Assignment 2. 83200211?

1.(a) $y' + 3x^2 y = x^2$

sol.

$\frac{dy}{dx} = -3x^2 y$

$\frac{dy}{y} = -3x^2 \cdot dx$

$\therefore \ln|y| = -x^3 + C \quad y \neq 0$

$y_c = C e^{-x^3}$

$y_p = u(x) \cdot e^{-x^3}$

$\frac{dy}{dx} = u'(x) e^{-x^3} - 3x^2 u(x) \cdot e^{-x^3}$.

$= x^2 - 3x^2 u(x) \cdot e^{-x^3}$. $\triangle$

$\therefore u'(x) e^{-x^3} = x^2$

$u'(x) = x^2 e^{x^3}$

$u(x) = \frac{1}{3} e^{x^3}$ ✓

$\therefore y_p = \frac{1}{3} \quad y_c = C e^{-x^3}$

So $y \begin{cases} = y_c + y_p = C \cdot e^{-x^3} + \frac{1}{3} \\ = 0 \end{cases}$

sol

(b) $\cos^2 x \sin \frac{dy}{dx} + \cos^3(x) y = \sin x$

$\frac{dy}{dx} + \frac{\cos x}{\sin x} \cdot y = \cos^{-2} x$

$y = C \cdot e^{-\int \frac{\cos x}{\sin x} dx} + e^{-\int \frac{\cos x}{\sin x} dx} (\cdot)$

and $(\cdot) = \int e^{\int \frac{\cos x}{\sin x} dx} \cdot \cos^{-2} x \, dx$

$\therefore y = \frac{C}{\sin x} + \tan x + \cot \cdot x$

2. a. $\frac{dy}{dx} = xy^5 - y$.

sol. $\frac{dy}{dx} + y = xy^5$.

$y^{-5} \frac{dy}{dx} + y^{-4} = x$ ①

Let $u = y^{-4}$

$\frac{du}{dx} = -4y^{-5} \frac{dy}{dx}$ ②

$\therefore -\frac{1}{4} \frac{du}{dx} + u = x$

$u = C \cdot e^{-\int -4dx} + e^{\int 4dx} \left(\int e^{\int -4dx} (-4) dx \right)$

So $y^{-4} = C \cdot e^{4x} + e^{-4}$

$\therefore y = \left(C e^{4x} + e^{-\frac{1}{4}} \right)^{-\frac{1}{4}}$.

b. $\frac{dy}{dx} + \frac{y}{x} = y^{\frac{1}{2}}$; $y(4) = \frac{1}{9}$

sol $y^{-\frac{1}{2}} \frac{dy}{dx} + \frac{y^{\frac{1}{2}}}{x} = 1$

Let $u = y^{\frac{1}{2}}$ $\quad dy = 2u$

So $\frac{1}{u} \frac{dy}{dx} + \frac{u}{x} = 1$

$\frac{2du}{dx} + \frac{u}{x} = 1$

$u = C \cdot e^{-\frac{1}{2}\int \frac{1}{x} dx} + e^{-\frac{1}{2}\int \frac{1}{x} dx} \cdot \int e^{\frac{1}{2}\int \frac{1}{x} dx} \cdot \frac{1}{2} dx$

$= C \cdot x^{-\frac{1}{2}} + \frac{x}{2} \int x^{\frac{1}{2}} dx$

$= C \cdot x^{-\frac{1}{2}} + \frac{1}{3} x = y^{\frac{1}{2}}$

$\therefore$ ~~$y = C \cdot x^{-1} + \frac{1}{9} x^2$~~

and $y(4) = \frac{1}{9} \Rightarrow \begin{cases} C_1 = -2 \\ C_2 = -\frac{10}{3} \end{cases}$

$\therefore y = (-2x^{-\frac{1}{2}} + \frac{1}{3} x)^2$

$y = (-\frac{10}{3} x^{-\frac{1}{2}} + \frac{1}{3} x)^2$

3.(a) $\frac{dy}{dx} = \tan^2(x+y)$

Sol. Let $x+y = u$

$$\frac{du}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{du}{dx} - 1 = \tan^2 u$$

$$\frac{du}{dx} = \frac{1}{\cos^2 u}.$$

$$\int \cos^2 u \cdot du = \int dx$$

So $2(y-x) + \sin(2x+2y) = C$

(b) $\frac{dy}{dx} = (x+y+1)^2$

Let $u = x+y+1$

$$\frac{du}{dx} = 1 + \frac{dy}{dx}$$

$\because \frac{du}{dx} - 1 = u^2,$

$\therefore \int \frac{du}{u^2+1} = \int dx$

$\therefore \tan u = x + C$

$\therefore x = \arctan(x+y+1) + C$

4.(a) separation of variables

(b) reduction to separation of variables

(c) linear first order

(d) Bernoulli