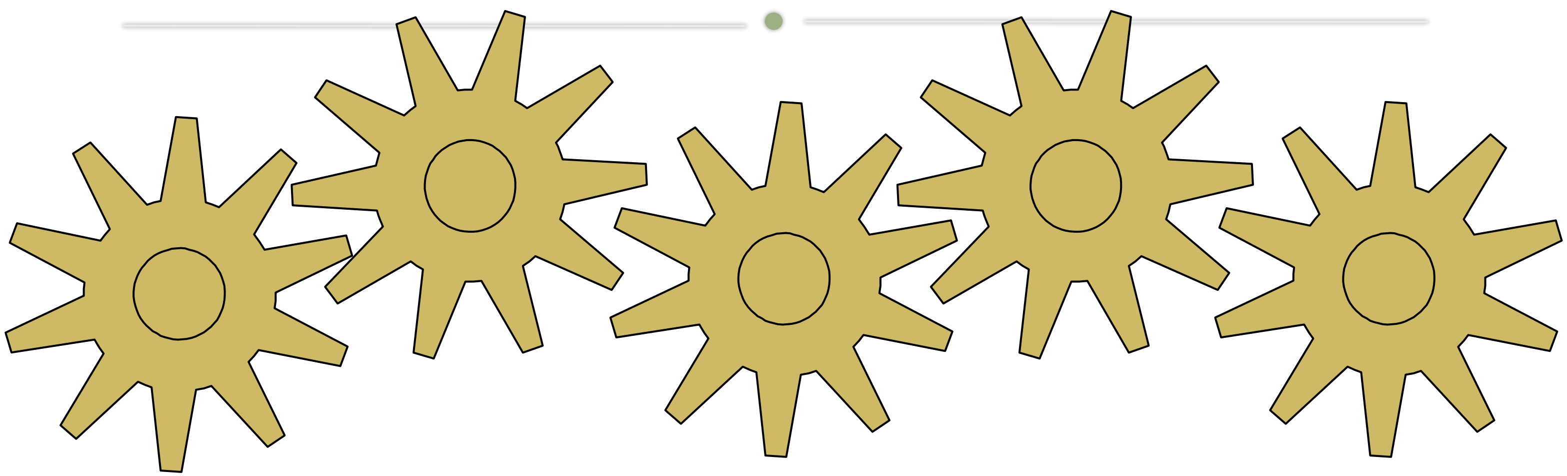
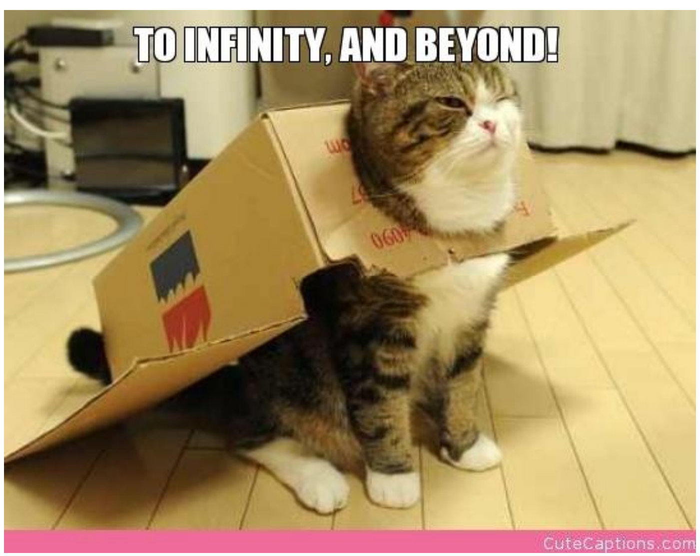
EE114
Intro to Systems & Control

Dr. Lachman Tarachand
Dr. Chen Zhicong
Prepared by Dr. Séamus McLoone
Dept. of Electronic Engineering



So far …

- We’ve modelled a range of systems – obtained differential equations & transfer function models …
- Used block diagram algebra to simplify complicated systems …
- Started to analyse our models – first we obtained their time response …



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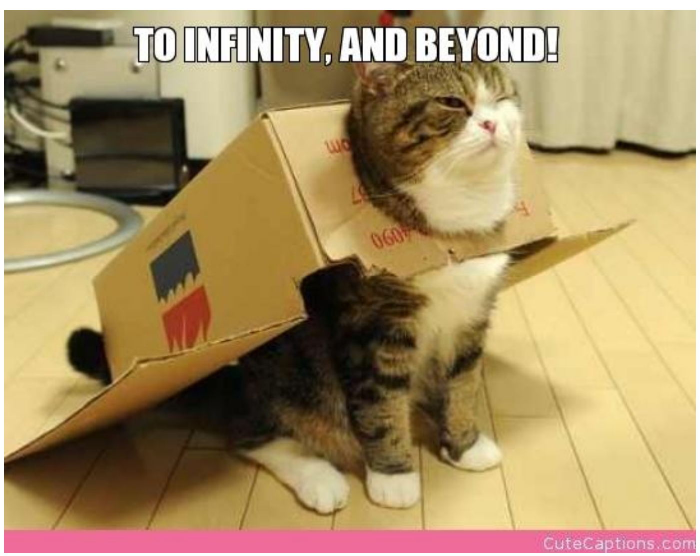

- **Today, we will look at their stability …**

Stability

- For the purposes of analysis and design of control systems, it's important that we consider and understand three key features of the underlying dynamical system, namely **stability, the transient response, and the steady-state output**.

- *We will examine the transient response and steady-state output in the next section of the notes*.

- Here, we will examine the concept of stability, and how we can determine the stability of a system from its transfer function representation.

- Firstly, we need to introduce the **concepts of poles and zeros.**

Poles and Zeros

Poles and Zeros

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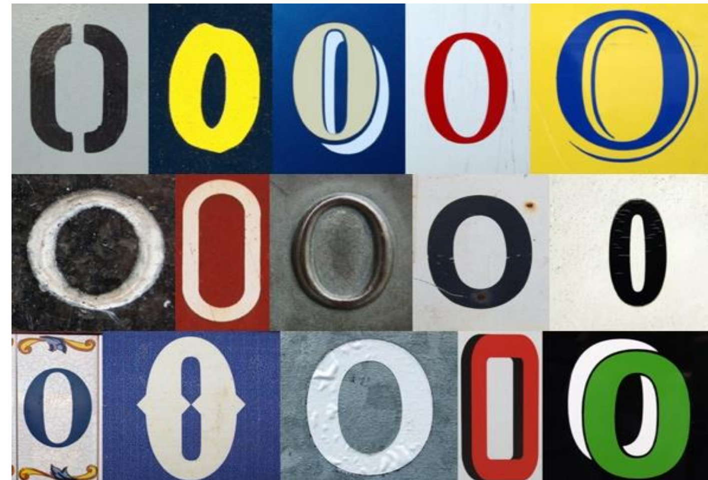

Poles and Zeros

- Given a continuous-time transfer function in the Laplace domain (i.e. the *s*-domain):

$$\frac{s + a_1}{s^2 + a_2 s + a_3}$$

Poles and Zeros

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  - a **zero** is any value of *s* such that the transfer function is 0

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Poles and Zeros

- Given a continuous-time transfer function in the Laplace domain (i.e. the *s*-domain):
  - a **zero** is any value of *s* such that the transfer function is 0
  - a **pole** is any value of *s* such that the transfer function is ∞

$$\frac{s+a_1}{s^2+a_2 s+a_3} = \mathbf{0}$$

Poles and Zeros

- In other words:
  - ***zeros are the roots of the numerator of the transfer function***

$$\frac{s+a_1}{s^2+a_2s+a_3} \quad = 0$$

Poles and Zeros

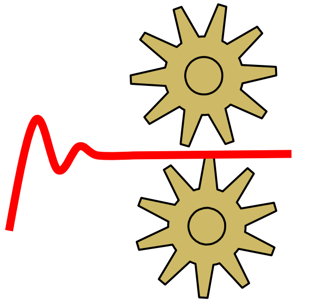
- In other words:
  - ***zeros are the roots of the numerator of the transfer function***
  - ***poles are the roots of the denominator of the transfer function***

$$\frac{s+a_1}{s^2+a_2s+a_3} = \mathbf{0}$$

Poles and Zeros

- These are typically plotted in the complex plane (known as the s-plane) with:

  zeros represented by '**O**' and poles represented by '**X**'



Poles and Zeros

- These are typically plotted in the complex plane (known as the s-plane) with:

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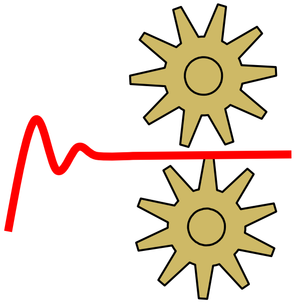
- A plot of a system's zeros and poles is referred to as the **pole-zero diagram**.



Poles and Zeros

- ***Ex 7.1 Determine the pole-zero diagram for the following systems:***

$$G(s) = \frac{2}{s+3}$$

$$G(s) = \frac{s-1}{s^2+3s+2}$$

$$G(s) = \frac{2s-4}{s(s^2+2s+4)}$$

Poles and Zeros

Solution:

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There is **no zero**, as the numerator does not contain any s term.

Setting the denominator to 0 gives: $s + 3 = 0 \quad \Rightarrow \boldsymbol{s = -3}$

Poles and Zeros

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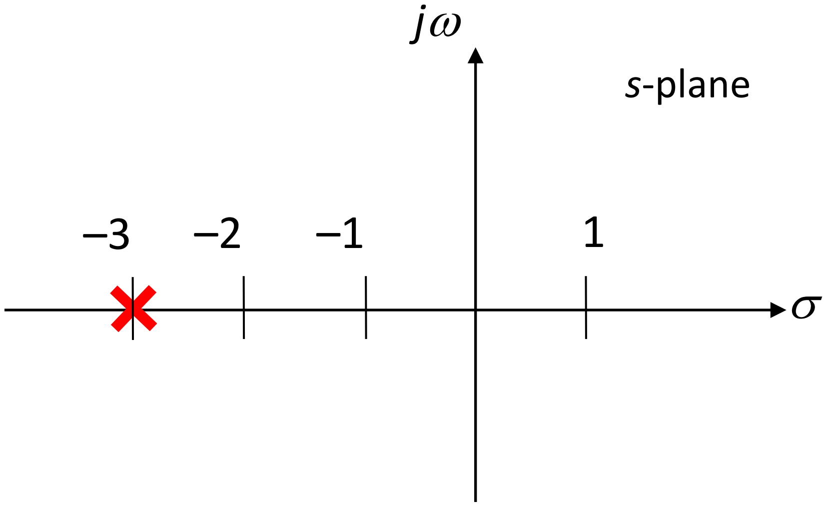
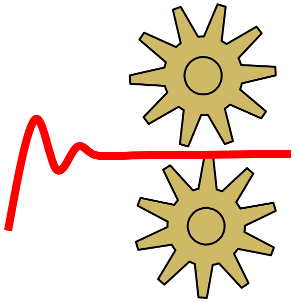
Setting the denominator to 0 gives: $s + 3 = 0 \quad \Rightarrow \boldsymbol{s = -3}$

Recall that:
$s = \sigma + j\omega$





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Poles: $s^2 + 3s + 2 = 0 \quad \Rightarrow (s+1)(s+2) = 0 \quad \Rightarrow \boldsymbol{s = -1,\ s = -2}$

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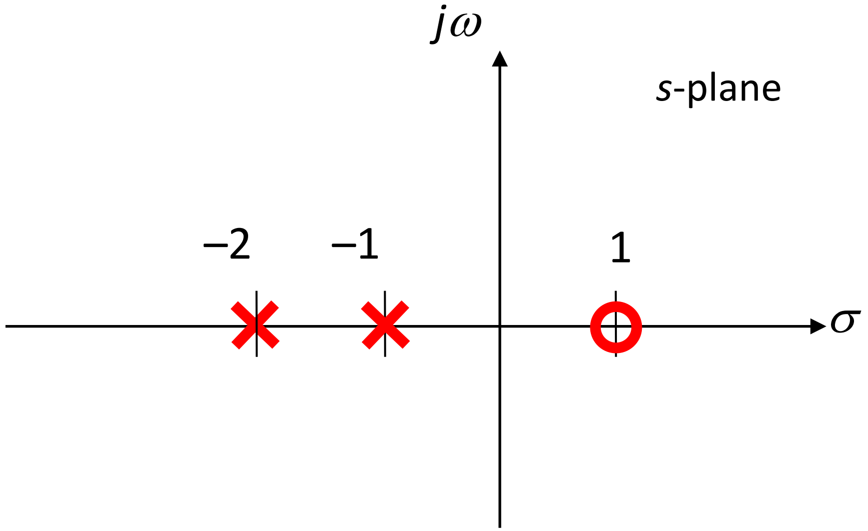

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Poles: $s(s^2+2s+4) = s(s+1-j\sqrt{3})(s+1+j\sqrt{3}) = 0 \qquad \Rightarrow \boldsymbol{s = 0, -1 \pm j\sqrt{3}}$

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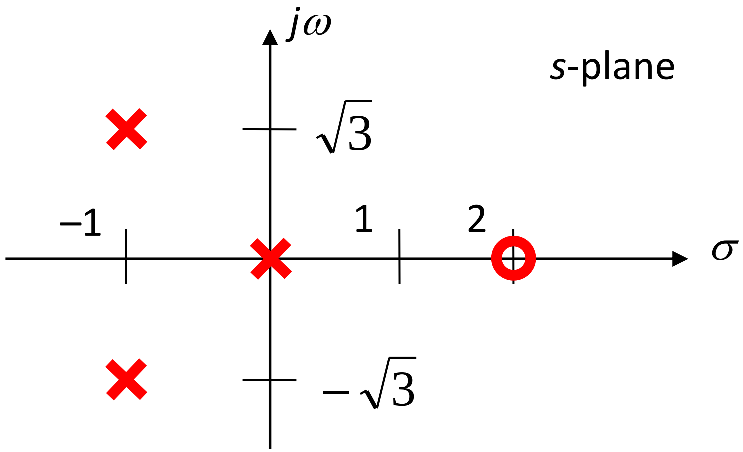
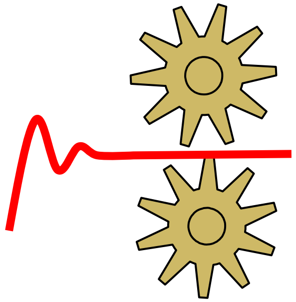




Poles and Zeros

- Previously we noted that the order of a system corresponds to the highest power of *s* in the denominator of the transfer function.

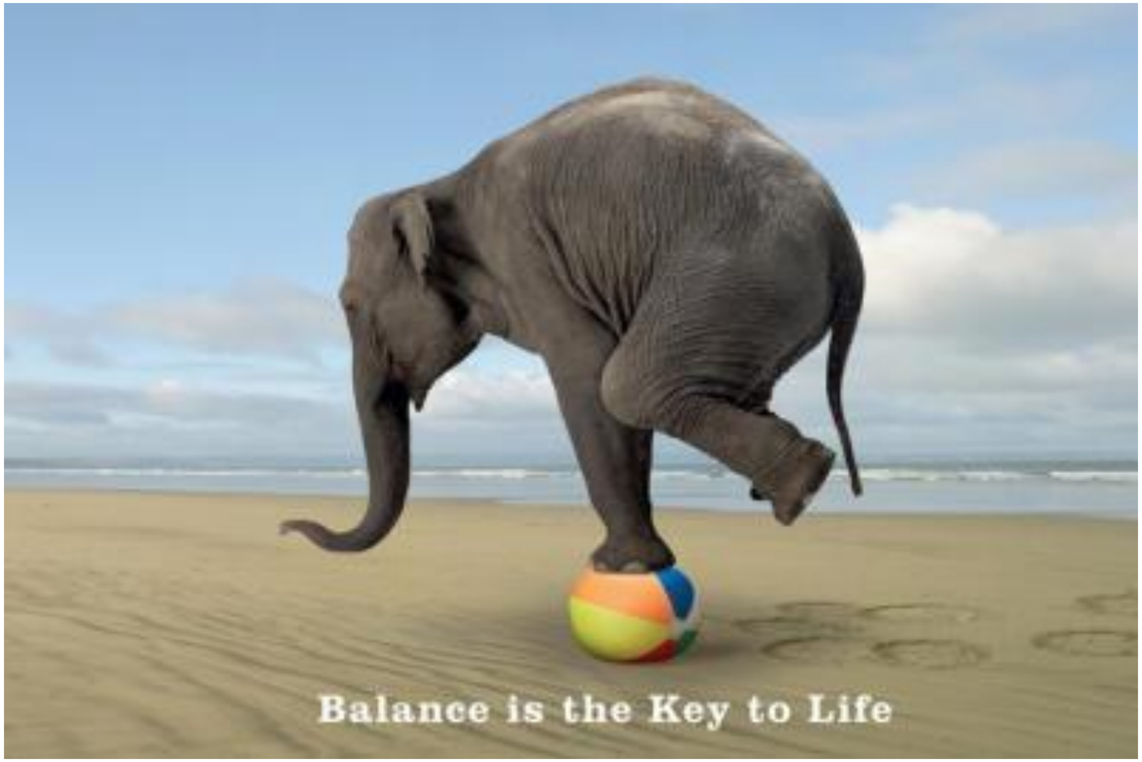
- This also equates to the number of poles in the system, as is clearly evident in the above solutions.

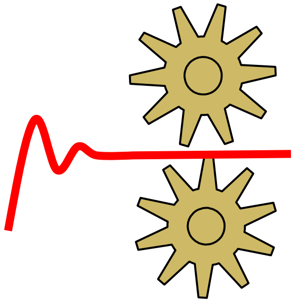
- Hence, we can also state that **the order of a system is determined by the number of poles it has and vice versa**.

- Pole-zero diagrams are important as they summarize the key features of transfer functions in terms of transient and steady-state responses, as we will see in the next section of the notes.

Poles and Zeros

- The stability of the system is also determined by the location of the poles, as we shall now illustrate.





Definitions of Stability

Definitions of Stability

- Stability is a very important requirement in most practical systems, i.e. systems should remain stable at all times.
- It is neither safe nor desirable for an airplane to go into an uncontrollable roll, a robotic arm to spin with ever increasing velocity or an elevator to crash through the floor or exit through the ceiling.
- Clearly, the need for stability is an absolute necessity in such systems.

Definitions of Stability

- Here, we are only considering continuous-time linear time-invariant (LTI) systems.

- Stability for such systems can be categorised as stable, unstable or marginally stable, each of which can be defined in terms of the system's natural response.

- *Recall from section 6.3 of the notes on solving first-order differential equations, the* ***natural response is defined as the output of a system for zero input.***

Definitions of Stability

- Hence, we have the following stability definitions.

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- A LTI system is:

  - **stable** if the natural response **decays to zero** as $t \to \infty$

  - **unstable** if the natural response **grows unbounded** as $t \to \infty$

Definitions of Stability

- Hence, we have the following stability definitions.

- A LTI system is:

  - **stable** if the natural response **decays to zero** as $t \to \infty$

  - **unstable** if the natural response **grows unbounded** as $t \to \infty$

  - **marginally stable** if the natural response neither decays nor grows but **remains constant or oscillates** as $t \to \infty$

Definitions of Stability

- Each of these concepts are illustrated below:

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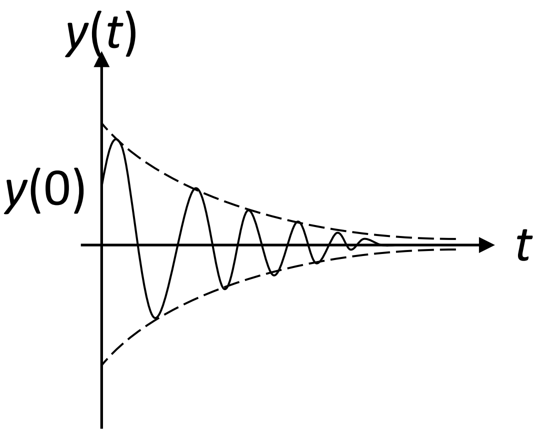

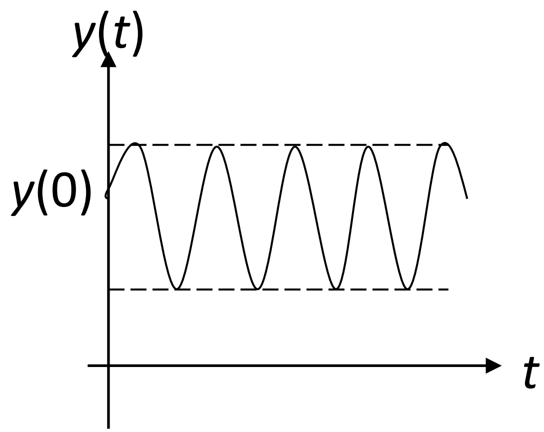

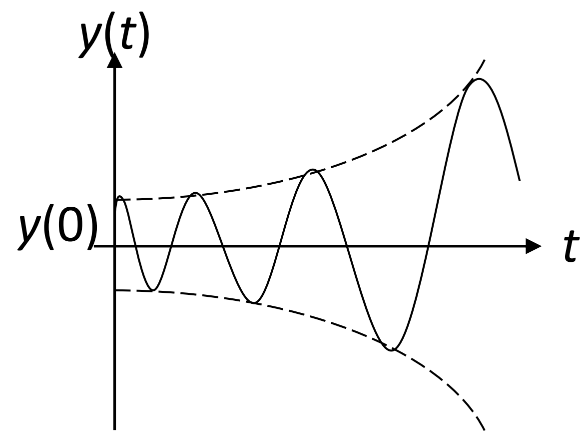

Stable

$y(t) \to 0$ as $t \to \infty$

Marginally stable

$y(t)$ neither decays nor grows

Unstable

$y(t) \to \infty$ as $t \to \infty$

Stability and the Transfer Function

- The following summarises how three different transfer functions (all representing **first order systems**) and their corresponding pole locations relate to stability:

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$$G(s) = \frac{k}{s+\alpha}$$

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$$G(s) = \frac{k}{s+\alpha} \longrightarrow g(t) = ke^{-\alpha t}$$

Stability and the Transfer Function

Note that $g(t)$ is obtained from the inverse Laplace transform of $G(s)$ and represents the **natural response** of the system, i.e. this is the output of the system with **no input applied**.

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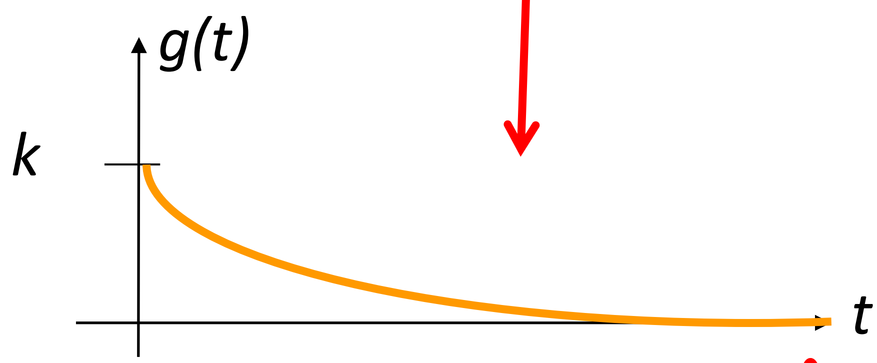

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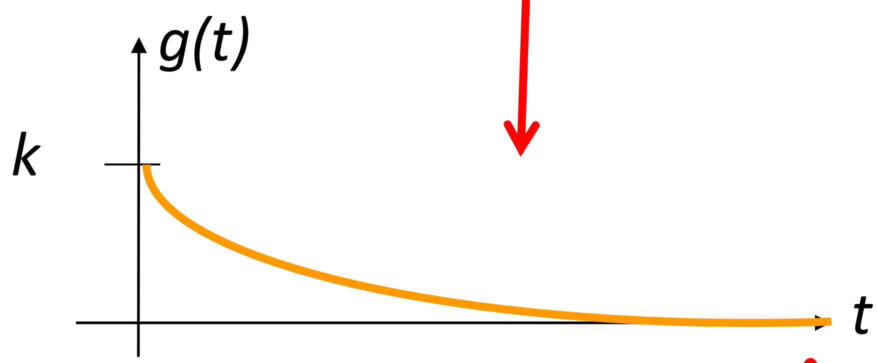

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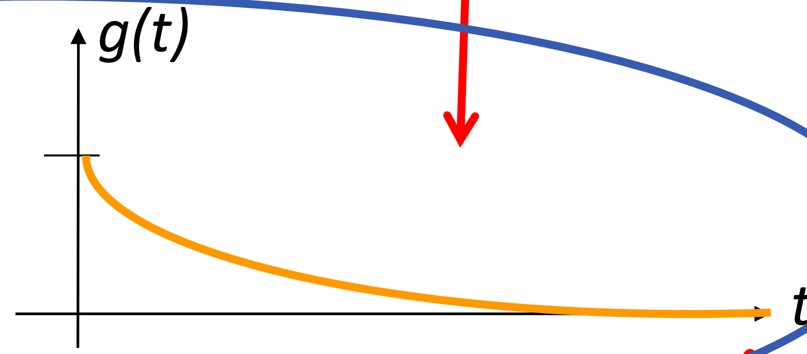

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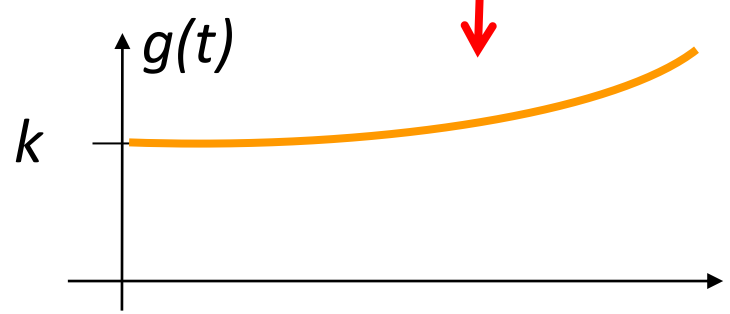

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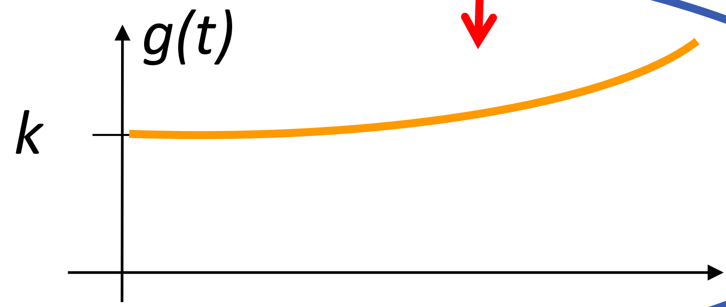

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$$G(s) = \frac{k}{s}$$

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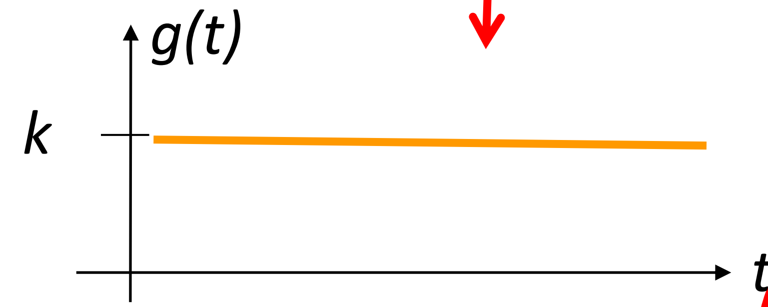

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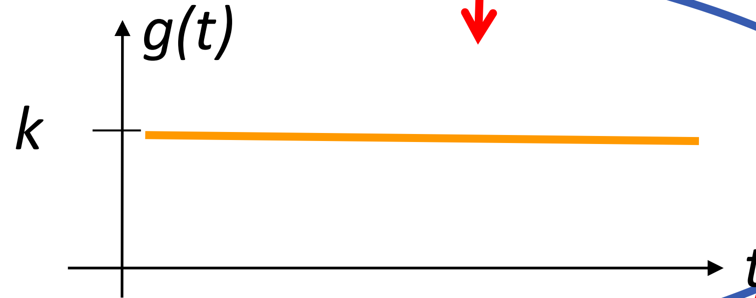

NOTE

Stability and the Transfer Function

- From the plots we can clearly see that:
  - the system with the **negative pole has a stable response**,
  - the system with the **positive pole has an unstable response** and
  - the system with a **pole at the origin (i.e. $s = 0$) has a marginally stable response**, as the output is constant.

Stability and the Transfer Function

- Now, let us consider a second-order system with the following transfer function:

$$G(s) = \frac{k}{(s+\alpha)(s+\beta)}$$

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- Now, consider the different possibilities for the parameters α and β.

Stability and the Transfer Function

$$G(s) = \frac{k}{(s+\alpha)(s+\beta)} \qquad g(t) = Ae^{-\alpha t} + Be^{-\beta t}$$

- α and β are both positive, i.e. $\alpha > 0$ and $\beta > 0$:

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Both system poles are negative, i.e.

$$s = -\alpha, \quad s = -\beta$$

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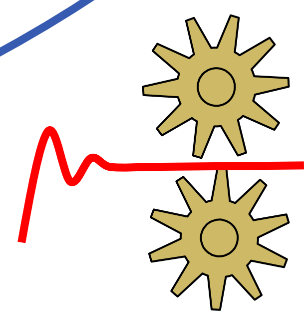

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$\Rightarrow$ The system is marginally stable.

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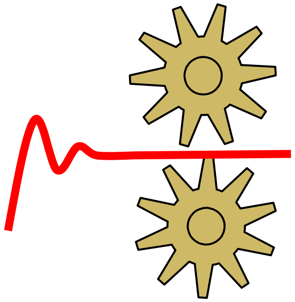
NOTE

Stability and the Transfer Function

- When we compare these results with those for the first-order system we can observe a common trend between the location of the poles and the stability of the system.

- The same pattern exists for higher-order systems.

- So, in brief, we can state that the **stability of a system depends on its poles, i.e. the roots of the denominator of $G(s)$**.



Stability and the Transfer Function

- The following stability conditions apply:
  - stable if *Re* $(s_i) < 0$ for ***all*** poles
  - marginally stable if *Re* $(s_i) = 0$ for some of the poles and *Re* $(s_i) < 0$ for the other poles
  - unstable if *Re* $(s_i) > 0$ for ***any*** of the poles.

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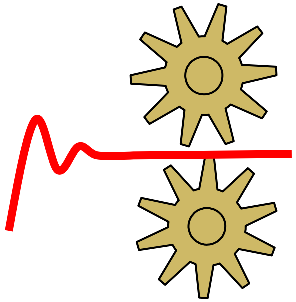
  - unstable if $Re\ (s_i) > 0$ for ***any*** of the poles.

- Note that $s = \sigma + j\omega$ is a complex variable.
- However the imaginary part does not impact in the stability of the system. ***All that matters is whether the poles are positive or negative.***

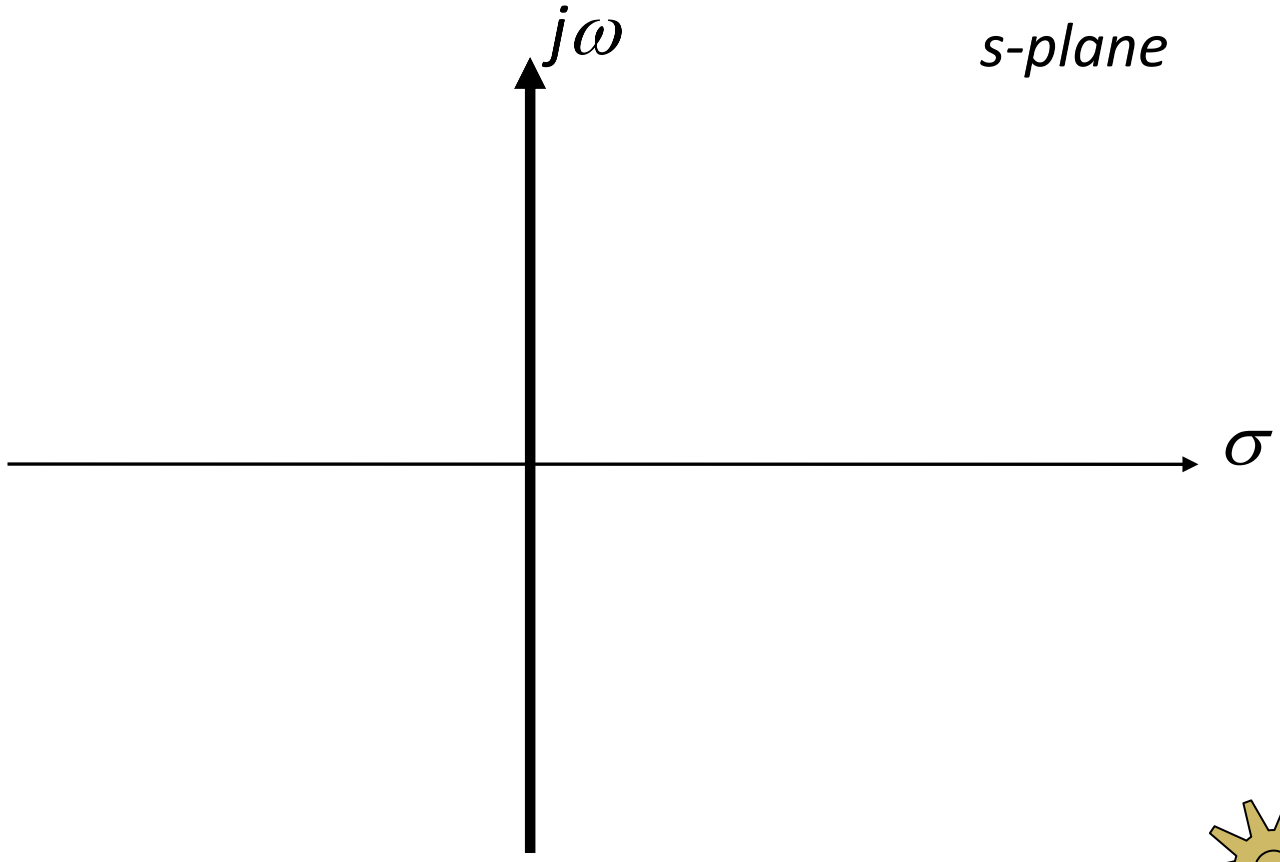


Stability and the Transfer Function

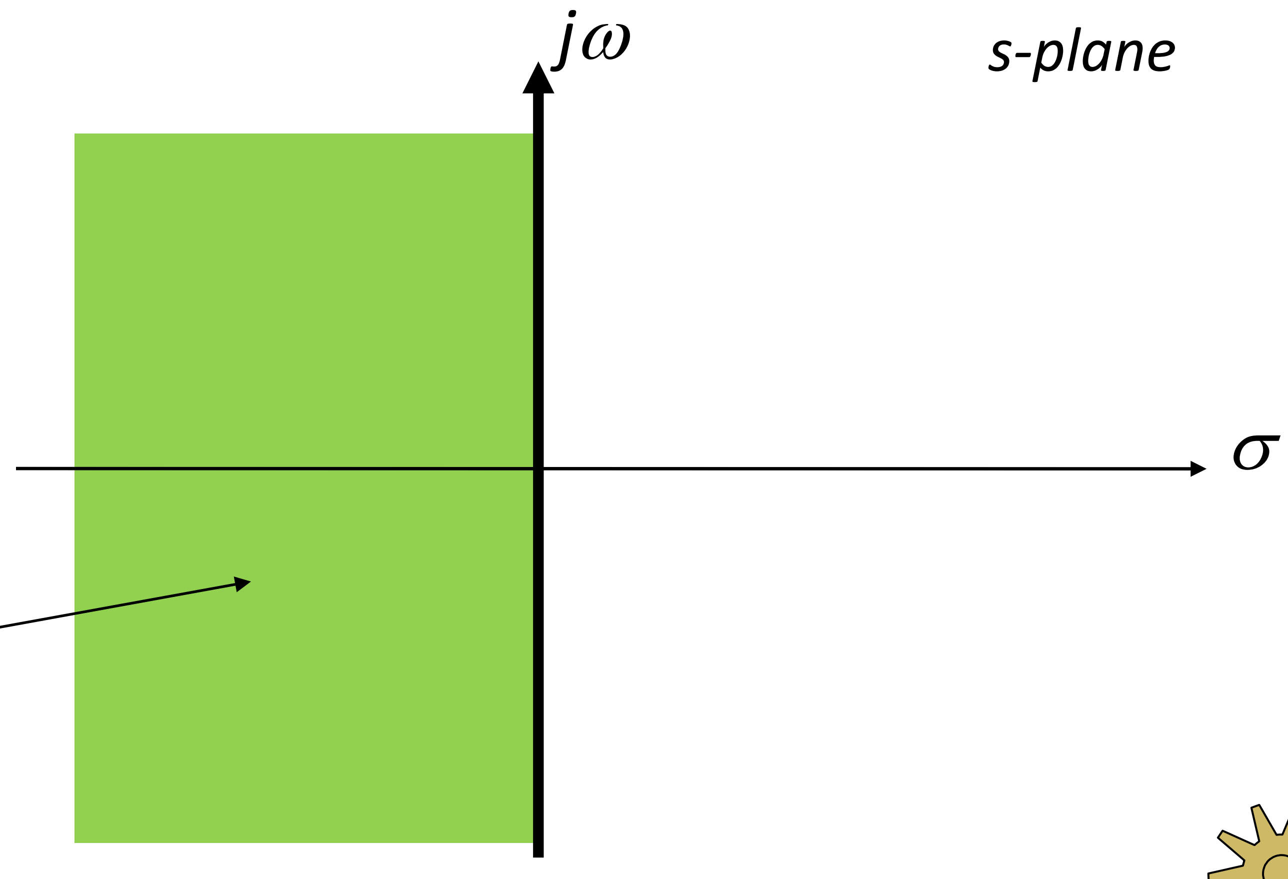
- Hence, all we need to consider in terms of stability is the sign of σ, i.e. the real part of *s* or simply *Re* (*s*).

- To make life even easier, we can visualise **the criteria for stability of continuous-time** systems by viewing these conditions **on the *s*-plane** as follows …

Stability and the Transfer Function

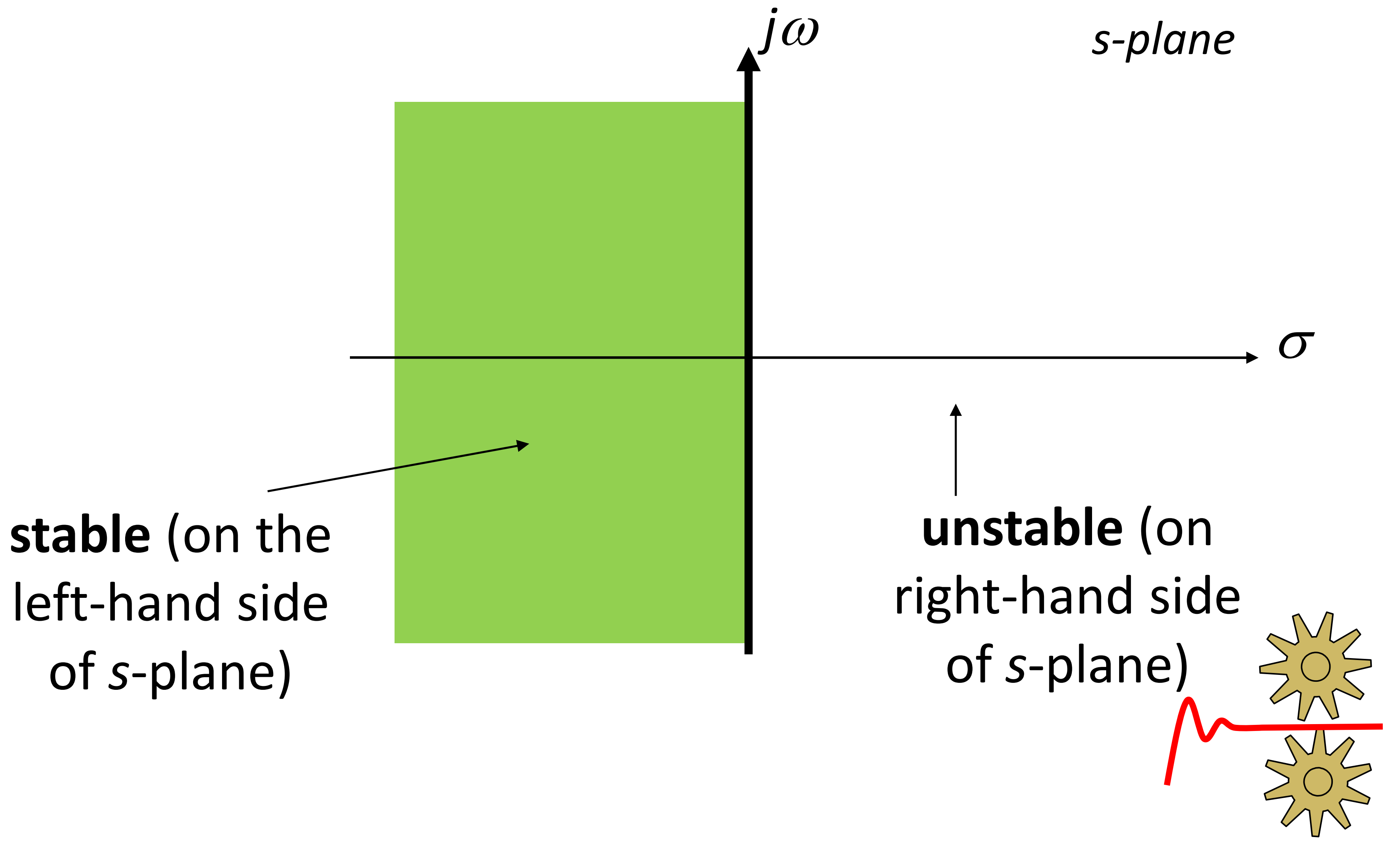


Stability and the Transfer Function

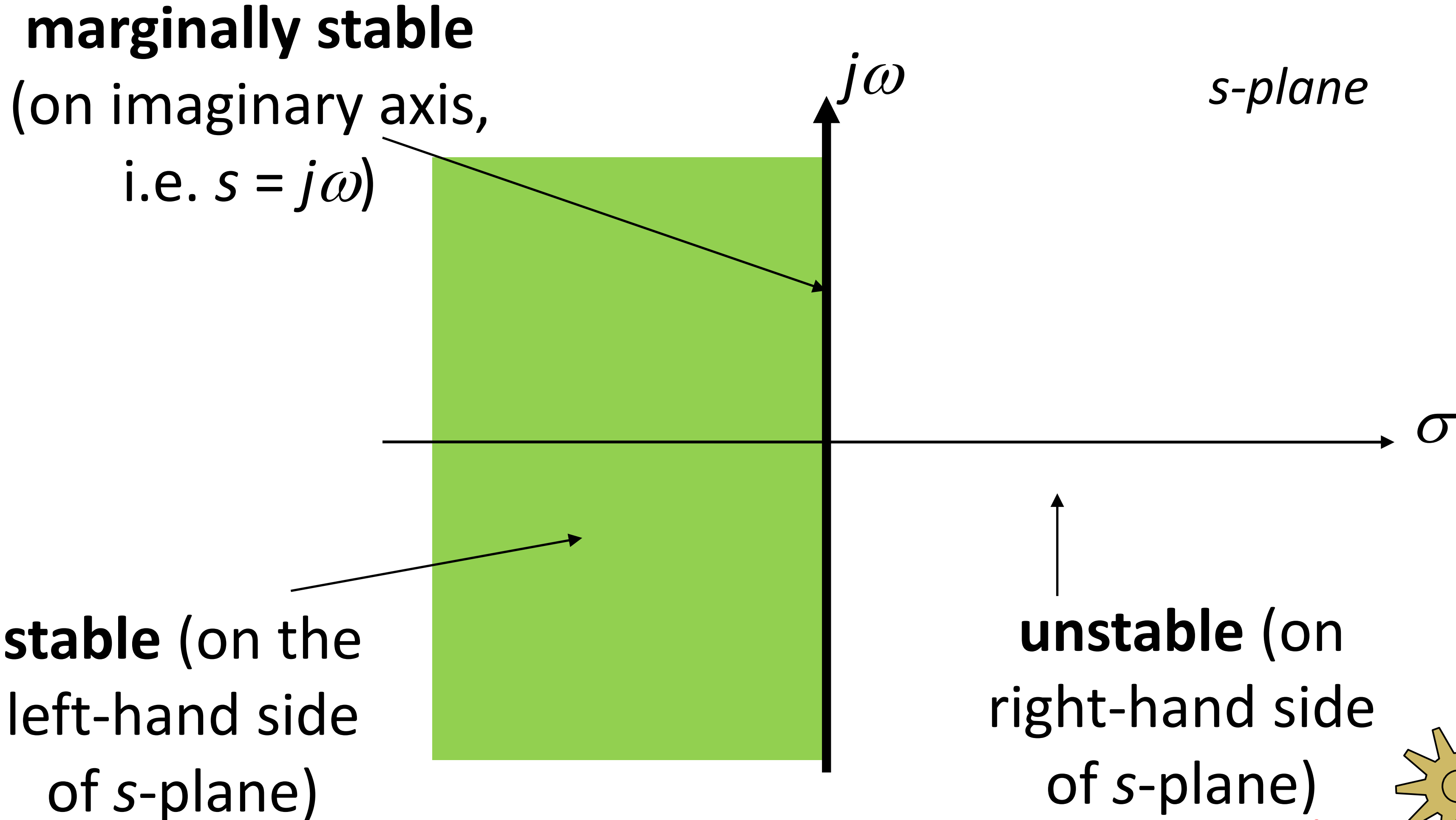


stable (on the left-hand side of *s*-plane)

Stability and the Transfer Function

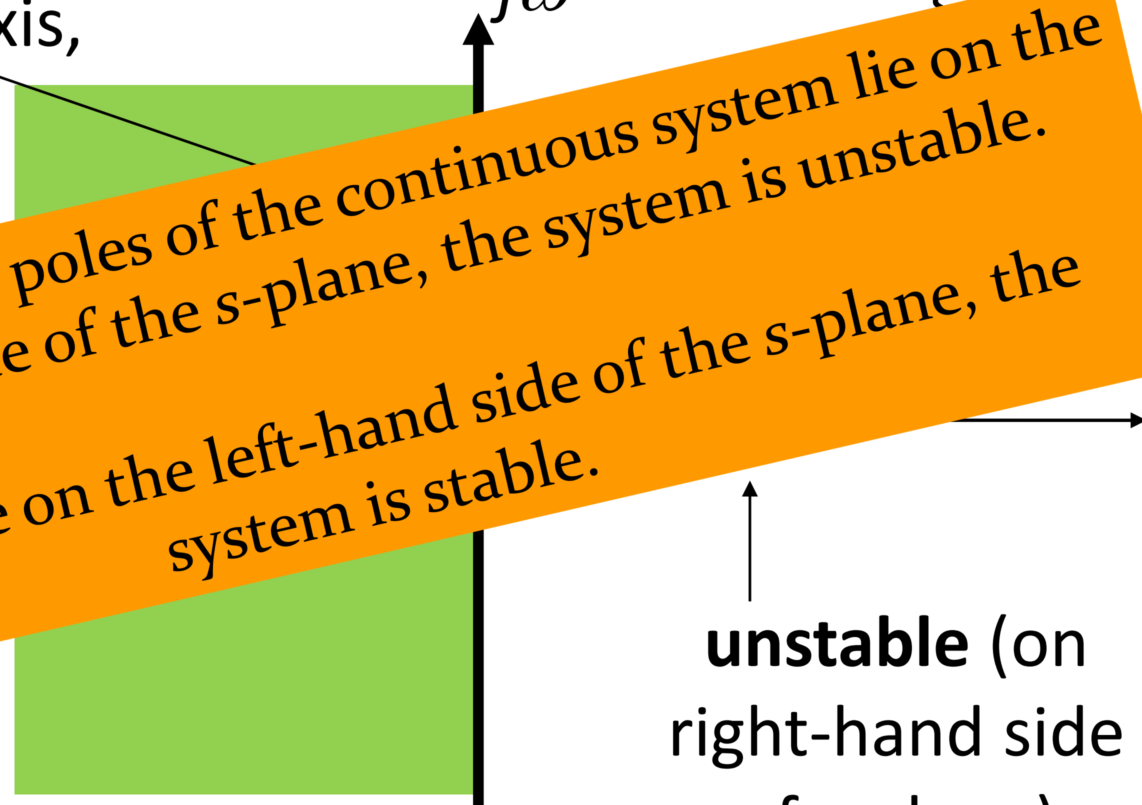


Stability and the Transfer Function



Stability and the Transfer Function

marginally stable
(on imaginary axis,
i.e. $s = j\omega$)

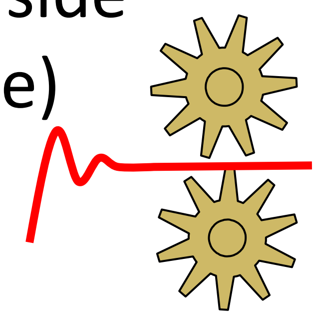


Simply put, if any poles of the continuous system lie on the right-hand side of the s-plane, the system is unstable.

If all poles lie on the left-hand side of the s-plane, the system is stable.

st... ...**ne**
left-hand side
of *s*-plane)

unstable (on
right-hand side
of *s*-plane)



Stability and the Transfer Function

- ***Ex 7.2 Determine the stability of the system described by the following transfer functions:***

$$G(s) = \frac{2}{s+3} \qquad G(s) = \frac{s-1}{s^2+3s+2}$$

$$G(s) = \frac{2s-4}{s(s^2+2s+4)} \qquad G(s) = \frac{s+1}{(s-2)(s+3)^2}$$

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{2}{s+3}$$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{2}{s+3}$$

Pole located at $s = -3$

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{2}{s+3}$$

Pole located at $s = -3$

... left-hand side of s-plane

$\Rightarrow$ system is **stable**

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{s-1}{s^2 + 3s + 2}$$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{s-1}{s^2+3s+2}$$

Poles: $s^2 + 3s + 2 = 0 \;\Rightarrow (s + 1)\,(s + 2) = 0 \;\Rightarrow \boldsymbol{s = -1,\ s = -2}$

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{s-1}{s^2+3s+2}$$

Poles: $s^2 + 3s + 2 = 0 \;\Rightarrow (s+1)(s+2) = 0 \;\Rightarrow \boldsymbol{s = -1, s = -2}$

Both poles on left-hand side of s-plane $\Rightarrow$ system is **stable**.

Stability and the Transfer Function

Solution …

$$G(s) = \frac{2s - 4}{s(s^2 + 2s + 4)}$$

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{2s-4}{s(s^2+2s+4)}$$

Poles: $s(s^2+2s+4) = s(s+1-j\sqrt{3})(s+1+j\sqrt{3}) = 0$

$$\Rightarrow s = 0,\ \ -1 \pm j\sqrt{3}$$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{2s-4}{s(s^2+2s+4)}$$

Poles: $s(s^2+2s+4) = s(s+1-j\sqrt{3})(s+1+j\sqrt{3}) = 0$

$$\Rightarrow s = 0,\ \ -1 \pm j\sqrt{3}$$

One pole at the origin (on the imaginary axis),
other poles on left-hand side of *s*-plane

$\Rightarrow$ system is **marginally stable**.

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{s+1}{(s-2)(s+3)^2}$$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{s+1}{(s-2)(s+3)^2}$$

Poles: $s = 2, \quad s = -3, \quad s = -3$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{s+1}{(s-2)(s+3)^2}$$

Poles: $s = 2, \quad s = -3, \quad s = -3$

The s = 2 pole is on the right-hand side of s-plane

$\Rightarrow$ system is **unstable**.

Stability and the Transfer Function

- ***Ex 7.3 Draw the pole-zero diagram for the system with the following transfer function and hence determine its stability:***

$$G(s) = \frac{s^2 + 2s + 1}{s(s^2 - s - 6)}$$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{s^2 + 2s + 1}{s(s^2 - s - 6)}$$

Stability and the Transfer Function

Solution ...

$$G(s) = \frac{s^2 + 2s + 1}{s(s^2 - s - 6)}$$

Zero: $s^2 + 2s + 1 = 0 \;\; \Rightarrow (s + 1)\,(s + 1) = 0 \;\; \Rightarrow \boldsymbol{s = -1,\ s = -1}$

Stability and the Transfer Function

Solution …

$$G(s) = \frac{s^2 + 2s + 1}{s(s^2 - s - 6)}$$

Zero: $s^2 + 2s + 1 = 0 \;\Rightarrow (s + 1)\,(s + 1) = 0 \;\Rightarrow \boldsymbol{s = -1,\ s = -1}$

Poles: $s\,(s^2 - s - 6) = 0 \;\Rightarrow s\,(s - 3)\,(s + 2) = 0 \;\Rightarrow \boldsymbol{s = 0,\ s = 3,\ s = -2}$

Stability and the Transfer Function

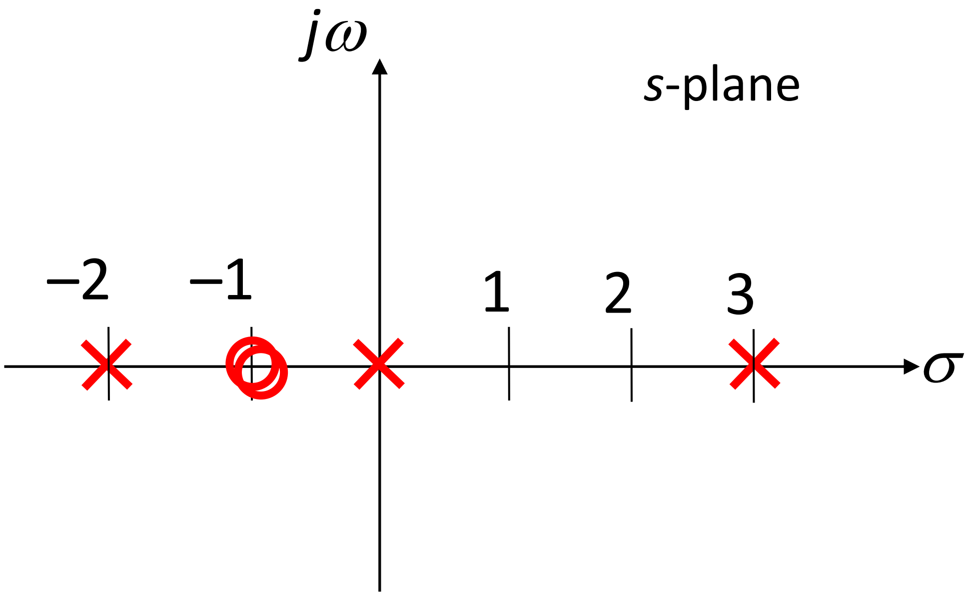
Solution ...

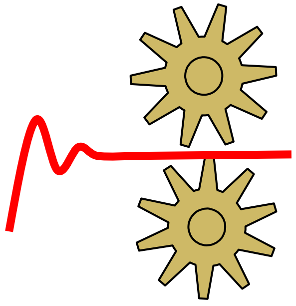
$$G(s)=\frac{s^2+2s+1}{s(s^2-s-6)}$$

Zero: $s^2 + 2s + 1 = 0 \;\Rightarrow (s + 1)\,(s + 1) = 0 \;\Rightarrow \boldsymbol{s = -1, s = -1}$

Poles: $s\,(s^2 - s - 6) = 0 \;\Rightarrow s\,(s - 3)\,(s + 2) = 0 \;\Rightarrow \boldsymbol{s = 0, s = 3, s = -2}$

Hence:





Stability and the Transfer Function

Solution …

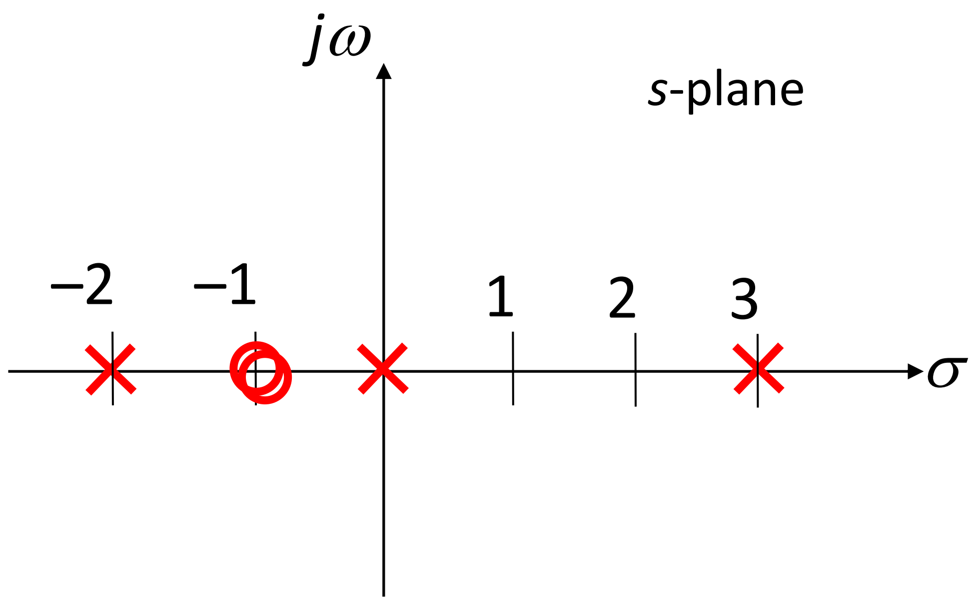
$$G(s) = \frac{s^2 + 2s + 1}{s(s^2 - s - 6)}$$

Zero: $s^2 + 2s + 1 = 0 \quad \Rightarrow (s + \ldots$

Stability: since this system has a positive pole (i.e. on the right-hand of the *s*-plane), it is **unstable**.

Po… $\ldots 3)(s + 2) = 0 \quad \Rightarrow$ $\boldsymbol{s = 0, s = 3, s = -2}$

Hence:



Time for an Engineering Joke …

Time for an Engineering Joke …

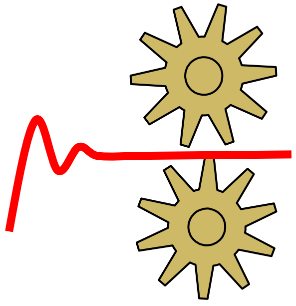
A plane is flying from Germany to Poland when it hits a patch of turbulence and starts to shake considerably.

The pilot looks worried, but fear not, the trusty co-pilot rushes to the public address system and asks for a show of hands from any passengers from Poland.

He then instructs all those who have raised their hands to please seat themselves on the left hand side of the plane.

Immediately the plane stops shaking and all is well once more.

The pilot is amazed and asks "what just happened?"



The co-pilot replies …

Time for an Engineering Joke …

… "every good engineer knows that in order to obtain stability, all poles must be on the left hand side of the plane!"

(I never said it was a good joke!)