Dear TA:

Special thanks for your patience in correcting my homework this semester, I sure appreciate it.
Hope you all the best in next term!
And wish you a happy new year in advance.

Regard,
Hanlin Cai

Date

Assign 10-P1 Hanlin-Cai

(a)

1. Sol

$x_k = (-1)^k$

$Z[x_k] = \frac{z}{z+1}$

$|z| > 1$

(b) $x_k = 3k + (-4)^{2k+1}$

$= 3k - 4(-4)^{2k}$

$= 3k - 4 \cdot 16^k$

$Z[x_k] = \frac{3z}{(z-1)^2} - \frac{4z}{(z-16)}$

2. (a) $\frac{F(z)}{z} = \frac{z}{(z+1)(z+2)(z+3)}$

$= -\left(\frac{\frac{1}{2}}{z+1} + \frac{-2}{z+2} + \frac{3/2}{z+3}\right)$

$F(z) = -\frac{1}{2}\frac{z}{z+1} + \frac{-2z}{z+2} + \frac{3}{2}\frac{z}{z+3}$

$\therefore F(z) = -\frac{1}{2} z\{-1^k\} - 2z\{-2^k\} + \frac{3}{2} z\{-3^k\}$

$Z^{-1} F(z) = \frac{1}{2}(-1)^{k+1} - (-2)^{k+1} + \frac{1}{2}(-3)^{k+1}$

(b) $F(z) = \frac{z(3z+1)}{(z-3)^2}$

$= \frac{3z}{z-3} + \frac{10z}{(z-3)^2}$

$= 3z\{3^k\} + \frac{10}{3} z\{k \cdot 3^k\}$

$\therefore Z^{-1} F(z)$

$= 3^{k+1} + 10 \cdot 3^{k-1} \cdot k$

3 (a) $\lim_{z\to 1}\left\{\frac{z-1}{z} F(z)\right\}$

$= \lim_{z\to 1}\left\{\frac{z-1}{z} \cdot \frac{3z^2-z}{2z^2-3z+1}\right\}$

$= \lim_{z\to 1} \frac{3z-1}{2z-1}$

$= 2$

4 (a) Sol

$\lim_{z\to\infty}\{F(z)\} = \lim_{z\to\infty} \frac{2z^2-z+1}{5-3z-7z^2}$

$= \lim_{z\to\infty} \frac{2-\frac{1}{z}+\frac{1}{z^2}}{\frac{5}{z^2}-3\frac{1}{z}-7} = -\frac{2}{7}$

(b) $\lim_{z\to\infty}\left\{\frac{2z^3+5z^2+2z-1}{6z^3-4z+2}\right\}$

$= \lim_{z\to\infty}\left\{\frac{2+5\frac{1}{z}+2\frac{1}{z^2}-\frac{1}{z^3}}{6-\frac{4}{z^2}+\frac{2}{z^3}}\right\}$

$= \frac{1}{3}$

5 (a) Sol

$x_{k+2} = 2x_k + x_{k+1}$ $\begin{cases} x_1 = 5 \\ x_0 = 2 \end{cases}$

$\begin{cases} x_2 = 2x_0 + x_1 \\ x_3 = 2x_1 + x_2 \\ x_4 = 2x_2 + x_3 \end{cases}$

$\therefore \{x_k\} = \{2, 5, 9, 19, 37 \ldots\}$

(b) $x_{k+2} = 3x_k - 2x_{k+1}$ $\begin{cases} x_0 = 1 \\ x_1 = 1 \end{cases}$

$\begin{cases} x_2 = 3x_0 - 2x_1 = 1 & (1) \\ x_3 = 2x_1 - 2x_2 = 1 & (2) \\ x_4 = 3x_2 - 2x_3 = 1 & (3) \end{cases}$

$\therefore \{x_k\} = \{1, 1, 1, 1, 1\}$

KOKUYO

No.

Date

sol.

6.(a) $a_{n+2} = a_n$

$F(z+2) = F(z)$

$z^2 F(z) - z^2 x_0 - z x_1 = F(z)$

$(z^2-1)\cdot F(z) = z$

$F(z) = \frac{z}{z^2-1} = z\left[\frac{1}{2}\left(\frac{1}{z-1} - \frac{1}{z+1}\right)\right]$

and $z > 1$

$Z^{-1}[F(z)] = \frac{1}{2}(1)^k - \frac{1}{2}(-1)^k$

6.(b) sol.

$a_{n+2} = 2a_n \quad \begin{cases} a_0 = 0 \\ a_1 = 2 \end{cases}$

$z^2 F(z) - z^2 \cdot x_0 - z x_1 = 2F(z)$

$(z^2-2)\cdot F(z) = z(x_0 + x_1)$

$F(z) = \frac{2z}{(z^2-2)} = \left(-\frac{\sqrt{2}}{2}\frac{z}{z+\sqrt{2}} + \frac{\sqrt{2}}{2}\frac{z}{z-\sqrt{2}}\right)$

$\therefore z^{-1} F(z) = -\frac{\sqrt{2}}{2}(-\sqrt{2})^k + \frac{\sqrt{2}}{2}(\sqrt{2})^k,$

$= \frac{1}{2}(-\sqrt{2})^{k+1} + \frac{1}{2}(\sqrt{2})^{k+1}$

(and $z > \sqrt{2}$)

6.(c) $a_{n+2} = a_n + a_{n+1}$

$z^2 F(z) - z^2 x_0 - z x_1 = F(z) + zF(z) - z x_0$

$F(z)[z^2 - z - 1] = z^2$

$F(z) = \frac{z^2}{z^2 - z - 1}$

7.(a) sol

$[z^2 F(z) - z^2 x_0 - z x_1 - 4(zF(z) - z x_0) + 4F(z)] = \frac{3z}{z-1}$

$\therefore (z^2 - 4z + 4)\cdot F(z) - z^2 + 4z = \frac{3z}{z-1}$

$\therefore \frac{F(z)}{z} = \frac{1}{z-1} - \frac{1}{(z-2)^2}$

$\therefore F(z) = \frac{z}{z-1} - \frac{z}{(z-2)^2}$

$\therefore z^{-1}[F(z)] = (1)^k - k(2)^k$

(b)

$[z^2 F(z) - z^2 x_0 - z x_1 + 5(zF(z) - z x_0) + 6F(z)] = \frac{4z}{(z-2)}$

$\therefore F(z)[z^2 + 5z + 6] - 4z = \frac{4z}{z-2}$

$\therefore F(z) = \frac{1}{5}\frac{z}{z-2} + 3\frac{z}{z+2} - \frac{16}{5}\frac{z}{z+3}$

$z^{-1} F(z) = \frac{1}{5}(2)^k + 3(-2)^k - \frac{16}{5}(-3)^k$

(c) $z^2 F(z) - z^2 x_0 - z x_1 - 9F(z) = \frac{2z}{(z-1)^2}$

$\therefore F(z)[z^2 - 9] = \frac{2z}{(z-1)^2} + z^2 + z$

$\frac{F(z)}{z} = \frac{1}{(z-1)^2(z^2-9)} + \frac{z+1}{z^2-9}$

$\therefore F(z) = \left[-\frac{32}{5}\frac{1}{z-1} - \frac{1}{8}\frac{1}{(z-1)^2} + \frac{1}{8}\frac{1}{z-3}\right.$
$\left. + \frac{1}{32}\cdot\frac{1}{z+3} + \frac{1}{3}\cdot\frac{1}{z+3} + \frac{2}{3}\cdot\frac{1}{z-3}\right]$

$\therefore z^{-1}F(z) = \left[-\frac{5}{32}(1)^k - \frac{1}{8}(k\cdot 1^k)\right.$

$\left. + \frac{17}{24}(3)^k + \frac{35}{96}(-3)^k\right]$

Assign 10 - P2

8. (a) Sol.

$\sin t = \left(\frac{e^{it} - e^{-it}}{2i}\right)$

$\sin(kT) = \left(\frac{e^{ikT} - e^{-ikT}}{2i}\right)$

$\therefore F(z) = \frac{1}{2j}\left(\frac{z}{z-e^{iT}} - \frac{z}{z-e^{-iT}}\right)$

$= \frac{1}{2j} \cdot \frac{ze^{jT} - z\cdot e^{-jT}}{z^2 - (e^{jT} + e^{-jT})z + 1}$

$= \frac{1}{2j} \cdot \frac{z(2j\sin T)}{z^2 - 2z\cos T + 1}$

$= \frac{z\cdot\sin T}{(z^2 - 2z\cos T + 1)}$

(b) $\sinh t = \frac{e^x - e^{-x}}{2}$

$\sinh(Tk) = \frac{e^{kT} - e^{-kT}}{2}$

$F(z) = \frac{1}{2}\left(\frac{z}{z-e^T} - \frac{z}{z-e^{-T}}\right)$

$= \frac{ze^T - ze^{-T}}{z^2 - z(e^T - e^{-T}) + 1}$,