1. Work done by a constant force

$$W = \vec{F} \cdot \vec{d} = F d \cos\varphi$$

3D space

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

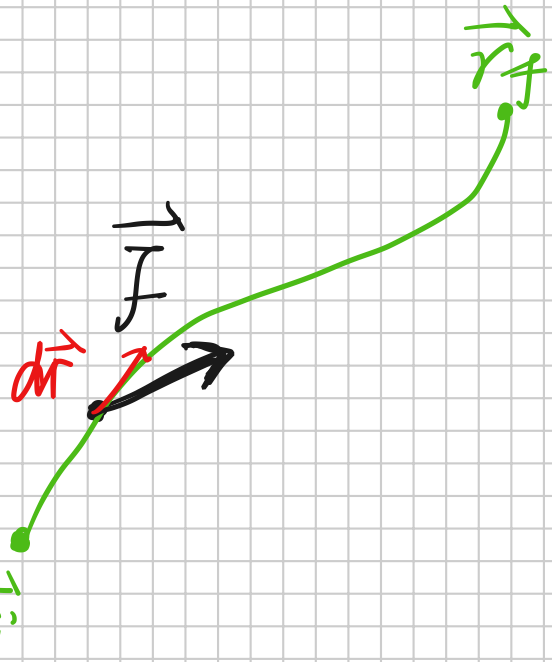
$$\vec{d} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

$$W = \vec{F} \cdot \vec{d} = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

2. work done by a variable force

1D case

$$W = \int_{x_i}^{x_f} F(x)\, dx$$



3D case

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$d\vec{r} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$dW = \vec{F} \cdot d\vec{r}$$

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy + \int_{z_i}^{z_f} F_z\, dz$$

Question

2D case $\vec{F} = F_x \hat{i} + F_y \hat{i}$

$\vec{d} = \Delta x \hat{i} + \Delta y \hat{j}$

$\Delta x > 0$, $W = F_x \Delta x > 0$, $F_x > 0$

$\Delta y > 0$, $W = F_y \Delta y < 0$, $F_y < 0$

Work - energy theorem

$$F_x = m\frac{dv_x}{dt}$$

$$W = \int_{x_i}^{x_f} F_x \, dx = m\int_{x_i}^{x_f} \frac{dv_x}{dt} dx = m\int_{v_i}^{v_f} v_x \, dv_x$$

$$= m\frac{1}{2}v_x^2 \Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Question

$$\vec{F} = (Cx - 3.00x^2)\hat{i}$$

$$\int_0^{3.00} \vec{F} \cdot dx\hat{i} = 11.0 - 20.0 = -9.0$$

$$= \left[\frac{C}{2}x^2 - \frac{3.00}{3}x^3\right]\Bigg|_0^{3.00}$$

$$= 4.50C - 27.0$$

$$C = 4.00\ N/m$$