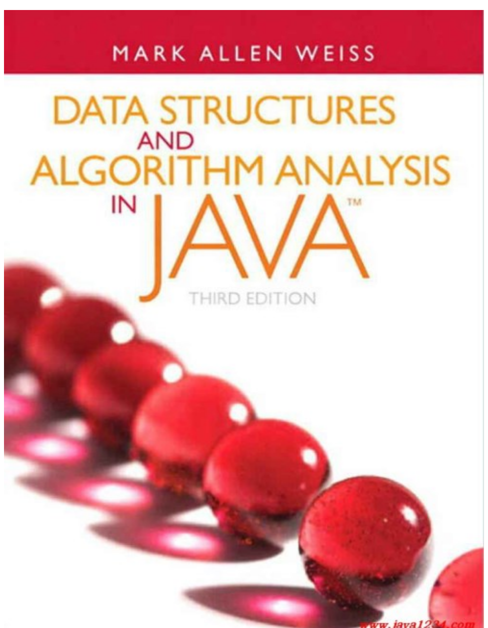
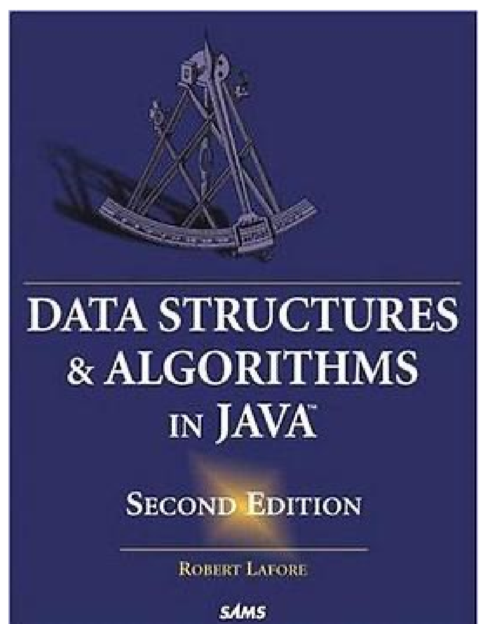
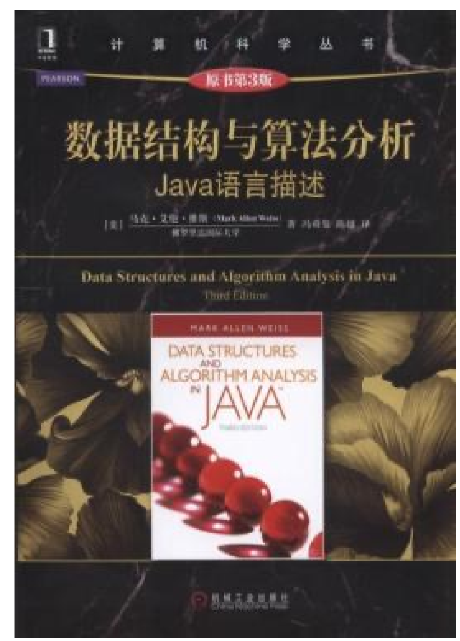
Topic 4 –
Arrays and Array Algorithms







Topics

- Introduction
- Programming Revision
- Methods and Objects
- **Arrays and Array Algorithms**
- Big O Notation
- Sorting Algorithms
- Stacks and Queues
- Linked Lists
- Recursion
- Bit Manipulation

Outline

- **Introduction to Arrays**
- 2D Arrays
- Algorithm implementation exercise: The Sieve of Eratosthenes
- Arrays: Randomization, swapping, search…
- Ordered Arrays
- Algorithm implementation exercise

Java program structure

- Java programs are built up of multiple files called **classes**

- There are advantages in splitting a program into distinct files rather than keeping it in one big chunk
  - The different components can be easily re-used
  - The same piece of code can be re-run as many times as you want without re-typing it
- **Classes** are made up of **variables** and **methods**
  - Variables store information
  - Methods are contained chunks of code which do a specific job and then return a result

Arrays

- Say that you are writing a program that reads in 100 numbers for data.
- Would you like to declare **100 variables** and write **100 input statements** to read in the data?
- Even if it was 6 numbers, it’s not too efficient to declare 6 separate variables, provided they are of the same type.
- The solution to grouping large numbers of variables together is to use arrays

Arrays

- An **array** is an object that is used to store a list of values
- It is made out of a contiguous block of memory that is divided into a number of "slots“
- Each slot holds a value, and **all the values are of the same type**.
- In the example array here, each slot holds an int

index	data
0	23
1	15
2	99
3	37
4	2
5	14

Arrays

- In the example array here, each slot holds an int
- Arrays have names, for example this one is called **data**
- The slots are indexed 0 through 5.
- Each slot can be accessed by using its **index**.
- For example, **data[0]** is the slot which is indexed by zero (which contains the value 23).
- data[5] is the slot which is indexed by 5 (which contains the value 14)

index	data
0	23
1	15
2	99
3	37
4	2
5	14

Arrays

- **Important:**
  - The slots are numbered sequentially **starting at 0**.
  - If there are **N slots** in an array, the indexes will be **0 through N-1**
  - If you write a for loop cycling through all of the slots in an array, make sure it **stops at N-1**
- Example.

```
public static void printArray (String arr) {
    for(int i = 0; i < arr.length; i++) {
        System.out.println(arr[i]);
    }
}
```

// arr.length is the length of array arr

Arrays

- For example, the array **data** is as follows

index	**data**
0	23
1	15
2	99
3	37
4	2
5	14

- Then **data**.length is **6.**
- The slots of **data** are indexed 0 through **5**.

Using Arrays

- Every slot of an array holds a value of the same type.
- For example, you can have an array of int, an array of double, and so on.
- This array holds data of type int.
- Every slot may contain only an int.
- A slot of this array can be used anywhere a variable of type int can be used.
- Example.

  data[3] = **55**;

index	data
0	23
1	15
2	99
3	**55**
4	2
5	14

Using Arrays

- Any of the array entries (or *elements*) can be used exactly the same way as a standard variable, including arithmetic expressions.

- For example, if x contains a 11, then
  (x + data[2]) / 5
  evaluates to (11+99) / 5 = 22

index	data
0	23
1	15
2	99
3	55
4	2
5	14

Using Arrays

- Here are some other legal statements:

```
data[0] = (x + data[2]) / 4 ;

data[2] = data[2] + 1;

x = data[3]++ ; // data in slot 3 is incremented

data[1] = data[5] / data[4];
```

Declaring Arrays

- Array declarations look like this:

type[] arrayName = new type[length];

- This names the type of data in each slot and the number of slots.
- Once an array has been constructed, the number of slots it has does **not** change.
- Example.
  - Construct a int array of length 3.

int[] arr1= new int[3];

Declaring Arrays

- Examples.

```
int[] myArray = new int[20];
double[] theArray = new double[5];
String[] words = new String[17];
char[] charArray = new char[256];
```

- Example.

```
int[] arr2 = {10, 20, 30};
// Here, we set data into this array directly
// That is arr2[0] = 10, arr2[1] = 20, and arr2[2] = 30
```

***Q**: What are values of myArray[0], theArray[0], words[0], and charArray[0]?*

Array Boundary Checking

- The **length** of an array is how many slots it has.
- An array of **length N** has slots indexed **0 … (N-1)**
- Indexes must be an integer type.

- It is OK to have spaces around the index of an array
- For example **data[1]** and **data[1]** are exactly the same as far as the compiler is concerned

- It is ***not legal*** to refer to a slot that does not exist

Array Boundary Checking

- Say that an array was declared:

```
int[] data = new int[10];
```

- Here are some elements of this array, are they valid?

```
data[ -1 ]
data[ 0 ]
data[ 1.5 ]
data[ 9 ]
data[ 10 ]
```

Array Boundary Checking

- Say that an array was declared:

```
int[] data = new int[10];
```

- Here are some elements of this array, are they valid?

data[-1]	always illegal, the index of array must be positive integer
data[0]	always OK
data[1.5]	always illegal
data[9]	always OK
data[10]	illegal, the index of array is must less then its length

Array Boundary Checking

Error line 17: ArrayIndexOutOfBoundsExceptionError

- This means you've overstepped the boundaries of the array
- You have used either an index less than 0, or greater than N-1, where N is the length of the array
- This problem is only revealed when you run the program, not when you compile
- Example.

```
public static void main(String[] args) {
    int arr[] = {13, 42};
    System.out.println(arr[2]);   // the length of arr[] is 2
}
```

Variables as Index Values

- The index of an array is always an integer type, i.e.,
- it can be any expression that evaluates to an integer.

- For example, the following are legal:

```
int[] values = new int[7];
int index = 0;
values[ index ] = 71; // put 71 into slot 0, that is values[0] = 71
index = 5;
values[ index ] = 23; // put 23 into slot 5, that is values[5] = 23

index = 3;
values[ 2+2 ] = values[ index-3 ]; //same as values[4] = values[0];
```

Variables as Index Values

- Using an **expression** for an array index is a very powerful tool
- Often a problem is solved by organizing the data into arrays, and then processing that data in a systematic way using variables as indexes

Variables as Index Values

- Using an **expression** for an array index is a very powerful tool
- Here are further examples:

```
double[] val = new double[4];
val[0] = 0.12;
val[1] = 1.43;
val[2] = 2.98;
int j = 2;
System.out.println("slot 2:" + val[j] );  // val[j] = val[2]
System.out.println("slot 1:" + val[j-1] );  //val[j-1]=val[1]
j = j-2;
System.out.println("slot 0:" + val[j] );  //val[j] = val[0]
```

Initial Values

- When array is created, all values are initialized depending on **array type**:
  - Numbers: 0
  - Boolean: false
  - Object References: null
- Example.

int[] val = new int[3];

  - We have

    val[0] = 0

    val[1] = 0

    val[2] = 0

Array initialization as a list

- You can declare, construct, and initializes the array all in one statement:

int[] data = {23, 38, 14, -3, 0, 14, 9, 103, 0, -56};

- This declares an array of int which is named data. Then it constructs an int array of **10** slots (indexed 0…9)
- Finally it puts the designated values into the slots.
- The first value in the **list** corresponds to index 0, the second value corresponds to index 1, and so on
- So in this example, **data[0]** gets the 23

Copying Arrays

- Say we have two arrays:

```
int[] array1 = {17,12,32,103,5};
int[] array2 = {22,57,13,203,15};
```

- How do we copy the contents of array1 into array2?
- Can we just do this?

```
array2 = array1;
```

Copying Arrays

array2 = array1;

- We just get two references to the same array

array1

index	value
0	23
1	15
2	99
3	55
4	2
5	14

array2

Copying Arrays

- **This does not work!**

~~array2 = array1;~~

- Never do this!
- Worst of all, it does not cause an error, so remember it!!!
- Arrays must be dealt with on an element by element basis

Copying Arrays

- If you do

```
array2 = array1;
```

- The problem is if you change the value of some slot of array1, then same slots of array2 is change too.
- In other words, you set an array two names, one is array1, and the other one is array2
- In fact, it doesn't make much sense
- It will only make your code error prone when it is executed

Copying Arrays

- You must copy all the elements **one by one**
- How about…

```
array2[0] = array1[0];
array2[1] = array1[1];
array2[2] = array1[2];
array2[3] = array1[3];
array2[4] = array1[4];
```

- This will work, but it’s a little inefficient, isn’t it?
- We can produce the same effect using a loop

Copying Arrays

- Arrays must be of the same type...

```
double[] array1 = {9, 8, 7, 6, 5, 4, 3, 2, 1, 0};
double[] array2 = new double[10];
for(int i = 0; i < array1.length; i++){
    array2[i] = array1[i];
}
```

```
array2[ 0] = array1[ 0];
array2[ 1] = array1[ 1];
array2[ 2] = array1[ 2];
array2[ 3] = array1[ 3];
...
array2[9] = array1[9];
```

```
// use loop to copy the contents of array1 into array2?
```

Printing Arrays

- To print any array, it's just the same…

```
for(int j = 0; j < array.length; j++) {
    System.out.println(array[j]);
}
```

// use loop to print each element of an array

Arrays and Loops

THINK OF FOR LOOPS!

- Why?
- Because for loops execute for an exact number of times, no more, no less
- This is tailor made for arrays which are always of a definite size

Array Length

- If we are uncertain about the size of an array, we can use **array.length** to get it
- Because arrays are a fundamental data type, we get the length using the statement

  int length = array.length;

- Because Strings are a class, when we get the length of a String we are calling a method and must provide brackets

  int length = message.length();

Exercise

- Write a program that:
  - takes the array size as input from the user,
  - creates an **int** array of that size,
  - populates it with values, prompting the reader for each value.
  - print out populated array

  - read an input number of the array from user to delete
  - print out deleted array

Program of Exercise

```
import java.util.Scanner;
public class loop {
    public static void main(String[] args) {
        Scanner kbinput = new Scanner(System.in);
        System.out.println("Please enter array size");
        int size = kbinput.nextInt();
        int[] array = new int[size];
        for(int i = 0; i < size; i++){
            System.out.println("Enter array value" +i);
            array[i] = kbinput.nextInt();
        }
    }
}
```

// takes the array size as input from the user

Program of Exercise

```java
import java.util.Scanner;
public class loop {
    public static void main(String[] args) {
        Scanner kbinput = new Scanner(System.in);
        System.out.println("Please enter array size");
        int size = kbinput.nextInt();
        int[] array = new int[size];
        for(int i = 0; i < size; i++){
            System.out.println("Enter array value" +i);
            array[i] = kbinput.nextInt();
        }
    }
}
```

// creates an **int** array of that size

Program of Exercise

```java
import java.util.Scanner;
public class loop {
	public static void main(String[] args) {
		Scanner kbinput = new Scanner(System.in);
		System.out.println("Please enter array size");
		int size = kbinput.nextInt();
		int[] array = new int[size];
		for(int i = 0; i < size; i++){
			System.out.println("Enter array value" +i);
			array[i] = kbinput.nextInt();
		}
	}
}
```

// populates it with values, prompting the reader for each value

Nested Loops

- This code uses nested for loops to print out each name in each slot, one character at a time
  - The outer loop selects a name in a particular slot
  - The inner loop prints out each character of that name, one at a time

```
String[] names = {"Peter", "Susan", “Keith"...};
for(int i = 0; i < names.length; i++){
    for(int j = 0; j < names[i].length(); j++){
        System.out.print(names[i].charAt(j) + " ");
    }
    System.out.println();
}
```

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- Introduction to Arrays
- **2D Arrays**
- Algorithm implementation exercise: The Sieve of Eratosthenes
- Arrays: Randomization, swapping, search…
- Ordered Arrays
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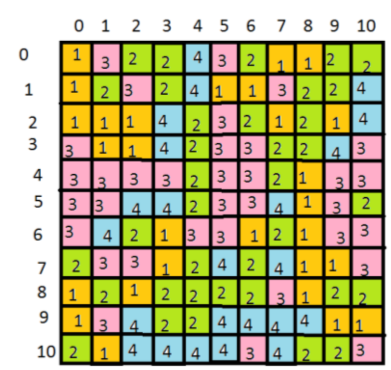
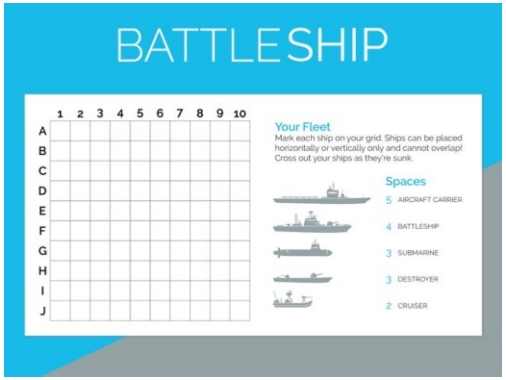
Two Dimensional (2D) Arrays

- Often data comes naturally in a two-dimensional form.
- For example, maps are two dimensional, the layout of a printed page is two dimensional, a computer-generated image (such as on your computer screen) is two dimensional, and so on.
- Think Battleships, or chess in a newspaper, or reading a map.
- It's always just rows by columns, **x by y**, whatever way you want to think of it…
- So, instead of one value to specify an array element or slot, we now need two





2D Arrays

- A single dimensional stores data as a list

$$[1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9]$$

- A two dimensional array stores data using two separate indices – like a rectangle

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

2D Arrays

```
int[][] gradeTable= new int[7][5];
```

```
gradeTable[0][1] //holds 42
gradeTable[3][4] //holds 94
gradeTable[6][2] //holds 78
```

Student	Week				
	0	1	2	3	4
0	99	42	74	83	100
1	90	91	73	88	95
2	88	61	74	89	96
3	61	89	82	98	94
4	93	73	75	78	99
5	50	65	92	87	94
6	43	98	78	56	99

2D Arrays

int**[][]** myArray = new int**[3][5]**;

- Will result in an array the same size as if we declared it as

  int[][] myArray = {{8,1,2,2,9},{1,9,4,0,3}, {0,3,0,0,7}};

  myArray[2][4] holds the value 7

  myArray[1][0] holds the value 1

- Remember, **row first**, then column

$$\begin{bmatrix} 8 & 1 & 2 & 2 & 9 \\ 1 & 9 & 4 & 0 & 3 \\ 0 & 3 & 0 & 0 & 7 \end{bmatrix}$$

Initializing 2D Arrays

- Usually, the number of **rows** and **columns** will be stored in variables
- Sometimes you will want to fill an array with default values
- Sometimes you will want to search through the whole array for a particular value
- It is common to use two nested loops when filling or searching a two-dimensional array:

```
for (int i = 0; i < rows; i++) {
    for (int j = 0; j < columns; j++) {
        board[i][j] = " ";
    }
}
```

Initializing 2D Arrays

- Lets say rows = 3 and columns = 3. Then this happens:

  board[0][0] = “ ”;
  board[0][1] = “ ”;
  board[0][2] = “ ”;
  board[1][0] = “ ”;
  board[1][1] = “ ”;
  board[1][2] = “ ”;
  board[2][0] = “ ”;
  board[2][1] = “ ”;
  board[2][2] = “ ”;

```
for (int i = 0; i < rows; i++) {
    for (int j = 0; j < columns; j++) {
        board[i][j] = " ";
    }
}
```

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The Sieve of Eratosthenes

- The Sieve of Eratosthenes is a famous method for obtaining prime numbers
- Eratosthenes was a famous Greek mathematician (276 BC – 194 BC) born in Libya
- A **prime integer** is any integer that is only divisible by itself and 1

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

- There is no simple way to predict which numbers are going to be prime without testing them using an algorithm

The Algorithm

- First step: create a boolean array with a size which corresponds to the range of numbers you want to check:

boolean[] sieve = new boolean[**12**];

- Put **all of the values equal to true** from 2 onwards

0	1	2	3	4	5	6	7	8	9	10	11
false	false	true	true	true	true	true	true	true	true	true	true

```
for(int i = 2; i < 12; i++){
    myArray[i] = true;
}
```

The Algorithm

- Starting with the element in slot 2, check if the value in that slot is true – if not skip it and go onto the next number
- If so, it is a prime number – print it out...

0	1	2	3	4	5	6	7	8	9	10	11
false	false	true	true	false	true	false	true	false	true	false	true

2 is prime

- Now loop through the remainder of the array and set to '**false**' every element whose slot number is a **multiple of that slot number**

The Algorithm

0	1	2	3	4	5	6	7	8	9	10	11
false	false	true	true	**false**	true	**false**	true	**false**	true	**false**	true

2 is prime

- Now loop through the remainder of the array and set to '**false**' every element whose slot number is a **multiple of that slot number**

0	1	2	3	4	5	6	7	8	9	10	11
false	false	true	true	**false**	true	**false**	true	**false**	**false**	**false**	true

3 is prime

The Algorithm

- For example, for the element in slot 2, all elements beyond 2 in the array that are multiples of 2 will be set to '**false**' (e.g. slot numbers 4, 6, 8, 10 etc.)
- For slot number 3, all elements beyond 3 in the array that are multiples of 3 will be set to '**false**' (e.g. slot numbers 6, 9, 12, 15 etc.)
- When you are finished, any slot which still contains '**true**' must be a prime number

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
false	false	true	true	false	true	false	true	false	false	false	true

Prime numbers

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Exercise

- Write a program that:
  - print out prime numbers
  - less than 100
  - using the Sieve of Eratosthenes method

Program of Exercise

```
boolean[] sieve = new boolean[100];
for(int i = 2; i < 100; i++) {
    myArray[i] = true;
}
for(int i = 2; i < 100; i++) {
    if(myArray[i] == true) {
        int temp = i;
        for(int j = 2i; j < 100; j = j + i) {
            myArray[j] = false;
        }
    }
}
```

0	1	2	3	4	5	6	7	8	9	10	11
false	false	true	true	false	true	false	true	false	true	false	true

2 is prime

Algorithm Structure

- Use a nested for **loop**
- The **outer loop** loops through all the numbers from two onwards checking if they are **true** or **false**
- The **inner loop** figures out all of the multiples and sets the contents of those slot numbers to **false**
- The inner loop goes to the end of the array and goes up in jumps of the multiple

```
for(int i = 2; i < 100; i++) {
        if(myArray[i] == true) {
                int temp = i;
                for(int j = 2i; j < 100; j = j + i) {
                        myArray[j] = false;
                }
        }
}
```

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Random Numbers

- **Math.random()** provides a random number that greater than and equal to 0 and smaller than 1

Math.random(): 0 ➔ 1

```
System.out.println("Here's one random number: " + Math.random());
System.out.println("Here's another random number: " + Math.random());
```

- The random number that is generated is of type double.
- If you need an int, you have to cast it by putting (int) in front

Random Numbers

- how about a random number in the range of 0 to 99.99…?

  double number = Math.random()*100;

- how about a random **int** in the range of 0 to 99?

  int number = (**int**)(Math.random()***100**);

- how about a random **int** in the range of **10** to 99?

  int number = (**int**)((Math.random()***90**)**+10**);

Exercise

- Write a program that:
  - Fill an integer array with length 100
  - using random numbers from 20 to 99

Fill an Array with Random Numbers

```
int[] randArray = new int [100];
for(int i = 0; i < randArray.length; i++) {
    randArray[i] = (int)(Math.random()*80.0)+20;
}
//Loops through 100 times and fills it in!
```

Randomize an Array

- Say we want to mix up all the elements in an array
- How can we swap all of the elements around in a random order? e.g. shuffling a deck of cards

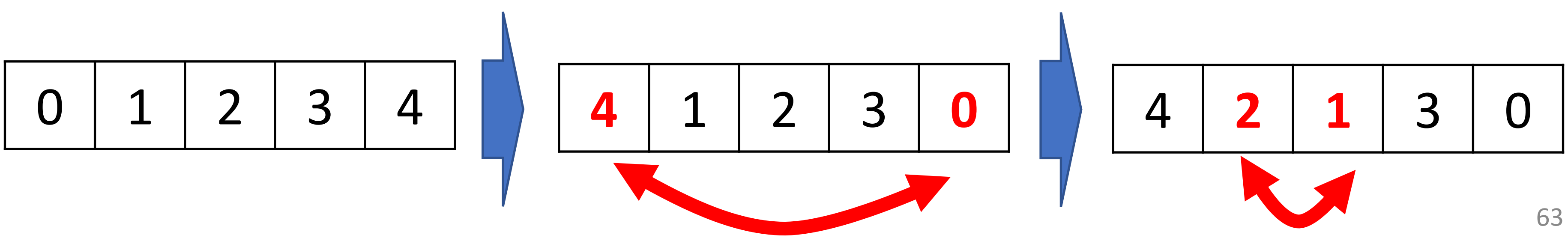
0	1	2	3	4

- Java lets us select a random number in a range
- **Math.random( )** returns a random floating point number between 0 and 1
- We can cast it into an **int** and then multiply by 5 to give us a number between 0 and 4

```
int randomNumber = (int)(Math.random()*5);
```

Random Swaps

- A good way to randomize the array is actually to go through each item and swap it with another random item
- Advantages:
    - We don't need to create another array and waste space
    - We only need to make one copy of a variable at a time in order to swap it
    - The final ordering will be completely random
- Disadvantages:
    - Some numbers may be swapped multiple times



Swapping

- Lets swap slot 1 with slot 4
  - Step 1. Backup slot 1 into temp

i.e., set temp = arr[1]

0	1	2	3	4

temp = 1

  - Step 2. Copy slot 4 into slot 1

0	4	2	3	4

temp = 1

  - Step 3. Copy temp into slot 4

0	4	2	3	1

temp = 1

Swapping

- In order to swap one variable with another in an array
  - Back-up variable #1 (the one that will be overwritten first) into a temporary variable
  - Overwrite variable #1 with the value of variable #2
  - Use the temporary variable to overwrite the value of variable #2 with the original value of variable #1

```
int temp = array[ i ];
array[ i ] = array[ random ];
array[ random ] = temp;
```

Array Linear Search

- To find an item in an array, start at the beginning and check every item

```
public int search (int searchKey) {
    for(int j = 0; j < array.length; j++) {
        if(array[j] == searchKey) {
            return j;
        }                    // searchKey is in array[]
    }
    return -1;           // searchKey is NOT in array[]
}
```

Counting matches

- Count the number of items with a searchKey greater than a specified threshold

```
public int countMatches (int threshold) {
    int count = 0;
    for(j = 0; j < array.length; j++) {
        if(array[j] > threshold){
            count++;
        }           // greater than a specified threshold
    }
    return count;
}
```

Finding the Maximum or Minimum

- Algorithm:
  - **Step1.** Initialize a candidate with the starting element
  - **Step2.** Compare candidate with remaining elements
  - **Step3.** Update it if you find a larger or smaller value

Find Biggest

- Find the biggest value in the array
- Go through every element and track biggest found so far

```
public int findMax() {
    int biggestSoFar = array[0];
    for(int j = 1; j < array.length; j++){
        if(array[j] > biggestSoFar){
            biggestSoFar = array[j];
        }
    }
    return biggestSoFar;
}
```

Find Biggest

- Find the biggest value in the array
- Go through every element and track biggest found so far

```
public int findMax() {
    int biggestSoFar = array[0];
    for(int j = 1; j < array.length; j++){
        if(array[j] > biggestSoFar){
            biggestSoFar = array[j];
        }
    }
    return biggestSoFar;
}
```

Step1. Initialize a candidate with the starting element

Step2. Compare candidate with remaining elements

Step3. Update it if you find a larger value

Inserting into an Array

- Arrays have fixed size and will usually not be filled to capacity
- Some slots will be filled whereas others will be empty

$$[4\ 6\ 2\ 7\ 9\ 8\ \_\ \_\ \_\ \_\ \_]$$

- When a new element is added, it makes sense to add it to the next available **free slot**

Inserting into an Array

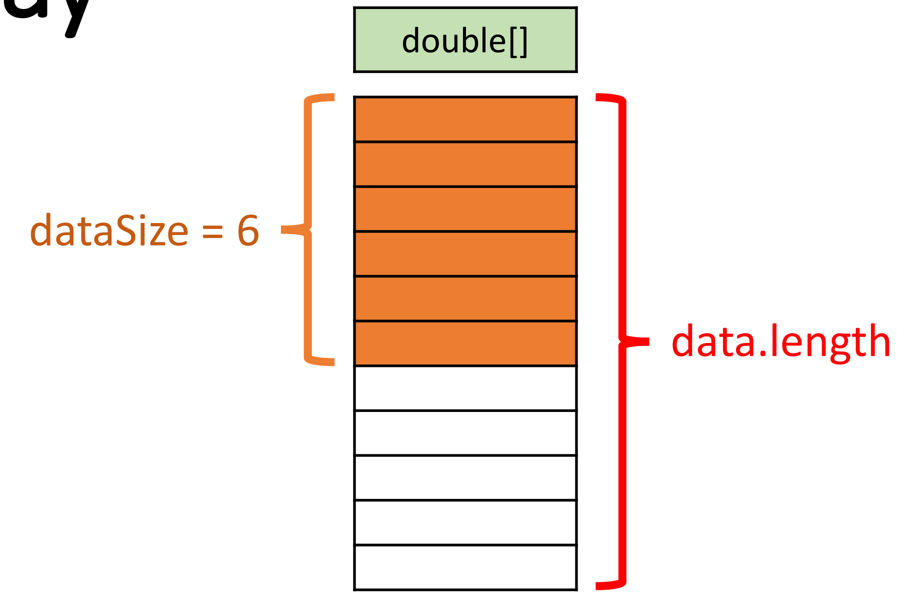
- If we know how many elements are in the array then we know what the next available slot number is
- We use a variable to track how many elements are currently in the array
- For example, if dataSize = 6 this means there are six elements in the array and the next available slot will be the seventh slot, namely data[6]

```
final int LENGTH = 100;
double[] data = new double[LENGTH];
int dataSize = 0;
```

Here, we use dataSize to record which slot is free now

Inserting into an Array

- **A Partially Filled Array**



- Next element inserted goes in slot [dataSize]
- Update dataSize as array is filled:

```
public void insert (int value){
    data[dataSize] = value;
    dataSize++;
}
```

Growing an Array

- If the array is full and you need more space, you can grow the array:
  - **Step 1.** Create a new, larger array

    ```
    double[] newData = new double[2 * data.length];
    ```
  - **Step 2.** Copy all elements into the new array

    ```
    for(int i = 0; i < data.length; i++) {
        newData[i] = data[i];
    }
    ```
  - **Step 3.** Change the reference to point to the new array

    ```
    data = newData;
    ```

Growing an Array

Problems with Arrays

You need to keep track of what slot in the array is the next free one

You need to write special code to search and delete a particular element

Every time you want to find an item, you have to check **EVERY** item

Every time you want to delete an item you have to check **EVERY** item

As the array gets bigger and bigger it will take longer and longer to find what you want

Imagine looking for a word in a dictionary and having to check every word!

Outline

- Introduction to Arrays
- 2D Arrays
- Algorithm implementation exercise: The Sieve of Eratosthenes
- Arrays: Randomization, swapping, search…
- **Ordered Arrays**
- Algorithm implementation exercise

Ordered Arrays

- When you have to check every item this is known as ***linear search***
- We notice that dictionaries and telephone directories are ***ordered***
- This makes it easier to find stuff we're looking for
- If information is ordered then you can use a ***binary search***

Ordered Arrays

- If an array is in order and we want to search for a particular entry then we just play the guessing game
- We try the **middle element** first (like guessing **50** for a number between **1** and **100**)

- If the middle element is smaller than the one we're looking for, we know that the element must be in the second half
- If the middle element is bigger then the one we're searching for must be in the first half

Ordered Arrays

- **Computer:** Guess my number between 1 and 100
- **You:** 50
- **Computer:** Too low!
- **You:** Aha, the number must be between 50 and 100. I guess 75
- **Computer:** Too high!
- **You:** Aha, the number must be between 50 and 75. I guess 63 which is in the middle again.
- **Computer:** Too low!
- **You:** Now I’m getting close. The number must be between 63 and 75. How about 69?
- **Computer:** Too high!
- **You:** 66
- **Computer:** You got the correct answer! There were 100 numbers but you guessed in only 5 guesses

What do we need?

- We keep dividing our search space and therefore need to keep track of the limits
  - Upper bound
  - Lower bound
- Example.
  - Our search space is between 15 and 49.
    - Upper bound: 49
    - Lower bound: 15
  - Then we try the **middle element** **27 = (15+49)/2**
  - If the number is bigger than 27 then 27 is the new lower bound
  - That is the new search space is between **27** and 49.
    - Upper bound: 49
    - Lower bound: **27**

Code

- In the following code we use the following variables:
  - **searchKey** is the number we're looking for
  - **nElems** is the number of elements in the array (it might not be full)
  - **lowerBound** and **upperBound** are used to track the range of our search
    - Here, we set **lowerBound** and **upperBound** being the slots which store the value of the lower bounded and the value of the upper bound.
  - **check** is used to store the slot number we are currently checking
  - **myArray** is the array we're searching through

Code

- **searchKey** is the number we're looking for
- **nElems** is the number of elements in the array (it might not be full)
- **lowerBound** and **upperBound** are used to track the range of our search
- **check** is used to store the slot number we are currently checking
- **myArray** is the array we're searching through

lowerBound upperBound

Slot	**9**	**10**	**11**	**…**	**18**	**19**	**20**	**…**	**29**
Data	11	13	15	…	43	51	62	…	99

check = (lowerBound + upperBound) / 2

Code

```
public int find(int searchKey) {
    int lowerBound = 0;
    int upperBound = nElems-1;
    int check;
    while(true) {
        check = (lowerBound + upperBound ) / 2;
        if(myArray[check] == searchKey) { return check; } // found it
        else if(lowerBound > upperBound) { return -1; } //can't find it
        else{ // divide range
            if(myArray[check] < searchKey) { lowerBound = check+1;}
            // it's in upper half
            else{ upperBound = check - 1; } // it's in lower half
        } // end else divide range
    } // end while
} // end find()
```

Keeping things ordered

- In order to be able to run a binary search like this, the array we're working with has to be sorted
- Now we need new algorithms to keep our array sorted

- Whenever a new number is inserted, it has to be inserted into the correct place
- Whenever a number is removed, the gap it leaves behind has to be filled

Inserting an element

- We need to insert an element according to its order
- This means we will have to move all the other elements up to make room

$$[2\ 4\ 6\ 7\ 8\ 9\ \_\ \_\ \_\ \_\ \_]$$

- Say we want to insert the number **5**
- This should go in the third slot (between 4 and 6)
- We need an algorithm that is going to shuffle all the elements from slot 2 onwards one space to leave a gap

Inserting an element

- Make a gap in the array by shifting everything up

$$[2\ \ 4\ \ 6\ \ 7\ \ 8\ \ 9\ \ \_\ \ \_\ \ \_\ \ \_\ \ \_]$$

```
//lets make space to insert something into slot 2
for(int j=dataSize;j>2;j--) {
    data[j]=data[j-1];
}
```

$$[2\ \ 4\ \ \_\ \ 6\ \ 7\ \ 8\ \ 9\ \ \_\ \ \_\ \ \_\ \ \_\ \ \_]$$

Full insertion method

```
public void insert(int value) {
    int j = 0;
    while(array[j] < value && j < nElems) { //find where it goes
        j++; //linear search
    }
    for(int k = nElems; k > j; k--) { // move bigger ones up
        a[k] = a[k-1];
    }
    a[j] = value; // insert it
    nElems++; // increment size
} // end insert()
```

Removing an Element

- Say we want to remove a particular element in our array
- Once we delete it there will be a gap left in our array
- If we don't keep track of these gaps, then the array will just fill up with holes like a **Swiss cheese**
- We need an algorithm that will move all the elements down to fill the gap that is created after one is removed

Removing an Element

- Removing – squishes an existing element by shifting everything down

$$[2\ \ 4\ \ 6\ \ 7\ \ 8\ \ 9\ \ \_\ \ \_\ \ \_\ \ \_\ \ \_]$$

```
//delete something from slot 2
for(int j = 2; j < dataSize; j++) {
    data[j] = data[j+1];
}
```

$$[2\ \ 4\ \ {\color{red}7}\ \ {\color{red}8}\ \ {\color{red}9}\ \ {\color{red}\mathbf{9}}\ \ \_\ \ \_\ \ \_\ \ \_]$$

Evaluation of Ordered Arrays

- Search process is much shorter
  - we can run a **binary search**
- **Insertion** takes **longer** because we have to move elements up to make room rather than just sticking a new element at the end
- **Deletion** is **slow** for both ordered and unordered arrays since you have to move items down to get rid of gaps
- Ordered arrays are useful in situations where searches are frequent, and insertions are not
  - Good for a shelf of books in a library
  - Not useful for a book jumble sale

How good is binary search?

- As arrays get bigger, using a binary search becomes more important
- A linear search would take ages!

Size of Array	Comparisons Needed of Binary Search (Worse Case)	Comparisons Needed of Linear Search (Worse Case)
10	4	10
100	7	100
1,000	**10**	**1,000**
10,000	14	10,000
100,000	17	100,000
1,000,000	20	1,000,000
10,000,000	24	10,000,000
100,000,000	27	100,000,000
1,000,000,000	**30**	**1,000,000,000**

Mathematically

- The number of steps needed to perform a binary search on an array of size N is the number of times that N can be halved
- If N is 16 then 4 steps will be needed
  - **Step 1:** narrow search space down to 8 slots
  - **Step 2:** narrow search space down to 4 slots
  - **Step 3:** narrow search space down to 2 slots
  - **Step 4:** narrow search space down to 1 slot
- Each iteration of the binary search algorithm halves the search space that needs to be considered
- In other words, each extra iteration allowed doubles the range you can search through

Mathematically

- Suppose that $\boldsymbol{N = 2^k}$.
- How many steps we need in worst case?

	Input search space	Output search space
Step 1.	2^k	2^{k-1}
Step 2.	2^{k-1}	2^{k-2}
Step 3.	2^{k-2}	2^{k-3}
...	$\dots$	$\dots$
Step $\boldsymbol{k}$.	$2^{k-(k-1)} = 2^1$	$2^0 = 1$
Step $\boldsymbol{k+1}$.	$2^0 = 1$ (We find the answer.)	

Analysis

- Suppose that $N = 2^k$.
- How many steps we need in worst case?

	Input search space	Output search space
Step 1.	2^k	2^{k-1}
Step 2.	2^{k-1}	2^{k-2}
Step 3.	2^{k-2}	2^{k-3}
…	$\dots$	$\dots$
Step k.	$2^{k-(k-1)} = 2^1$	$2^0 = 1$
Step $k+1$.	$2^0 = 1$ (We find the answer.)	

- We stop at step $\boldsymbol{k}$.
- $N = 2^k$ ➔ $k = \log_2 N$
- That is we need $\log_2 N$ steps at most.

A log relationship

- Each step halves the size, so the number of iterations needed to search through an array using a binary search is the number of times the size of the array can be halved

$$\mathbf{size} = \mathbf{2}^{\mathbf{iterations}}$$

- The opposite of raising something to a power is to take its log

$$\mathbf{iterations} = \mathbf{log_2\, size}$$

- Number of steps required increases very slowly compared to increases in size – logarithmically as opposed to linearly
- We express this log type relationship between array size and number of steps required by saying that the complexity of binary search is $\boldsymbol{O(\log n)}$
- **Note. In this class, we always set $\mathbf{log}$ as $\boldsymbol{log_2}$**

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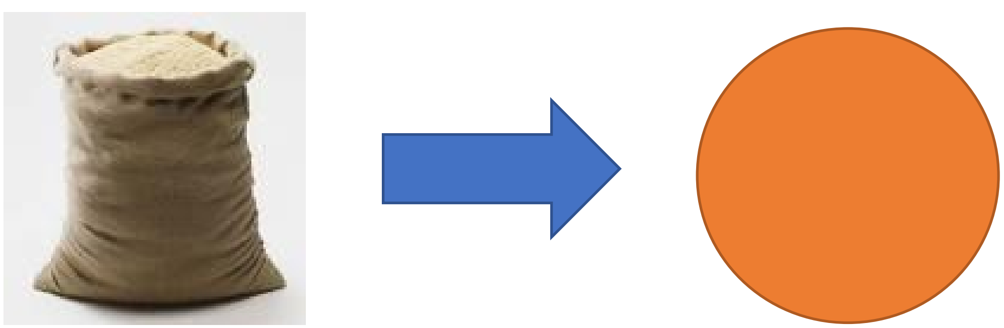
Monte Carlo method

- Often it is complicated to work out precise mathematical formulae which describe how a system works
- The lazier solution is simply to model the system and run the simulation many times **randomly**
- You base the probability on what you observe, letting the simulation do the hard work

Monte Carlo method

- Even the value for **PI** can be calculated using a Monte Carlo method
- Draw a square on the ground and inscribe a circle in it
- Scatter some grains of rice randomly throughout the square (or count rain drops falling into it etc.)
- The ratio of grains of rice in the square to grains of rice in the circle will be the ratio of their areas, or **PI/4**



Monte Carlo Example: Get PI

```
public class Test {
    public static void main(String[] args) {
        int n_sim = 10000000;
        double x, y;
        int count = 0;
        for (int i=0; i<n_sim; i++) {
            x = Math.random();
            y = Math.random();
            if (x*x+y*y <= 1) {
                count++;
            }
        }
        System.out.println(4.*count/n_sim);
    }
}
```

Check Coprime

- Euclidean Algorithm to check the greatest common divisor (GCD)

$$A \div B = Q_1 \text{ remainder } R_1$$
$$B \div R_1 = Q_2 \text{ remainder } R_2$$
$$R_1 \div R_2 = Q_3 \text{ remainder } R_3$$

- Continue this process until the remainder is 0 then stop.
- The divisor in the final step will be the greatest common factor.

Check Coprime

- Euclidean Algorithm to check the greatest common divisor (GCD)

$78 \div 66 = 1$ remainder 12 $(78 = 66 \times 1 + 12)$

$66 \div 12 = 5$ remainder 6 $(66 = 12 \times 5 + 6)$

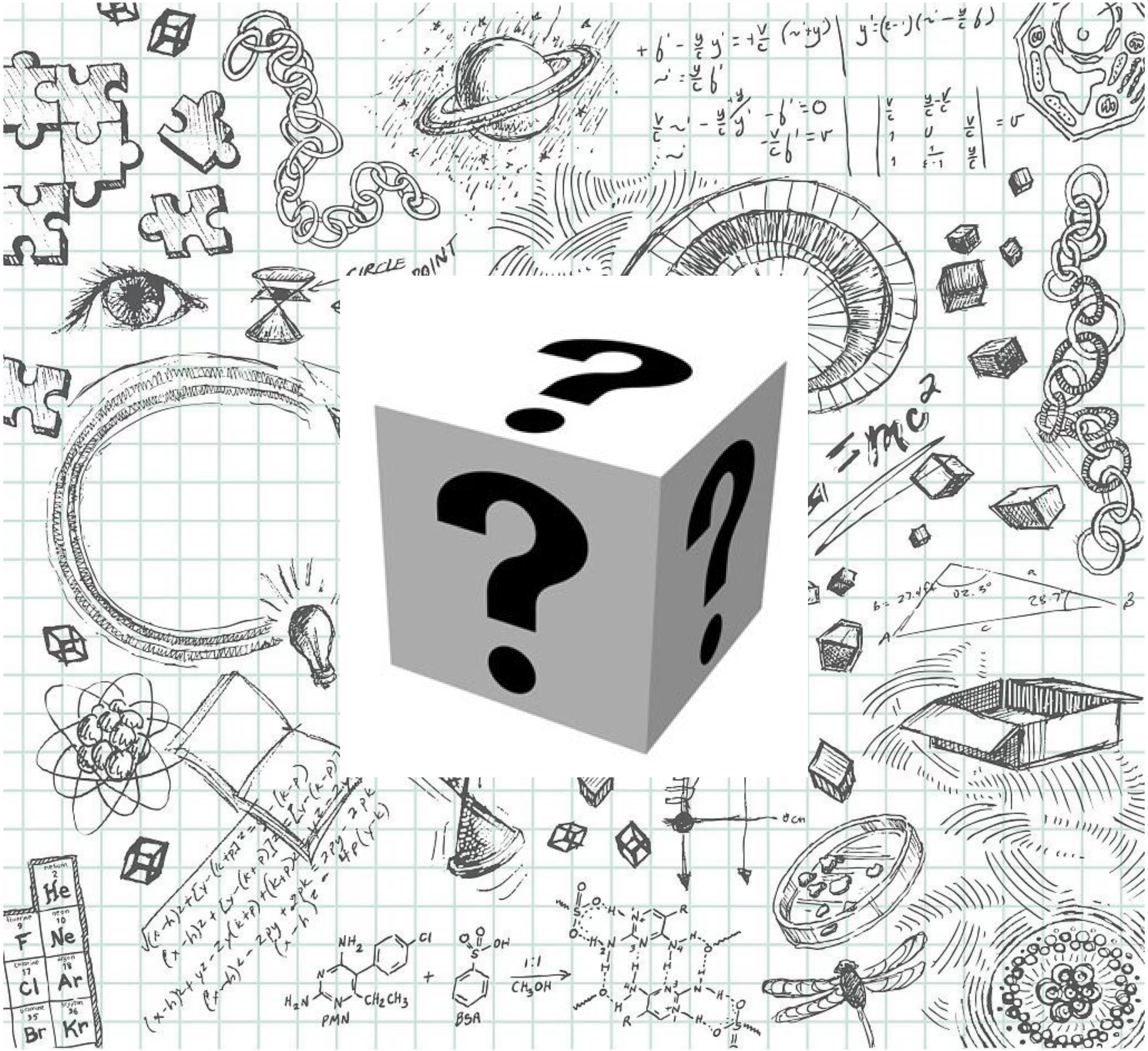
$12 \div 6 = 2$ remainder 0 $(12 = 6 \times 2 + 0)$

6 = **Greatest Common Factor**

https://www.inchcalculator.com/euclidean-algorithm-calculator/

Exercise

- Write a function to check whether two input numbers are coprime
  - Input: two numbers
  - Output: true if they are coprime, false otherwise


CIRCLE
POINT
He
F
Ne
Cl
Ar
Br
Kr
PMN
BSA