

Simple Harmonic Motion

① Spring Oscillator

$$ma = F = -kx$$

$$\Rightarrow m \frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{d^2x}{dt^2} + \underbrace{\left(\frac{k}{m}\right)}_{\omega^2} x = 0$$

2nd order linear homogenous differential equation
with constant coefficient

$$x = A \cos(\omega t + \varphi), \quad \omega = \sqrt{k/m}$$

2. Simple pendulum

$$-mg \sin\theta = mL\ddot{\theta} = mL\alpha \quad \leftarrow \text{angular acceleration}$$

small angle only $\sin\theta \doteq \theta$

$$-mg\theta = mL \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \underbrace{\left(\frac{g}{L}\right)}_{\omega^2} \theta = 0$$

$$\omega^2 = g/L$$