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1. (a) $\mathcal{L}\{t^n\} = \int_0^{\infty} t^n \cdot e^{-st} dt$

$= -\frac{1}{s} e^{-st} \cdot t^n - \int_0^{\infty} n \cdot t^{n-1} \cdot (-\frac{1}{s} \cdot e^{-st}) dt$

$= -\frac{1}{s} t^n \cdot e^{-st} \Big|_{t=0}^{\infty} + \frac{n}{s} \int_0^{\infty} t^{n-1} \cdot e^{-st} dt$

$= -\frac{1}{s} t^n \cdot e^{-st} \Big|_0^{\infty} + \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$= \lim_{t \to \infty} (-\frac{1}{s} t^n \cdot e^{-st}) + \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$\because \lim_{t \to \infty} (-\frac{1}{s} t^n \cdot e^{-st}) = 0$

$\therefore \mathcal{L}\{t^n\} = \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$\mathcal{L}\{1\} = \int_0^{\infty} 1 \cdot e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_0^{\infty} = \lim_{t \to \infty} -\frac{1}{s} e^{-st} + \frac{1}{s} = \frac{1}{s}$

$\therefore \mathcal{L}\{t^n\} = \frac{n}{s} \cdot \frac{n-1}{s} \cdot \cdots \cdot \frac{1}{s} \cdot \mathcal{L}\{t\}$

$= \frac{n!}{s^n} \cdot \mathcal{L}\{1\} = \frac{n!}{s^{n+1}}$

(b). $F(s) = \int_0^{\infty} (2\sinh 3t + \cos 2t) e^{-st} dt$

$= 2\int_0^{\infty} \sinh 3t \cdot e^{-st} dt + \int_0^{\infty} \cos 2t \cdot e^{-st} dt$

We let $IF_1 = \int_0^{\infty} \sinh 3t \cdot e^{-st} dt$

$= \frac{1}{3} \cosh 3t \cdot e^{-st} \Big|_0^{\infty} - \int_0^{\infty} -s e^{-st} \cdot \frac{1}{3} \cosh 3t \, dt$

$= -\frac{1}{3} + \frac{s}{3} \int_0^{\infty} \cosh 3t \cdot e^{-st} dt$

$\int_0^{\infty} \cosh 3t \cdot e^{-st} dt = \frac{1}{3} \sinh 3t \cdot e^{-st} \Big|_0^{\infty} - \int_0^{\infty} -s e^{-st} \cdot \frac{1}{3} \sinh 3t \, dt$

$= \frac{s}{3} IF_1$

$IF_1 = -\frac{1}{3} + \frac{s^2}{9} IF_1 \quad \therefore IF_1 = \frac{3}{s^2-9}$

We let $IF_2 = \int_0^{\infty} \cos 2t \cdot e^{-st} dt = \frac{1}{2} \sin 2t \cdot e^{-st} \Big|_0^{\infty} - \int_0^{\infty} \frac{1}{2} \sin 2t \cdot (-s \cdot e^{-st}) dt$

$= \frac{s}{2} \int_0^{\infty} \sin 2t \cdot e^{-st} dt$

$\int_0^{\infty} \sin 2t \cdot e^{-st} dt = -\frac{1}{2} \cos 2t \cdot e^{-st} \Big|_0^{\infty} - \int_0^{\infty} -\frac{1}{2} \cos 2t \cdot (-s \cdot e^{-st}) dt$

$= \frac{1}{2} - \frac{s}{2} \int_0^{\infty} \cos 2t \cdot e^{-st} dt = \frac{1}{2} - \frac{s}{2} IF_2$

$IF_2 = \frac{s}{4} - \frac{s^2}{4} IF_2 \quad IF_2 = \frac{s}{s^2+4}$

$\therefore F(s) = 2IF_1 + IF_2 = \frac{6}{s^2-9} + \frac{s}{s^2+4}$

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2. (a) $\mathcal{L}^{-1}\left\{\frac{6}{s^2+36s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{6}\left(\frac{1}{s}-\frac{1}{s+36}\right)\right\}$

$= \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+36}\right\} = \frac{1}{6}-\frac{1}{6}e^{-36t}$

(b) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-5)(s-7)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{2}{15}}{s-2}+\frac{-\frac{5}{6}}{s-5}+\frac{\frac{7}{10}}{s-7}\right\}$

$= \frac{2}{15}\mathcal{L}\left\{\frac{1}{s-2}\right\} - \frac{5}{6}\mathcal{L}^{-1}\left\{\frac{1}{s-5}\right\} + \frac{7}{10}\mathcal{L}^{-1}\left\{\frac{1}{s-7}\right\}$

$= \frac{2}{15}e^{2t} - \frac{5}{6}e^{5t} + \frac{7}{10}e^{7t}$

(c) $\mathcal{L}^{-1}\left\{\frac{(s-1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3-3s^2+3s-1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{3}{s^2}+\frac{3}{s^3}-\frac{1}{s^4}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\}$

$= 1-3t+\frac{3}{2}t^2-\frac{1}{6}t^3$

3. (a) $(s^2Y(s)-sy(0)-y'(0)) + 5(sY(s)-y(0)) + 4Y(s) = 0$

$(s^2+5s+4)Y(s) - s - 5 = 0$ $\quad Y(s) = \frac{s+5}{(s+1)(s+4)} = \frac{\frac{4}{3}}{s+1} + \frac{-\frac{1}{3}}{s+4}$

$\therefore y = \mathcal{L}^{-1}\{Y(s)\} = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$

(b) $2(sY(s)-y(0)) - Y(s) = 0$ $\quad (2s-1)Y(s) = 10$ $\quad Y(s) = \frac{10}{2s-1} = \frac{5}{s-\frac{1}{2}}$

$y = \mathcal{L}^{-1}\{Y(s)\} = 5e^{\frac{t}{2}}$

(c) $sY(s) - y(0) - Y(s) = 2\cdot\frac{s}{s^2+36}$ $\quad Y(s) = \frac{2s}{(s^2+36)(s-1)} = \frac{-\frac{2}{37}s+\frac{72}{37}}{s^2+36} + \frac{\frac{2}{37}}{s-1}$

$y = \mathcal{L}^{-1}\{Y(s)\} = -\frac{2}{37}\cos 6t + \frac{12}{37}\sin 6t + \frac{2}{37}e^t$ $\quad = -\frac{2}{37}\frac{s}{s^2+36} + \frac{12}{37}\cdot\frac{6}{s^2+36} + \frac{2}{37}\cdot\frac{1}{s-1}$

(d) $s^2Y(s) - sy(0) - y'(0) - 10(sY(s)-y(0)) + 25Y(s) = 3\frac{1}{s-3}$

$(s-5)^2Y(s) + 1 = \frac{3}{s-3}$ $\quad Y(s) = \frac{6-s}{(s-3)(s-5)^2} = \frac{\frac{1}{2}}{(s-5)^2} - \frac{3}{4}\left(\frac{1}{s-5}-\frac{1}{s-3}\right)$

$\therefore y = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{2}t\cdot e^{5t} - \frac{3}{4}e^{5t} + \frac{3}{4}e^{3t}$

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4. (a) $\mathcal{L}\{\cosh(t)\cos(t)\} = \mathcal{L}\{\frac{e^{t}+e^{-t}}{2}\cdot\cos t\} = \frac{1}{2}\mathcal{L}\{e^{t}\cdot\cos t\} + \frac{1}{2}\{e^{-t}\cdot\cos t\}$

$= \frac{1}{2}F(S-1) + \frac{1}{2}F(S+1)$

$= \underbrace{\frac{1}{2}\cdot\frac{S-1}{(S-1)^2+1} + \frac{1}{2}\frac{S+1}{(S+1)^2+1}}$

(b) $\mathcal{L}^{-1}\{\frac{(S-1)^2}{(S+2)^4}\} = \mathcal{L}^{-1}\{\frac{((S+2)-3)^2}{(S+2)^4}\} = \mathcal{L}^{-1}\{\frac{1}{(S+2)^2} - \frac{6}{(S+2)^3} + \frac{9}{(S+2)^4}\}$

$= \mathcal{L}^{-1}\{\frac{1}{(S+2)^2}\} - 3\mathcal{L}^{-1}\{\frac{2}{(S+2)^3}\} + \frac{3}{2}\mathcal{L}^{-1}\{\frac{6}{(S+2)^4}\}$

$= \mathcal{L}^{-1}\{F(S+2)\} - 3\mathcal{L}^{-1}\{F(S+2)\} + \frac{3}{2}\{F(S+2)\}$

$= \underline{e^{-2t}(t - 3t^2 + \frac{3}{2}t^3)}$

5. (a) $\mathcal{L}\{(3t+1)\mathcal{U}(t-1)\} = \mathcal{L}\{(3(t-1)+4)\mathcal{U}(t-1)\}$

$= 3\mathcal{L}\{(t-1)\mathcal{U}(t-1)\} + 4\mathcal{L}\{\mathcal{U}(t-1)\}$

$= 3e^{-S}\mathcal{L}\{t\} + 4\cdot\frac{e^{-S}}{S}$

$= \underline{3e^{-S}\cdot\frac{1}{S^2} + 4\cdot\frac{e^{-S}}{S} = e^{-S}(\frac{3}{S^2}+\frac{4}{S})}$

(b) $\mathcal{L}\{\cos(4t-8)\mathcal{U}(t-2)\} = \mathcal{L}\{\cos[4(t-2)]\,\mathcal{U}(t-2)\}$

$= e^{-2S}\mathcal{L}\{\cos 4t\} = \underline{e^{-2S}\cdot\frac{S}{S^2+16}}$

(c) $\mathcal{L}^{-1}\{\frac{(1+e^{-S})^2}{S+3}\} = \mathcal{L}^{-1}\{\frac{1+2e^{-S}+e^{-2S}}{S+3}\} = \mathcal{L}^{-1}\{\frac{1}{S+3}\} + 2\mathcal{L}^{-1}\{\frac{1}{S+3}\cdot e^{-S}\} + \mathcal{L}^{-1}\{\frac{1}{S+3}e^{-2S}\}$

$= \underline{e^{-3t} + 2e^{-3(t-1)}\cdot\mathcal{U}(t-1) + e^{-3(t-2)}\cdot\mathcal{U}(t-2)}$