EE113FZ
Solid State Electronics
Lecture 6: Conduction Models

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What is to be Discussed Today?

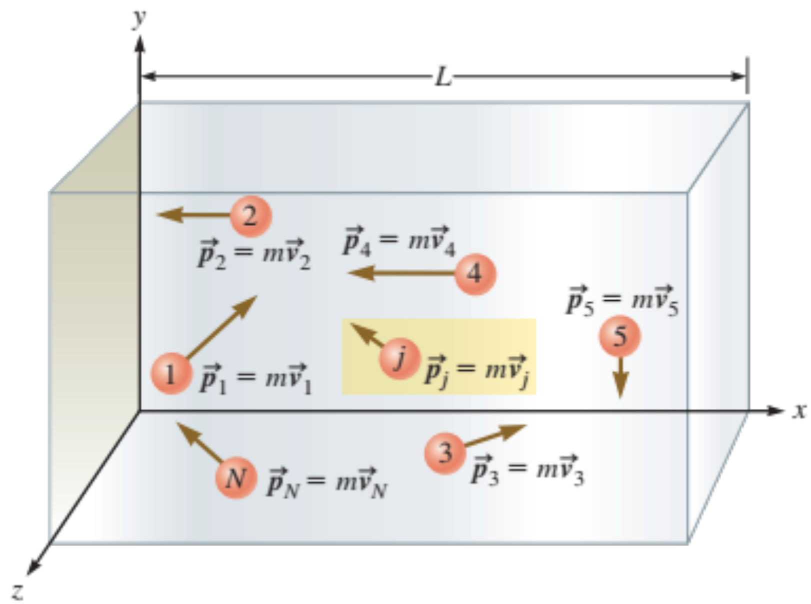
- How does conduction actually happen?
- Going to look at three main conduction models, for completeness sake!
- Drude model (classical free electron model, assumptions, current density & conductivity, carrier mobility, the Hall effect, the Wiedemann-Franz law, and shortcomings);
- Sommerfeld model (Free electron model + Fermi-Dirac distribution);
- Bloch model (Bloch's theorem and some basic ideas about energy bands).

Drude Model (Pinball Model)

- Developed in 1900 by Paul Drude (German physicist);
- It is to explain the transport properties of electrons in materials (especially metals);
- The model, which is an application of the kinetic theory of gases, assumes that the microscopic behaviour of electrons in a solid may be treated classically and looks much like a pinball machine, with a sea of constantly jittering electrons bouncing and re-bouncing off heavier, relatively immobile positive ions;
- It is a classical free electron model (FEM).

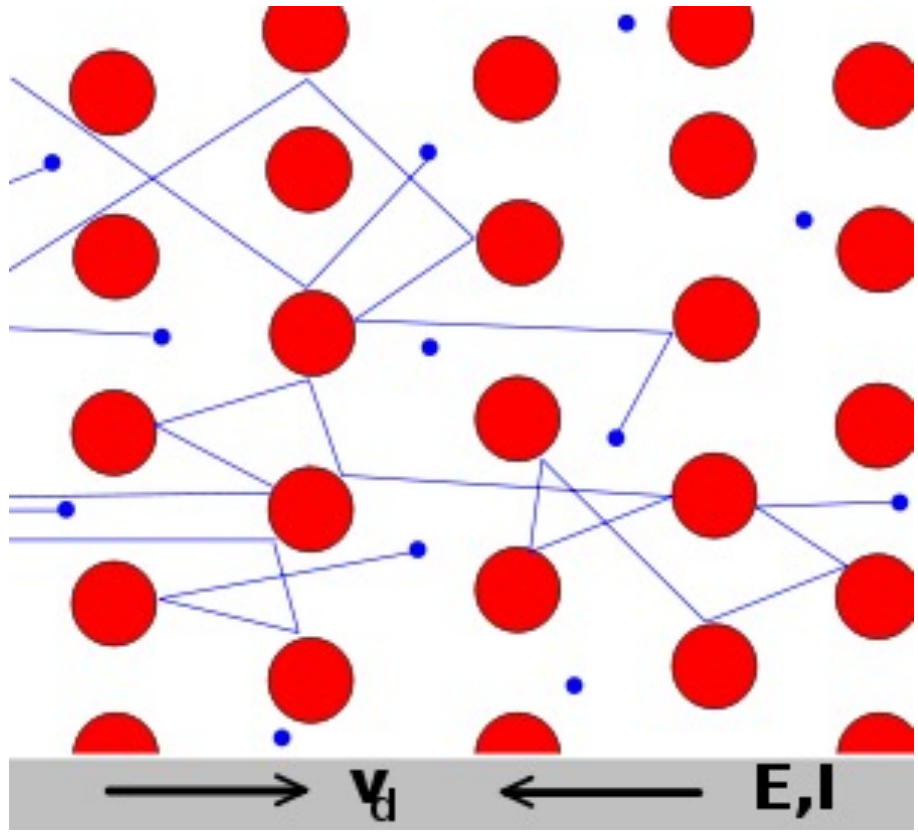
What is the Kinetic Theory of Gases?

- A gas consists of a very large number of identical particle-like molecules or atoms;
- The particles in an ideal gas do not interact with one another except when they collide;
- The particles are free to move in any direction at any speed and they make short elastic collisions with each other and with the walls of the container.



Drude Model

Paul Karl Ludwig Drude (1863 – 1906)
German Physicist



https://en.wikipedia.org/wiki/Drude_model#cite_note-:1-19

Core Assumptions in the Drude Model

- Independent electron approximation: Electrons in a solid behave like a classical ideal gas. There is no Coulomb interaction between them but they also do NOT collide with each other;
- Free electron approximation: Positive charge is located on immobile ion cores. Electrons can collide with ion cores but do not otherwise interact with them;
- Collision assumption: Electrons reach thermal equilibrium with the lattice through collisions. Their mean kinetic energy is $\frac{1}{2}m_e {v_{th}}^2 = \frac{3}{2}k_B T$;
- Relaxation time approximation: Electrons collide (with ions) with a probability of $1/\tau$ per unit time. The relaxation time (τ) is independent of the position and velocity of electrons.

Terms in the Drude Model: An Explanation

- Conduction electron density (n): Number of conduction electrons per unit volume measured in m^{-3};
- Relaxation time (τ): Average time between two successive collisions of an electron. The probability of collision per unit time is $1/\tau$;
- Thermal velocity (v_{th}): Typical velocity due to thermal motion of electrons in the free electron gas;
- Mean free path ($\lambda = v_{th}\tau$): The average distance an electron travels between two successive collisions;
- Drift velocity (v_d): Additional velocity conduction electrons acquired in an electric field;
- Carrier mobility (μ): The ability of charged particles to move in an electric field measured in $m^2/(V \cdot s)$.

Drift Velocity of Conduction Electrons

- The motion of an electron under an electric field (E) can be described as (this is simply Newton's 2nd Law):

$$m_e \frac{dv}{dt} = -eE$$

- Its solution is $v(t) = v_0 + \frac{-eEt}{m_e}$. We already know that the average time between successive collisions is the relaxation time, τ. Hence, the drift velocity can be defined as*

$$v_d = \frac{-eE\tau}{m_e}$$

This is the average velocity of the electrons moving in a direction set by the applied electric field.

*This conclusion is not as straight-forward as it first appears. But let's ignore all the complications for now.

Current Density & Ohm's Law

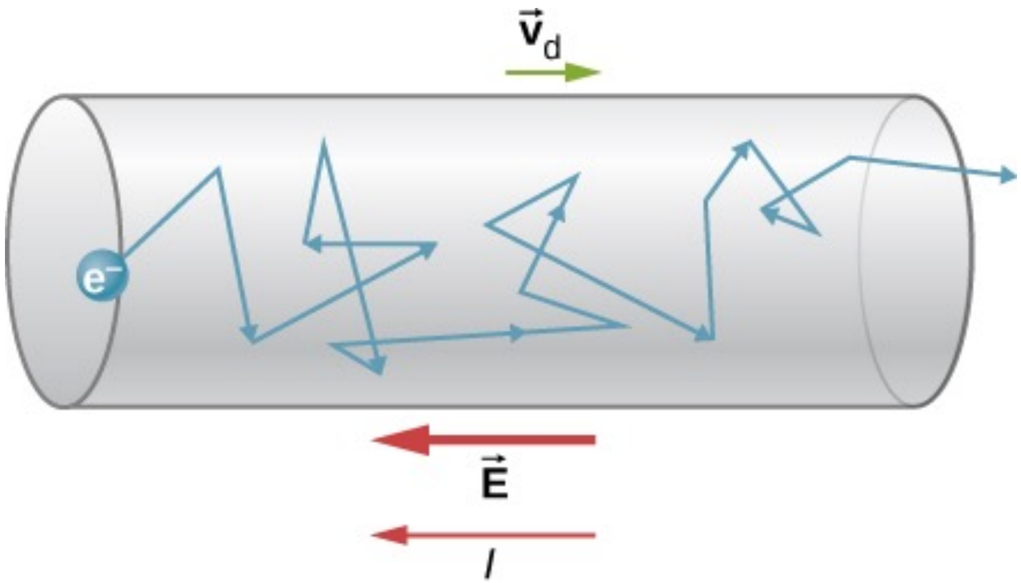
- For a conductor with a cross-sectional area A perpendicular to the electric field, the number of electrons going through this area per unit time is nv_dA and the charge is therefore $-env_dA$;
- Current density:

$$j = -env_d = \frac{ne^2\tau}{m_e}E = \sigma E$$

in which $\sigma = \frac{ne^2\tau}{m_e} = 1/\rho$ is the conductivity (S/m, siemens per metre) and ρ is the resistivity (ohm·m). This is simply the Ohm's law.

Watch out! The conductivity/resistivity does not depend on the sign of the charge carriers.

Diagram: Drift Velocity & Current Density



https://openstax.org/books/university-physics-volume-2/pages/9-2-model-of-conduction-in-metals

- At RT, $v_{th} = \sqrt{3k_B T/m_e} \approx 1.2{\times}10^5$ m/s.

$$|v_d| = \frac{eE\tau}{m_e} \approx 1.7{\times}10^{-2} \text{ m/s}$$

(using E = 10 V/m and a typical τ of 10^{-14} s).

$v_d << v_{th}$.

Carrier Mobility

- One defines the carrier mobility as

$$\mu = e\tau/m_e$$

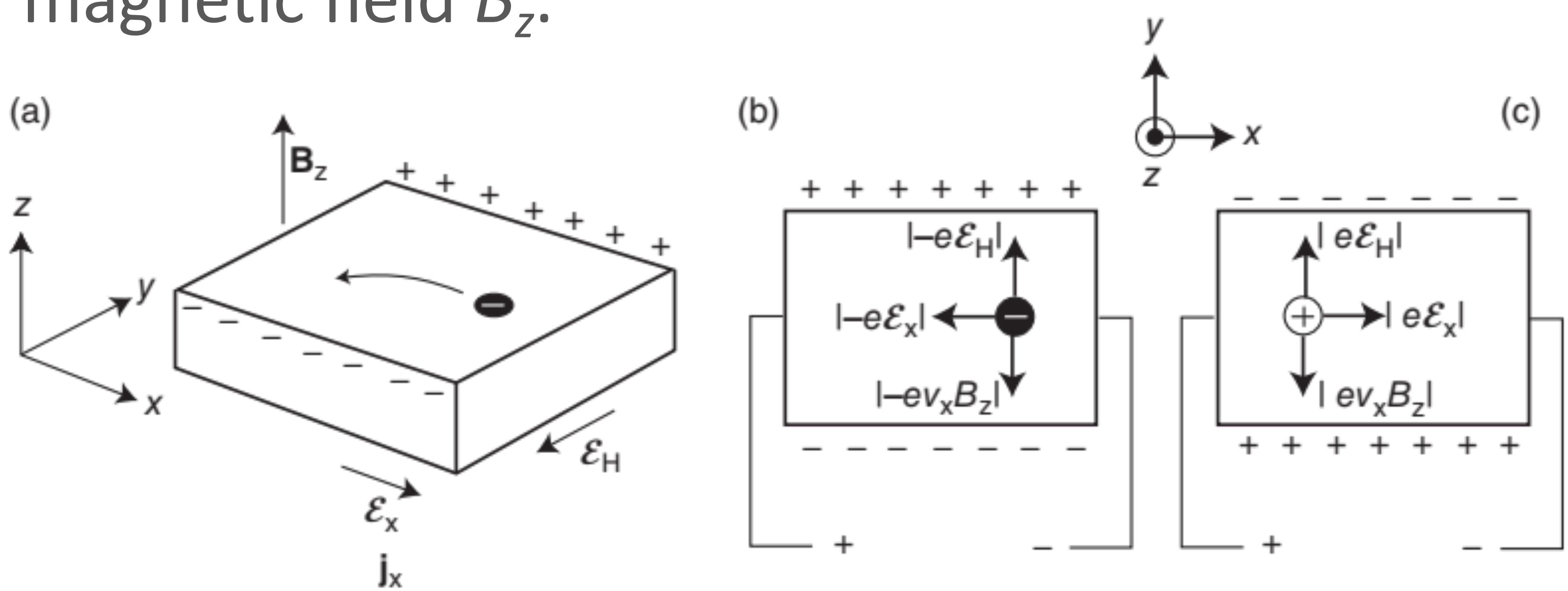
- The conductivity and resistivity can then be expressed as

$$\sigma = n\mu e \text{ and } \rho = 1/(n\mu e)$$

- Mobility is the ratio of the magnitude of the drift velocity and the applied electric field ($|v_d|/E$). Mobility does NOT depend on the carrier concentration (n). If one can control the carrier concentration without changing the relaxation time, one can control the conductivity and the resistivity of a material.

Hall Effect (1879)

- An electric field E_H is built up that is perpendicular to both the magnetic field and the current density;
- The magnitude of the Hall electric field is proportional to both the current density j_x and the magnetic field B_z.



Drude Model & Hall Effect

- Drude model can be used to (at least qualitatively) explain the Hall effect;
- For an electron to pass through the sample, the Lorentz force (due to the magnetic field) and the electric force (due to the Hall electric field) have to be of the same magnitude but point in opposite directions:

$$|-eE_H| = |-ev_d B_z|$$

- Applying the expression of $j_x = -nev_d$ in the Drude model:

$$E_H = v_d B_z = \frac{j_x}{-en} B_z = R_H j_x B_z$$

in which $R_H = -1/(ne)$ is called the Hall coefficient.

Measuring the Hall effect can lead to information on the type and concentration of charge carriers.

Drude Model & Thermal Conductivity

- It is known that the ratio of thermal (k) to electronic conductivity (σ) is a constant for all metals at a given temperature. This is the Wiedemann-Franz law (1853):

$$\frac{k}{\sigma} = LT$$

in which L is called the Lorentz number.

- Electrons in metals not only carry current but also heat;
- One of the major successes of the Drude model is that it can give a qualitative explanation of the Wiedemann-Franz law. However, the Lorentz number given by the Drude model, $L = (\frac{3}{2})(\frac{k_B}{e})^2$ is in fact a factor of 2 too small!

Electronic Molar Specific Heat Capacity

- Heat capacity is the amount of heat to be provided to an object to produce a unit rise of its temperature while molar specific heat capacity is the heat capacity per mole of substances;
- In a conducting solid, both the ion cores and the electrons ought to contribute to the heat capacity;
- However, it is experimentally known that at room temperature, the molar specific heat capacity for a large number of materials is approximately $(3/2)R$ (the Dulong-Petit law). This is only the molar specific heat capacity of the ion cores (lattice) and the electronic molar specific heat capacity is hardly observable.

Shortcomings of the Drude Model

- The most critical failure of the Drude model is that it fails (almost completely) to explain the molar specific heat capacity of conduction electrons.

  Drude model: $c_{e,v} = \left(\frac{3}{2}\right) R$

  Experimentally: $c_{e,v} \approx 10^{-4} RT = 0.03R$ at RT.
- In the Drude model, all free electrons contribute to the specific heat. However, the electron specific heat is much much less in reality (many free electrons cannot absorb thermal energy).

In fact, with what you have learnt in the last lecture on electronic materials, you can already take a qualitative guess on where the inconsistency lies.

Sommerfeld Model

- The Sommerfeld model is a quantum mechanical free electron model. It assumes that electrons interact with neither the ion cores nor other electrons;
- Possible energy levels of electrons are found by solving the Schrödinger equation under certain boundary conditions (don’t have to care about it if you don’t understand what it really means). As you may have expected, these energy levels are ‘quantised’;
- Electrons fill up these permitted energy levels according to the Fermi-Dirac distribution.

Fermi-Dirac Distribution

- According to the Fermi-Dirac distribution (it is a result of the Pauli exclusion principle), the occupation possibility of an electron energy state in metal is:

$$f(E,T) = \frac{1}{e^{(E-E_F)/k_B T}+1}$$

- E_F is called the Fermi level. It can be understood as the energy level where there is a 50% chance of finding an electron (you can see it for yourself by setting $E = E_F$ in the above equation).

We will come back to some of these concepts later in this course module. For now, you only need to know that these concepts exist.

Bloch Model

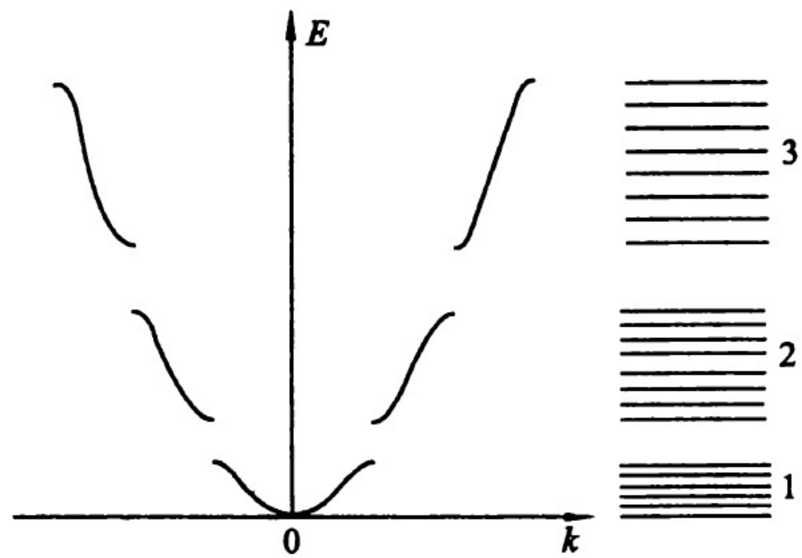
- In 1928, Bloch (Heisenberg's first graduate student) considered the effect periodic potential (of ions) on the movement of electrons;
- He proved that the wave function of an electron moving in a periodic potential satisfies the following equations:

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\boldsymbol{k}}(\boldsymbol{r})$$
$$u_{\boldsymbol{k}}(\boldsymbol{r}+\boldsymbol{R}) = u_{\boldsymbol{k}}(\boldsymbol{r})$$

- This is called Bloch's theorem. Here, ψ is the wavefunction of the electron, and u is a periodic function, which has the same period of the crystal potential.

Bloch Model & Band Structure

- Based on Bloch's theorem, Ralph Kronig and William Penney proposed the Kronig-Penney model in 1931. The model represents the periodic ion potential as an infinite periodic array of regular potential barriers;
- One of the key findings of the model is that there are certain values of energy for which no eigenfunction (solution) of the Schrödinger equation exists. These values constitute the band gap.



Summary: Conduction Models

Drude
Classical Free Electron
Model

Independent electron approximation
Free electron approximation
Collision assumption
Relaxation time approximation

Sommerfeld
Quantum Mechanical
Free Electron Model

Independent electron approximation
Free electron approximation
No collision
Quantum statistics: Fermi-Dirac Distribution

Energy band theory

Free Electron Model
+
Periodical potential field