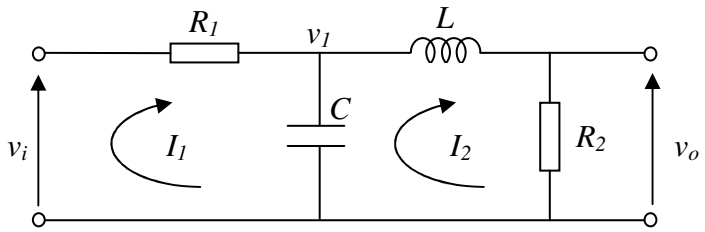
Tutorial Sheet 3 - State-Space Representation

Q1 Consider the circuits given below. In each case:

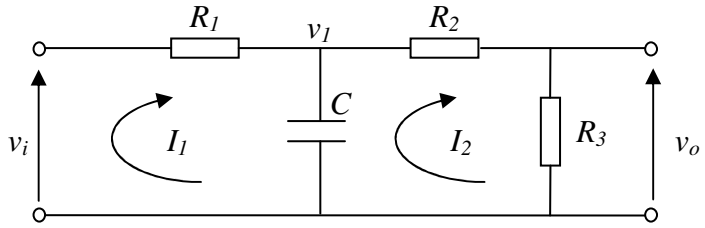
(i) Using mesh analysis, write the system's equations in state-space form (matrix form) using the currents as states.

(ii) Using nodal analysis, write the system's equations in state-space form (matrix form) using the voltages as states.

(iii) Using the transfer function v_o / v_i from Q2 on Tutorial Sheet 2, determine the minimal state realization.

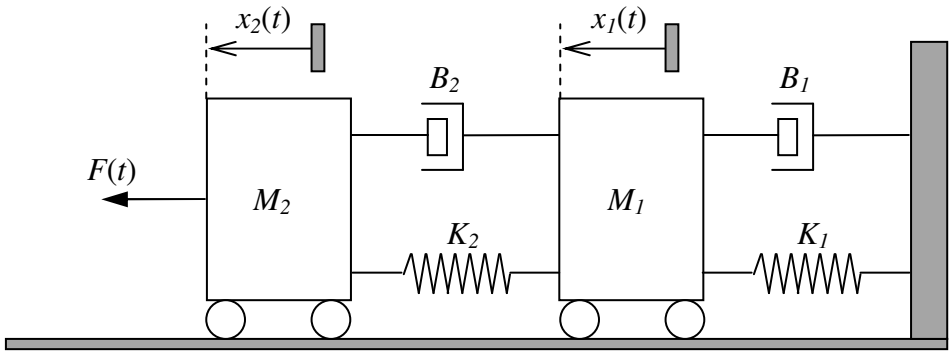


Circuit A



Circuit B

Q2 Consider the following mechanical system:



(i) Develop a state equation for the system using the position and velocity of each mass as states.

(ii) Determine the output equation for the system if the outputs of interest are the position of mass M_1 and the relative velocity of mass M_2 with respect to mass M_1.

Q3 The data in the table below are samples of the input and output of an unknown dynamical system.

Sample instant (*k*)	**1**	**2**	**3**	**4**	**5**	**6**
input *u*(*k*)	0.065	0.065	–0.15	–0.15	–0.1	0.11
output *y*(*k*)	0.098	0.109	0.117	–0.019	–0.128	–0.1

Experimental data from an unknown process

(i) Using the 2nd order transfer function from Q5 on Tutorial Sheet 2, develop a discrete-time state-space model for the 2nd order system.

(ii) Show that the 2nd order transfer function model can be obtained directly from the state-space model.

Q4 Consider the mechanical system shown below, with $M = 0.1$ kg, $K_1 = 6$ N/m, $K_2 = 4$ N/m and $B = 0.4$ Ns/m. Distances y and z are measured with respect to the equilibrium point of the system.

(i) Determine the equations of motion for this system.

(ii) Choosing the states as $x_1 = y, x_2 = \dot{y}$ and $x_3 = z$ and the output as y, derive a state-space model for the system.

(iii) Determine the transfer function model for the system (a) directly from the equations of motion in part (i) and (b) from the state-space model in part (ii)

(iv) Derive the canonical state-space model for the system from the transfer function obtained in part (iii). Verify your answer by transforming the system back to transfer function form.