CS 162FZ: Introduction to Computer Science II

Lecture 10

Turing Machines

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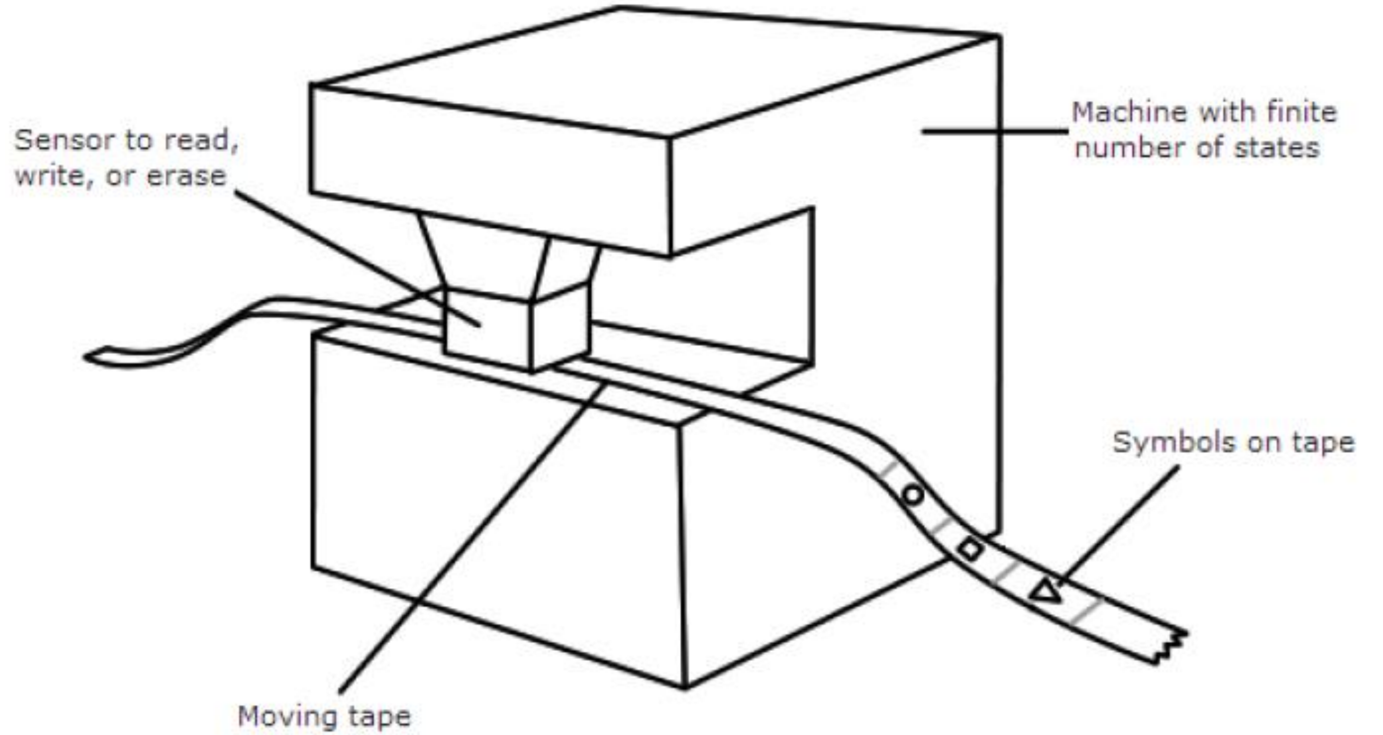
Introduction

- A Turing Machine is a theoretical (universal) computer.
- It is a **mathematical model** of computation that can be used to **simulate any computer algorithm**, no matter how complicated it is.
- The Turing machine was invented in 1936 by British Computer Scientist **Alan Turing.**

Example of Turing Machine



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Overview of a Turing Machine

- A Turing Machine (TM) is an idealised computing device consisting of a read/write head with a paper tape passing through it.
- The tape is of **unbounded length.**
- The tape is divided into squares, each square bearing a single symbol - '0' or '1', for example.
- The tape acts as the machine's general purpose storage medium, serving both as the **means of input and output** and also as a **working memory** for storing the results of intermediate steps of the computation
- The machine needs to keep **track of the previous state** it was in when it moves to a new state.

Overview of a Turing Machine

- There must be **a finite number of symbols** used in the alphabet that the Turing machine can recognise.
- The read/write head is **programmable.**
- To compute with the device, **you program it**, write the input on the tape, place the head over the square containing the **leftmost input symbol**, and set the machine in motion.
- Once the **computation is completed**, the machine will come to a halt with the head positioned over the square containing the **leftmost symbol** of the output (or elsewhere if so programmed).

Overview of a Turing Machine

There are just **six types of fundamental operation** that a Turing machine performs in the course of a computation. These are to:

- **read the symbol** that the head is currently over
- **write a symbol** on the square the head is currently over it will need to **clear** the symbol currently here, if any
- **move the tape left one position**
- **move the tape right one position**
- **change state**
- **halt**

Overview of a Turing Machine

- A program or 'instruction table' for a Turing machine is a **finite collection of instructions**, each calling for certain operations to be performed **if certain conditions** are met.

- Every instruction is of the form:

  If the current state is *n* and the symbol under the head is *x*, then write *y* on the square under the head, go to state *m*, and move one square **left or right**

Example of Instruction Table

An example of one such table might be:

Current State	Current Symbol	Print Symbol	Move Tape	Next State
a	1	1	L	B
a	0	*	R	S
b	1	0	R	A
...				

Example of Instruction Table

- There are **three special states**: **start state, accept state** and **reject state.**
- The Turing Machine computes until it produces an output:
- It either **accepts or rejects** by entering **designated halt states.**
- If it never enters an accepting or rejecting state the Turing Machine **goes on forever, never halting.**

Current State	Current Symbol	Print Symbol	Move Tape	Next State
a	1	1	L	B
a	0	*	R	S
b	1	0	R	A
...				

Example Turing Machine

- Describe a TM *M1* that multiplies an integer number by 10. If the input on the tape is:

...	0	1	2	Δ	Δ	...

The output should be:

...	0	1	2	0	Δ	...

- The alphabet of this TM is: Γ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, Δ} where **Δ is the empty symbol.**

- The input alphabet (what the TM **can write** on the tape) is: Σ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

Example Turing Machine

- The instruction table for this TM might be:

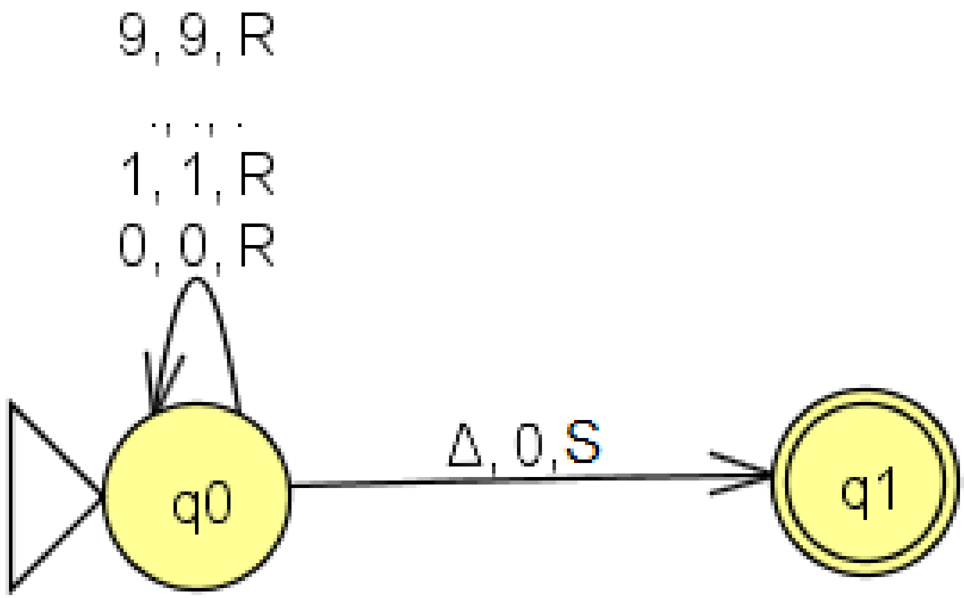
Current State	Current Symbol	Print Symbol	Move Tape	Next State
q_0	0	0	R	q_0
q_0	1	1	R	q_0
q_0	2	2	R	q_0
q_0	3	3	R	q_0
q_0	4	4	R	q_0
q_0	5	5	R	q_0
q_0	6	6	R	q_0
q_0	7	7	R	q_0
q_0	8	8	R	q_0
q_0	9	9	R	q_0
q_0	Δ	0	S	q_1

Example Turing Machine

- **q0** is the starting state for this TM.
- We will stay in this state reading symbols and moving right until we come across the first empty symbol Δ.
- Once we encounter this, we want to change this Δ symbol to a 0 to represent multiplying the number by 10
- We move to state **q1** which is the accepting state for this TM and halt (represented by the movement **S (Stop)**).

Example Turing Machine

The graphical representation of this TM is:



Example Turing Machine

- Design a TM that will perform the unary addition of two numbers. Unary representation can be defined as follows: 1 = 1, 2= 11, 3 = 111, 4 = 1111, 5 = 1111, …
- Unary represents a number x by using x 1's – written as 1^x.
- The unary addition of 2(11) and 4 (1111) is:

11 + 1111 = 111111 (Equivalent to: $1^2 + 1^4 = 1^6$)

A sample input for the TM is:

…	1	+	1	1	Δ	…

The output should be:

…	1	1	1	Δ	Δ	…

Example Turing Machine

- What is the alphabet? What is the input alphabet? What is the instruction table? Can you design the graphical representation?
- The simplest way to solve this is to find the + symbol and change it to a 1.
- We must then move to the last 1 to the right and replace it with a Δ.
- The alphabet is: $\Gamma = \{1, +, \Delta\}$
- The input alphabet (what the TM **can write** on the tape) is: $\Sigma = \{1, \Delta\}$

Example Turing Machine

- Let us assume that we start at the first one of the first number to the left.
- Let us assume that this is state q0 – we will stay in q0 while we keep encountering 1's, writing 1's to the tape and moving right with each step.
- When we encounter the + symbol we need to first change this symbol from a + to a 1, move right to the next symbol and move to state q1.
- We move to this new state as we no longer need to worry about the first number or the + symbol.

Example Turing Machine

- We now know we are at the start of the second number.
- We will keep reading 1's, writing 1's to the tape and moving **right** (all the time staying in q1) until we encounter a Δ symbol.
- When we encounter the Δ symbol we write a Δ to the tape and move back **left** – we need to get to the last 1 to remove it from the tape.
- We will change state again to q2.
- We now know that we should encounter a 1 which needs to be overwritten with a Δ.
- This is the unary addition complete and we move to state q3 which is the **halting state.**

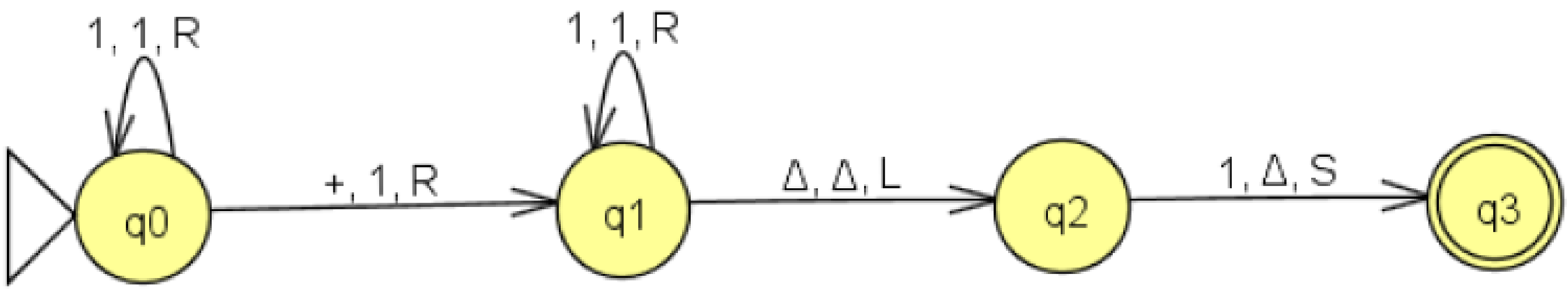
Example Turing Machine

- The instruction table for this TM might be:

Current State	Current Symbol	Print Symbol	Move Tape	Next State
q_0	1	1	R	q_0
q_0	+	1	R	q_1
q_1	1	1	R	q_1
q_1	Δ	Δ	L	q_2
q_2	1	Δ	S	q_3

Example Turing Machine

The graphical representation of this TM is:



Church-Turing Thesis

Any real-world computer can be simulated by a Turing machine.

- Proposed independently by Alonzo Church and Alan Turing.
- “Everything computable is computable by a Turing Machine”.

Turing Machine: Summary

- A Turing Machine (TM) is a theoretical (universal) computer. It is a mathematical model of computation that can be used to simulate any computer algorithm
- There are six types of fundamental operation that a TM performs in the course of a computation.
- These are to:
  - Read
  - Write
  - Move the tape to one left position
  - Move the tape to one right position
  - Change state
  - Halt
- Every TM can be represented by an 'instruction table' which is a finite collection of operating instructions

Turing Machine: Summary

- There are three special states of a TM: start state, accept state and reject state
- The TM computes until it produces an output: it either accepts or rejects by entering designated halt states.
- If a TM never enters an accepting or rejecting state the TM goes on forever, never halting.
- Any real-world computer can be simulated by a Turing machine