EE213 Assignment 2: Sampling and Aliasing

In this assignment, you will complete the following tasks:

- Compute and plot the Fourier Transform of a given analogue signal.
- Generate the sampled version of the analogue signal.
- Observe the sampled signal in the time domain.
- Compute the Fourier Transform of the sampled signal and relate it to the Fourier Transform of the analogue signal.
- Change the rate at which the analogue signal is sampled and observe the effect that this has in the frequency domain.

The objective of the assignment is to give you a feel for the effect that sampling a signal has on its frequency content - as indicated by its frequency spectrum, which is produced by the Fourier Transform.

The prerequisite theory for completing this assignment can be found in **Lecture 6: Frequency Analysis** and **Lecture 7: Analogue to Digital Conversion**.

You should upload **a single PDF file** to Moodle. This PDF file should include your MATLAB code, figures, and answers to all the tasks and the questions therein. Make an effort to communicate your answers clearly, concisely, and with precision. Ensure that all figures have appropriate titles, labels, and units. Marks will be deducted if these components are absent.

Introduction: Fourier Transform of a Continuous Signal

In Assignment 1, you plotted the two-sided exponential decay signal $x_{\mathrm{a}}(t) = e^{-a|t|}$ for various values of a. In this case, the subscript a indicates that $x(t)$ is an analogue signal.

The Fourier Transform of $x_{\mathrm{a}}(t)$ is given by

$$X_{\mathrm{a}}(\omega) = \frac{2a}{a^2 + \omega^2}$$

We will verify this before we proceed. We know that, in general, the Fourier Transform of a signal is given by

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

For the purposes of computing the Fourier Transform of $x_{\mathrm{a}}(t)$, we must rewrite it as a piecewise fuction.

$$x_{\mathrm{a}}(t) = \begin{cases} e^{-at} & t \geq 0 \\ e^{at} & t < 0 \end{cases}$$

This piecewise representation aids us in evaluating the Fourier Transform integral by allowing us to break it up into the sum of two integrals. The limits of each integral match the domain of that piece of the function.

$$X_{\mathrm{a}}(\omega) = \int_{-\infty}^{0} e^{at}\ e^{-j\omega t} dt + \int_{0}^{+\infty} e^{-at}\, e^{-j\omega t} dt$$

We now evaluate these integrals analytically. Thus, we have

$$X_{\mathrm{a}}(\omega) = \frac{1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{2a}{a^2 + \omega^2}$$

Task 1: Plotting a Frequency Spectrum

Set $a = 1000$ and plot $X_{\mathrm{a}}(\omega)$ for $\omega \in [-8000{,}8000]$ radians.

Task 2: Sampling

The objective of this task is to generate a sampled i.e. discrete version of $x_{\mathrm{a}}(t)$. Sampling is the process that maps an analogue signal to a discrete signal. In keeping with our subscript convention, we will call this digital signal $x_{\mathrm{d}}[n]$.

- Set the sampling rate (or sampling frequency) $F_s = 5000$ samples per second.
- This implies a sampling period $T_s = \frac{1}{F_s} = 0.2$ ms.
- Compute the sampled signal $x_{\mathrm{d}}[n]$ for $n \in [-50{,}50]$.
- Plot the sampled signal $x_{\mathrm{d}}[n]$. Recall that the stem function should be used to plot discrete signals.

The code will look like:

```
a = 1000;
n = -50:50;
Fs = 5000;
Ts = 1/Fs;

t = n*Ts;    % This generates a discrete time vector.

xd = exp(-a*abs(t));

stem(t,xd)
```

Task 3: Discrete Time Fourier Transform

In Task 1, you were asked to plot the Continuous Time Fourier Transform (CTFT) of the analogue (or continuous) signal $x_\mathrm{a}(t)$. In this task, the objective is to plot the Discrete Time Fourier Transform (DTFT) of the discrete signal $x_\mathrm{d}[n]$.

Recall the definition of the DTFT

$$X_\mathrm{d}(\omega) = \sum_{n=-\infty}^{+\infty} x_\mathrm{d}[n] e^{-j\omega n}$$

In order to observe the periodicity of $X_\mathrm{d}(\omega)$, you should plot $X_\mathrm{d}(\omega)$ for $\omega \in [-3\pi, 3\pi]$. Your code will resemble the following:

```
omega = linspace(-3*pi,3*pi,500);
Xd = zeros(1,length(omega));
for k = 1:length(omega)
    Xd(k) = sum(xd.*exp(-1i*omega(k)*n));
end
plot(omega,abs(Xd))
```

In your submission, comment on the periodicity of $X_\mathrm{d}(\omega)$.

Hint: To answer this question, you should think about how the DTFT spectrum relates to the CTFT spectrum. Refer to Lecture 7 for more information.

Task 4: Varying the Sampling Rate

- Repeat Task 2 and Task 3 for a different sampling rate, namely $F_\mathrm{s} = 500$ samples per second.
- Comment on the sampled signal.
- Comment on the shape of the DTFT of the samples signal in this case. In particular, compare it with the Fourier Transform of the continuous signal.