Lecture 5: Elementary Signals

EE213 - Introduction to Signal Processing

Semester 1,2019

Outline

- Study simple models of a continuous time signal. These are also known as elementary signals
  - Sinusoidal signals
  - Exponential signals
  - Unit step and unit ramp
  - Impulse functions

- We mainly focus on continuous time signals. The discrete-time signal can be easily obtained by sampling

Roles of Elementary Signals

- Serveral elementary signals feature prominently in the study of signals and systems (e.g.Sinusoidal Signals, Exponential Signals, the step function, the impluse function and the ramp function).
- A signal in reality is often complex → difficult to analyse directly.

  Elementary signals serve as building blocks to construct more complex signals.
- They are used to model many physical models in nature.

In what follows, we will describe these elementary signals, one by one.

Sinusoidal Signals

- A sinusoidal signal is of the form

$$x(t) = A\cos(wt + \theta)$$

where the amplitude is A, the angular frequency is w, which has the units of radians/s.

- It is also commonly written as

$$x(t) = A\cos(2\pi ft + \theta)$$

where *f* is the frequency in Hertz.

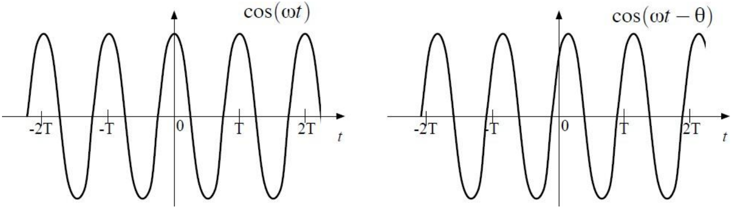
Sinusoidal Signals...

- The period of the sinusoid is

$$T = \frac{1}{f}$$

  with the units of seconds.

- The phase shift of the signal is θ, given in radians, all known as initial phase.



Exponential Signals

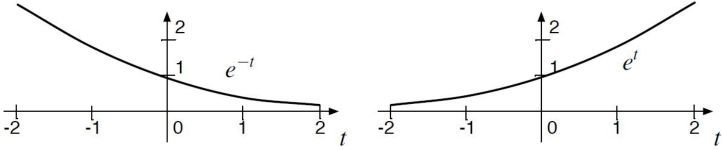
- An exponential signal is given by

$$x(t) = Ae^{\sigma t}$$

where both A and σ are real parameter.

The parameter A is the amplitude of the exponential signal measured at time t=0.

- We differentiate two cases:
  - If $\sigma < 0$ this is exponential decay.
  - If $\sigma > 0$ this is exponential growth.

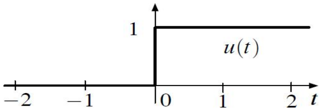


Unit Step Functions

- The continuous-time version of the unit step function $u(t)$ is deined as

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}.$$

- Depicted as follows



- It is said to exhibit a discontinuity at t=0, since the value of $u(t)$ changes intantaneously from 0 to 1 when t=0. That is, u(0) is undefined.
- The unit-step function $u(t)$ is a particularly simple signal to apply. Electrically, a battery or DC source is applied at t = 0 by, for example, closing a switch.
- As a test signal, the unit-step function is useful because the output of a system due to a step input reveals a great deal about how quickly the system responds to an abrupt change in the input signal.

Unit Step Functions. . .

- Unit step functions can be used to extract part of another signal.

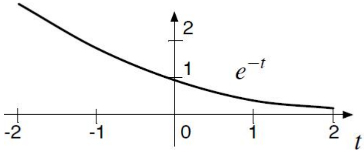
Example

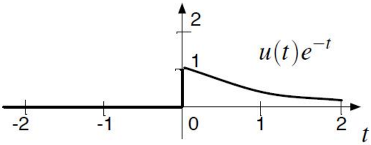
The piecewise-deined signal

$$x(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

can be written as

$$x(t) = u(t)e^{-t}$$



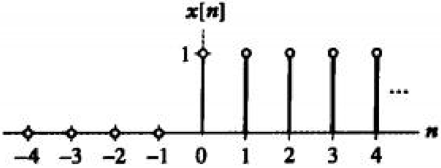


Unit Step Sequence

- The unit step sequence (or also known as discrete-time unit step)u[n] is deined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- illustrated in following Fig:

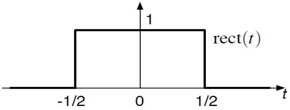


Unit Rectangle

- Unit rectangle signal

$$x(t) = \mathrm{rect}(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

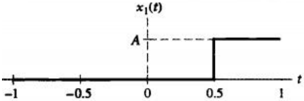
where |t| denotes the magnitude of time t.

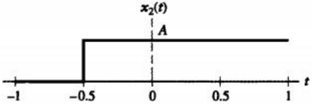


- The rectangular pulse $x(t)$ is represented as the difference of two time-shifted step functions, $x_1(t)$ and $x_2(t)$. That is, we may express $x(t)$ as

$$x(t) = Au\left(t + \frac{1}{2}\right) - Au\left(t - \frac{1}{2}\right),$$

where A=1.





Unit Rectangle

Example

Any rectangular signal

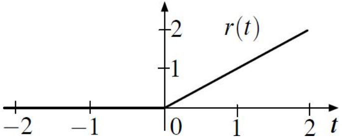
$$x(t) = \begin{cases} 0 & t > t_1 \\ A & t_0 \leq t \leq t_1 \\ 0 & t < t_0 \end{cases}$$

can be written as ??

Ramp Function

- The unit ramp is defined as

$$x(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

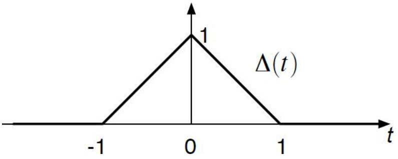


- As a test signal, the ramp function enables us to evaluate how a continuous-time system would respond to a signal that increases linearly with time.
- We create the unit ramp signal from the unit step signal, using an integrator circuit.

- Unit Triangle Signal

$$x(t) = \triangle(t) = \begin{cases} 1 - |t| & t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Shown as follow:



Impulsive / Delta signal

- The continuous-time impulsive signal is defined by:

$$\delta(t) = 0 \quad \text{for} \quad t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1.$$

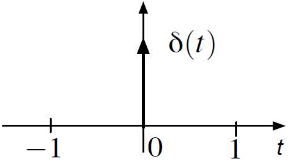
- Definition says that:

  1) the impulse $\delta(t)$ is zero everywhere except at the origin;

  2) the total area under the impulsive signal is one.

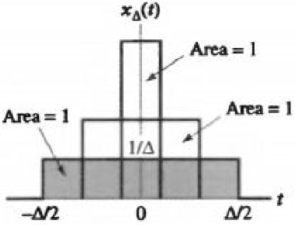
- $\delta(t)$ does not exist in reality.

- $\delta(t)$ is shown as a solid arrow:



Impulsive signals

- One way to visualize $\delta(t)$ is to view it as the limiting form of a rectangular pulse of unit area, as illustrated in Fig.(a).
- Specifically, the duration of the pulse is decreased, and its amplitude is increased, such that the area under the pulse is maintained constant at unity.



(a)

- As the duration decreases to approach 0, the rectangular pulse approximates the impulsive signal.

Impulsive signals…

- the step function u(t) is the integral of the impulse $\delta(t)$ with respect to time t:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

- $\delta(t)$ is the derivative of u(t) with respect to time t:

$$\delta(t) = \frac{d}{dt} u(t).$$

- Let $x(t)$ be a continuous-time function, we have

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

The operation extracts out the value $x(t_o)$ of the function $x(t)$ at time $t = t_0$.

Impulsive signals

- For discrete signals (unit sample or unit impulse), we have

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$