1. (a) $A(\alpha)=\int_0^{\pi} \sin x\cdot\cos\alpha x\,dx = \left(\frac{\sin\alpha x}{\alpha}\cdot\sin x\right)\Big|_0^{\pi} - \int_0^{\pi}\frac{\sin\alpha x}{\alpha}\cdot(-\cos x)\,dx$

$= \frac{1}{\alpha}\int_0^{\pi}\cos x\cdot\sin\alpha x\,dx$

$= \frac{1}{\alpha}\left[\left(-\frac{\cos\alpha x}{\alpha}\cdot\cos x\right)\Big|_0^{\pi} - \int_0^{\pi}\frac{\cos\alpha x}{\alpha}\cdot\sin x\,dx\right]$

$\alpha I = \frac{(-1)^{\alpha}}{\alpha}+\frac{1}{\alpha}-\frac{1}{\alpha}I \qquad A(\alpha)=I=\frac{1+(-1)^{\alpha}}{\alpha^2+1}$

$B(\alpha)=0$

so $\underline{f(x)=\frac{1}{\pi}\int_0^{\pi}\frac{1+(-1)^{\alpha}}{\alpha^2+1}\cdot\cos\alpha x\,d\alpha}$

(b) $A(\alpha)=\int_{\pi}^{2\pi}3\cos\alpha x\,dx = \frac{3\sin\alpha x}{\alpha}\Big|_0^{\pi}=0$

$B(\alpha)=\int_{\pi}^{2\pi}3\sin\alpha x\,dx = -\frac{3\cos\alpha x}{\alpha}\Big|_0^{\pi} = \frac{3[1-(-1)^{\alpha}]}{\alpha}$

So $\underline{f(x)=\frac{1}{\pi}\int_{\pi}^{2\pi}\frac{3[1-(-1)^{\alpha}]}{\alpha}\sin\alpha x\,d\alpha}$

(c) $A(\alpha)=\int_{-\pi}^{\pi}\frac{2}{3}x\cdot\cos\alpha x\,dx = \frac{2}{3}\left(\frac{\sin\alpha x}{\alpha}\cdot x+\frac{\cos\alpha x}{\alpha^2}\right)\Big|_{-\pi}^{\pi}=0$

$B(\alpha)=\int_{-\pi}^{\pi}\frac{2}{3}x\cdot\sin\alpha x\,dx = \frac{2}{3}\left(-\frac{\cos\alpha x}{\alpha}x+\frac{\sin\alpha x}{\alpha^2}\right)\Big|_{-\pi}^{\pi}$

$= -2\pi\cdot\frac{(-1)^{\alpha}}{\alpha}$

So $\underline{f(x)=\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{(-1)^{\alpha+1}}{\alpha}\cdot(2\pi)\cdot\sin\alpha x\,d\alpha}$

2. $A(\alpha)=\int_0^{\infty}e^{-x/2}\cdot\cos\alpha x\,dx$

$=\left(\frac{\sin\alpha x}{\alpha}\cdot e^{-x/2}\right)\Big|_0^{\infty}-\int_0^{\infty}\frac{\sin\alpha x}{\alpha}\cdot\left(-\frac{1}{2}e^{-x/2}\right)dx$

$=\frac{1}{2\alpha}\int_0^{\infty}e^{-x/2}\cdot\sin\alpha x\,dx$

$=\frac{1}{2\alpha}\left[\left(-\frac{\cos\alpha x}{\alpha}\cdot e^{-x/2}\right)\Big|_0^{\infty}-\int_0^{\infty}\left(-\frac{\cos\alpha x}{\alpha}\right)\left(-\frac{1}{2}e^{-x/2}\right)dx\right]$

$=\frac{1}{2\alpha}\left(\frac{1}{\alpha}-\frac{1}{2\alpha}\int_0^{\infty}e^{-x/2}\cdot\cos\alpha x\,dx\right)$

$2\alpha\cdot I=\frac{1}{\alpha}-\frac{1}{2\alpha}I \qquad A(\alpha)=I=\frac{2}{4\alpha^2+1}$

$B(\alpha)=\int_0^{\infty}e^{-x/2}\cdot\sin\alpha x\,dx$

$=\left(-\frac{\cos\alpha x}{\alpha}\cdot e^{-x/2}\right)\Big|_0^{\infty}-\int_0^{\infty}\left(-\frac{\cos\alpha x}{\alpha}\right)\left(-\frac{1}{2}e^{-x/2}\right)dx$

$=\frac{1}{\alpha}-\frac{1}{2\alpha}\int_0^{\infty}e^{-x/2}\cdot\cos\alpha x\,dx$

$=\frac{1}{\alpha}-\frac{1}{2\alpha}\cdot\frac{2}{4\alpha^2+1}=\frac{4\alpha}{4\alpha^2+1}$

so $\underline{f(x)=\frac{2}{\pi}\int_0^{\infty}\frac{2}{4\alpha^2+1}\cdot\cos\alpha x\,d\alpha}$

$\underline{f(x)=\frac{2}{\pi}\int_0^{\pi}\frac{4\alpha}{4\alpha^2+1}\cdot\sin\alpha x\,d\alpha}$

3. (a) $F(\lambda)=\int_{-1}^{1} e^{2ix}\cdot e^{-i\lambda x}dx=\int_{-1}^{1} e^{i(2-\lambda)x}dx$

$$=\frac{1}{(2-\lambda)i}e^{i(2-\lambda)x}\Big|_{-1}^{1}=\frac{e^{i(2-\lambda)}-e^{i(\lambda-2)}}{(2-\lambda)i}$$

$$=\frac{2\sin(2-\lambda)}{2-\lambda}$$

(b) $F(\lambda)=\int_{-\infty}^{0} e^{x}\cdot e^{-i\lambda x}dx+\int_{0}^{\infty}e^{-x}\cdot e^{-i\lambda x}dx$

$$=\frac{1}{1-i\lambda}e^{(1-i\lambda)x}\Big|_{-\infty}^{0}-\frac{1}{1+i\lambda}e^{-(1+i\lambda)x}dx\Big|_{0}^{\infty}$$

$$=\frac{1}{1-i\lambda}+\frac{1}{1+i\lambda}=\frac{2}{1+\lambda^2}$$

(c) $F(\lambda)=\int_{-1}^{1}x\cdot e^{-i\lambda x}dx$

$$=\left(-\frac{1}{i\lambda}e^{-i\lambda x}\cdot x\right)\Big|_{-1}^{1}-\int_{-1}^{1}\left(-\frac{1}{i\lambda}e^{i\lambda x}\right)dx$$

$$=-\frac{e^{-i\lambda}+e^{i\lambda}}{i\lambda}+\frac{1}{i\lambda}\cdot\left(-\frac{1}{i\lambda}e^{-i\lambda x}\right)\Big|_{-1}^{1}$$

$$=-\frac{2\cos\lambda}{i\lambda}+\frac{1}{\lambda^2}\cdot\left(e^{-i\lambda}-e^{i\lambda}\right)=-\frac{2\cos\lambda}{i\lambda}-\frac{2i\sin\lambda}{\lambda^2}$$

$$=\frac{2i\cos\lambda}{\lambda}-\frac{2i\sin\lambda}{\lambda^2}$$