Electrostatics

Introduction to Charge

Electricity may be described as the phenomenon of electric charge in motion.

To better understand the simplest of electric circuits, it is necessary to discuss what we mean by 'electric charge'.

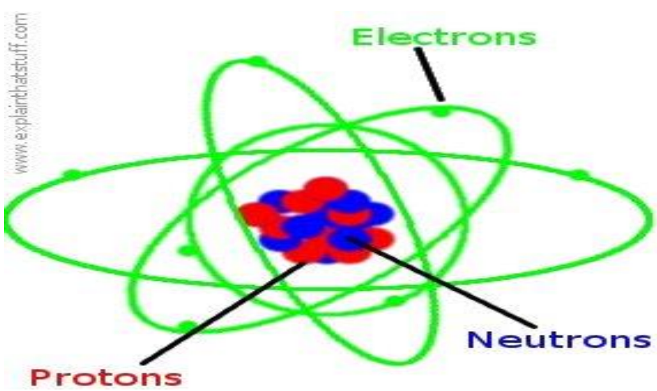
What is charge?

Charge is an electrical property of the atomic particles of which matter consists and it is measured in coulombs (C).

- All matter is made of fundamental building blocks known as <u>atoms.</u>
- Each atoms consists of <u>electrons</u>, <u>protons</u> and <u>neutrons</u>.
- The charge of an electron (**e**) is negative and equal in magnitude to 1.602 x 10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron.

$$e = -\ 1.602 \times 10^{-19}\ C$$

What is the value of charge for a proton and for a neutron?



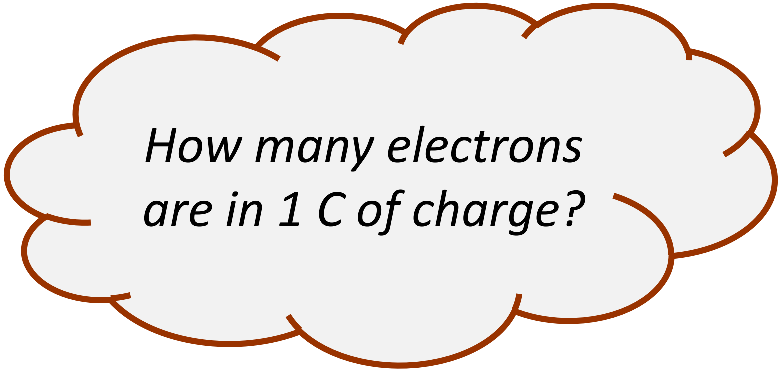
Some facts about charge (1/2)

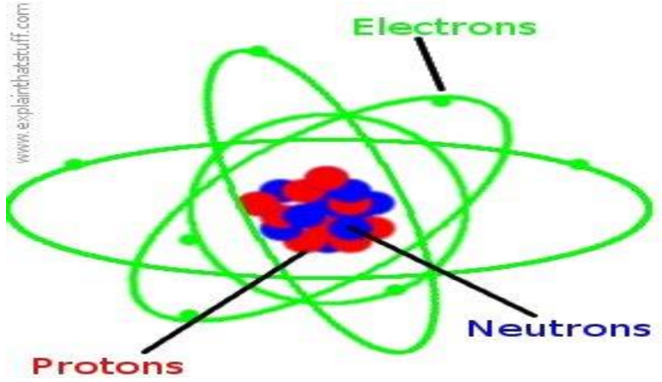
- The presence of equal numbers of protons and electrons leaves an atom *neutrally* charged.

- The net (or total) charge (and hence forces) arises from an **imbalance** in the proton/electron balance.

- Only electrons can move. So, a loss of electrons results to *positive charge*, while an excess of electrons results to *negative charge*.

$e = -1.602 \times 10^{-19}\ C$





Some facts about charge (2/2)

- Charge *is conserved*. It may move but it cannot be destroyed.
- Charge *is quantised* and it comes in units of electron charge, which means that we can't break an electron.

Conductors (eg metal) are very good at facilitating the movement of electrons.

Insulators (eg plastic) resist the movement of electrons.

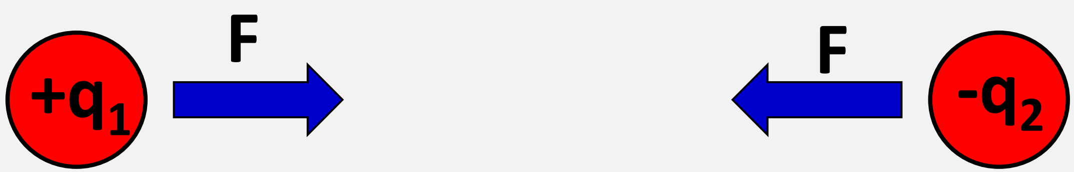
Forces between Charges

Charges exert forces on each other.

The force is vectorised, it has a magnitude and a direction.

Like/Similar charges repel

Unlike/Opposite charges attract

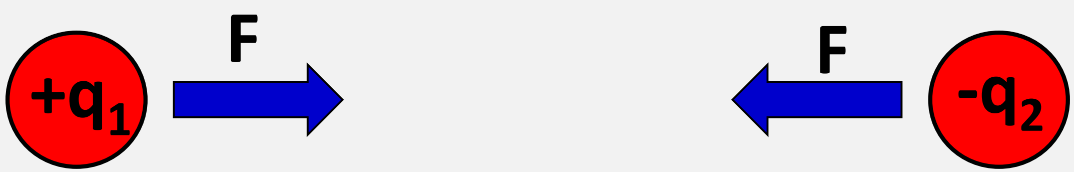


Forces between Charges

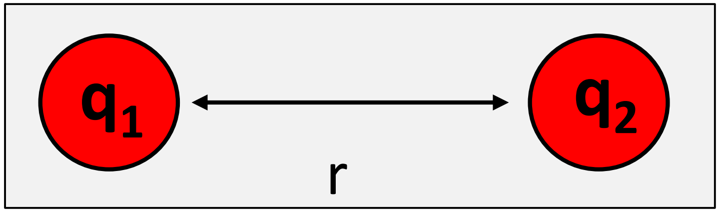
What happens if I bring the charges closer?

Like/Similar charges repel

Unlike/Opposite charges attract



Coulomb's Law



The electric force acting on a point charge q_1 as a result of the presence of a second point charge q_2 is proportional to their product and inversely proportional to the square of the distance. This proportionality constant is called Coulomb's Constant and is inversely related to the permittivity of the intervening space.

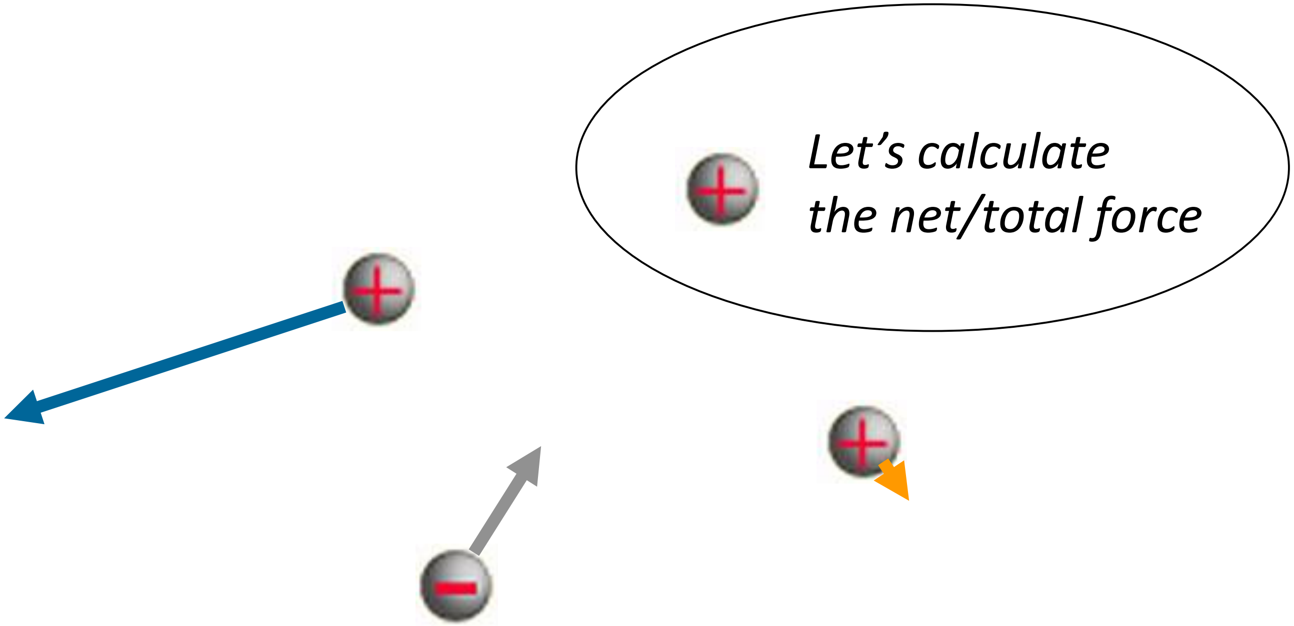
$$F = k\frac{q_1 q_2}{r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$\mathrm{k} = 8.987 * 10^{9}\ Nm^2C^{-2} \qquad \text{Coulomb's Constant}$$

$$\varepsilon_0 = 8.854 * 10^{-12}\ Fm^{-1} \qquad \text{Free Space Permitivity}$$

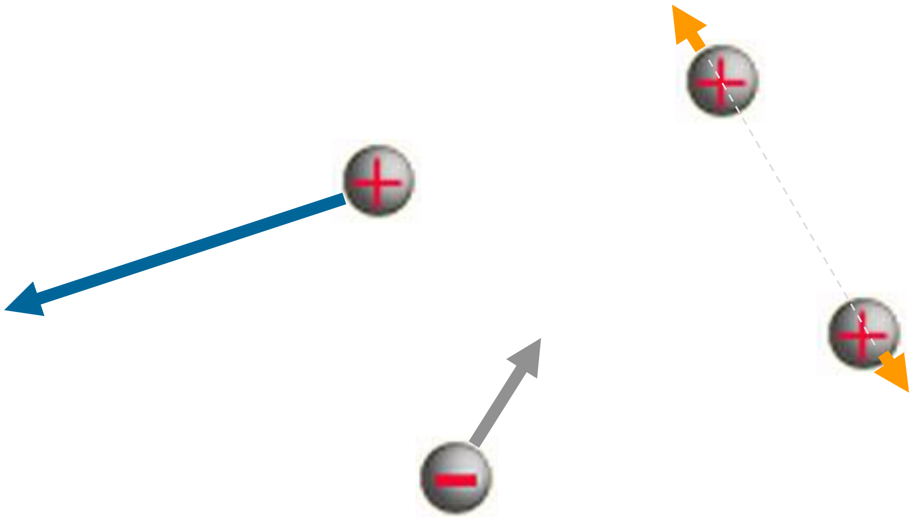
Forces between Charges

The effect of multiple charges **are added in vectorised form**.



Forces between Charges

The effect of multiple charges are added in *vectorised* form.



Keep in mind that: Similar charges repel, while opposite charges attract!

Forces between Charges

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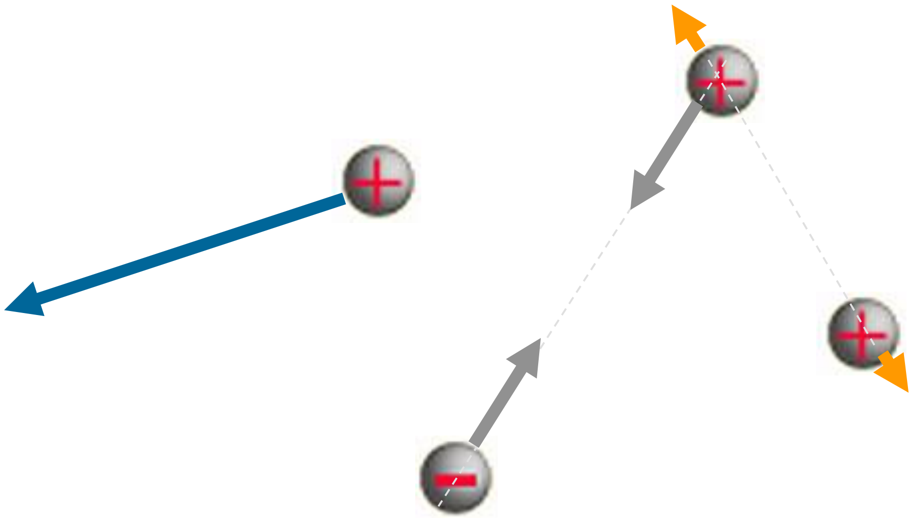

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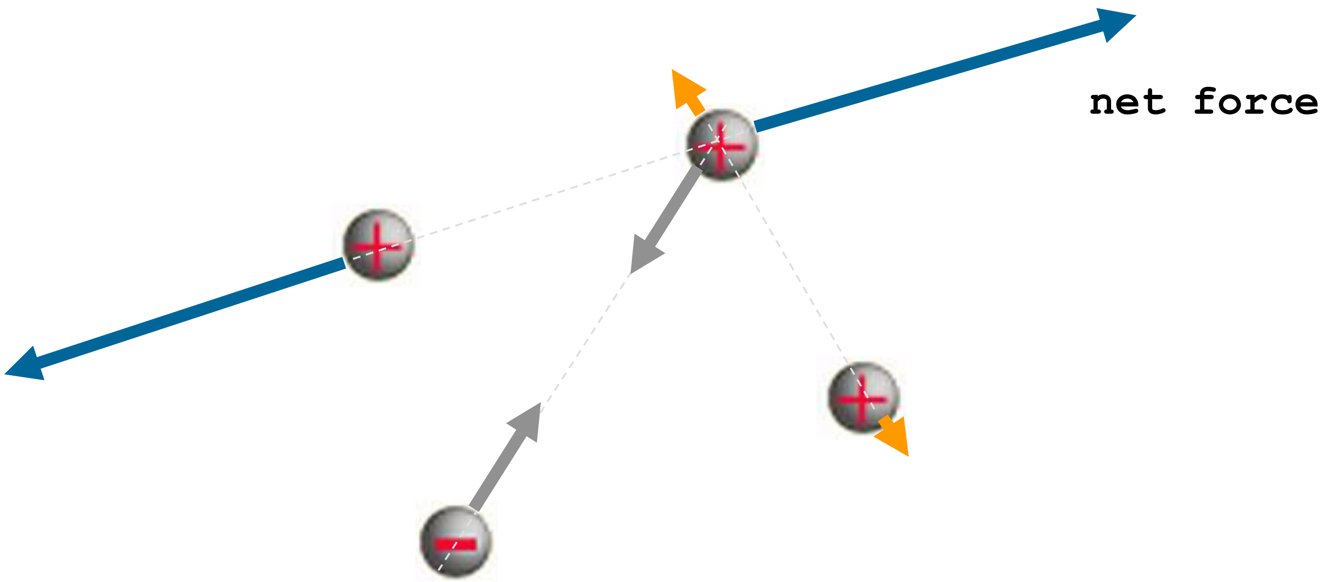

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Forces between Charges

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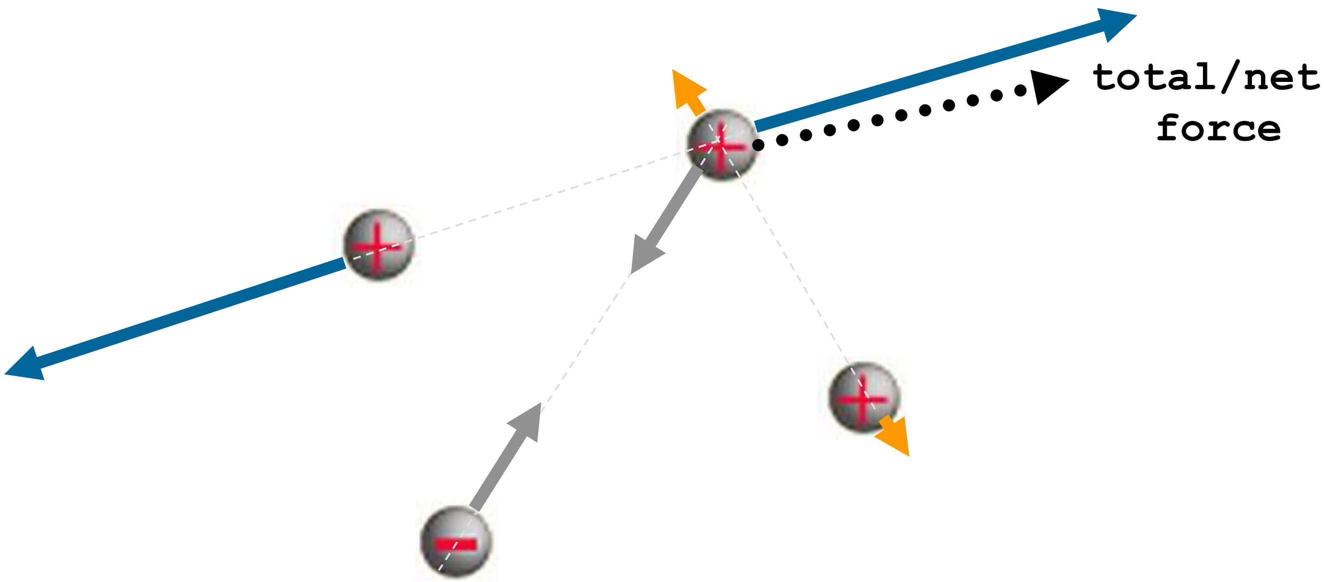
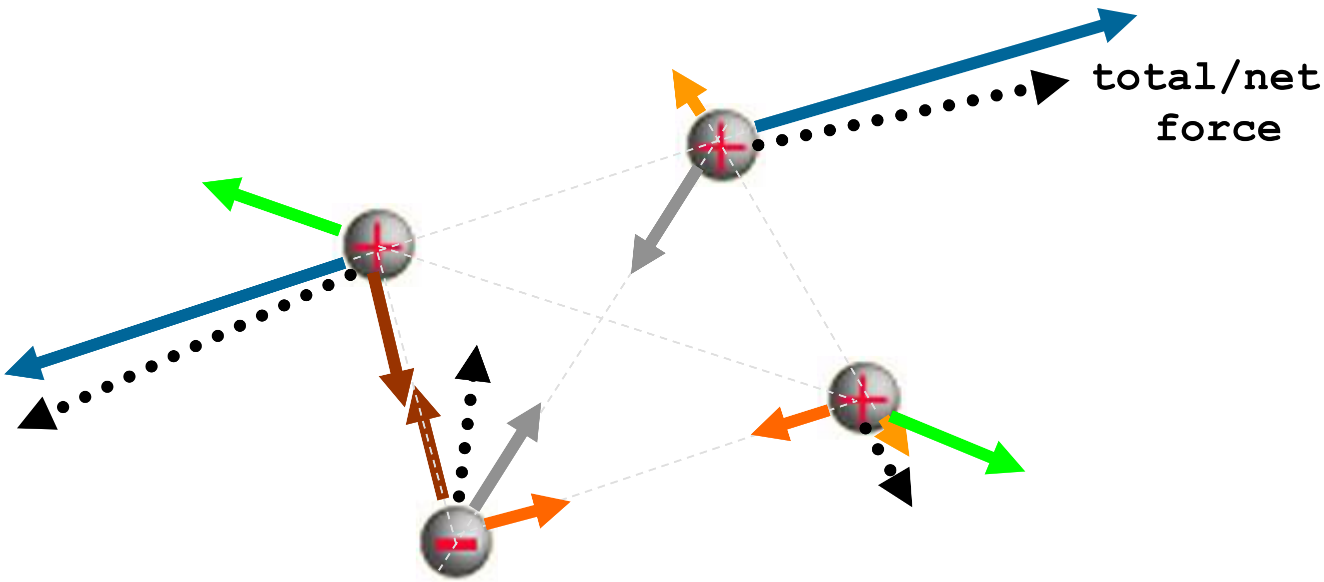


Keep in mind that: Similar charges repel, while opposite charges attract!

Forces between Charges

Actually all forces act on each other!



Keep in mind that: Similar charges repel, while opposite charges attract!

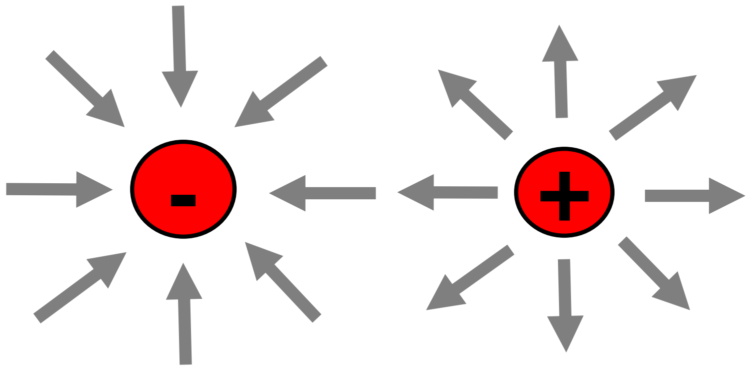
Electric Field

We mentioned earlier that similar charges repel and opposite charges attract, but how is this possible?

This is because each charge creates an electric field all around it all times, either there are other charges nearby or not. The charges 'talk to each other' through the electric field.

Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

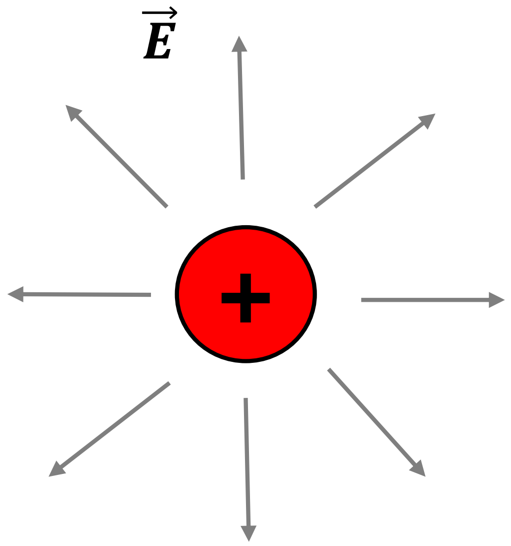
$$\vec{E} = \frac{\vec{F}}{q} \qquad \text{Newtons/Coulomb}$$

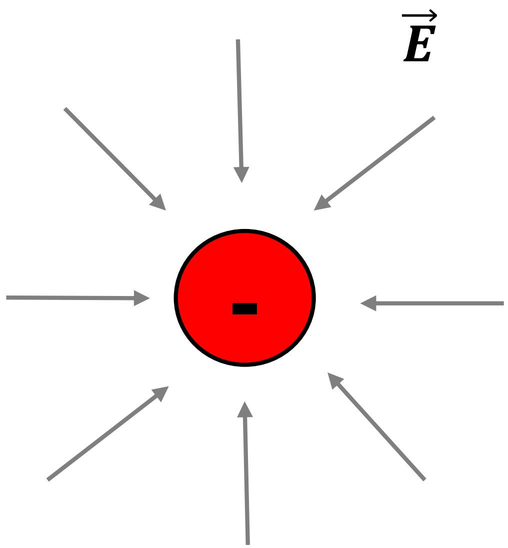


Electric Field

The electric field is radially outward from a positive charge and radially in toward a negative point charge.

The electric field is inversely proportional to the square of the distance.





Electric Field

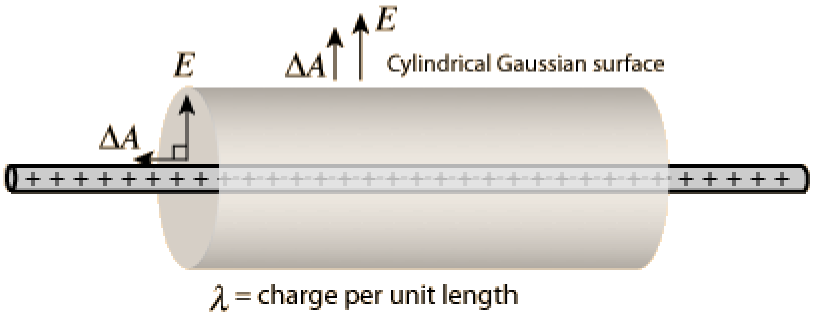
$$\vec{E} = \frac{\vec{F}}{q} \quad \text{Newtons/Coulomb}$$

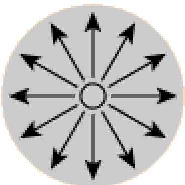
From the formula above, we can notice that the direction of the electric field is the same as the direction of the electric force if q is positive. This means that they have the same direction, not the same *magnitude*.

Do the electric field and the electric force have the same direction when q is negative?

Different Electric Fields

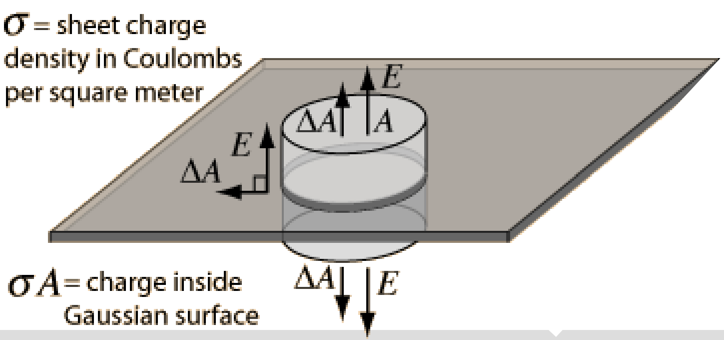
An infinitely long charged wire



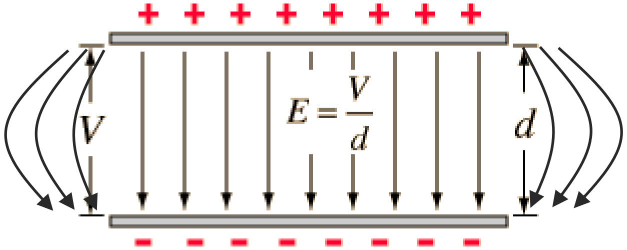


$$E = \frac{q}{2\pi\varepsilon_0 r}$$

A charged sheet



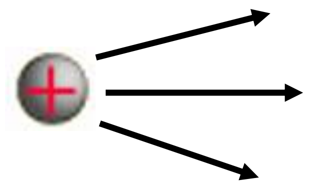
Parallel Plates



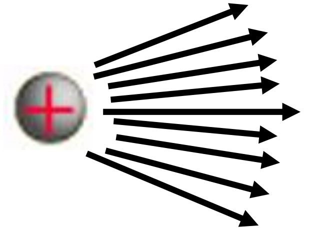
Electric Flux (Φ)

The electric flux through a planar area is defined as the electric field times the component of the area perpendicular to the field. If the area is not flat or the field is not perpendicular to the surface, then the evaluation of the flux generally requires a complex area integral.

Visually imagine the electric field being represented by lines. The more lines, the greater the electric field. The electric flux is the number of lines that pass through the surface of a region enclosing a charge.



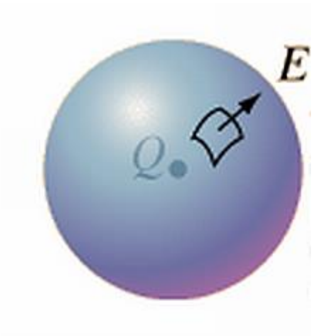
weaker field



Stronger field

Electric Flux

For a point charge, the electric field radiate evenly from that point. If we define a sphere enclosing the point, then all electric field lines are perpendicular to the surface. Then all you need to do is calculate the field and multiply by the area.



$$E = \frac{F}{q_0} = \frac{q_0 Q}{4\pi\varepsilon_0 r^2}\frac{1}{q_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$surface\ area = 4\pi r^2$$

$$Total\ Electric\ Flux = area * electric\ field$$

$$\Phi = \left(4\pi r^2\right) * \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)$$

$$= \frac{Q}{\varepsilon_0}$$

The total electric flux is independent of sphere radius and is only dependent on Q. This can be generalised to Gauss' Law.

Gauss' Law

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

- The shape of the surface doesn't matter. If it curves and bends, then we'll have different vector components that will add and subtract. Normally we like symmetric surfaces as they are mathematically easy to use.
- If you can calculate the electric flux (which might be easy), then the electric field is found by dividing by the unit area.
- It also doesn't say how the charge is stored internally… just that the electric flux is dependent solely on the level of charge enclosed.
- If the net charged enclosed is zero, then the electric flux is zero. This is something that is used a lot in practice.

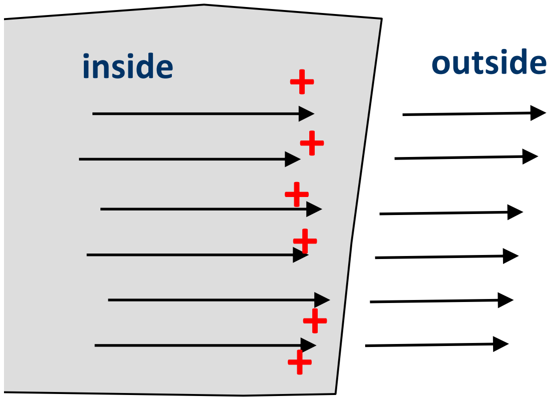
Gauss' Law

the hand-wavy version

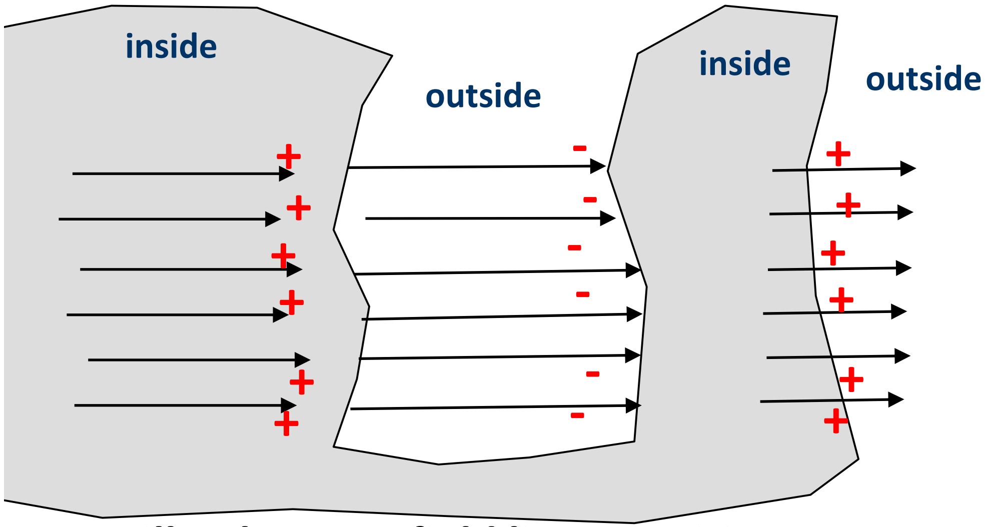
Charge creates an electric field (imagine it as a certain number of field lines leaving the charge).

It doesn't matter what the shape is, any shape enclosing that charge will have the same **NET** number of field lines leaving that space.

The shape won't create new field lines, no matter what the shape is



6 field line transitions



Still only 6 NET field line transitions

Gauss' Law – How to use it

$$area * electric\ \ field = \Phi = \frac{Q}{\varepsilon_0}$$

$$area * E = \Phi = \frac{Q}{\varepsilon_0}$$

So if we can construct a shape that is easy to use and where the E field is constant, then it becomes very easy.

Step 1: construct your shape

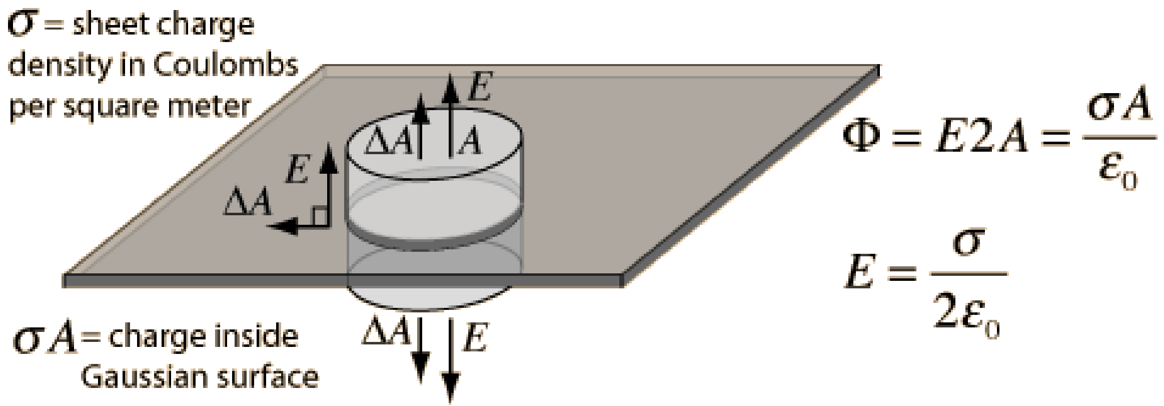
Step 2: calculate the charge enclosed by your shape

Step 3: calculate the E field by dividing by the area of the shape

The best shapes are picked when you sort of know the answer already… ie which way the arrows are pointing.

Example : A charged sheet

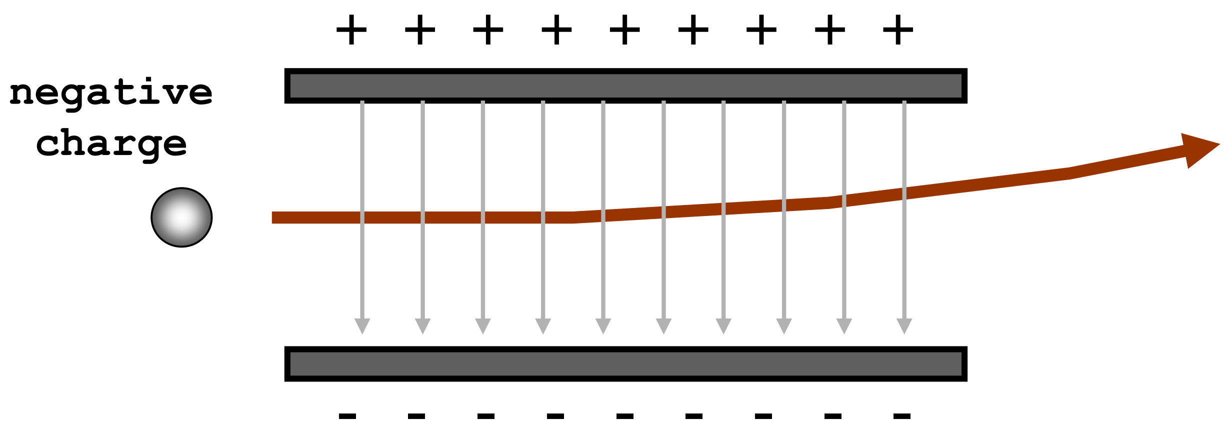
Consider an infinitely large flat sheet that is charged with $+\sigma$ coulombs/m^2. Identify the direction of the electric field and its magnitude.



For an infinite sheet of charge, the electric field will be perpendicular to the surface. Therefore only the ends of a cylindrical gaussian surface will contribute to the electric flux (perpendicularity). The resulting flux is the area of the cylinder (A) multiplied by σ (charge) divided by the area of the two ends (2*A), giving a flux of $\sigma A/\varepsilon_o$ and an electric field of $\sigma/2\varepsilon_o$

Uses of Electric Field

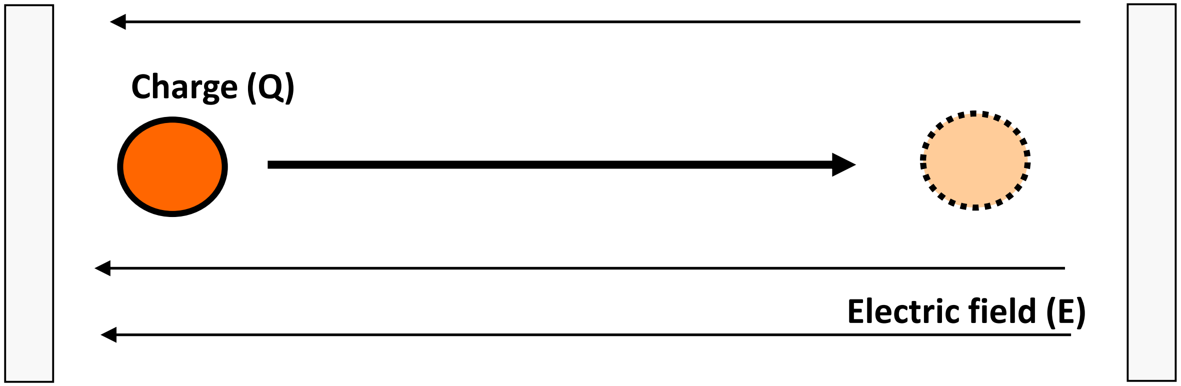
Imagine that I have a charged object (an electron) and I set it moving in an electric field, what will happen to it?



The electric field will apply a force to the object and that will deflect it from it's original path.

Electro-potential Difference

Imagine an electric field **E**, a distance of 1 metre, and a charge of 1 Coulomb



To move the charge Q against the “**electric field**” requires work.

As I push it up the electric field, I’m giving *potential energy* to the charge.

Question

An electron in near-Earth space is accelerated Earthward by an electric field of 0.01 NC^{-1}. Find its speed when it strikes air molecules in the atmosphere after travelling 3 Earth radii (19 000 km).

The electron experiences a force:

$$F = ma = qE$$

$$a = {}^{qE}/_{m}$$

For motion at constant acceleration:

$$v^2 = u^2 + 2as = 2as$$

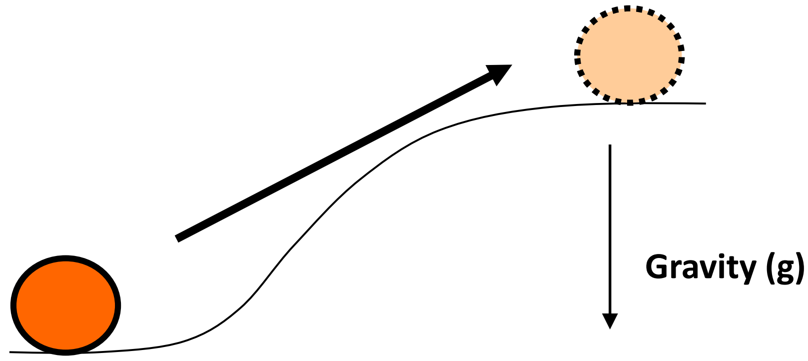
$$v = 2.6 \times 10^8 \ ms^{-1}$$

i.e. 0.8 x speed of light (of course it'd begin to hit relativistic speed issues at this point)

PS: this electric field is unreasonable (why???)

Electro-potential Difference

Imagine gravity, a 1 kg ball and a 1 metre tall hill



To move that ball up the hill required me to do some work. I had to push it against gravity. Using different terms, there is a "**gravitational field**" all around us (pointing down). To move against the gravitational field requires a force to be applied. That force adds energy to the object which could later be released.

The equation for the energy added/work done =

energy transferred or work done = Force * distance
= Mg * distance
= 1*9.8*1 = 9.8 Joules

We call this in gravity potential energy and the potential difference is the difference in height (as the force is constant).

Electro-potential Difference

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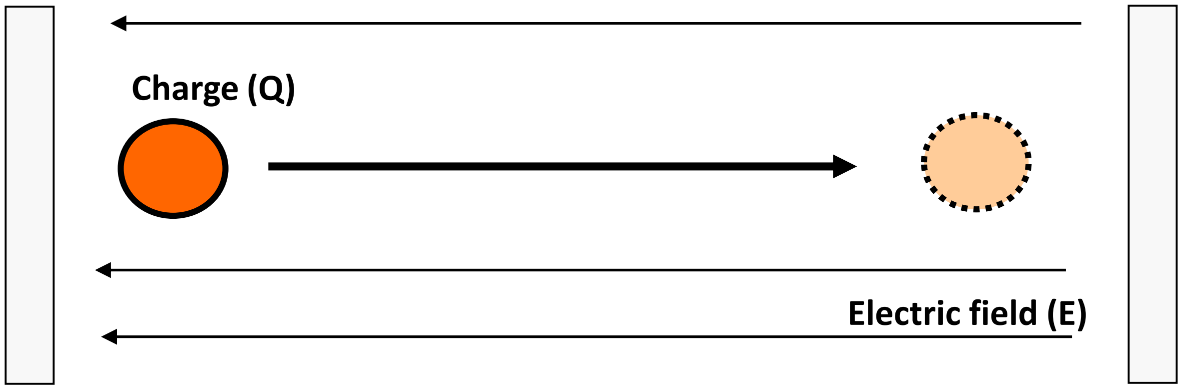

The equation for the energy added/work done:

Energy transferred / Work done = Force * distance
= E*Q * distance
= potential difference $*Q$

$$\vec{E} = \frac{\vec{F}}{Q}$$

The potential difference here is defined as *E*distance* and we use the term "voltage" for that.

Electro-potential Difference

Electrical "potential difference" or voltage between two points in an electrical field is defined as the work per unit charge required to move between those two points.

$$voltage = \frac{work}{Q}$$

$$voltage = electric\ field * distance$$

To work out the potential difference (voltage), you need to integrate the force over distance between the two points.

$$voltage_{AB} = \int_{A}^{B} (E)dx$$

Where the electric field is constant, then the voltage is the product of the electric field by distance.

$$voltage_{AB} = \int_{A}^{B} (E)dx = E * dAB$$

Review...voltage

Voltage on it's own means little from a practical perspective. *What is important in any electrical system is the difference in voltage because that is the measure of energy that can be extracted/involved.*

If you know the electric field between two points, you can calculate the voltage (potential difference) between the two points.