

$$\begin{aligned}
 1. (a) \quad A(\omega) &= \int_0^{\lambda} \sin x \cdot \cos 2x dx = \left( \frac{\sin 2x}{2} \cdot \sin x \right) \Big|_0^{\lambda} - \int_0^{\lambda} \frac{\sin 2x}{2} \cdot (-\cos x) dx \\
 &= \frac{1}{2} \int_0^{\lambda} \cos x \cdot \sin 2x dx \\
 &= \frac{1}{2} \left[ \left( -\frac{\cos 2x}{2} \cdot \cos x \right) \Big|_0^{\lambda} - \int_0^{\lambda} \frac{\cos 2x}{2} \cdot \sin x dx \right] \\
 2I &= \frac{(-1)^2}{2} + \frac{1}{2} - \frac{1}{2}I \quad A(\omega) = I = \frac{1+(-1)^2}{2^2+1}
 \end{aligned}$$

$$B(\omega) = 0$$

$$\text{so } f(x) = \frac{1}{\lambda} \int_0^{\lambda} \frac{1+(-1)^2}{2^2+1} \cdot \cos 2x dx$$

$$\begin{aligned}
 (b) \quad A(\omega) &= \int_{-\lambda}^{\lambda} 3 \cos 2x dx = \frac{3 \sin 2x}{2} \Big|_{-\lambda}^{\lambda} = 0 \\
 B(\omega) &= \int_{-\lambda}^{\lambda} 3 \sin 2x dx = -\frac{3 \cos 2x}{2} \Big|_{-\lambda}^{\lambda} = \frac{3[1-(-1)^2]}{2} \\
 \text{So } f(x) &= \frac{1}{\lambda} \int_{-\lambda}^{\lambda} \frac{3[1-(-1)^2]}{2} \sin 2x dx
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad A(\omega) &= \int_{-\lambda}^{\lambda} \frac{2}{3} x \cdot \cos 2x dx = \frac{2}{3} \left( \frac{\sin 2x}{2} \cdot x + \frac{\cos 2x}{2^2} \right) \Big|_{-\lambda}^{\lambda} = 0 \\
 B(\omega) &= \int_{-\lambda}^{\lambda} \frac{2}{3} x \cdot \sin 2x dx = \frac{2}{3} \left( -\frac{\cos 2x}{2} x + \frac{\sin 2x}{2^2} \right) \Big|_{-\lambda}^{\lambda} \\
 &= -2\lambda \cdot \frac{(-1)^2}{2} \\
 \text{So } f(x) &= \frac{1}{\lambda} \int_{-\lambda}^{\lambda} \frac{(-1)^{2+1}}{2} \cdot (2\lambda) \cdot \sin 2x dx
 \end{aligned}$$

$$\begin{aligned}
 2. \quad A(\omega) &= \int_0^{\infty} e^{-x/2} \cdot \cos 2x dx \\
 &= \left( \frac{\sin 2x}{2} \cdot e^{-x/2} \right) \Big|_0^{\infty} - \int_0^{\infty} \frac{\sin 2x}{2} \cdot \left( -\frac{1}{2} e^{-x/2} \right) dx \\
 &= \frac{1}{2\omega} \int_0^{\infty} e^{-x/2} \cdot \sin 2x dx \\
 &= \frac{1}{2\omega} \left[ \left( -\frac{\cos 2x}{2} \cdot e^{-x/2} \right) \Big|_0^{\infty} - \int_0^{\infty} \left( -\frac{\cos 2x}{2} \right) \cdot \left( -\frac{1}{2} e^{-x/2} \right) dx \right] \\
 &= \frac{1}{2\omega} \left( \frac{1}{2} - \frac{1}{2\omega} \int_0^{\infty} e^{-x/2} \cdot \cos 2x dx \right) \\
 2\omega \cdot I &= \frac{1}{2} - \frac{1}{2\omega} I \quad A(\omega) = I = \frac{2}{4\omega^2+1}
 \end{aligned}$$

$$\begin{aligned}
 B(\omega) &= \int_0^{\infty} e^{-x/2} \cdot \sin 2x dx \\
 &= \left( -\frac{\cos 2x}{2} \cdot e^{-x/2} \right) \Big|_0^{\infty} - \int_0^{\infty} \left( -\frac{\cos 2x}{2} \right) \cdot \left( -\frac{1}{2} e^{-x/2} \right) dx \\
 &= \frac{1}{2} - \frac{1}{2\omega} \int_0^{\infty} e^{-x/2} \cdot \cos 2x dx \\
 &= \frac{1}{2} - \frac{1}{2\omega} \cdot \frac{2}{4\omega^2+1} = \frac{4\omega}{4\omega^2+1}
 \end{aligned}$$

$$\text{so } f(x) = \frac{2}{\lambda} \int_0^{\infty} \frac{2}{4\omega^2+1} \cdot \cos 2x d\omega$$

$$f(x) = \frac{2}{\lambda} \int_0^{\lambda} \frac{4\omega}{4\omega^2+1} \cdot \sin 2x d\omega$$

$$\begin{aligned}
 3. \quad (a) \quad \bar{f}(\omega) &= \int_{-1}^1 e^{2ix} \cdot e^{-i\omega x} dx = \int_{-1}^1 e^{i(2-\omega)x} dx \\
 &= \frac{1}{(2-\omega)i} e^{i(2-\omega)x} \Big|_{-1}^1 = \frac{e^{i(2-\omega)} - e^{-i(2-\omega)}}{(2-\omega)i} \\
 &= \frac{2 \sin(2-\omega)}{2-\omega}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \bar{f}(\omega) &= \int_{-\infty}^0 e^x \cdot e^{-i\omega x} dx + \int_0^{\infty} e^{-x} \cdot e^{-i\omega x} dx \\
 &= \frac{1}{1-i\omega} e^{(1-i\omega)x} \Big|_{-\infty}^0 - \frac{1}{1+i\omega} e^{-(1+i\omega)x} \Big|_0^{\infty} \\
 &= \frac{1}{1-i\omega} + \frac{1}{1+i\omega} = \frac{2}{1+\omega^2}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad \bar{f}(\omega) &= \int_{-1}^1 x \cdot e^{-i\omega x} dx \\
 &= \left( -\frac{1}{i\omega} e^{-i\omega x} \cdot x \right) \Big|_{-1}^1 - \int_{-1}^1 \left( -\frac{1}{i\omega} e^{-i\omega x} \right) dx \\
 &= -\frac{e^{-i\omega} + e^{i\omega}}{i\omega} + \frac{1}{i\omega} \cdot \left( -\frac{1}{i\omega} e^{-i\omega x} \right) \Big|_{-1}^1 \\
 &= -\frac{2 \cos \omega}{i\omega} + \frac{1}{\omega^2} \cdot (e^{-i\omega} - e^{i\omega}) = -\frac{2 \cos \omega}{i\omega} - \frac{2i \sin \omega}{\omega^2} \\
 &= \frac{2i \cos \omega}{\omega} - \frac{2i \sin \omega}{\omega^2}
 \end{aligned}$$