Assign 8.

1. (a) sol (Let $x > 0$)

$f(x) = x \cdot \cos x$

$f(-x) = -x \cos(-x) = -x \cos x$

$\therefore f(x) = -f(-x)$ & $f(0) = 0$

$\therefore$ odd

(b) sol

$f(-x) = x^4 + 4x$

$f(x) \neq f(-x)$ & $f(x) \neq -f(-x)$

$\therefore$ neither

(c) sol.

$f(-x) = e^{-x} - e^{x}$

$f(x) = -f(-x)$ & $f(0) = 1 - 1 = 0$

$\therefore$ odd

(d) sol. $f(x) = |x|^5$ let. $x > 0$

$f(-x) = |x|^5 = f(x)$ (where $|-x| = |x|$)

$\therefore$ even

(e) $f(x) = -(-x) + 5 = x + 5 = f(x)$

So it's even.

Notice! Thanks!

2. (c)

$A(\alpha) = 2\int_1^2 \cos\alpha x \, dx = 2\left(\frac{\sin 2\alpha - \sin\alpha}{\alpha}\right)$

$B(\alpha) = \int_{-2}^{-1} \sin\alpha x \, dx + \int_1^2 \sin\alpha x \, dx$

$= 0$

$f(x) = \sum_{\alpha=1}^{\infty} 2\left(\frac{\sin 2\alpha - \sin\alpha}{\alpha}\right) \cos\alpha x$

2. (a)

$A(\alpha) = \int_{-\pi}^{\pi} x \cdot \cos 2x \, dx = 0$

$B(\alpha) = \int_{-\pi}^{\pi} x \cdot \sin\alpha x \, dx = 2\int_0^{\pi} x \cdot \sin\alpha x \, dx$

$= 2\left(-\frac{\cos 2x}{2} \cdot x \Big|_0^{\pi} + \frac{1}{2}\int_0^{\pi} \cos\alpha x \, dx\right)$

$= -\frac{2\cos\alpha\pi}{\alpha}$

$f(x) = \sum_{\alpha=1}^{\infty} -\frac{2\cos\alpha\pi}{\alpha} \sin\alpha x$

$f(x) = \sum_{\alpha=1}^{\infty} -\frac{2\cos\alpha\pi}{\alpha} \sin\alpha \cdot x$

2. (b)

$A(\alpha) = \int_{-1}^{0} -x^2 \cdot \cos\alpha x \, dx + \int_0^1 x^2 \cos\alpha x \, dx$

$= 0$

$B(\alpha) = 2 \cdot \int_0^1 x^2 \cdot \sin\alpha x \, dx$

$= 2\left(-x^2 \frac{\cos\alpha x}{\alpha} + \frac{2}{\alpha^2}\sin\alpha x + \frac{2}{\alpha^3}\cos\alpha x\right)\Big|^1$

$= -\frac{2}{\alpha}\cos\alpha + \frac{4}{\alpha^2}\sin\alpha + \frac{4}{\alpha^3}\cos\alpha - \frac{4}{\alpha^3}$

$f(x) = \sum_{\alpha=1}^{\infty}\left(-\frac{2}{\alpha}\cos\alpha + \frac{4}{\alpha^2}\sin\alpha + \frac{4}{\alpha^3}\cos\alpha - \frac{4}{\alpha^3}\right)\sin\alpha x$

3.

$A(\alpha) = \int_0^2 (2x) \cdot x \cdot \cos\alpha x \, dx = 2\int_0^2 x \cdot \cos\alpha x \, dx - \int_0^2 x^2 \cdot \cos\alpha x \, dx$

$= 2\left(\frac{\sin\alpha x}{\alpha} x + \frac{\cos\alpha x}{\alpha^2}\right)\Big|_0^2 - \left(x^2 \frac{\sin\alpha x}{\alpha} + \frac{2}{\alpha^2}\cos\alpha \cdot x^2 - \frac{2}{\alpha^3}\sin\alpha x\right)\Big|_0^2$

$= \frac{2}{\alpha^3}\sin 2\alpha - \frac{2}{\alpha^2}\cos 2\alpha - \frac{2}{\alpha^2}.$

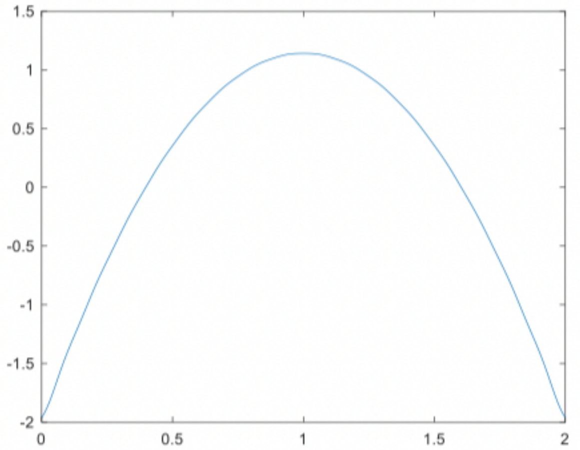
$B(\alpha) = \int_0^2 (2-x) \cdot x \cdot \sin\alpha x \, dx$

$= 2\int_0^2 x \cdot \sin\alpha x \, dx - \int_0^2 x^2 \cdot \sin\alpha x \, dx$

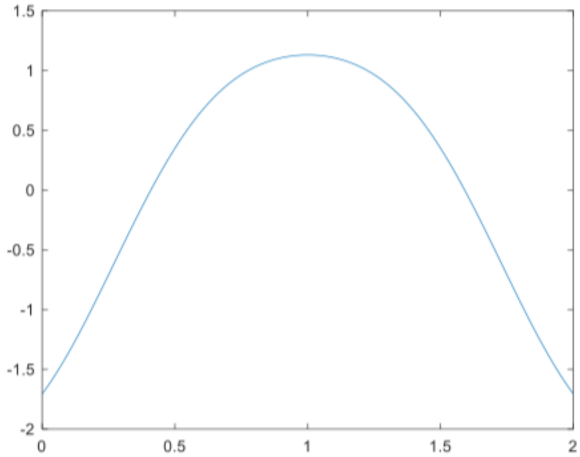
$= \frac{2}{\alpha^3} - \frac{2}{\alpha^2}\sin 2\alpha - \frac{2}{\alpha^3}\cos 2\alpha$

The graph of Q3.

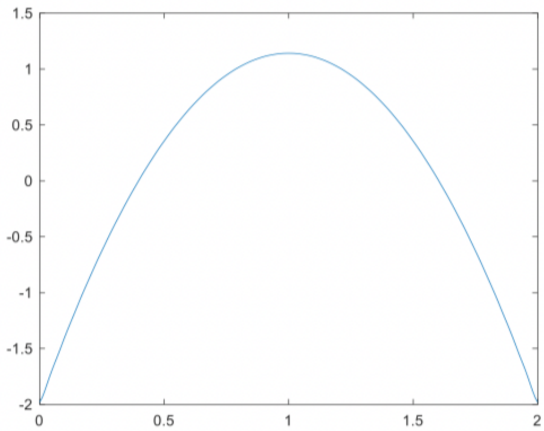
When a=5



When a=50



When a=100



4. (1) Sol

$\cancel{C_{(\alpha)} =}$

$$C_{(\alpha)} = \int_{-\infty}^{\infty} f(x)\, e^{-j\alpha x}\, dx = \int_{0}^{2} e^{-j\alpha\cdot x} dx$$
$$= -\frac{e^{j\alpha x}}{\alpha j}\Big|_0^2$$
$$= -\frac{1}{\alpha j}\left(e^{-2j\alpha} - 1\right)$$

$$\therefore f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{\alpha j}\left(e^{-2\alpha j} - 1\right) e^{j\alpha\cdot x} \cancel{dx}\, d\alpha$$

4. (2)

$$C_{(\alpha)} = \int_0^{\pi} x\, e^{-j\alpha\cdot x} dx$$
$$= -\frac{e^{-\alpha j x}}{\alpha j}\cdot x \Big|_0^{\pi} - \int_0^{\pi} -\frac{e^{-2j\alpha\cdot x}}{\alpha j} dx$$
$$= -\frac{\pi \cos\alpha\pi}{\alpha j} + \frac{1}{\alpha j}\left(-\frac{e^{-\alpha j x}}{\alpha j}\right)\Big|_0^{\pi}$$
$$= \frac{\cos\alpha\pi - 1}{\alpha^2} + \frac{\pi\cos\alpha\pi}{\alpha} j$$

$$\therefore f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\cos\alpha\pi - 1}{\alpha^2} + \frac{\pi\cos\alpha\pi}{\alpha} j\right)\cdot e^{j\alpha\cdot x}\, d\alpha$$