Tutorial Sheet 3 – *Minimisation using Karnaugh Maps*

1. Using Karnaugh Maps, obtain the simplified expressions in sum of products for the following Boolean functions:

(i) $f_{(A,B,C)} = \sum (1, 3, 5, 7)$

(ii) $f_{(A,B,C)} = \sum (1, 2, 5, 6)$

(iii) $f_{(A,B,C)} = \sum (0, 1, 4, 5, 6)$

2. Using Karnaugh Maps, simplify the following Boolean functions, expressing the answers as sum of products.

(i) $f_{(A,B,C,D)} = \sum (0, 2, 4, 5, 6, 7\ 8, 10, 12, 14)$

(ii) $f_{(A,B,C,D)} = \sum (5, 12, 13, 15)$

(iii) $f_{(A,B,C,D)} = \sum (0, 2, 7, 8, 9, 10)$

(iv) $f_{(A,B,C,D)} = \sum (0, 2, 7, 8, 9, 10) + \sum d(3, 5, 6)$

(v) $f_{(A,B,C,D)} = \sum (0, 1, 4) + \sum d(5, 6)$

3. Find the minimal *sum of products* expression for each of the following functions using 5-variable Karnaugh Maps:

(i) $f_{(A,B,C,D,E)} = \sum (5, 7, 13, 15, 16, 20, 25, 27, 29, 31)$

(ii) $f_{(A,B,C,D,E)} = \sum (4, 6, 7, 9, 11, 12, 13, 14, 15, 20, 22, 25, 27, 28, 30) + \sum d(1, 5, 29, 31)$

(iii) $f_{(A,B,C,D,E)} = \prod (4, 5, 9, 13, 16, 17, 21, 25, 29)$

4. Find a minimal *product of sums* expression for each of the Boolean functions given in Q3 above. (*Hint: Group the zeros, invert the resulting output and apply De Morgan's).*

5. The Boolean expression $BE + \overline{B}D\overline{E}$ is a simplified version of the expression:

$$\overline{A}BE + BCDE + B\overline{C}\overline{D}E + \overline{A}\overline{B}D\overline{E} + \overline{B}\overline{C}D\overline{E}$$

Are there any don't care conditions? If so, what are they?

6. Consider the following simple alarm system, used in a wildlife sanctuary. There are five different zones, namely A, B, C, D, and E, each with its own sensor. These sensors are used to detect the presence of zebras within their allocated zone. Zebras can only be located in specific zones at any instant in time. The following combinations are not permitted:

- zebras in zones D only
- zebras in zones D and E only
- zebras in zones C, D and E only
- zebras in zones A and D only
- zebras in zones A, C, and D only
- zebras in zones A, C, D and E only.

If any of the above combinations are detected then the alarm system is triggered. Use Karnaugh Maps to obtain a minimal SOP expression that indicates when the alarm system is triggered.

ANSWERS

1 (i) C
(ii) $\overline{B}C + B\overline{C}$
(iii) $\overline{B} + A\overline{C}$

2 (i) $\overline{D} + \overline{A}B$
(ii) $ABD + AB\overline{C} + B\overline{C}D$
(iii) $\overline{B}\overline{D} + A\overline{B}\overline{C} + \overline{A}BCD$
(iv) $\overline{B}\overline{D} + A\overline{B}\overline{C} + \overline{A}C$
(v) $\overline{A}\overline{C}$

3 (i) $\overline{A}CE + ABE + A\overline{B}\overline{D}\overline{E}$
(ii) $\overline{A}C + BE + C\overline{E}$
(iii) $\overline{A}\overline{B}\overline{C} + AC\overline{E} + B\overline{E} + D$

4 (i) $(A + C)(A + E)(\overline{B} + E)(\overline{D} + E)(\overline{A} + B + \overline{E})$
(ii) $(B + C)(C + E)(\overline{A} + B + \overline{E})$ or $(B + C)(C + E)(\overline{A} + \overline{C} + \overline{E})$
(iii) $(A + B + \overline{C} + D)(\overline{A} + B + C + D)(\overline{B} + D + \overline{E})(\overline{A} + D + \overline{E})$
or $(A + B + \overline{C} + D)(\overline{A} + B + C + D)(\overline{B} + D + \overline{E})(\overline{C} + D + \overline{E})$

5 Yes, at least 3: $A\overline{B}CD\overline{E}$, $ABC\overline{D}E$, $AB\overline{C}DE$

6 $\overline{B}\overline{C}D\overline{E} + \overline{A}\overline{B}DE + A\overline{B}CD$ (or $\overline{B}CDE + \overline{A}\overline{B}\overline{C}D + A\overline{B}D\overline{E}$)