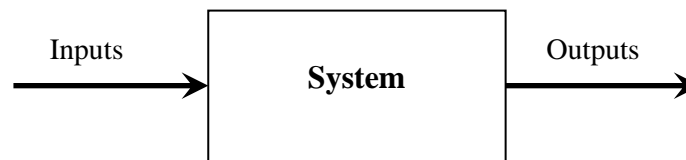
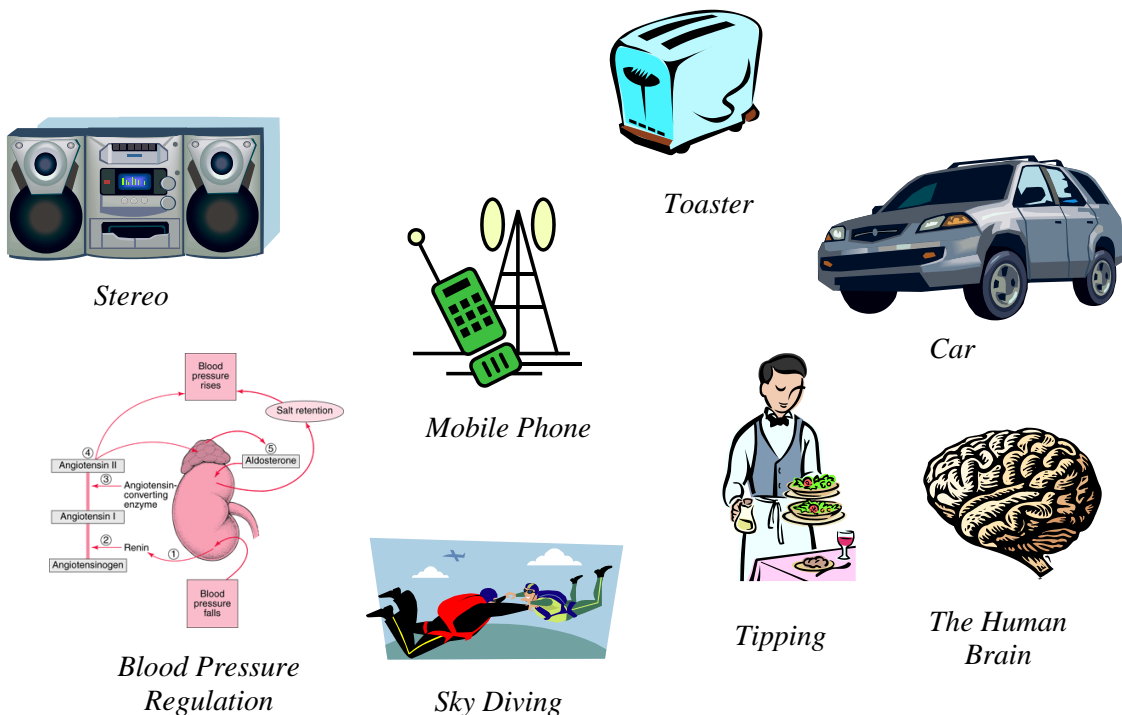

2. Introduction to Systems

2.1 What is a system?

- As stated in the first section, a **system** is a collection of components acting together to perform a specific task – it is anything with inputs and outputs.



- A **component** is a single functioning unit of the system.
- Systems are not always physical – they can extend to more abstract phenomena, for example: economical, transportation, population, biological, etc.
- Example systems include:



- Most systems are sub-systems of other larger systems. For example:
 - a car is a system with many subsystems including brakes, steering, suspension, etc.
 - electricity supply grid - transmission lines, transformers, consumer appliances, etc.
 - human body - breathing, blood pressure, heart rate, sleeping etc.

2.2 Dynamical and static systems

-
- A **dynamical system** is where the current output depends on past inputs. As such, a dynamical system is also referred to as a system with memory.
 - On the other hand, a **static system** is where the current output depends only on the current input. It cannot depend on past values and, as such, these systems are referred to as systems with no memory.
 - In a static system, the output remains constant if the input does not change. In a dynamical system the output can change with time even though the input is no longer changing.
 - Examples of dynamical systems include:
 - steering a car
 - accelerating a car
 - aircraft altitude/ship steering autopilot
 - domestic central heating
 - the currency exchange rate
 - blood pressure control
 - the weather
 - the human body
 - a cup of tea !



- *Can you give some examples of systems that are not dynamical (i.e. static systems)?*

- The world is full of dynamical systems. In fact, the world itself is a dynamical system!

2.3 Categorizing systems

- Systems can be categorized as:
 - linear or nonlinear
 - time-invariant or time-varying
 - continuous or discrete

2.3.1 Linear v Nonlinear

- **Linear** systems possess two important properties, namely *superposition* and *homogeneity*. If these properties are not withheld, then the system is said to be **nonlinear**.
- *The principle of superposition* - this states that the response produced by a combination of different inputs is the sum of the individual responses produced by each of the different inputs acting alone.
- In other words, the response of a multi-input linear system can be determined by dealing with each input in turn and then adding the results.

-
- *Homogeneity (or scaling)* – this states that if an input $u(t)$ produces an output $y(t)$ then, for a linear system, an input $ku(t)$ will produce an output $ky(t)$, where k is a scalar.
 - We can easily demonstrate these properties with some trivial examples.
 - Consider the following function with output y and inputs u_1 and u_2 .

$$y(t) = 3u_1(t) + 5u_2(t)$$

- If, at an arbitrary point in time, $u_1(t) = 2$ and $u_2(t) = 3$, then:

$$y(t) = 3(2) + 5(3) = 21$$

- Now, let's obtain the output $y(t)$ by considering each input separately, as follows:

$$\text{First, let } u_2(t) = 0, \text{ then } y_1(t) = 3(2) + 5(0) = 6$$

$$\text{Now, let } u_1(t) = 0, \text{ then } y_2(t) = 3(0) + 5(3) = 15$$

$$\text{Overall, } y(t) = y_1(t) + y_2(t) = 6 + 15 = 21 \quad \dots \text{ as above}$$

- Hence, this very simple system obeys the principle of superposition.
- If we multiply both inputs by a factor of 2 (for example) then:

$$y(t) = 3(4) + 5(6) = 42 \quad = \quad 2 \times 21$$

- Thus, the new output is also multiplied (i.e. scaled) by a factor of 2. Hence the system possesses the homogeneity property and thus this system is linear.
- Now consider another very simple system:

$$y(t) = 3u_1(t) + 5u_2(t) + 4$$

- Taking an arbitrary point in time, where $u_1(t) = 2$ and $u_2(t) = 3$, then:

$$y(t) = 3(2) + 5(3) + 4 = 25$$

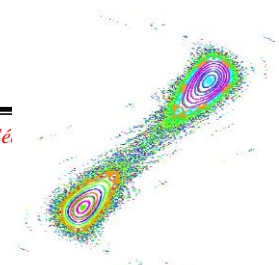
- Once again, let's obtain the output $y(t)$ by considering each input separately, as follows:

$$\text{First, let } u_2(t) = 0, \text{ then } y_1(t) = 3(2) + 5(0) + 4 = 10$$

$$\text{Now, let } u_1(t) = 0, \text{ then } y_2(t) = 3(0) + 5(3) + 4 = 19$$

$$\text{Overall, } y(t) = y_1(t) + y_2(t) = 10 + 19 = 29 \quad \dots \text{ different to above}$$

- Hence, this system is not linear as it does not obey the principle of superposition.
- **Ex2.1 Is the following system linear or nonlinear?**



$$y(t) = 3u_I(t)^2$$

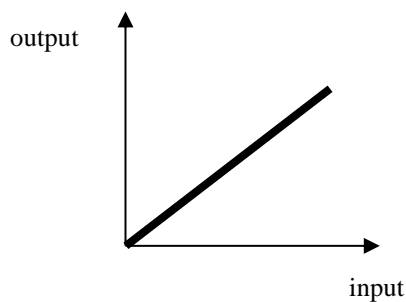
- Take the arbitrary value, $u_I(t) = 2$ then:

$$y(t) = 3(2)^2 = 3(4) = 12$$

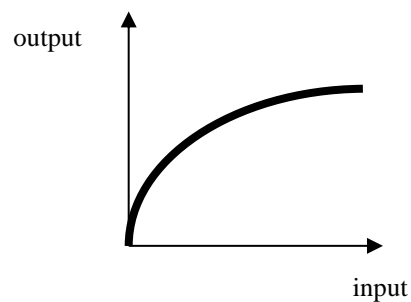
- Now, let's scale the input by a factor of 2, i.e. $2 \times u_I(t)$, then:

$$y(t) = 3(4)^2 = 3(16) = 48 \quad \neq \quad 2 \times 12$$

- Hence, this system is nonlinear, as it does not possess the homogeneity property.
- Graphically the difference between linear and nonlinear systems can be illustrated as follows:

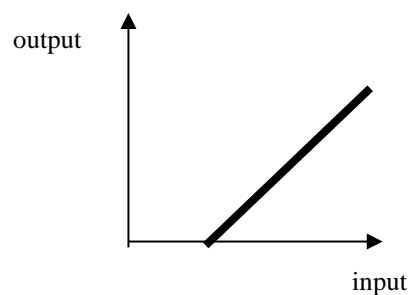


Linear system

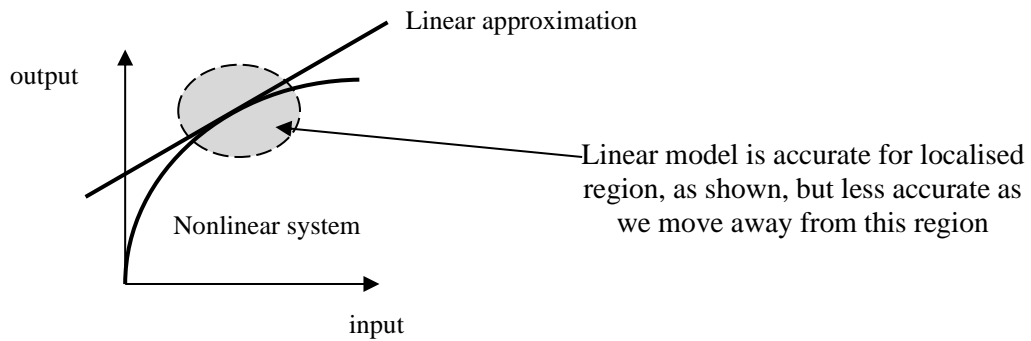


Nonlinear system

- *Question – is the following representative of a system that is linear or nonlinear? Why?*



- In general, analyzing nonlinear systems can be complicated and can require difficult mathematical calculations.
- It is, therefore, common practice to *linearize* such systems about specific operating points. Thus, simpler, linear mathematical techniques can now be applied to the linear approximation of the nonlinear system.
- Graphically, we can demonstrate the concept of linearization as follows:



- You will study linearization in more detail in EE211 System Dynamics.

2.3.2 Time-invariant v Time-varying

- The output of a **time-invariant** system does not explicitly depend on *time* whereas the output of a **time-varying** system does.
- For example, consider the following systems with output y and input u :

System 1: $y(t) = 3u(t)$
 and
 System 2: $y(t) = \sin(t) u(t)$

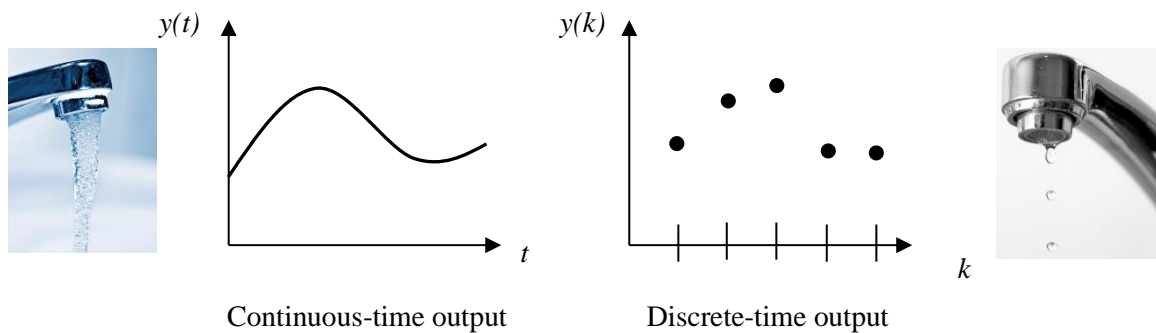
- Here, system 1 is time-invariant but system 2 is time-varying as it depends explicitly on the time parameter t .
- Note, the input function (or signal) $u(t)$ clearly depends on time, as it is a function of time.
- However, it is the relationship between the input $u(t)$ and the output $y(t)$, i.e. the system, that we are referring to and the question becomes does this contain a parameter directly affected by time?
- In system 1, the system is simply a constant value, i.e. 3, and independent of time. In system 2, the system is $\sin(t)$ and clearly dependent on time and hence time-varying.
- A resistor is a time-invariant system (recall Ohm's Law) while an aircraft can be regarded as a time-varying system, as its dynamics are effective by parameters of the system that change with time, such as weight change due to fuel consumption.



2.3.3 Continuous v Discrete

- Continuous-time** systems involve signals that are continuous in time while **discrete-time** systems involve variables that only change at discrete instants of time.

- This concept is simply captured in the following illustration:



- *In this module, we will focus on continuous- time, linear invariant (LTI) system and analysis.*



2.4 Mathematical modelling (an overview)

- In order to design a controller for a system, we must first model the system, so that we can analyse its response and predict its performance.
- The model of the system is typically based on a mathematical description of the dynamical characteristics of the system.
- This mathematical description is referred to as a **mathematical model**.
- For many practical systems, useful mathematical models are generally described in terms of **differential equations**, for continuous-time systems, or **difference equations**, for discrete-time systems.
- For example, a sky diver falling under gravity g and encountering a resistance proportional to his velocity v can be modelled by the following first order *differential equation*, where k is a constant:

$$\frac{dv}{dt} + kv = g$$



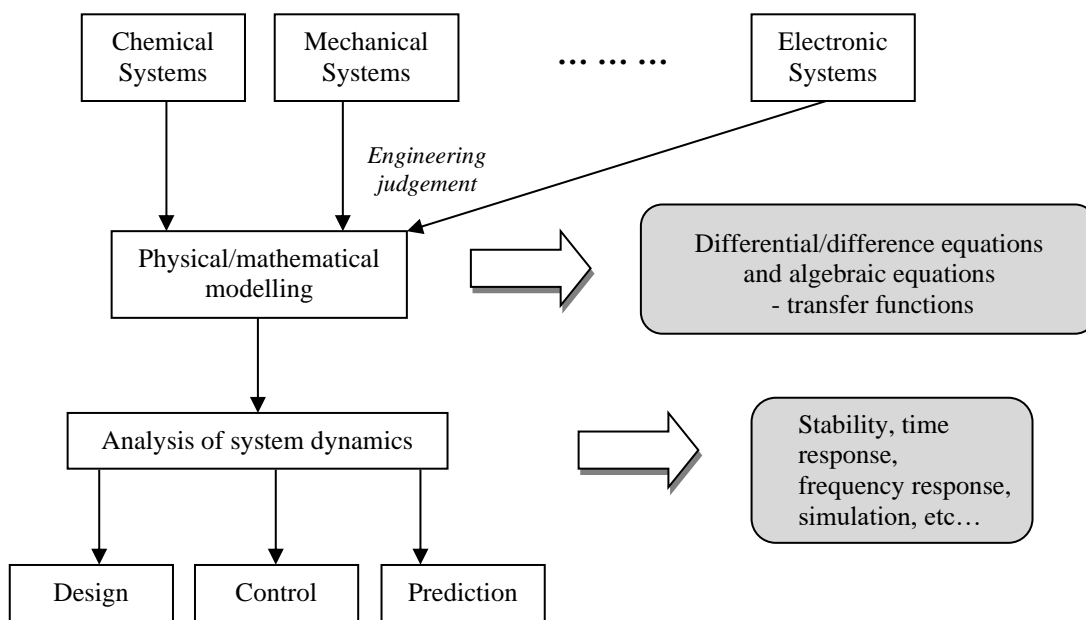
- In contrast, given that a country's population in year k is $x(k)$ and that there is a constant birth rate b and a constant death rate d , then the country's population growth can be modelled by the *difference equation*:



$$x(k+1) = x(k) + bx(k) - dx(k)$$

- While these equations can be solved in their current form, it is easier to work with them in an alternative format known as the **transfer function** format.
- **Laplace transforms** are used to convert differential equations into algebraic form, while **Z-transforms** are used to convert difference equations.
- *Later in this module, we will introduce and examine the transfer function format for continuous-time systems and, consequently, introduce and briefly examine Laplace transforms.*
- *Laplace Transforms are also covered in more detail in the mathematics modules EE112 and (more significantly) EE206.*
- *Z-transforms will not be covered in this module but will be studied in detail in other modules later in the programme, including EE206, EE211 and EE213.*

2.5 The big picture



- System dynamics involves developing a mathematical model of a dynamical system with a view to analyzing the system's performance.
- This, in turn, can lead to better design, better control and better prediction of the system.
- Ultimately, the success of all these depends on the validity of the underlying mathematical model. In other words, the validity of a prediction depends largely on how accurately the mathematical model captures the behaviour of the actual system.

-
- This concept of modelling, analysis and design/control/prediction is summarized in the ‘big picture’ diagram given previously.
 - *In terms of this module, we will be focus primarily on some basic modelling, with a key emphasis on the transfer function representation for continuous-time systems.*

