EE114
Intro to Systems & Control

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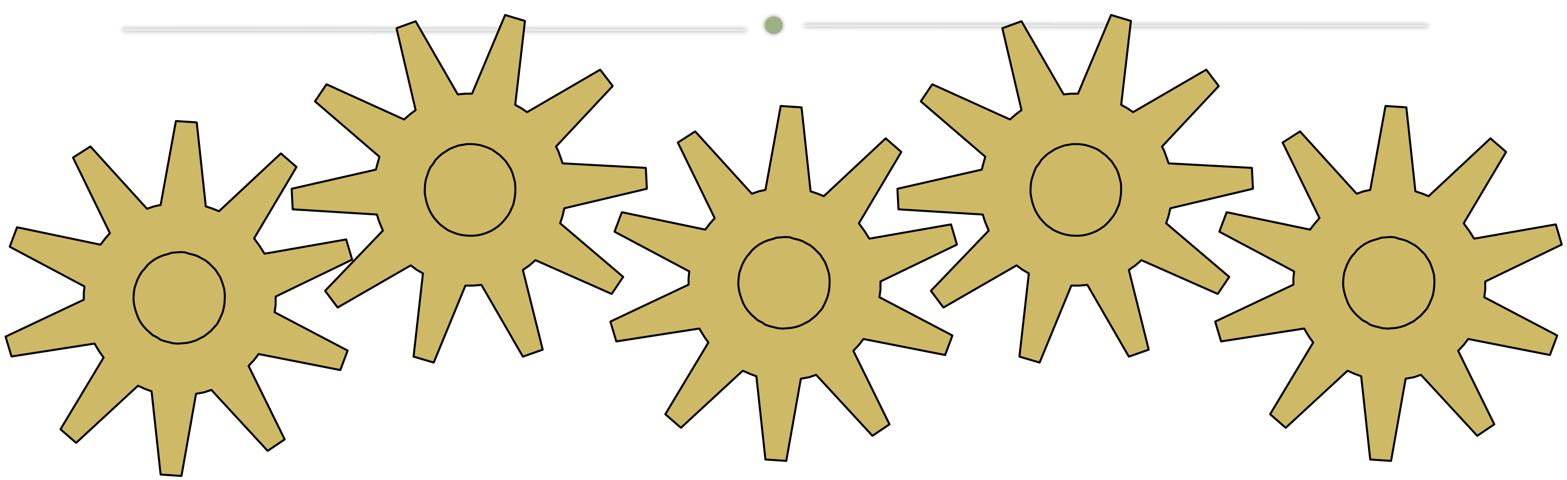
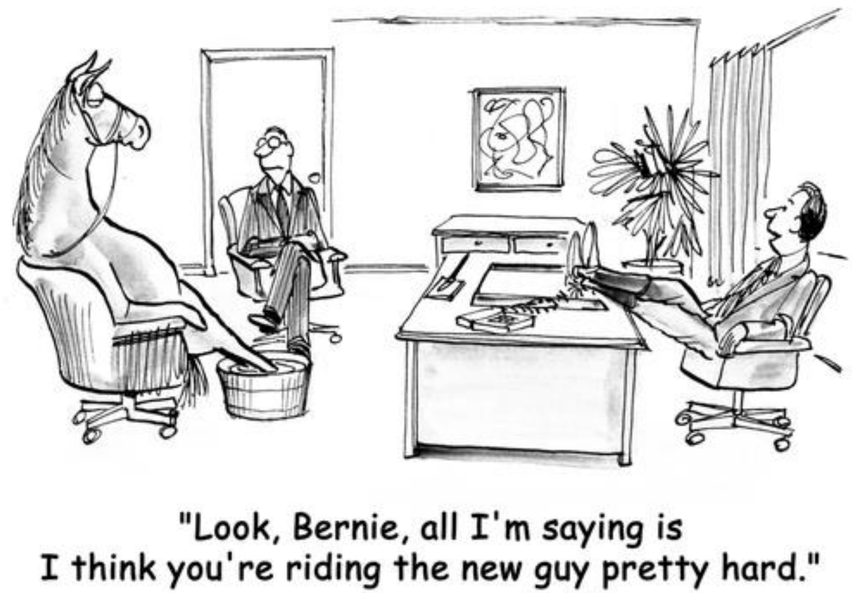


So far …

- We've modelled a range of systems – obtained differential equations & transfer function models

- We've started to analyse these systems – RC circuit …



- **Today, we will analyse the bicycle and the water tank …**

Solving the transfer function model

- ***Ex. 6.1 Obtain a solution for h(t) for the single tank system whose model is given by:***

$$\frac{H(s)}{F(s)} = \frac{1}{sA + k}$$

Solution:

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Output height is given by:

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Here, the input is a constant flow rate, say *fin*, hence:

$$F(s) = \frac{fin}{s}$$

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Thus:

$$H(s) = fin\left(\frac{1}{s\,(sA + k)}\right)$$

Solution:

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Here, the input is a constant flow rate, say *fin*, hence: $$F(s)=\frac{fin}{s}$$

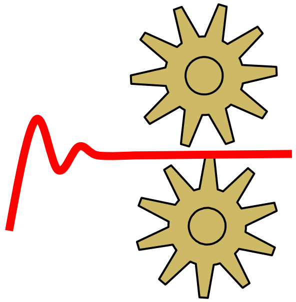
Thus:

$$H(s)=fin\left(\frac{1}{s\,(sA+k)}\right)$$

Using the method of partial fractions:

$$\frac{1}{s\,(sA+k)} \equiv \frac{X}{s}+\frac{Y}{sA+k} = \frac{X\,(sA+k)+Ys}{s\,(sA+k)}$$

$$=\frac{s(XA+Y)+Xk}{s\,(sA+k)}$$



Solution:

Equating the coefficients of s gives:

$$Xk = 1 \quad \Rightarrow X = \frac{1}{k} \qquad\qquad XA + Y = 0 \quad \Rightarrow Y = -\frac{A}{k}$$

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Hence:

$$H(s) = fin\left(\frac{\frac{1}{k}}{s} - \frac{\frac{A}{k}}{sA+k}\right)$$

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Hence:

$$H(s) = fin\left(\frac{\frac{1}{k}}{s} - \frac{\frac{A}{k}}{sA+k}\right)$$

$$= \frac{fin}{k}\left(\frac{1}{s} - \frac{A}{sA+k}\right)$$

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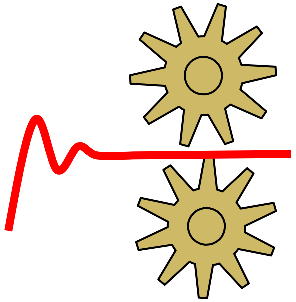
Hence:

$$H(s) = fin\left(\frac{\frac{1}{k}}{s} - \frac{\frac{A}{k}}{sA+k}\right)$$

$$= \frac{fin}{k}\left(\frac{1}{s} - \frac{A}{sA+k}\right)$$

$$= \frac{fin}{k}\left(\frac{1}{s} - \frac{1}{s+\frac{k}{A}}\right)$$



Solution:

Using the Inverse Laplace transform, we obtain:

$$h(t) = L^{-1}\left(\frac{fin}{k}\left(\frac{1}{s} - \frac{1}{s + \frac{k}{A}}\right)\right)$$

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$$h(t) = L^{-1}\left(\frac{fin}{k} \left(\frac{1}{s} - \frac{1}{s + \frac{k}{A}} \right) \right)$$

$$= \frac{fin}{k} \left(1 - e^{\frac{-kt}{A}} \right)$$

A few noteworthy points about this solution:

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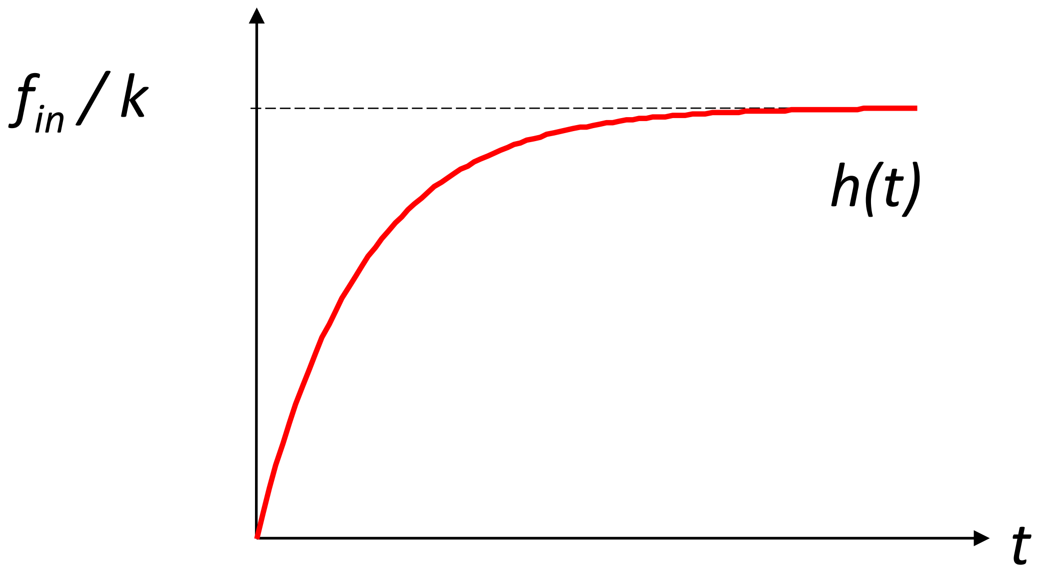
A sketch of the change in height with time is:



A few noteworthy points about this solution:

Recall that $h = k\,f_{out}$, hence, if we let $k = 1$, then $h = f_{out}$, the output flow and vice versa, $f_{out} = h$.

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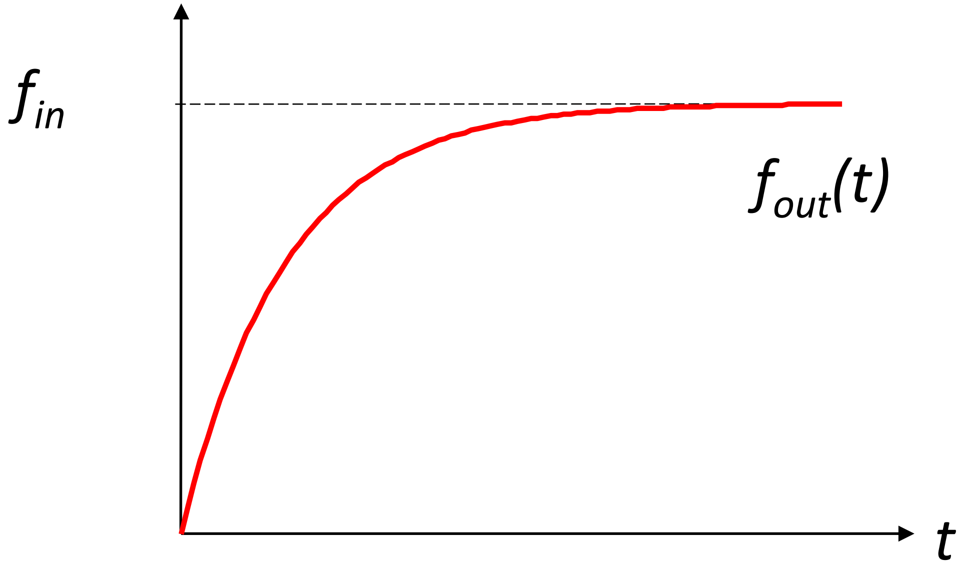
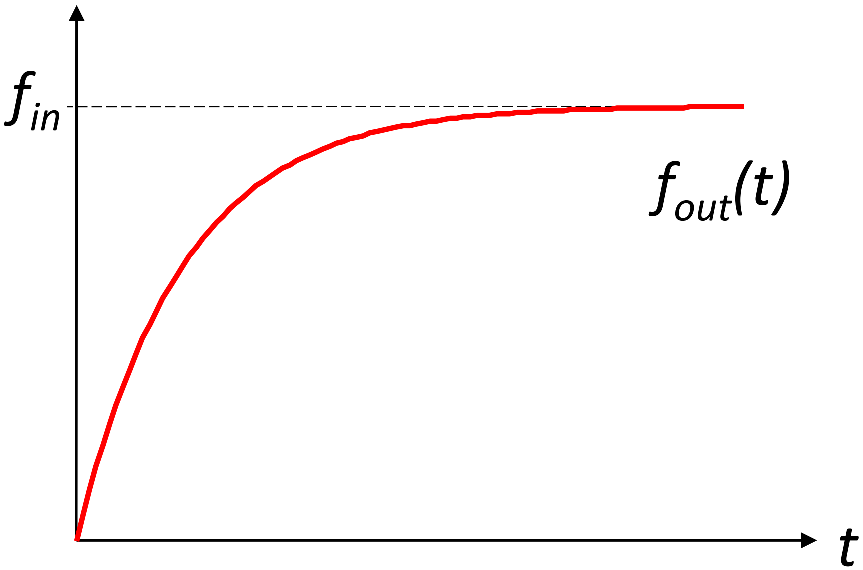
Recall that $h = k\, f_{out}$, hence, if we let $k = 1$, then $h = f_{out}$, the output flow and vice versa, $f_{out} = h$.

Hence we can sketch the relationship between the output flow and the input flow as:

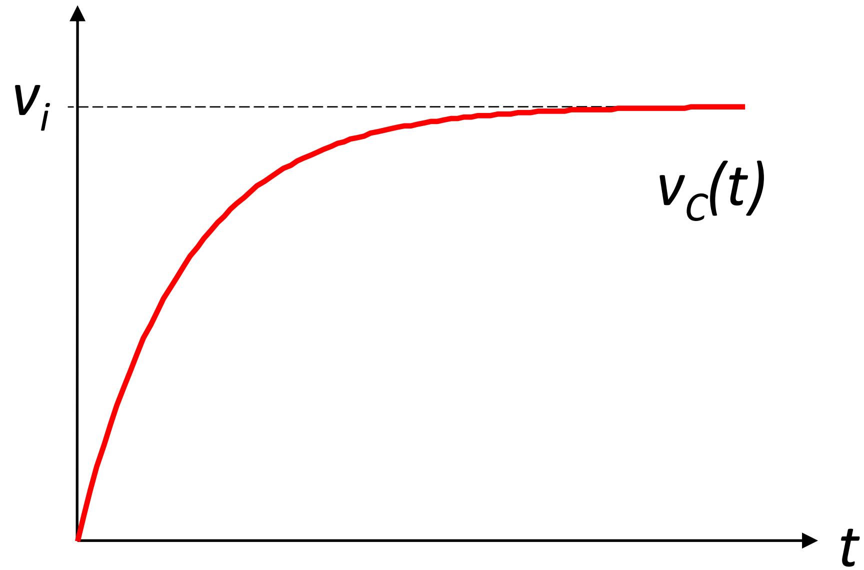


A few noteworthy points about this solution:

This is identical to the relationship between the output voltage and input voltage of the RC circuit :



Single tank system



RC Circuit

A few noteworthy points about this solution:

Also, the charge q(t) in a capacitor is given by:

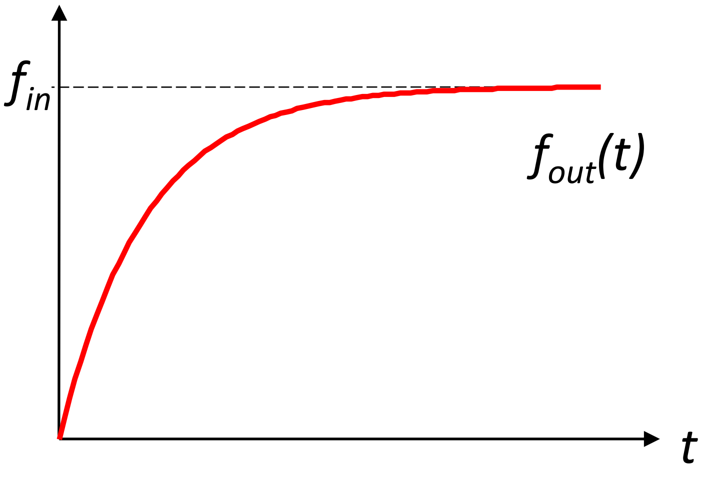
$$q(t) = Cv_C(t)$$

where C is the capacitance (farads).

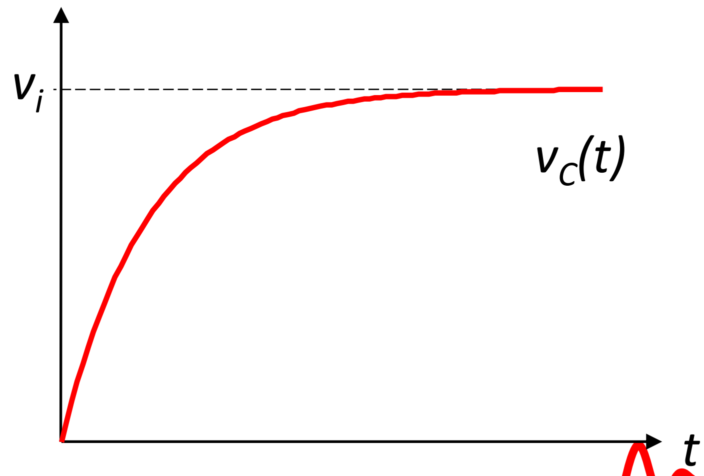
This is similar to the tank system where:

$$h(t) = k\,f_{out}(t)$$

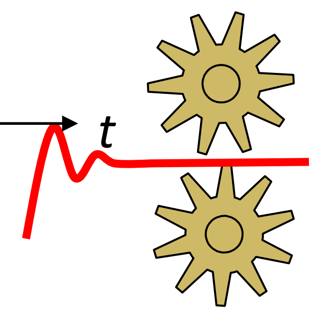
A few noteworthy points about this solution:



Single tank system



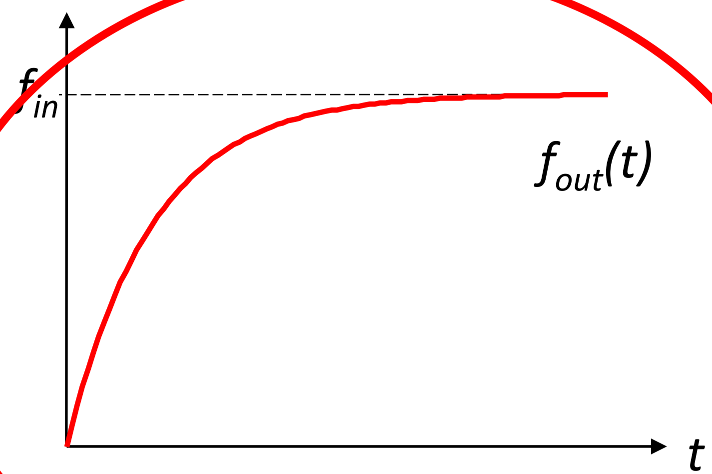
RC Circuit



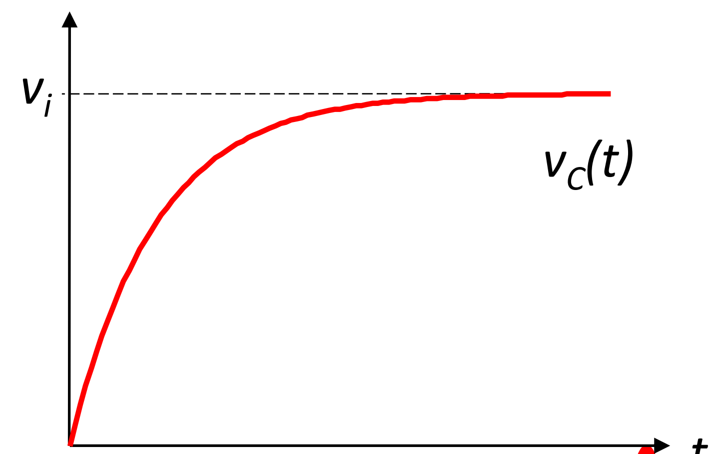
A few noteworthy points about this solution:

In the tank system, the output flow is initially zero and increases from time $t = 0$ as the height of the water in the tank increases.

At some point the pressure due to the volume of water (directly related to the height) causes the output flow to match the input flow (for k = 1) and the system settles at this point, i.e. the height of the water no longer changes.



Single tank system

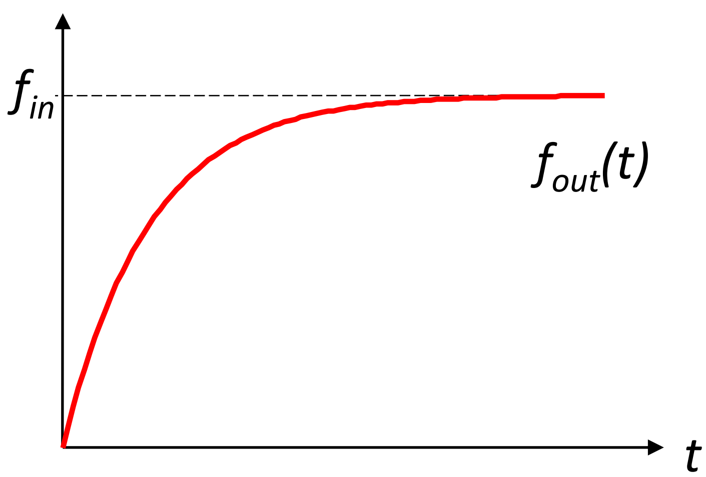


RC Circuit

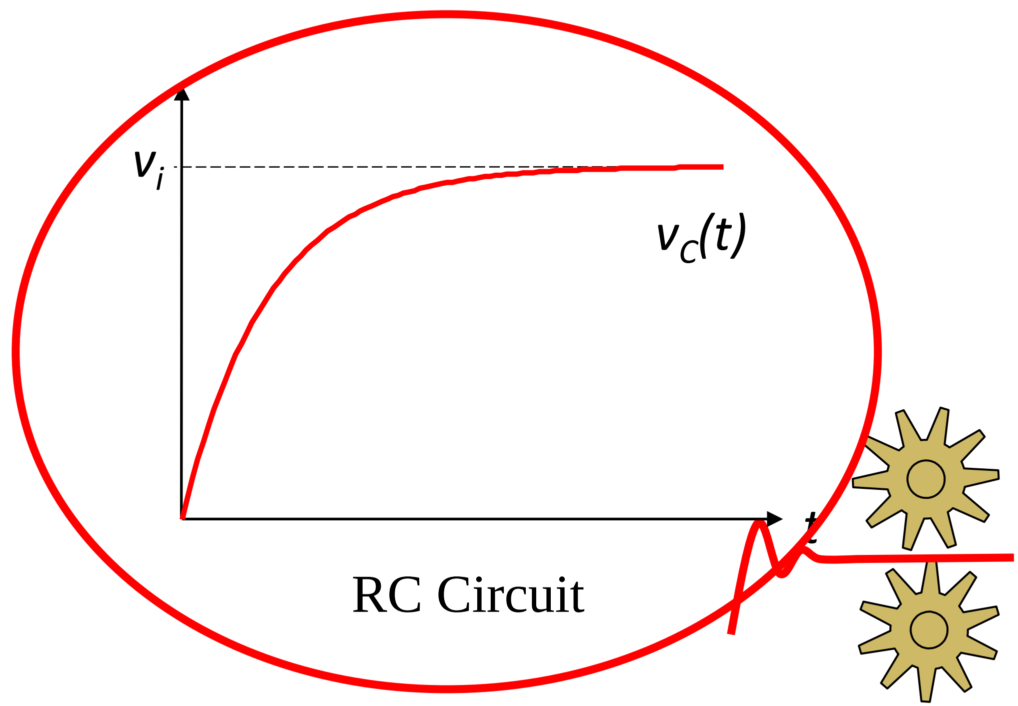
A few noteworthy points about this solution:

Similarly, in the RC circuit, the voltage across the capacitor is initially zero and from time t = 0 as the capacitor charges.

At some point the capacitor becomes fully charged, the current stops flowing and the system settles at this point, i.e. the charge no longer changes and the output voltage matches the input.

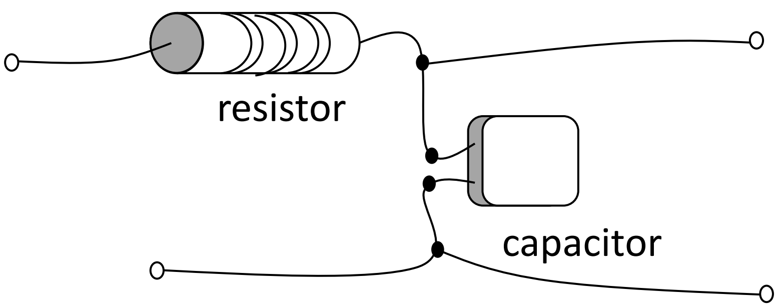


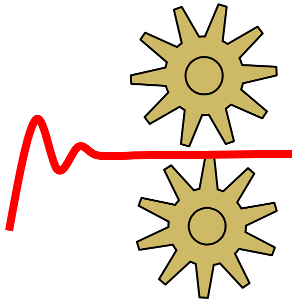
Single tank system



RC Circuit

So, in brief, although we have two very different physical systems (one a single tank and one an RC circuit) they are both first order systems and exhibit identical dynamical behaviour and thus can effectively be represented by the same mathematical model!





Solving the transfer function model

- ***Ex. 6.2 Obtain a solution for x(t) for the mass-spring-damper system whose model is:***

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

- ***given that the input force is 1N, M = 1kg, B = 5Ns/m and K = 6N/m.***

Solution:

Solution:

Output position is given by:

$$X(s)=\frac{F(s)}{Ms^{2}+Bs+K}$$

Solution:

Output position is given by:

$$X(s) = \frac{F(s)}{Ms^2 + Bs + K}$$

Here, the input is a constant force of value 1N, hence:

$$F(s) = \frac{1}{s}$$

Solution:

Output position is given by:

$$X(s) = \frac{F(s)}{Ms^2 + Bs + K}$$

Here, the input is a constant force of value 1N, hence:

$$F(s) = \frac{1}{s}$$

Thus, substituting for the parameters, we obtain:

$$X(s) = \frac{1}{s\left(s^2 + 5s + 6\right)}$$

Solution:

Using the method of partial fractions:

$$\frac{1}{s\left(s^2+5s+6\right)}=\frac{1}{s\,(s+2)\left(s+3\right)} \equiv \frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$

$$\equiv \frac{A(s+2)(s+3)+Bs(s+3)+Cs(s+2)}{s\,(s+2)\left(s+3\right)}$$

Solution:

Using the method of partial fractions:

$$\frac{1}{s\left(s^2+5s+6\right)}=\frac{1}{s\,(s+2)(s+3)}\equiv\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$

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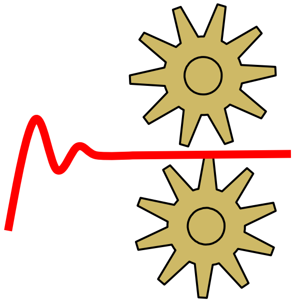
$$\frac{1}{s\left(s^2+5s+6\right)}=\frac{1}{s\,(s+2)(s+3)}\equiv\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$

$$\equiv\frac{A(s+2)(s+3)+Bs(s+3)+Cs(s+2)}{s\,(s+2)(s+3)}$$

Setting s = 0: $1=A(2)(3)\ \Rightarrow \boldsymbol{A}=\frac{\boldsymbol{1}}{\boldsymbol{6}}$



Solution:

Using the method of partial fractions:

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$$\equiv\frac{A(s+2)(s+3)+Bs(s+3)+Cs(s+2)}{s\,(s+2)(s+3)}$$

Setting s = 0: $1=A(2)(3)\ \Rightarrow \boldsymbol{A}=\frac{\boldsymbol{1}}{\boldsymbol{6}}$

Setting s = −2: $1=B\,(-2)(1)\ \Rightarrow \boldsymbol{B}=-\frac{\boldsymbol{1}}{\boldsymbol{2}}=-\frac{\boldsymbol{3}}{\boldsymbol{6}}$

Solution:

Using the method of partial fractions:

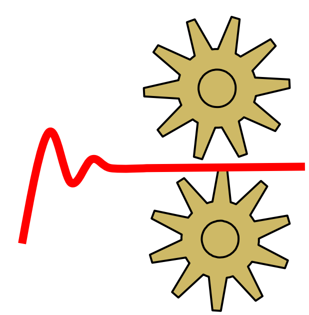
$$\frac{1}{s\left(s^2+5s+6\right)}=\frac{1}{s\,(s+2)\left(s+3\right)}\equiv\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$

$$\equiv\frac{A(s+2)(s+3)+Bs(s+3)+Cs(s+2)}{s\,(s+2)\left(s+3\right)}$$

Setting s = 0: $1=A(2)(3)\ \Rightarrow \boldsymbol{A}=\dfrac{\mathbf{1}}{\mathbf{6}}$

Setting s = −2: $1=B\,(-2)(1)\Rightarrow \boldsymbol{B}=-\dfrac{\mathbf{1}}{\mathbf{2}}=-\dfrac{\mathbf{3}}{\mathbf{6}}$

Setting s = −3: $1=C\,(-3)(-1)\Rightarrow \boldsymbol{C}=\dfrac{\mathbf{1}}{\mathbf{3}}=\dfrac{\mathbf{2}}{\mathbf{6}}$



Solution:

Hence:

$$X(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{1}{6}\left(\frac{1}{s} - \frac{3}{s+2} + \frac{2}{s+3}\right)$$

Setting s = 0: $1 = A(2)(3) \Rightarrow A = \frac{1}{6}$

Setting s = −2: $1 = B(-2)(1) \Rightarrow B = -\frac{1}{2} = -\frac{3}{6}$

Setting s = −3: $1 = C(-3)(-1) \Rightarrow C = \frac{1}{3} = \frac{2}{6}$

Solution:

Hence:

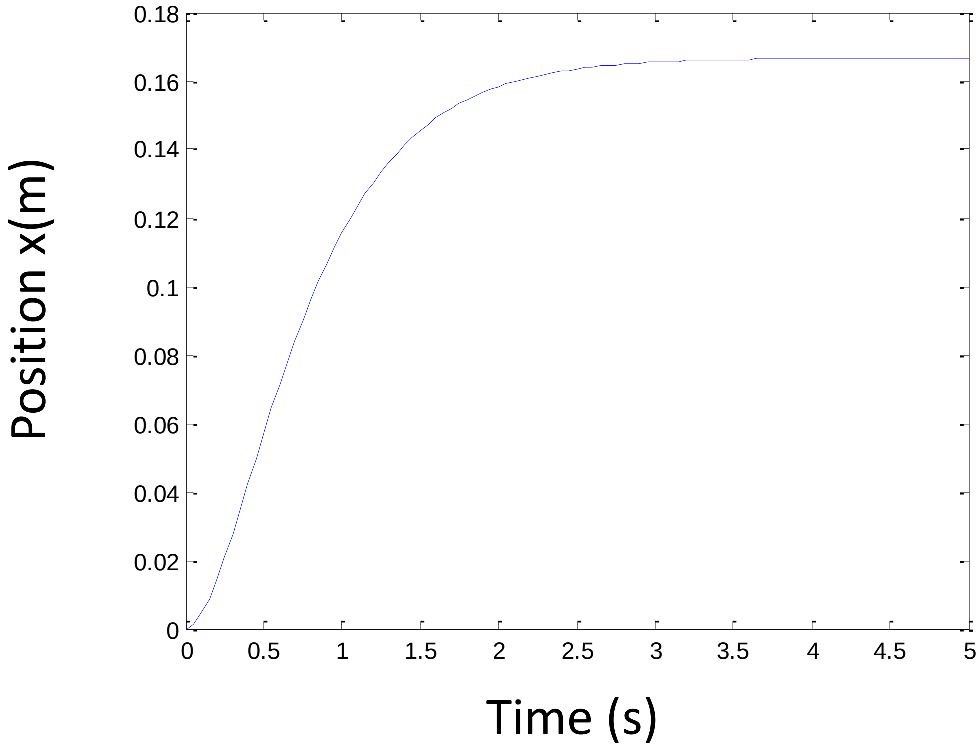
$$X(s)=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}=\frac{1}{6}\left(\frac{1}{s}-\frac{3}{s+2}+\frac{2}{s+3}\right)$$

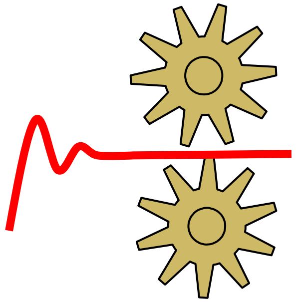
Finally:

$$x(t)=L^{-1}\left(X(s)\right)=\frac{1}{6}\left(1-3e^{-2t}+2e^{-3t}\right)$$

A few noteworthy points about this solution:

A plot of $x(t)$ looks like:





A few noteworthy points about this solution:

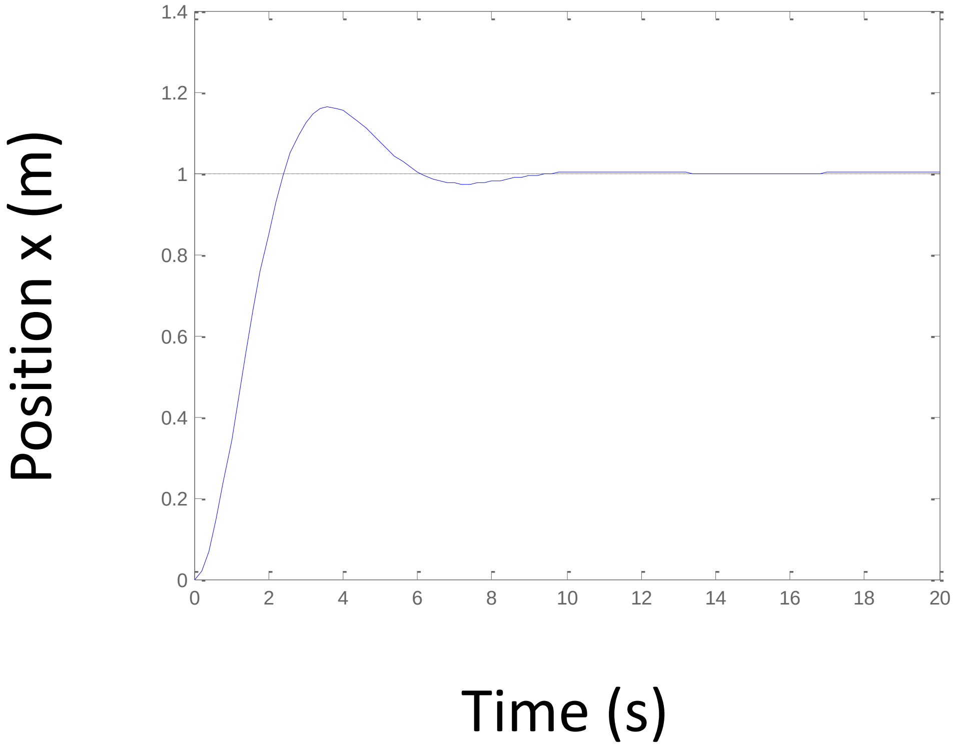
While this response is similar in nature to those obtained from the previous systems, this is simply due to the parameters chosen for this example.

Consider a mass-spring-damper system given by the transfer function:

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + s + 1}$$

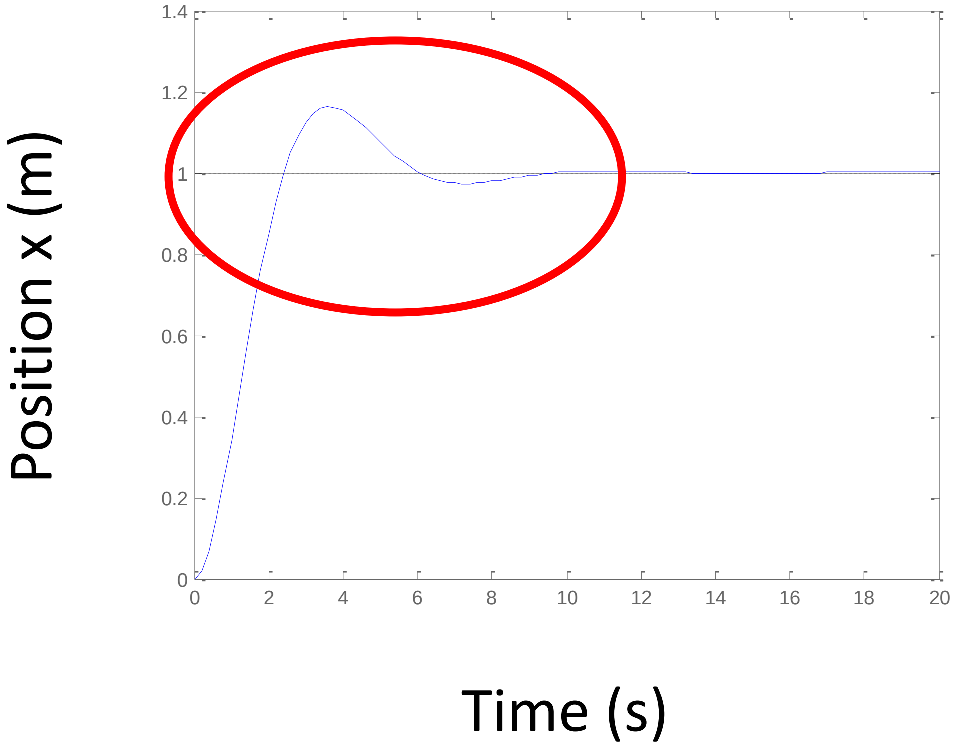
A few noteworthy points about this solution:

This produces the following response:



A few noteworthy points about this solution:

Clearly, we can see that is very different to our previous systems, with more complicated dynamics.



A few noteworthy points about this solution:

- This is to be expected, however, as we are now dealing with a 2nd order system as opposed to a 1st order system, as in the previous examples.
- Try solving this latter transfer function using inverse Laplace transforms … you will find that it is not quite as straightforward as the previous example!
- Thankfully, we don’t need to worry about solving more complicated systems.
- Instead we can use suitable software to simulate the systems for us and we can use such simulations to analyse the given system.

System Simulation

- In this module, we use **Simulink**, a graphical modeling and simulation environment that can be used to design, simulate, implement and test mathematical models of real systems.

- Simulink is integrated with **Matlab**, the latter being a high-level language and interactive environment for numerical computation, data analysis and visualisation, etc.

- **You have been introduced to, and learned to use both of, these environments in our labs.**

System Simulation

- *It is important to learn to use these environments well, as you will use them (Matlab in particular) for a range of activities, across numerous different modules in the BE programme, and possibly even …*

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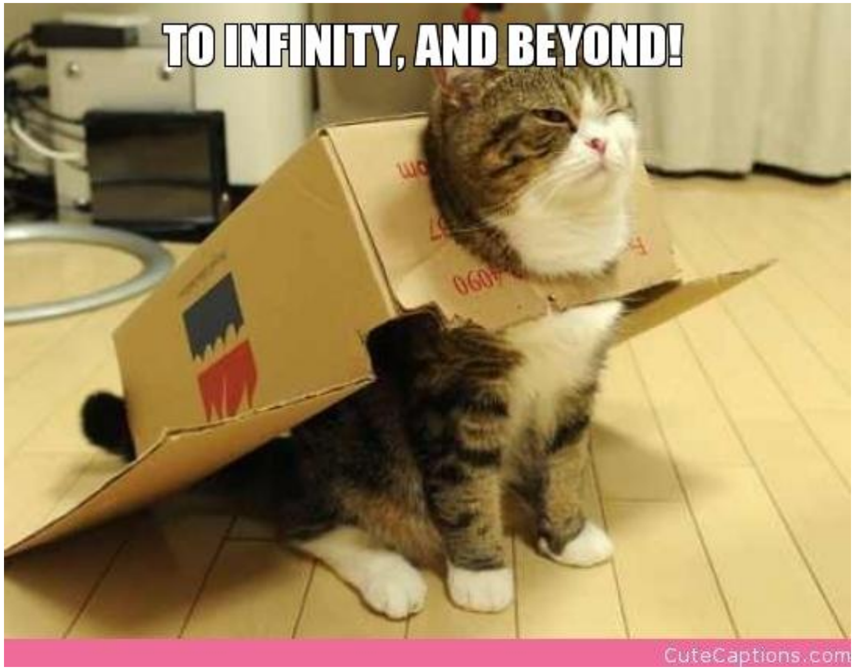

Sorry, I couldn't resist!