EE311FZ Control System Design — Project 1

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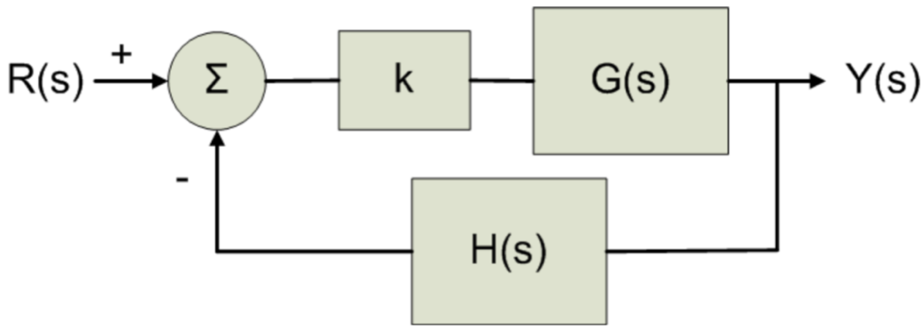

Figure 1 The Target Feedback System

The components of this feedback system are as follows:

$$G(s) = \frac{6}{s(s+3)}$$

$$H(s) = \frac{1}{s+1}$$

Procedure 1 — Routh criterion

As Figure 1 shown, we can easily get the closed-loop transfer function of the system:

$$C(s) = \frac{k \cdot G(s)}{1 + k \cdot G(s) \cdot H(s)} \tag{1}$$

So,

$$C(s) = \frac{6k \cdot (s+1)}{s(s+1)(s+3) + 6k} = \frac{6k \cdot (s+1)}{s^3 + 4s^2 + 3s + 6k} \tag{2}$$

And the characteristic equation of the system is,

$$D(s) = s^3 + 4s^2 + 3s + 6k \tag{3}$$

We can draw the Routh table of this characteristic equation, as shown in Table 1.

Table 1 Routh Table of the equation

$D(s)$	$s^3+4s^2+3s+6k$	
s^3	1	3
s^2	4	$6k$
s^1	(6-3k)/2	
s^0	6k	

To make sure the system can be stable, we need:

$$\begin{cases} 6-3k>0 \\ k>0 \end{cases} \tag{4}$$

So, we get that the range of the proportional gain k should be:

$$0<k<2 \tag{5}$$

Procedure 2 — Nyquist Stability Criterion

When the proportional gain $k=1$, the open-loop transfer function of the system is:

$$G(s)H(s)=\frac{6}{(s)\cdot(s+1)\cdot(s+3)}=\frac{6}{s^3+4s^2+3s} \tag{6}$$

In this case, the Nyquist plot for the system is shown in Figure 2,

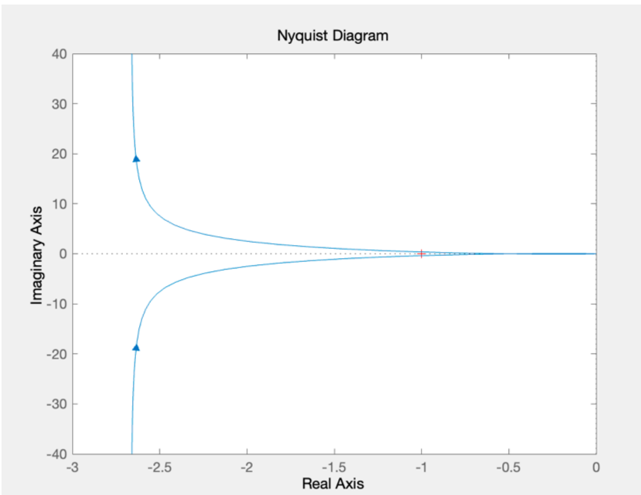

Figure 2 The Nyquist Plot of the System (when k=1)

The following Table 2 shows the MATLAB program used to draw the Nyquist Plot:

Table 2 MATLAB Program 1

Table 2: MATLAB Program 1
This program is used to draw the Nyquist Plot

```
%% Q2 Nyquist Plot
num2 = [6];
den2 = [1,4,3,0];
sys2 = tf(num2,den2);
nyquist(sys2);
```

As illustrated in the Figure 3, the Nyquist Plot of the system (when k=1) does not enclose (-1,0). So, we can get that:

$$N = 0 \tag{7}$$

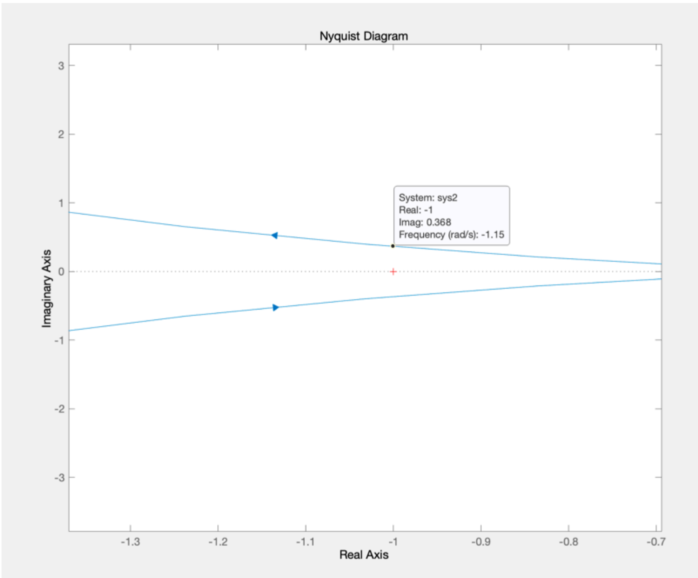

Figure 3 The Nyquist Plot of the System (After partial magnification)

And, considering the roots of $G(s)H(s)$,

$$\begin{cases} p_1 = 0 \\ p_2 = -1 \\ p_3 = -3 \end{cases} \tag{8}$$

All of the roots of $G(s)H(s)$ meet $p \leq 0$, then we know

$$P = 0 \tag{9}$$

We have already known that $Z = P - N$, so we can get:

$$Z = 0 \tag{10}$$

To summarize, due to $Z = 0$, we can confirm the system is stable.

Procedure 3 — Bode Plot

When the proportional gain $k = 1$, the open-loop transfer function of the system is:

$$G(s)H(s) = \frac{6}{(s) \cdot (s+1) \cdot (s+3)} = 1 \cdot \frac{1}{s} \cdot \frac{6}{(s+1)(s+3)} \tag{11}$$

So, this function $C(s)$ possesses a constant of 6. And two real pole at $p = -1$ and $p = -3$, and a pole at the origin.

$$\begin{cases} p_1 = -3 \\ p_2 = -1 \\ p_3 = 0 \end{cases} \tag{12}$$

Therefore, we know the system have the following terms:

$$\begin{cases} \text{Constant: } K = 6.02 \text{ dB} \\ \text{Real Pole: } \omega_{p1} = 1.00 \\ \text{Real Pole: } \omega_{p2} = 3.00 \\ \text{Pole at Origin} \end{cases} \tag{13}$$

In this case, we can draw the Bode plots of each of the individual terms enumerated above, as shown in Figure 4 and Figure 5. The bule line represents the constant, and the red line represents the pole at origin. The yellow line represents the real pole at 3.00, while the purple line represents the real pole at 1.00.

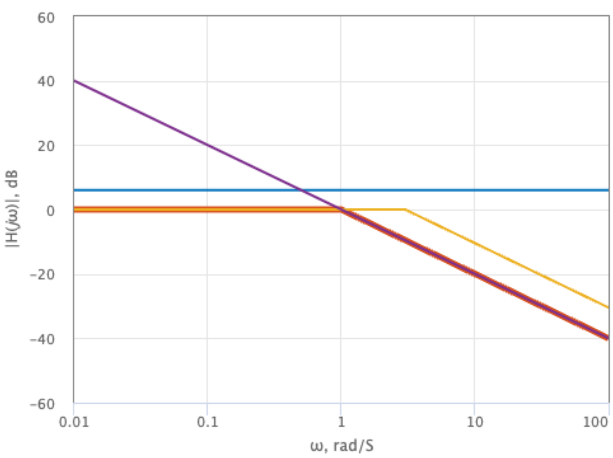

Figure 4 The Bode Plot of Each of the Individual Terms (Magnitude)

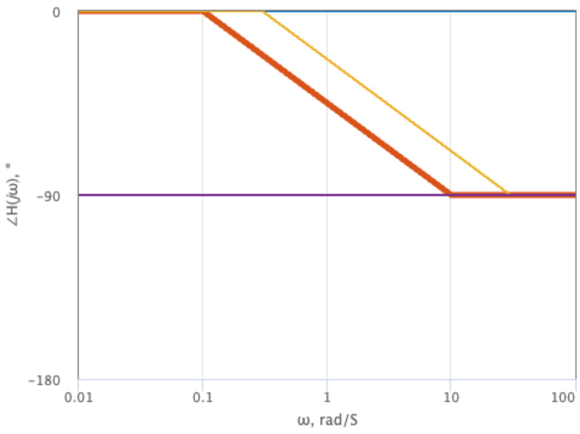

Figure 5 The Bode Plot of Each of the Individual Terms (Phase)

then, we can get the key points of the asymptotic approximation:

$$\begin{cases} (1.00\ \mathrm{rad/s}, 6.02\ \mathrm{dB}) \\ (3.00\ \mathrm{rad/s}, -13.1\ \mathrm{dB}) \end{cases} \tag{14}$$

and we can also calculate the key points of the standard magnitude plot:

$$\begin{cases} (1.00\ \mathrm{rad/s}, 2.54\ \mathrm{dB}) \\ (3.00\ \mathrm{rad/s}, -16.5\ \mathrm{dB}) \end{cases} \tag{15}$$

Finally, we can draw the Bode Diagram using the asymptotic approximation and the MATLAB calculation as following Figure 6, where the red lines represent the asymptotic approximation, while the blue lines represent the standard Bode Plot [1].

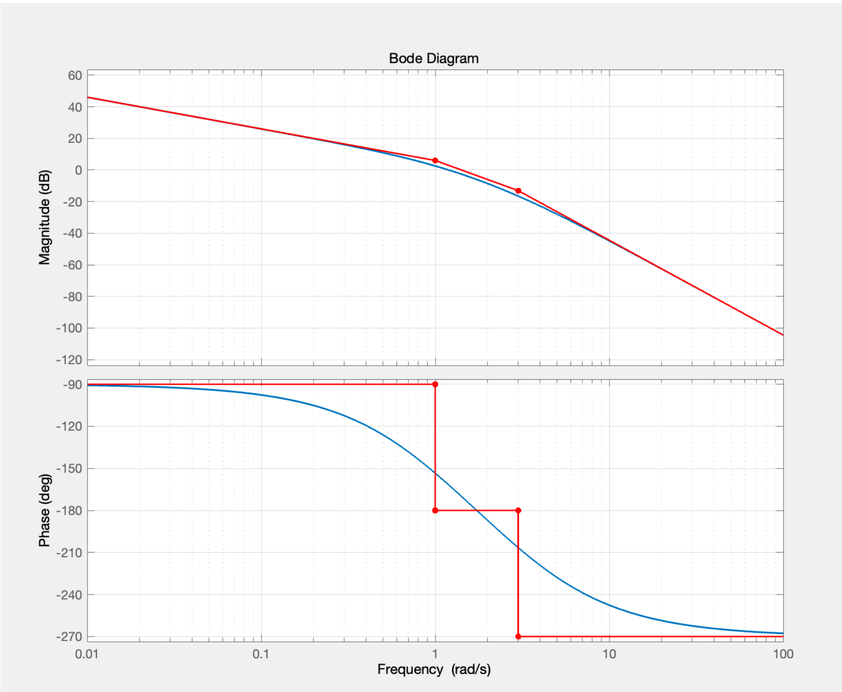

Figure 6 The Asymptotic Approximation (Red) and the Standard Bode Plot (Blue)

Table 3 MATLAB Program 2

Table 3: MATLAB Program 2
This program is used to draw the Bode Plot

```
%% Q3 Bode Plot
num3 = [6];
den3 = [1,4,3,0];
sys3 = tf(num3,den3);
asymp(sys3);
% the asymp() function is used to draw the red line of asymptotic approximation, which can be accessed in Ref [1].
```

Procedure 4 — Gain Margin Calculation

We know the open-loop transfer function of the system is

$$G(s)H(s) = \frac{6k}{(s)\cdot(s+1)\cdot(s+3)} \tag{16}$$

Then, the $GH(j\omega)$ would be:

$$GH(j\omega) = \frac{-6k}{\omega^3 j + 4\omega^2 - 3\omega j} = \frac{-6k}{(\omega^3 - 3\omega)j + (4\omega^2)} \tag{17}$$

To find the gain margin, we make the phase $\emptyset = -180°$, therefore, we know

$$(-\omega^3 + 3\omega) = 0 \tag{18}$$

So, the phase crossover frequency is

$$\omega_{pc} = \sqrt{3} \approx 1.732 \tag{19}$$

Then, we can calculate

$$C(j\omega_{pc}) = \frac{-6k}{12} \tag{20}$$

We know the gain margin is $GM = 6$,

$$GM = -20log|C(j\omega)| = -20log\left|\frac{-k}{2}\right| = 6dB \tag{21}$$

Finally, we achieve the result of k,

$$k = 1.0024 \tag{22}$$

In this case, the specific $G(s)H(s)$ would be

$$GH(s) = \frac{6.014}{s^3 + 4s^2 + 3s} \tag{23}$$

The corresponding Nyquist and Bode plots when $k = 1.0024$ are shown in Figure 7 and Figure 8, which agree.

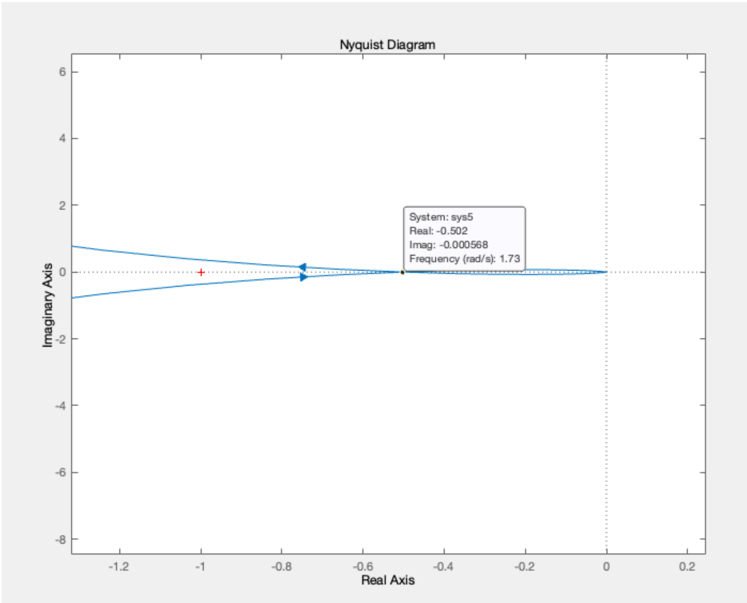

Figure 7 The Corresponding Nyquist Plot (when k=1.0024)

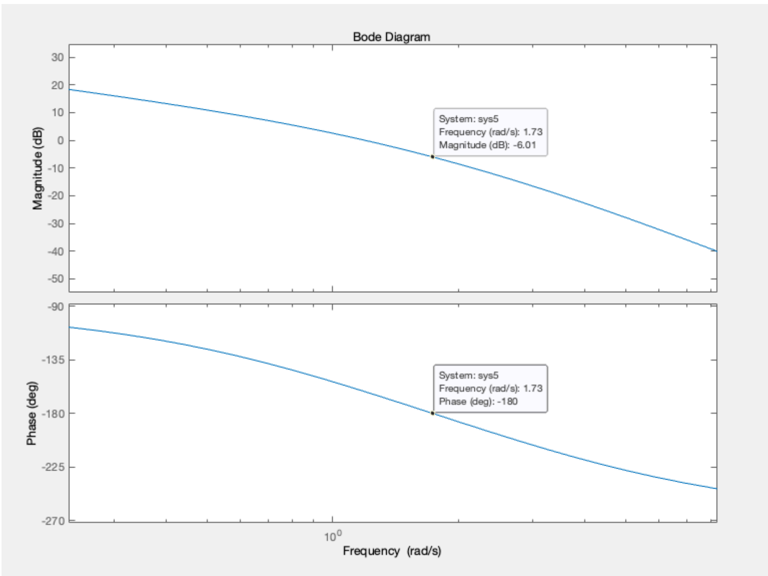

Figure 8 The Corresponding Bode Plot (when k=1.0024)

Now, to perform the gain margin for the Nyquist plot,

$$|GH(j\omega)| = (6.014) \cdot \frac{1}{\sqrt{\omega^2}} \cdot \frac{1}{\sqrt{1+\omega^2}} \cdot \frac{1}{\sqrt{9+\omega^2}} \tag{24}$$

So, at $\omega_{pc} = \sqrt{3}$,

$$|GH(j\omega)| = 0.502 = \frac{1}{K_g} \tag{25}$$

Then, the gain margin is

$$K_g = 2.08 \approx 6 \text{ dB} \tag{26}$$

To summarize, the answer of Nyquist and Bode plot are consistent. And the program utilized to draw the diagrams and calculate the gain margin is shown in Table 4, which further verify the effectiveness of the results.

Table 4 MATLAB Program 3

Table 4: MATLAB Program 3

This program is used to calculate the gain margin (Gm) and phase margin (Pm)

```
%% Q4 Gain Margin
syms K;
K = 1.0024;
num = [6*K];
den = [1,4,3,0];
sys = tf(num,den);
% nyquist(sys);
% bode(sys);
[Gm,Pm,Wcg,Wcp] = margin(sys)
% Gm =  1.9953 ≈ 2 = 6 dB
% Pm =  18.1988
```

Procedure 5— Root Locus Diagram

We know the open-loop transfer function of the system is

$$G(s)H(s) = \frac{6k}{(s)\cdot(s+1)\cdot(s+3)} \tag{27}$$

Figure 9 shows the Root Locus Diagram of the system $GH(s)$, the important points have been annotated as follows. And the MATLAB program is shown in Table 5.

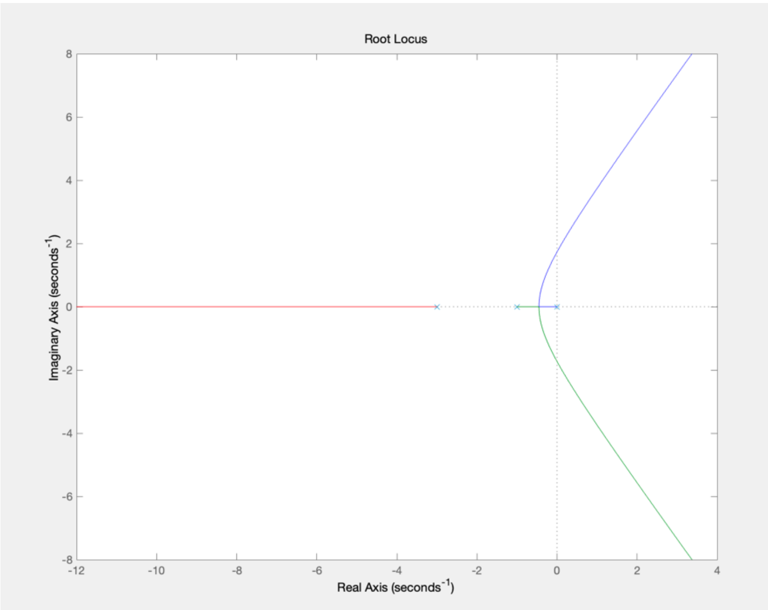

Figure 9-1 The Root Locus Diagram

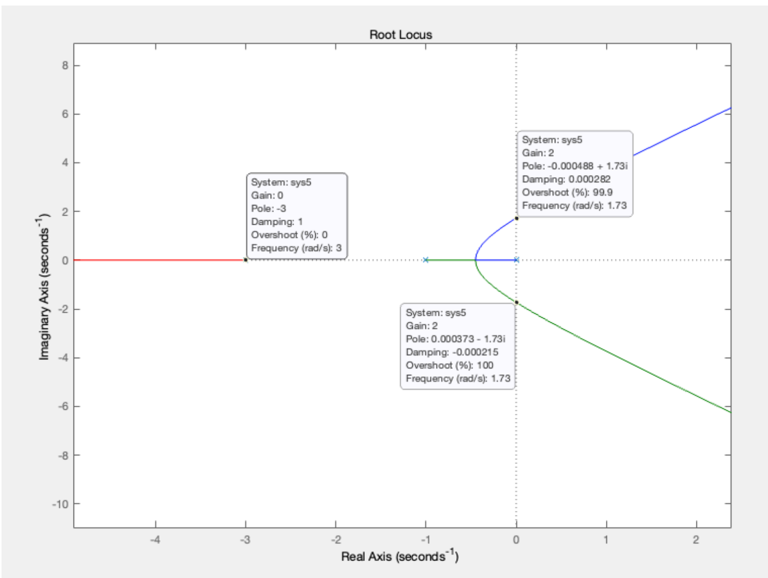

Figure 9-2 The Root Locus Diagram (with annotation)

Table 5 MATLAB Program 4

Table 3: MATLAB Program 4
This program is used to draw the Root Locus Diagram

```
%% Q5 Root Locus Diagram
syms K;
K = 6;
num5 = [K];
den5 = [1,4,3,0];
sys5 = tf(num5,den5);
rlocus(sys5);
```

Moreover, we can also calculate the important points by hand, for the separation points,

$$\frac{1}{d}+\frac{1}{d+1}+\frac{1}{d+3}=0 \tag{28}$$

So the three separation point in the real axis are respectively:

$$\begin{cases} (0,0) \\ (1,0) \\ (3,0) \end{cases} \tag{29}$$

And, as for the intersections of the curve with the imaginary axis. Considering the Rough Table of the characteristic equation $D(s)$, we know

$$4s^2+12=0 \tag{30}$$

Then we get $s=\pm\sqrt{3}\approx\pm1.732$, so the intersections are respectively:

$$\begin{cases} (0,+1.732) \\ (0,-1.732) \end{cases} \tag{31}$$

To summarize, the results are consistent with the Figure 9.

Procedure 6 — Design Specifications

We know the closed-loop transfer function of the system is

$$C(s)=\frac{6k\cdot(s+1)}{s(s+1)(s+3)+6k}=\frac{6k\cdot(s+1)}{s^3+4s^2+3s+6k} \tag{32}$$

We are required to design a system which meets the following specifications:

$$\begin{cases} \sigma\approx 10\% \\ t_s<8s \\ \text{find } (t_r)_{\min} \end{cases} \tag{33}$$

As for the overshoot σ, the overshoot is the peak value of the response curve measured from unity, which is defined by

$$\sigma=\frac{c(t_p)-c(\infty)}{c(\infty)}\times 100\% \tag{34}$$

As for the settling time t_s, is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2%). Finally, for the rise time t_r, is the time required for the response to rise from 10% to 90% of its final value.

Considering the above feedback system $C(s)$ is a high-order (3rd) system, it is not practical to calculate the results directly. In this context, we can simulate this system utilizing MATLAB and try to explore a reasonable solution. Table 6 shows the program used to calculate the performance metrics of the 3rd system [2].

Table 6 MATLAB Program 5

Table 6: MATLAB Program 5

This program is used to calculate the performance metrics of the 3rd system

```
clear;clc;
syms K;
% K = 0.244
% K = 1.0024
num = [6*K,6*K];
den = [1,4,3,6*K];
sys = tf(num,den);
stepinfo(sys);
% the stepinfo() function is used to calculate the performance
metrics of the system, which can be accessed in Ref [2].
G = step(sys);
plot(G);
hold on;
```

Then, the following Table 7 and Figure 10 show the numerical results of simulations utilizing MATLAB.

Table 7 Results of MATLAB Simulations

k	overshoot	Settling time	Rise time
0.216	9.871%	8.6483	2.4609
0.216905	**10.00%** (ζ =)	8.6253	2.4481
0.242898	13.72%	**8.0000**	2.1277
0.244	13.879%	7.9759	2.1162
0.252	15.039%	7.7979	2.0365

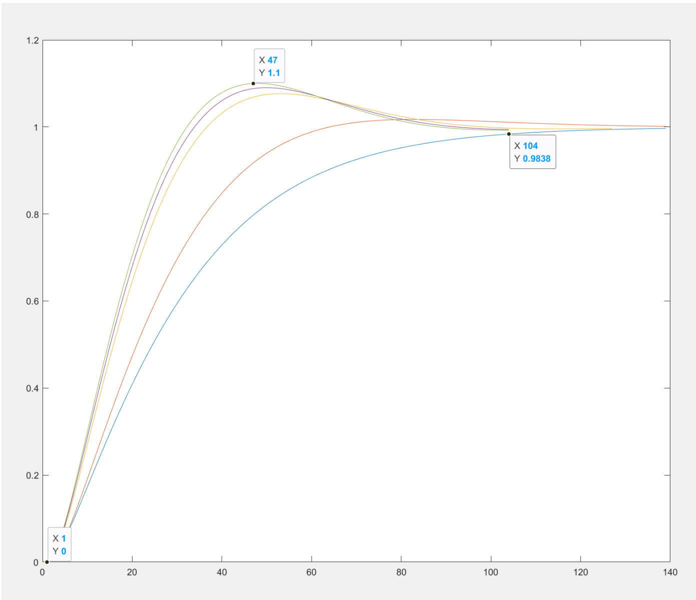

Figure 10 The Step Response of Simulations

Noted that when the overshoot $\sigma = 10\%$, the gain $k = 0.216905$, and the closed-loop transfer function would be,

$$C(s) = \frac{1.301s + 1.301}{s^3 + 4s^2 + 3s + 1.301} = \frac{1.301(s+1)}{(s + 3.187)(s^2 + 0.8132s + 0.4084)} \tag{35}$$

We can draw the Pole-Zero Map of this system, as shown in Figure 11. So, we can see the two complex poles will be the **predominant poles** while the real pole will be the non-dominant pole. In this case, the closed-loop transfer function can be simplified to:

$$C(s) = \frac{1.301(s+1)}{(s + 3.187)(s^2 + 0.8132s + 0.4084)} \Leftrightarrow \frac{1.301(s+1)}{(s^2 + 0.8132s + 0.4084)} \tag{36}$$

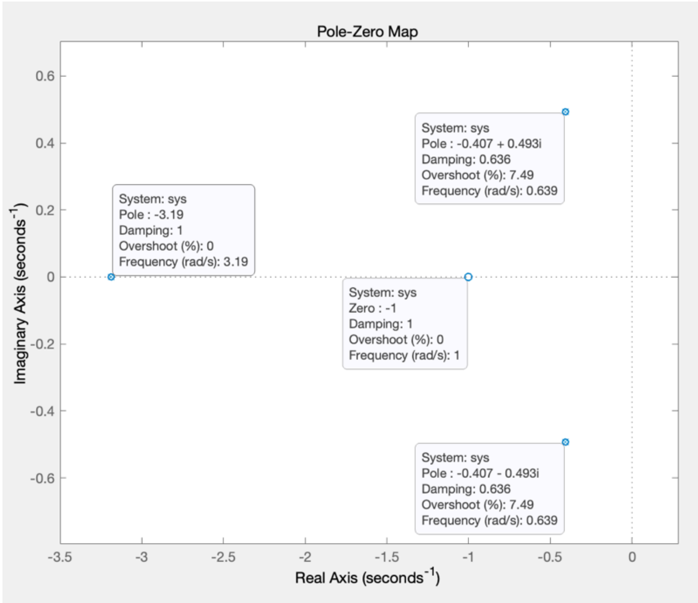

Figure 11 Pole-Zero Map (when k=0.216905)

So, when the overshoot $\sigma = 10\%$, the gain $k = 0.216905$, while the corresponding damping ratio $\boldsymbol{\zeta} = \mathbf{0.63624}$.

Moreover, through simulations, we find that no matter what value of k is, the specifications in the equation (33) **cannot perfectly satisfy**. In this case, we can try to pick a balanced k to meet the design specifications in general.

Compared in Table 7, the value of $\boldsymbol{k} = \mathbf{0.244}$ will be a relatively reasonable value. In this case, the system performance can reach a satisfactory balance. Table 8 shows the performance metrics of the system when $k = 0.244$.

Table 8 Performance Metrics of the System

k	overshoot	Rise time	Peak time	Settling time
0.244	13.879%	2.1162	4.7389	7.9750

As shown in Table 8, when the value of gain $k = 0.244$, the feedback system reaches a balance condition, where the overshoot is around 13.88% and the settling time is about 7,98s which is less than 8.00s, while the rise time is about 2.1162s.

Procedure 7 — System Simulation

As shown in Figure12, we have simulated the feedback system in MATLAB Simulink.

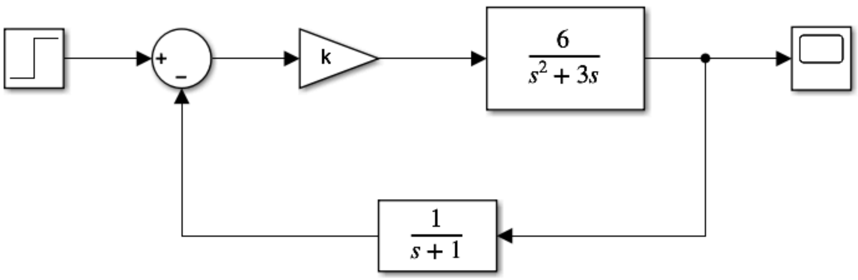

Figure 12 The Simulink Model of the System

We can compare the two different system (when $k = 1.0024$ and $k = 0.244$) through encapsulating the system, as shown in Figure 13.

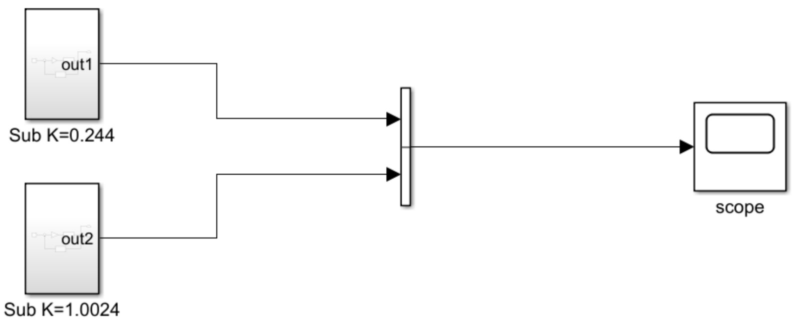

Figure 13 Encapsulation of the two System

The following Figure 14 and Figure 15 compare the step responses of the two systems. And the Table 9 compares the performance metrics of the two system.

Table 9 Performance Metrics of the two System

k	overshoot	Rise time	Peak time	Settling time
0.244	**13.879%**	2.1162	4.7389	**7.9750**
1.0024	104.06%	**0.5655**	**1.9537**	22.2449

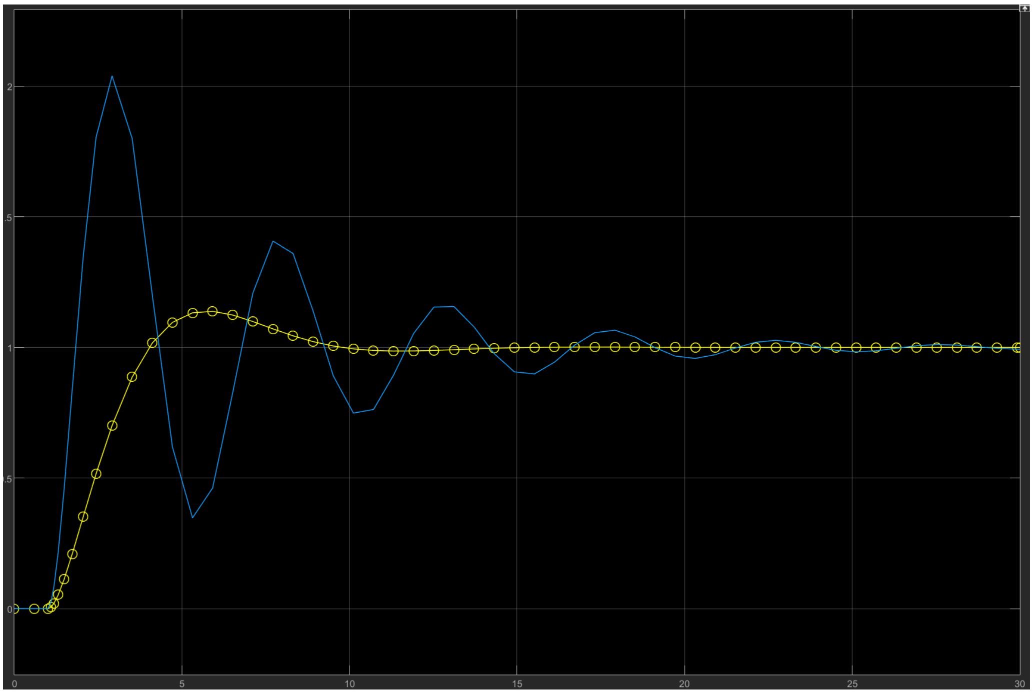

Figure 14 Step Responses of the two Systems (Bule: 1.0024, Yellow: 0.244)

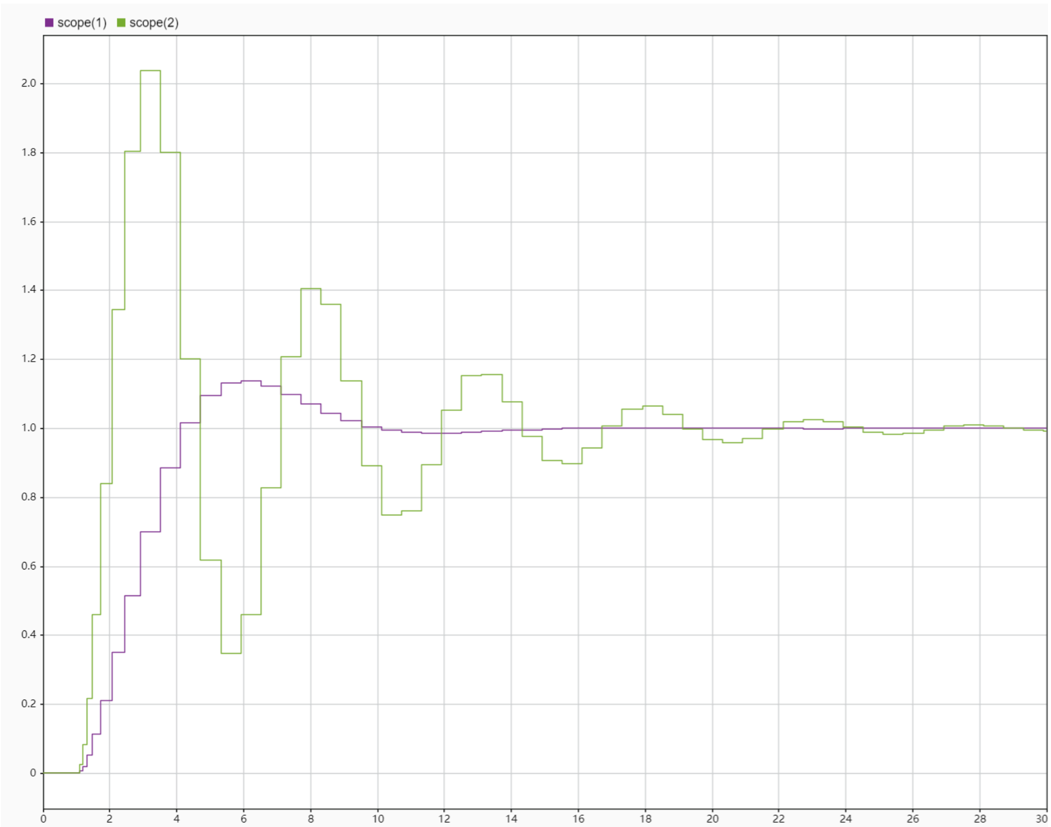

Figure 15 Step Responses of the two Systems (Green: 1.0024, Purple: 0.244)

Procedure 8 — Brief Conclusion

Generally speaking, either the time-domain method utilizing transient response specifications and root locus, or the frequency-domain method using Nyquist and Bode plot, all of these approaches are useful to analyze the performance of a specific system. And each of these analysis approaches possess different advantages and disadvantages. The following Table 10 illustrates my personal understanding of the relative merits of these control system design methodologies.

Table 10 Summary of different control system design methods (by Hanlin CAI)

Analysis Methods	Application Objects	Stability Analysis	Performance Metrics	Design Approaches
Time-domain Method	Accurate and specific mathematical equations (Intuitive and Clear)	Root Locus, Rough Criterion	Damping ratio, Delay time, Rise time, Peak time, Max overshoot, Settling time	PID Control, Root Locus, Rough Criterion, Optimum Control
Frequency-domain Method	Available for linear time-invariant systems and nonlinear time-invariant systems (Adaptive and Propagable)	Bode Plot, Nyquist Plot, Nichols Plot	Gain margin, Phase margin, Phase-Crossover Frequency, Gain-Crossover Frequency	Lag-Lead Compensation, Parallel Compensation

In a nutshell, the time-domain method is clearer and more intuitive, which is easy to explain because we live in a time-domain world and time is a real-world metric. While the frequency-domain method can be thought of as a mathematical tool to analyze the complicated system and possesses a stronger adaptability and scalability. Both of these analysis methodologies can play a significant role in the appropriate usage scenarios.

Ultimately, the key to system control is balance [3]. In the virtual control environment, we may achieve an optimal solution. However, we can never grasp every detail of the system in the real-world control scenarios. In this context, figuring out how to reach a balance is the most challenging task every engineer faces.

And that is what I strike for.

Acknowledgements

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References

[1] Andresen T. Bode plot with asymptotes. Available online URL: https://www.mathworks.com/matlabcentral/fileexchange/10183-bode-plot-with-asymptotes [Z]. 2020

[2] MathWorks Groups, stepinfo function. Available online: https://www.mathworks.com/help/ident/ref/lti.stepinfo.html [Z]. 2022

[3] Ogata K. Modern control engineering [M]. Prentice hall Upper Saddle River, NJ, 2010.