Assignment 5 1103

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1.(a) $\mathcal{L}\{t^n\} =$

$\int_0^\infty t^n e^{-st} \cdot dt$

$= -\frac{1}{s} e^{-st} - \int_0^\infty n \cdot t^{n-1} \cdot (-\frac{1}{s} e^{-st}) dt$

$= -\frac{1}{s} t^n e^{-st} \Big|_{t=0}^{\infty} + \frac{n}{s} \int_0^\infty t^{n-1} \cdot e^{-st} dt$

$= -\frac{1}{s} t^n e^{-st} \Big|_{t=0}^{\infty} + \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$= \lim_{t\to\infty} (-\frac{1}{s} t^n \cdot e^{-st}) + \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$= 0 + \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

$\therefore \mathcal{L}\{t^n\} = \frac{n}{s} \mathcal{L}\{t^{n-1}\}$

And $\mathcal{L}\{1\} = \int_0^\infty e^{-st} dt = \frac{-1}{s} e^{-st} \Big|_0^\infty$

$= \lim_{t\to\infty} \frac{-1}{s} e^{-st} + \frac{1}{s} = 0 + \frac{1}{s}$

$\therefore \mathcal{L}\{t^n\} = \frac{n}{s} \cdot \frac{n-1}{s} \cdot \ldots \cdot \frac{1}{s} \mathcal{L}\{t^0\}$

$= \frac{n!}{s^n} \mathcal{L}\{1\} = \frac{n!}{s^{n+1}}$

So $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

1.(b) sol. $f(t) = 2\sinh(3t) + \cos(2t)$

$f(t) = \int_0^\infty (2\sinh 3t + \cos 2t) e^{-st} dt$

$= 2\int_0^\infty \sinh 3t \cdot e^{-st} dt + \int_0^\infty \cos 2t \cdot e^{-st} dt$

Let $\begin{cases} IF_1 = \int_0^\infty \sinh 3t \cdot e^{-st} dt \\ IF_2 = \int_0^\infty \cos 2t \cdot e^{-st} dt \end{cases}$

① $IF_1 = \frac{1}{3} \cosh 3t \cdot e^{-st} \Big|_0^\infty - \int_0^\infty -s e^{-st} \frac{1}{3} \cosh 3t \cdot dt$

$= -\frac{1}{3} + \frac{s}{3} \int_0^\infty \cosh 3t \cdot e^{-st} \cdot dt$

$= -\frac{1}{3} + \frac{s}{3} (\frac{1}{3} \sinh 3t e^{-st} \Big|_0^\infty - \int_0^\infty -s e^{-st} \cdot \frac{1}{3} \sinh 3t \cdot dt$

$= -\frac{1}{3} + \frac{s}{3} (\frac{s}{3} IF_1)$

$\therefore IF_1 = \frac{3}{(s^2-9)}$

② similarly, we can get $IF_2 = \frac{s}{4} - \frac{s^2}{4} IF_2$

$\therefore IF_2 = \frac{s}{(s^2+4)}$

$\therefore F(s) = \frac{6}{s^2-9} + \frac{s}{s^2+4}$, Date

$36C - B = 0$

2.(a). $\mathcal{L}^{-1}\left\{\frac{6}{s^2+36s}\right\}$

sol

$= \mathcal{L}^{-1}\left\{\frac{1}{6}\left(s - \frac{1}{s+36}\right)\right\}$

$= \frac{1}{6}\mathcal{L}^{-1}\{s\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+36}\right\}$

$= \frac{1}{6} - \frac{1}{6}e^{-36t}$

sol (b) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-5)(s-7)}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{\frac{2}{15}}{s-2} + \frac{-\frac{5}{6}}{s-5} + \frac{\frac{7}{10}}{s-7}\right\}$

$= \frac{2}{15}e^{2t} - \frac{5}{6}e^{5t} + \frac{7}{10}e^{7t}$

(c) $\mathcal{L}^{-1}\left\{\frac{(s-1)^3}{s^4}\right\}$ let $I = \frac{(s-1)^3}{s^4}$

sol $I = \frac{s^3-3s^2+3s-1}{s^4} = \frac{1}{s} - 3\frac{1}{s^2} + \frac{3}{s^3} - \frac{1}{s^4}$

$\therefore \mathcal{L}^{-1}\{I\} = 1 - 3t + \frac{3}{2}t^2 - \frac{1}{6}t^3$.

3.(a) sol. $y'' + 5y' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$

$(s^2+5s+4)Y(s) = s+5$

$Y(s) = \frac{(s+5)}{(s^2+5s+4)} = \frac{s+5}{(s+1)(s+4)}$

$= \frac{\frac{4}{3}}{s+1} + \frac{-\frac{1}{3}}{s+4}$ $\therefore y = \left(\frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}\right)$

(b) $2\frac{dy}{dt} - y = 0$, $y(0) = 5$

sol. $2(sY(s) - y(0)) - Y(s) = 0$

$(2s-1)Y(s) = 10$

$\therefore Y(s) = \frac{10}{2s-1} = \frac{5}{s-1/2}$

$\therefore y = 5 \cdot e^{\frac{1}{2}t}$

3.(c) $y' - y = 2\cos 6t$ $y(0) = 0$

sol. $sY(s) - y(0) - Y(s) = \mathcal{L}\{2\cos 6t\}$

$\therefore (s-1)Y(s) = 2\frac{s}{s^2+36}$

$Y(s) = \frac{2s}{(s^2+36)(s-1)} = \frac{As+B}{s^2+36} + \frac{C}{s-1}$

$\therefore As^2 + Bs - As - B + Cs^2 + 36C = 2s$

$\begin{cases} A = -\frac{2}{37} \\ B = \frac{12}{37} \\ C = \frac{2}{37} \end{cases}$ $\therefore Y(s) = \frac{-\frac{2s}{37} + \frac{12}{37}}{s^2+36} + \frac{\frac{2}{37}}{(s-1)}$

$\therefore y = \mathcal{L}^{-1}\left\{-\frac{2}{37}\frac{s}{s^2+36} + \frac{12}{37}\frac{6}{s^2+36} + \frac{2}{37}\cdot\frac{1}{s-1}\right\}$

$\therefore y = -\frac{2}{37}\cos 6t + \frac{12}{37}\sin 6t + \frac{2}{37}e^{t}$

3.(d) $y'' - 10y' + 25y = 3e^{3t}$

$y(0) = 0$ $y'(0) = -1$.

sol. $(s-5)^2 Y(s) = \frac{6-s}{s-3}$

So $Y(s) = \frac{-s+6}{(s-5)^2(s-3)}$

$= \frac{\frac{1}{2}}{(s-5)^2} - \frac{3}{4}\left(\frac{1}{s-5} - \frac{1}{s-3}\right)$

$\therefore y = \mathcal{L}^{-1}\left\{\frac{1}{2}\frac{1}{(s-5)^2} - \frac{3}{4}\frac{1}{s-5} + \frac{3}{4}\frac{1}{s-3}\right\}$

$\therefore y = \left(\frac{1}{2}te^{5t} - \frac{3}{4}e^{5t} + \frac{3}{4}e^{3t}\right)$

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Assign 5. Page 2.

4. (a) Sol. $\mathcal{L}\{\cosh(t)\cos(t)\}$

$= \mathcal{L}\{\frac{e^{t}+e^{-t}}{2}\cdot\cos t\}$

$= \mathcal{L}\{\frac{1}{2}e^{t}\cdot\cos t\} + \mathcal{L}\{\frac{1}{2}e^{-t}\cdot\cos t\}$

$= \frac{1}{2}F(s-1) + \frac{1}{2}F(s+1)$

$= \frac{1}{2}\left(\frac{s-1}{(s-1)^2+1} + \frac{s+1}{(s+1)^2+1}\right)$

4(b) sol. $\mathcal{L}^{-1}\{\frac{(s-1)^2}{(s+2)^4}\}$.

$= \mathcal{L}^{-1}\{\frac{[(s+2)-3]^2}{(s+2)^4}\}$

$= \mathcal{L}^{-1}\{\frac{1}{(s+2)^2} - \frac{6}{(s+2)^3} + \frac{9}{(s+2)^4}\}$

$= \mathcal{L}^{-1}\{\frac{1}{(s+2)^2}\} - 3\mathcal{L}^{-1}\{\frac{2}{(s+2)^3}\} + \frac{3}{2}\mathcal{L}^{-1}\{\frac{6}{(s+2)^4}\}$

$= \mathcal{L}^{-1}\{F_1(s+2)\} - 3\mathcal{L}^{-1}\{F_2(s+2)\} + \frac{3}{2}\mathcal{L}^{-1}\{F_3(s+2)\}$.

$= e^{-2t}(t - 3t^2 + \frac{3}{2}t^3)$

5(b) $\mathcal{L}\{\cos(4t-8)\mathcal{U}(t-2)\}$

$4(t-2)$,

Sol. $= e^{-2s}\mathcal{L}\{\cos 4t\}$

$= e^{-2s}\cdot\frac{s}{s^2+16}$

5. (a) $\mathcal{L}\{(3t+1)\mathcal{U}(t-1)\}$

Sol. $= \mathcal{L}\{[3(t-1)+4]\,\mathcal{U}(t-1)\}$

$= 3\mathcal{L}\{(t-1)\,\mathcal{U}(t-1)\} + 4\mathcal{L}\{\mathcal{U}(t-1)\}$

$= 3e^{-s}\frac{1}{s^2} + 4\frac{e^{-s}}{s}$.

$= e^{-s}(\frac{3}{s^2} + \cancel{4}\frac{4}{s})$

5(c) $\mathcal{L}^{-1}\{\frac{(1+e^{-s})^2}{s+3}\}$,

sol.

$= \mathcal{L}^{-1}\{\frac{1+2e^{-s}+e^{-2s}}{(s+3)}\}$

$= \mathcal{L}^{-1}\{\frac{1}{s+3}\} + \mathcal{L}^{-1}\{\frac{2e^{-s}}{s+3}\} + \mathcal{L}^{-1}\{\frac{e^{-2s}}{s+3}\}$

$= e^{-3t} + 2e^{-3(t-1)}\mathcal{U}(t-1) + e^{-3(t-2)}\mathcal{U}(t-2)$