Lecture 4: Basic Operations on Signals

EE213 - Introduction to Signal Processing

Semester 1, 2019

Outline

- Mathematical models of some basic operations on signals
- Their implementation in analogue and digital domains.

Amplitude Scaling

- Continuous signals

$$y(t) = cx(t)$$

- Analogue implementation:

A voltage divider

Inverting Amplifier

Non-inverting Amplifier

- Digital signals

$$y[n] = cx[n]$$

- Digital implementation: floating point operations in digital computing systems.

Analogue Amplitude Scaling

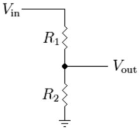


Figure 1: A voltage divider.

$$V_{\text{out}} = \frac{V_{\text{in}}}{R_1 + R_2} \times R_2 = \frac{R_2}{R_1 + R_2} \times V_{\text{in}}$$

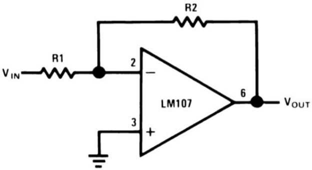


Figure 2: Inverting Amplifier.

$$V_2 = V_3,\ V_3 = 0$$

$$\frac{V_{\text{out}}}{R_2} = -\frac{V_{\text{in}}}{R_1} \Rightarrow V_{\text{out}} = -\frac{R_2}{R_1} V_{\text{in}}$$

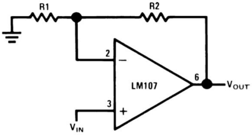


Figure 3: Non-inverting Amplifier.

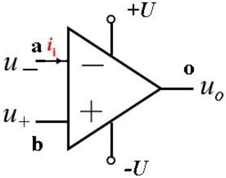
$$V_2 = V_3 = V_{in}$$

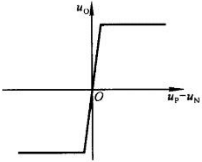
$$\frac{V_{\text{out}} - V_{\text{in}}}{R_2} = \frac{V_{\text{in}}}{R_1} \Rightarrow \frac{V_{\text{out}}}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{\text{in}}$$

$$\Rightarrow V_{\text{out}} = \frac{R_1 + R_2}{R_1} V_{\text{in}}$$

Preliminary about Operational Amplifier

- Operational Amplifier





$u_o = A(u_+ - u_-)$ A is a very big factor

- Visual Short

$$\left.\begin{aligned} u_o &= A_o(u_+ - u_-) \\ u_+ - u_- &= \frac{u_o}{A_o} \approx 0 \\ A_o &\approx \infty \end{aligned}\right\} u_+ \approx u_-$$

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- Visual Open

$$\left.\begin{aligned} i_i &= \frac{u_- - u_+}{r_{id}} \approx 0 \\ r_{id} &\approx \infty \end{aligned}\right\} i_i \approx 0$$

Addition

- Continuous signals

$$y(t) = x_1(t) + x_2(t)$$

- Analogue implementation

Inverting Summing Amplifier

Non-inverting Summing Amplifier

- Digital signals

$$y[n] = x_1[n] + x_2[n]$$

- Digital implementation: Adder

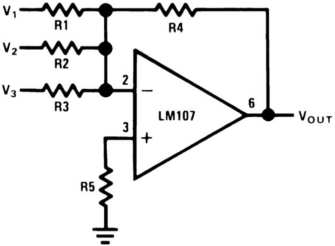


Figure 4: Inverting Summing Amplifier.

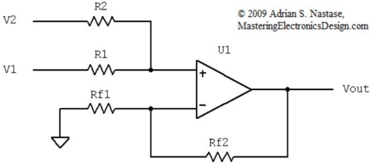


Figure 5: Non-inverting Summing Amplifier.

$$V_2 = V_3 = 0$$

Part1: $V_{out} = -\dfrac{V_1}{R_1} \times R_4$

Part2: $V_{out} = -\dfrac{V_2}{R_2} \times R_4$

Part3: $V_{out} = -\dfrac{V_3}{R_3} \times R_4$

SUM: $V_{\text{out}} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$

$$V_{\text{out}} = \left(1 + \frac{Rf_2}{Rf_1}\right) \left(\frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2 \right)$$

Multiplication: Digital Signals

- Digital signals

$$y[n] = x_1[n]x_2[n]$$

- Digital implementation: floating-point algorithms in digital processors.

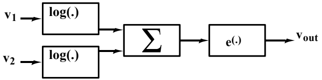
Multiplication

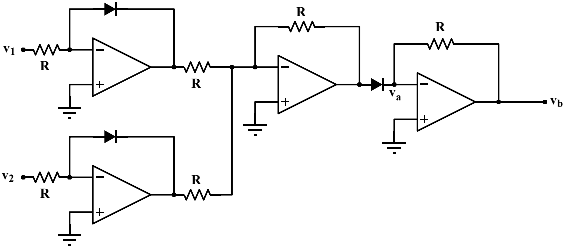
- Continuous signals

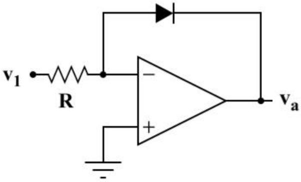
$$y(t) = x_1(t)x_2(t)$$

  - Used in audio mixer, analogue modulation.
  - Implementation: commonly through the log domain:

$$\log(y(t)) = \log(x_1(t)) + \log(x_2(t))$$



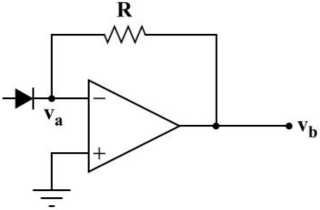




Logarithmic Converter

$$\frac{v_1}{R} = I_S = e^{aV_A}$$

$$v_A = \frac{1}{a}\ln\left(\frac{v_1}{R}\right)$$



Antilogarithmic Converter

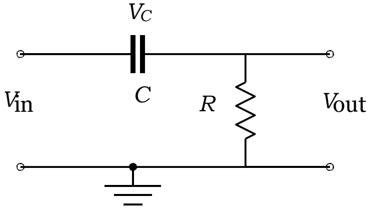
$$e^{av_A} \cdot R = v_b$$

Differentiation

- Applied to continuous signals

$$y(t) = \frac{d}{dt}x(t) = x'(t)$$

- Implementation:

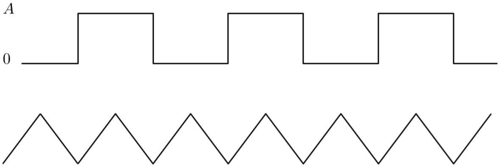


$$I = C\frac{dV_C}{dt} = \frac{V_{out}}{R}$$

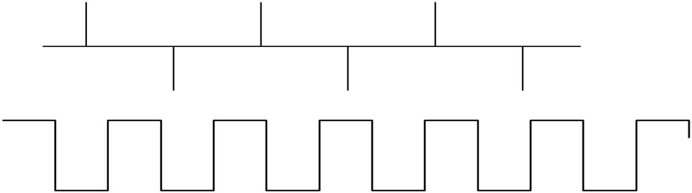
$$V_{out} = RC\frac{dV_C}{dt} \approx RC\frac{dV_{in}}{dt}$$

Exercise

Exercise: Compute the differentiated signals for the signals plotted in Fig. 4.



Solution: The differentiated signals are given in the figure below.



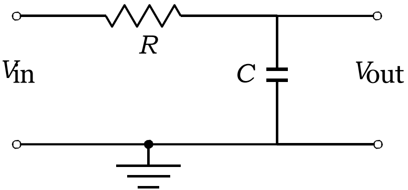
Note that we don’t define the differentiation for discrete signals. The reason is simple. Discrete signals are not continuous, so the definition of differentiation does not exist.

Integration

- Applied to continuous signal

$$y(t) = \int_{-\infty}^{t} x(u)\,du$$

- Implementation



$$I = \frac{V_{\text{in}}}{R} \qquad V_{\text{out}} = \frac{1}{C}\int I dt = \frac{1}{RC}\int V_{\text{in}} dt$$

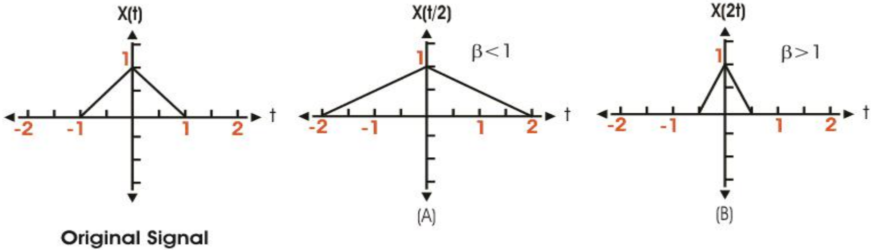
$$V_{\text{out}} = \frac{1}{C}\int I dt = \frac{1}{RC}\int V_R dt \approx \frac{1}{RC}\int V_{\text{in}} dt$$

Time Scaling: Continuous Signals

- For continuous signals

$$y(t) = x(at)$$

  where a is a positive constant.
- Two cases
  - $a > 1$: compression
  - $0 < a < 1$: expansion



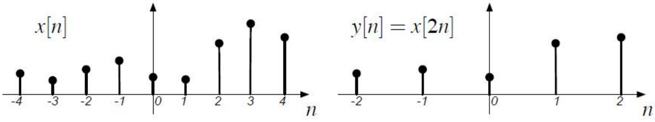
Time Scaling: Discrete Signals

- For discrete signals, the **compressed signal** is generated as

$$y[n] = x[kn]$$

where k is an **integer number**.

  - This extracts every kth sample of $x[n]$
  - Intermediate samples are lost
  - The sequence is shorter.



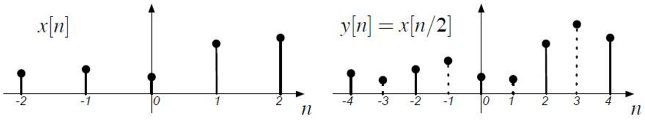
This is called **downsampling.**

- For a discrete signal $x[n]$, the **expanded signal** is generated as

$$y[n] = x[n/k]$$

where k is an **integer number**.

  - The intermediate samples must be synthesized (set to zero, or interpolated)
  - The sequence is longer

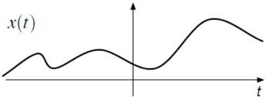


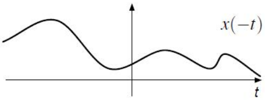
This is called **upsampling**

Reflection

- Also known as **time reversal.**
- Continuous signals

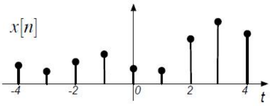
$$y(t) = x(-t) \tag{1}$$

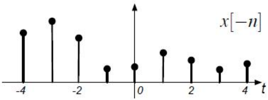




- Discrete signals

$$y[n] = x[-n] \tag{2}$$





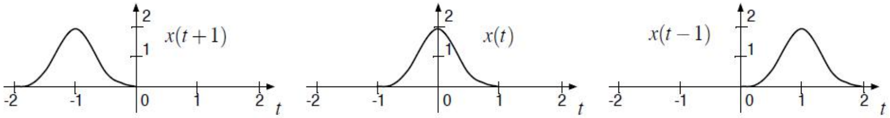
- Same as time scaling with $a = -1$.

Time Shifting

- For a continuous-time signal $x(t)$ and a time t_0, the time shifted signal of $x(t)$ is expressed as

$$y(t) = x(t - t_0) \tag{3}$$

- Consider two cases
  - $t_0 > 0$: the signal $y(t) = x(t - t_0)$ is shifted to the right, which gives rise to a delayed signal.
  - $t_0 < 0$: the signal $y(t) = x(t - t_0)$ is shifted to the left, which gives rise to an advanced signal.

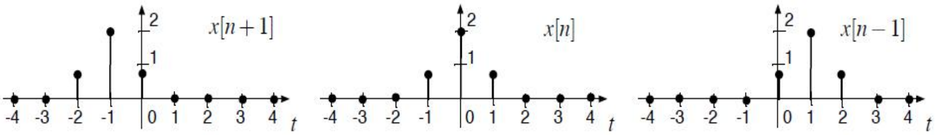


Time Shifting...

- For a discrete time signal $x[n]$ and an integer n_0, the time shifted signal of $x[n]$ is expressed as

$$y[n] = x[n - n_0] \tag{4}$$

- Consider two cases
  - $n_0 > 0$: the signal $y[n] = x[n - n_0]$ is shifted to the right, which gives rise to a delayed signal.
  - $n_0 < 0$: the signal $y[n] = x[n - n_0]$ is shifted to the left, which gives rise to an advanced signal.



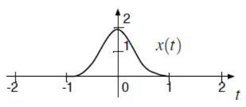
Combination of Time Scaling and Time Shifting

- Time scaling, shifting, and reversal can all be combined
- Operation can be performed in any order, but **care is required**
- This may cause confusion

Combinations of Operations...

Example

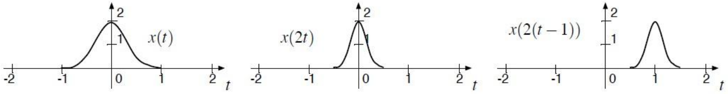
Given the signal $x(t)$ below



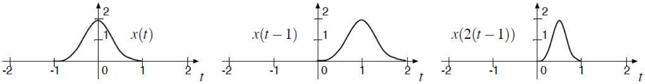
create the signal $x(2t - 2)$.

Example (continued)

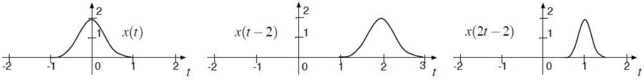
→Scale first, then shift : Compress by 2, shift by 1



→Shift first, then scale: **Shift by 1, compress by 2 → incorrect**



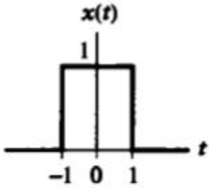
→Shift first, then scale: Shift by 2, scale by 2



Example

Consider the rectangular pulse $x(t)$ of unit amplitude and a duration of 2 time units, depict in (a). Find

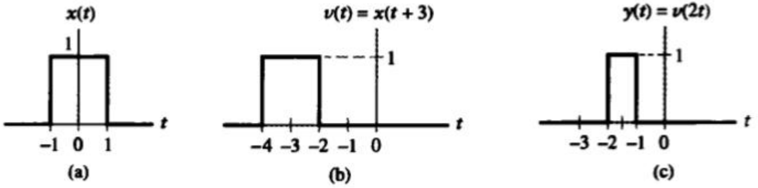
$$y(t) = x(2t+3)$$

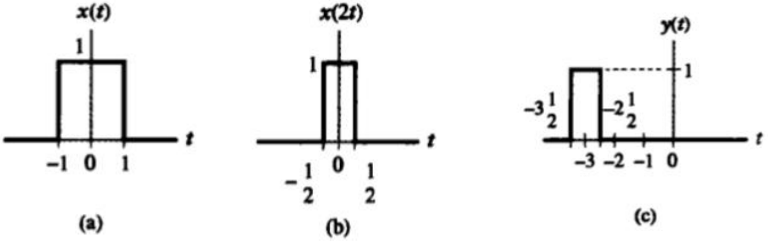


(a)

Combinations of Operations...

Answer:





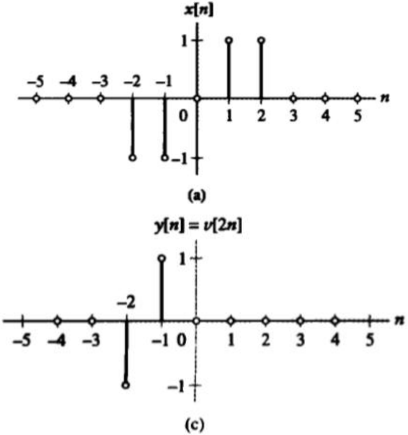
Example

Consider a discrete-time signal

$$X[n]=\begin{cases} 1, & 1 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & otherwise \end{cases}$$

Find $y[n] = x[3n - 2]$

Answer:



$x[n]$
1
−1
−5 −4 −3 −2 −1
0 1 2 3 4 5
n
(a)
$v[n] = x[n+3]$
1
−1
−5 −4
−3 −2 −1 0 1 2 3 4 5
n
(b)
$y[n] = v[2n]$
1
−1
−2
−5 −4 −3 −1 0 1 2 3 4 5
n
(c)