

EE206 Assignment 7 *

Due 26th Nov.

1. Show the given functions are orthogonal on the indicated interval

(a) $f_1(x) = \cos x$, $f_2 = \sin^3 x$; $[0, \pi]$

(b) $f_1(x) = e^x$, $f_2(x) = \sin x$; $[\frac{\pi}{4}, \frac{5\pi}{4}]$

- (c) The set

$$\left\{ \sin\left(\frac{n\pi x}{p}\right); \quad n = \pm 1, 2, 3, \dots \right\}; \quad [0, p]$$

Hint: Integrate $\int_0^p \sin\left(\frac{n\pi x}{p}\right) \sin\left(\frac{m\pi x}{p}\right) dx$

2. Verify by direct integration that the functions are orthogonal with respect to the indicated weight functions on the given interval.

(a) $f_1(x) = 1$, $f_2(x) = \frac{1}{2}x^2 - 2x + 1$; $w(x) = e^{-x}$, $[0, \infty)$

(b) $f_1(x) = 1$, $f_2(x) = -x + 1$; $w(x) = e^{-x}$, $[0, \infty)$

3. Find the Fourier series of $f(x)$ on the given interval

(a) $f_1(x) = \begin{cases} -1 - x/2, & -2 \leq x < 0 \\ 1 - x/2, & 0 < x \leq 2 \end{cases}$

(b) $f_2(x) = \begin{cases} 0, & -1 < x < 0 \\ \frac{e^{-10x} - e^{-10}}{1 - e^{-10}}, & 0 \leq x < 1 \end{cases}$

(c) $f_3(x) = \begin{cases} (x+1)^2, & -1 \leq x \leq 0 \\ (x-1)^2, & 0 \leq x \leq 1 \end{cases}$

(d) $f_4(x) = \begin{cases} 0, & -2 \leq x < 0 \\ x, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$

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