Lance 83200211

Assign 6

1.(a) sol. $f(t) = te^{-t}\cos(2t)$

$L\{t\cdot e^{-t}\cos 2t\} = -\frac{d}{ds}\cdot F(s)$

$= -\frac{d}{ds}L\{e^{t}\cdot\cos 2t\}$

$= -\frac{d}{ds}\left(\frac{s+1}{(s+1)^2+4}\right) = \frac{(s+1)^2-4}{[(s+1)^2+4]^2}$

2. (a) $y''+y' = e^{-t}\cos t$ $\begin{cases} y(0)=0 \\ y'(0)=0 \end{cases}$

sol

$s^2Y(s) - sy(0) - y'(0) + sY(s) - y(0) = \frac{s+1}{(s+1)^2+1}$

$Y(s) = \frac{1}{s}\cdot\frac{1}{(s+1)^2+1} = \left[\frac{1}{2}\cdot\frac{1}{s} - \frac{(\frac{1}{2}s+1)}{(s+1)^2+1}\right]$

$y = \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)$

(b) sol.

$sY(s) - y(0) + 2Y(s) = L\{f(t)\} = L\{1-2u(t-1)\}$

$(s+2)Y(s) = \frac{1}{s} - 2\frac{e^{-s}}{s}$.

$Y(s) = \frac{1}{s(s+2)} - \frac{2}{s(s+2)}\cdot e^{-s}$.

$y = L^{-1}\{Y(s)\} = \frac{1}{2}(1-e^{-2t}) - (1-e^{-2(t-1)})$

(c) sol.

$sY(s) - y(0) + 3Y(s) = L\{1-u(t-1)\}$

$(s+3)Y(s) = \frac{1}{s} - \frac{e^{-s}}{s}$.

$Y(s) = \frac{1}{s(s+3)} - \frac{1}{s(s+3)}\cdot e^{-s}$

$y = y(t) = \frac{1}{3}(1-e^{3t}) - \frac{1}{3}(1-e^{-3(t-1)})$

3.(a) sol $f(t) = t^3 * t\cdot e^{-t}$.

$F(s) = L\{t^3 * t\cdot e^{-t}\}$

$= \int_0^\infty \left[\int_0^t \tau^3 (t-\tau)e^{-(t-\tau)}d\tau\right] e^{-st}dt$

$= \cancel{\int_0^\infty}$

Let $t-\tau = u$, so $t = u+\tau$

$dt = du + d\tau = du$

$F(s) = \int_0^\infty \left[\int_0^\infty e^{-s\tau}\cdot\tau^3 d\tau\right]\left[\int_0^\infty u\cdot e^{-u}\cdot e^{-su}du\right]$

$= L\{t^3\}\cdot L\{t\cdot e^{-t}\} = \frac{6}{s^4}\cdot\frac{1}{(s+1)^2} = \frac{6}{s^4(s+1)^2}$.

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3.(b)

$F(s) = \mathcal{L}\{e^{2t} * \sin 3t\}$

$= \mathcal{L}\{e^{2t}\} \cdot \mathcal{L}\{\sin 3t\}$

$= \frac{1}{s-2} \cdot \frac{3}{s^2+9}$

$= \frac{3}{(s-2)(s^2+9)}.$

4.(a) $\mathcal{L}\{\int_0^t \tau \sin\tau \, d\tau\}$

sol. $= (t \cdot \sin t) * 1$

$\therefore F(s) = \mathcal{L}\{t \cdot \sin t\} \cdot \mathcal{L}\{1\}$

$= -\frac{d}{ds}\mathcal{L}\{\sin t\} = \frac{2s}{(s^2+1)^2}.$

$\therefore F(s) = \frac{2}{(s^2+1)^2}.$

(b) $\mathcal{L}\{\int_0^t 2\sin\tau \cos(t-\tau)\, d\tau\}$

sol $= 2\sin t * \cos t$

$\therefore F(s) = \mathcal{L}\{2\sin t\} \cdot \mathcal{L}\{\cos t\}$

$= \frac{2}{s^2+1} \cdot \frac{s}{s^2+1}$

$= \frac{2s}{(s^2+1)^2}.$

5.(a) $F(s) + \mathcal{L}\{f(t) * 1\} = \frac{1}{s}.$

$F(s) + F(s) \cdot \frac{1}{s} = \frac{1}{s}$

$\therefore F(s) = \frac{1}{s+1}$ $\therefore f(t) = e^{-t}$

(b) sol. $s^2 Y(s) - s y(0) - y'(0) + 9Y(s) = \frac{s}{s^2+9}$

$\therefore (s^2+9)Y(s) - s - 4 = \frac{s}{s^2+9}$

$\therefore Y(s) = \frac{s}{(s^2+9)^2} + \frac{s+4}{(s^2+9)}.$

$\therefore y = \frac{1}{6} t \sin 3t + \cos 3t + \frac{4}{3}\sin 3t.$

5.(c) $s^2 Y(s) - s y(0) - y'(0) + 4(sY(s) - y(0))$

sol. $+ 13 Y(s) = e^{-\pi s} + e^{-3\pi s}$

$\therefore (s^2+4s+13)Y(s) = (\cdots) + (s+4)$

$\therefore Y(s) = \frac{(\quad,\quad)}{(s+2)^2+9} + \frac{(s+4)}{(s+2)^2+9}$

$\therefore y(t) = e^{-2t}\cos 3t + \frac{3}{2} e^{-2t} \sin 3t +$
$\frac{1}{3} e^{-2(t-\pi)} \sin 3(t-\pi)\, u(t-\pi) +$
$\frac{1}{3} e^{-2(t-3\pi)} \sin 3(t-3\pi)\, u(t-3\pi)$

5.(d)

sol.

$s^2 Y(s) - s y(0) - y'(0) + 2(sY(s) - y(0)) = e^{-s}$

$\therefore (s^2+2s)Y(s) = 1 + e^{-s}$

$Y(s) = \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2} + \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)e^{-s}$

$\therefore y(t) = \frac{1}{2}(1 - e^{-2t}) + \frac{1}{2}(1 - e^{-2(t-1)}) \cdot u(t-1)$

$\therefore$

$y(t) = \frac{1}{2}(1 - e^{-2t}) + \frac{1}{2}(1 - e^{-2(t-1)}) \cdot u(t-1)$

Campus