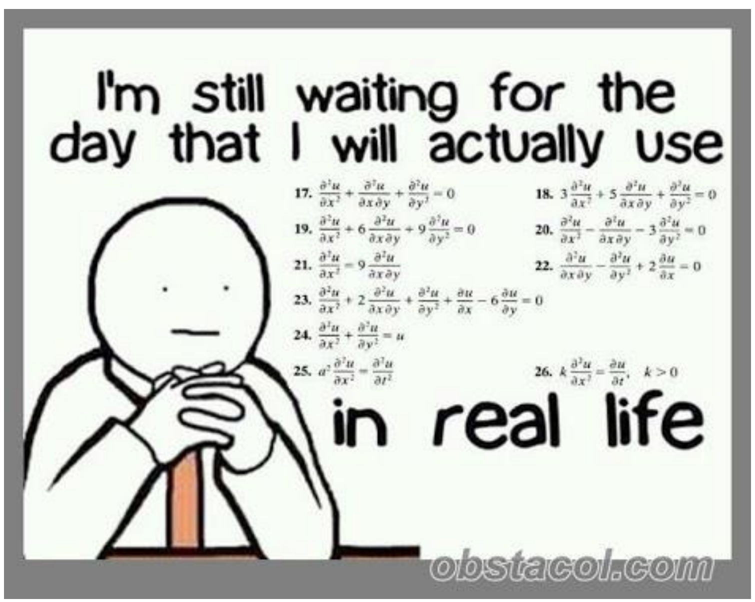


- **Today, we are going to look at groups of transfer functions and how to reduce these to just one …**

Block Diagram Analysis

- In section 3 of the notes, we examined the modelling of various static and dynamical systems and ended up with mathematical models in the form of linear differential equations.
- In section 4 we effectively described the same systems as transfer function models, using the Laplace transform.
- Large, complicated systems may consist of many components, each of which may be represented by a transfer function.
- A **block diagram** shows these transfer functions and illustrates the functional relationship between them.

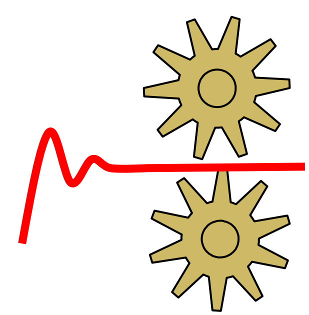
Block Diagram Analysis

- In general, a block diagram consists of a specific configuration of four types of elements:

Block Diagram Analysis

- In general, a block diagram consists of a specific configuration of four types of elements:

  - blocks,
  - summing points,
  - take-off points and
  - arrows representing unidirectional signal flow.

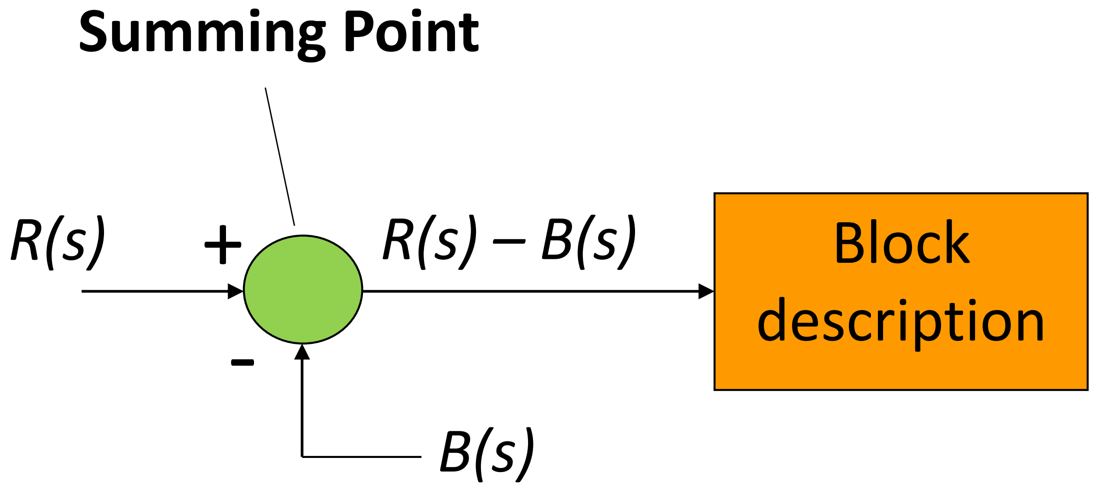


Block Diagram Analysis

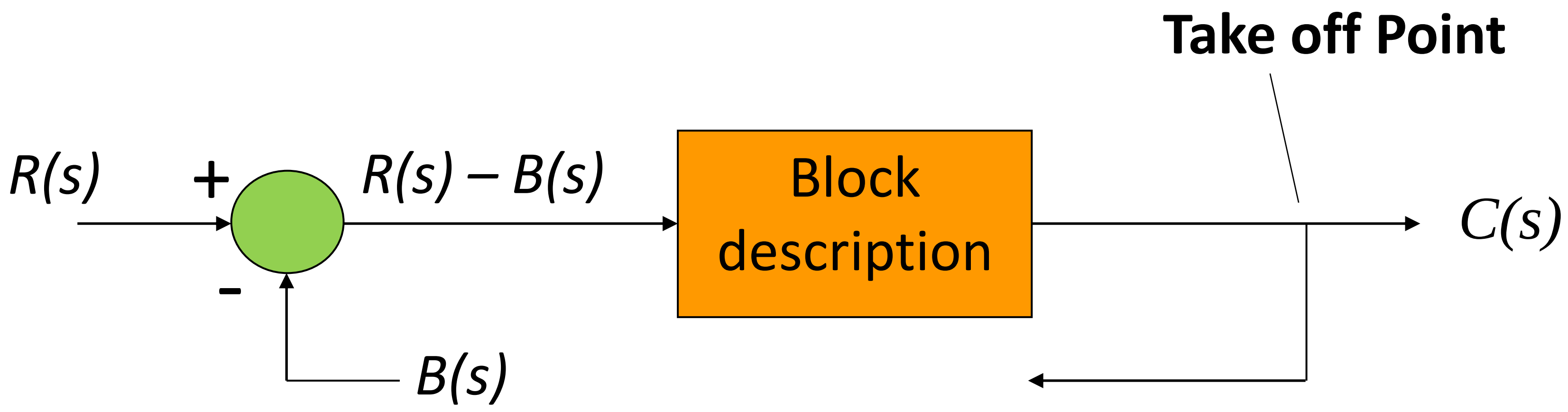
Block

Block
description

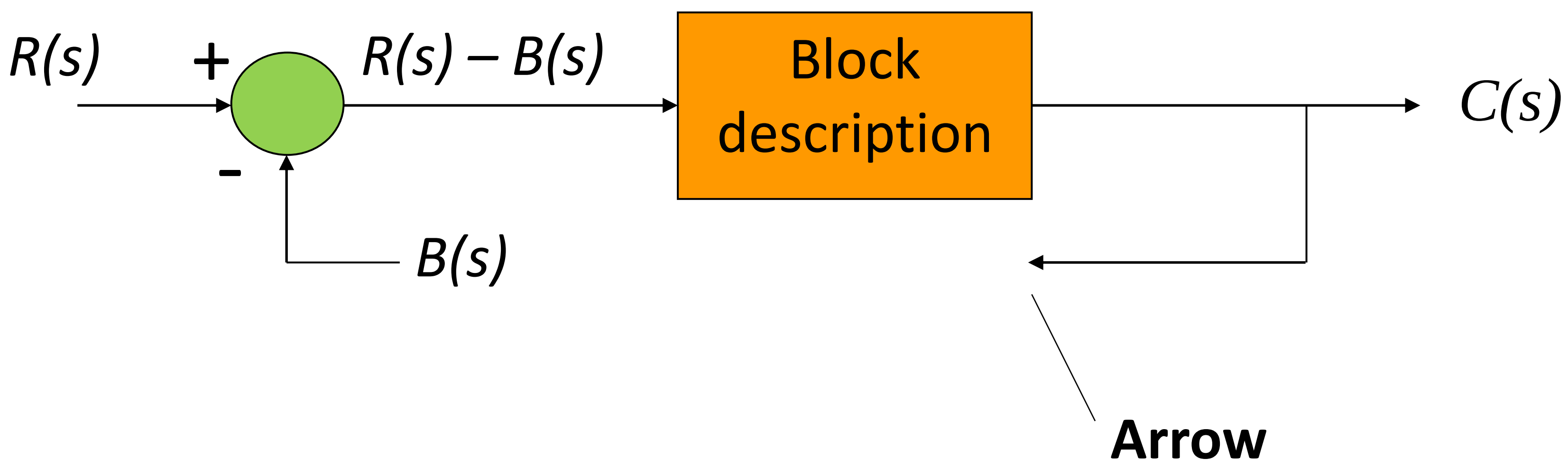
Block Diagram Analysis



Block Diagram Analysis



Block Diagram Analysis

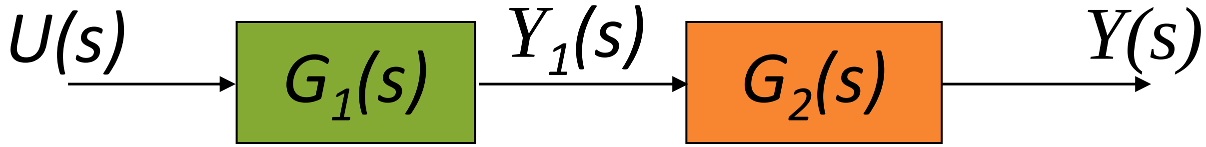


Block Diagram Algebra

- Complicated block diagrams can be simplified by combining different blocks together in a step-by-step fashion using block diagram algebra.
- There are 3 basic operations, depending on the type of connection between blocks.
- Series, parallel and feedback connections are possible.
- We will now look at each of these in turn.

Series (or Cascade) Connection

- **Series (or cascade) connection** – the blocks are connected as shown below.



Series (or Cascade) Connection

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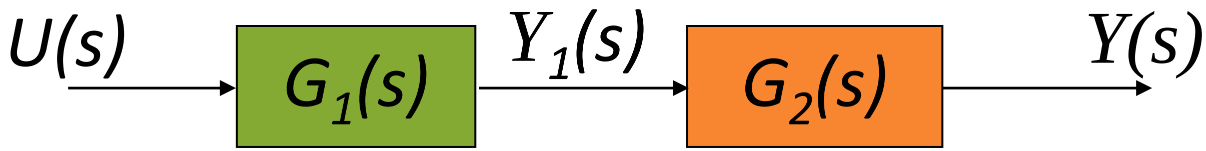
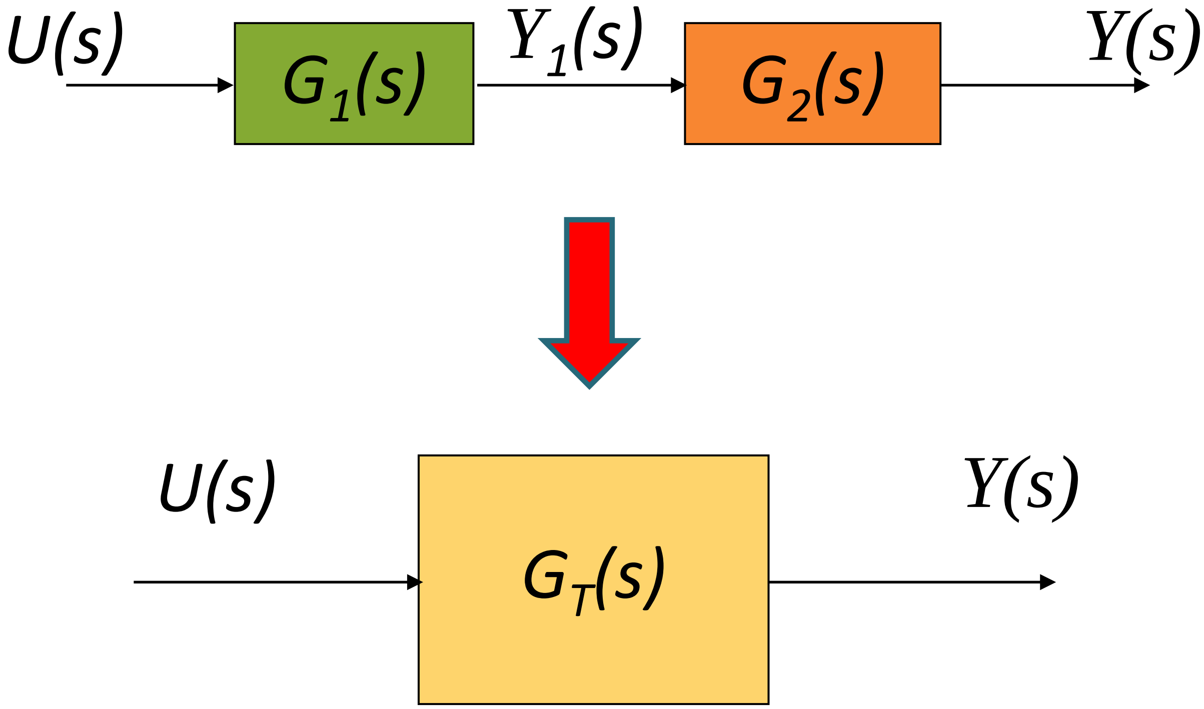


- **Blocks in series** are simply combined by **multiplying** them together.

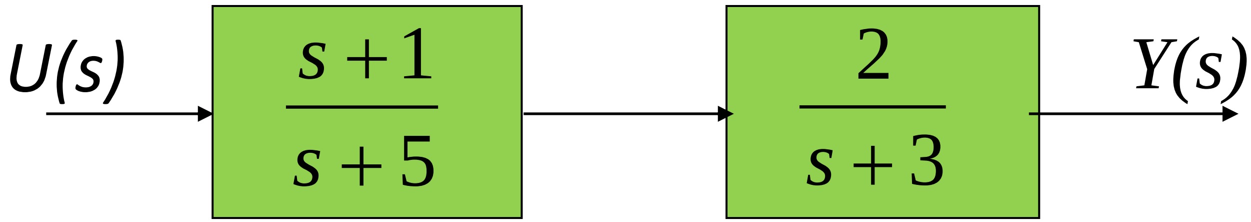
Series (or Cascade) Connection



$$G_T(s) = G_1(s).G_2(s)$$

Series (or Cascade) Connection

- ***Ex. 5.1 Simplify the following system:***



Series (or Cascade) Connection

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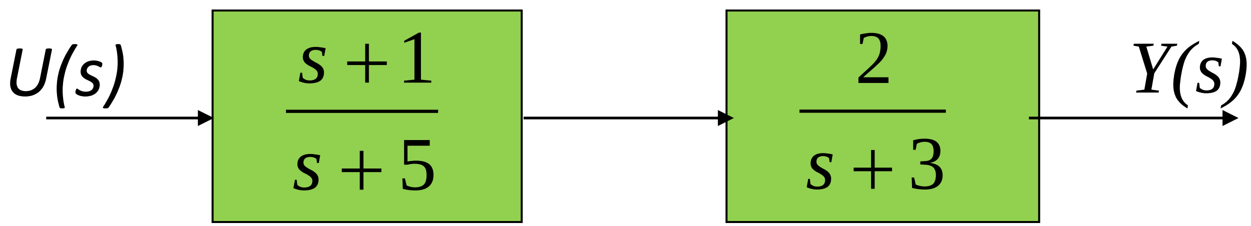
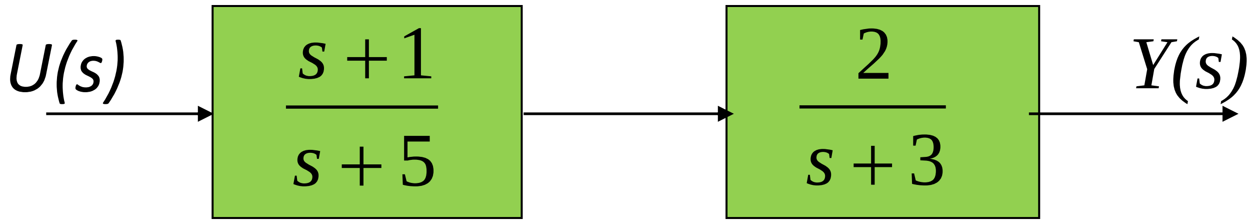


Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right)\left(\frac{2}{s+3}\right)$$

Series (or Cascade) Connection

- *Ex. 5.1 Simplify the following system:*



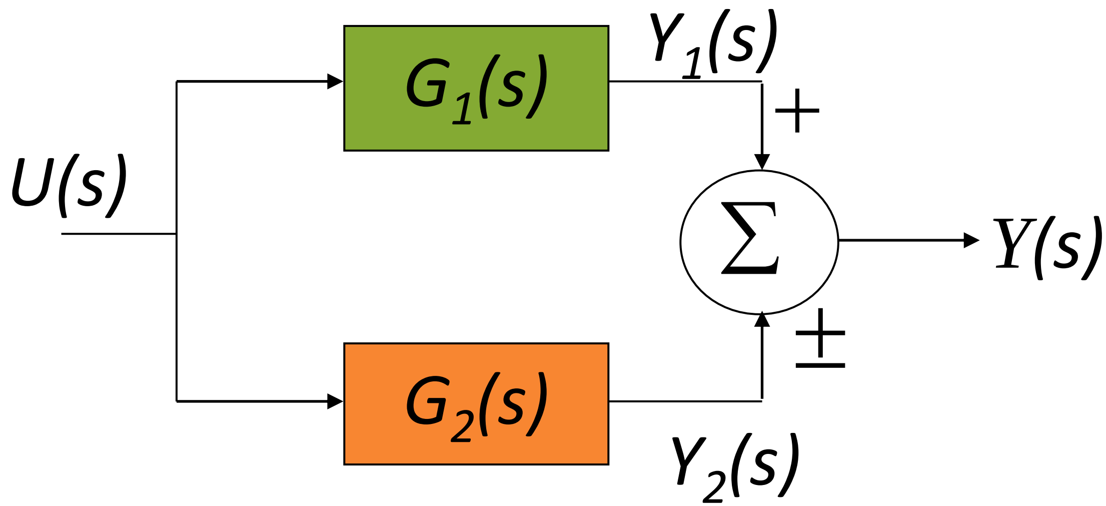
Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right)\left(\frac{2}{s+3}\right)$$

$$= \frac{2(s+1)}{(s+3)(s+5)} = \frac{2s+2}{s^2+8s+15}$$

Parallel Connection

- **Parallel connection** – the blocks are connected as shown below.



Parallel Connection

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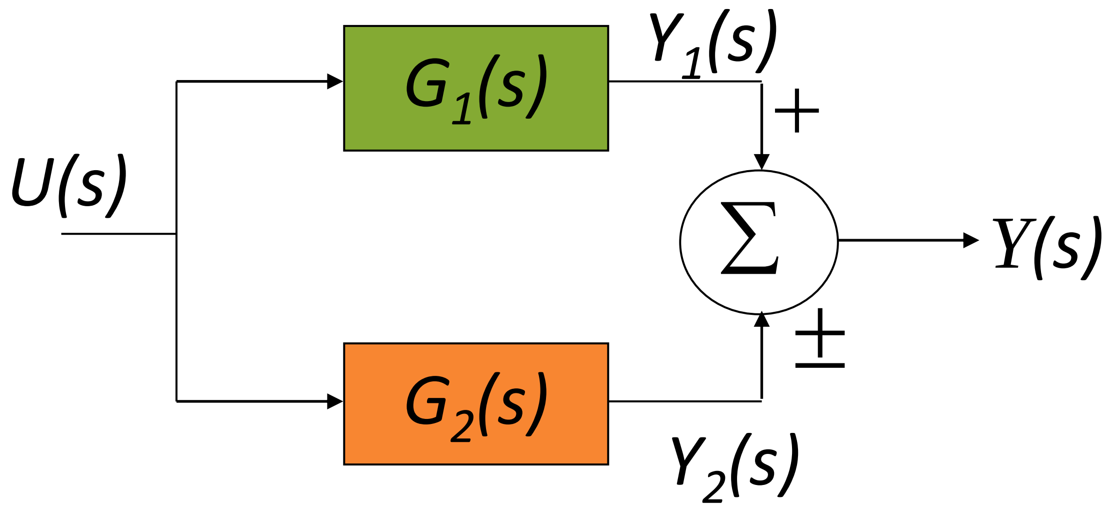
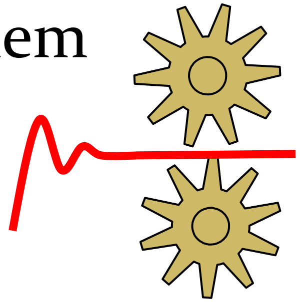
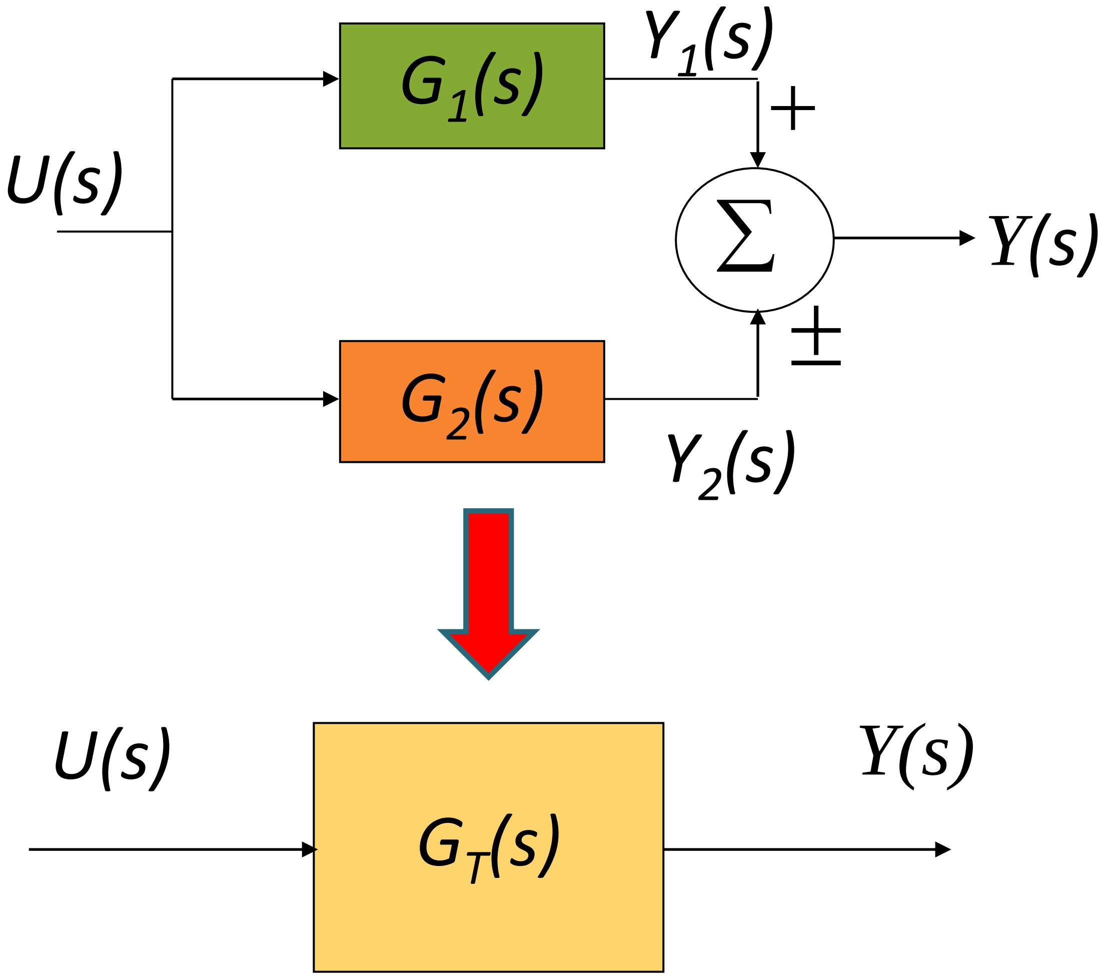


- **Blocks in parallel** are simply combined by **adding** them together.



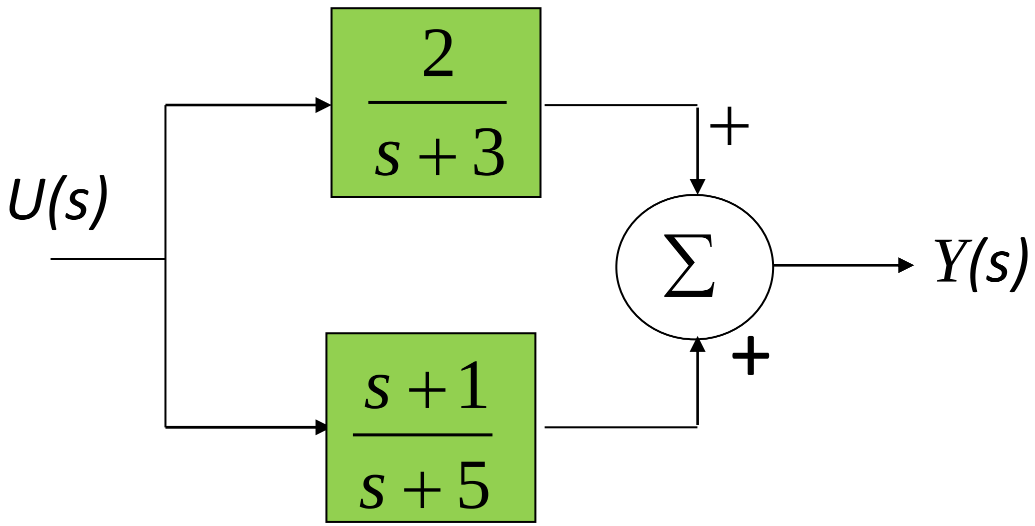
Parallel Connection



$$G_T(s) = G_1(s) \pm G_2(s)$$

Parallel Connection

- *Ex. 5.2 Simplify the following system:*



Parallel Connection

Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right) + \left(\frac{2}{s+3}\right)$$

Parallel Connection

Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right) + \left(\frac{2}{s+3}\right) \qquad = \frac{(s+1)(s+3)+2(s+5)}{(s+3)(s+5)}$$

Parallel Connection

Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right) + \left(\frac{2}{s+3}\right) \qquad = \frac{(s+1)(s+3)+2(s+5)}{(s+3)(s+5)}$$

$$= \frac{s^2+4s+3+2s+10}{s^2+8s+15}$$

Parallel Connection

Solution:

$$\frac{Y(s)}{U(s)} = \left(\frac{s+1}{s+5}\right) + \left(\frac{2}{s+3}\right) = \frac{(s+1)(s+3)+2(s+5)}{(s+3)(s+5)}$$

$$= \frac{s^2+4s+3+2s+10}{s^2+8s+15} = \frac{s^2+6s+13}{s^2+8s+15}$$

Feedback Connection

- **Feedback connection** – when the output of a block is fed back to the input of an earlier block in the block diagram, a feedback mechanism is introduced.

- This is usually referred to as ***closing the loop***.

"The good news is we're getting a lot of feedback.
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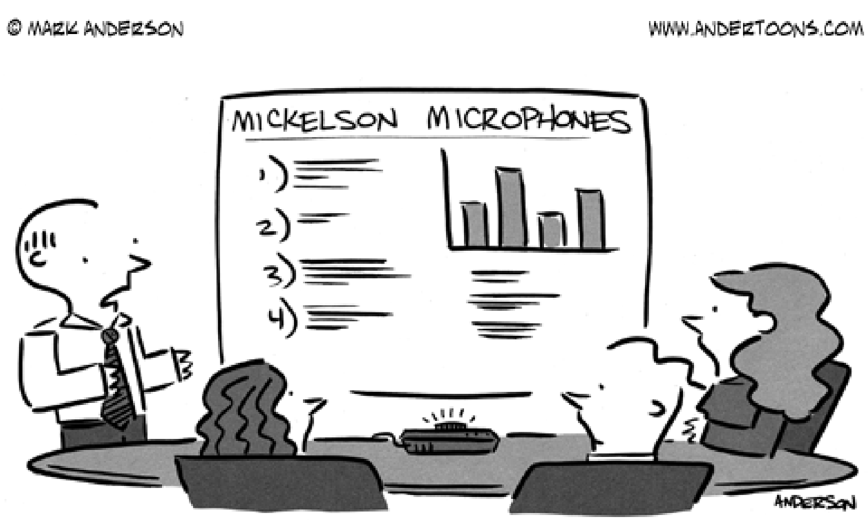
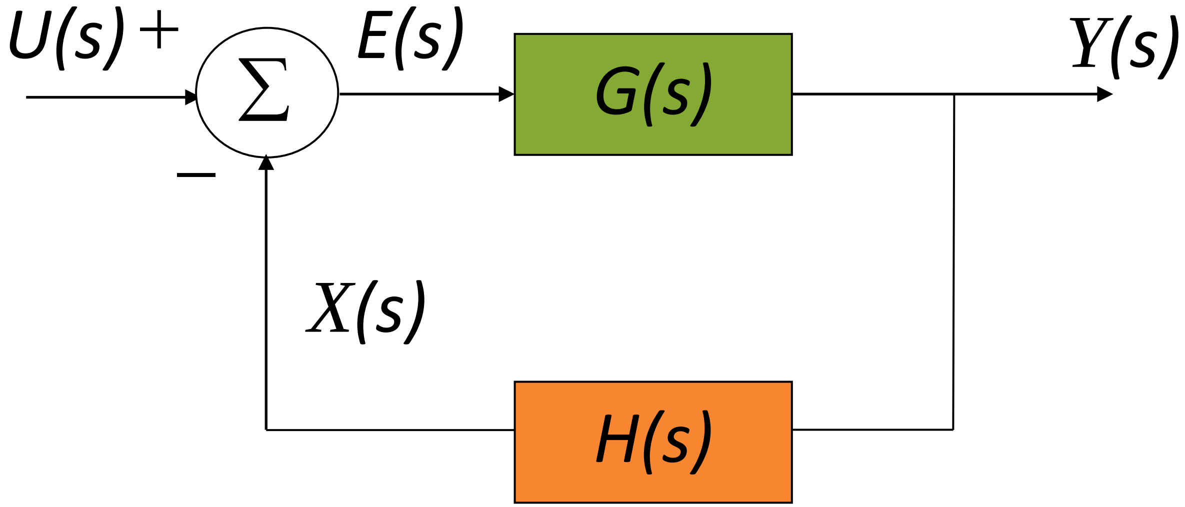


"The good news is we're getting a lot of feedback. The bad news is we're getting a lot of feedback."

- The resulting transfer function is called the **closed loop transfer function**.

Feedback Connection

- The fundamental closed loop diagram for a feedback system is as follows:



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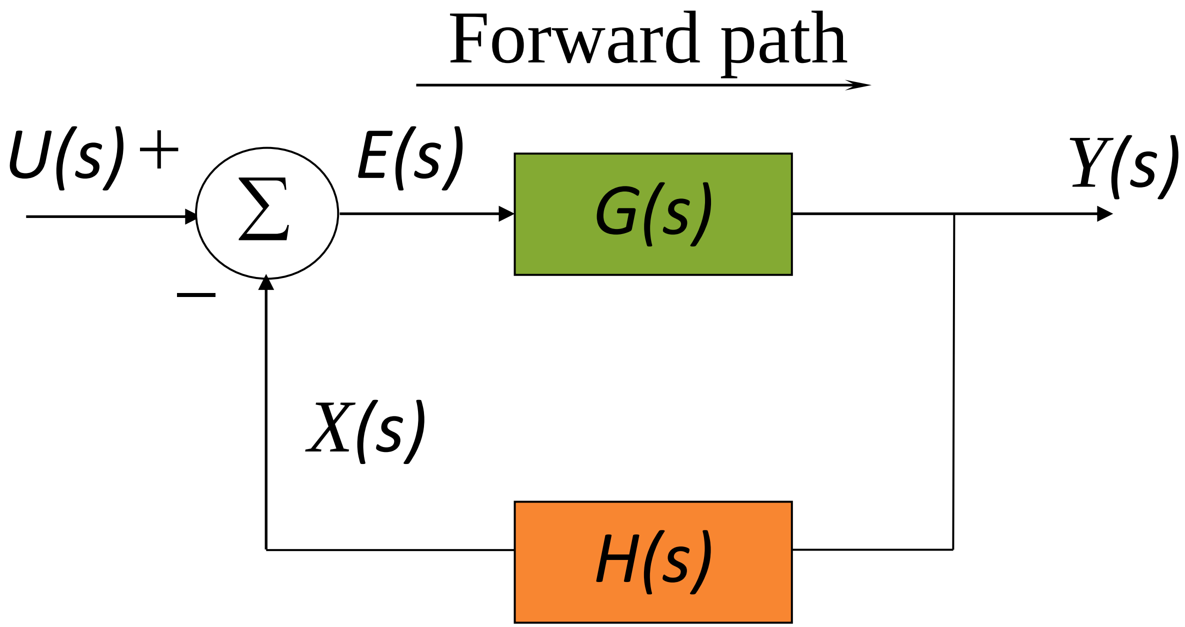

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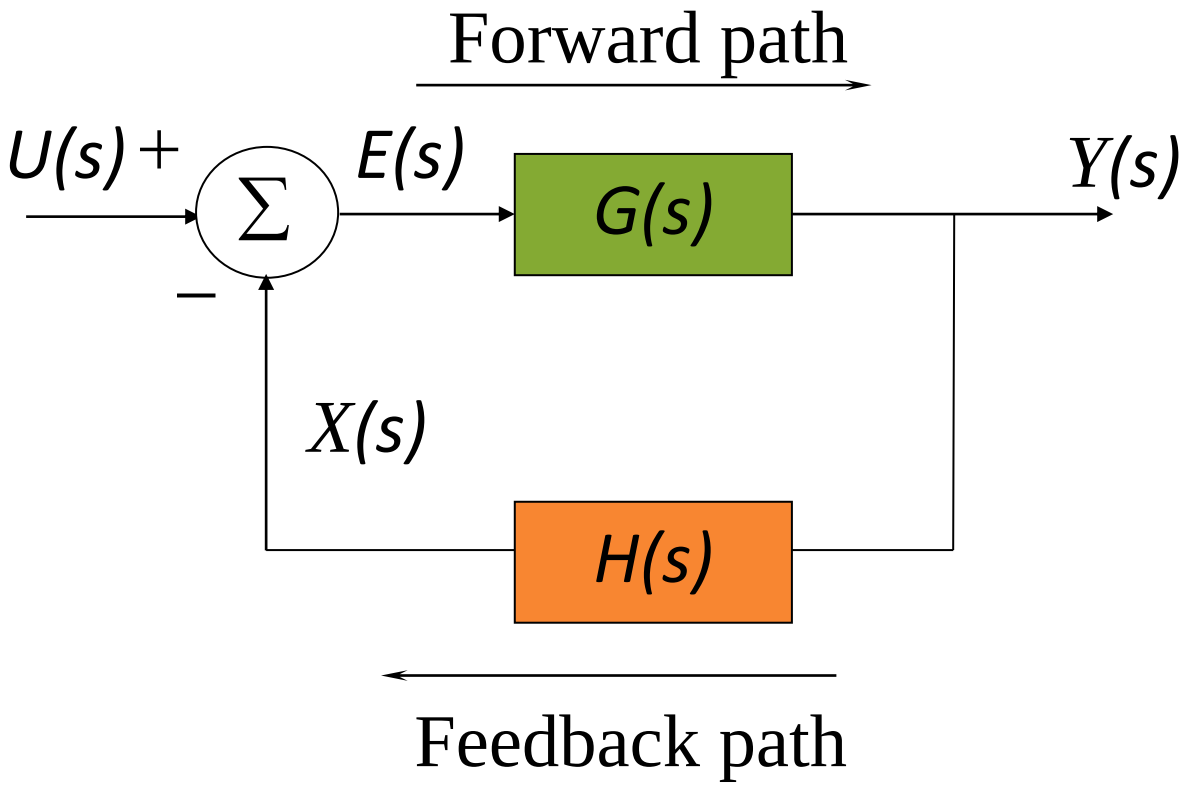
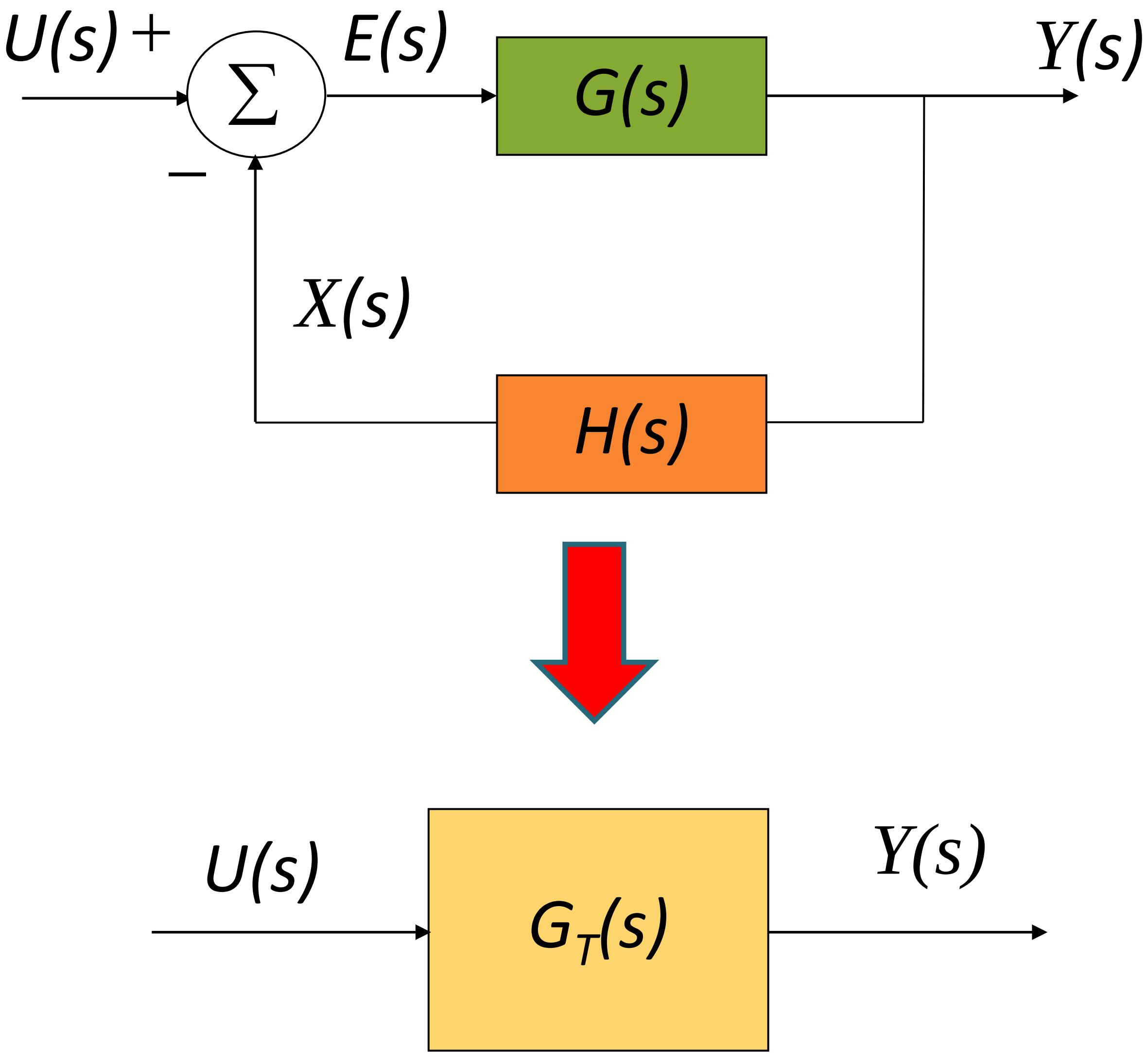
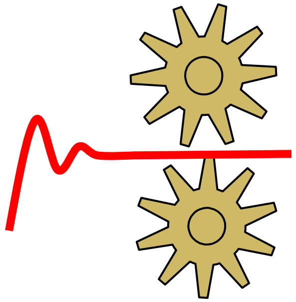


Feedback Connection





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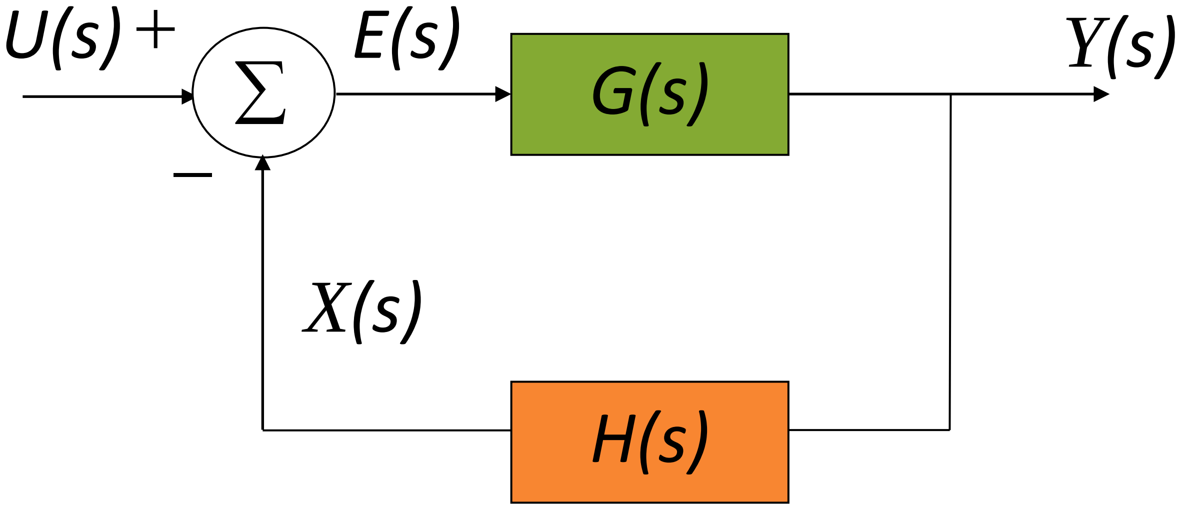
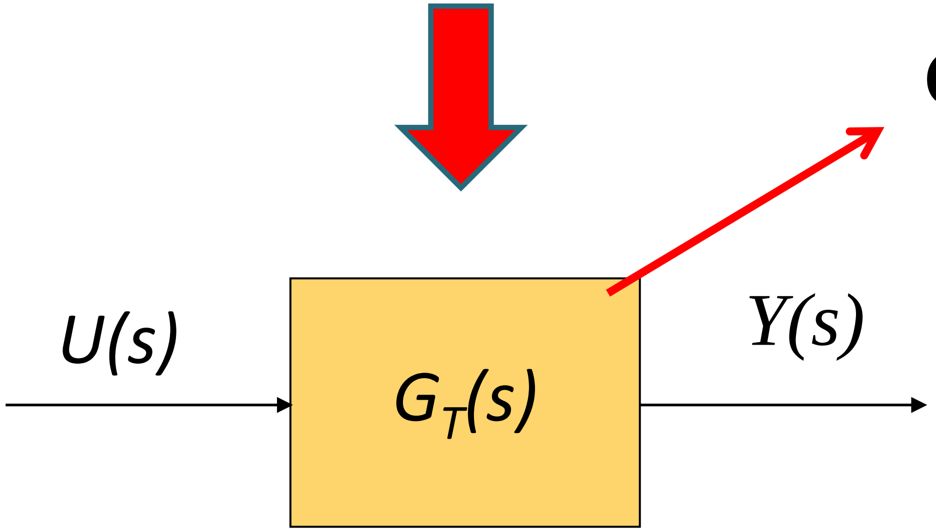
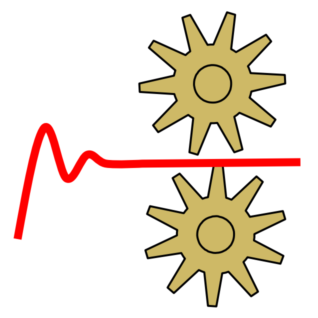




$$G_T(s) = \frac{G}{1 + GH}$$



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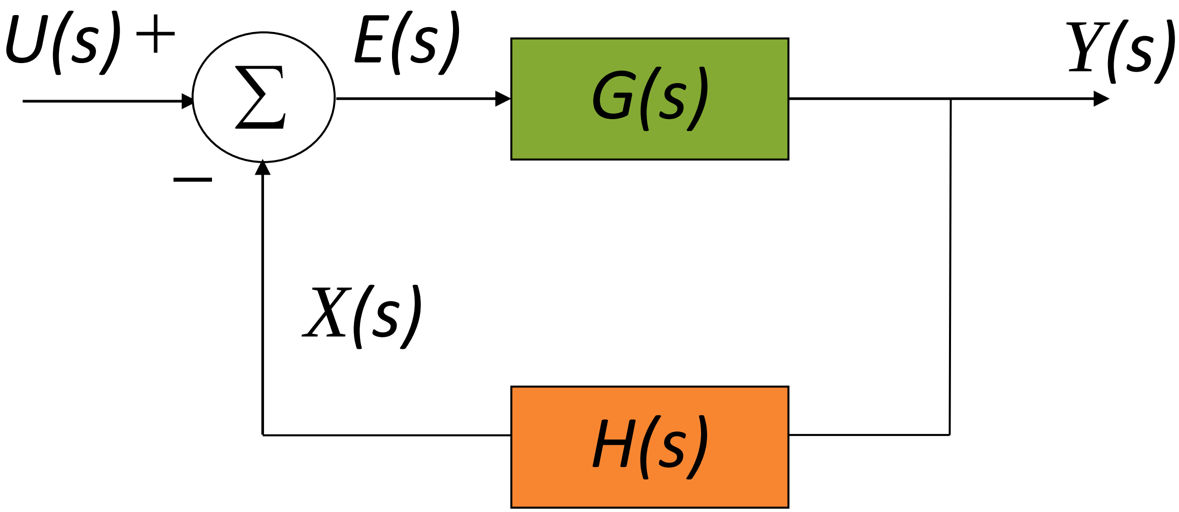
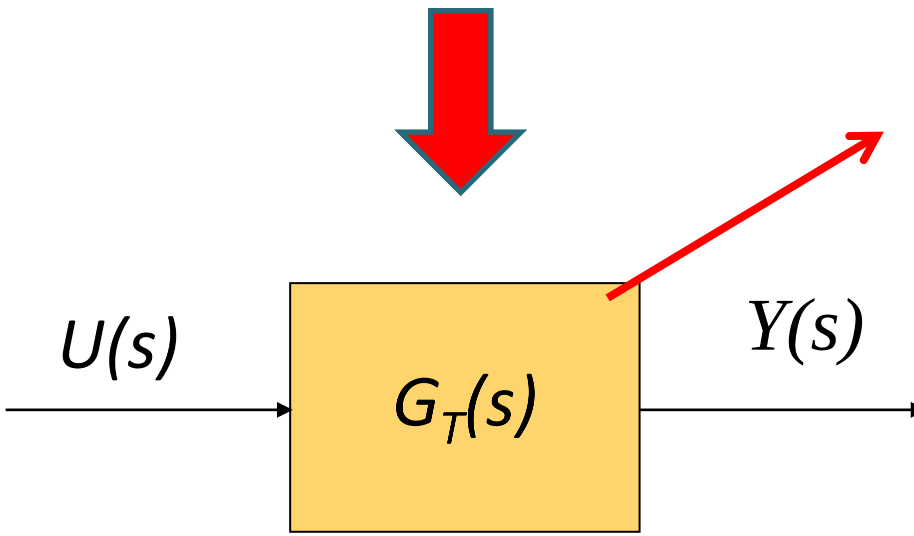


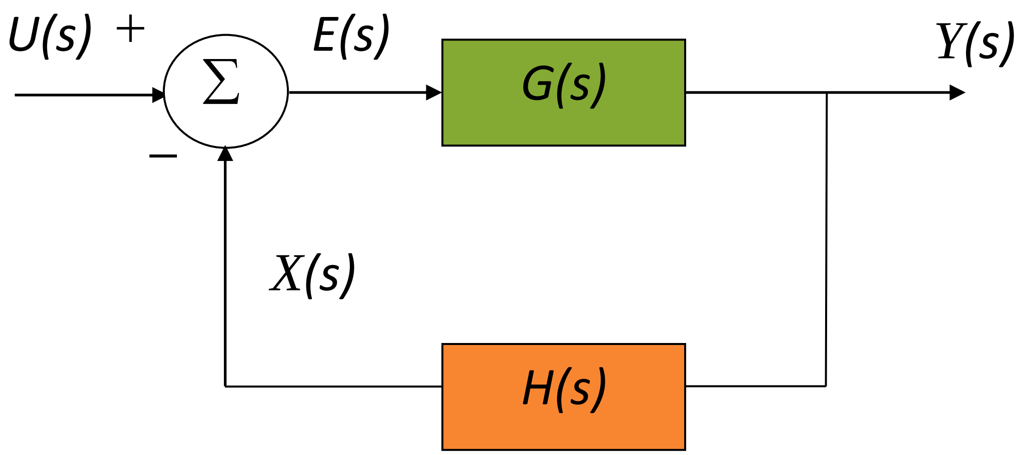
Closed Loop Transfer Function (CLTF)

$$G_T(s) = \frac{G}{1 + GH}$$



Feedback Connection

- This is known as the **canonical block diagram** and is characterised by:



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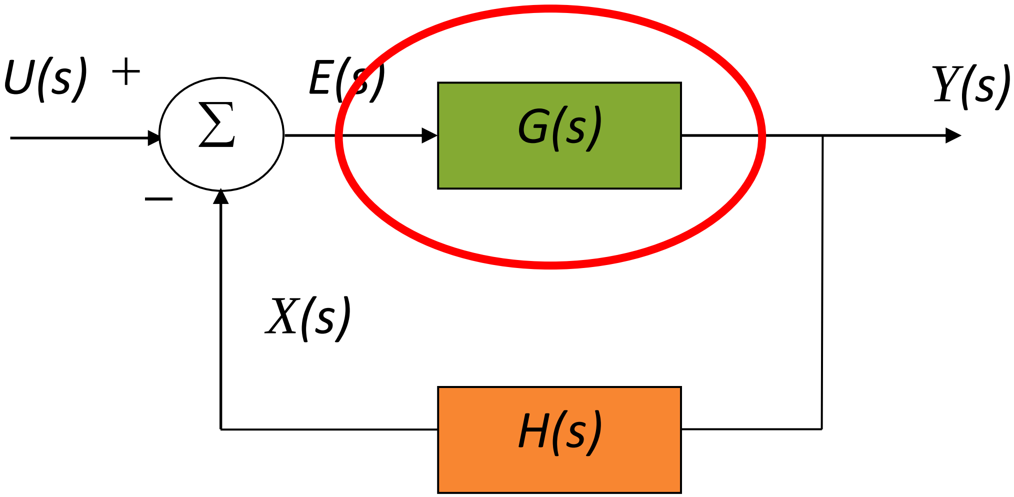

G(s) is the direct or forward path transfer function.

Feedback Connection

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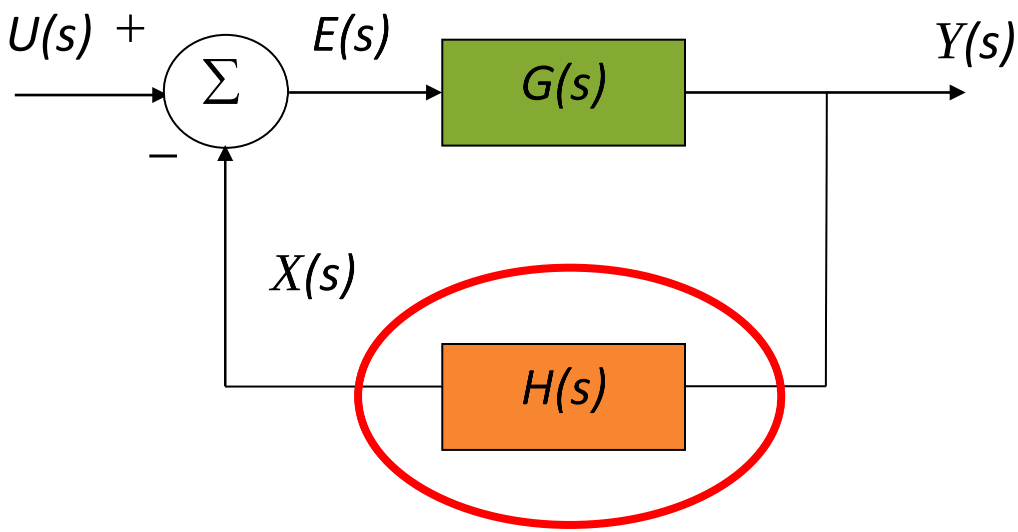

H(s) is the feedback transfer function
(a unity feedback system has *H(s)* = 1).

Feedback Connection

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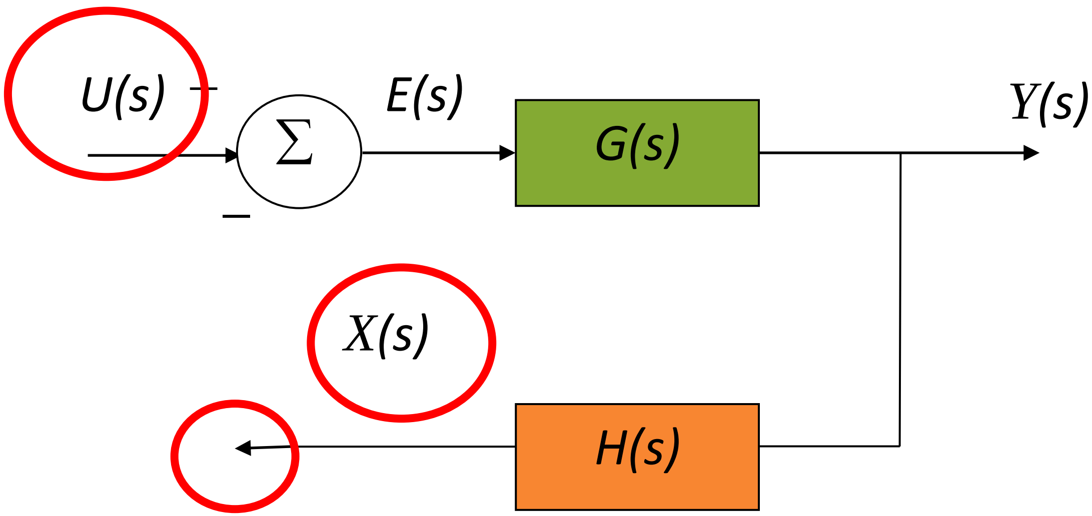
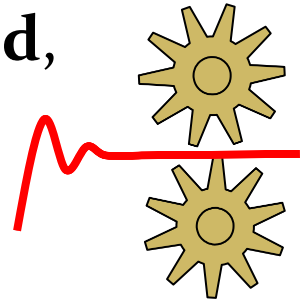


$$\frac{X(s)}{U(s)} = G(s)H(s)$$

with feedback disconnected, is the open-loop transfer function (OLTF)



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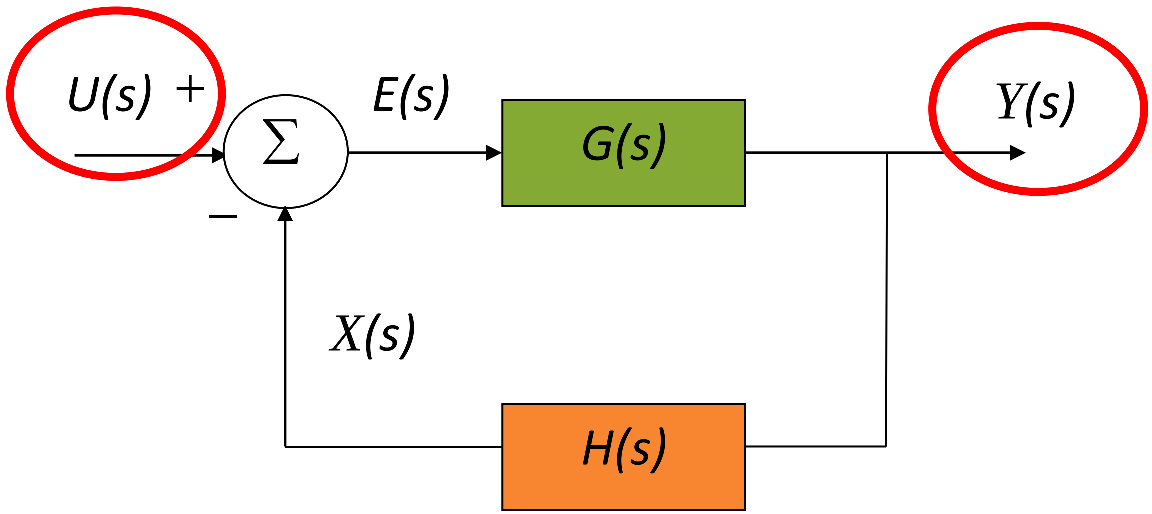
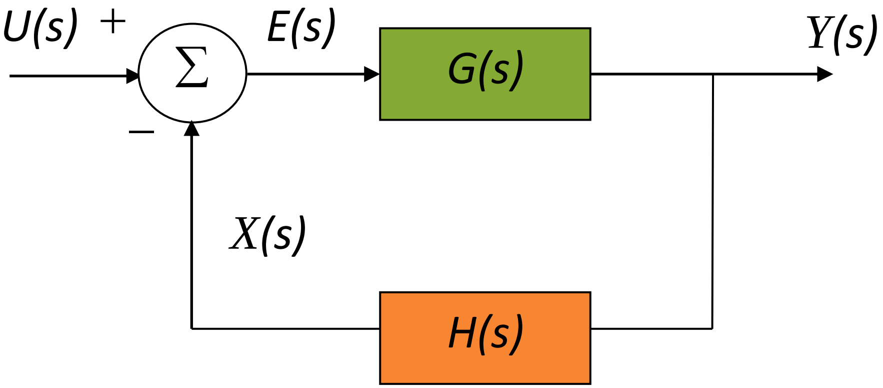


$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$ is the **closed-loop transfer function (CLTF).**

Feedback Connection

- Looking at the block diagram, we can derive the CLTF as follows:



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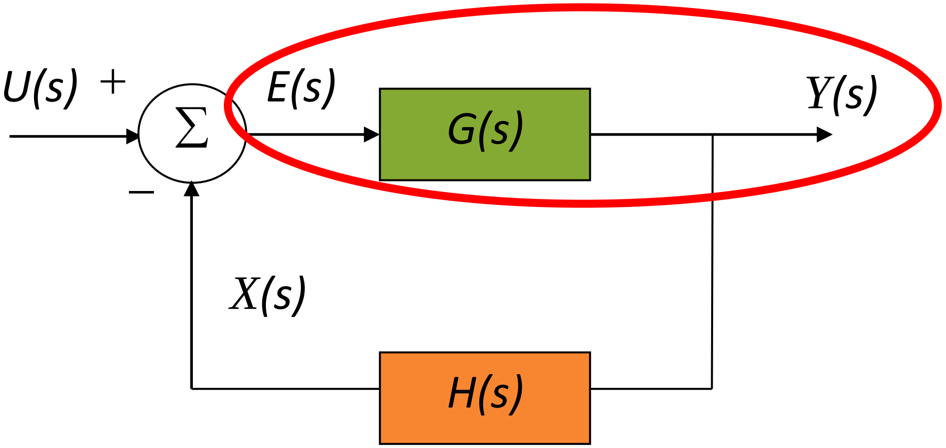
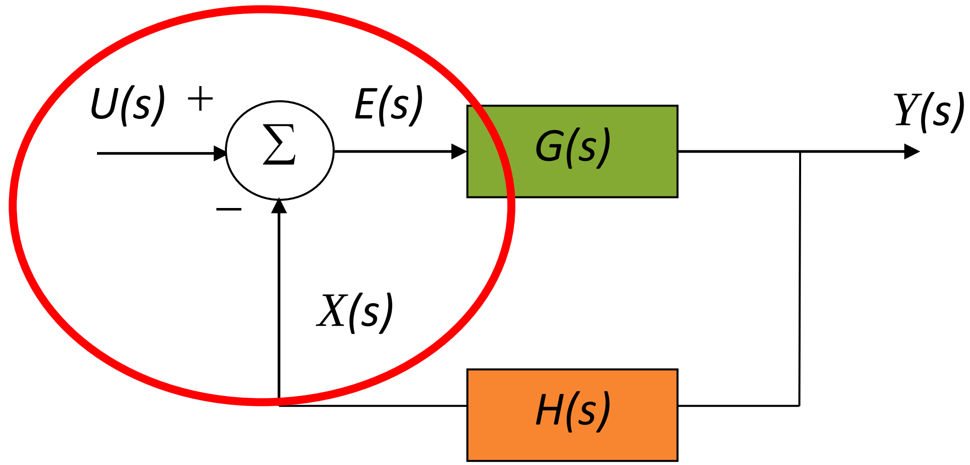


- Looking at the block diagram, we can derive the CLTF as follows:

$$Y(s) = G(s)E(s)$$

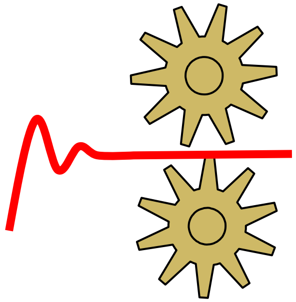
Feedback Connection



- Looking at the block diagram, we can derive the CLTF as follows:

$$Y(s) = G(s)E(s)$$

$$E(s) = U(s) - X(s)$$



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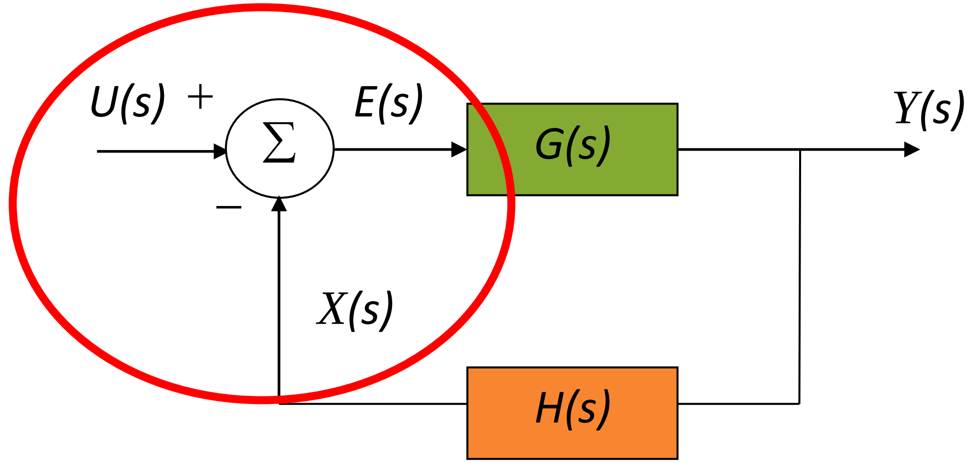
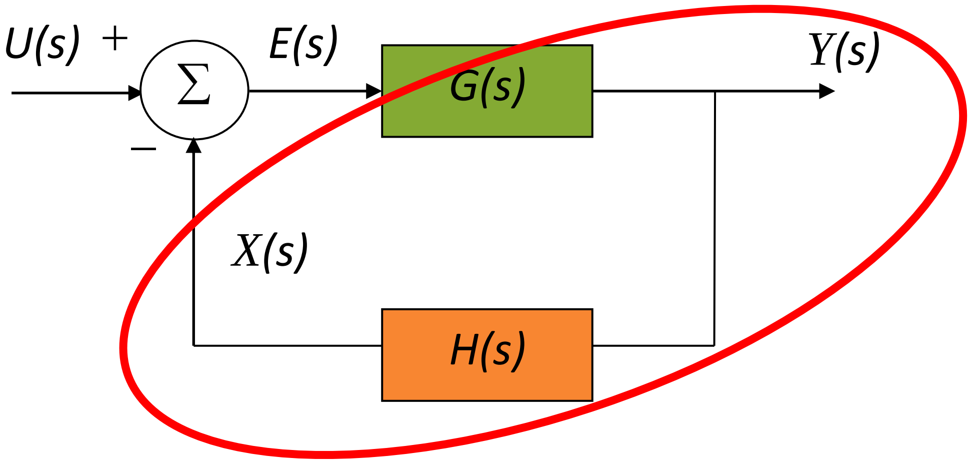


$$Y(s) = G(s)E(s)$$

$$E(s) = U(s) - X(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - X(s)\big)$$

Feedback Connection



- Looking at the block diagram, we can derive the CLTF as follows:

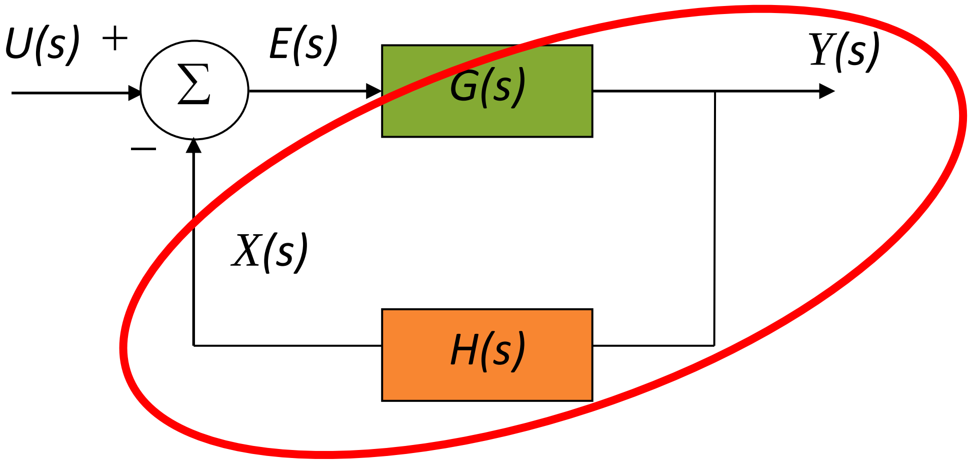
$$Y(s) = G(s)E(s)$$

$$E(s) = U(s) - X(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - X(s)\big)$$

$$X(s) = H(s)Y(s)$$

Feedback Connection

- Looking at the block diagram, we can derive the CLTF as follows:



$$Y(s) = G(s)E(s)$$

$$E(s) = U(s) - X(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - X(s)\big)$$

$$X(s) = H(s)Y(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - H(s)Y(s)\big)$$

Feedback Connection

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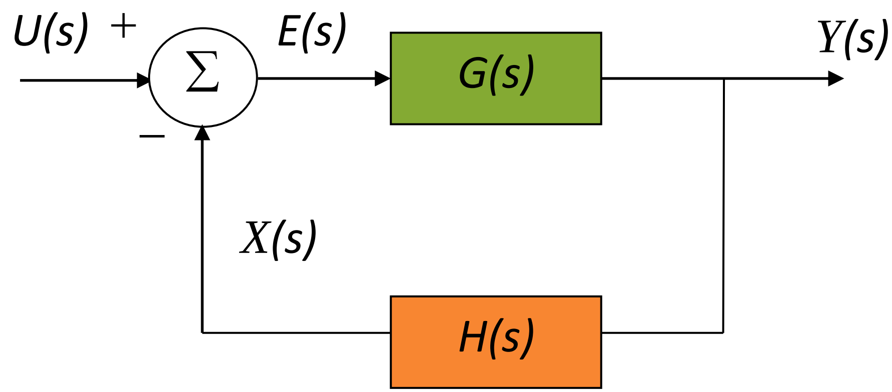


$$Y(s) = G(s)E(s)$$

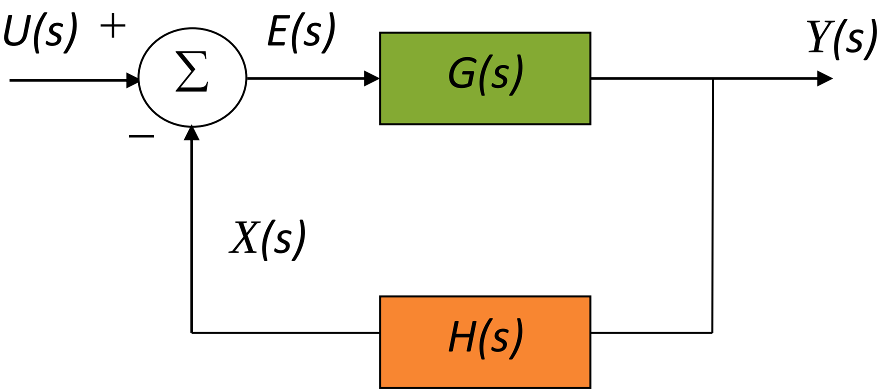
$$E(s) = U(s) - X(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - X(s)\big)$$

$$X(s) = H(s)Y(s) \quad \Rightarrow Y(s) = G(s)\big(U(s) - H(s)Y(s)\big)$$

$$\Rightarrow Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

Feedback Connection

- Looking at the block diagram, we can derive the CLTF as follows:



$$Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

Feedback Connection

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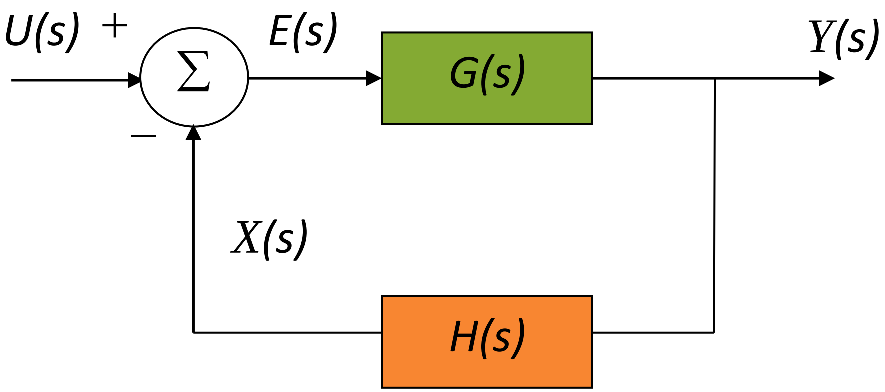
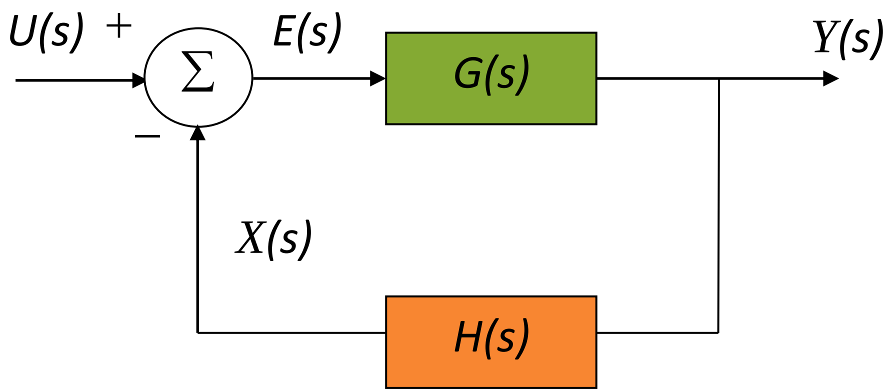


$$Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

$$\Rightarrow Y(s)\left(1 + G(s)H(s)\right) = G(s)U(s)$$

Feedback Connection

- Looking at the block diagram, we can derive the CLTF as follows:



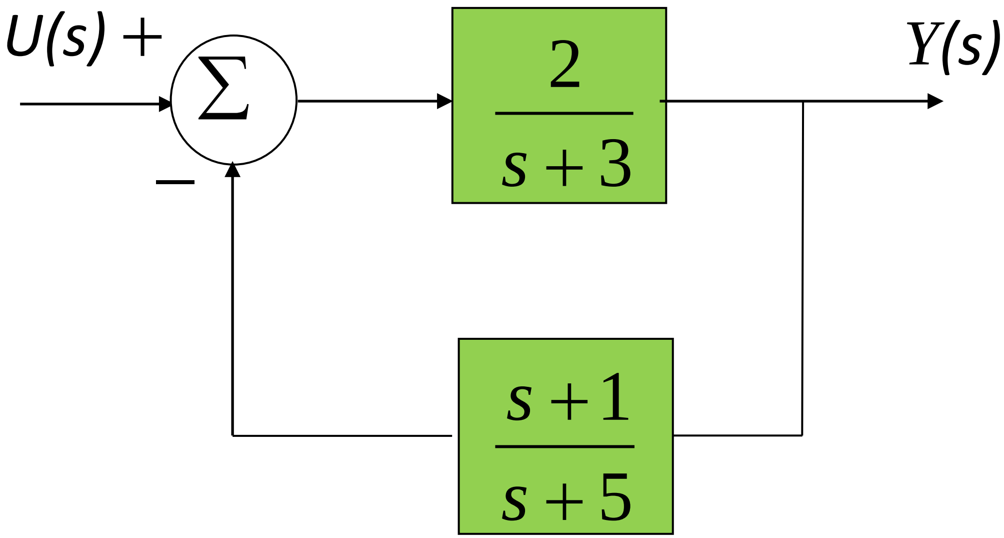
$$Y(s) = G(s)U(s) - G(s)H(s)Y(s)$$

$$\Rightarrow Y(s)\left(1 + G(s)H(s)\right) = G(s)U(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)} = \mathrm{CLTF}$$

Feedback Connection

- *Ex. 5.3 Simplify the following system:*



Feedback Connection

Solution:

$$\frac{Y(s)}{U(s)} = \frac{\left(\dfrac{2}{s+3}\right)}{1+\left(\dfrac{2}{s+3}\right)\left(\dfrac{s+1}{s+5}\right)}$$

Feedback Connection

Solution:

$$\frac{Y(s)}{U(s)} = \frac{\left(\dfrac{2}{s+3}\right)}{1+\left(\dfrac{2}{s+3}\right)\left(\dfrac{s+1}{s+5}\right)}$$

Multiplying up and down by $(s+3)(s+5)$ gives:

$$\frac{2(s+5)}{(s+3)(s+5)+2(s+1)}$$

Feedback Connection

$$\frac{2(s+5)}{(s+3)(s+5)+2(s+1)}$$

Feedback Connection

$$\frac{2(s+5)}{(s+3)(s+5)+2(s+1)}$$

$$=\frac{2s+10}{s^2+8s+15+2s+2}$$

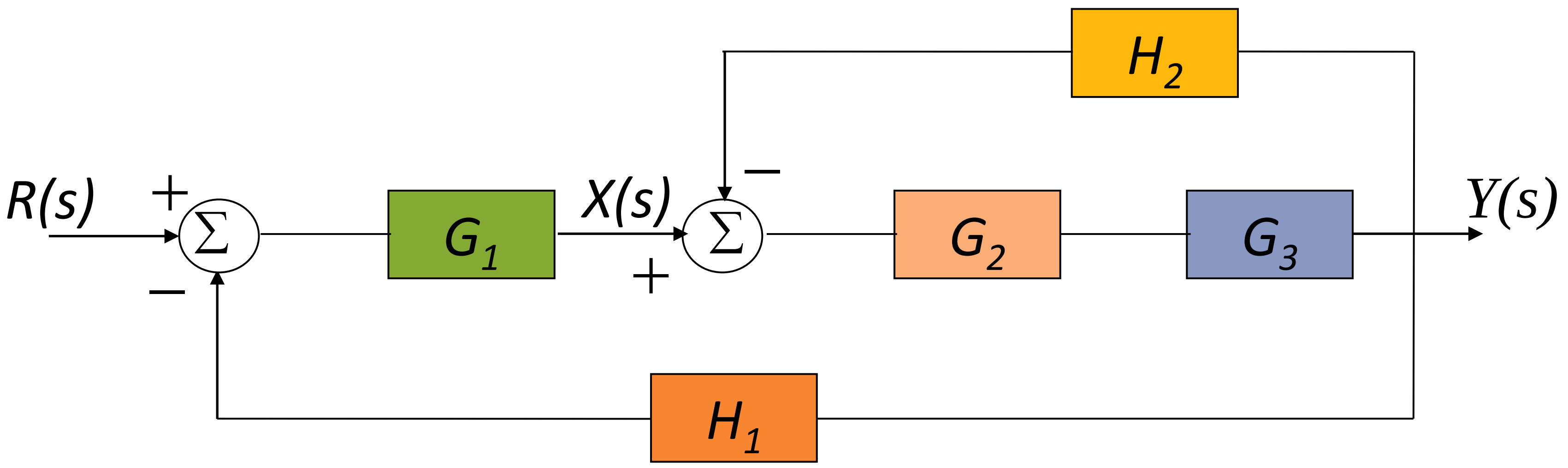
Feedback Connection

$$\frac{2(s+5)}{(s+3)(s+5)+2(s+1)}$$

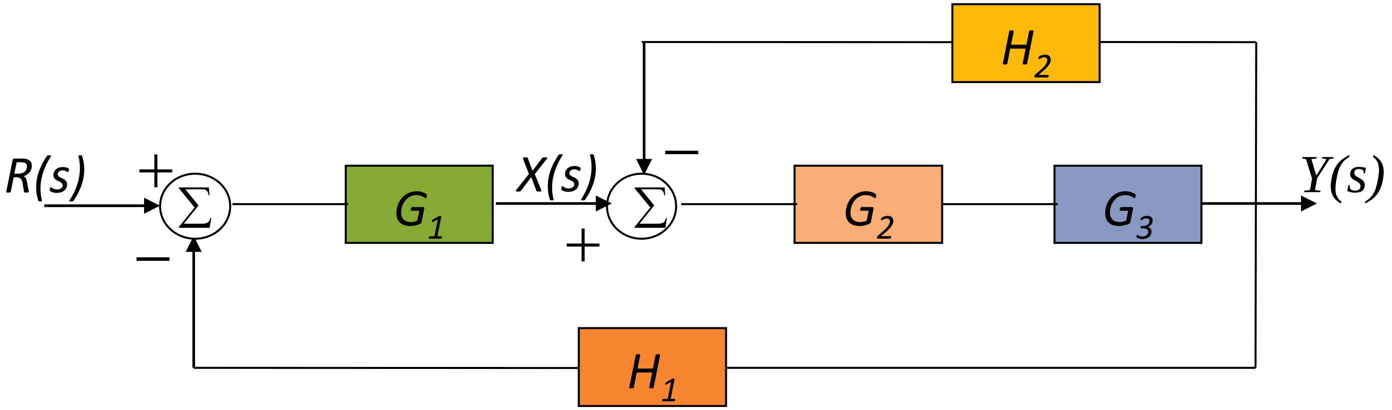
$$=\frac{2s+10}{s^2+8s+15+2s+2}$$

$$=\frac{2s+10}{s^2+10s+17}$$

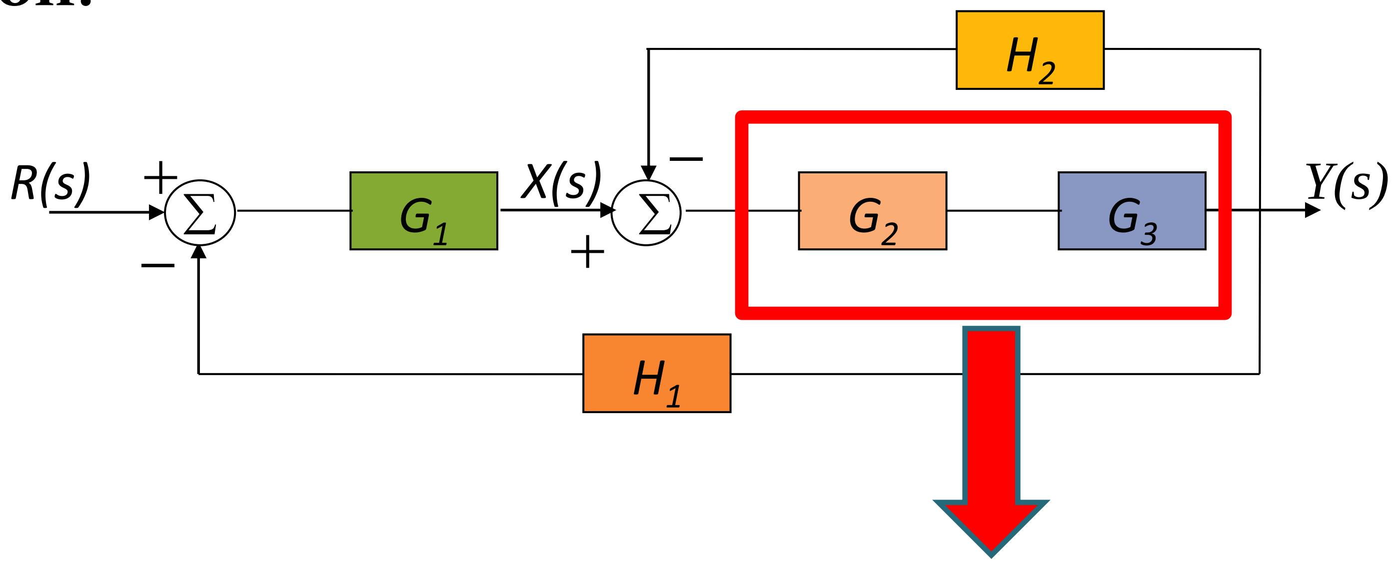
- ***Ex. 5.4 Find the closed-loop transfer function (CLTF) for the following system:***



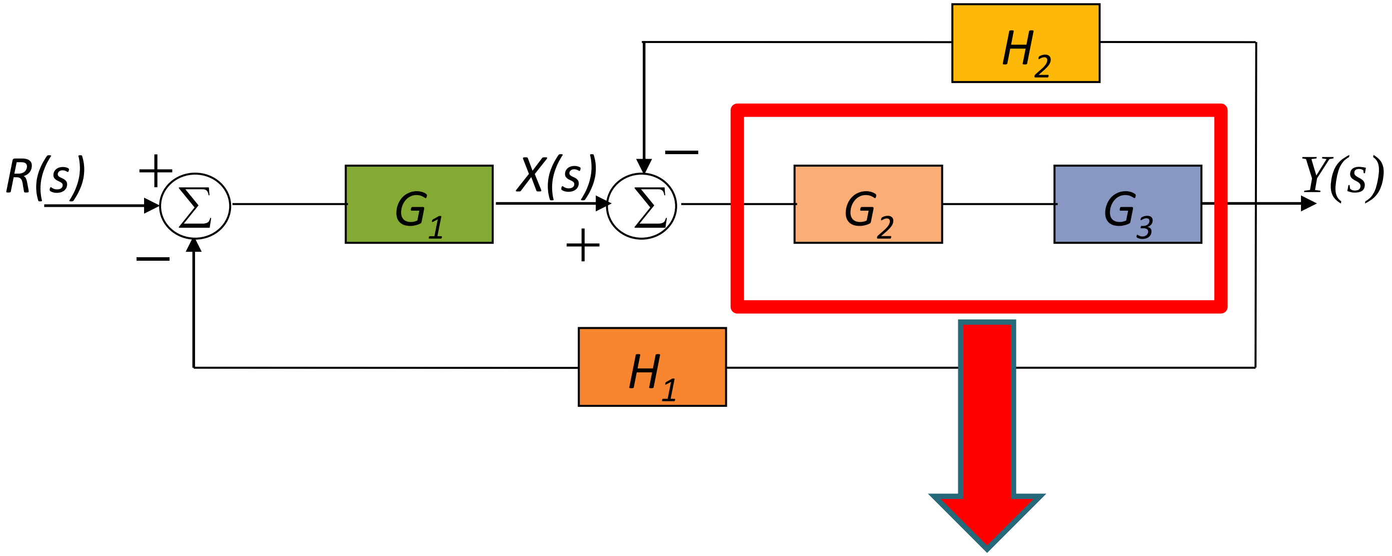
Solution:



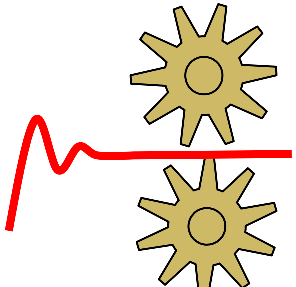
Solution:



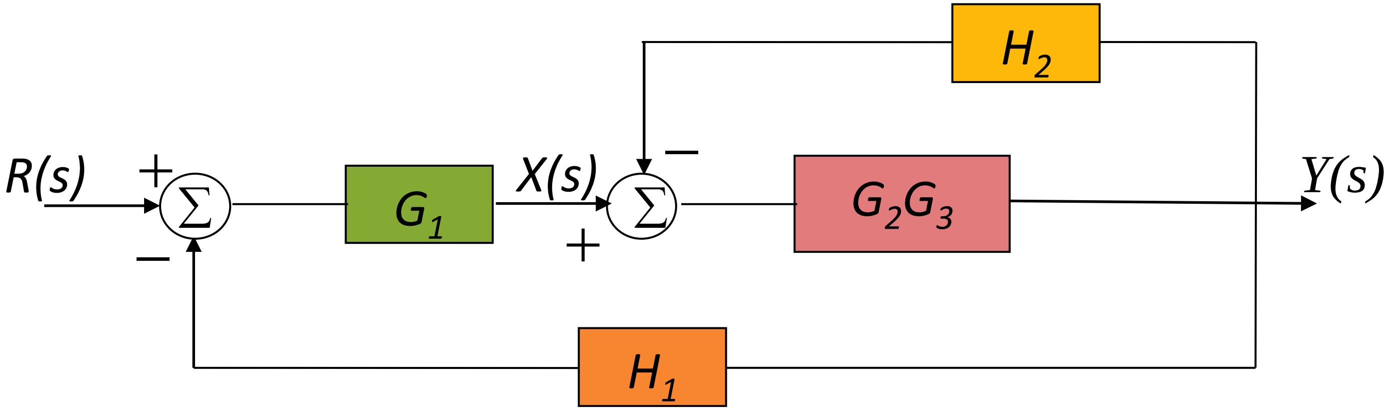
Solution:



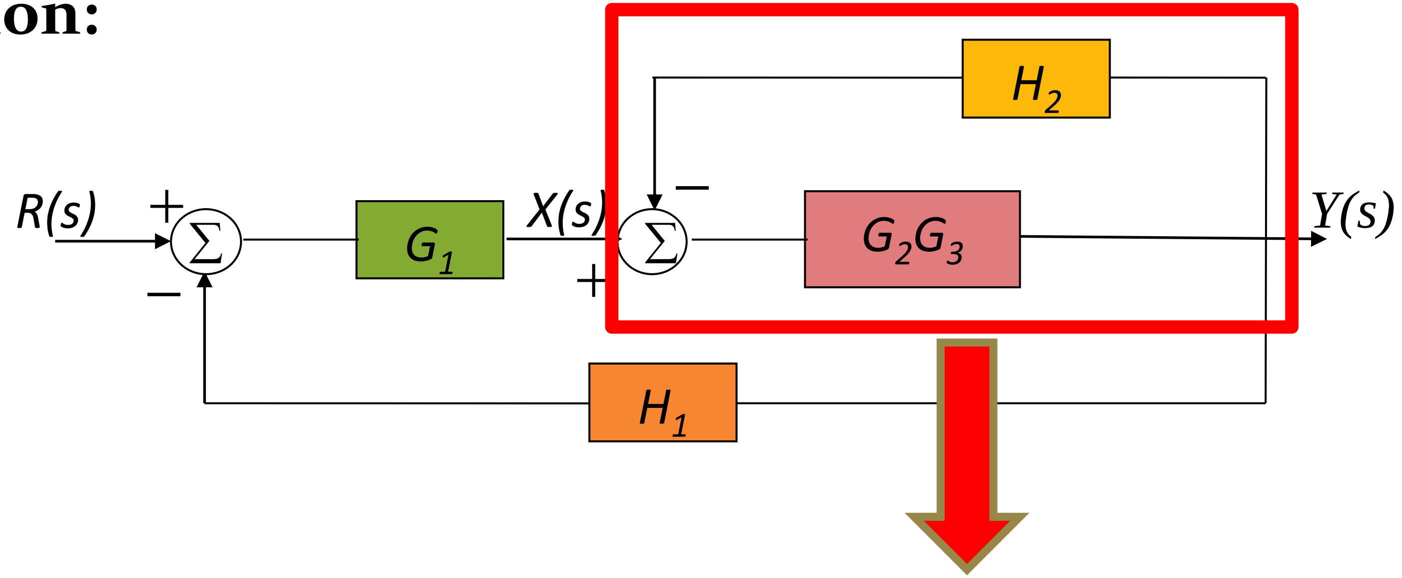
$$G_2G_3$$



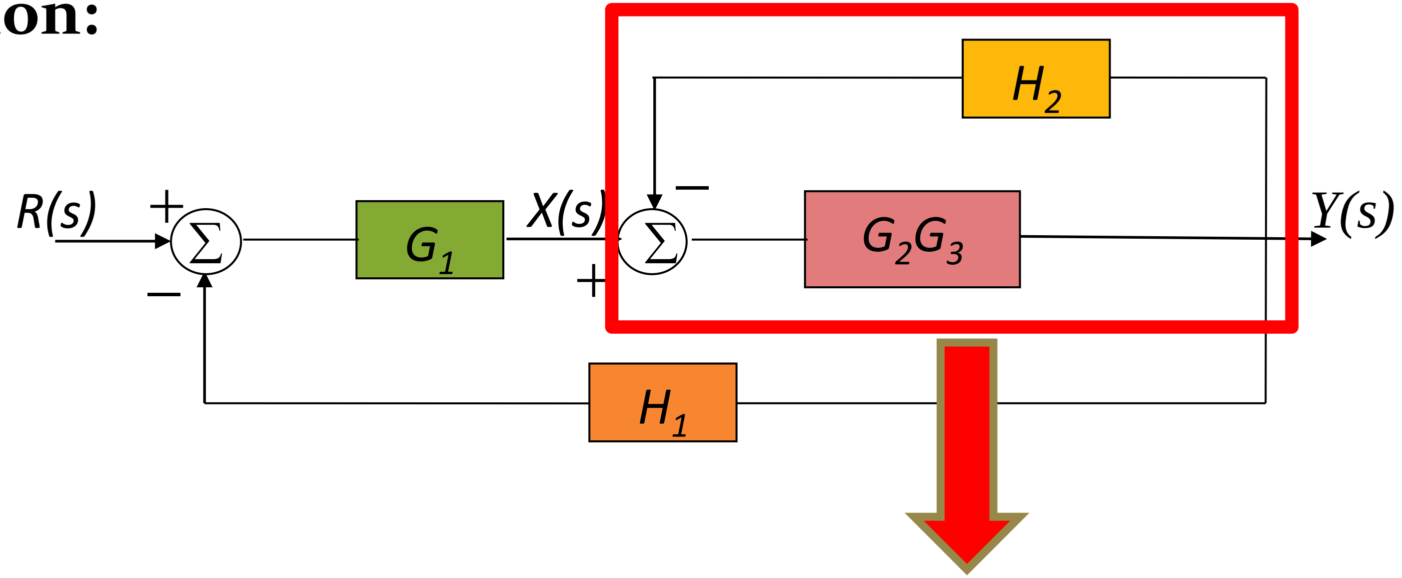
Solution:



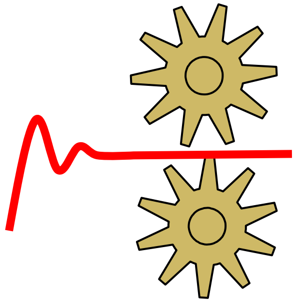
Solution:



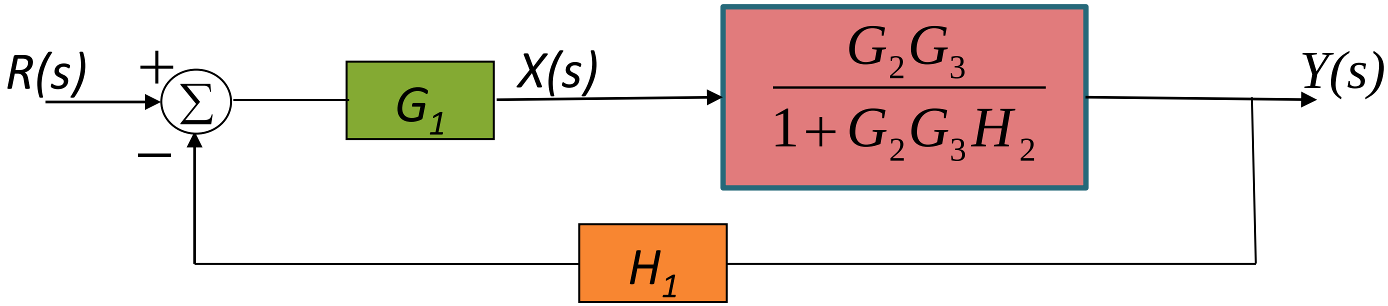
Solution:



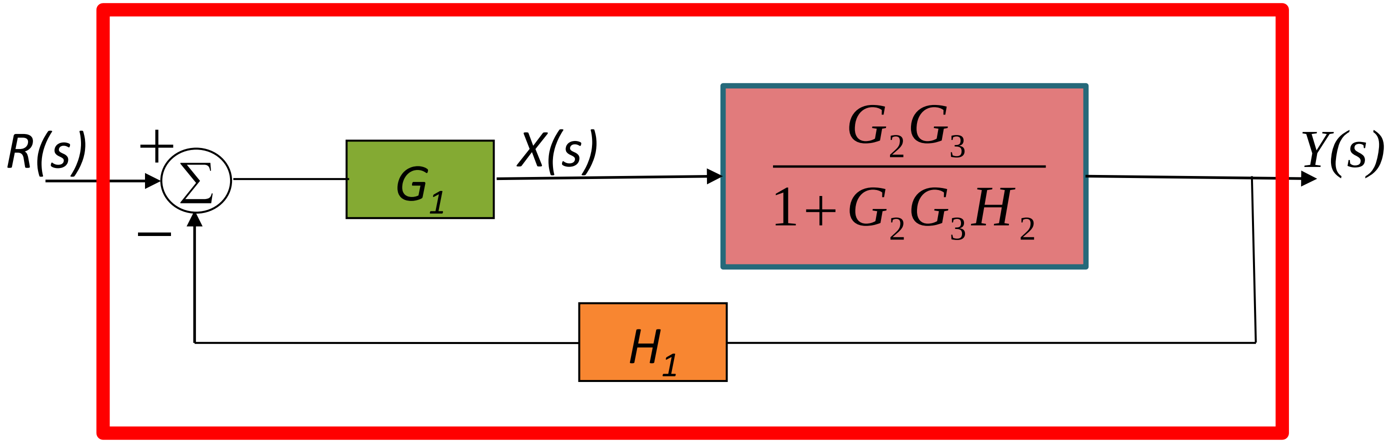
$$\frac{Y(s)}{X(s)} = \frac{G_2G_3}{1+G_2G_3H_2}$$

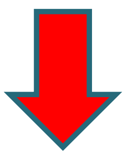


Solution:

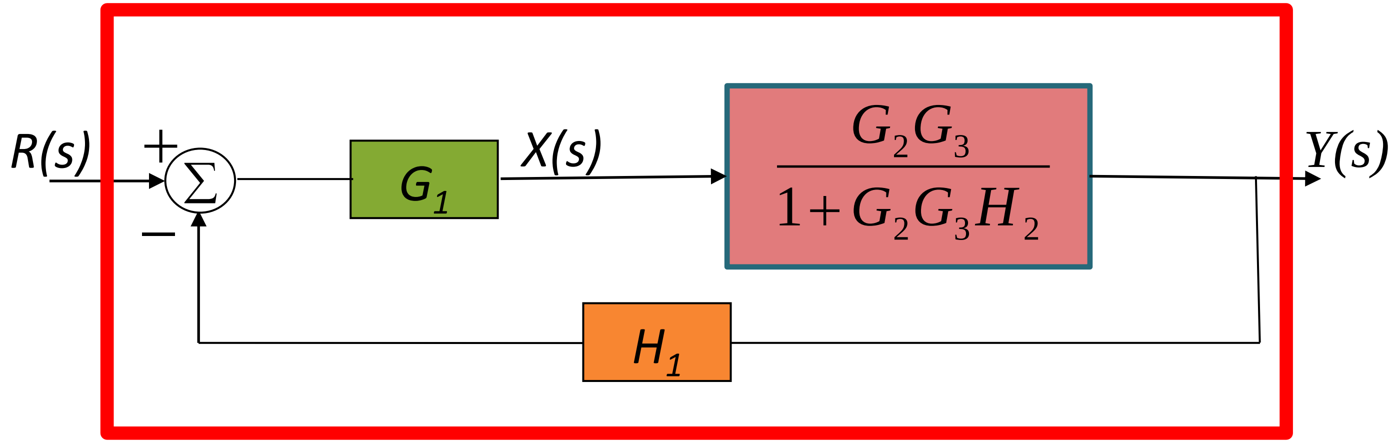


Solution:

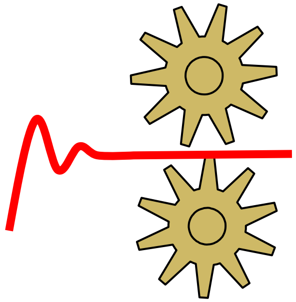




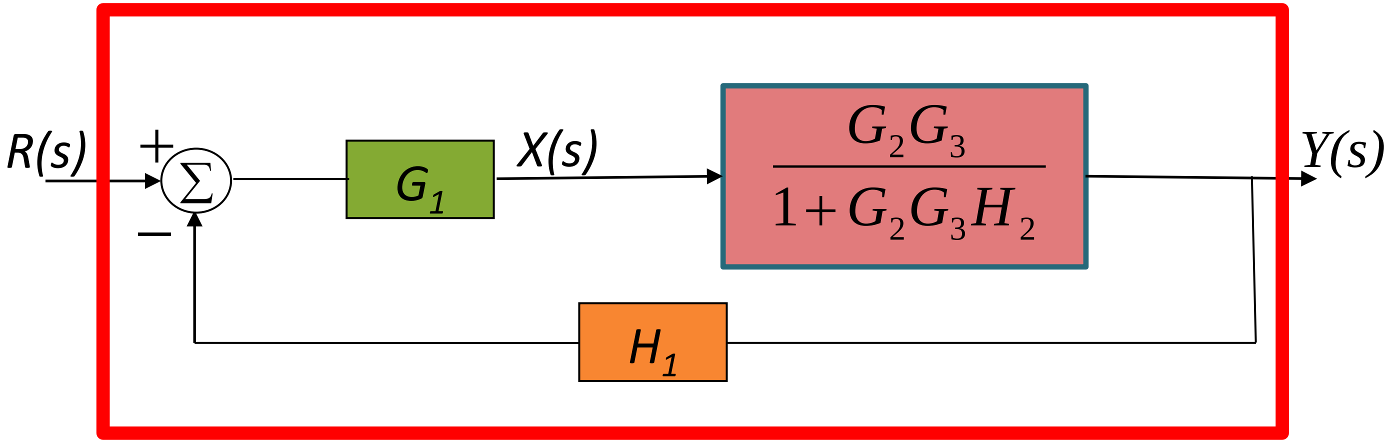
Solution:



$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\left(\dfrac{G_1G_2G_3}{1+G_2G_3H_2}\right)H_1}$$



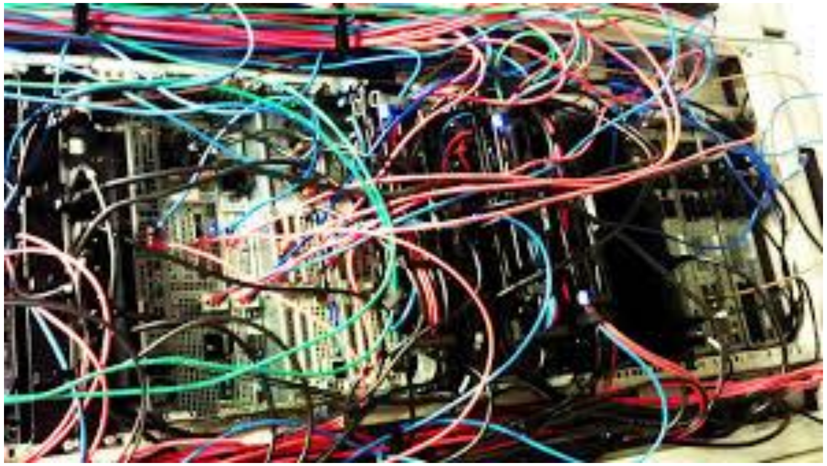
Solution:



$$\frac{Y(s)}{R(s)}=\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\left(\dfrac{G_1G_2G_3}{1+G_2G_3H_2}\right)H_1}$$

$$=\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2G_3H_1}$$

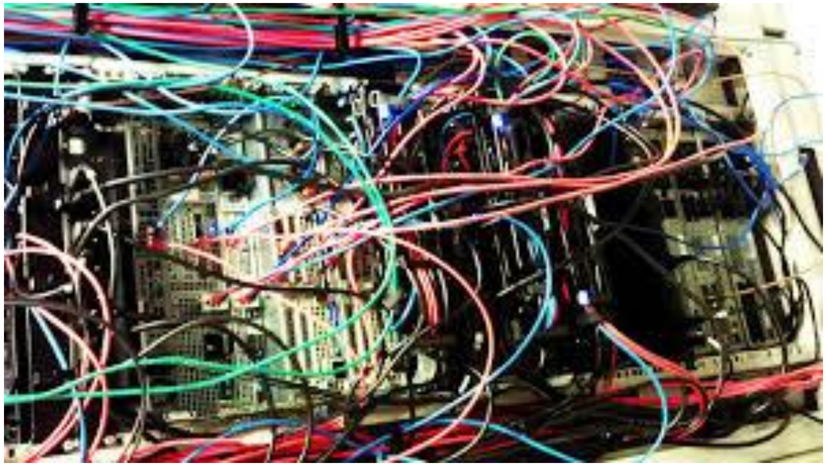
Dealing with Multiple Inputs



Dealing with Multiple Inputs

- When multiple inputs are present in a linear system, the *superposition theorem* is used.

- In other words, each input is treated independently of the others and the resultant output is given by the sum of the outputs due to each input separately.

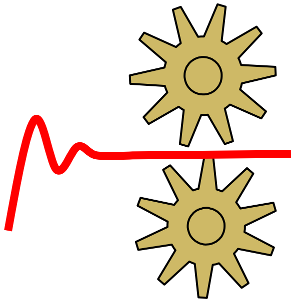


Dealing with Multiple Inputs

- When multiple inputs are present in a linear system, the *superposition theorem* is used.

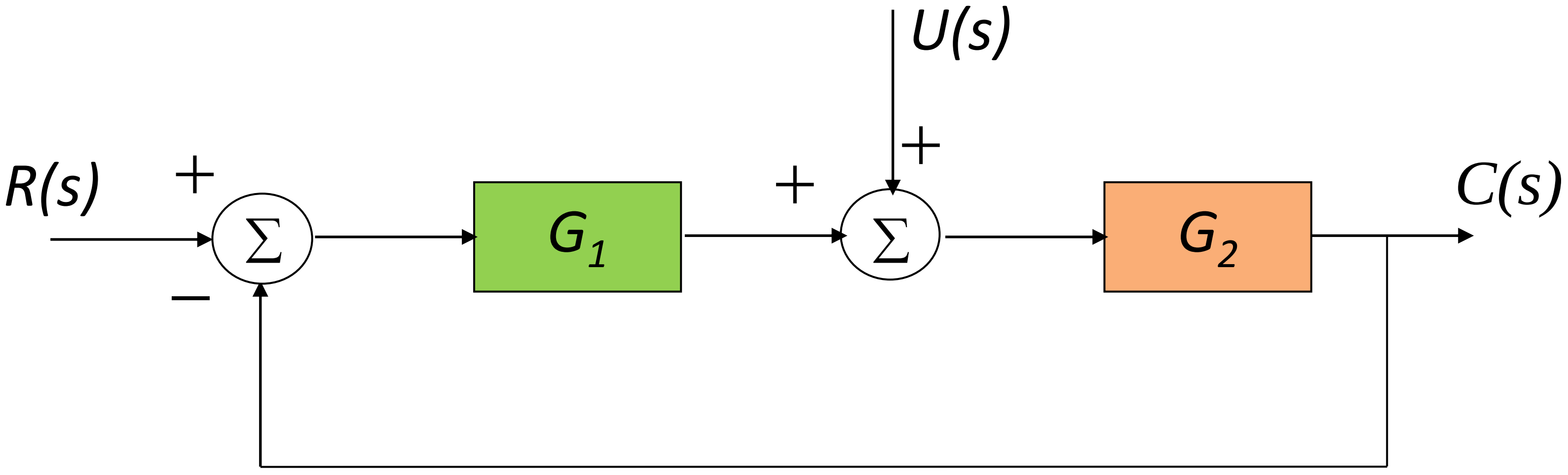
- In other words, each input is treated independently of the others and the resultant output is given by the sum of the outputs due to each input separately.

- The procedure is as follows:
  - Set all inputs to zero, except for one.
  - Calculate the response due to the chosen input acting alone.
  - Repeat the above two steps for all inputs.
  - Algebraically add all of the responses (outputs) determined.

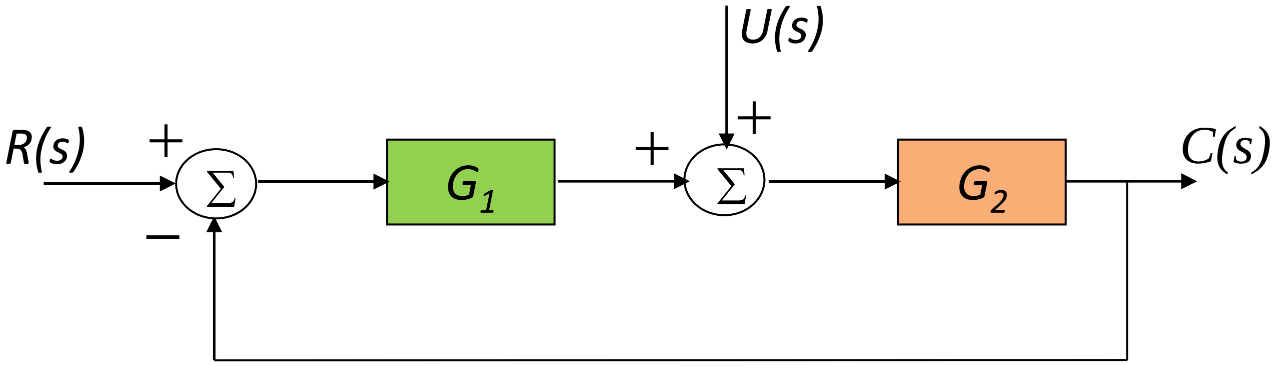


Dealing with Multiple Inputs

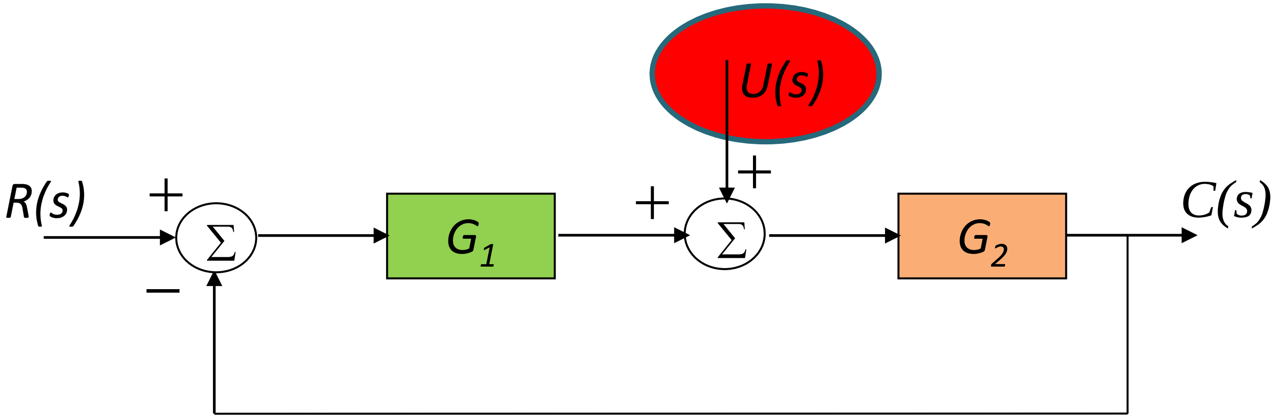
- ***Ex 5.5 Simplify the following system:***



Solution:

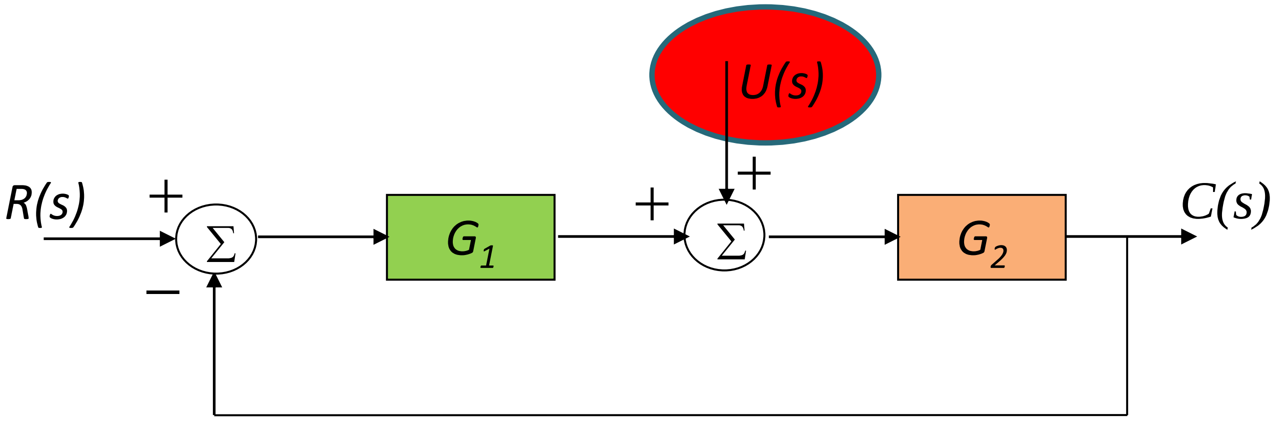


Solution:

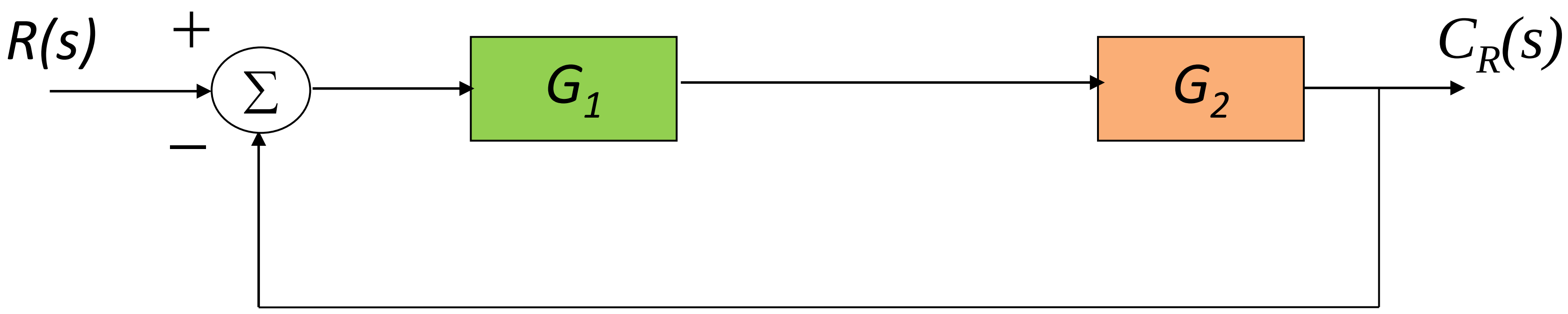


Firstly, set $U(s) = 0$. Hence ...

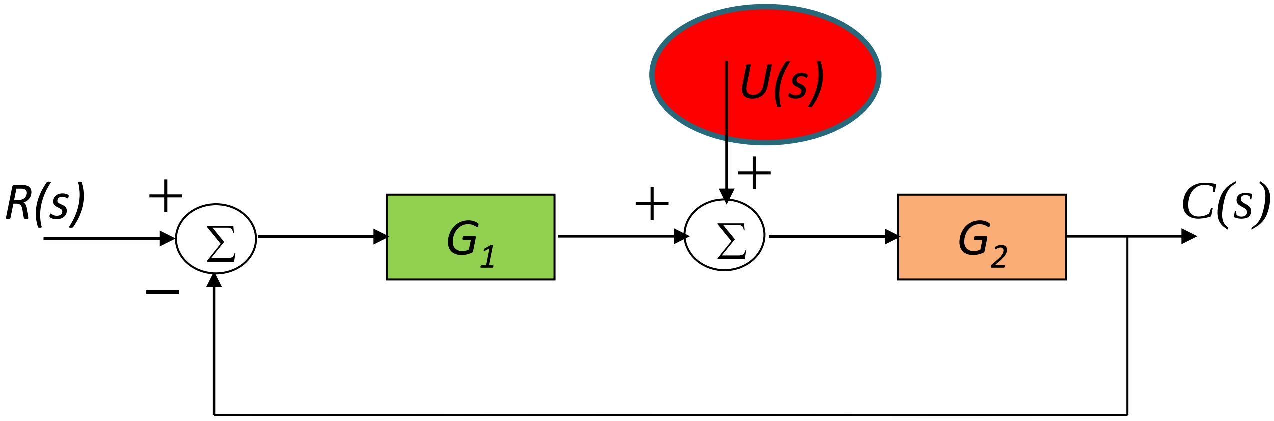
Solution:



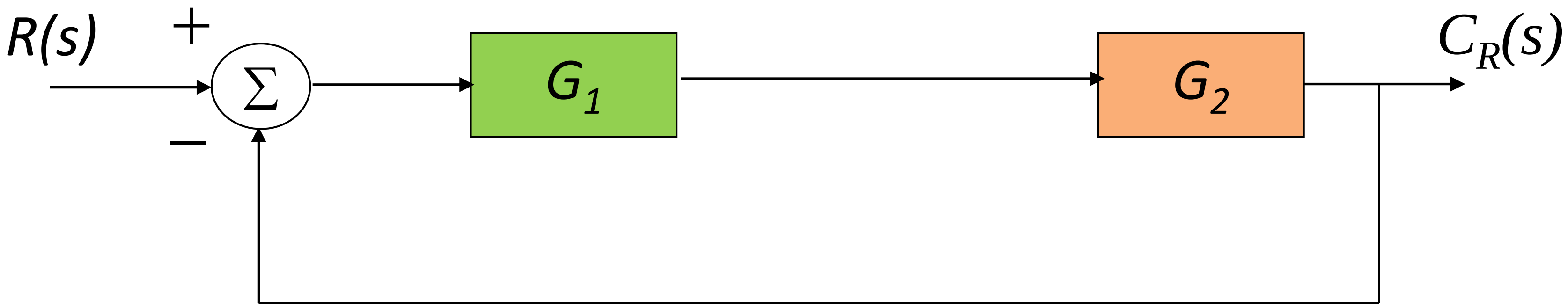
Firstly, set $U(s) = 0$. Hence ...



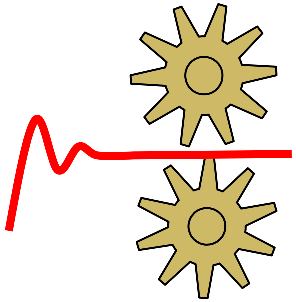
Solution:



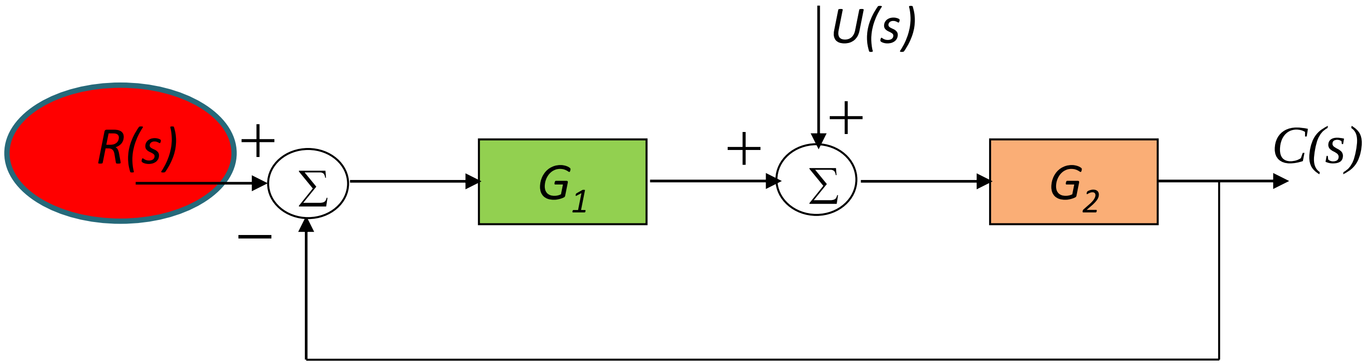
Firstly, set $U(s) = 0$. Hence ...



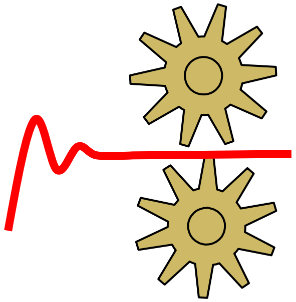
$$C_R(s) = \frac{G_1 G_2}{1 + G_1 G_2} R$$



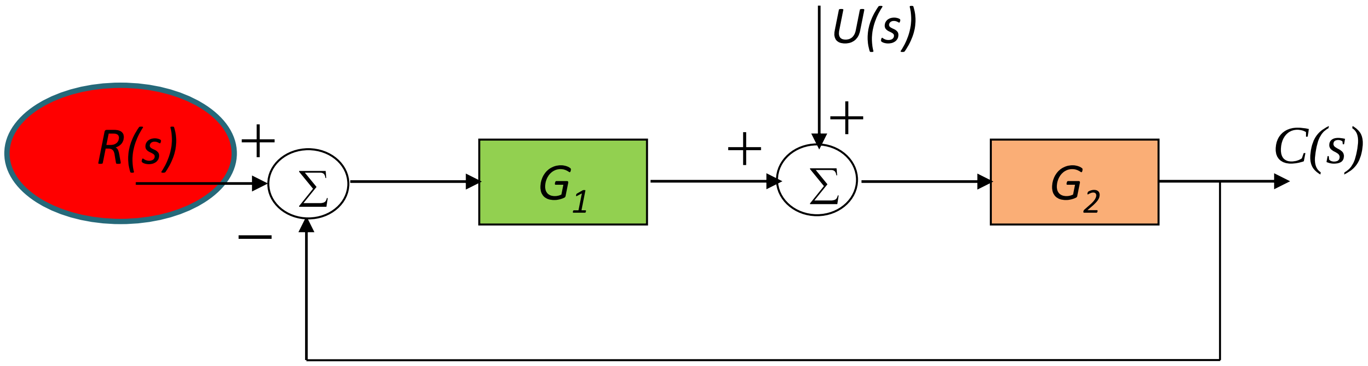
Solution:



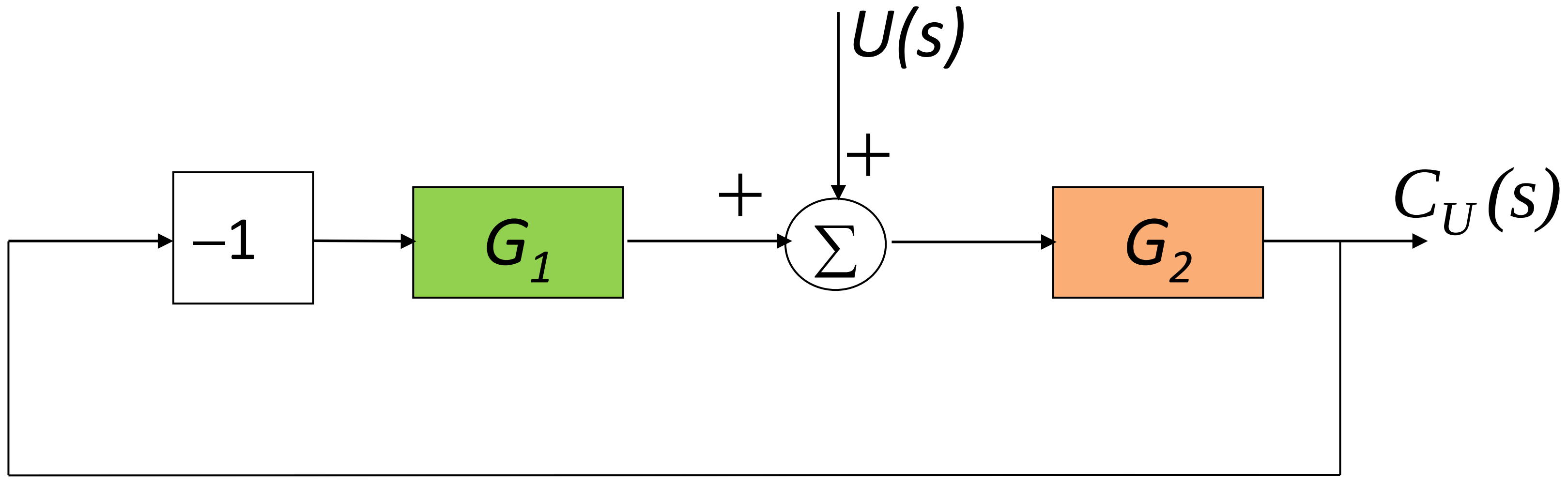
Next, set R(s) = 0. Hence ...



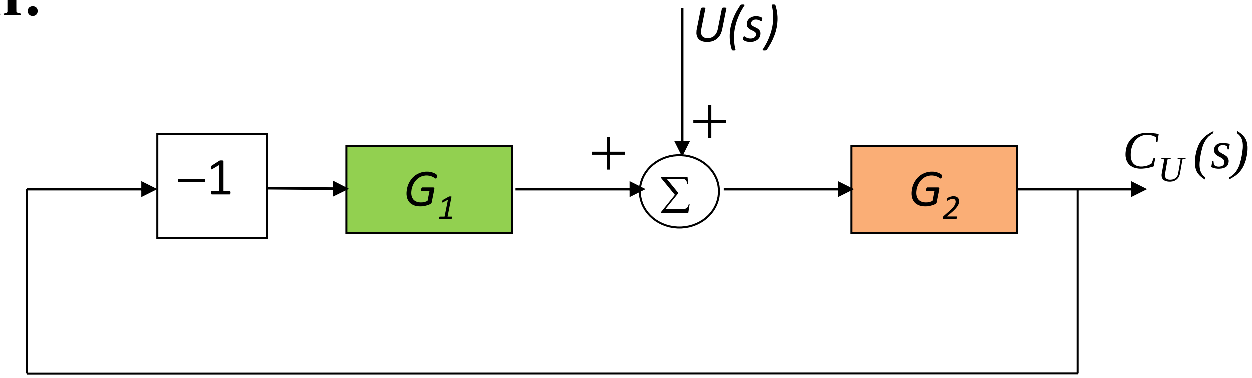
Solution:



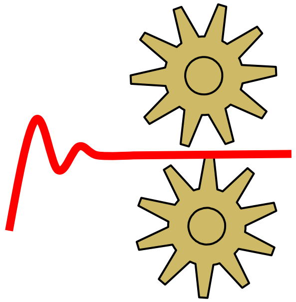
Next, set R(s) = 0. Hence ...



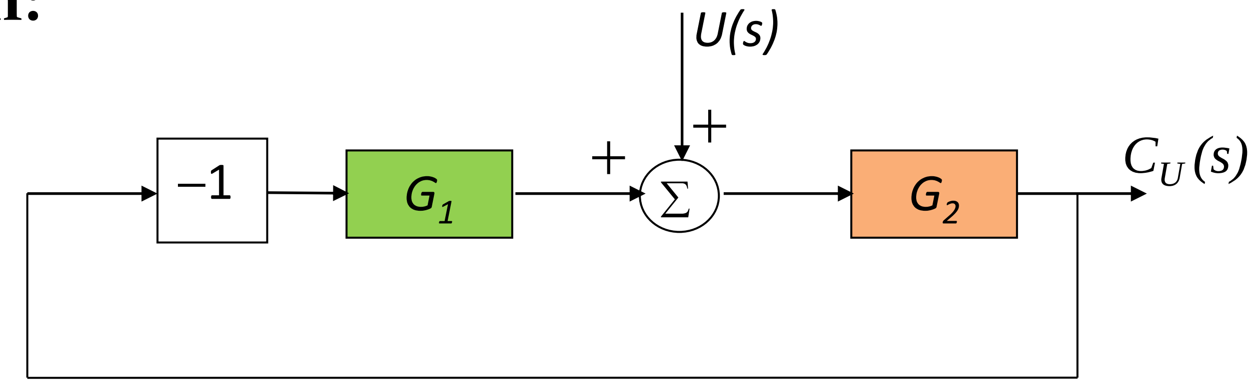
Solution:



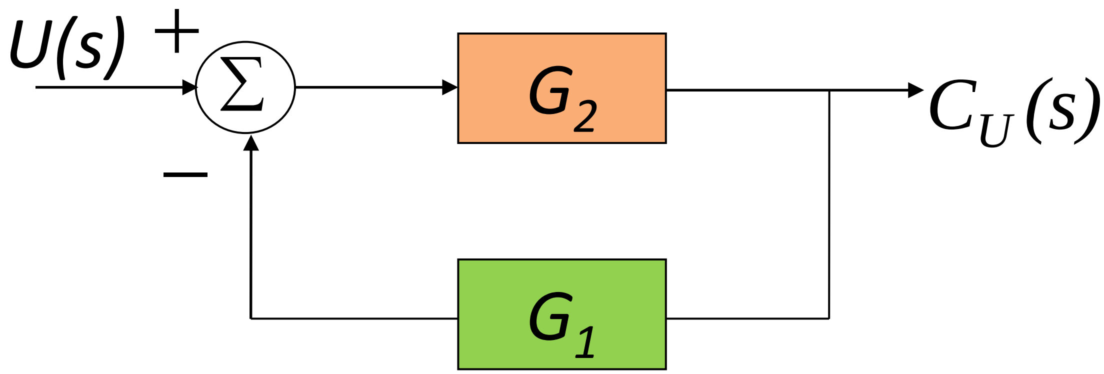
This can be redrawn as ...



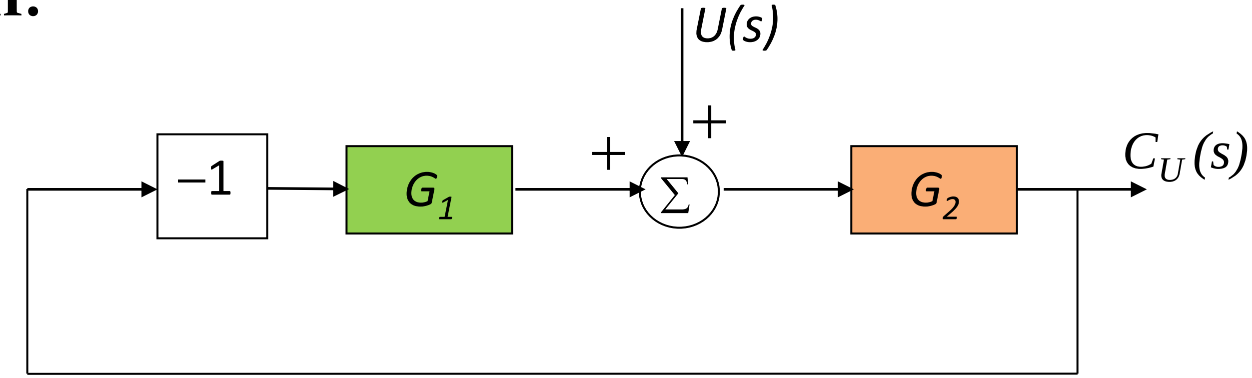
Solution:



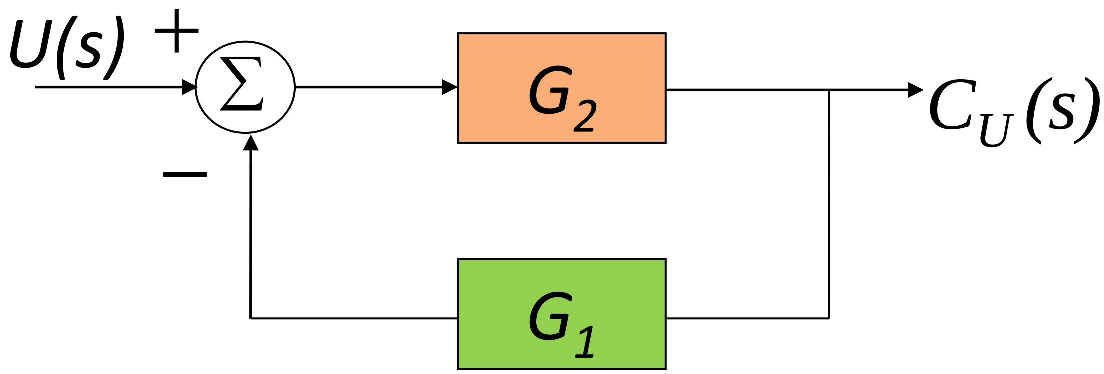
This can be redrawn as ...



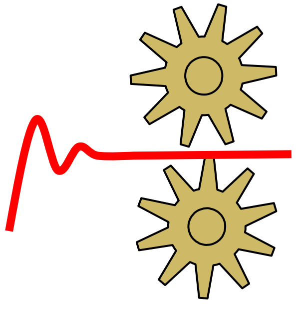
Solution:



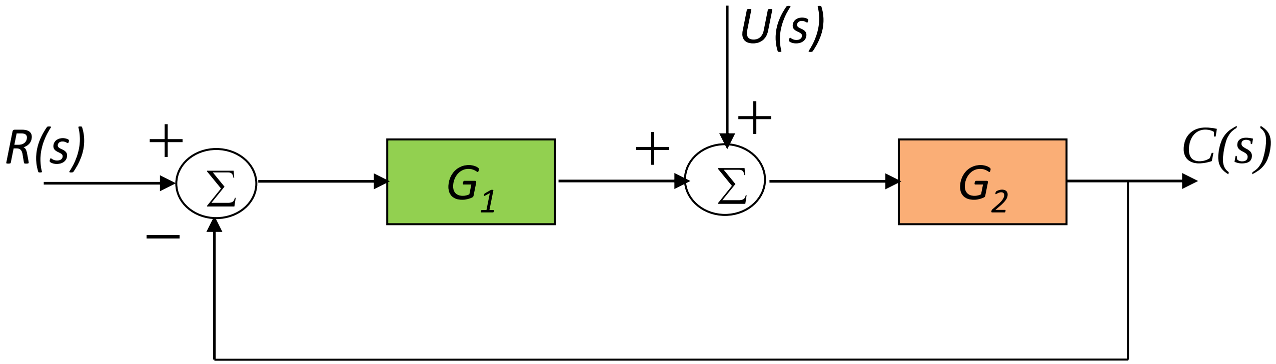
This can be redrawn as …



$$C_U(s) = \frac{G_2}{1 + G_1 G_2} U$$

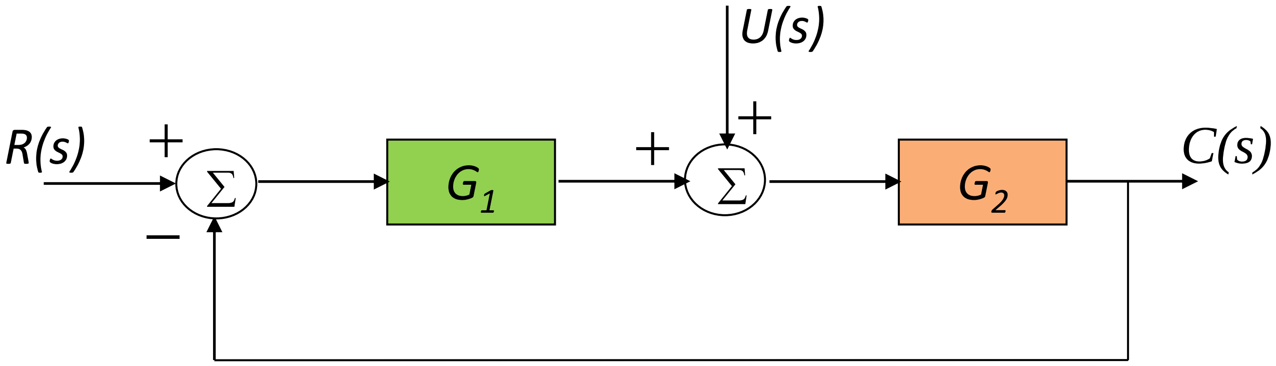


Solution:



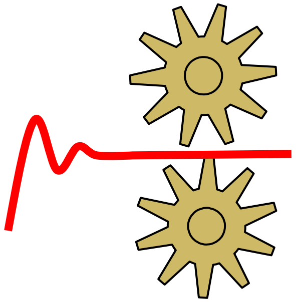
Hence ...

Solution:

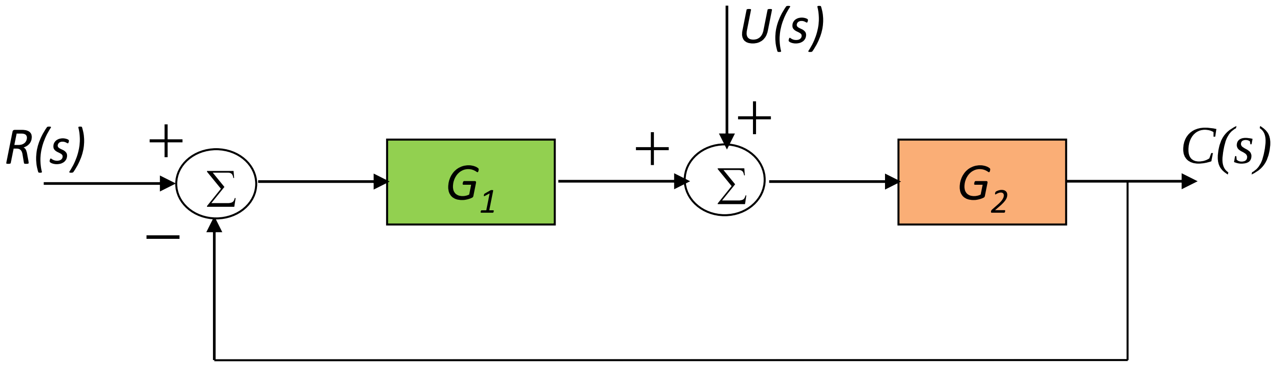


Hence ...

$$C(s) = C_R(s) + C_U(s) = \frac{G_1G_2}{1+G_1G_2}R + \frac{G_2}{1+G_1G_2}U$$



Solution:



Hence ...

$$C(s) = C_R(s) + C_U(s) = \frac{G_1 G_2}{1 + G_1 G_2} R + \frac{G_2}{1 + G_1 G_2} U$$

$$= \frac{G_1 G_2 R(s) + G_2 U(s)}{1 + G_1 G_2}$$

Block Diagram Reduction

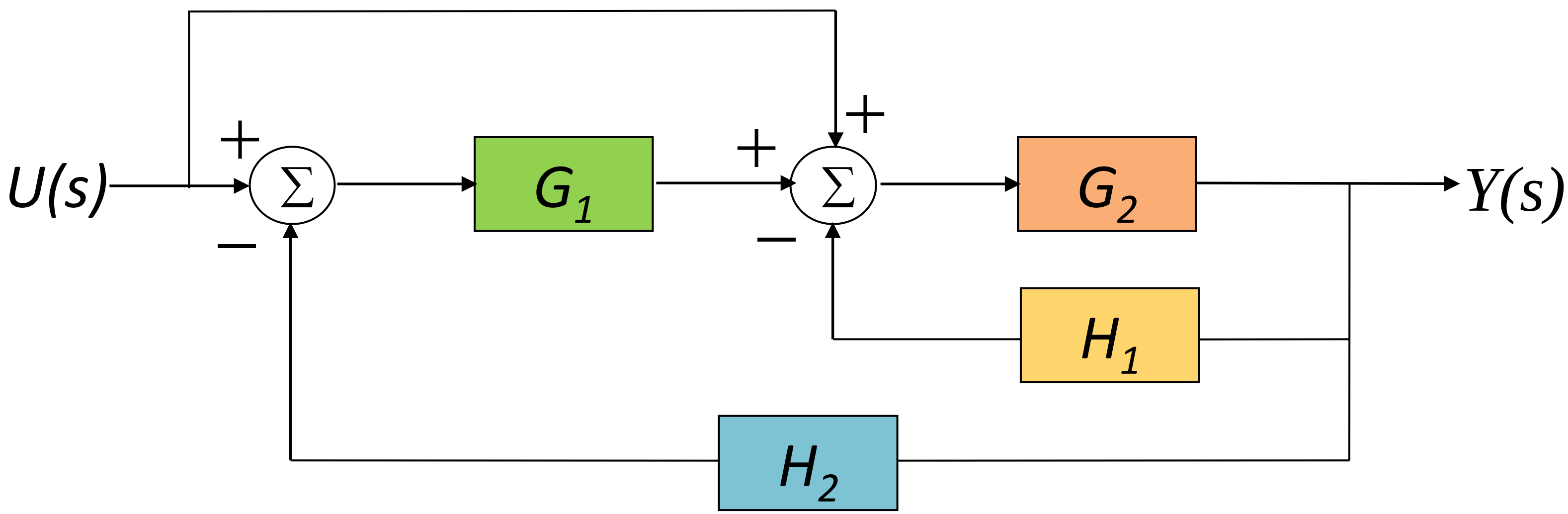
Block Diagram Reduction

- The procedure for block diagram reduction is:

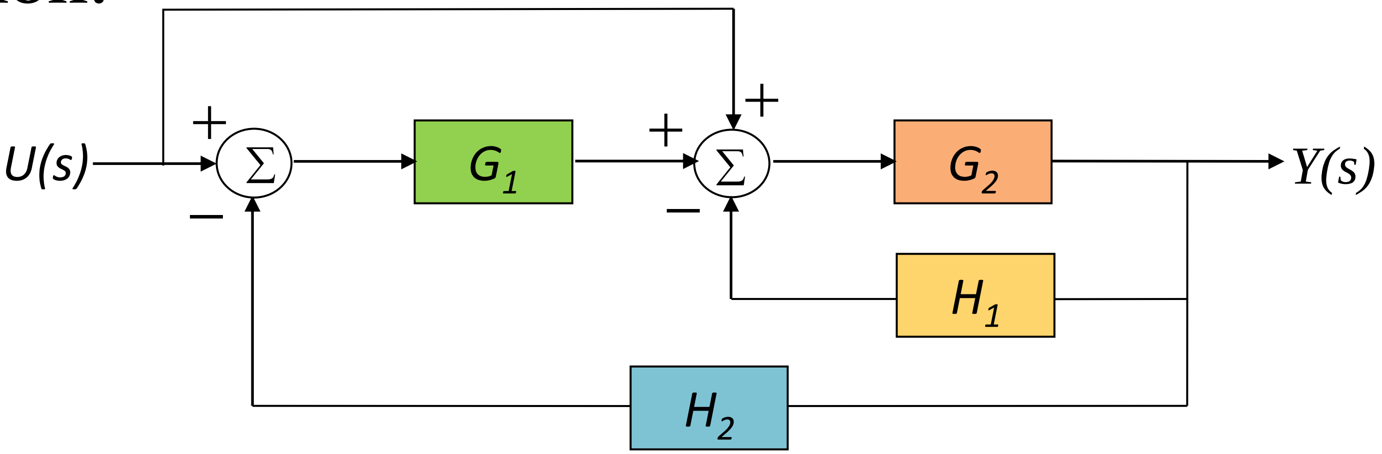
  - Assign intermediate variables to signals between blocks.
  - Write a complete set of equations for the system.
  - Eliminate the intermediate variables.
  - Where possible, use standard results (series, parallel, feedback) to simplify analysis.

Block Diagram Reduction

- ***Ex 5.6 Determine the CLTF for the system represented by the following block diagram:***

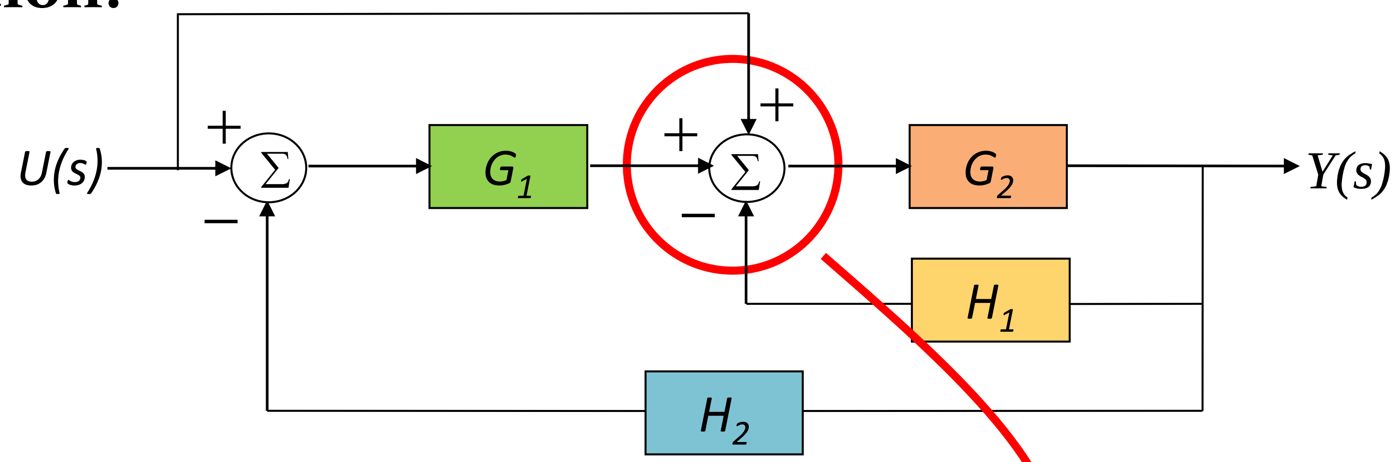


Solution:

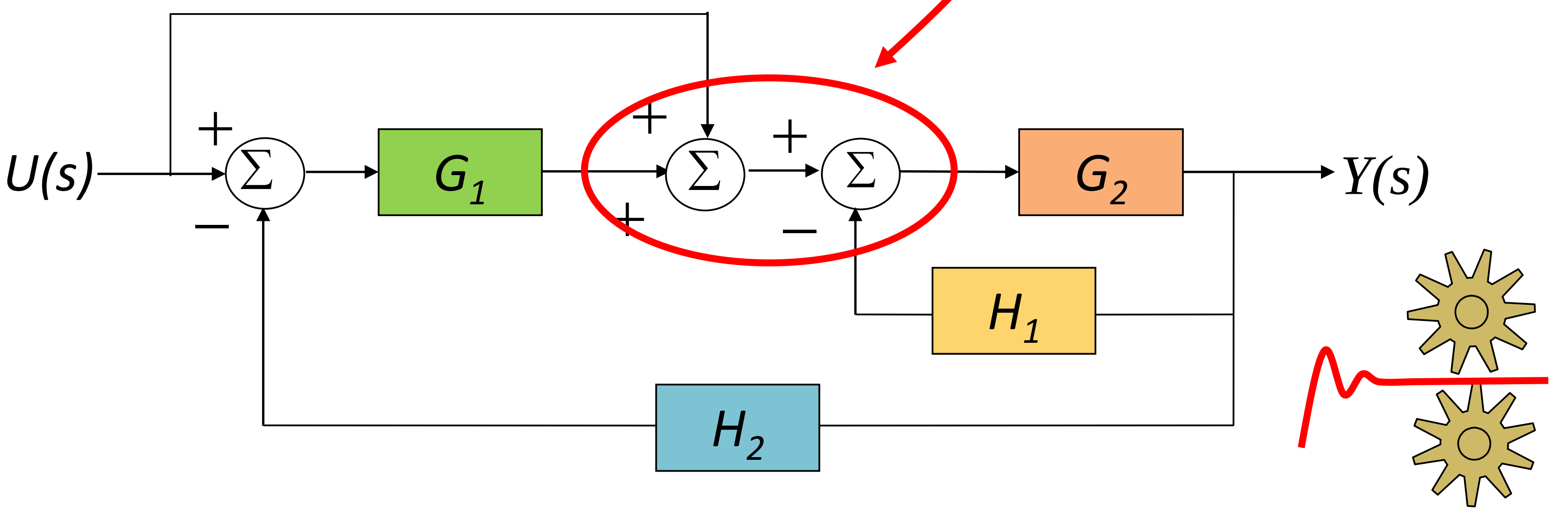


For clarity, we can redraw as follows ...

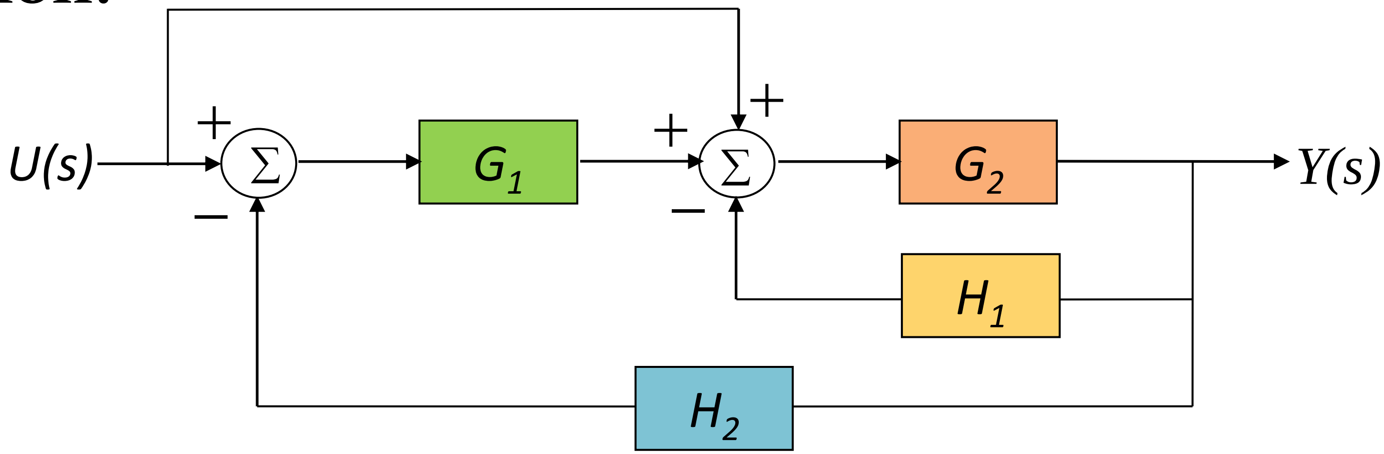
Solution:



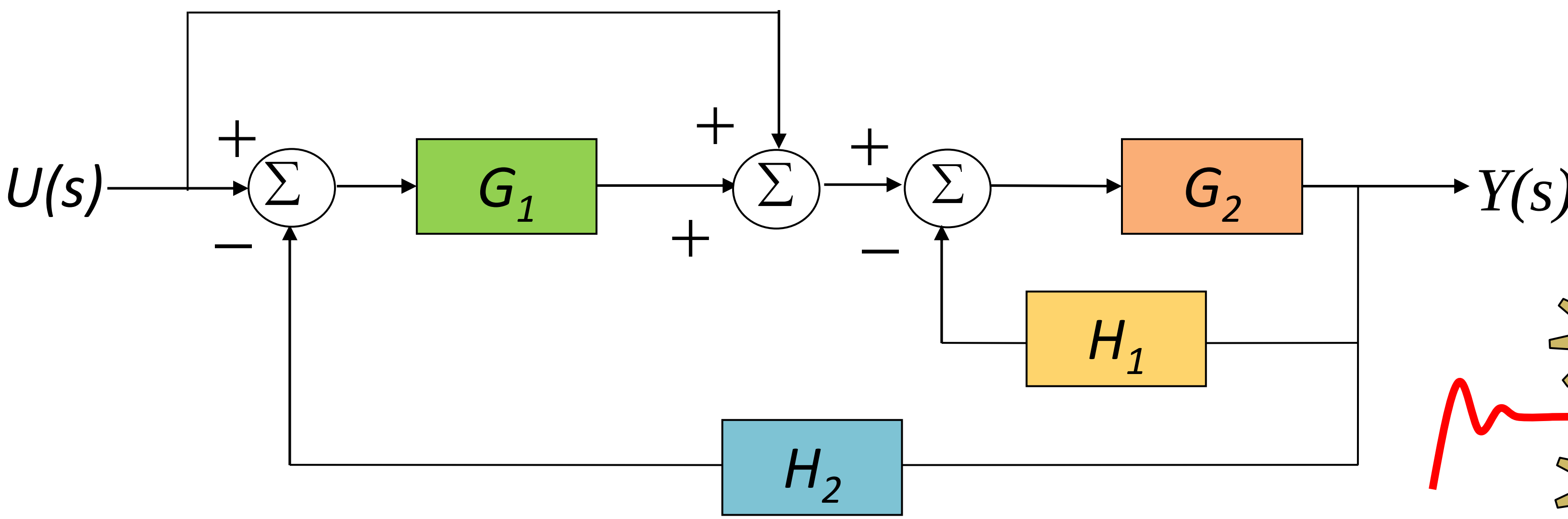
For clarity, we can redraw as follows ...



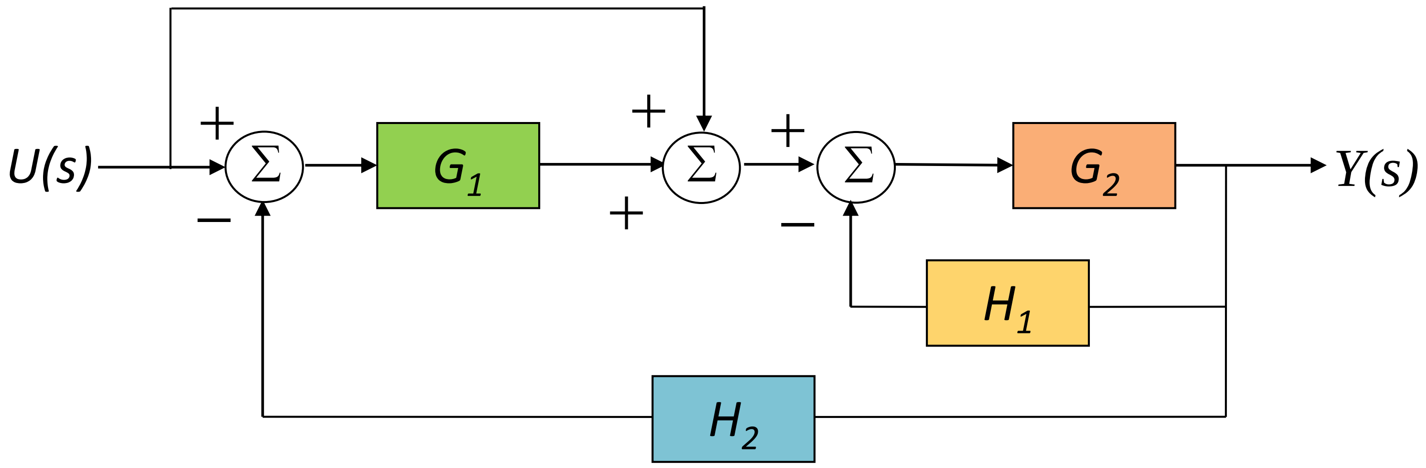
Solution:



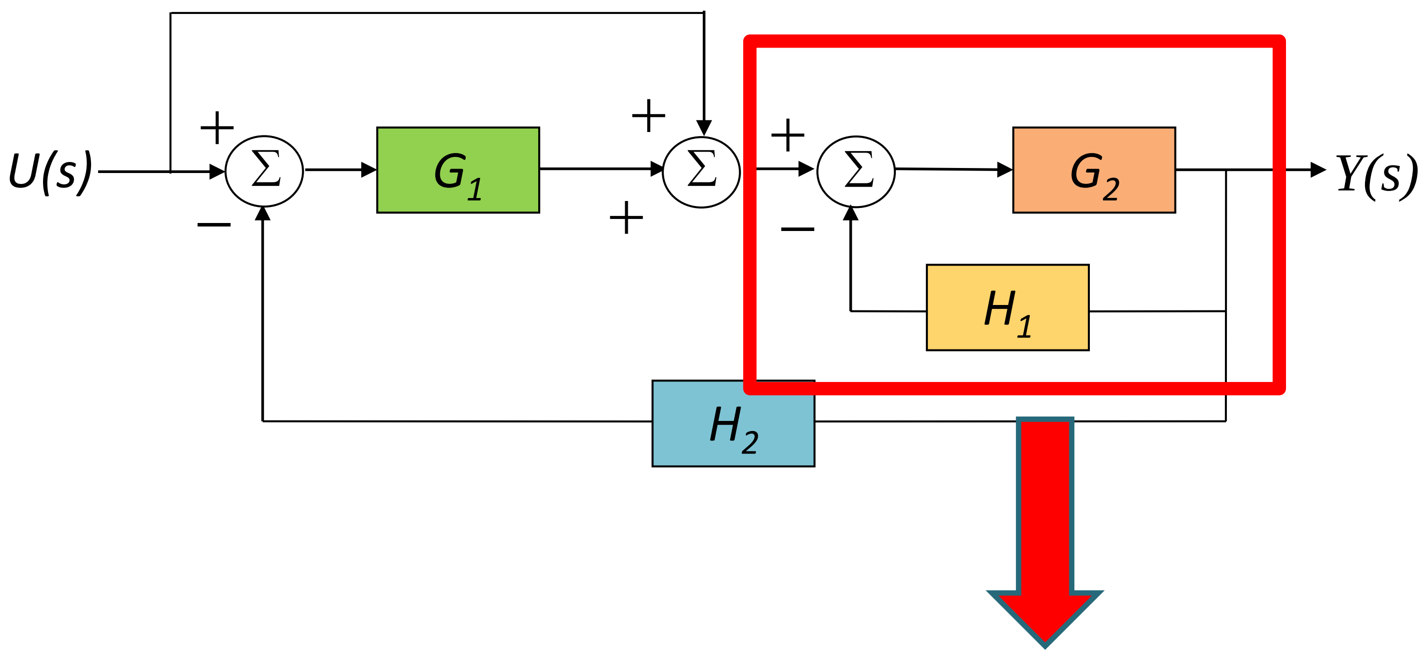
For clarity, we can redraw as follows ...



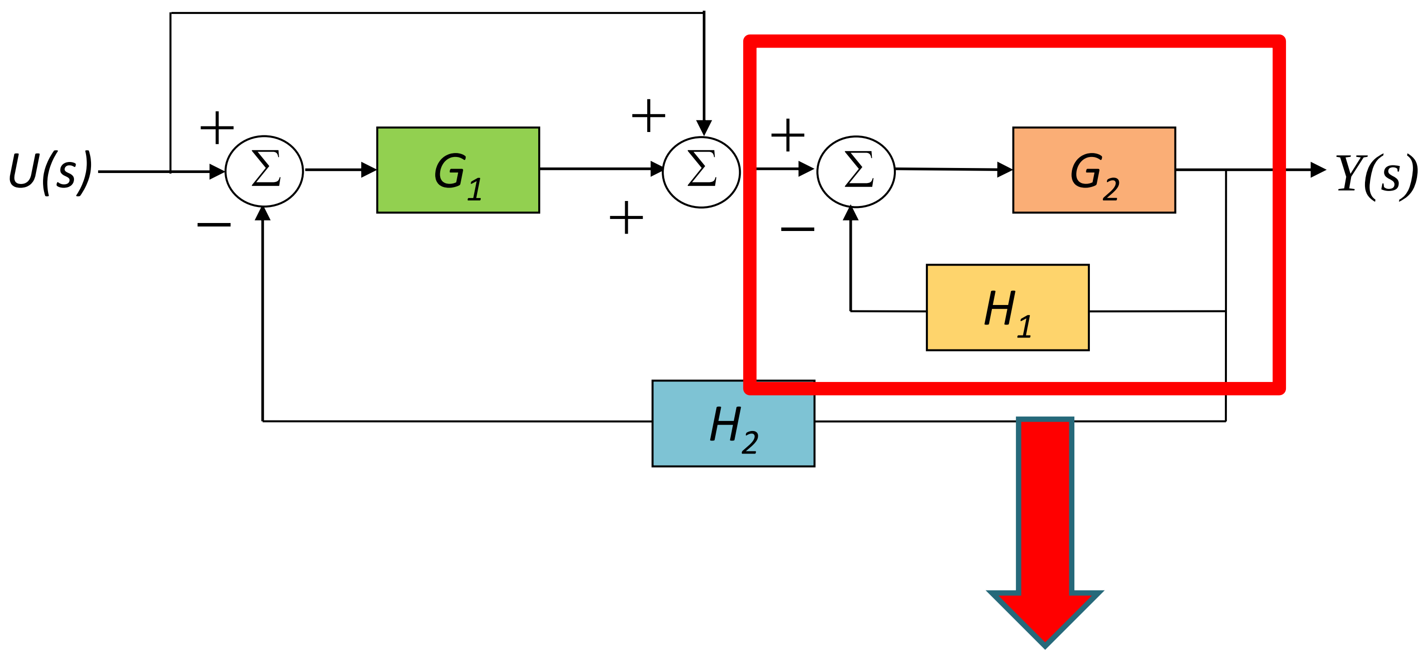
Solution:



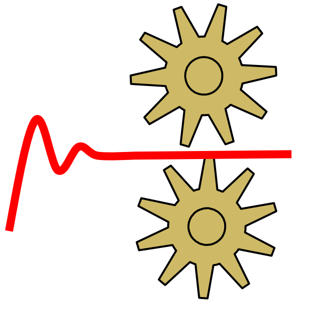
Solution:



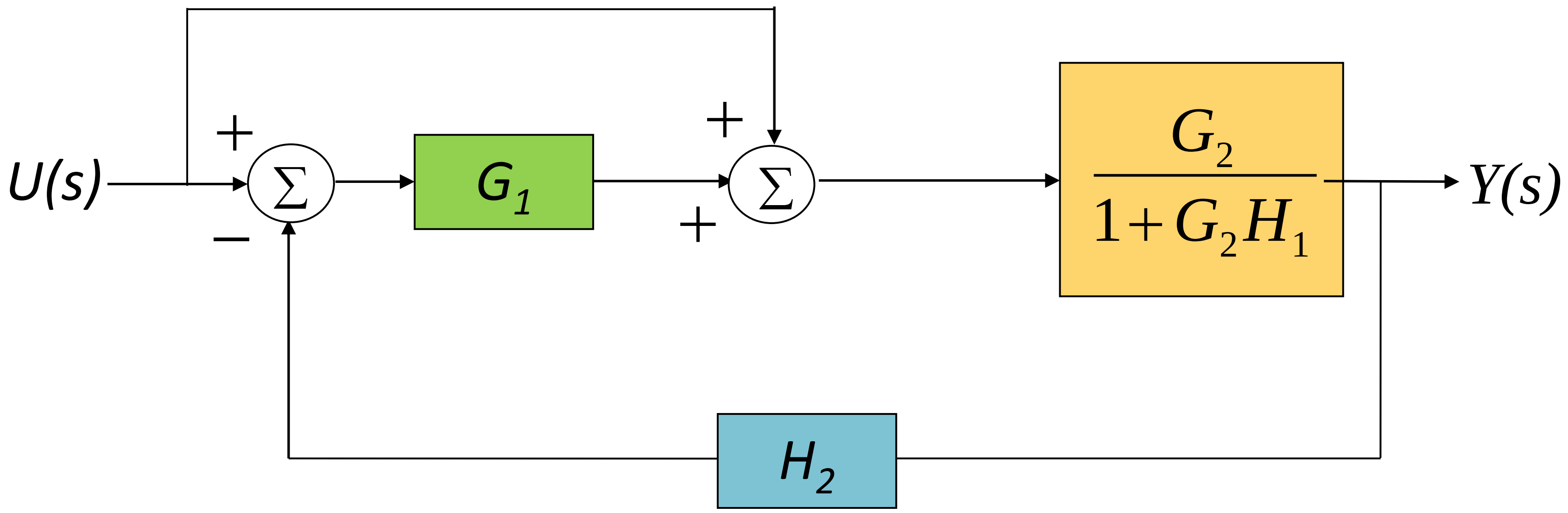
Solution:



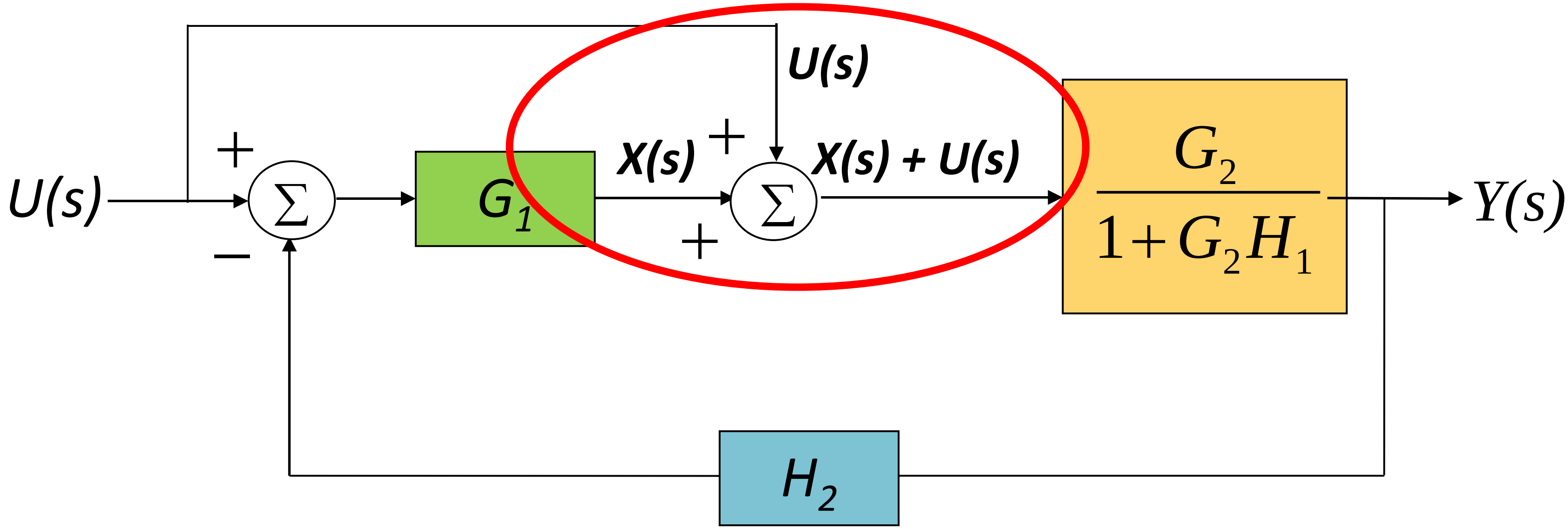
$$\frac{G_2}{1+G_2H_1}$$



Solution:

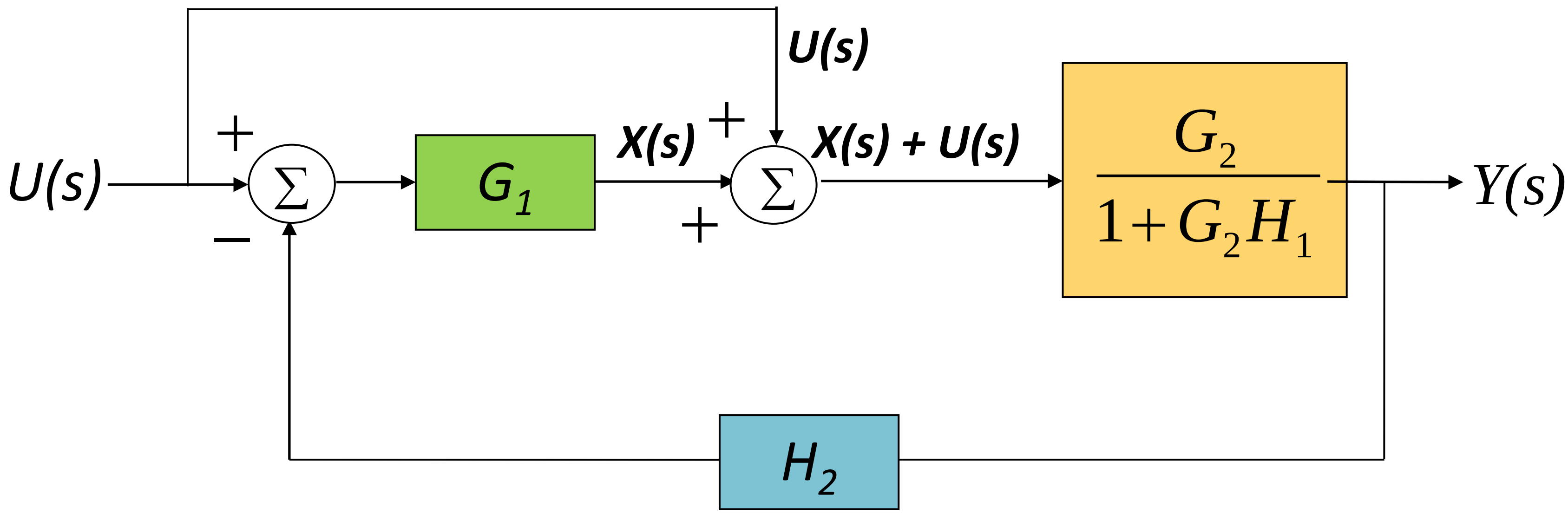


Solution:



Assign the intermediate variable $X(s)$ as shown, hence ...

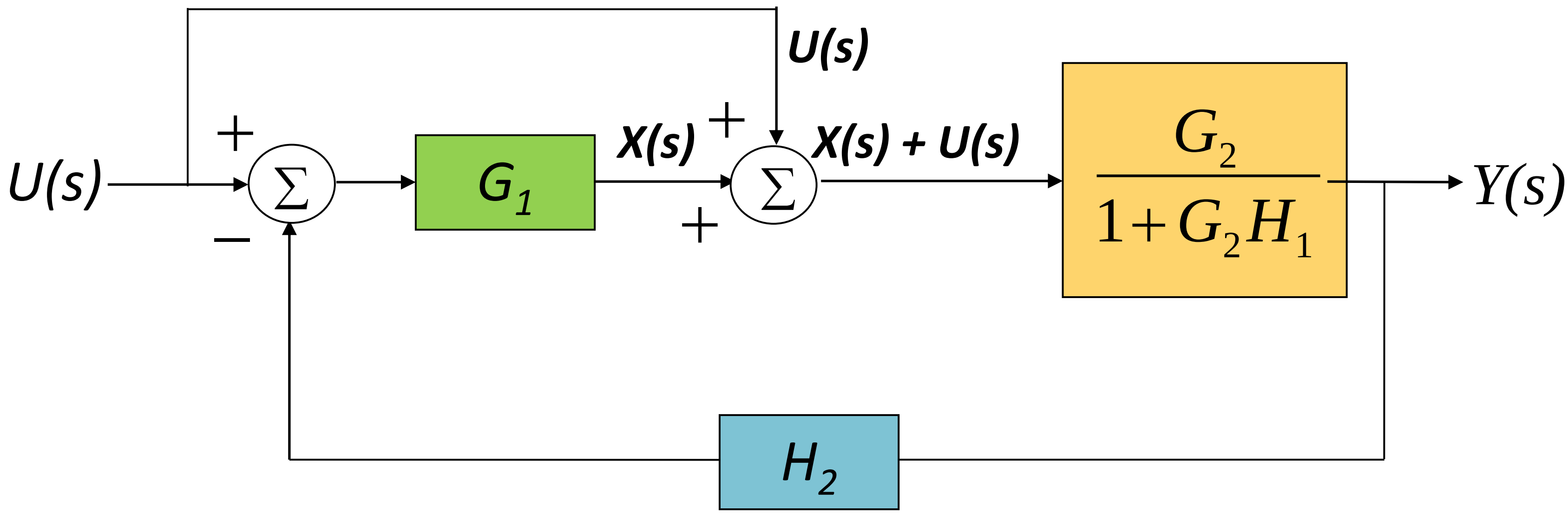
Solution:



Assign the intermediate variable $X(s)$ as shown, hence ...

$$Y(s) = \frac{G_2}{1 + G_2 H_1}\left(X(s) + U(s)\right)$$

Solution:



Assign the intermediate variable $X(s)$ as shown, hence ...

$$Y(s) = \frac{G_2}{1 + G_2 H_1}\left(X(s) + U(s)\right)$$

$$X(s) = G_1\left(U(s) - H_2 Y(s)\right)$$

Solution:

$$Y(s)=\frac{G_2}{1+G_2H_1}\big(X(s)+U(s)\big) \qquad X(s)=G_1\big(U(s)-H_2Y(s)\big)$$

Solution:

$$Y(s) = \frac{G_2}{1+G_2H_1}\left(X(s)+U(s)\right) \qquad \underline{X(s) = G_1\left(U(s) - H_2Y(s)\right)}$$

Need to eliminate variable $X(s)$ as follows ...

Solution:

$$Y(s)=\frac{G_2}{1+G_2H_1}\left(X(s)+U(s)\right) \qquad X(s)=G_1\left(U(s)-H_2Y(s)\right)$$

Need to eliminate variable $X(s)$ as follows ...

$$Y(s)=\frac{G_2}{1+G_2H_1}\left(G_1\left(U(s)-H_2Y(s)\right)+U(s)\right)$$

Solution:

$$Y(s)=\frac{G_2}{1+G_2H_1}\big(X(s)+U(s)\big) \qquad X(s)=G_1\big(U(s)-H_2Y(s)\big)$$

Need to eliminate variable $X(s)$ as follows ...

$$Y(s)=\frac{G_2}{1+G_2H_1}\Big(G_1\big(U(s)-H_2Y(s)\big)+U(s)\Big)$$

$$\Rightarrow Y(s)=\frac{G_1G_2}{1+G_2H_1}U(s)-\frac{G_1G_2H_2}{1+G_2H_1}Y(s)+\frac{G_2}{1+G_2H_1}U(s)$$

Solution:

$$Y(s)=\frac{G_1G_2}{1+G_2H_1}U(s)-\frac{G_1G_2H_2}{1+G_2H_1}Y(s)+\frac{G_2}{1+G_2H_1}U(s)$$

Solution:

$$Y(s)=\frac{G_1G_2}{1+G_2H_1}U(s)-\frac{G_1G_2H_2}{1+G_2H_1}Y(s)+\frac{G_2}{1+G_2H_1}U(s)$$

$$\Rightarrow Y(s)\left(1+\frac{G_1G_2H_2}{1+G_2H_1}\right)=U(s)\left(\frac{G_1G_2}{1+G_2H_1}+\frac{G_2}{1+G_2H_1}\right)$$

Solution:

$$Y(s)=\frac{G_1G_2}{1+G_2H_1}U(s)-\frac{G_1G_2H_2}{1+G_2H_1}Y(s)+\frac{G_2}{1+G_2H_1}U(s)$$

$$\Rightarrow Y(s)\left(1+\frac{G_1G_2H_2}{1+G_2H_1}\right)=U(s)\left(\frac{G_1G_2}{1+G_2H_1}+\frac{G_2}{1+G_2H_1}\right)$$

$$\Rightarrow \frac{Y(s)}{U(s)}=\frac{\left(\dfrac{G_1G_2}{1+G_2H_1}+\dfrac{G_2}{1+G_2H_1}\right)}{\left(1+\dfrac{G_1G_2H_2}{1+G_2H_1}\right)}$$

Solution:

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{G_1 G_2 + G_2}{1 + G_2 H_1 + G_1 G_2 H_2} = \text{CLTF}$$

"Look, Bernie, all I'm saying is
I think you're riding the new guy pretty hard."