Tutor 7-1

Q1. sol

(i) $\frac{1+2j}{2-j} = \frac{\sqrt{5}\angle \tan^{-1}(2)}{\sqrt{5}\angle \tan^{-1}(-\frac{1}{2})} = 1\angle \tan^{-1}(2)-\tan^{-1}(-\frac{1}{2}) = 1\angle 90^\circ = j$

(ii) $\frac{j(1-3j)}{(3-2j)(5+4j)} = \frac{1\sqrt{10}}{\sqrt{13}\cdot\sqrt{14}}\cdot \angle 90^\circ+\tan^{-1}(-3)-\tan^{-1}(-\frac{2}{3})-\tan^{-1}(\frac{4}{5})$,

$= 0.137\angle 13.47^\circ$

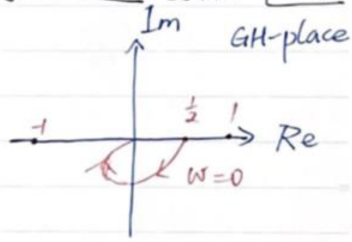
Q2 sol

(i) $G(s) = \frac{K}{(s+1)(s+2)}$, $H(s)=1$, $K=1$

$s \to j\omega \Rightarrow GH(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)} = \frac{1\angle 0^\circ}{\sqrt{1+\omega^2}\angle\tan^{-1}(\omega)\ \sqrt{4+\omega^2}\cdot\angle\tan^{-1}(\frac{\omega}{2})}$

① $\omega = 0 \iff \begin{cases} |GH(j0)| = \frac{1}{2} \\ \angle GH(j0) = 0^\circ \end{cases}$

② $\omega \to \infty \iff \begin{cases} |GH(j\infty)| = \frac{1}{\omega^2} = 0 \\ \angle GH(j\infty) = -2\tan^{-1}(\infty) = -180^\circ \end{cases}$



③ $\omega = -90^\circ$

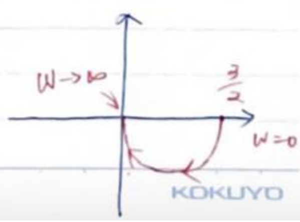
(ii) sol

$G(s) = \frac{K(s+3)}{(s+1)(s+2)}$, $H(s)=1$, $K=1$

$\therefore GH(j\omega) = \frac{1\cdot(3+j\omega)}{(1+j\omega)(2+j\omega)} \iff \frac{1\angle 0^\circ\ \sqrt{9+\omega^2}\angle\tan^{-1}(\frac{\omega}{3})}{\sqrt{1+\omega^2}\angle\tan^{-1}(\omega)\sqrt{4+\omega^2}\angle\tan^{-1}(\frac{\omega}{2})}$, $= \frac{\sqrt{\cdots}}{\sqrt{\cdots}\sqrt{\cdots}}\angle\text{---}$

① $\therefore \omega = 0 \quad GH(j0) = \frac{3}{(1)(2)}\angle 0^\circ = \frac{3}{2}\angle 0^\circ$

② $\omega \to \infty \quad GH(j\omega) = \frac{1}{\omega}\angle -90^\circ = 0\angle -90^\circ$



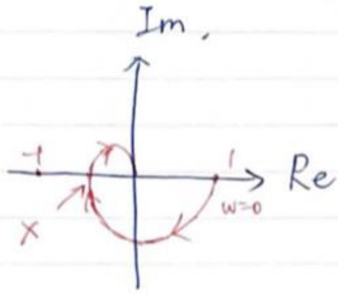
Q3 sol $\quad GH(s) = \dfrac{27k}{(s+3)^3}$, $k=1$

$$GH(j\omega) = \frac{27}{(3+j\omega)^3} = \frac{27\angle 0^\circ}{(\sqrt{\omega^2+3^2})^3 \angle 3\tan^{-1}(\frac{\omega}{3})} = \frac{27}{(\sqrt{\omega^2+3^2})^3}\angle -3\tan^{-1}(\tfrac{\omega}{3})$$

① $\omega = 0 \Longleftrightarrow \quad GH(j0) = 1\angle 0^\circ$

② $\omega \Rightarrow \infty \longleftrightarrow \quad GH(j\infty) = 0\angle -270^\circ$

Nyquist sketch:



(i) $\quad G(s) = \dfrac{k}{(s+1)(s+2)(s+3)}$, $\quad H(s) = \dfrac{1}{s+3} \quad k = 10, 40, 100$

Q4. sol

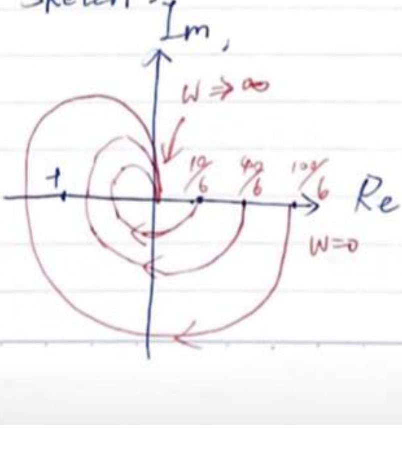
$$GH(j\omega) = \frac{k}{(\ )(\ )(\ )} = \frac{k\angle 0^\circ}{\sqrt{\ }\angle\ \sqrt{\ }\angle\ \sqrt{\ }\angle}$$

$$= \frac{k}{\sqrt{\cdots}\sqrt{\cdots}\sqrt{\cdots}}\angle -\tan^{-1}(\omega) - \tan^{-1}(\tfrac{\omega}{2}) - \tan^{-1}(\tfrac{\omega}{3})$$

① $\omega = 0 \Longleftrightarrow \dfrac{k}{1\cdot 2\cdot 3}\angle 0^\circ = \dfrac{k}{6}\angle 0^\circ$

② $\omega \to \infty \Longleftrightarrow \quad = 0\angle 270^\circ$

Nyquist Sketch:



Tutor T-2.

Q4 (ii) sol

$$GM = \frac{1}{GH(jw)}\Big|_{w=\sqrt{11}}$$

$$= \left(\frac{1}{\frac{k}{\sqrt{\ }\sqrt{\ }\sqrt{\ }}}\right) = \frac{\sqrt{\ }\sqrt{\ }\sqrt{\ }}{k}\Big|_{\sqrt{11}}$$

$$= \frac{\sqrt{12}\cdot\sqrt{15}\cdot\sqrt{20}}{k} = \frac{60}{k},$$

For $k = 10 \quad 40 \quad 100$.

then $GM = \underbrace{6 \quad 1.5}_{\text{stable}} \quad \underset{\downarrow\ \text{unstable}}{0.6}$

(iii)

$$|GH(jw)| = \frac{k}{\sqrt{1+w^2}\sqrt{4+w^2}\sqrt{9+w^2}}.$$

(1). $k=10 \quad w=1 \iff \frac{10}{\sqrt{2}\sqrt{5}\sqrt{10}} = 1$

(2). $k=40 \quad w=2.73 \iff \frac{40}{\sqrt{\ }\sqrt{\ }\sqrt{\ }} = 1$

(3). $k=100 \quad w=4.14 \iff 0.9988 \simeq 1$

Q5. sol. $GH(s) = \frac{1}{(s+1)(s+2)}$,

(Q2) (i) $\therefore GH(j\omega) = \frac{1}{(1+j\omega)\cdot 2(1+j\frac{\omega}{2})}$. $\Rightarrow \frac{1}{1+j\omega} \cdot \frac{1}{1+j\omega/2} \quad \frac{1}{2}$

$20 \log_{10} \frac{1}{2} = -6\ dB.$

(ii) $GH(s) = \frac{k(s+3)}{(s+1)(s+2)} \quad k=1$

$GH(j\omega) = \frac{1}{1+j\omega} \cdot \frac{1}{1+j\frac{\omega}{2}} \cdot \frac{1}{2} \cdot 3\ (1+j\frac{\omega}{3}).$

$20 \log_{10}(3) = 9.54\ dB$

(See the Pic-1 in next Page).

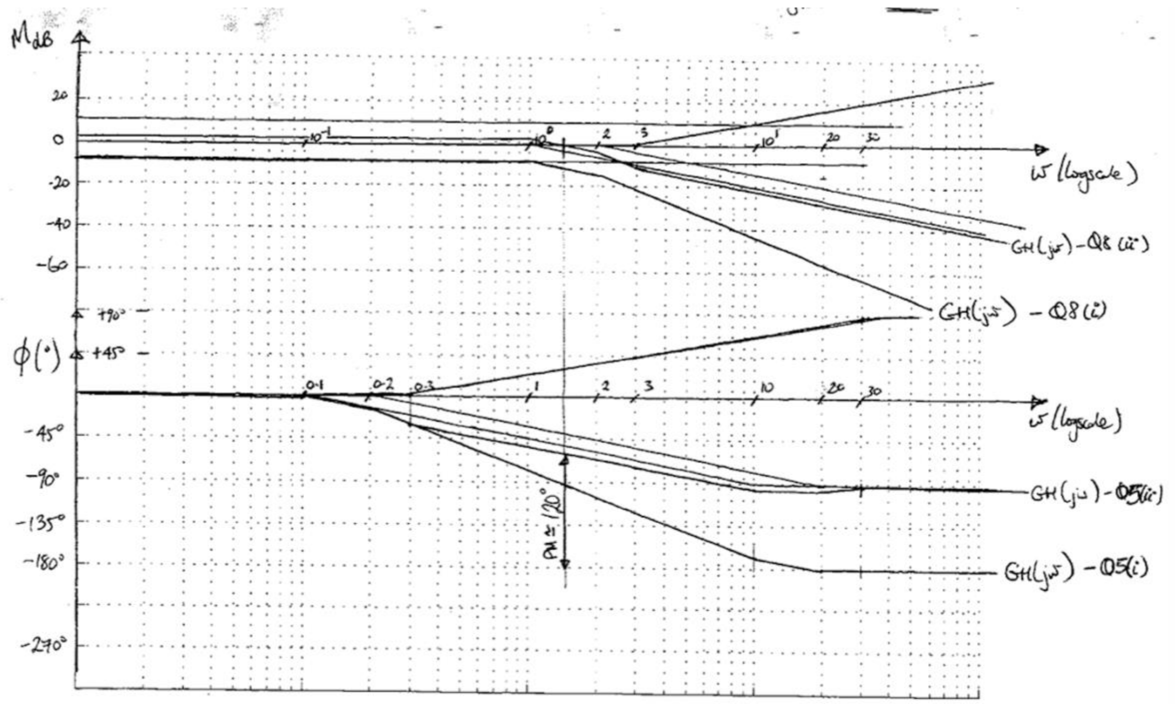
Q6. (Q3) $GH(j\omega) = \frac{27}{(j\omega+3)^3} \Rightarrow \frac{1}{(1+j\frac{\omega}{3})^3}$.

$PM = 180^\circ \quad GM = 20\ dB$

(See the Pic-2 in next page)

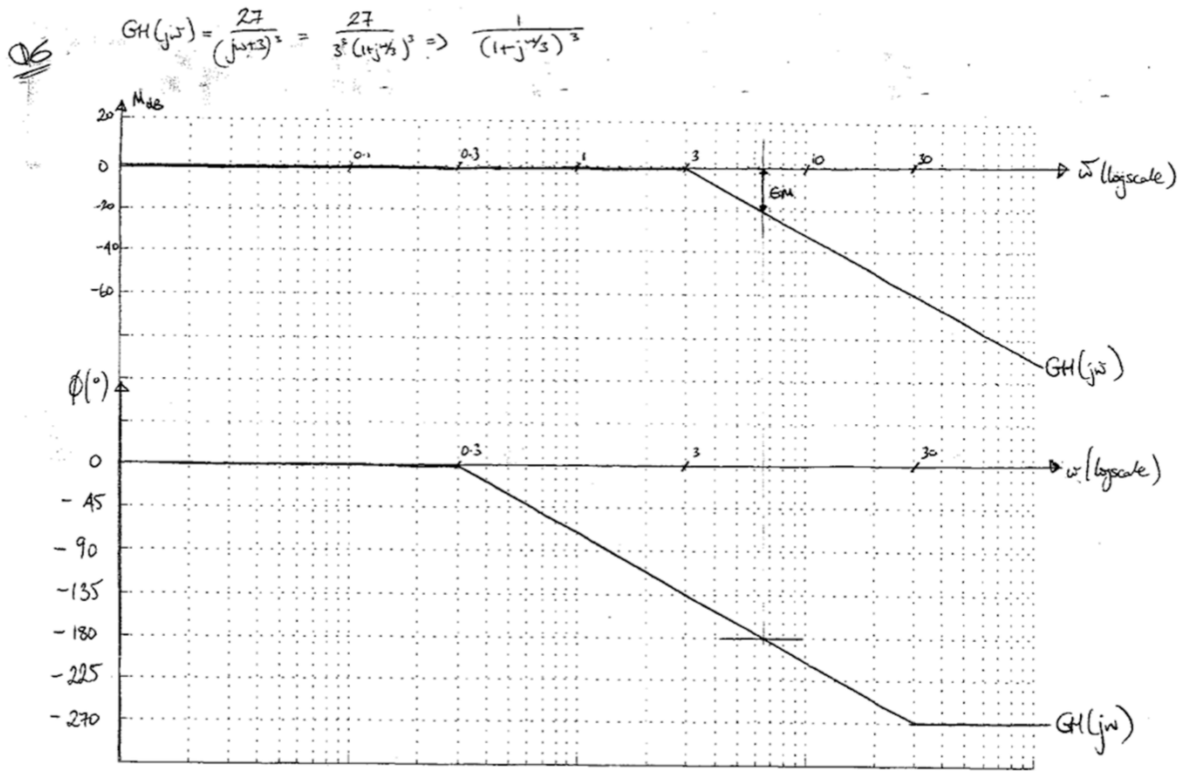
Q7. (See the Pic-3 in next page).

For Q5



M_{dB}
20
0
-20
-40
-60
10^{-1}
10^{0}
2
3
10^{1}
20
30
ω (logscale)
GH(jω) – Q8 (ii)
GH(jω) – Q8 (i)
$\phi(°)$
+90°
+45°
-45°
-90°
-135°
-180°
-270°
0.1
0.2
0.3
1
2
3
10
20
30
ω (logscale)
PM ≈ 120°
GH(jω) – Q5(ii)
GH(jω) – Q5(i)

For Q6



Q6
$GH(j\omega) = \frac{27}{(j\omega+3)^3} = \frac{27}{3^3(1+j\omega/3)^3} \Rightarrow \frac{1}{(1+j\omega/3)^3}$
M_{dB}
20
0
-20
-40
-60
0.1
0.3
1
3
10
30
ω (logscale)
GM
GH(jω)
$\phi(°)$
0
-45
-90
-135
-180
-225
-270
0.3
3
30
ω (logscale)
GH(jω)

For Q7

Q7

(i), (ii)

$GH(j\omega) = \frac{K}{(1+j\omega)(2+j\omega)(3+j\omega)} \Rightarrow \frac{K}{1+j\omega} \cdot \frac{K}{1+j\omega/2} \cdot \frac{K}{1+j\omega/3} \cdot \frac{K}{6}$

$20\log_{10}\left(\frac{1}{6}\right) = -15.56\,dB$ ($K=1$)

$K=10$ gives: $4.4\,dB$, $K=40$ gives: $16.5\,dB$

and $K=100$ gives: $24.4\,dB$