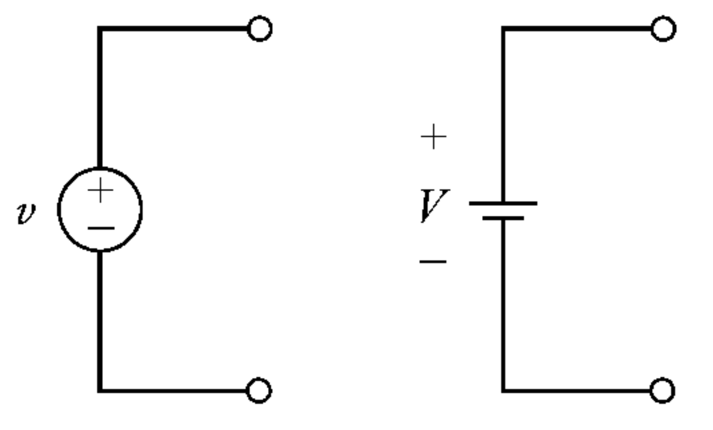
The basic circuit analysis theorems

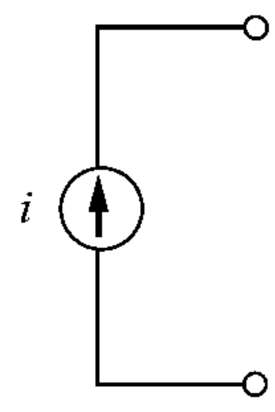
DEPENDENT & INDEPENDENT SOURCES

Independent Sources

An *independent* current source provides a certain current, and an independent voltage source provides a certain voltage, both independently of any other voltage or current.



Independent voltage sources



Independent current source

SOURCE TRANSFORMATION

Source Transformations

Source transformation is a tool for simplifying circuits analysis.

During our circuits analysis, it might be useful to:

- transform voltage sources to *equivalent* current sources, or
- transform current sources to *equivalent* voltage sources

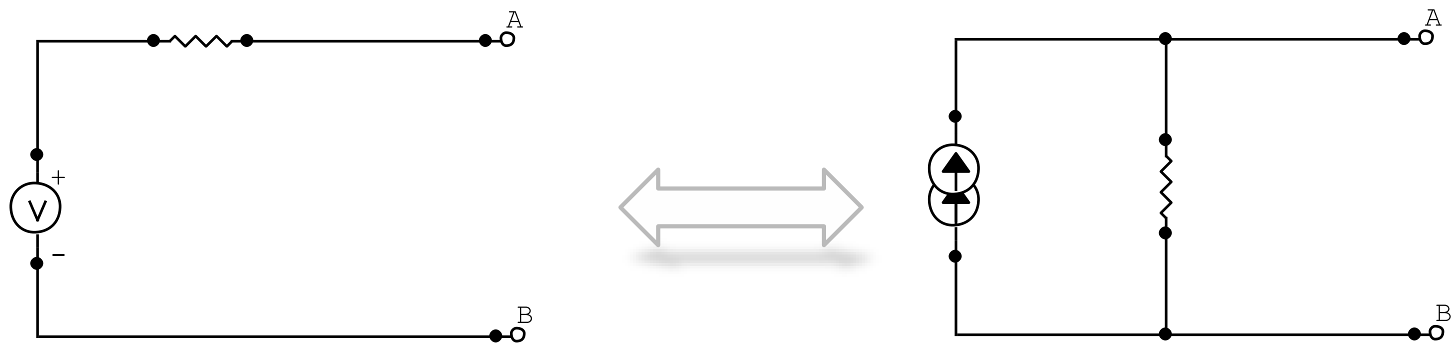
(*an equivalent circuit is one whose voltage-current characteristics are identical with the original circuit*).

We must keep in mind that:

- Each **voltage source** should have **a series internal resistance**
- Each **current source** should have **a parallel internal resistance**

Source Transformations

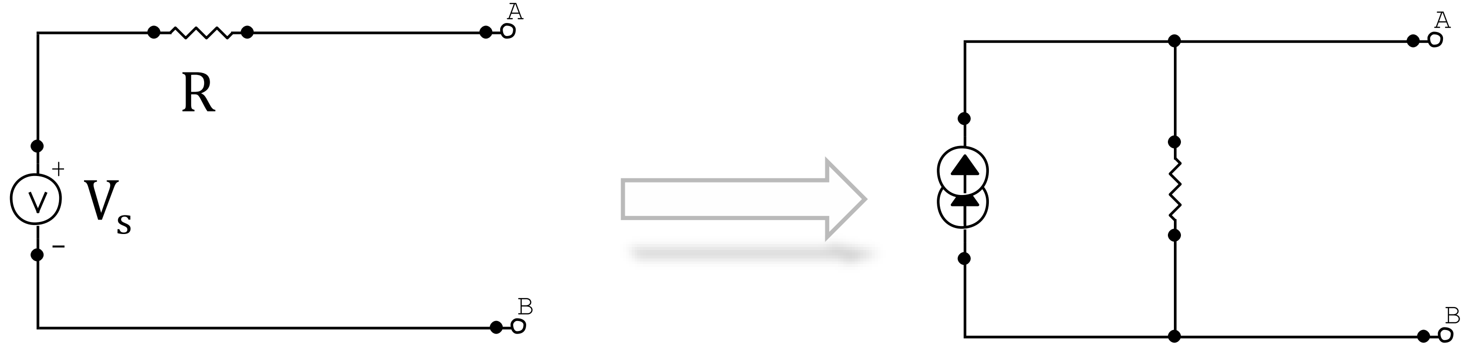
Any combination of voltage source and resistor in series can be replaced by an *equivalent* combination of a current source and a resistor in parallel… and visa versa. This is called **source transformation** and becomes very useful when we are trying to analyse circuits.



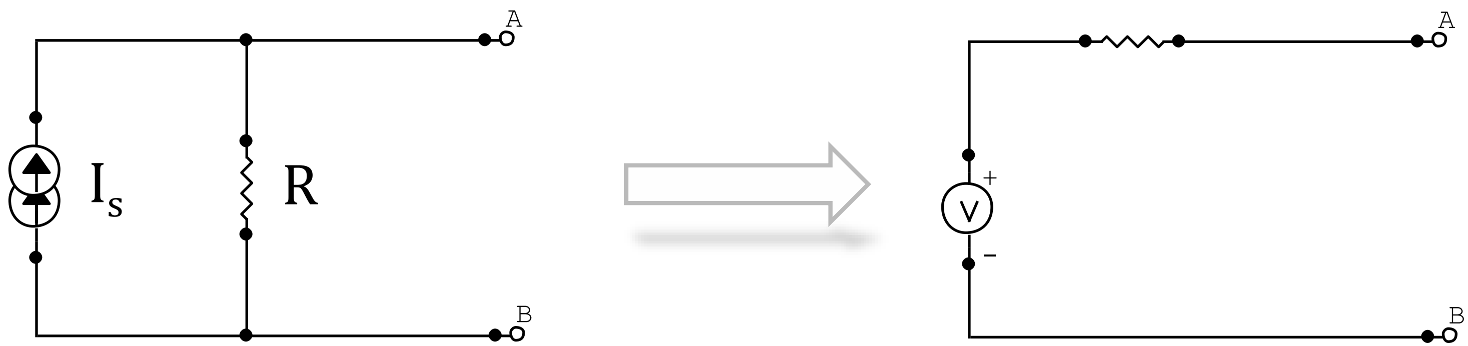
The two circuits are *equivalent* provided they have the same voltage-current relation at terminals A-B.

Source Transformations

Transformation from a voltage source to an ***equivalent*** current source

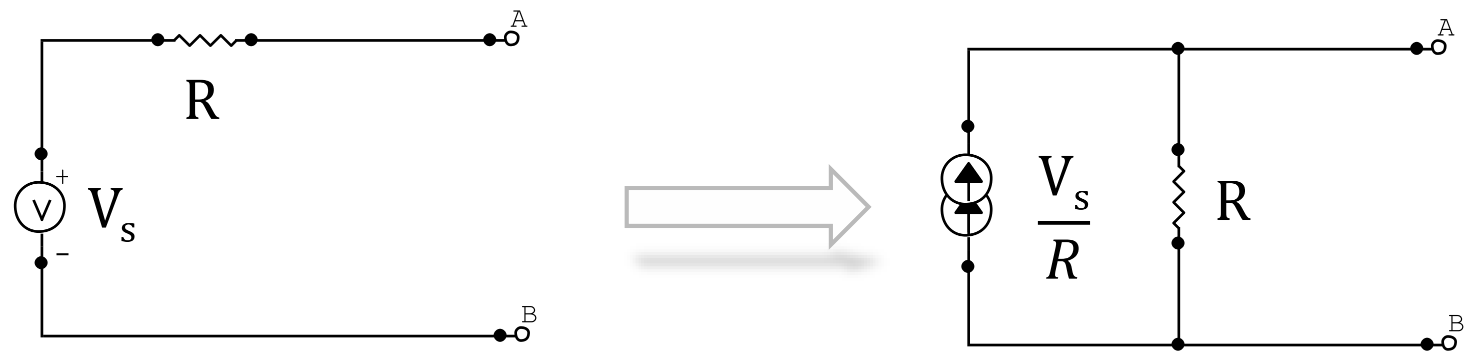


Transformation from a current source to an ***equivalent*** voltage source

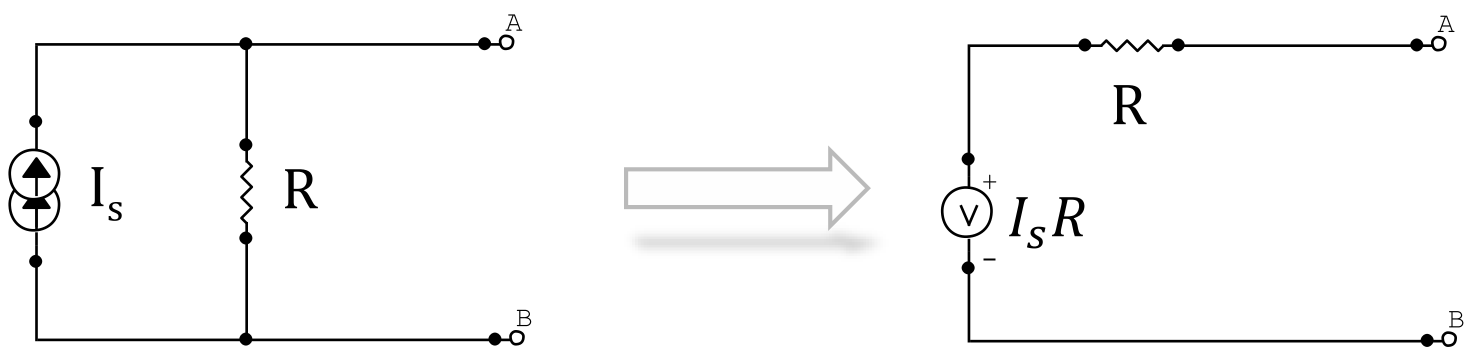


Source Transformations

Transformation from a voltage source to an ***equivalent*** current source

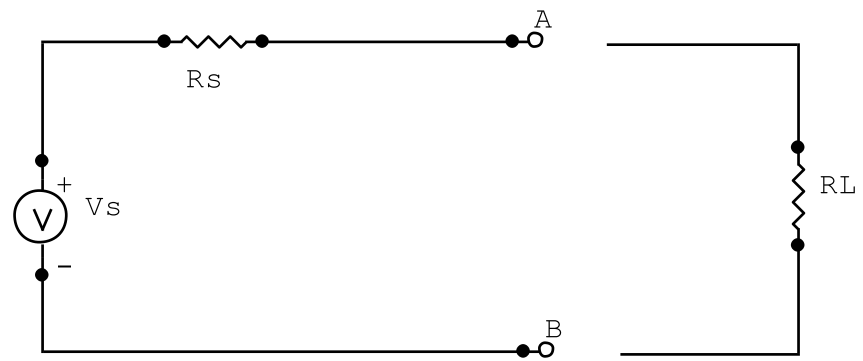


Transformation from a current source to an ***equivalent*** voltage source



Source Transformations

First, let's analyse this circuit assuming that we have added a resistance R_L.
What we wish to do is work out the voltage and current for R_L.
(Key thing here is that the resistors are in **series**).

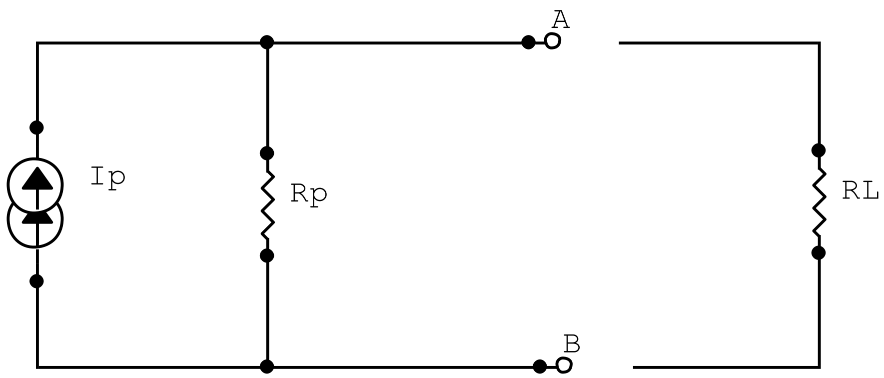


$$I_L = \frac{V_S}{R_S + R_L}$$

$$V_L = V_S\left(\frac{R_L}{R_S + R_L}\right)$$

Source Transformations

Then, let's analyse this circuit assuming that we have added a resistance R_L.
What we wish to do is to work out the voltage and current for R_L .
(Key thing here is that the resistors here are in **parallel**).



$$I_L = \frac{V_L}{R_L} = I_P\left(\frac{R_P}{R_P + R_L}\right)$$

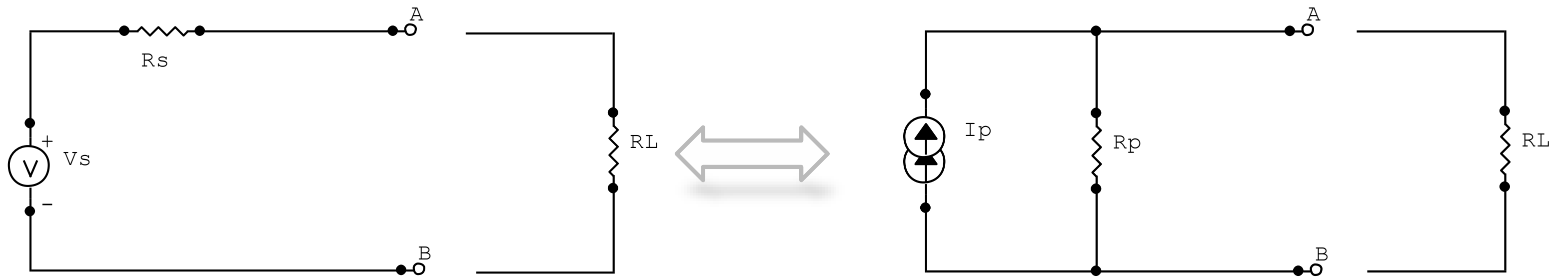
$$V_L = I_P\left(\frac{R_L R_P}{R_P + R_L}\right)$$

$$\frac{1}{R_{||}} = \frac{1}{R_L} + \frac{1}{R_P}$$

$$R_{||} = \frac{R_L R_P}{R_P + R_L}$$

Source Transformations

Let's compare now:



$$V_L = V_S\left(\frac{R_L}{R_S + R_L}\right) \qquad V_L = I_P R_P\left(\frac{R_L}{R_P + R_L}\right)$$

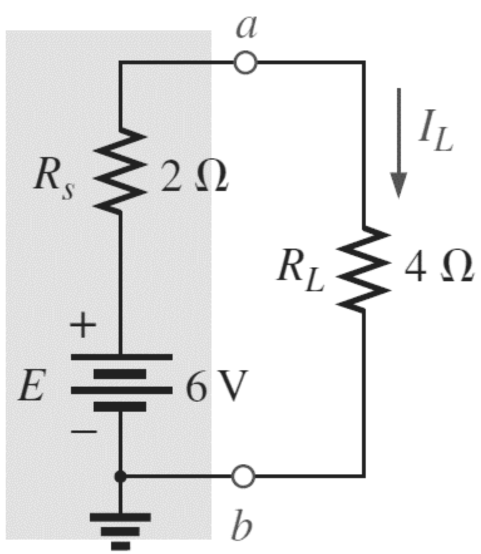
$$I_L = V_S\left(\frac{1}{R_S + R_L}\right) \qquad I_L = I_P R_P\left(\frac{1}{R_P + R_L}\right)$$

If I let R_S equal to R_P and call this the equivalent source resistance, R_{SOURCE} then the results from the two types of sources are identical if:

$$V_S = I_P R_{SOURCE}$$

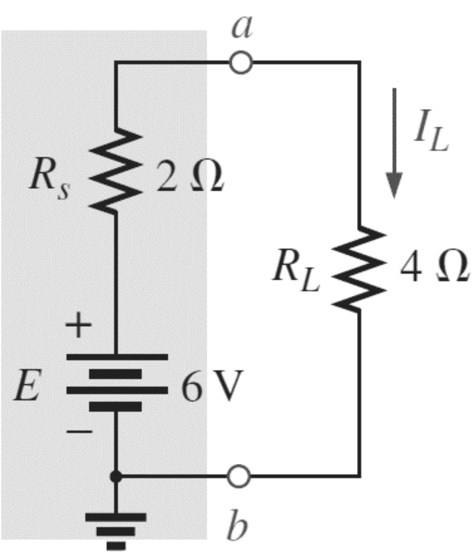
From a circuit analysis perspective (not a build perspective), the two systems will behave identically and can be swapped between the two as needed.

Let's see an example…



(a) Determine the current I_L.
(b) Convert the voltage source to a current source.
(c) Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

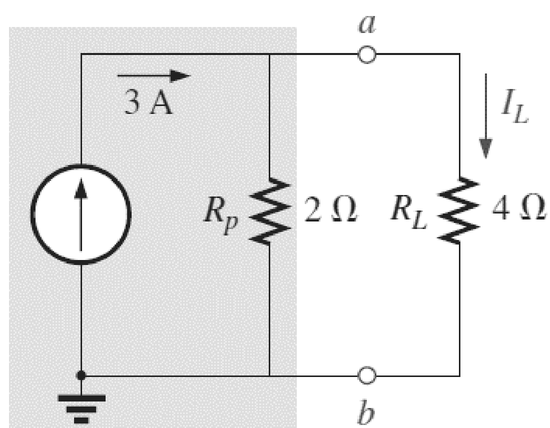
Let's see an example…



(a) Determine the current I_L.
(b) Convert the voltage source to a current source.
(c) Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

(b)

$$I = \frac{E}{R_s} = 3\ \text{A}$$



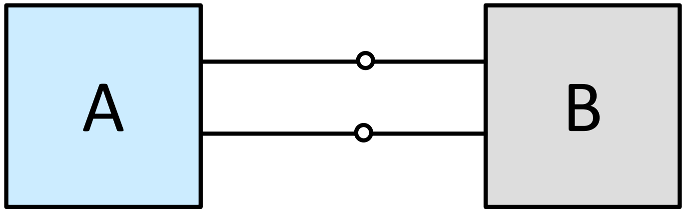
Source Transformations

The concept of source transformation (as we will see later in detail) is a great tool for Thévenin’s and Norton’s theorems.

THEVENIN'S THEOREM

Thevenin's & Norton's theorems

For the application of Thevenin's and Norton's theorems, a network is divided into two parts (A and B). One part must be linear and bilateral (*let's say part A*), but the other part can be anything.



bilateral – means that the devices in the circuit must behave in the same fashion irrespective of current flow (so resistors, capacitors, inductors are good).

linear – means that we have y=mx+c type equations, so again resistors, capacitors, inductors, but not diodes, transistors, or anything complicated.

Thevenin's & Norton's theorems

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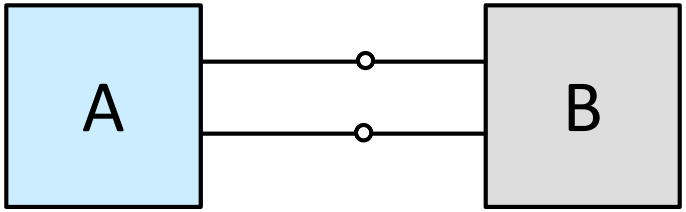
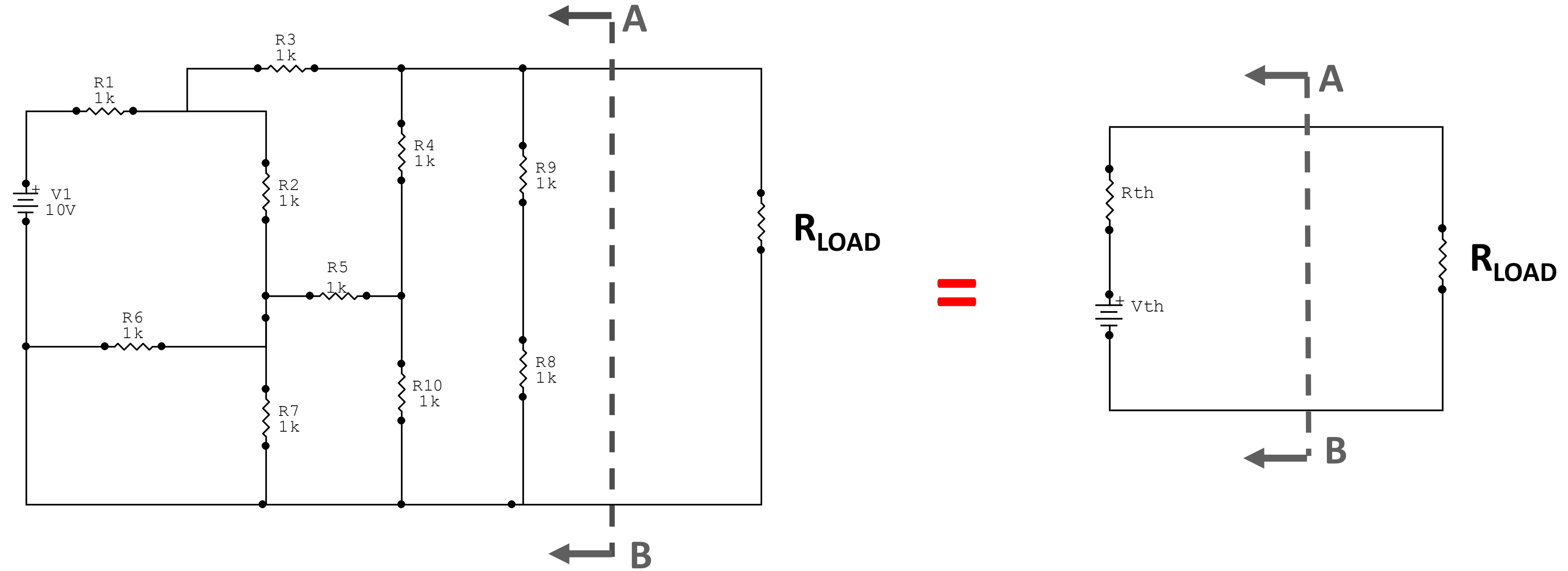


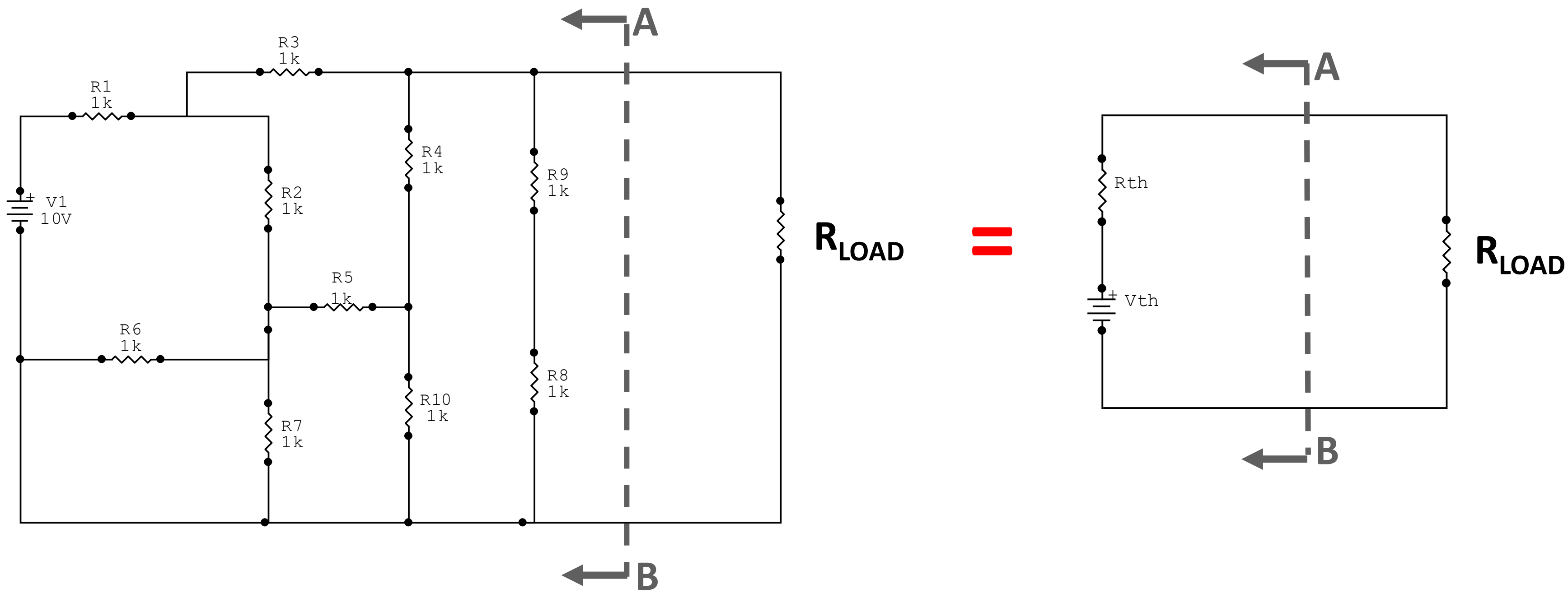
Thevenin's theorem specifies that the linear, bilateral part can be replaced by a Thevenin *equivalent* circuit consisting of a voltage source and a resistor in series, without any change in voltages and currents in part B.

Thevenin's Theorem

In other words, Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a **single voltage source and series resistance** connected to a load.

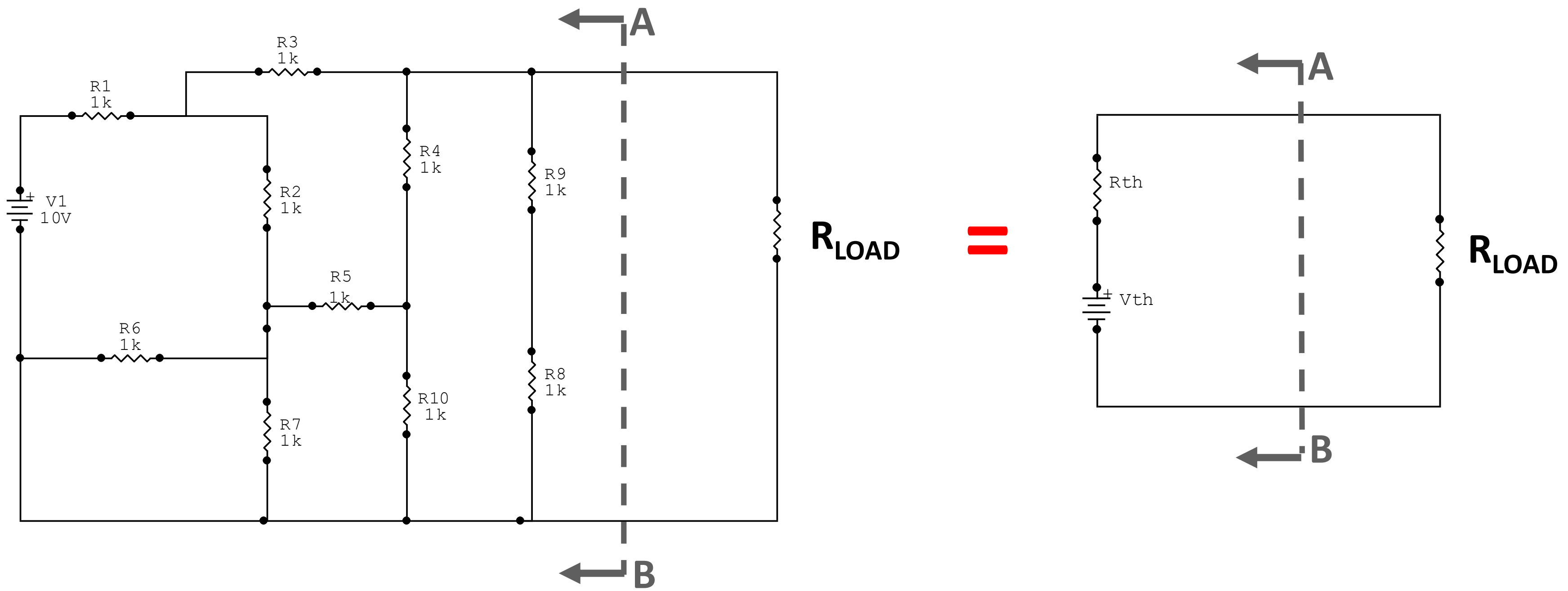


Thevenin's Theorem



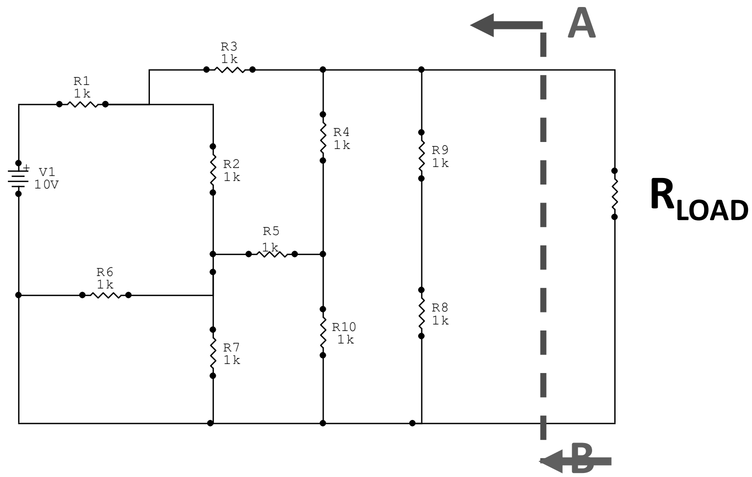
This transformation is very valuable and can simplify your analysis greatly. However it reveals **no information** about what is happening **inside** the part of the circuit that is "transformed". You use it to simplify a part of the circuit you are not interested in, so you can focus on the bit that you are interested in.

Thevenin's Theorem



Depending on where you put your imaginary divider (A-B), you will get **different Thevenin equivalent circuits**. There is not one "magic" equivalent circuit… it depends on what you want to do.

Finding the Equivalent Circuit…



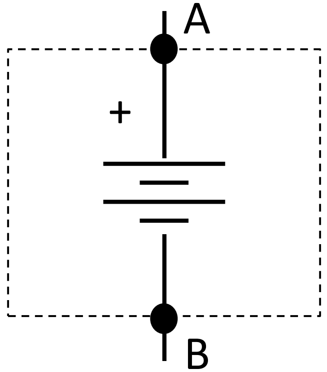
1 First, **remove the load resistor** (or the interesting part of the circuit) from the original circuit.

2 Then, **calculate the Thevenin Voltage** (V_{TH}) by calculating the voltage across the open connection points where the load resistor used to be (also called the open-circuit voltage).

3 Finally, **find the Thevenin resistance** (R_{TH}) by *zeroing* all power sources in the original circuit (*voltage sources shorted* and *current sources open*) and calculating the resistance between the open connection points.

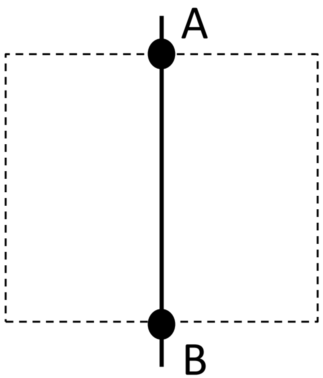
Let's see now a simple example….

a zeroed source?

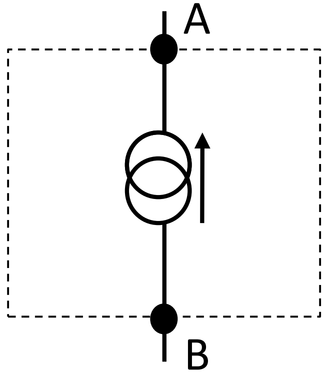


A voltage source says that the voltage between 2 points (A,B) is a certain value and that current can flow.

So if the voltage between 2 points is ZERO but current can flow, well that is the definition of a wire.

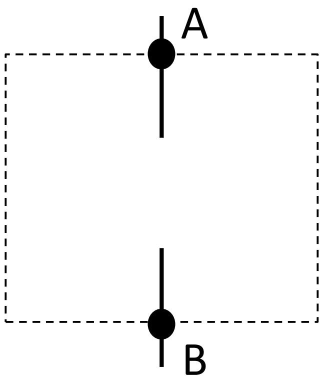


when zeroed



A current source says that the current flowing in a wire (B to A) is a certain value and that there may be a voltage difference.

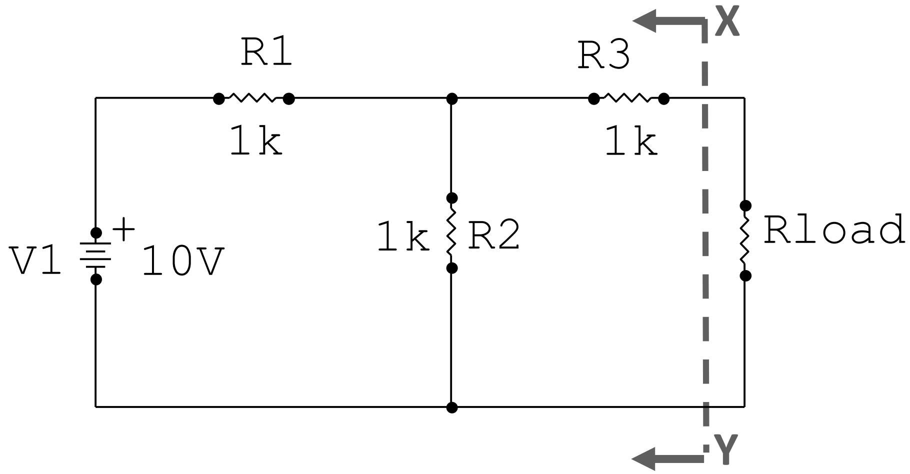
So if the current flowing must be ZERO irrespective of voltage, then that must mean there is a gap.



when zeroed

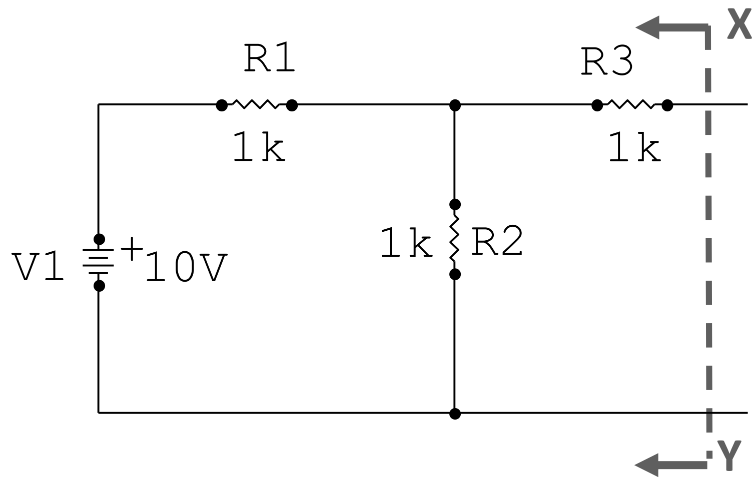
Thevenin Example (simple)

Find the Thevenin equivalent circuit to the left of the XY divider. Then calculate the current that would flow through the load resistance if the load resistance were 1 kΩ and 10 kΩ



Thevenin Example (simple)

1 First, we remove the part of the circuit to the right of the divider.



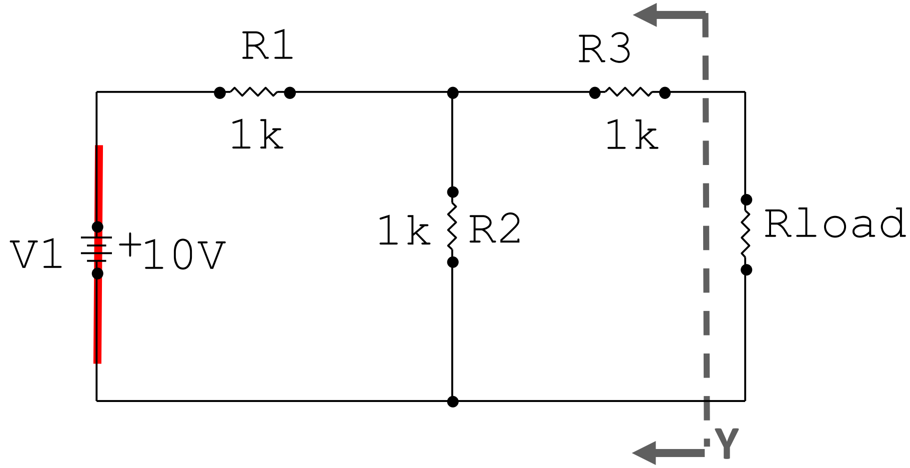
2 Then, we calculate the voltage dropped between X and Y.
(note: no current flows through R3 as there is no loop)

$$V_{XY} = V_1\left(\frac{R_2}{R_1+R_2}\right) = 10\left(\frac{1k}{1k+1k}\right)$$

$$\boldsymbol{V_{TH} = 5Volts}$$

Thevenin Example (simple)

3 Finally, we set the sources to zero (short the voltage source) and find the resistance (note: R1 is in parallel with R2, and R3 is then in series with them):

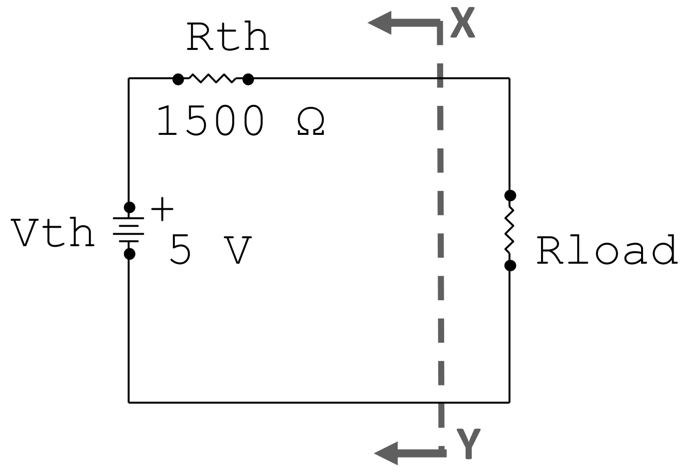


$$R_{1||2} = \left(\frac{1}{R_1} + \frac{1}{R_1}\right)^{-1} = \left(\frac{1}{1k} + \frac{1}{1k}\right)^{-1} = 500\Omega$$

$$\boldsymbol{R_{TH} = R_3 + R_{1||2} = 500 + 1000 = 1500\Omega}$$

Thevenin Example (simple)

Let's now draw the new "equivalent circuit"



If I have a 1 kΩ and 10 kΩ options for the load, what is the current that could flow?

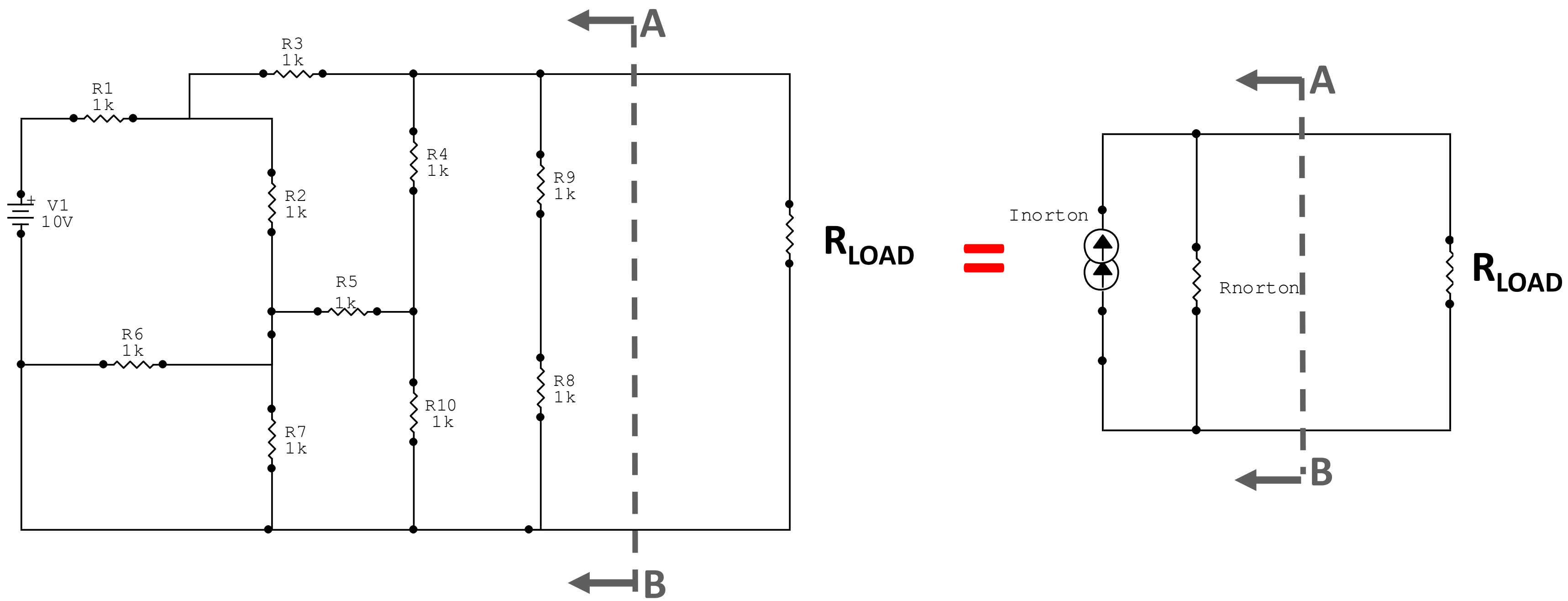
$$I_{10k} = V/R_{total} \\ = 5/(1500 + 1000) \\ = 2mA$$

$$I_{10k} = V/R_{total} \\ = 5/(1500 + 10000) \\ = 0.43mA$$

NORTON’S THEOREM

Norton's Theorem

Norton's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a **single current source and a parallel resistance** connected to a load.



The Norton equivalent circuit can be derived by applying a transformation to the Thevenin's equivalent circuit.

SUPERPOSITION

Superposition

The superposition theorem for electrical circuits states that the response (Voltage or Current) in any branch of a bilateral linear circuit having more than one **independent** sources equals the algebraic sum of the responses caused by each independent source acting alone, while all other independent sources are replaced by their internal impedances.

or

The total current/voltage in any part of a linear circuit **equals the algebraic sum** of the currents/voltages produced by **each source separately**.

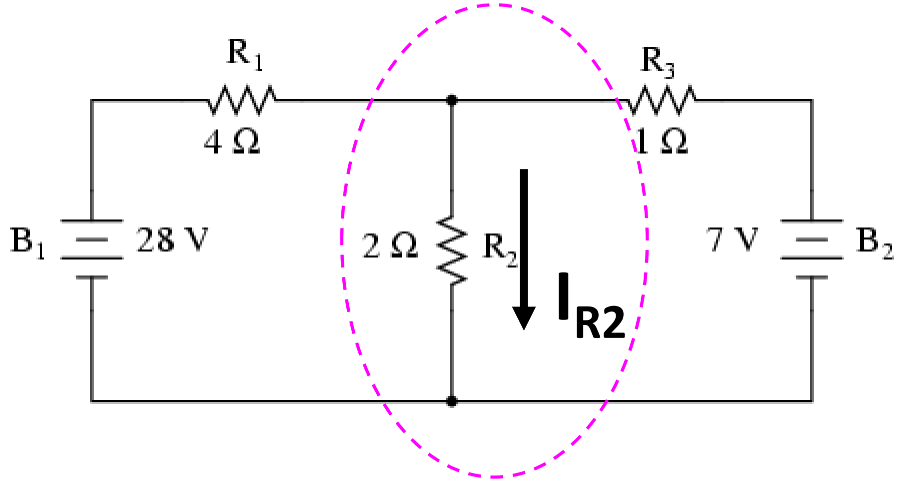
Note: *impedance is a more general term for resistance but includes normal resistance and imaginary resistance figures (which we get from capacitors and inductors… We will discuss about this later).*

Superposition

The superposition theorem for electrical circuits states that the response (Voltage or Current) in any branch of a **bilateral** **linear** circuit having more than one independent source equals the algebraic sum of the responses caused by each **independent source** acting alone, while all other independent sources are replaced by their **internal impedances**.

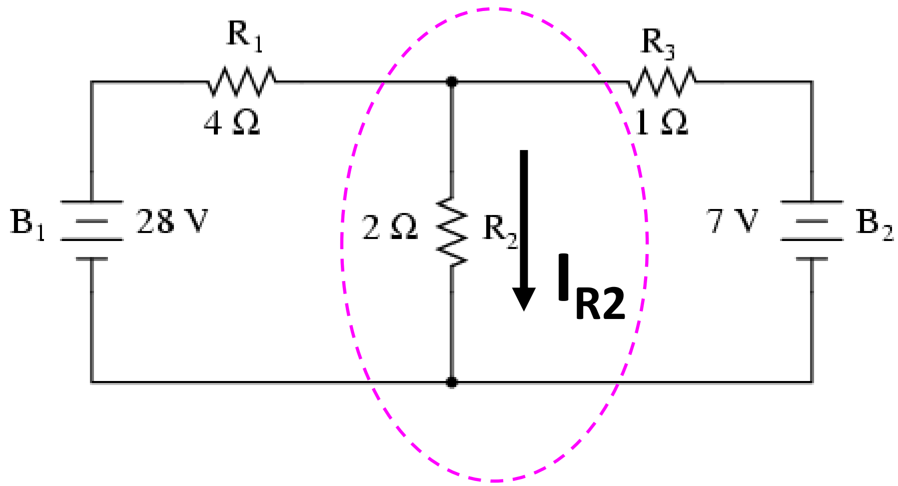
bilateral – means that the devices in the circuit must behave in the same fashion irrespective of current flow (so resistors, capacitors, inductors are good).
independent sources – means that one energy supply (voltage/current) does not have any affect or relationship with the other sources.
linear – means that we have y=mx+c type equations, so again resistors, capacitors, inductors, but not diodes, transistors, or anything complicated.
internal impedances – for the batteries, this is their source resistance (if the source resistance doesn't have an imaginary component).

Superposition - An example



$I_{R2} = ?$

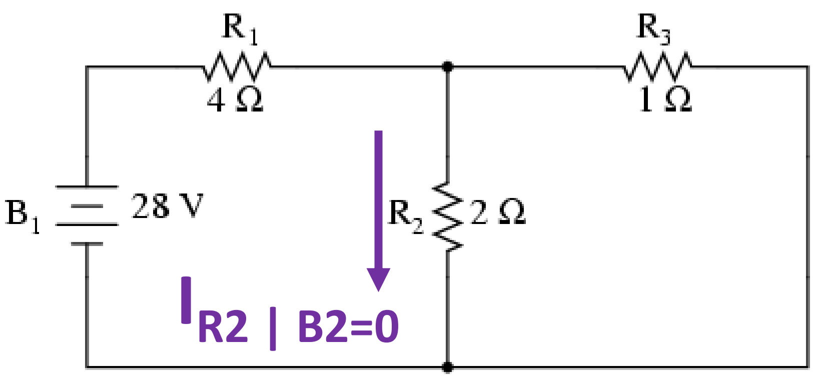
Superposition - An example

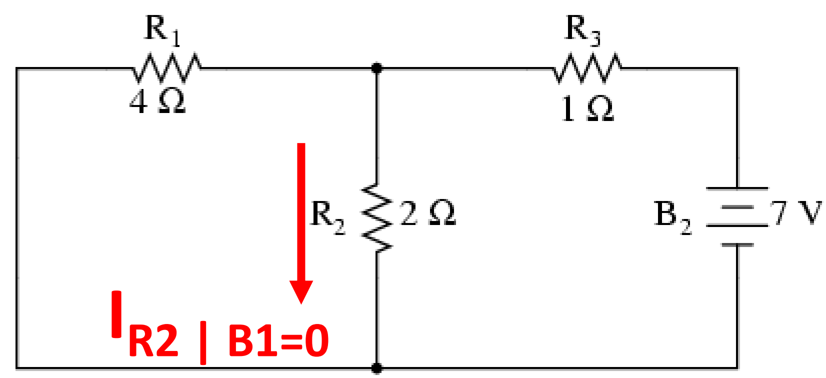


$$I_{R2} = I_{R2\ |\ B2=0} + I_{R2\ |\ B1=0}$$

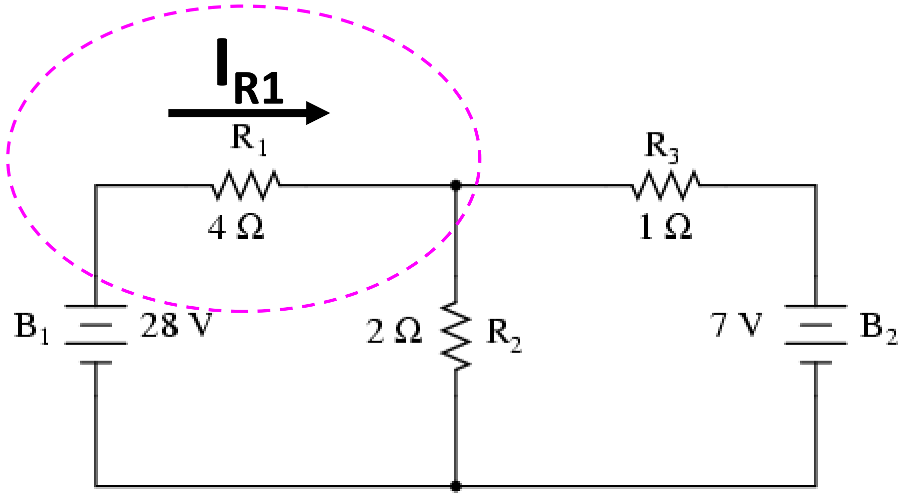
Current when B2 is deactivated.

Current when B1 is deactivated.



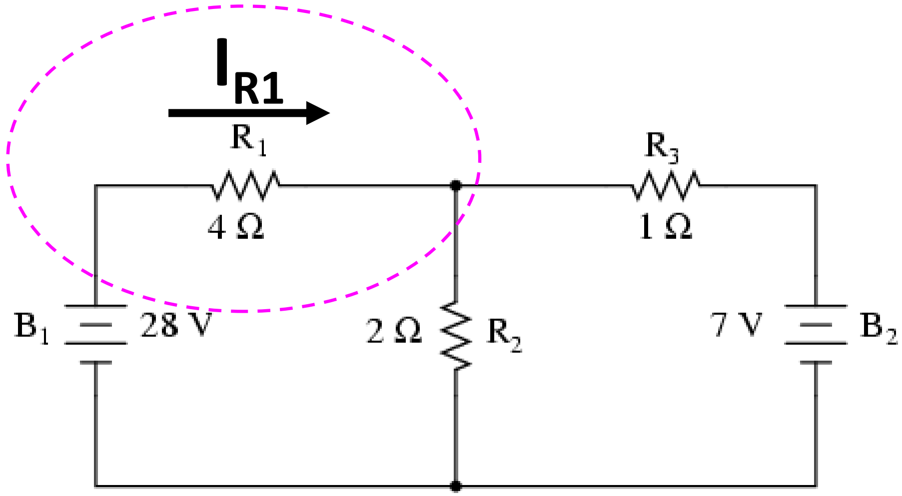


Superposition – Another Example

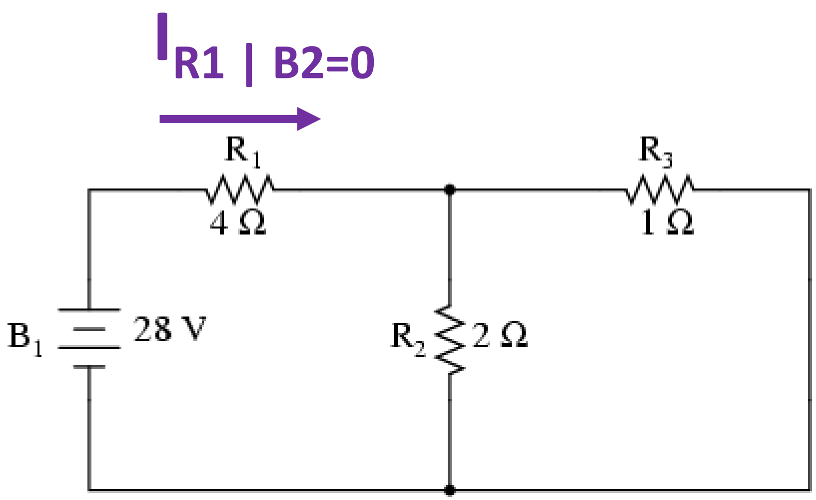


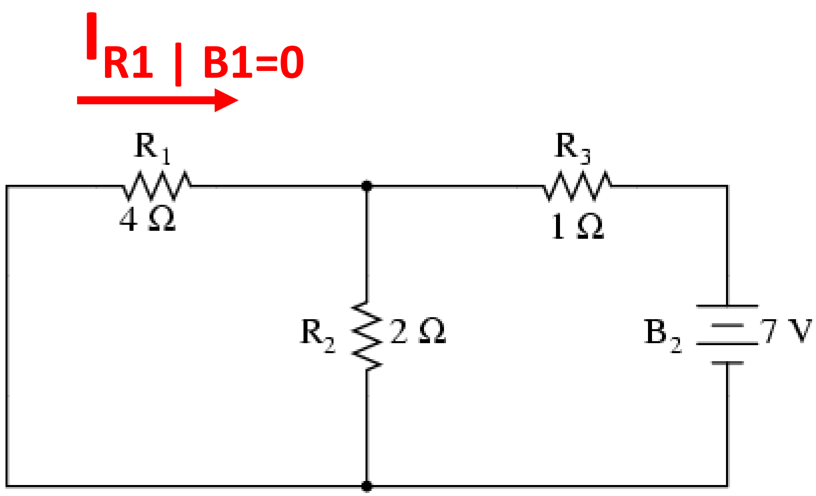
$I_{R1} = ?$

Superposition – Another Example



$$I_{R1} = I_{R1\,|\,B2=0} + I_{R1\,|\,B1=0}$$





Superposition – the technique

step 1: identify all the voltage/current sources.

step 2: identify the branch/node that you wish to solve for (may be everything).

step 3: pick a source, and set all the other sources to zero. ***This is a zero-resistance wire for a voltage source*** and ***an infinite resistance gap (open) for a current source.***

step 4: simplify the circuit as necessary and calculate the currents and voltages using whatever technique is needed.

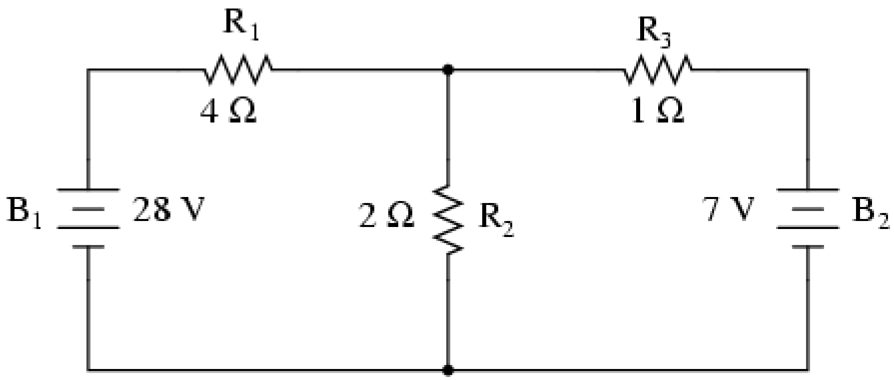
step 5: repeat step 3 and step 4 for each source.

step 6: combine the results to find the aggregate response.

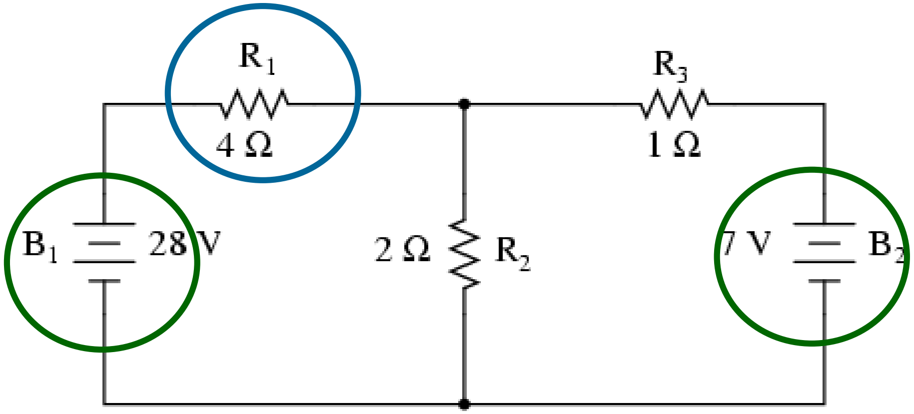
Note: because we are doing an algebraic sum, it is very important that you keep the directions of your currents and voltage drops consistent.

Superposition – an example

Find the *current and voltage flowing through R1* in the circuit below.



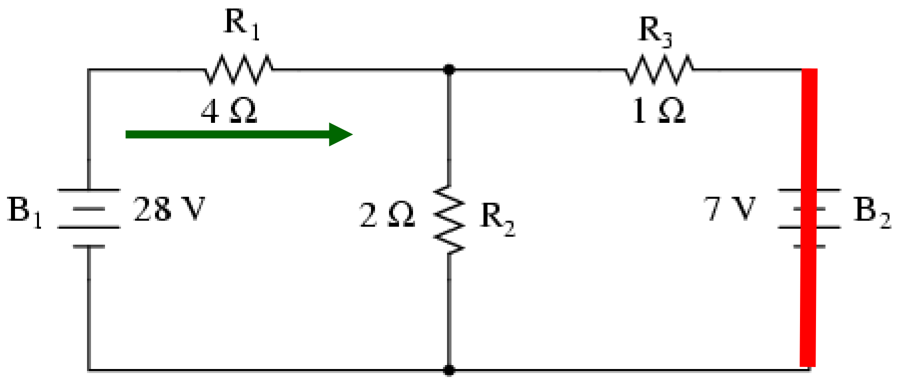
step 1: identify all the voltage/current sources.
step 2: identify the branch/node that you wish to solve for.



Superposition – an example

step 3: pick a source *(Battery 1)* and set all the other sources to zero. This is a zero-resistance wire for a voltage source and an infinite resistance gap(open) for a current source.

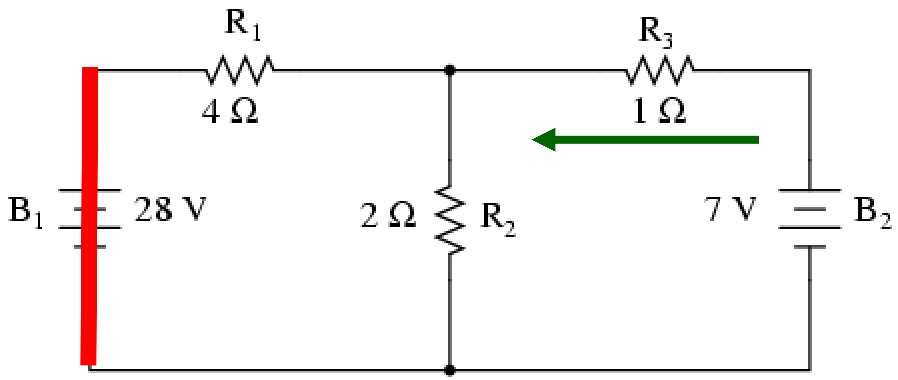
step 4: simplify the circuit as necessary and calculate the currents and voltages using whatever technique is needed.



In this case, R3 and R2 are in parallel, producing a combined resistance of 0.66 Ω. This is in then in series with R1, so total resistance seen by the battery is 4.66 Ω, the current through R1 is 28/4.66 = 6 Amps

Superposition – an example

step 5: repeat step 3 and step 4 for each source (*Battery 2*) .

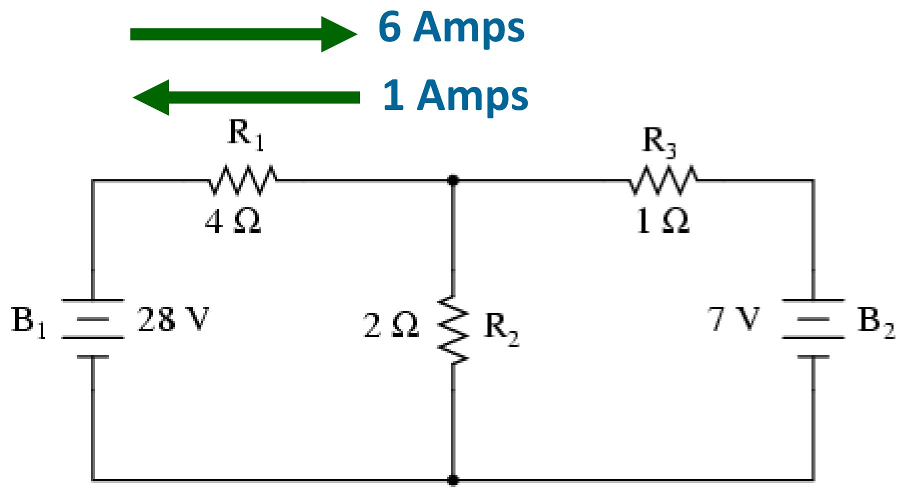


In this case, R1 and R2 are in parallel, producing a combined resistance of 1.33 Ω. This is then in series with R3, so total resistance seen by the battery is 2.33 Ω, current through R1||R2 is 7/2.33 = 3 Amps.

So if I multiply this current by the combined resistance, I get the voltage drop across the parallel combination (R1||R2) to be 4 Volts (1.333 * 3 = 4) volts. So if there is 4 volts, then if R1=4, then the current through R1 is 1 Ampere, going from right to left.

Superposition – an example

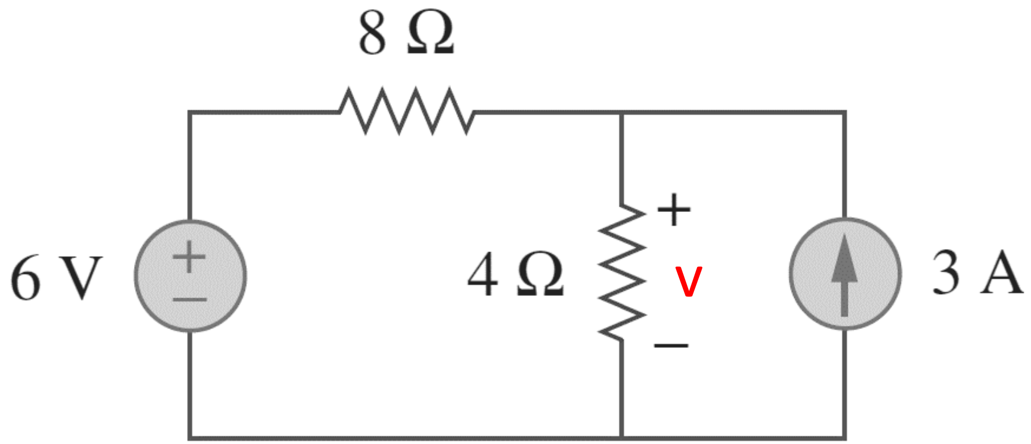
step 6: combine the results to find the aggregate response.



The net current is 5 amps from left to right

Superposition – More Examples

Use the superposition theorem to find v in the circuit.



Answer=10 V

Superposition – More Examples

Using the superposition theorem, determine the current through resistor R2. Demonstrate that the superposition theorem is not applicable to power levels.