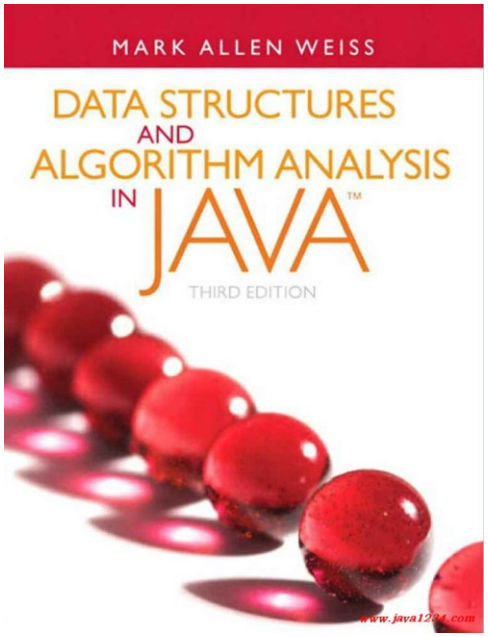
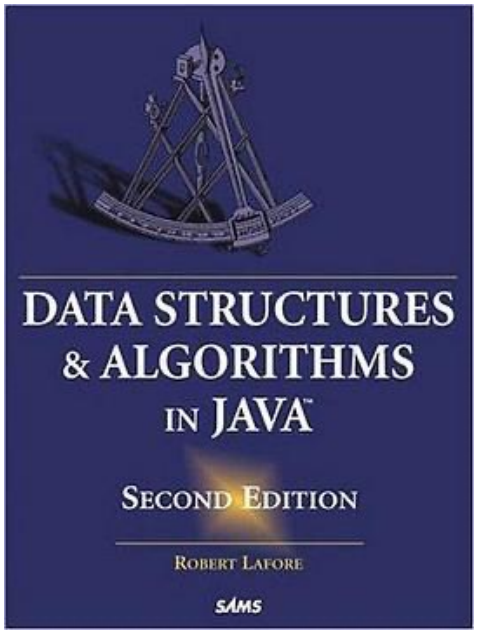
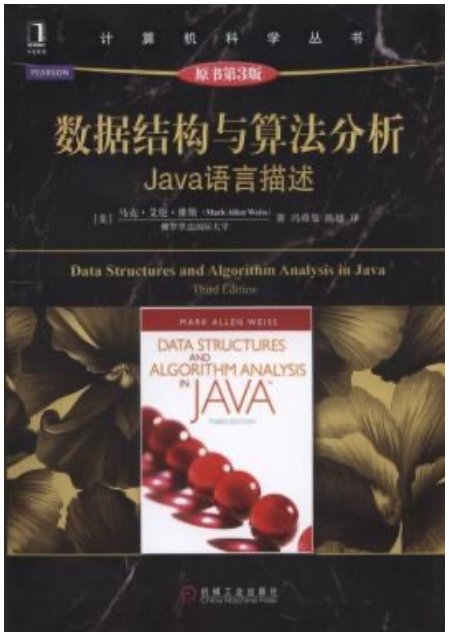
Topic 7 – Stacks and Queues







Topics

- Introduction
- Programming Revision
- Methods and Objects
- Arrays and Array Algorithms
- Big O Notation
- Sorting Algorithms
- **Stacks and Queues**
- Linked Lists
- Recursion
- Bit Manipulation

Abstract Data Types (ADTs)

- Stacks and queues are abstract data types – they are more conceptual in nature than concrete data types such as arrays

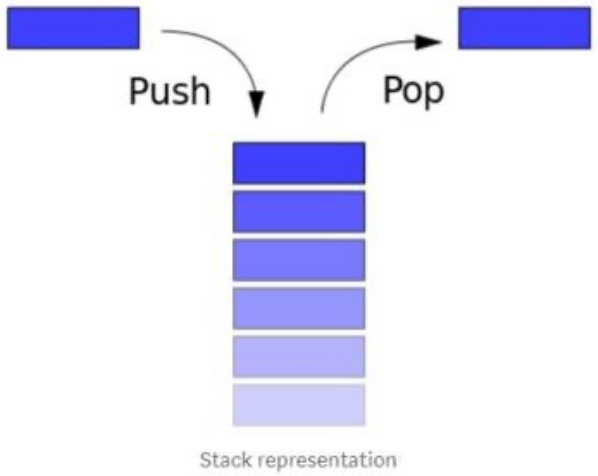
- The ideas of stacks and queues are described by their interface – we're not interested in how they're actually implemented

- Underlying mechanism is typically not visible to the user – we just want to know how to use them

Stacks

Stack Interface

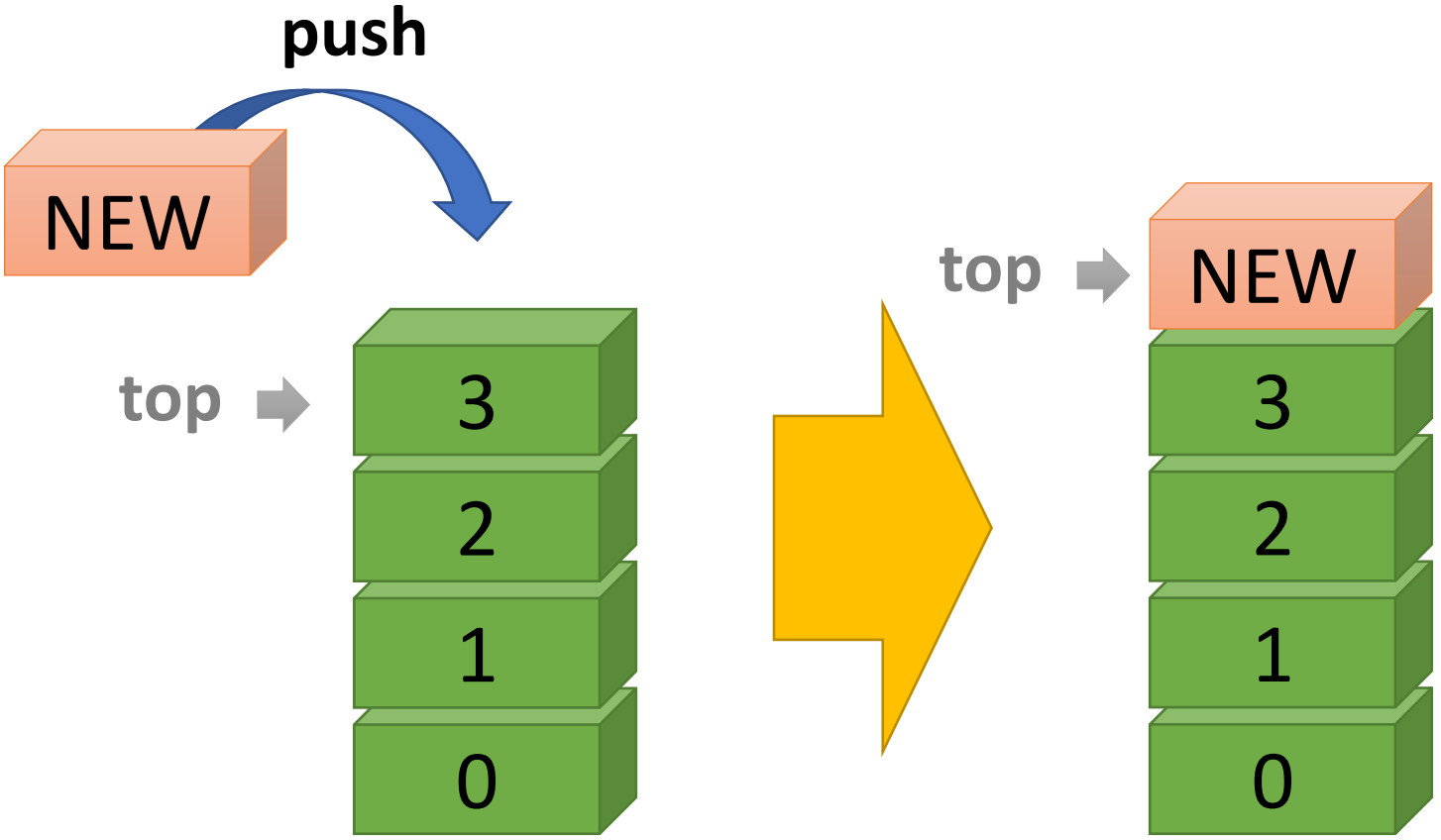
- The stack mechanism is known as **Last-In-First-Out (LIFO)** because the last item inserted is the first one removed
- Stacks are also referred to as
  - pushdown stack
  - pushdown list



- There are several things you can do with a stack
  - **pop ( )** – pop the top item off the stack
  - **push ( )** – put another item onto the top
  - **peek ( )** – look at the top item and copy it

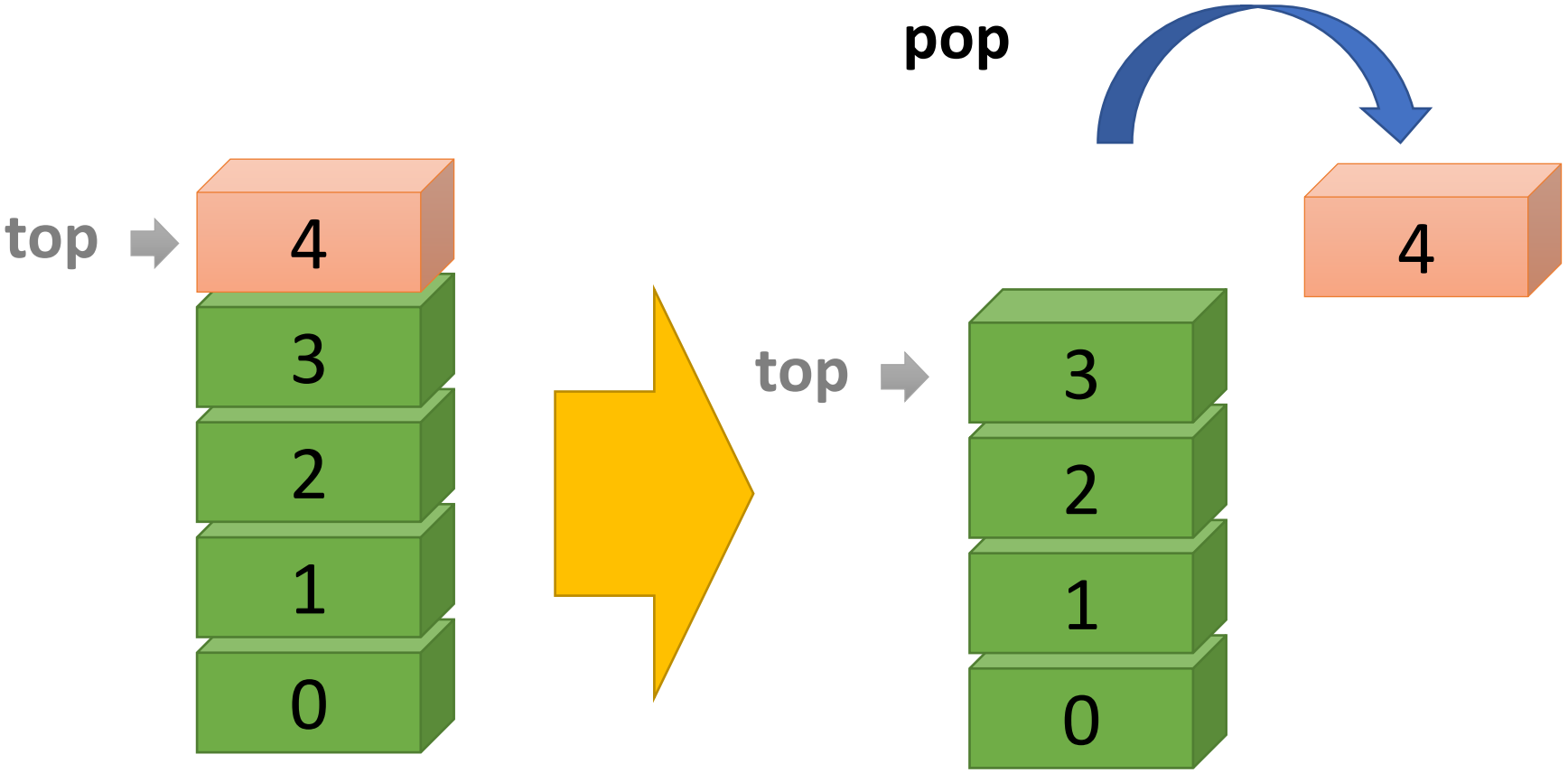
Push

- push (element)
  - **insert** an element at the top of the stack



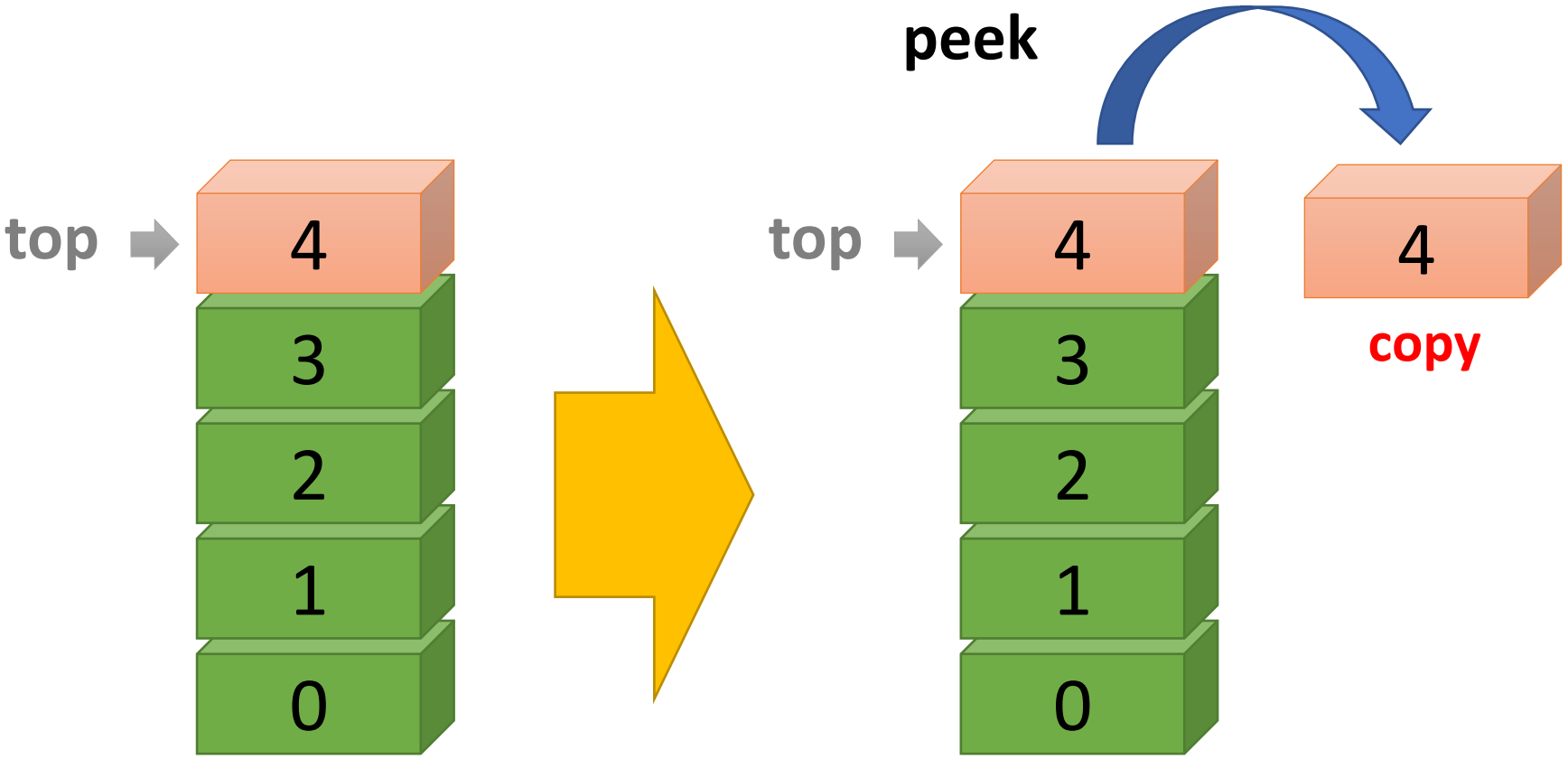
Pop

- pop ( )
  - **remove** an element from the top of the stack



Peek

- peek ( )
  - look at the top item and copy it



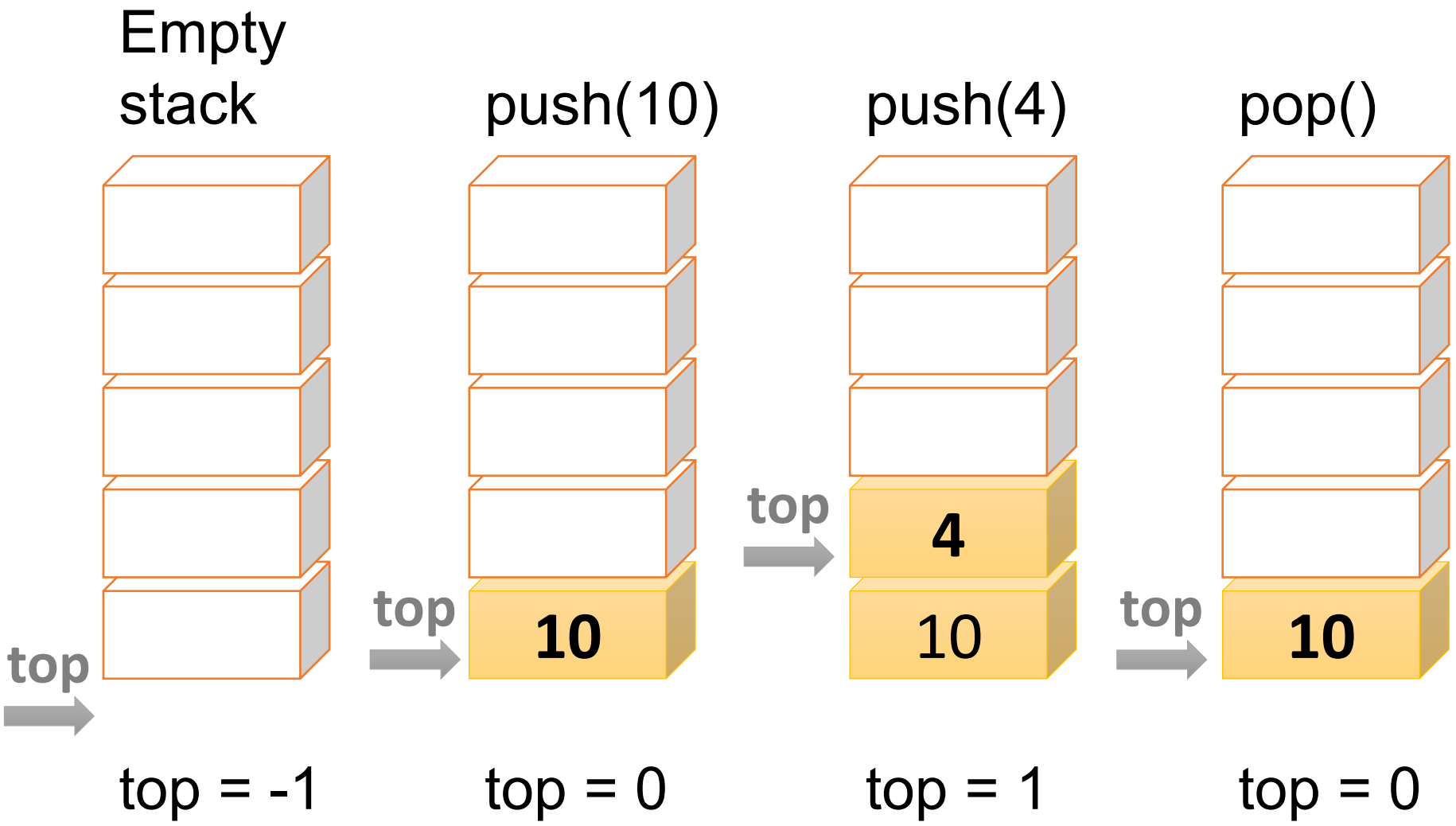
Other Stack Operations

- makeEmpty( )
  - Remove all items from the stack
- isEmpty( )
  - True if stack is empty, false otherwise
- isFull( )
  - True if stack is full, false otherwise

Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Push and Pop


Empty stack
push(10)
push(4)
pop()
top
top
10
top
4
10
top
10
top = -1
top = 0
top = 1
top = 0

Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a **FullStackException**
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Implementing a Stack

- Stack object will need several instance fields
  - ***maxSize*** to store the size of the array
  - an array of numbers ***stackArray*** to store the stack
  - a variable called ***top*** to track where the top of the stack is
- Constructor will take in size of array, initialize it and set ***top*** **to -1** since there's nothing in the stack to start with

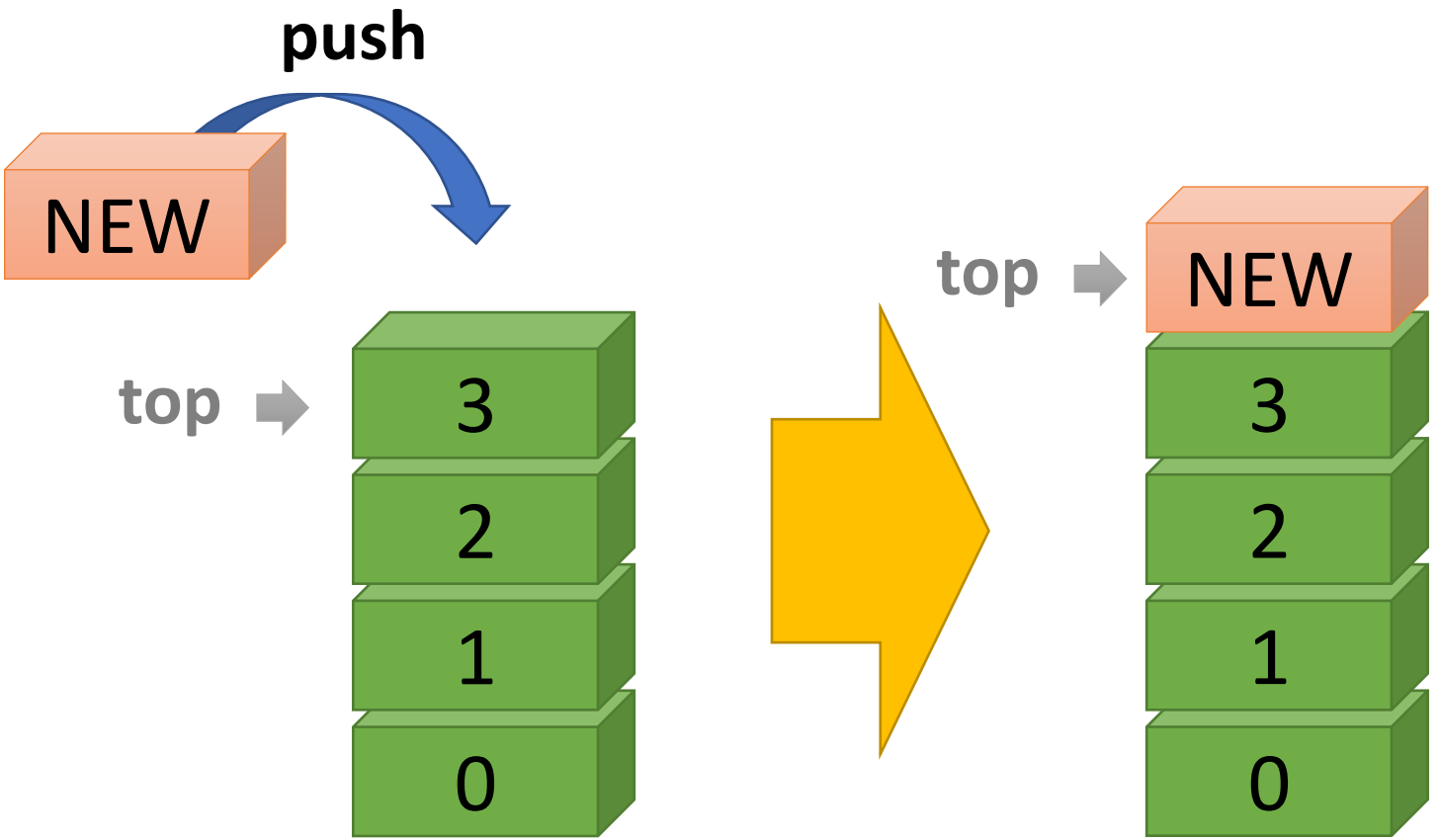
Java Implementation

```
public class Stack {
    private int maxSize;   // size of stack array
    private long[] stackArray;
    private int top;   // top of stack

    public Stack(int s) {  // constructor
        maxSize = s;   // set array size
        stackArray = new long[maxSize];   // create array
        top = -1;   // no items yet
    }
}
```

Methods

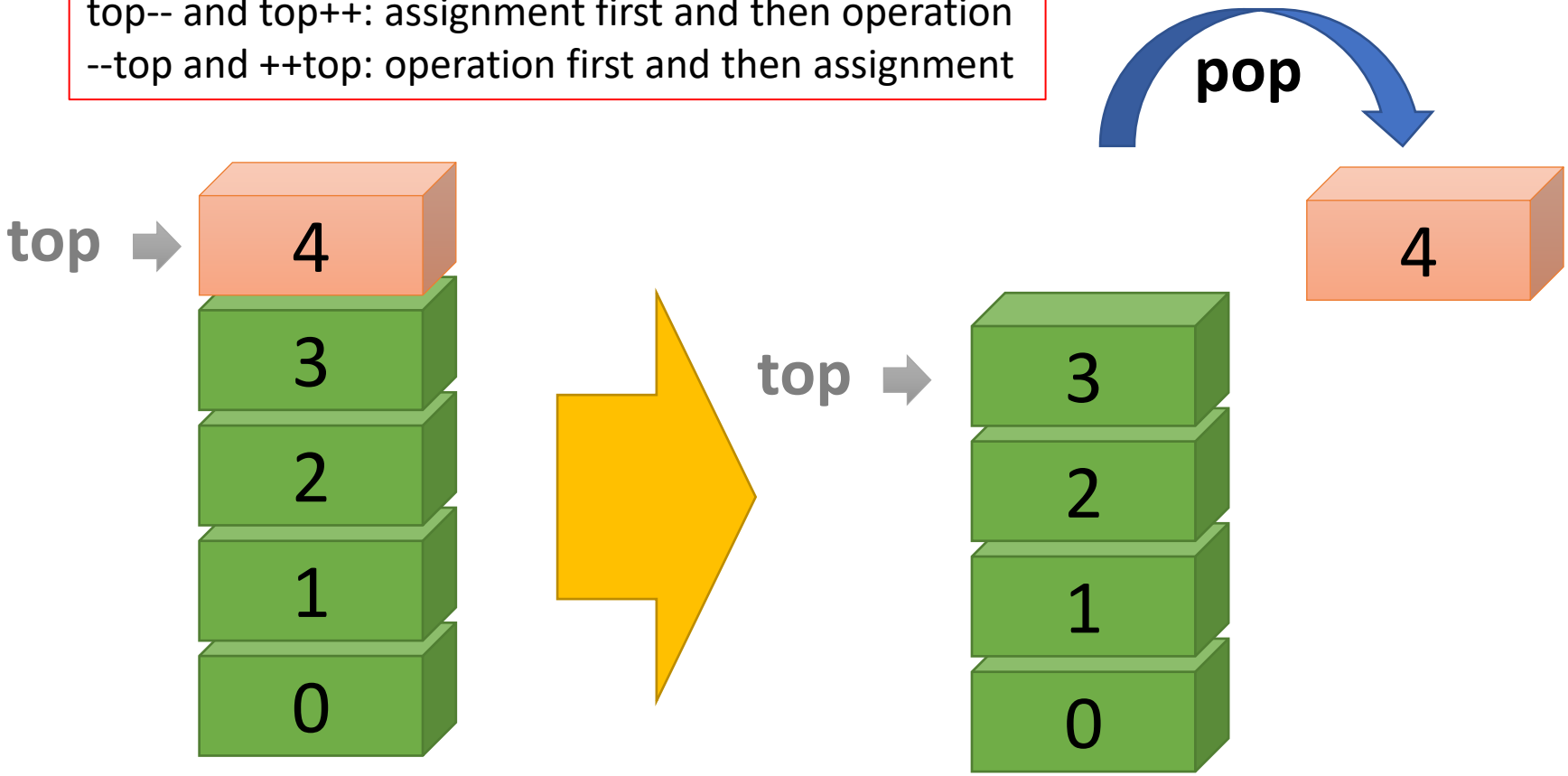
```
public void push(long j) { // put item on top of stack
    top++;
    stackArray[top] = j; // increment top, insert item
}
```



Methods

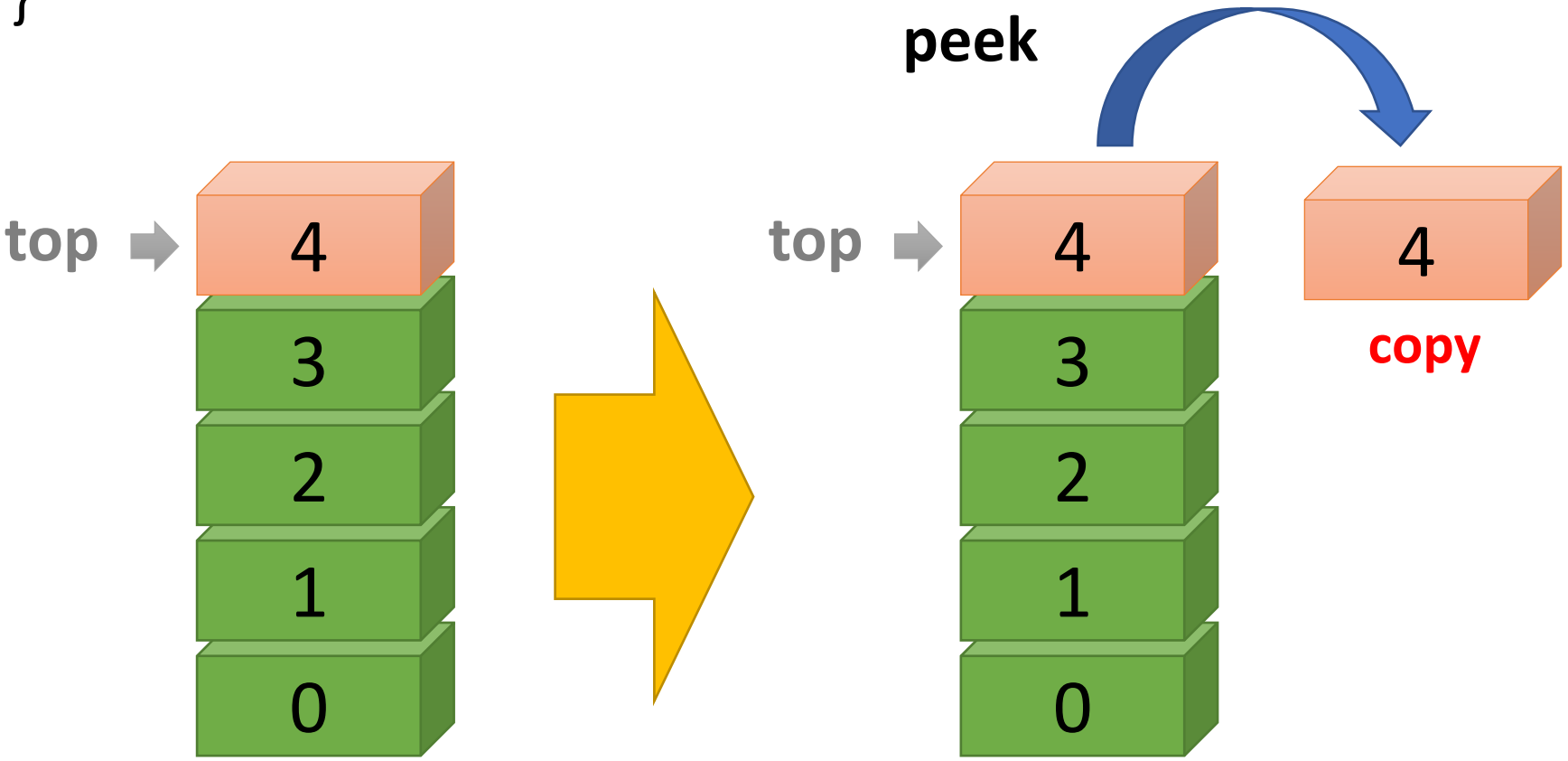
```
public long pop() {     // take item from top of stack
    return stackArray[top--]; //access item, decrement top
}
```

Note.
top-- and top++: assignment first and then operation
--top and ++top: operation first and then assignment



Methods

```
public long peek() { // peek at top of stack
    return stackArray[top];
}
```



Methods

```
public boolean isEmpty() { // true if stack is empty
    return (top == -1);
}

public boolean isFull() { // true if stack is full
    return (top == maxSize - 1);
}

public void makeEmpty() { // empty stack
    top = -1;
}
```

// We just need to reset the index of top as -1.

Result

```
Stack theStack = new Stack(10); // make new stack
// push items onto stack
theStack.push(20);
theStack.push(40);
theStack.push(60);
theStack.push(80);

while (!theStack.isEmpty()){
   System.out.println(theStack.pop());
}
```

Output would be :

```
================
80
60
40
20
```

Stack Structures

- In which of the following situations could you feasibly use a stack?
  - Storing fruit
  - Storing milk cartons
  - Storing cans
- Can you think of anything else using a stack structure?
  - Web browsers
  - Undo sequence in a text editor
  - Java Virtual Machine

Method Stack in the JVM

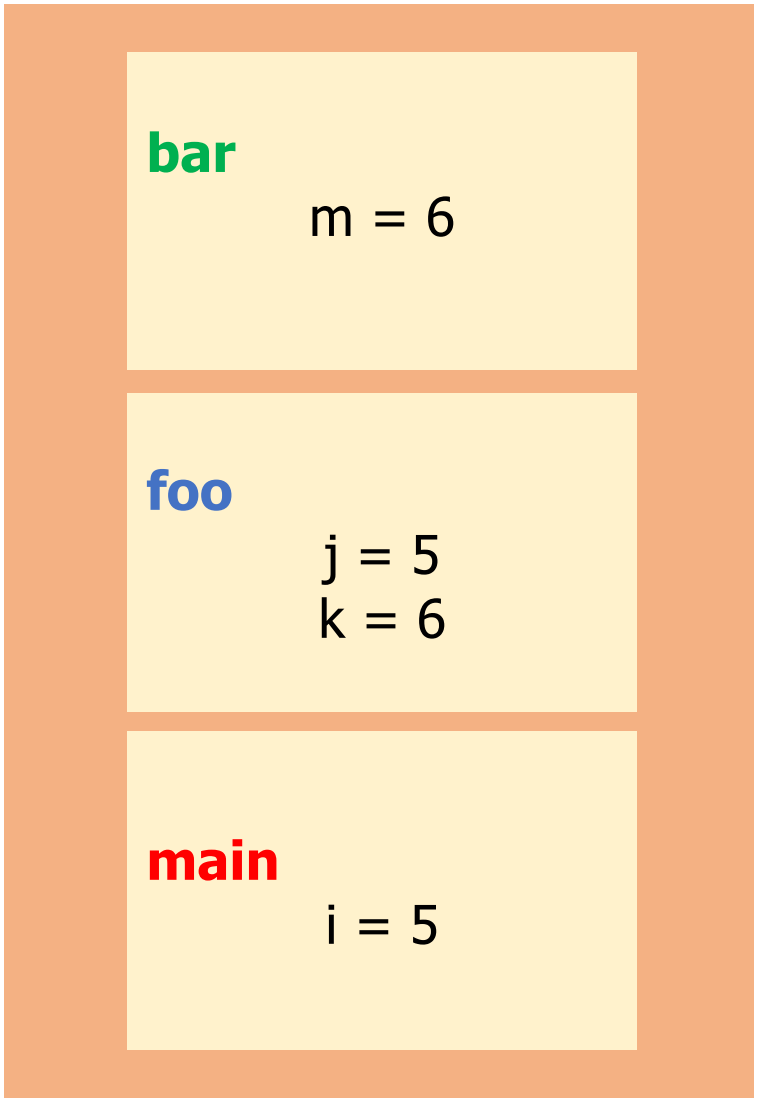
- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack

- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed

- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack

Method Stack in the JVM

```
main() {
    int i = 5;
    foo(i);
}
foo(int j) {
    int k;
    k = j+1;
    bar(k);
}
bar(int m) {
    …
}
```

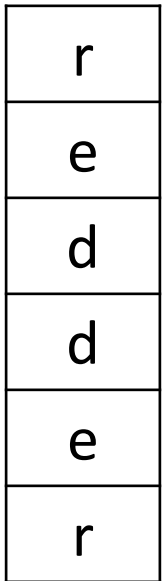


What can we use stacks for?

- The LIFO principle can be used in **reversing a word**
- If we push “h”, “e”, “l”, “l”, “o” and then pop the contents of the stack
- Output is “o”, “l”, “l”, “e”, “h”

Checking for palindromes

- The LIFO principle can be used to check if a word is a palindrome
- Say we have a word like **redder, madam, racecar**
- Push the the word onto the stack
- Pop it off the stack
- It has now been reversed – check if it is the same as the original



Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( ))(( )){([( )])})
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (
- Start from the beginning of the sequence
- Opening brackets are placed on a stack
- When the program comes across a closing bracket it pops from the ‘opening bracket stack’ and this should match
- When the program comes to the end, the ‘opening bracket stack’ must be empty

Performance and Limitations

- Performance
  - Let $\boldsymbol{n}$ be the number of elements in the stack
  - The space used is $\boldsymbol{O(n)}$
  - Each operation runs in time $\boldsymbol{O(1)}$ (e.g. pop, push)

- Limitations for array-based stacks
  - The maximum size of the stack must be defined ***a priori*** and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Exercise

- Write your Stack class, and implement methods using stack for
  - Checking for palindromes
  - Parentheses Matching

Queues

- A queue means to line up for something

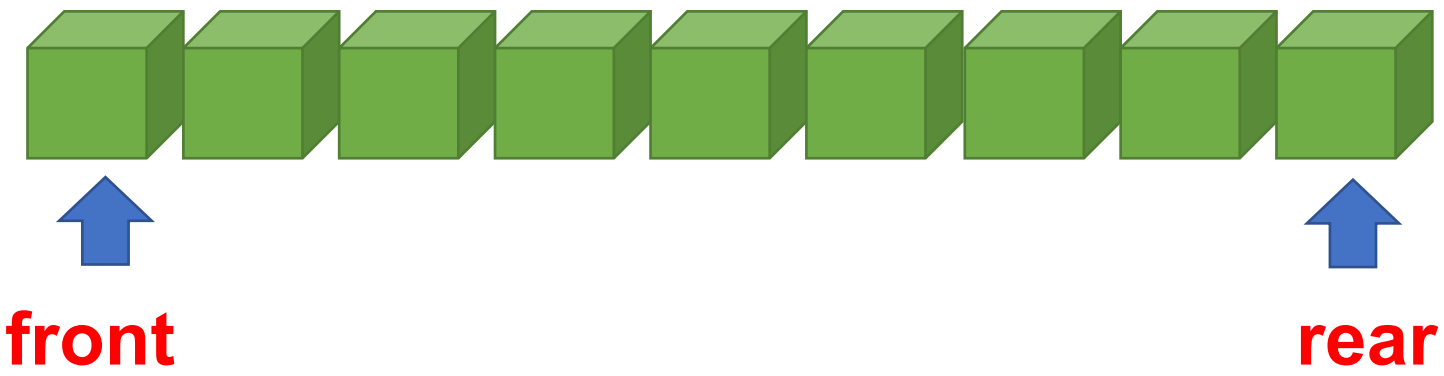
Queues

- Queues use the **First-In-First-Out system (FIFO)**
- Rather than piling items up, the one that has been in the queue the longest is the one that is popped
- Insertions are made at one end, the back of the queue
- Deletions take place at the other end, the front of the queue
- So, the last one added is always the last one available for deletion

Queues

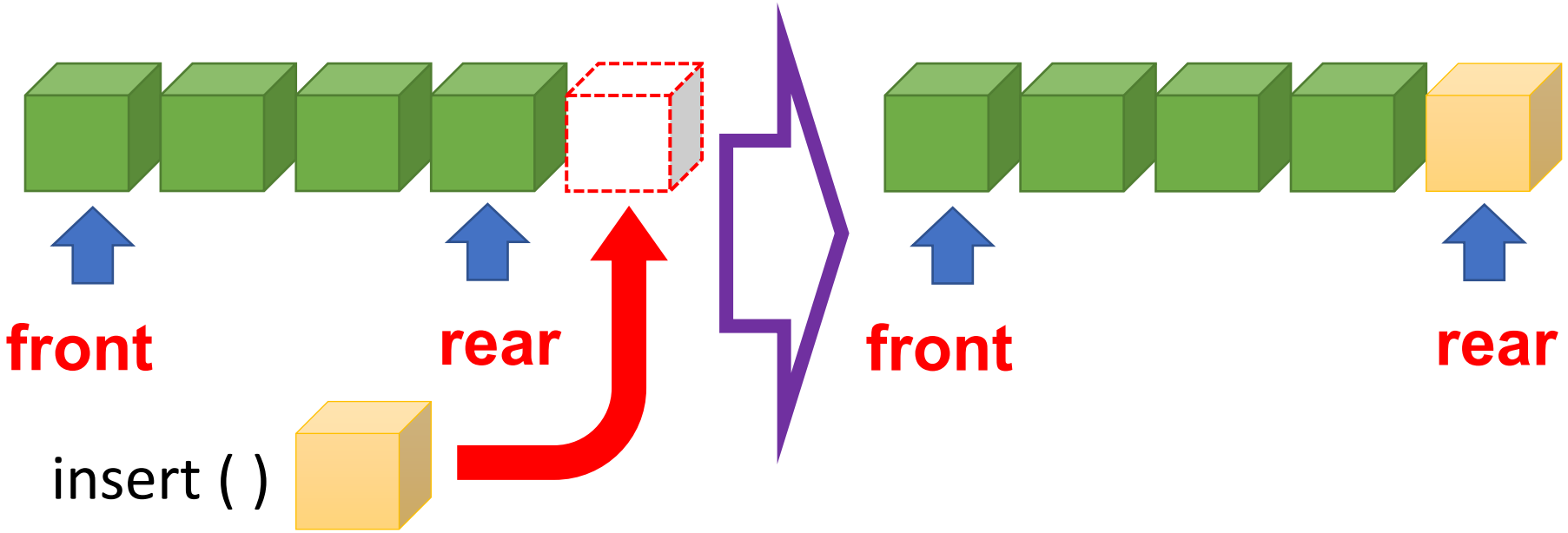
- Queues are used everywhere e.g. printer queue, multitasking
  - push( ) is called insert, put, add or enqueue!
  - pop( ) is called remove, delete, get or dequeue!

- The front and back of the queue are called the front and rear



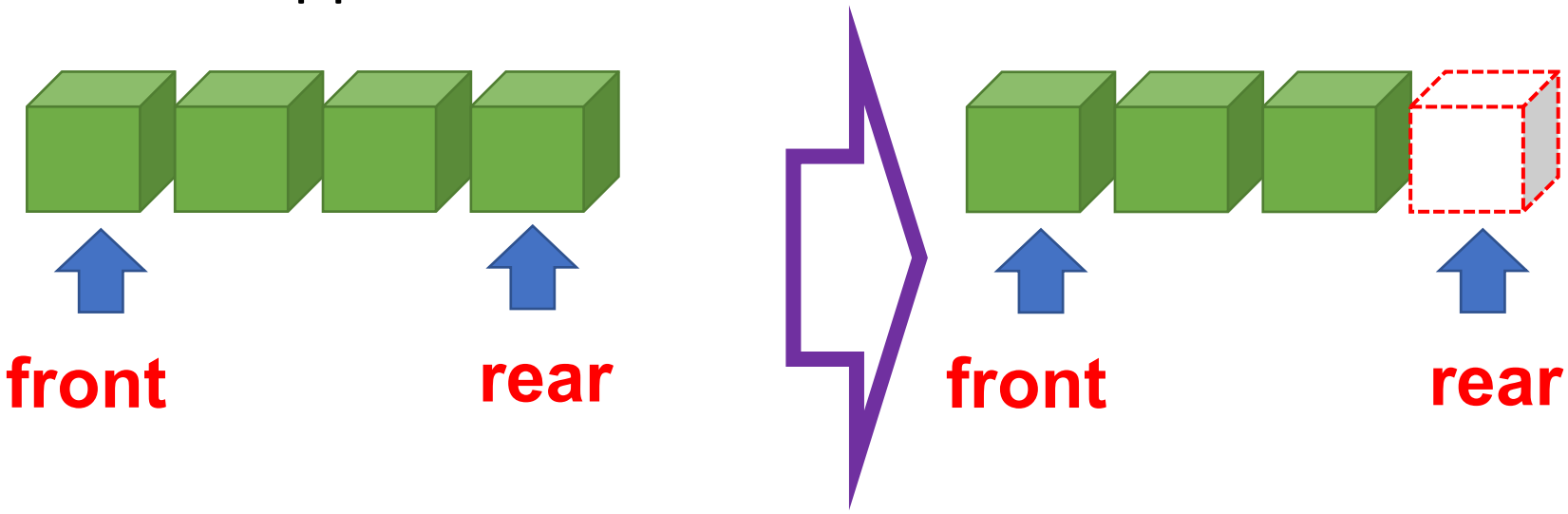
Insert

- insert ( ) method
  - Assumes the queue is not full
  - Inserts at rear
  - If rear is at the top of the array then it wraps around to the bottom of the array



Remove

- remove ( ) method
  - Assumes queue is not empty
  - Obtains the value at the front
  - Increments the front variable
  - If front goes beyond the end of the array it must be wrapped around to 0



Other Methods

- peek ( )
  - Returns the value at the front
- size ( )
  - Assumes queue not empty
  - Returns total number in queue
- isFull ( )
  - Returns true if queue is full
- isEmpty ( )
  - Returns true if queue is empty

Variables

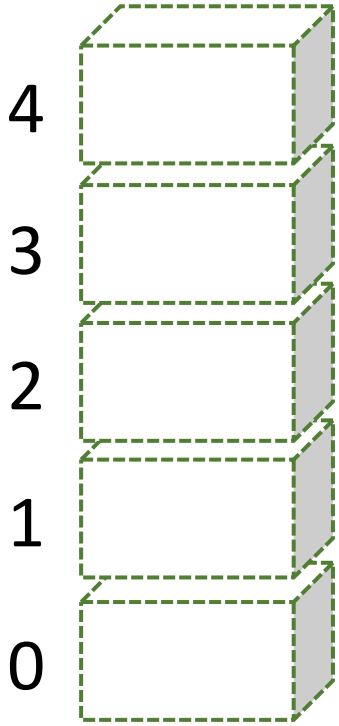
- We need to maintain some variables with our Queue class
  - Size of the array
  - The array itself
  - Variables for tracking **front** and **rear**

- Empty Queue:
  - front = 0, rear = -1

- 3 items in queue:
  - front = 0, rear = 2

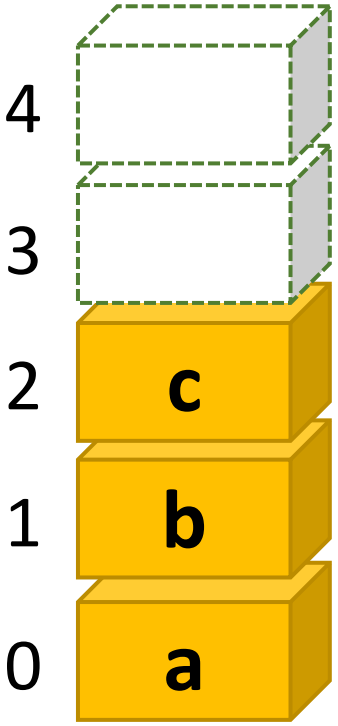
Examples

Empty Queue



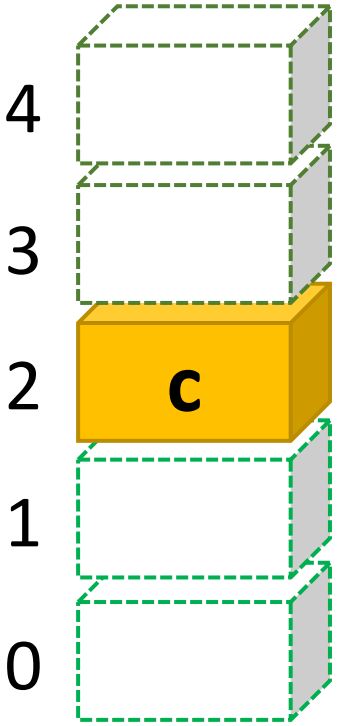
front: 0
rear: -1

3 Elements



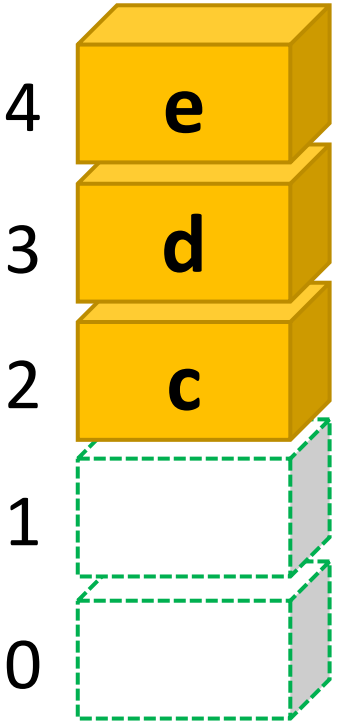
front: 0
rear: 2

Remove 2 Elements



front: 2
rear: 2

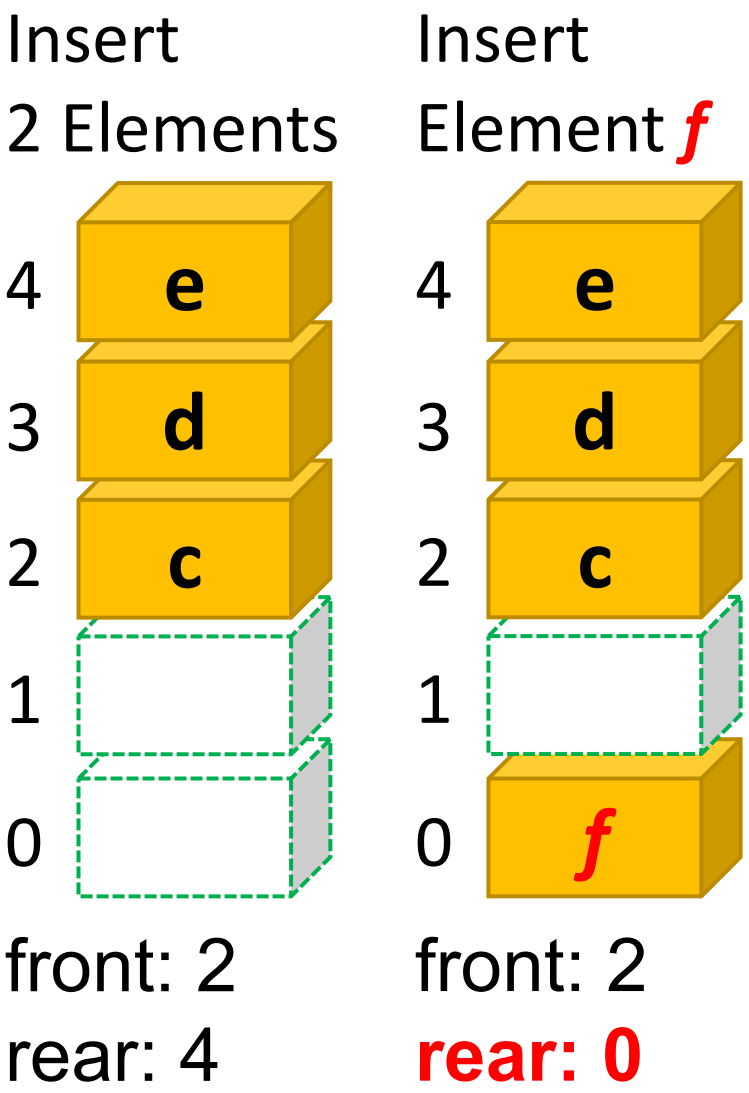
Insert 2 Elements



front: 2
rear: 4

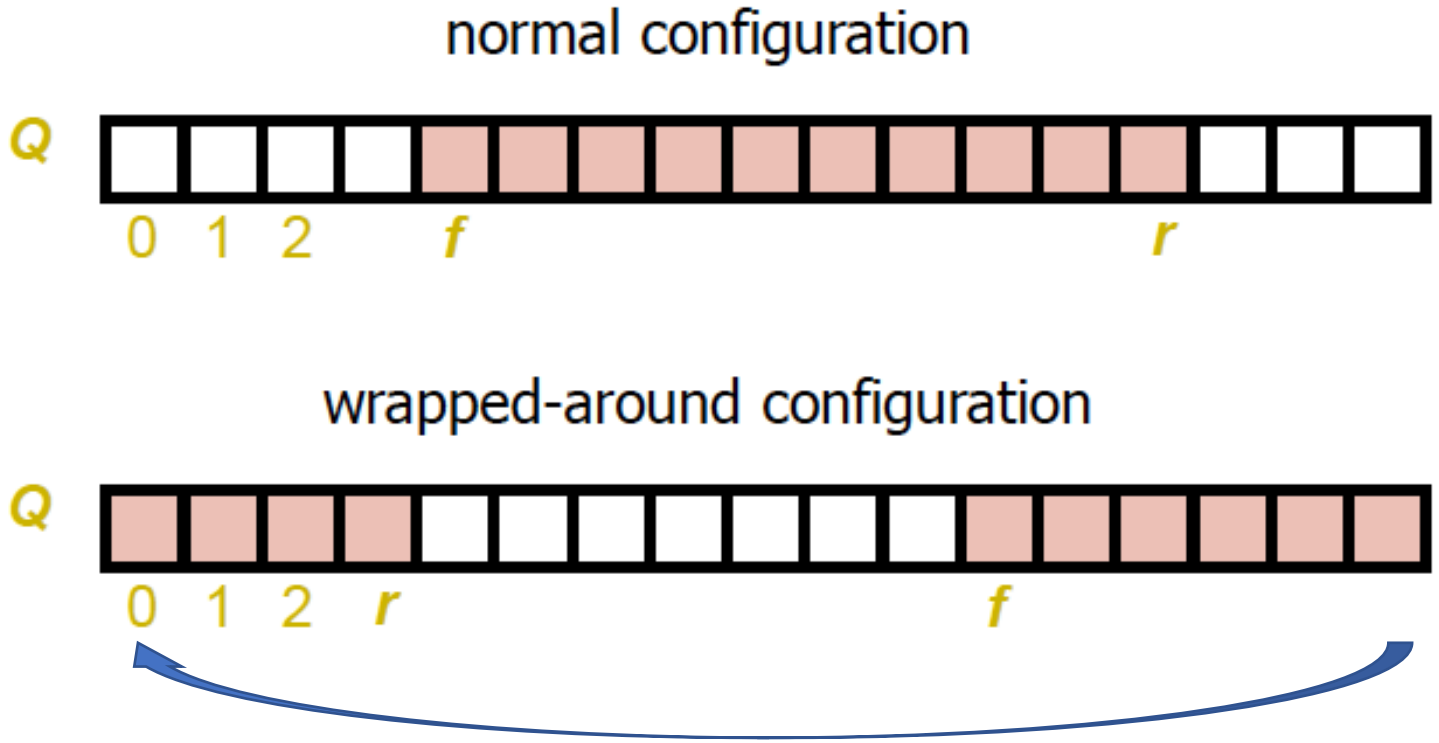
Examples

- We implement **wraparound**
  - when either back or front reach the end of the array, we reset it to the beginning.
  - To insert element ***f***, we reset back to the start of the array and place ***f*** there
    - rear = (4+1) % 5



Array-based Queue

- Use an array of size ***N*** in a **circular** fashion
- Two variables keep track of the front and rear
  - ***front*** index of the front element
  - ***rear*** index of the last element



Queue Class

```
public class Queue {
    private int maxSize;
    private long[] queArray;
    private int front;
    private int rear;
    private int nItems;
    public Queue(int s) { // constructor
        maxSize = s;
        queArray = new long[maxSize];
        front = 0;
        rear = -1;
        nItems = 0;
    }
```

Method of Insert

```
public boolean insert(long j) { // put item at rear of queue
    if(isFull())
        return false; //don't insert if full

    if(rear == maxSize-1) // deal with wraparound
        rear = -1;

    rear++;
    queArray[rear] = j; // increment rear and insert
    nItems++; // one more item
    return true; //successfully inserted
}
```

Method of Remove

```
public Long remove() { // take item from front of queue

    if(isEmpty())
        return null; //don't remove if empty

    long temp = queArray[front];// get value and incr front
    front++;

    if(front == maxSize) // deal with wraparound
        front = 0;

    nItems--; // one less item
    return temp;
}
```

Method of peekFront and check empty

```
public long peekFront() { // peek at front of queue
    return queArray[front];
}

public boolean isEmpty() { // true if queue is empty
    return (nItems==0);
}
```

Mothod of check Full and return its size

```
public boolean isFull() { // true if queue is full
    return (nItems==maxSize);
}

public int size() { // number of items in queue
    return nItems;
}
```

Queue Example

Operation	Output	front ← Q ← rear
insert(5)	–	(5)
insert(3)	–	(5, 3)
remove()	5	(3)
insert(7)	–	(3, 7)
remove()	3	(7)
peekFront()	7	(7)
remove()	7	()
remove()	“error”	()
isEmpty()	true	()
insert(9)	–	(9)
insert(7)	–	(9, 7)
size()	2	(9, 7)
insert(3)	–	(9, 7, 3)
insert(5)	–	(9, 7, 3, 5)
remove()	–	(7, 3, 5)

Performance and Limitations

- Performance
  - Let $\boldsymbol{n}$ be the number of elements in the queue
  - The space used is $\boldsymbol{O(n)}$
  - **Each** operation runs in time $\boldsymbol{O(1)}$

- Limitations for array-based queues
  - The maximum size of the queue must be defined a priori and cannot be changed
  - Trying to insert a new element into a full queue causes an implementation-specific exception

Exercise

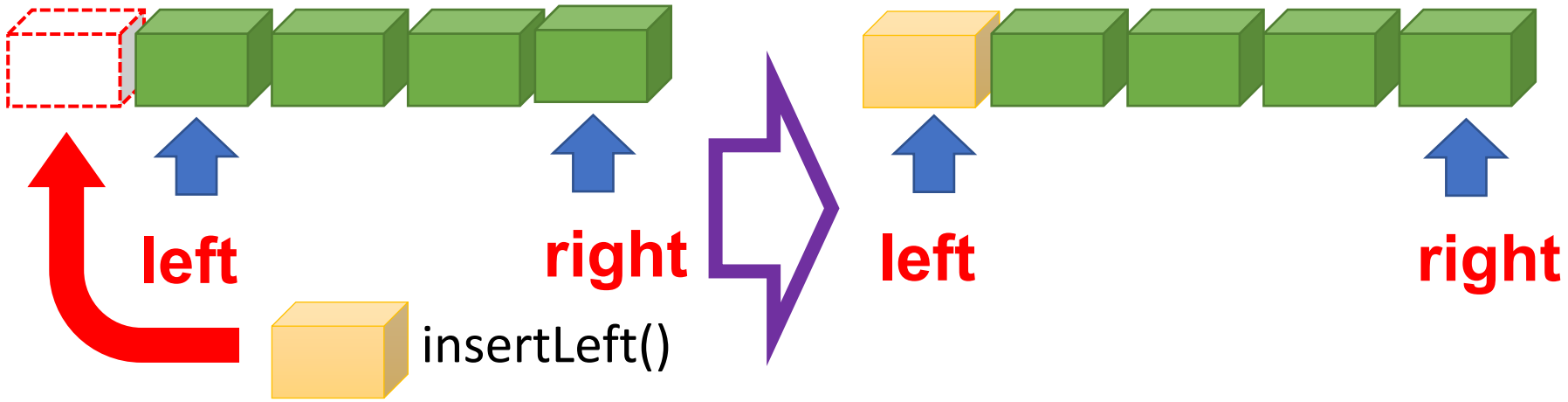
- Write your Queue class with relevant methods such as:
  - insert
  - remove
  - isEmpty
  - isFull
  - …

Deque

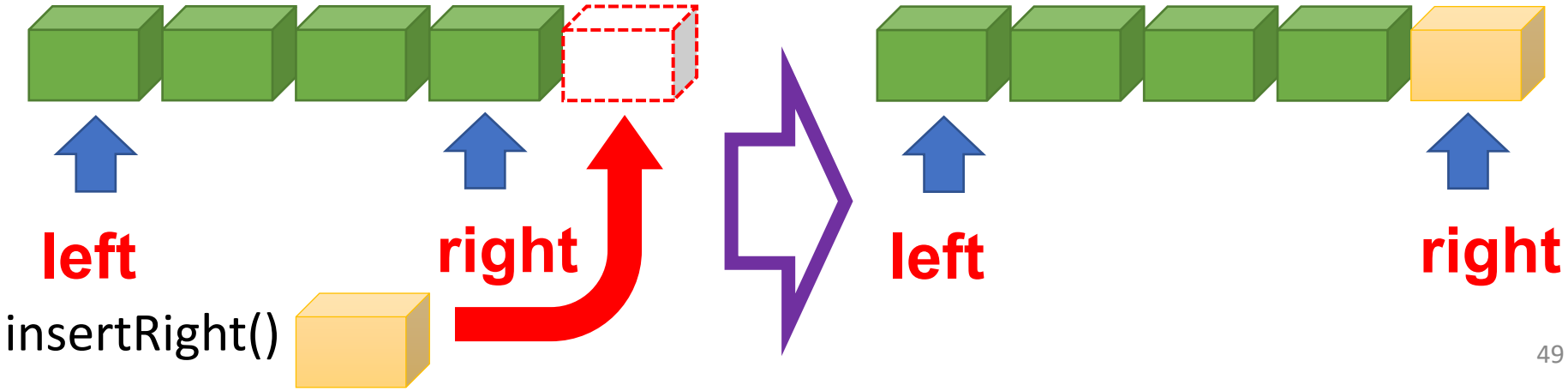
- A deque is a **double-ended queue**
- This means **you can insert items at either end and delete them at either end**
- Essentially, there is no longer a front and rear, simply two ends
- Use methods called
  - insertLeft( )
  - insertRight( )
  - removeLeft( )
  - removeRight( )

insertLeft( ) and insertRight( )

- **insertLeft( )**

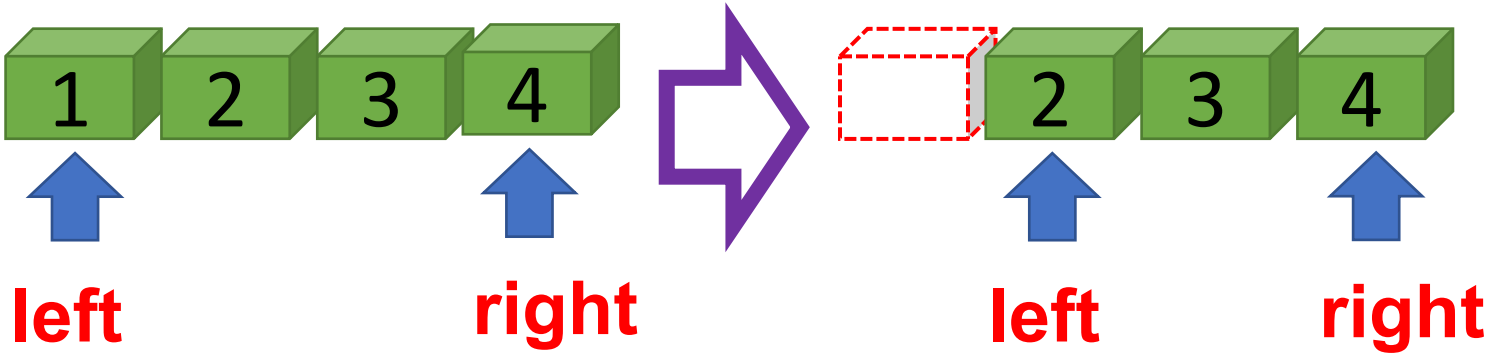


- **insertRight( )**

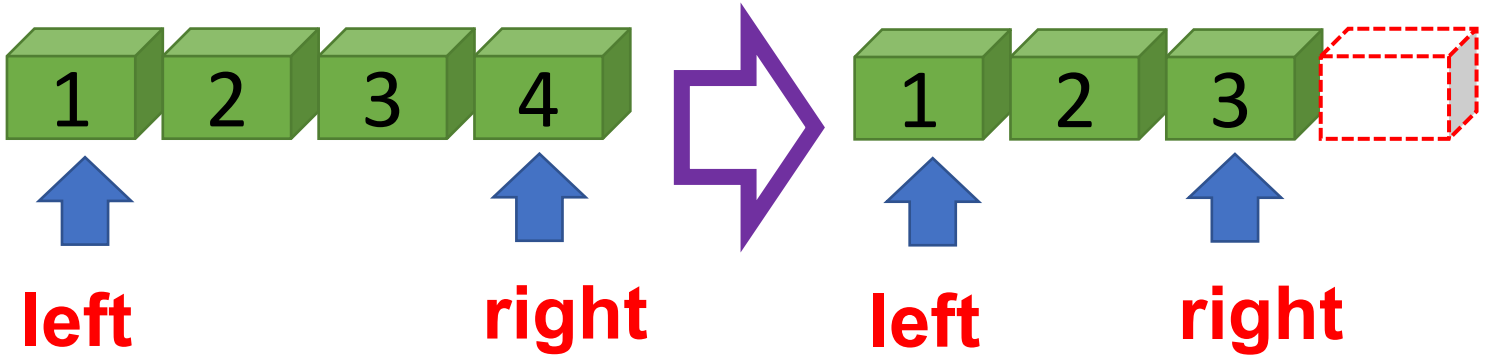


removeLeft() and removeRight()

- **removeLeft()**



- **removeRight()**



Deques, Stacks and Queues

- A stack is actually a deque with only the methods
  - insertRight( )
  - removeRight( )

- A queue is a deque with only the methods
  - insertRight( )
  - removeLeft( )

- A deque is actually a more versatile data structure than either a stack or a queue but is not used as often

Priority Queue

- A **priority queue** is a queue where items don't just join at the rear, they are slotted into the queue according to their priority

- Imagine a stack of mail which are sorted according to priority
  - Each time a new letter is added, you slot it in according to its **priority**
  - Every time you pick up a letter to read, you are picking the most important one of the **pile**

What's Different?

- Don't really need to track front and rear as the rear always stays put at slot 0
- It looks kind of similar to a **stack** because we only need to track the top - would be better named as a "priority stack"
- We shift elements up to make space rather than just putting the element at the rear
- Insert method has a for loop that shifts elements up
- Remove method simply removes the **top** (highest priority) element

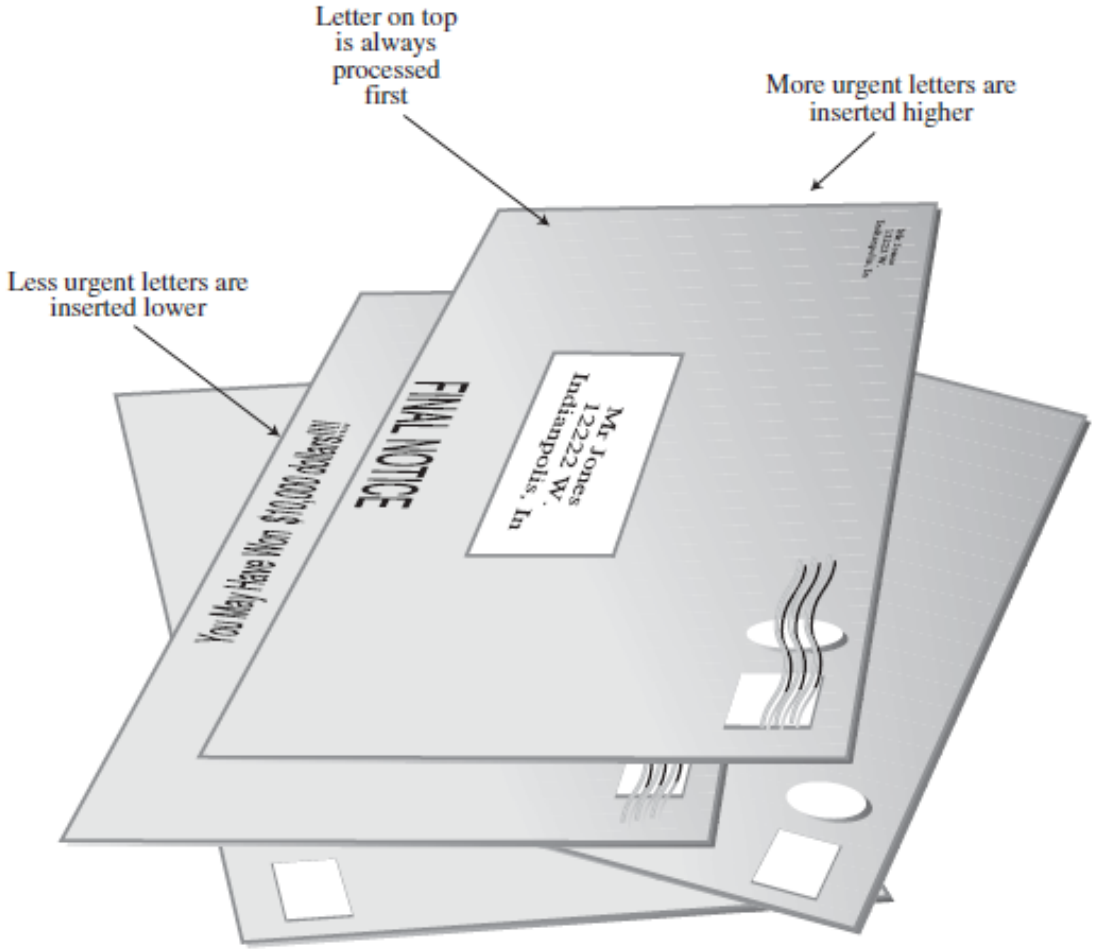


FIGURE 4.10 Letters in a priority queue.

Priority Queue Insert ( )

```
public void insert(long item) { // insert item
      if(nItems == 0){ // if no items,
            queArray[0] = item; // insert at 0
      }
      else{ // if some items,
            int j = nItems; // start at end
            while(j > 0 && queArray[j-1] > item){ // while new item larger
                  queArray[j] = queArray[j-1]; // shift upward
                  j--; // decrement j
            }
            queArray[j] = item; // insert it
      }
      nItems++; // increase items
}
```

Priority Queue

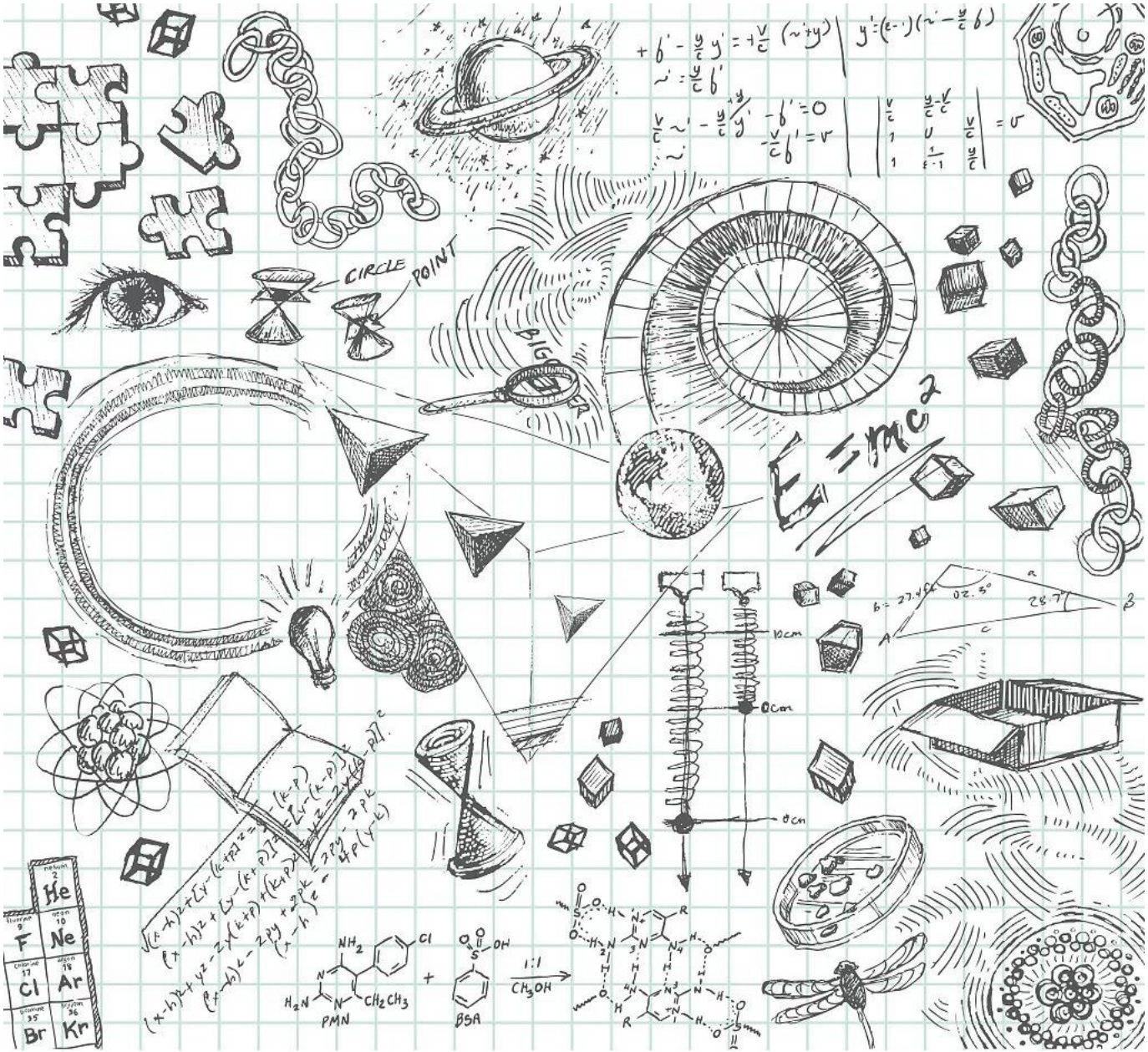
- **Insertion is $O(n)$ while deletion is $O(1)$**
- What output do we get following from the following (assuming higher numbers have highest priority?)

```
PQ thePQ = new PQ(10); // make new priority queue
thePQ.insert(60); // slot items into queue
thePQ.insert(20);
thePQ.insert(80);
thePQ.insert(40);
while (!thePQ.isEmpty()){
    System.out.println(thePQ.remove());
}
```

Output: 80 60 40 20

Exercise

- Write your PriorityQueue class by updating the insert/pop method of your Stack class


CIRCLE
POINT
BIGGER
$E=mc^2$
PMN
BSA
1:1
CH_3OH
He
Ne
Ar
Kr
Br
Cl
F