EE206 Assignment 3 *

Due 15th Oct.

Read Programmes 14-17 (Page 731-820) and answer the following questions

1. If $z = \tan(x^2 - y^2)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$$z = \tan(x^2 - y^2)$$

$$\frac{\partial z}{\partial x} = 2x \cdot \sec^2(x^2 - y^2)$$

$$\frac{\partial z}{\partial y} = -2y \cdot \sec^2(x^2 - y^2)$$

2. If $z = \dfrac{1}{x^2 + y^2 - 1}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -2z(1 + z)$

$z = \frac{1}{x^2+y^2-1}$ $\quad \therefore \frac{\partial z}{\partial x} = -\frac{2x}{(x^2+y^2-1)^2}$, $\frac{\partial z}{\partial y} = -\frac{2y}{(x^2+y^2-1)^2}$

$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -\frac{2x^2+2y^2}{(x^2+y^2-1)^2}$

$$-2z(1+z) = -2\cdot\frac{1}{x^2+y^2-1}\cdot\left(1+\frac{1}{x^2+y^2-1}\right)$$

$$= -\frac{2}{x^2+y^2-1}\cdot\frac{x^2+y^2-1+1}{x^2+y^2-1}$$

$$= -\frac{2x^2+2y^2}{(x^2+y^2-1)^2}$$

$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = -2z(1+z)$

3. If $z = e^x(x\cos y - y\sin y)$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

$z = e^x(x\cos y - y\sin y)$

$\therefore \frac{\partial z}{\partial x} = e^x[(x+1)\cos y - y\sin y]$, $\frac{\partial z}{\partial y} = e^x[-(x+1)\sin y - y\cos y]$

$\therefore \frac{\partial^2 z}{\partial x^2} = e^x[(x+1)\cos y - y\sin y + \cos y] = e^x[(x+2)\cos y - y\sin y]$

$\frac{\partial^2 z}{\partial y^2} = e^x[-(x+1)\cos y - \cos y + y\sin y] = e^x[-(x+2)\cos y + y\sin y]$

$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^x[(x+2)\cos y - y\sin y - (x+2)\cos y + y\sin y] = 0$

4. Determine the following

(a) $\int x^2 \ln x dx$ (Hint: integration by parts)

(b) $\int \frac{x+1}{x^2-3x+2} dx$ (Hint: integration by partial fractions)

(c) $\int \cos^4 x dx$ (Hint: Integration of trigonometric functions)

(d) $\int \frac{dZ}{Z^2+A^2}$

(e) $\int \frac{dZ}{\sqrt{Z^2+A^2}}$

4. (a). $\int x^2 \ln x \, dx$

$= \frac{1}{3}\int \ln x \, dx^3$

$= \frac{1}{3}[x^3 \cdot \ln x] - \frac{1}{3}\int x^2 dx$

$= \frac{x^3}{3} \cdot \ln x - \frac{1}{9}x^3 + C$ (C is any constant)

(b). $\int \frac{x+1}{x^2-3x+2} dx$

$= -2\int \frac{1}{x-1} dx + 3\int \frac{1}{x-2} dx$

$= -2\ln|x-1| + 3\ln|x-2| + C$

(c) $\int \cos^4 x \, dx$

$= \frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x \, dx$

$= \frac{1}{8}\int \cos 4x + 4\cos 2x + 3 \, dx$

$= \frac{1}{32}(\sin 4x + 8\sin 2x + 12x) + C$

(d) $\int \frac{dz}{z^2+A^2}$

Let $\tan\theta = \frac{z}{A} \Rightarrow z = A\tan\theta$

$\therefore \frac{dz}{d\theta} = A(1+\tan^2\theta)$

$\therefore \int \frac{A(1+\tan^2\theta)}{A^2\tan^2\theta + A^2} = \frac{1}{A}\tan^{-1}\left(\frac{z}{A}\right) + C$

(e) $\int \frac{dz}{\sqrt{z^2+A^2}}$

Let $z = A\sinh\theta$

$\frac{dz}{d\theta} = A\cosh\theta$

$\sqrt{z^2+A^2} = \sqrt{A^2\sinh^2\theta + A^2} = A\cosh\theta$

$\therefore \int \frac{A\cosh\theta \, d\theta}{A\cosh\theta} = \int d\theta = \theta + C = \sin^{-1}\left(\frac{z}{A}\right) + C$

Sinh^(−1)