Lecture 2: Classification of Signals

E213 - Introduction to Signal Processing

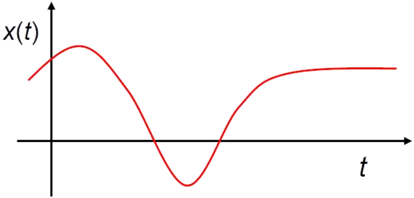
Semester 1, 2021

Classification of Signals

- Continuous and Discrete signals
- Periodic and Nonperiodic Signals
- Even and Odd Signals
- Energy and Power Signals
- Deterministic and Random Signals
- Complex Exponential signal

Continuous and Discrete signals

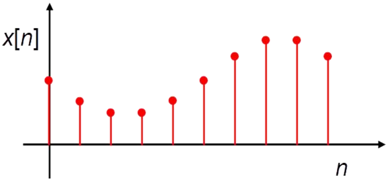
- Continuous (Time) signals:
  - Defined for all 'time' $\rightarrow x(t)$.
  - Arise naturally, when a physical waveform is converted into an electrical signal.



- Some Continuous Time signals:
  - Both t and $x(t)$ are continuous.
  - Also known as analogue signal.

Continuous and Discrete Signals...

- Discrete (time) signals:
  - Defined only at some discrete instants of time → $x[n]$.
  - Often derived from analogue signals by sampling.



- n is called the sampled index or simply index.
- $x[n]$ is the value of the discrete signals at index n.

Periodic and Nonperiodic Signals

Periodic continuous-time signals:

- $x(t)$ is **periodic** if there exists a number $T > 0$, such that $x(t+T) = x(t)$, for all t.
- **Fundamental period**: *smallest* positive T, such that the above holds
- **Fundamental frequency**:

$$f = 1/T \text{ (Hz or cycles per second)} \tag{1}$$

- **Angular frequency**

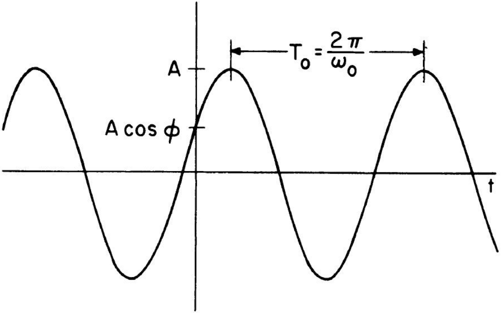
$$\omega = \frac{2\pi}{T} \text{ (\textbf{radians})} \tag{2}$$

$x(t)$ is **nonperiodic** (or **aperiodic**) if there is no T satisfying the above condition

Periodic and Nonperiodic Signals - Example

Example (sinusoidal signals)

$A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$ where A is real, $\omega_0 > 0$ is real, ϕ is real, and t is the time.



Periodic and Nonperiodic Signals - Example

- Periodic with the fundamental period $T_0 = \frac{2\pi}{\omega_0}$
- ω_0 is the angular frequency (radians/s)
- $f_0 = \frac{\omega_0}{2\pi}$ is frequency (Hz), i.e., the number of cycles per unit time (large f_0 means more oscillations)
- |A| is the amplitude
- ϕ is the size of the phase shift

Periodic and Nonperiodic Signals

Periodic discrete-time signals

- $x[n]$ is **periodic** if there exists a number $N > 0$, such that $x[n+N] = x[n]$, for all n.
- **Fundamental period**: *smallest* positive integer N, such that the above holds
- **Fundamental frequency:**

$$f = 1/N \text{ (\textbf{cycles per sample})} \tag{3}$$

- **Angular frequency:**

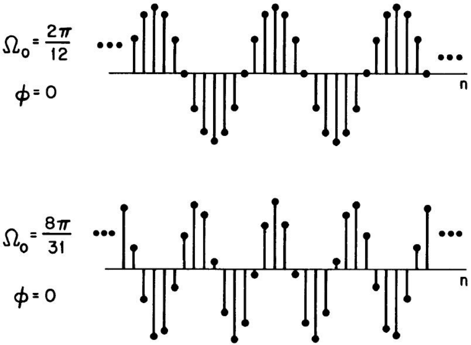
$$\Omega = 2\pi/N \text{ (radians)} \tag{4}$$

- $x[n]$ is **nonperiodic** (or **aperiodic**) if there is no integer $N > 0$ satisfying the above condition

Periodic and Nonperiodic Signals - Example...

Example (Discrete sinusoidal signals)

$A\cos(\Omega_0 n + \phi)$ where A is real, ϕ is real, and n is the sample index.



- ✓ Are they periodic?
- ✓ What is the fundamental period N?

Periodic and Nonperiodic Signals - Example...

Example (Discrete sinusoidal signals)

$A\cos(\Omega_0 n + \phi)$ where A is real, ϕ is real, and n is the sample index.

- $A\cos(\Omega_0 n + \phi)$ is periodic $\Leftrightarrow$ $\frac{\Omega_0}{2\pi}$ is rational
- If $\frac{\Omega_0}{2\pi} = \frac{m}{M}$ for some integers m and M which have **no common factors**, then the fundamental period is $M = \frac{2\pi m}{\Omega_0}$
- $|A|$ is the amplitude
- ϕ is the size of the phase shift

$$x[n] = x[n+N]$$

$$A\cos(\Omega_0 n + \emptyset) = A\cos(\Omega_0 (n+N) + \emptyset)$$

$$\Omega_0 N = 2\pi k$$

$$N = \frac{2\pi k}{\Omega_0}$$

Nonperiodic Signals

- The definition of a nonperiodic signal is quite simple. If a continuous or discrete signal is not periodic, it is said to be nonperiodic.
  More specifically, a continuous signal x(t) is nonperiodic if there is no T > 0 such that x(t +T) = x(t).
  In the same way, a discrete signal x[n] is nonperiodic if there is no integer N > 0 such that x[n +N] = x[n]

Exercise: Show that the continuous signal $x(t) = e^{-4t}$ is nonperiodic

Even and Odd Signals

- In continuous time a signal is even if

$$x(t) = x(-t)$$

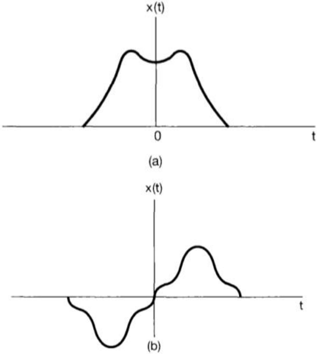
while a discrete-time signal is even if

$$x[n] = x[-n]$$

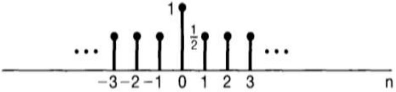
- A signal is referred to as odd if

$$x(-t) = -x(t)$$

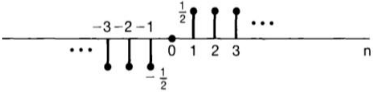
$$x[-n] = -x[n]$$



$$\mathcal{E}v\{x[n]\} = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



$$\mathcal{O}d\{x[n]\} = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



Even and Odd Signals

- An important fact is that any signal can be broken into a sum of two signals, one of which is even and one of which is odd.

That is, $$x(t) = x_e(t) + x_0(t)$$

✓ How?

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_0(t) = \frac{1}{2}[x(t) - x(-t)]$$

Energy and Power Signals

- Before we present the definition of energy and power signals, let us recall the following well-known relationship between the two terms energy and power:

$$power = \frac{Energy}{Time}$$

or equivalently

$$Energy = Power \cdot Time$$

Energy and Power Signals

- if v(t) and i(t) are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

The total energy expended over the time interval $t_1 \le t \le t_2$ is

$$\int P(t)dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

and the average power over this time interval is

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} P(t)dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

✓ With a unit *R*, what is the formulation of *P* and *E*?

Energy and Power Signals

- Now consider a continuous signal real-valued x(t). we can define the energy of x(t) over the time interval t ∈ [T1, T2] as:

$$E_{(t_1 \cdot t_2)} = \int_{T_1}^{T_2} |x[t]|^2 dt$$

In the above equation we implicitly assume that T2 > T1.

- The energy of a discrete signal x[n] over the interval [N1,N2] is given by

$$E_{[N_1 \cdot N_2]} = \sum_{k=N_1}^{N_2} |x[k]|^2$$

Energy and Power Signals

- The total energy of a continuous signal is its energy over the interval t ∈ (−∞,∞). Similarly, the total energy of a discrete signal is its energy over the interval k ∈ (−∞,∞). Thus, the total energy of a signal is given by

continuous signal $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Discrete signal $$E_x = \sum_{k=-\infty}^{\infty} |x[k]|^2$$

✓ Highlight: infinity time range

Energy and Power Signals

- Energy:
  - CT signals: $E_x = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$
  - DT signals: $E_x = \sum_{k=-\infty}^{\infty} |x[k]|^2$
- Average power:
  - CT signals: $P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt.$
  - DT signals: $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[k]|^2$

Definition (Energy signals)

A signal $x(t)$, or $x[k]$, is called an energy signal if the total energy E_x has a non-zero finite value, i.e.$0 < E_x < \infty$.

Definition (Power signals)

A signal is called a power signal if it has non-zero finite power, $0 < P < \infty$.

- A signal **cannot** be both an energy signal and a power signal.

Energy and Power Signals - Examples

Determine if the following signal is an energy or power signal

$$x(t) = \begin{cases} 8 & |t| < 5 \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

✓ Time-limited Signal

Energy and Power Signals - Examples...

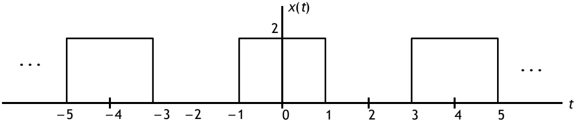
Determine if the following signal is an energy or power signal

$$x(t) = \begin{cases} e^{at} & t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

✓ Decays with time or not

Energy and Power Signals - Examples...

Determine if the following signal is an energy or power signal



✓ Periodic Signal

Energy and Power Signals...

- Most periodic signals are typically power signals
- The average power is calculated from one period of the signal
  - CT signals: $P_x = \frac{1}{T_0}\int_{<T_0>} |x(t)|^2 dt = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt.$
  - DT signals: $Px = \frac{1}{N_0}\sum_{<N_0>} |x[n]|^2 = \frac{1}{N_0}\sum_{n_0}^{n_0+N_0-1} |x[n]|^2$

Note

In the above equations t_0 is an arbitrary real number and n_0 is an arbitrary integer. In practice choosing t_0 and n_0 properly may simplify the computation.

Deterministic and Random Signals

- Deterministic signals

  Known its value at any time.

  Can be modelled as a completely specified function of time.

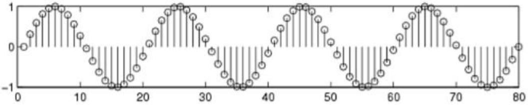
- Random Signals

  Take random values at any given time.

  Characterized statistically by a probability distribution function.

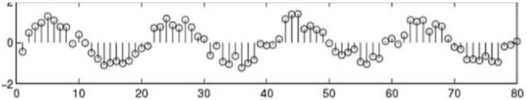
  Usually referred to as random processes

Deterministic and Random Signals



Deterministic

$y = \sin(\Omega n)$



Random

Probability density function, like:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-u)^2}{2\sigma^2}\right)$$

Complex Exponential Signals

- A second important class of complex exponentials is obtained by constraining a to be purely imaginary. Specifically, consider

$$x(t) = e^{jw_0 t}$$

x(t) will be periodic with period T if

$$e^{jw_0 t} = e^{jw_0 (t+T)}$$

it follows that for periodicity, we must have

$$e^{jw_0 T} = 1$$

$$T_0 = \frac{2\pi}{|w_0|}$$

$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$\cos\alpha = \mathrm{Re}\left[e^{j\alpha}\right]$$

$$\sin\alpha = \mathrm{Im}\left[e^{j\alpha}\right]$$

Discrete-Time Complex Exponential signal

- Similarly , the Discrete-Time Complex Exponential

$$x[n] = e^{jw_0 n}$$

x(n) will be periodic with period N if

$$e^{jw_0 n} = e^{jw_0 (n+N)}$$

or equivalently,

$$e^{jw_0 N} = 1$$

$$e^{jw_0 n} = \cos w_0 n + j\sin w_0 n$$

$$w_0 N = 2\pi m$$

$$N = \frac{2\pi m}{w_0}$$