Assignment 3   832002117

1. $z = \tan(x^2 - y^2)$

Sol $\frac{\partial z}{\partial x} = 2x \sec^2(x^2 - y^2)$

$\frac{\partial z}{\partial y} = -2y \sec^2(x^2 - y^2)$

2. $z = \frac{1}{x^2+y^2+1}$ show ...

sol.

$x\frac{\partial z}{\partial x} = -(2x^2)(x^2+y^2+1)^{-2}$

$y\frac{\partial z}{\partial y} = -(2y^2)(x^2+y^2+1)^{-2}$

$-2z(1+z) = -\frac{2}{(\ )}\left[\frac{x^2+y^2+2}{x^2+y^2+1}\right]$

$= \frac{-2(x^2+y^2)}{(x^2+y^2-1)^2}$

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{-2(x^2+y^2)}{(x^2+y^2-1)^2}$ ..

So RHS = LHS

3. sol

$z = e^x(x\cos y - y\sin y)$

$\frac{\partial z}{\partial x} = e^x x\cos y + e^x\cos y - e^x y\sin y$ ①

$\frac{\partial z}{\partial y} = -e^x x\sin y - e^x\sin y - e^x y\cos y$ ②

$①^{12} + ②^{12} = e^x \cdot x\cos y + 2e^x\cos y - e^x y\sin y$ ③

$- e^x x\cos y - 2e^x\cos y + e^x\sin y = 0$

∴ LHS = RHS

4. (a) $\int x^2 \ln x\, dx$

$u = \ln x$ & $v = \frac{x^3}{3}$

$du = \frac{1}{x}dx$ $dv = x^2 dx$

$I = \frac{1}{3}x^3 \cdot \ln x - \frac{1}{3}\int x^2\, dx$

$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

(b) $\int \frac{x+1}{x^2-3x+2}\, dx$

$= \int\left[\frac{x-1}{(x-1)(x-2)} + \frac{2}{(x-1)(x-2)}\right]dx$

$= \int\left[\frac{1}{x-2} + 2\left(\frac{1}{x-1} - \frac{1}{x-2}\right)\right]dx$

$= \ln|x-2| - 2\ln\left|\frac{x-1}{x-2}\right| + C$

$= 3\ln|x-2| - 2\ln|x-1| + C$

(c) $\int \cos^4 x\, dx = \int (\cos^2 x)^2\, dx$

$= \int\left[(1+\cos 2x)\frac{1}{2}\right]^2 dx$

$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$

$= \frac{1}{4}\int\left[1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right]dx$

$= \frac{1}{4}\int\left(\frac{\cos 4x}{2} + 2\cos 2x + \frac{3}{2}\right)dx$

$= \frac{1}{32}\sin 4x + \frac{1}{4}\sin 2x + \frac{3}{8}x + C$

Date

(d) $\int \frac{dz}{z^2+A^2}$

sol.

$$= \frac{1}{A}\int \frac{d\frac{z}{A}}{(\frac{z}{A})^2+1} \quad \text{Let } u = \frac{z}{A}$$

$$I = \frac{1}{A}\int \frac{du}{u^2+1} = \frac{1}{A}(\arctan u + c)$$

$$= \frac{1}{A}(\arctan \frac{z}{A} + c)$$

(e) $\int \frac{dz}{\sqrt{z^2+A^2}}$ let $z = A\tan x$

$\frac{dz}{dx} = A\sec^2 x$

sol

$$= \int \frac{A\sec^2 x\, dx}{\sqrt{A^2\sec^2 x}}$$

$$= \int \sec^{\cancel{2}} x\, dx$$

$$= \ln|\tan x + \sec x| + C$$

$$= \ln\left|\frac{z}{A} + \sec(\arctan\frac{z}{A})\right| + C$$

$$= \ln\left|\frac{z + \sqrt{A^2+z^2}}{A}\right| + C$$