Tutorial 2

Q1 A continuous-time signal $x(t)$ is shown in Figure 1. Sketch and label each of the following signals.

(a) $x(t)u(1-t)$

(b) $x(t)[u(t)-u(t-1)]$

sketch

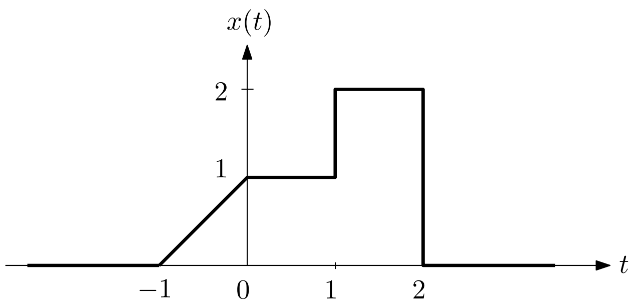


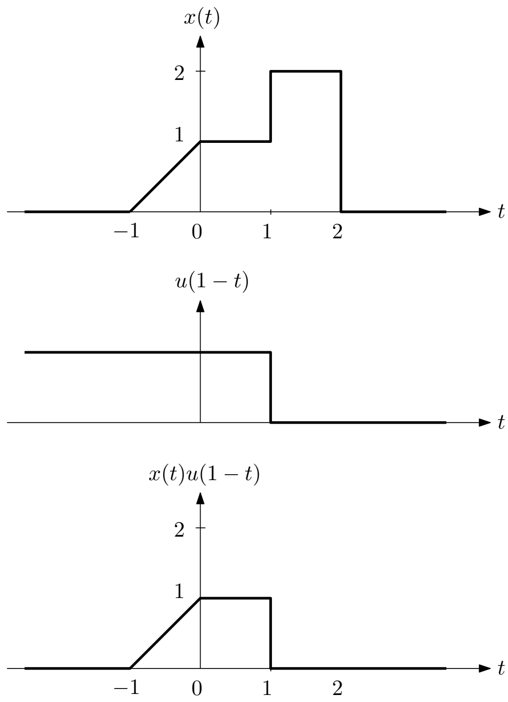
Figure 1

Solution:

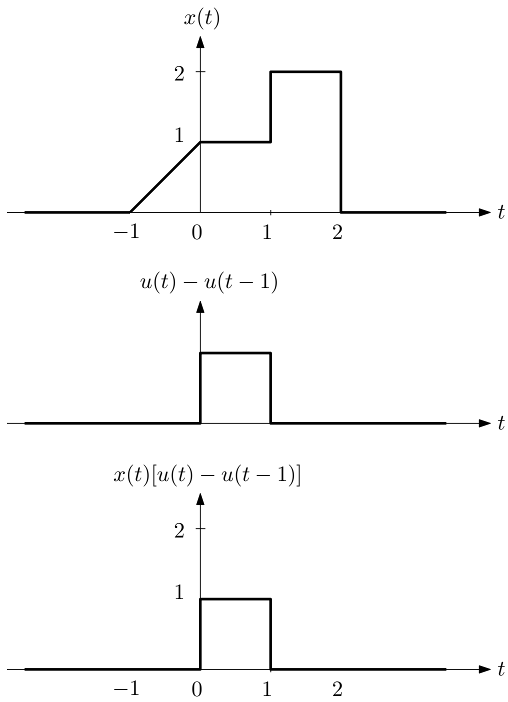
(a) We first need to general $u(1-t)$. Recall that $u(t)$ is the unit step function. In particular $u(1-t)$ is obtained by

- Shift $u(t)$ to the left by 1.
- Fold the resulting signal in the above step around the vertical axis (the y-axis).

The signal $u(1-t)$ and $x(t)u(1-t)$ are plotted in the following figure.



(b) The signal $u(t) - u(t-1)$ and $x(t)[u(t) - u(t-1)]$ are plotted in the following figure. We can easily check that the signal $u(t - t_0) - u(t - t_1)$ is equal to 1 for t from t_0 to t_1, and 0 otherwise.



Q2 A continuous-time signal $x(t)$ is shown in Figure 2. Sketch and label each of the following signals.

(a) $x(t-2)$

(b) $x(2t)$

(c) $x(t/2)$

(d) $x(-t)$

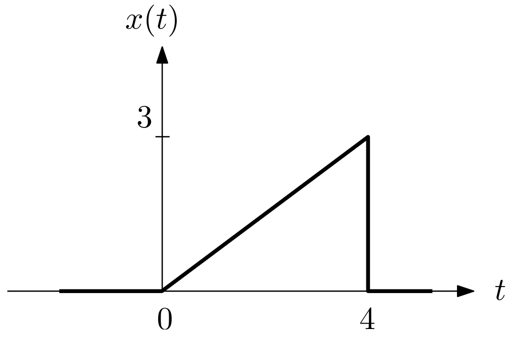
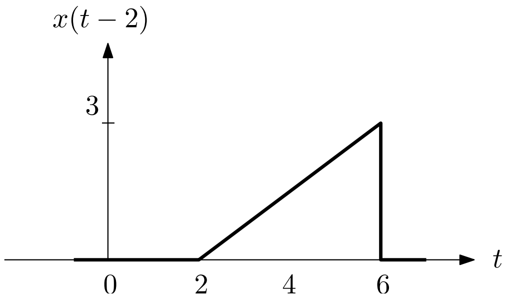


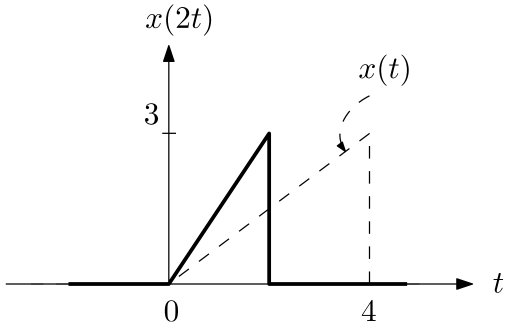
Figure 2

Solution:

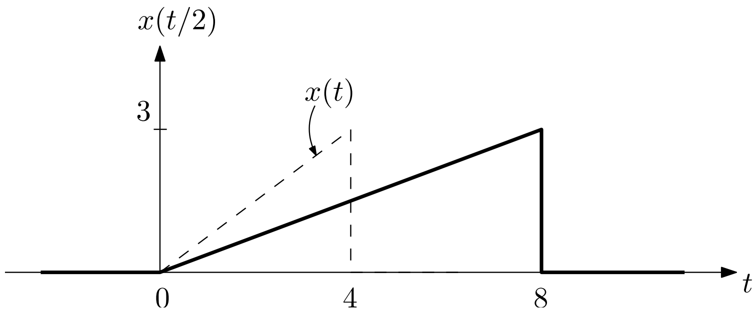
(a) $x(t-2)$ is simply the delayed version of $x(t)$ by 2 time unit.



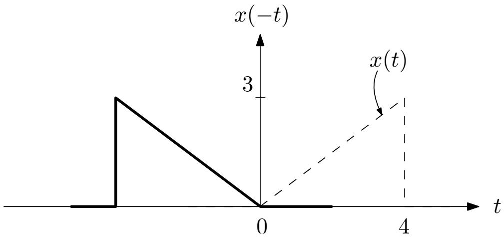
(b) $x(2t)$ is time compressed version of $x(t)$ by a factor of 2.



(c) $x(t/2)$ is a time expanded version of $x(t)$ by a factor of 2.



(d) $x(-t)$ is a time reversed version of $x(t)$.



Q3 Express the signals shown in Figure 3 in terms of unit step functions.

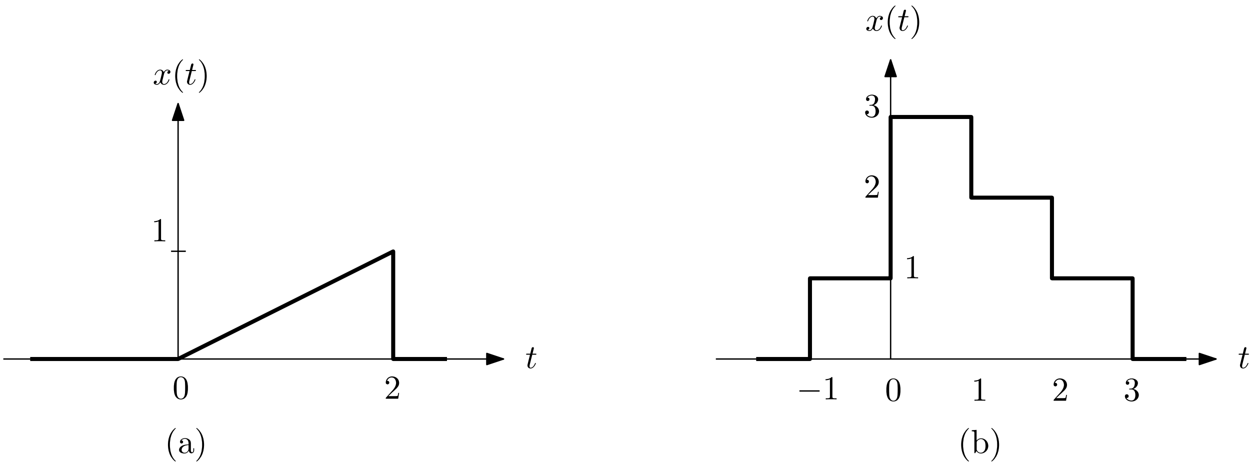
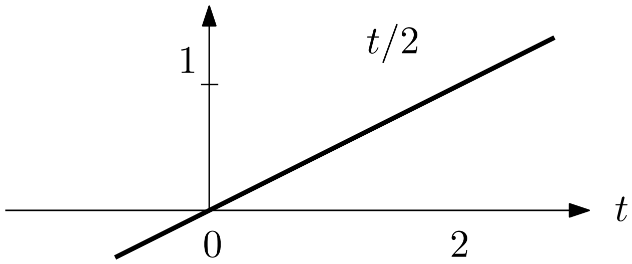
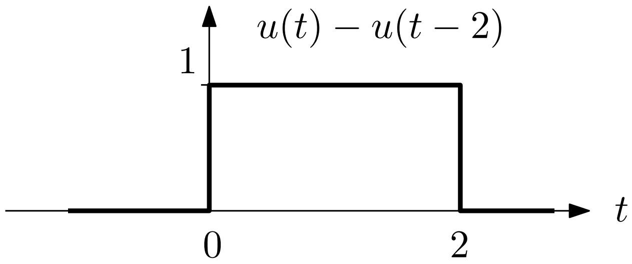


Figure 3

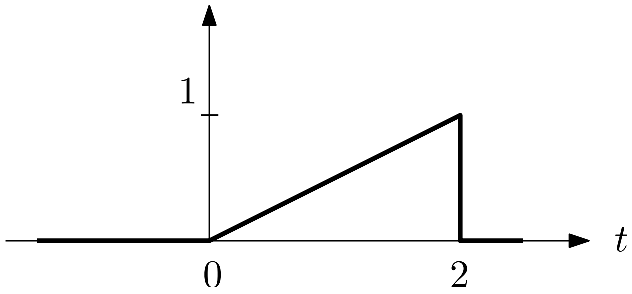
Solution:

We can see that the signal in Fig. 3(a) has non-zero value for $t \in [0, 2]$, and over this period the signal $x(t)$ is given by $x(t) = t/2$. Thus, we can write $x(t) = \frac{t}{2}[u(t) - u(t-2)]$. The graphical explanation is shown in the figure below.





$$x(t) = \frac{t}{2}[u(t) - u(t-2)]$$



Q4 A discrete-time signal $x[n]$ is shown in Figure 4. Sketch and label each of the following signals.

(a) $x[n]u[1-n]$

(b) $x[n]\{u[n+2]-u[n]\}$

(c) $x[n]\delta[n-1]$

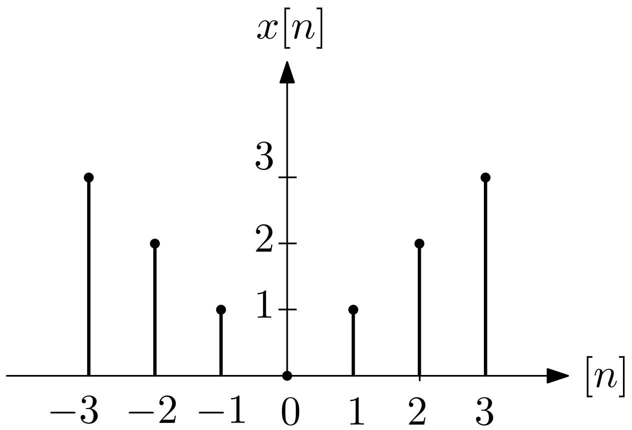


Figure 4

Q5 Given the sequence $x(n) = (6-n)[u(n) - u(n-6)]$, make a sketch of

(a) $y_1[n] = x[4-n]$.

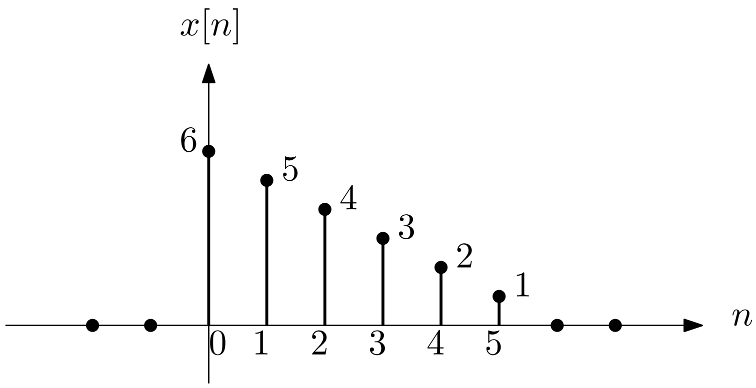
(b) $y_2[n] = x[2n-3]$.

Solution:

(a) First we need to determine the sequence $x[n]$. By looking at the definition of $x[n]$ we consider 3 cases as follows

- $n < 0$: In this interval both $u[n]$ and $u[n-6]$ are zero, and thus $x[n]$.
- $n \geq 6$: In this interval both $u[n]$ and $u[n-6]$ are 1, and thus $x[n] = 0$.
- $0 \leq n < 6$: In this interval both $u[n] = 1$ and $u[n-6]$ are 0, and thus $x[n] = 6-n$.

The sketch of $x[n]$ is given below.

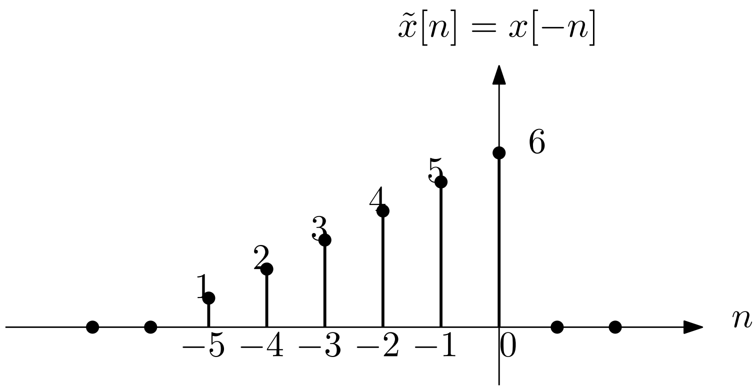


Now we can write $y_1[n]$ as

$$y_1[n] = x[4-n] = x[-(n-4)] \tag{1}$$

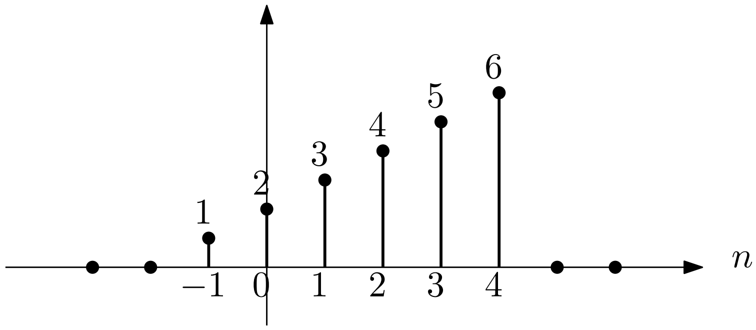
The above equation implies that $y_1[n]$ can be obtained from $x[n]$ by two operations

- Fold $x[n]$ around the origin. This creates the time-reversed signal of $x[n]$: $\tilde{x}[n] = x[-n]$ which is shown below.



- Shift the time-reversed signal $\tilde{x}[n]$ to **the right** by 4 samples. This creates the signal $\tilde{x}[n-4]$, which is equivalent to $x[-(n-4)] = y_1[n]$. Thus the sketch of $y_1[n]$ is given in the following figure.

$$y_1[n] = \tilde{x}[n-4] = x[-(n-4)]$$



Let us do some simple calculation to make sure if we have achieved a correct answer. For example when $n = 1$ we need to have $y_1[1] = x[4-1] = x[3]$. By looking at the figures for $x[n]$ and $y_1[n]$ above, we can see that $x[3] = y_1[1] = 3$.

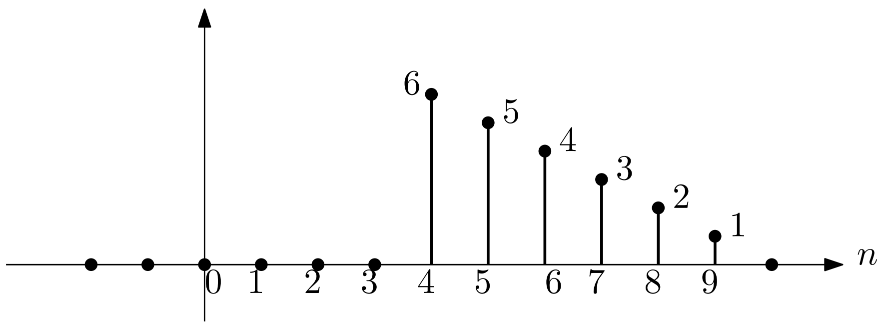
We can also obtain $y_1[n]$ by perform two above operations in an reverse order as follow

- Shift $x[n]$ to **the left** by 4samples. This creates the time-reversed signal of $x[n]$: $\tilde{x}[n] = x[n+4]$.
- Fold the resulting signal $\tilde{x}[n]$ around the origin to achieve $y_1[n]$.

IMPORTANT NOTE:

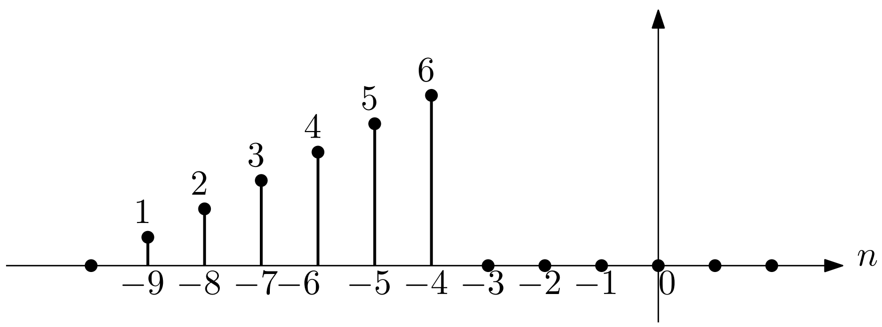
If you first shift $x[n]$ by 4 samples to the right, and then fold the resulting signal, you will have a **WRONG** answer. By shifting $x[n]$ to the right by 4 samples, you will have an intermediate signal $\tilde{x}[n] = x[n-4]$ shown in the following figure.

$$\tilde{x}[n] = x[n-4]$$



Now if $\tilde{x}[n]$ is time -reversed, then the resulting signal is plotted below

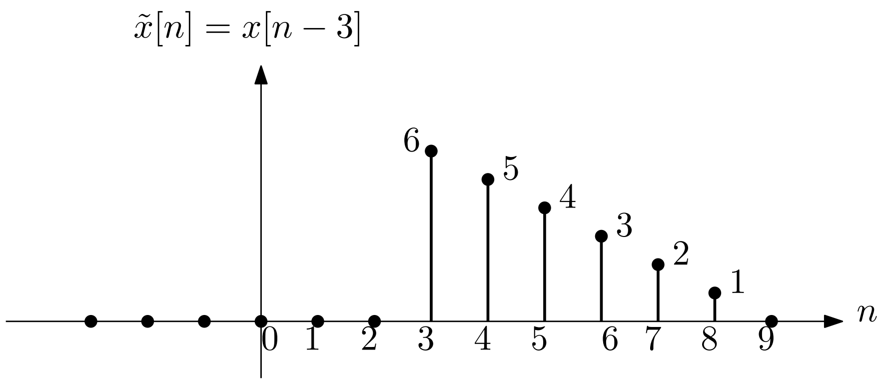
$$\tilde{y}[n] = \tilde{x}[-n] = x[-n-4]$$



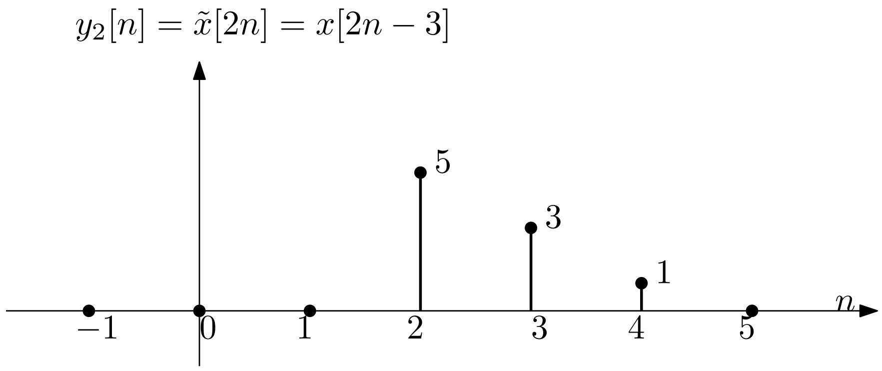
The obtained signal is actually $\tilde{y}[n] = x[-n-4]$, not $x[-n+4]$ as supposed to derive.

(b) The signal $y_2[n] = x[2n-3]$ is a combination of time shifting, downsampling operations on $x[n]$. For this simple case we can simply compute $y_2[n]$ from the definition. In particular $y_2[0] = x[-3] = 0$, $y_2[1] = x[-1] = 0$, $y_1[2] = x[1] = 5$, $y_2[3] = x[3] = 3$, $y_2[4] = x[5] = 1$, $y[5] = x[7] = 0$. From these values we can sketch $y_2[n]$. A better way to attain $y_2[n]$ is to perform the following time reversal and downsampling operations

- First time-shift $x[n]$ by 3 samples to the **right**. (If you are supposed to find the signal $x[2n+3]$, then time-shift $x[n]$ by 3 samples to the left), The resulting signal is plotted below.



- Then downsample the above signal by a factor of 2 to obtain $y_2[n]$ which is sketched below.

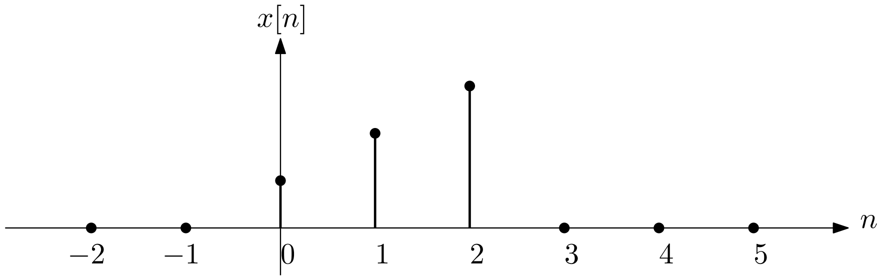


Q6 Express the sequence

$$x[n] = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 0 & \text{else} \end{cases}$$

as a sum of scaled and shifted **unit steps**.

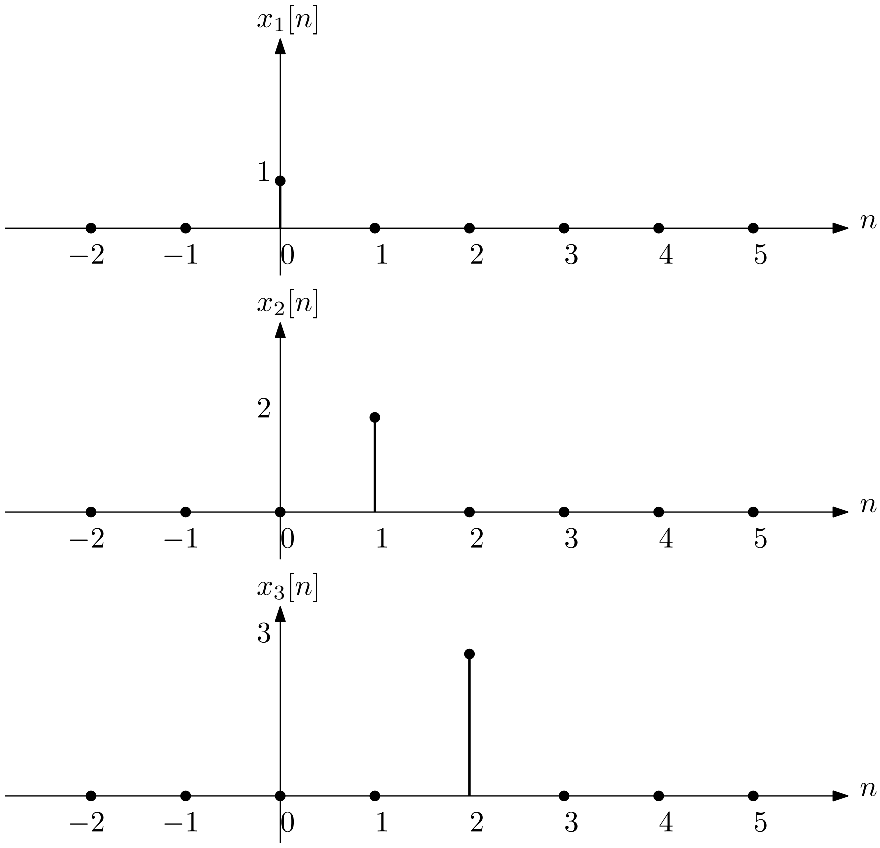
Solution: This problem demonstrates how a discrete signal can be decomposed as a sum of unit step functions. The sketch of $x[n]$ is given below.



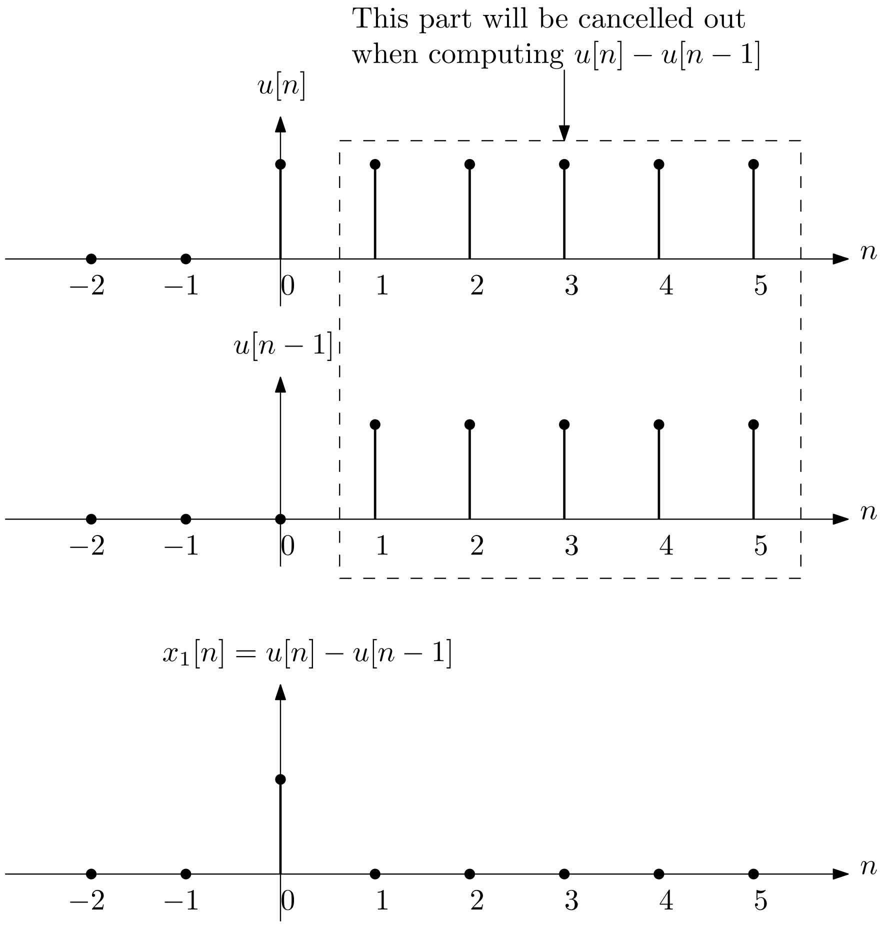
From the figure above, we can write $x[n]$ as

$$x[n] = x_1[n] + x_2[n] + x_3[n] \quad (2)$$

where $x_1[n]$, is are shown in the following figure



Let us first express $x_1[n]$ in terms of unit step function. Note that $x_1[n]$ has a non-zero value only at $n = 0$, and thus we can write $x_1[n] = u[n] - u[n-1]$. This can be easily verified by plotting $u[n]$ and $u[n-1]$, and then performing the subtraction as illustrated in the figure below.



Similarly we can write $x_2[n] = 2\big(u[n-1-u[n-2]\big)$ and $x_3[n] = 3\big(u[n-2]-u[n-3]\big)$. Thus

$$\begin{aligned} x[n] &= x_1[n] + x_2[n] + x_3[n] \\ &= u[n] - u[n-1] + 2\big(u[n-1-u[n-2]\big) + 3\big(u[n-2] - u[n-3]\big) \\ &= u[n] + u[n-1] + u[n-2] - 3u[n-3] \end{aligned} \tag{3}$$

From the procedure described above we can express any discrete sequence as as a sum of scaled and shifted unit steps. This is of very significant importance in signal processing. Suppose we want to find the response of a system to a discrite signal $x[n]$. Instead of directly computing the response of the system to $x[n]$ as a whole which is in many cases very difficult, we can do it for each unit step function and then combine the results to determine the overall response of the system to $x[n]$.

Q7 Express the sequence in **Q6** as a sum of scaled and shifted **unit samples**.

Solution: From the solution of **Q6**, it is easy to see that

$$x[n] = \delta[n] + 2\delta[x-1] + 3\delta[n-2] \tag{4}$$