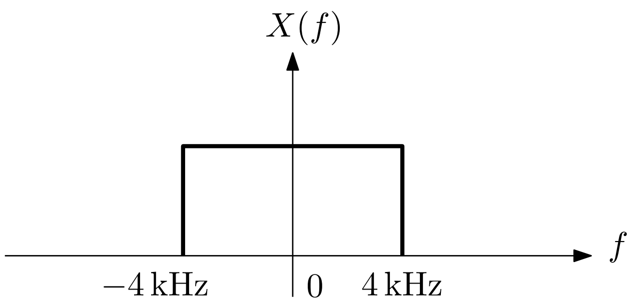
Tutorial 5

1. Consider the continuous-time signal $x(t) = \sin(20t) + \cos(40t)$.
   (a) Find the fundamental period of $x(t)$.
   (b) If $x(t)$ is sampled with a sampling period T to obtain the discrete-time signal
   $$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$
      1. Determine a choice of T consistent with this information.
      2. Is your choice of T in the previous question (i.e., Part (1.)) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

2. The continuous-time signal $x(t) = v_1(t) \times v_2(t)$ is sampled with an impulse train
   $$p_T(t) = \sum_{-\infty}^{\infty} \delta(t - nT)$$
   where T is the sampling interval.
   (a) Assuming $v_1(t)$ and $v_2(t)$ are band-limited to 100 Hz and 300 Hz, respectively, compute the minimum value of the sampling rate f_s that does not introduce any aliasing.
   (b) Repeat part (2a) for $v_1(t) = \text{sinc}(200t)$ and $v_2(t) = \text{sinc}(500t)$. Assuming that a sampling interval of $T = 3$ ms is used to sample $x(t) = v_1(t) \times v_2(t)$, can $x(t)$ be accurately recovered from its samples?
   (c) Repeat part (2b) for a sampling interval of $T = 0.1$ ms.

3. A signal $x(t)$ has the Fourier transform $X(f)$ shown in the following figure



   The signal $x(t)$ is sampled by an ideal uniform sampling process with a sampling rate f_s (Hz or samples/sec). The sampled signal is denoted by $x_s(t)$.

(1) Sketch the Fourier transform of $x_s(t)$ for $f_s = 9000$.

(2) Repeat (1) for $f_s = 6000$ and give your comments on this case.

4. Consider a uniform quantiser in the range of $(-1.5, 1.5)$ with 4 levels.

   (a) Sketch the 4 quantisation levels of the quantiser.

   (b) Compute the step size and the maximum quantisation error of the quantiser.

   (c) Compute the quantiser output for the input sequence $\{1.2, -0.2, 0.4, -0.89\}$.

   (d) Assume the input signal of the quantiser is uniformly distributed, compute the signal-to-quantisation noise.

5. Explain how you would (approximately) measure the system impulse response of an LTI system, without knowing its components.

6. Suppose that the input is a continuous non-periodic signal $x(t)$ and its continuous-time Fourier transform $X(\omega)$ is given. The system impulse response is denoted by $h(t)$ and its frequency response is $H(\omega)$. Describe two methods to find the output of the system.

7. Consider an LTI system having an impulse response $h(t) = u(t)$. Find the output signal of the system if the input signal is $x(t) = e^{-2t}u(t+2)$.

8. Consider a continuous-time LTI system with impulse response $h(t) = u(t) - u(t-1)$ and input $x(t) = u(t) - u(t-2)$. Compute the output signal of the system.

9. A discrete-time LTI system has the impulse response given below

$$h[n] = \begin{cases} 1 & n = -1 \\ 3 & n = 0 \\ 2 & n = 1 \\ -1 & n = 2 \end{cases}$$

   Given the input $x[n] = u[n] - u[n-3]$, determine the system output $y[n]$.