Lecture 2 Kinematics in One Dimension

Some Definitions

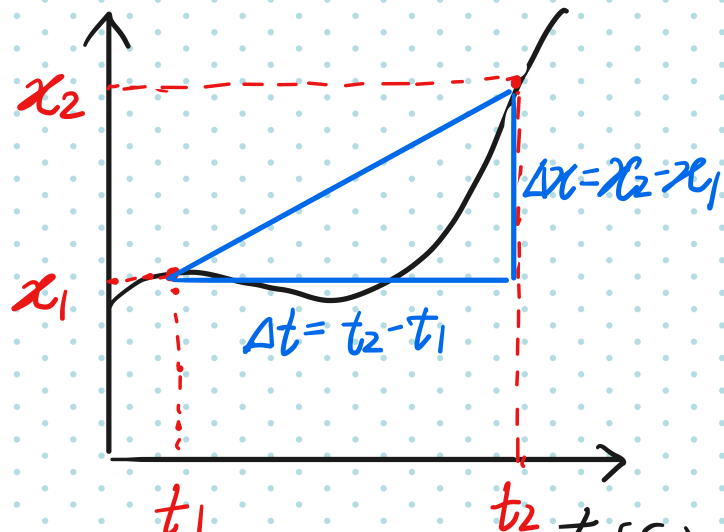
1. $\vec{x}$ position vector

x (+, −)

$\Delta\vec{x}$ displacement

$$\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$$

$$\Delta x = x_2 - x_1$$



$$V_{avg} = \frac{\Delta x}{\Delta t}$$

2. $\vec{v}$ velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\Delta x}{\Delta t}$$ average velocity

instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt} = \frac{dx}{dt}$$

3. $$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v}{\Delta t}$$ average acceleration

instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Question $x = 9t - t^3$ (m)

$$\begin{cases} x_1 = 9.0 - 1.0 = 8.0 \\ x_2 = 36 - 64 = -28 \end{cases}$$

$t_1 = 1.0s$, $t_2 = 4.0s$.

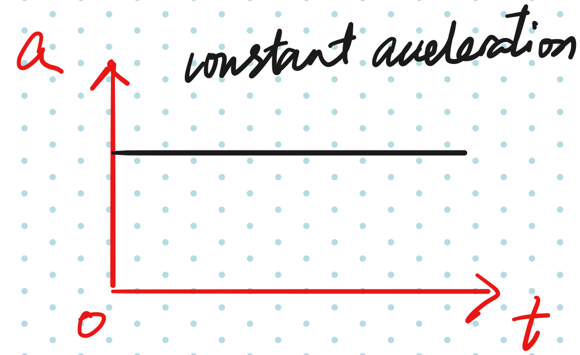
$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{\Delta t} = \frac{-28 - 8.0}{4.0 - 1.0} = \frac{-36}{3}$$

$$= -12 \, m/s$$

1 Dimensional Kinematics

with constant acceleration

constant acceleration



$$a = \frac{dv(t)}{dt}$$

$\Downarrow$

$$\int^{t=t_f} a \, dt \qquad \int^{v(t_f)} dv(t) = v(t_f) - v(t_i) = V - V_0$$

$$\int_{t=t_i} a\,dt = \int_{v(t_i)} dv(t) = v(t_f) - v(t_i) = v_f - v_0$$

$$= a\int_{t_i}^{t_f} dt = a(t_f - t_i) = a(t - t_0)$$

$$\Rightarrow a(t - t_0) = v_t - v_0$$

$$\Rightarrow v_t = v_0 + a(t - t_0)$$

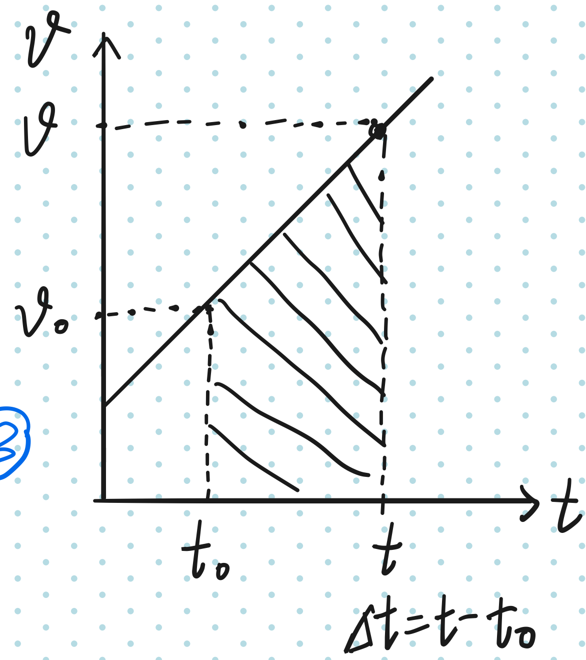
when $t_0 = 0$, $\boxed{v_t = v_0 + at}$ ①

$$\frac{dx}{dt} = v_t = v_0 + at$$

$$\int_0^s \frac{dx}{\cancel{dt}} \cancel{dt} = \int (v_0 + at)\,dt$$

$$S = \int_0^t v_0\,dt + \int_0^t at\,dt = v_0 t + \frac{1}{2}at^2$$

$$\boxed{S = v_0 t + \frac{1}{2}at^2}$$ ②



$$(v_0 + v)(t - t_0)/2 = x$$

when $t_0 = 0$, $\boxed{x = \frac{1}{2}(v_0 + v)t}$ ③

$$v - v_0$$

$a = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t}$ when $t_0 = 0$

$\Rightarrow$

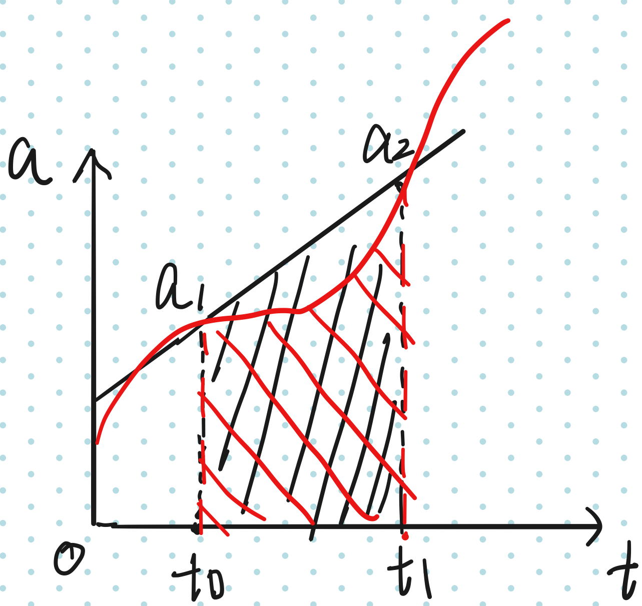
$at = v - v_0$ $\quad\quad$ $2x = (v_0 + v)t$

$$2ax\cancel{t} = (v + v_0)(v - v_0)\cancel{t}$$
$$= v^2 - v_0^2$$

$$\boxed{v^2 = v_0^2 + 2ax} \quad (4)$$

Graphical Representations

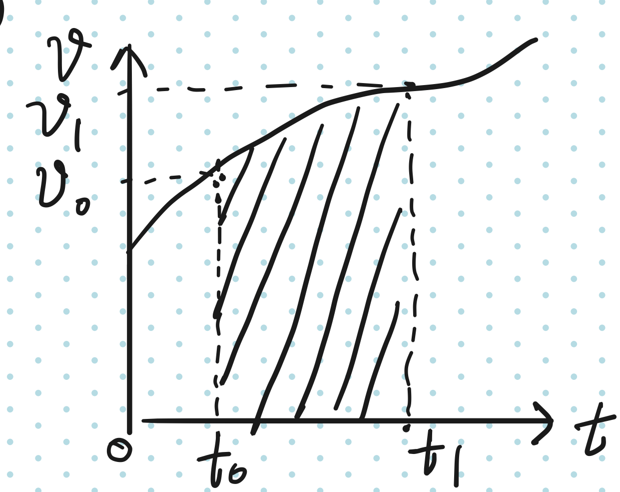
(1)



$$\int_{t_0}^{t_1} a(t)\,dt = v(t_1) - v(t_0)$$

change in velocity

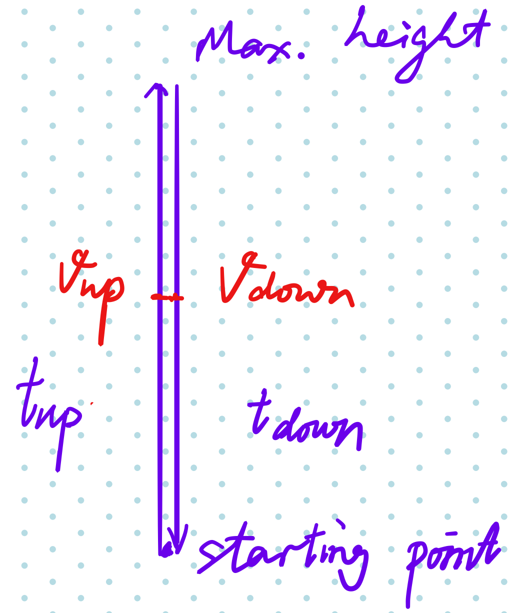
(2)



$$\int_{t_0}^{t_1} v(t)\,dt = x(t_1) - x(t_0)$$

change in position

Free Falling Objects



① Time Symmetry

$$t_{up} = t_{down}$$

② Speed Symmetry

$$V_{up} = V_{down}$$