Date

Assign 9

1.(a) sol

$A(\alpha) = \int_0^{\pi} \sin x \cos\alpha x \, dx$

$= \left(\frac{\sin\alpha x}{\alpha} \cdot \sin x\right)_0^{\pi} - \int_0^{\pi} \frac{\sin\alpha x}{\alpha}(-\cos x) dx$

$= \frac{1}{\alpha} \int_0^{\pi} \cos x \sin\alpha x \, dx$

$= \frac{1}{\alpha}\left[\left(-\frac{\cos\alpha x}{\alpha} \cdot \cos x\right)_0^{\pi} - \int_0^{\pi} \frac{\cos\alpha x}{\alpha} \sin x dx\right]$

$\alpha I = \frac{(-1)^2}{\alpha} + \frac{1}{\alpha} - \frac{1}{\alpha} I.$

$\begin{cases} A(\alpha) = \frac{1+(-1)^{\alpha}}{\alpha^2+1} \\ B(\alpha) = 0 \end{cases}$

$\therefore f(x) = \frac{1}{\pi}\int_0^{\pi} \frac{1+(-1)^2}{\alpha^2+1} \cos\alpha x \, d\alpha.$

(b)

$A(\alpha) = \int_{\pi}^{2\pi} 3\cos\alpha x dx = \frac{3\sin\alpha x}{\alpha}\Big|_{0}^{\pi} = 0$

$B(\alpha) = \int_{\pi}^{2\pi} 3\sin\alpha x dx = -\frac{3\cos\alpha x}{\alpha}\Big|_0^{\pi}$

$= \frac{3[1-(-1)^{\alpha}]}{\alpha}$

So $f(x) = \frac{1}{\pi}\int_{\pi}^{2\pi} \frac{3[1-(-1)^{\alpha}]}{\alpha} \sin\alpha x d\alpha$

(c)

$A(\alpha) = \int_{-\pi}^{\pi} \frac{2}{3}x \cdot \cos\alpha x \, dx$

$= \frac{2}{3}\left(\frac{\sin\alpha x}{\alpha} x + \frac{\cos\alpha x}{\alpha^2}\right)_{-\pi}^{\pi} = 0$

$\cancel{B(\alpha) = \int_{-\pi}^{\pi} \frac{2}{3} \sin\alpha x \, dx}$

$B(\alpha) = \int_{-\pi}^{\pi} \frac{2}{3} x \cdot \sin\alpha x \, dx$

$= \frac{2}{3}\left(-\frac{\cos\alpha x}{\alpha} \cdot x + \frac{\sin\alpha x}{\alpha^2}\right)_{-\pi}^{\pi}$

$= -2\pi \cdot \frac{(-1)^{\alpha}}{\alpha}$

So $f(x) = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{(-1)^{\alpha+1}}{\alpha}(2\pi) \sin\alpha x \, d\alpha.$

sol

2 $A(\alpha)$

$= \int_0^{\infty} e^{-x/2} \cdot \cos\alpha x \, dx$

$= \left(\frac{\sin\alpha x}{\alpha} \cdot e^{-\frac{x}{2}}\right)\Big|_0^{\infty} - \int_0^{\infty} \frac{\sin\alpha x}{\alpha}\left(-\frac{1}{2}e^{-\frac{x}{2}}\right) dx$

$= \frac{1}{2\alpha}\int_0^{\infty} e^{-\frac{x}{2}} \sin\alpha x \, dx$

$= \frac{1}{2\alpha}\left(\frac{1}{\alpha} - \frac{1}{2\alpha}\int_0^{\infty} e^{-x/2} \cos\alpha x \, dx\right)$

$2\alpha I = \frac{1}{\alpha} - \frac{1}{2\alpha} \cdot I$

$A(\alpha) = I = \frac{2}{(4\alpha^2+1)}.$

$B(\alpha) = \int_0^{\infty} e^{-x/2} \sin\alpha x \, dx$

$= \frac{1}{\alpha} - \frac{1}{2\alpha} \cdot \frac{2}{4\alpha^2+1}$

$= \frac{4\alpha}{4\alpha^2+1}$

So $f(x) = \frac{2}{\pi}\int_0^{\infty} \frac{2}{4\alpha^2+1} \cos\alpha x \, d\alpha$

$f(x) = \frac{2}{\pi}\int_0^{\pi} \frac{4\alpha}{4\alpha^2+1} \sin\alpha x \, d\alpha$

KOKUYO

3.(a)

$$F(\alpha)=\int_{-1}^{1} e^{2ix}\cdot e^{-i\alpha x}\,dx$$
$$=\int_{-1}^{1} e^{i(2-\alpha)x}\,dx$$
$$=\frac{1}{(2-\alpha)i}\, e^{i(2-\alpha)x}\Big|_{1}^{-1}$$
$$=\frac{e^{i(2-\alpha)}-e^{i(\alpha-2)}}{(2-\alpha)i}.$$
$$=\frac{2\sin(2-\alpha)}{2-\alpha}.$$

(b)

$$F(\alpha)=\int_{-\infty}^{0} e^{x}\cdot e^{-i\alpha x}\,dx+\int_{0}^{\infty} e^{-x}\cdot e^{-i\alpha x}\,dx$$
$$=\frac{1}{1-i\alpha}\, e^{(1-i\alpha)x}\Big|_{-\infty}^{0} - \frac{1}{1+i\alpha}\, e^{-(1+i\alpha)x}\, dx\Big|_{0}^{\infty}$$
$$=\frac{1}{1-i\alpha}+\frac{1}{1+i\alpha}$$
$$=\frac{2}{1+\alpha^2}.$$

(c)

$$F(\alpha)=\int_{-1}^{1} x\cdot e^{-i\alpha x}\,dx$$
$$=\left(-\frac{1}{i\alpha}e^{-i\alpha x}\cdot x\right)_{-1}^{1}-\int_{-1}^{1}\left(-\frac{1}{i\alpha}e^{i2x}\right)dx$$
$$=-\frac{e^{-i\alpha}+e^{i\alpha}}{i\alpha}+\frac{1}{i\alpha}\cdot\left(-\frac{1}{i\alpha}e^{-i\alpha x}\right)_{-1}^{1}$$
$$=-\frac{2\cos\alpha}{i\alpha}+\frac{1}{\alpha^2}\left(e^{-i\alpha}-e^{i\alpha}\right)$$
$$=\frac{2i\cos\alpha}{\alpha}-\frac{2i\sin\alpha}{\alpha^2}.$$