Task1

$$X_n = \frac{1}{2}\int_0^2 x(t)\, e^{-j\pi n t}\, dt$$

$$= \frac{1}{2}\left(\int_0^{0.5} 2t\, e^{-j\pi n t} + \int_{0.5}^{1.5} (2-2t)\, e^{-j\pi n t}\, dt + \int_{1.5}^{2} 2(t-2)\, e^{-j\pi n t}\, dt\right)$$

$$= \int_0^{0.5} t\, e^{-j\pi n t}\, dt + \int_{0.5}^{1.5} e^{-j\pi n t}\, dt - \int_{0.5}^{1.5} t\, e^{-j\pi n t}\, dt + \int_{1.5}^{2} t\, e^{-j\pi n t}\, dt - \int_{1.5}^{2} e^{-j\pi n t}\, dt$$

$$= \left(-2e^{-0.5\pi j n} + 2e^{-1.5\pi j n} - e^{-2\pi j n} + 1\right) \frac{1}{-\pi^2 n^2}$$

$$\because e^{-j\theta} = \cos\theta - j\sin\theta$$

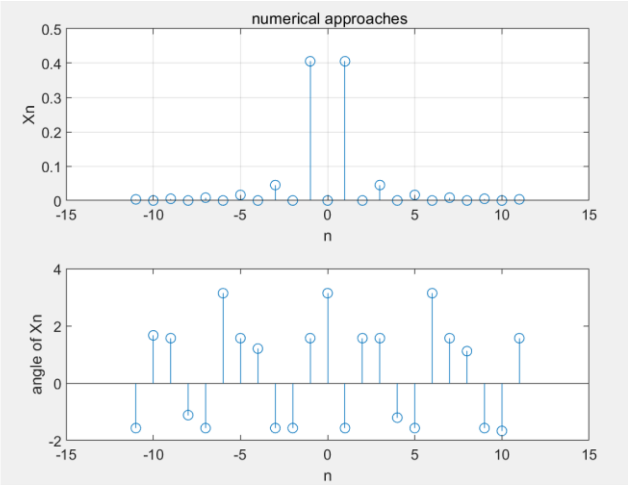
$$= \left[2\left(\cos\frac{\pi}{2}n - \cos\frac{3}{2}\pi n\right) + 2j\left(\sin\frac{3}{2}\pi n - \sin\frac{\pi}{2}n\right)\right] \frac{1}{\pi^2 n^2}$$

$$X_n = \begin{cases} 0 & n = \text{even} \\ j \times (-1)^{\frac{n+1}{2}} \times \frac{4}{\pi^2 n^2} & n \text{ is odd.} \end{cases}$$

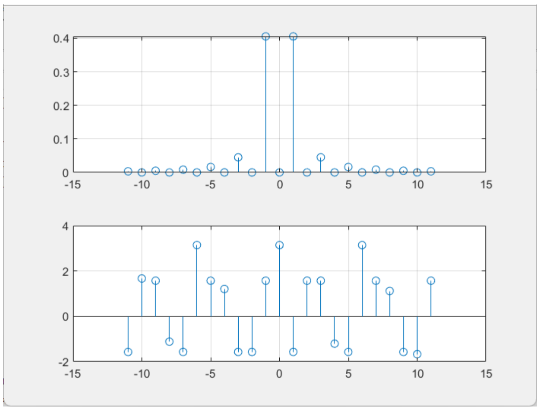
Task2

We could make the following graph by codes

```
N = 11;
  n = -N:N;
  for k = 1:length(n)
          a = (-1)*i*pi*n(k);
        f1 = (@(t)(2*t).*exp(a*t));
        f2 = (@(t)(-2*(t-1)).*exp(a*t));
        f3 = (@(t)(2*(t-2)).*exp(a*t));
Xn(k) = 0.5*quadgk(f1,0,0.5) + 0.5*quadgk(f2,0.5,1.5) + 0.5*quadgk(f3,1.5,2);
x1(k) = conj (x(k));
end
  subplot(211)
stem(n,abs(Xn))
  title(' numerical approaches')
  xlabel('n')
ylabel('Xn')
grid on
subplot(212)
stem(n,angle(x))
  xlabel('n')
  ylabel('angle of Xn')
```



subplot(211)
stem(n,abs(x1))
grid on
subplot(212)
stem(n,angle(x1))
grid on

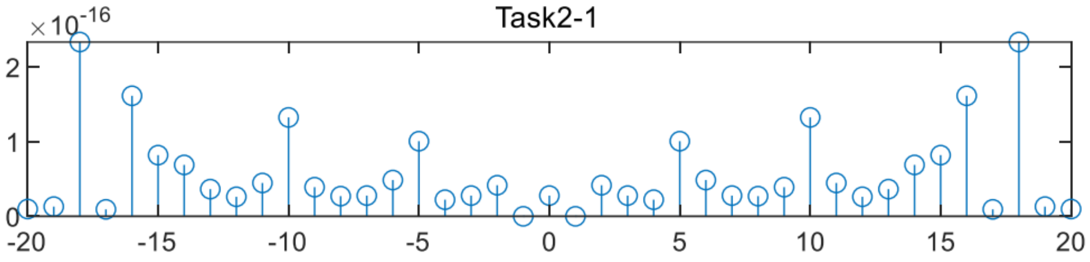


Comment: There is no difference between the two graphs above, so we verify that $X_n = X^*_{-n}$

To compare with mathematically approach I write the another codes

```
N = 11;
n = -N:N;
for k = 1:length(n)
    if (mod(n(k),2)==1)
    Xm(k) = i*(-1)^((n(k)+1)/2)*(4/((pi*n(k))^2));
end
subplot(211)
stem(n,abs(x))
 subplot(211)
stem(n,abs(x))
title(' mathematical approaches')
xlabel('n')
```

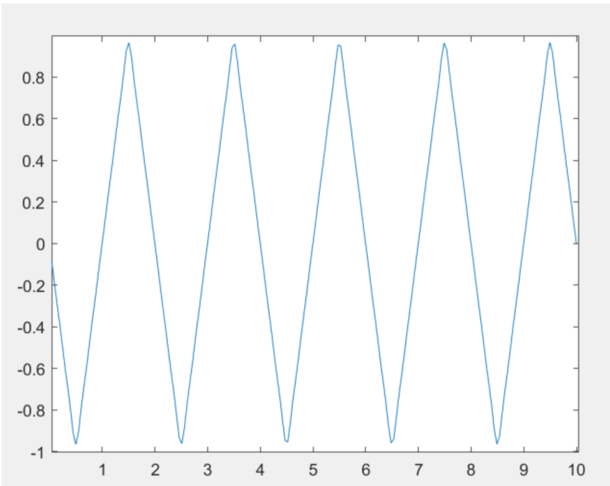
```
ylabel('Xm')
grid on
xlabel('n')
ylabel('angle of Xm')
plot(n,abd(abs(Xm)-abs(Xn)));
```



Comment: We can see that the magnitude of the difference between two approaches is 10^{-16}which is so small that we could ignore it. So, we conclude that the two approaches are same

Task3

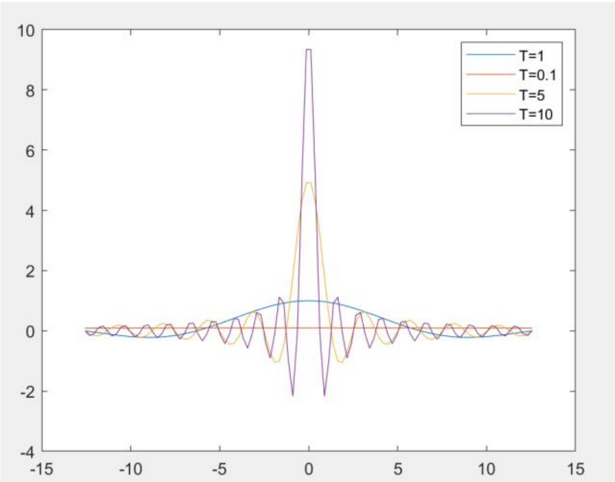
```
t = linspace(0,10,200);
N = 11;
n = -N:N;
x = 0;
for k = 1:length(n)
    a = -i*pi*n(k);
    if (mod(n(k),2)==1)
        temp = i*(-1)^((n(k)+1)/2)*(4/((pi*n(k))^2))*exp(a*t);
        x = x + temp;
end
end
plot(t,x)
```



Tassk4

We could conclude from mathematical approach X=T*sin(T/2*w)

```
freqs = linspace(-4*pi,4*pi,100);
T = 1;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
 T = 0.1;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
T = 5;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
T = 10;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
```

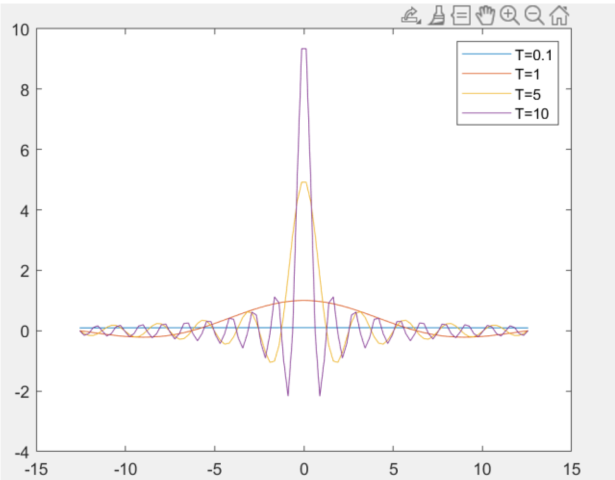


From this graph, as T become larger and larger, the oscillation of function become bigger and the highest point become higher.

Tesk5

```
freqs = linspace(-4*pi,4*pi,100);
T =0.1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on
T =1;
```

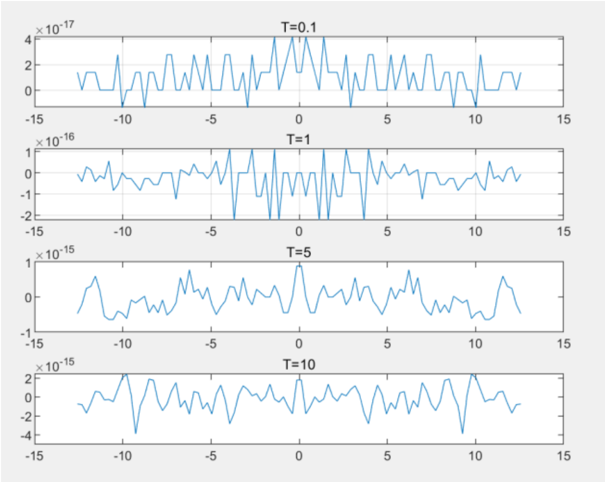
```
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on
T =5;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on
T =10;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on
```



In order to get the difference between two ways, we write the following code

```
freqs = linspace(-4*pi,4*pi,100);
T =0.1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(411)
plot(freqs,diff)
title('T=0.1')
```

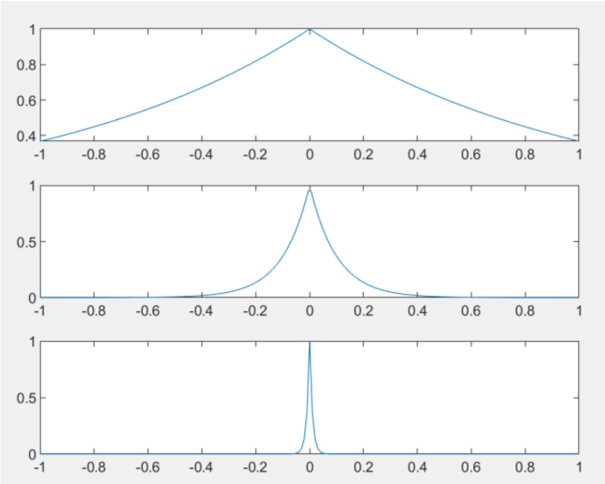
```
  grid on
freqs = linspace(-4*pi,4*pi,100);
T =1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
  subplot(412)
  plot(freqs,diff)
title('T=1')
grid on
freqs = linspace(-4*pi,4*pi,100);
T =5;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(413)
  plot(freqs,diff)
  title('T=5')
freqs = linspace(-4*pi,4*pi,100);
T =10;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(414)
  plot(freqs,diff)
  title('T=10')
```



We can see that the difference between two ways is very small, the magnitude is 10^{-17} - 10^{-15}. So small that we could ignore it and conclude that the two approaches have the same effects.

Task6

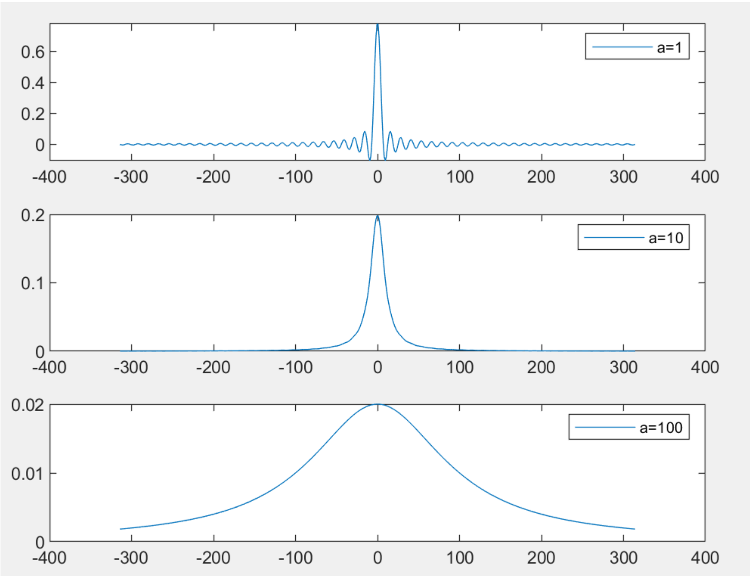
```
a = 1;
t = linspace(-1,1,200);
xa = exp(-a*abs(t));
subplot(311)
plot(t, xa)
a = 10;
t = linspace(-1, 1,200);
xa = exp(-a*abs(t));
subplot(312)
plot(t, xa)
a = 100;
t = linspace(-1, 1,199);
xa = exp(-a*abs(t));
subplot(313)
plot(t, xa)
```



As a become bigger, the x decreases faster, but when t=0, x has the same number 1,

Task7

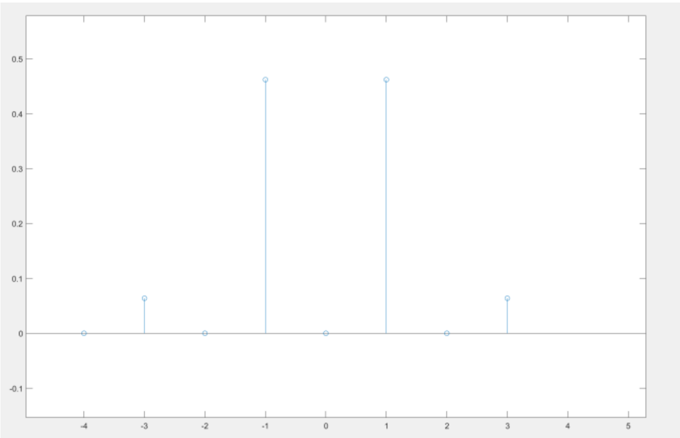
```
N = 8;
n = -4:3;
for k = 1:length(n)
w = pi*n(k)*2;
a = -i*w /N;
f1 = (@(t)(-pi/4).*exp(-a*t));
f2 = (@(t)(pi/4).*exp(-a*t));
x(k) = 1/N*quadgk(f1,-4,-1) + 1/N*quadgk(f2,0,3);
end
stem(n,abs(x))
```



Comment: When a become more and more larger , the FT function of Xa(t) is value become smaller . What's more the function 's slope will be more flat .

Task8

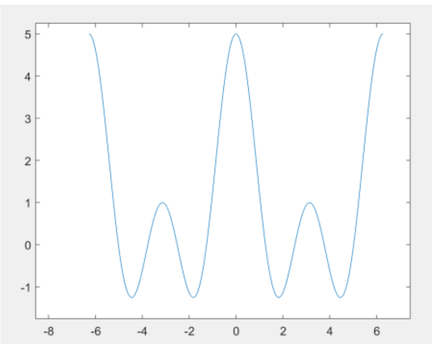
```
N = 8;
n = -4:3;
for k = 1:length(n)
w = pi*n(k)*2;
a = -i*w /N;
f1 = (@(t)(-pi/4).*exp(-a*t));
f2 = (@(t)(pi/4).*exp(-a*t));
x(k) = 1/N*quadgk(f1,-4,-1) + 1/N*quadgk(f2,0,3);
end
stem(n,abs(x))
```



Comment: we could get the same graph like the lecture 6 which we want

Task9

```
fre=linspace(-2*pi,2*pi,200);
n=-2:2;
x=0;
for    k=1:length(n)
x=x+exp(-1*i*fre*n(k));
end
>> plot(fre,x)
```



we got the same graph like lecture that we want