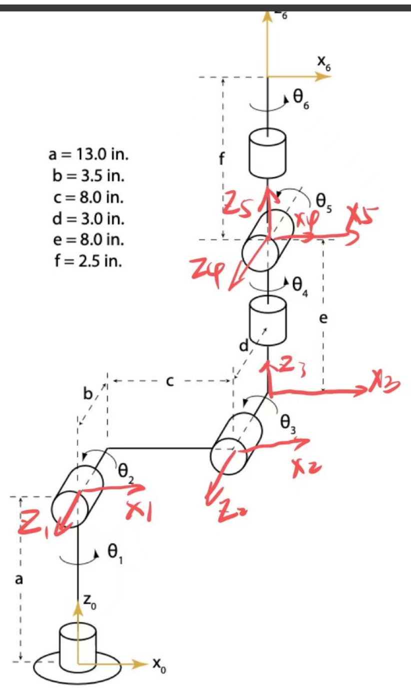
832002125 lizheng 李政



	θ	**d**	**a**	α
0-1	θ_1	a	0	90
1-2	θ_2	-b	c	0
2-3	θ_3	-d	0	-90
3-4	θ_4	e	0	90
4-5	θ_5	0	0	-90
5-6	θ_6	f	0	0

$$H_1 = \begin{bmatrix} C1 & 0 & S1 & 0 \\ S1 & 0 & -C1 & 0 \\ 0 & 1 & 0 & a \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2 = \begin{bmatrix} C2 & -S2 & 0 & cC2 \\ S2 & C2 & 0 & cS2 \\ 0 & 0 & 1 & -b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} C3 & 0 & -S3 & 0 \\ S3 & 0 & C3 & 0 \\ 0 & -1 & 0 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_4 = \begin{bmatrix} C4 & 0 & S4 & 0 \\ S4 & 0 & -C4 & 0 \\ 0 & 1 & 0 & e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

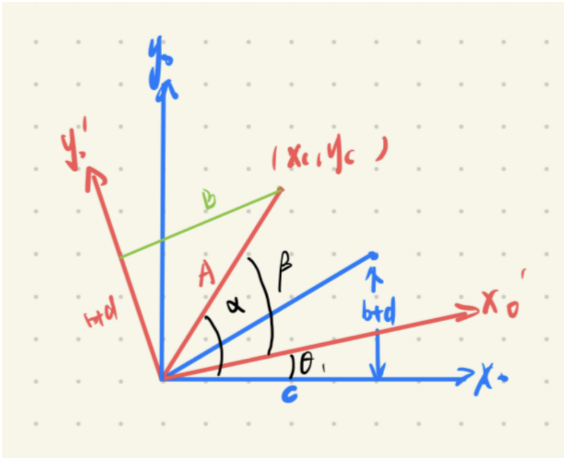
$$H_5 = \begin{bmatrix} C5 & 0 & -S5 & 0 \\ S5 & 0 & C5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_6 = \begin{bmatrix} C6 & -S6 & 0 & 0 \\ S6 & C6 & 0 & 0 \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First we need to calculate the first three angles$\theta_1, \theta_2, \theta_3$ (**Inverse Position**) via the location of wrist centre

$$o_c^0 = o - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_c - d_6 r_{13} \\ o_c - d_6 r_{23} \\ o_c - d_6 r_{33} \end{bmatrix}$$

For theθ_1, we could draw the following garh and notations, We can solve it by combining the following equations



$$\alpha = \arctan(\frac{y_c}{x_c})$$

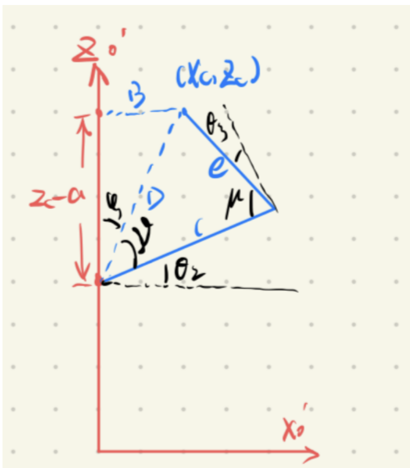
$$\beta = \arctan(\frac{b+d}{B})$$

$$A = \sqrt{{x_c}^2 + {y_c}^2}$$

$$B = \sqrt{A^2 + (b+d)^2}$$

$$\theta_1 = \alpha - \beta$$

For the$\theta_1, \theta_2, \theta_3$, we could draw the following garh and notations, We can solve it by combining the following equations



$$D = \sqrt{{x_c}^2 + (z_c - a)^2}$$

$$\cos\vartheta = \frac{D^2 + C^2 - e^2}{2Dc}$$

$$\xi = \arctan(\frac{B}{z_c - a})$$

$$\theta_2 = 90^\circ - \xi - \vartheta$$

$$\cos\mu = \frac{e^2 + c^2 - D^2}{2ec}$$

$$\theta_3 = 90^\circ - \mu$$

Now, we are focusing on **Inverse Orientation** to solve the$\theta_4, \theta_5, \theta_6$, what we need to do is solve the following equations

$$R_3^0 = A_0^1 A_2^1 A_3^2 = \begin{bmatrix} c1c2c3 - c1s2s3 & -s1 & -c1c2s3 - c1s2c3 \\ s1c2c3 - s1s2s3 & c1 & -s1c2s3 - s1s2c3 \\ s2c3 + c2s3 & 0 & -s2s3 + c2c3 \end{bmatrix}$$

$$R_6^3 = A_4^3 A_5^4 A_6^5 = \begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & -c4s5 \\ s4c5c6 - c4s6 & -s4c5s6 - c4c6 & -s4s5 \\ s5s6 & -s5s6 & c5 \end{bmatrix}$$

$$R_6^3 = (R_3^0)^T R_6^0$$

$$\begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & -c4s5 \\ s4c5c6 - c4s6 & -s4c5s6 - c4c6 & -s4s5 \\ s5s6 & -s5s6 & c5 \end{bmatrix} = \begin{bmatrix} c1c2c3 - c1s2s3 & -s1 & -c1c2s3 - c1s2c3 \\ s1c2c3 - s1s2s3 & c1 & -s1c2s3 - s1s2c3 \\ s2c3 + c2s3 & 0 & -s2s3 + c2c3 \end{bmatrix}^T \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\begin{gathered} -c4s5 = (c123 - c1s2s3)r13 + (-s1)r23 + (-c1c2s3 - c1s2c3)r33 \\ -s4s5 = (s1c2c3 - s123)r13 + (c1)r23 + (-s1c2s3 - s1s2c3)r33 \\ c5 = (s2c3 + c2s3)r13 + (c2c3 - s2s3)r33 \\ s5c6 = (s2c3 + c2s3)r11 + (c2c3 - s2s3)r31 \\ -s5s6 = (s2c3 + c2s3)r12 + (c2c3 - s2s3)r32 \end{gathered}$$

so

$$\begin{gathered} \theta_4 = a\tan 2(-c4s5, -s4s5) \\ \theta_5 = a\tan 2(\mathrm{c}5, \pm\sqrt{1 - c5^2}) \\ \theta_6 = a\tan 2(s5c6, s5s6) \end{gathered}$$

I just leave the process of substituting variables into functions, then we could totally get all $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$

Mr. Zhan, Thanks for your attention, and have a good time! 🙂

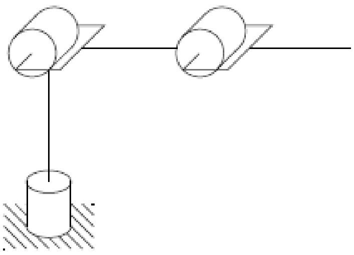
Homework 2

I choose to use **Modified DH,**so the matrix is that

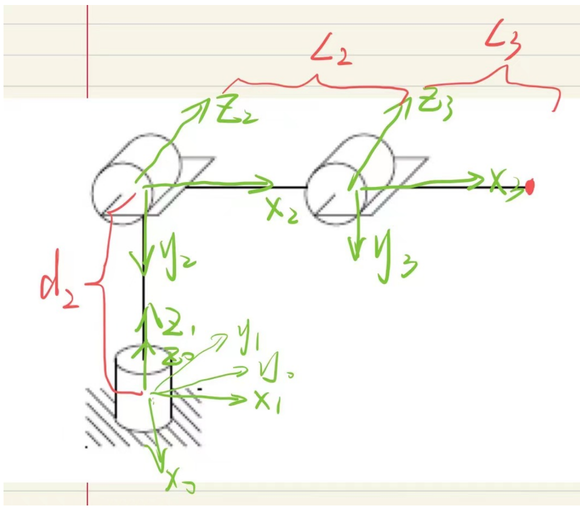
$$^{n-1}T_n = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & a_{n-1} \\ \sin\theta_n\cos\alpha_{n-1} & \cos\theta_n\cos\alpha_{n-1} & -\sin\alpha_{n-1} & -d_n\sin\alpha_{n-1} \\ \sin\theta_n\sin\alpha_{n-1} & \cos\theta_n\sin\alpha_{n-1} & \cos\alpha_{n-1} & d_n\cos\alpha_{n-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1,

The original joints are there



And I construct the frames as following:



and the table is

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90°	0	d_2	θ_2
3	0	L_2	0	θ_3

SO we use c_1 to represent $cos\theta_1$. s_1 to represent $sin\theta_1$

$$^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4 = \begin{bmatrix} c1c2c3 - c1s2s3 & -c1c2c3 - c1s2c3 & -s1 & c1c2c3L_3 - c1s2s3L_3 + c1c2L_2 - s_1d_2 \\ s1c2c3 - s1s2s2 & -s1c2s3 - s1s2c3 & c1 & s1c2c3L_3 - s1s2s3L_3 + s1c2L_2 + c1d_2 \\ -s2c3 - c2s3 & s2s3 - c2c3 & 0 & -s2c3L_3 - c2s3L_3 - s2L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2

NOTES AND REFERENCES 105

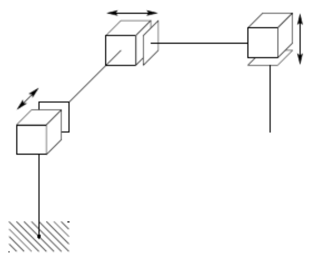


Fig. 3.28 Three-link cartesian robot

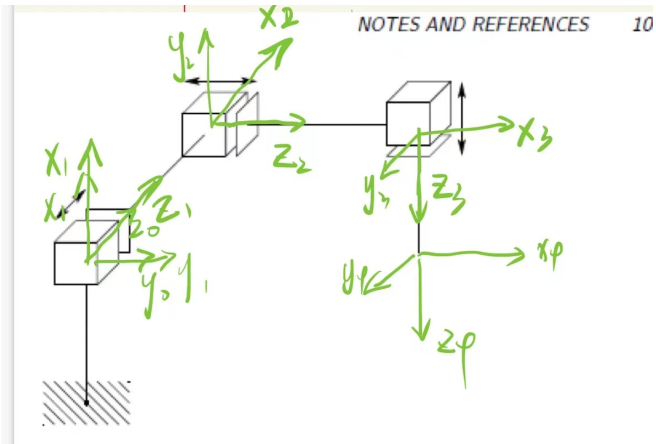


Fig. 3.28 Three-link cartesian robot

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	-90°	0	d_2	-90°
3	90°	0	d_3	90°

$$
{}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1T_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2T_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^3T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_4 = {}^0T_1 \times {}^1T_2 \times {}^2T_3 \times {}^3T_4 = \begin{bmatrix} 0 & 0 & -1 & d_3 - d_4 \\ 1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Li Zheng

FZU 832002125

MCU 20123302

1. Three-link Cylindrical Manipulator

(a)Use the position of the end-effector in the base frame to calculate the 33 linear velocity Jacobian J_v for the three-link cylindrical manipulator of Figure 1.

we could get the formula of the robot

$$\begin{cases} x = -d3s1 \\ y = d3c1 \\ z = d2 \end{cases}$$

Then we take differentiation and get the following result

$$J_v = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} & \frac{\partial f_1}{\partial q_3} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} & \frac{\partial f_2}{\partial q_3} \\ \frac{\partial f_3}{\partial q_1} & \frac{\partial f_3}{\partial q_2} & \frac{\partial f_3}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -c1d3 & 0 & -s1 \\ -s1d3 & 0 & c1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b)Use the positions of the origins o_i and the orientations of the z-axes zi to calculate the 33 linear velocity Jacobian J_v for the same robot. You should get the same answer as before.

For **Prismatic Joints**, the linear velocity is

$$J_{v_i} = z_{i-1}$$

For **Revolute Joints**, the linear velocity is

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1})$$

The homogeneous transform function will be calculated

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1A_2A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1d_3 \\ s_1 & 0 & c_1 & c_1d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1A_2 = \begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & d1 + d2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For this Three-link Cylindrical Manipulator, because the first joint is revolute and the second and third ones are prismatic joints we have

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1+d_2 \end{bmatrix}, o_3 = \begin{bmatrix} -s1d3 \\ c1d3 \\ d1+d2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, z_2 = \begin{bmatrix} -s1 \\ c1 \\ 0 \end{bmatrix}$$

$$J_v = \begin{bmatrix} -c1d3 & 0 & -s1 \\ -s1d3 & 0 & c1 \\ 0 & 1 & 0 \end{bmatrix}$$

We could see that the Jacobian matrixs from (a) and (b) are same, so the answer is right

(c) Find the 3X3 angular velocity Jacobian J_ω for the same robot.

For angular velocity we know that

$$J_{\omega_i} = \begin{cases} z_{i-1} & revoluate \\ 0 & \text{prismatic} \end{cases}$$

So we could calculate the J_ω as follows:

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(d)Find this robot's 6 3 Jacobian J.

we just need to combine the J_v and J_ω

$$J = \begin{bmatrix} -c1d3 & 0 & -s1 \\ -s1d3 & 0 & c1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(e) We just need to substitute the data into the above formula and get the linear velocity answer**

$$v_3^0 = J_v \dot{q} = \begin{bmatrix} 0 & 0 & -1 \\ -0.3 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0.05 \\ -0.03 \\ 0.25 \end{bmatrix} m/s$$

(f) We just need to substitute the data into the above formula and get the angular velocity answer**

$$\omega_3^0 = J_\omega \dot{q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.25 \\ -0.05 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} rad/s$$

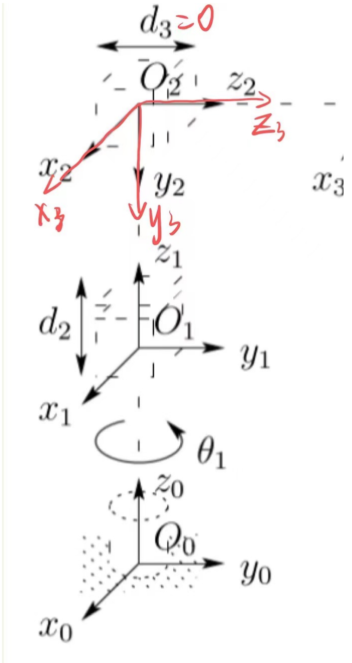
(e)derive the singular configurations

$$\det(J) = \det \begin{bmatrix} -c1d3 & 0 & -s1 \\ -s1d3 & 0 & c1 \\ 0 & 1 & 0 \end{bmatrix} = c1^2d3 + s1^2d3 = d3$$

therefore if $d_3 = 0$ there is a singularity, we must be very cautious about this

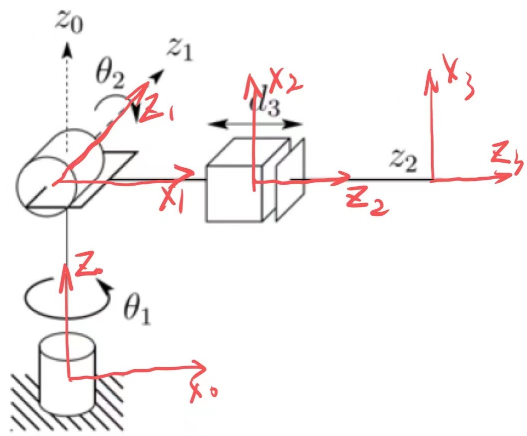
(h) Sketch the cylindrical manipulator in each singular configuration

We could clearly see from the graph that if we set $d_3 = 0$ which means no matter how the joint 0 turn around, the end effector will not take any movements.



2. Three-link Spherical Manipulator

We first establish the joint coordinate frames using the DH convention as shown.



Link	d_i	a_i	α_i	θ_i
1	d_1	0	-90	θ_1
2	0	0	-90	θ_2-90
3	d_3	0	0	0

$$A_1 = \begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} s2 & 0 & c2 & 0 \\ -c2 & 0 & s2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c1s2 & -s1 & c1c2 & 0 \\ s1s2 & c1 & s1c2 & 0 \\ c2 & 0 & -s2 & d1 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c1c2 & -s1 & -c1s2 & c1c2d3 \\ s1c2 & c1 & -s1s2 & s1c2d3 \\ c2 & 0 & -s2 & d1 - s2d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and we could get the data that

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad o_3 = \begin{bmatrix} c1c2d3 \\ s1c2d3 \\ d1 - s2d3 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} -s1 \\ c1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} c1c2 \\ s1c2 \\ -s2 \end{bmatrix}$$

(a) Because the first and second joints are reevaluate joints and the third is prismatic joint, so the linear velocity is

$$J_v = \begin{bmatrix} -s1c2d3 & -c1s2d3 & c1c2 \\ c1c2d3 & -s1s2d3 & s1c2 \\ 0 & -c2d3 & -s2 \end{bmatrix}$$

(b) For angular velocity we know that

$$J_{\omega_i} = \begin{cases} z_{i-1} & revoluate \\ 0 & \text{prismatic} \end{cases}$$

Substitute the data we would get that

$$J_\omega = \begin{bmatrix} 0 & -s1 & 0 \\ 0 & c1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(c) Combine the linear and angular velocity we could get the result

$$J = \begin{bmatrix} -s1c2d3 & -c1s2d3 & c1c2 \\ c1c2d3 & -s1s2d3 & s1c2 \\ 0 & -c2d3 & -s2 \\ 0 & -s1 & 0 \\ 0 & c1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(d) we could substitute the data and get the angular velocity functions of variables

$$\omega_3^0 = J_\omega \dot{q} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\dot{\theta}_2 \\ \frac{\sqrt{2}}{2}\dot{\theta}_2 \\ \dot{\theta}_1 \end{bmatrix} rad/s$$

(e) we could substitute the data and get the linear velocity functions of variables

$$v_3^0 = J_v\dot{q} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ \frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ -\frac{\sqrt{2}}{2}\dot{\theta}_2 \end{bmatrix}$$

(f)substitute the $v_3^0 = [0m/s \quad 0.5m/s \quad 0.1m/s]^T$ into angular velocity and we could get

$$\begin{bmatrix} 0m/s \\ 0.5m/s \\ 0.1m/s \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ \frac{\sqrt{2}}{2}\dot{\theta}_1 + \frac{\sqrt{2}}{2}\dot{d}_3 \\ -\frac{\sqrt{2}}{2}\dot{\theta}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{4}rad/s \\ \frac{\sqrt{2}}{10}rad/s \\ \frac{\sqrt{2}}{4}m/s \end{bmatrix}$$

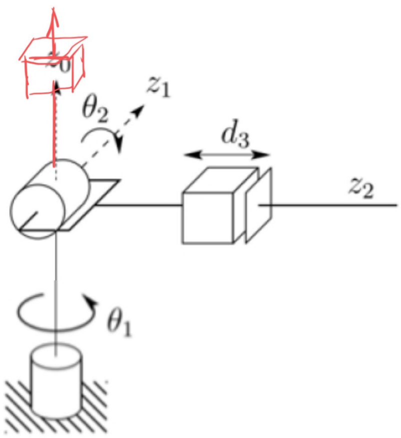
(g) We should make determinant be 0 to find the singularity configuration

$$\det(J) = \det \begin{bmatrix} -s1c2d3 & -c1s2d3 & c1c2 \\ c1c2d3 & -s1s2d3 & s1c2 \\ 0 & -c2d3 & -s2 \end{bmatrix} = -c2d_3^2$$

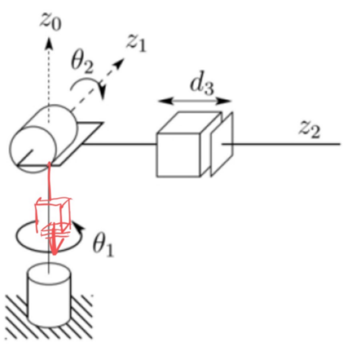
so we clearly know that $cos(\theta_2) = 0$ that is $\theta_2 = \pm 90^\circ$ and $d_3 = 0$ will lead to singularity

(h) When $cos(\theta_2) = 0$ that is $\theta_2 = \pm 90^\circ$ or $d_3 = 0$, in order to move the end effector other joint will have a relatively large speed and a obvious movement and the degree of this moment will decrease 1

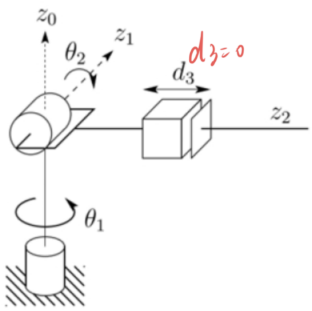
$\theta_2 = 90^\circ$



and when $\theta_2 = -90^\circ$



when $d_3 = 0$



(i) Singular configurations will not be different from the previous one because the singularity of the robot is objectively exist. So no matter what direction and zero configurations, We cannot avoid them by redesign a direction or zero configuration.

Through this homework I learned many knowledge and promote my calculating skills! 😀