

# **EE204: Analog Electronics**

## **Models and Biasing**

# What is a model?

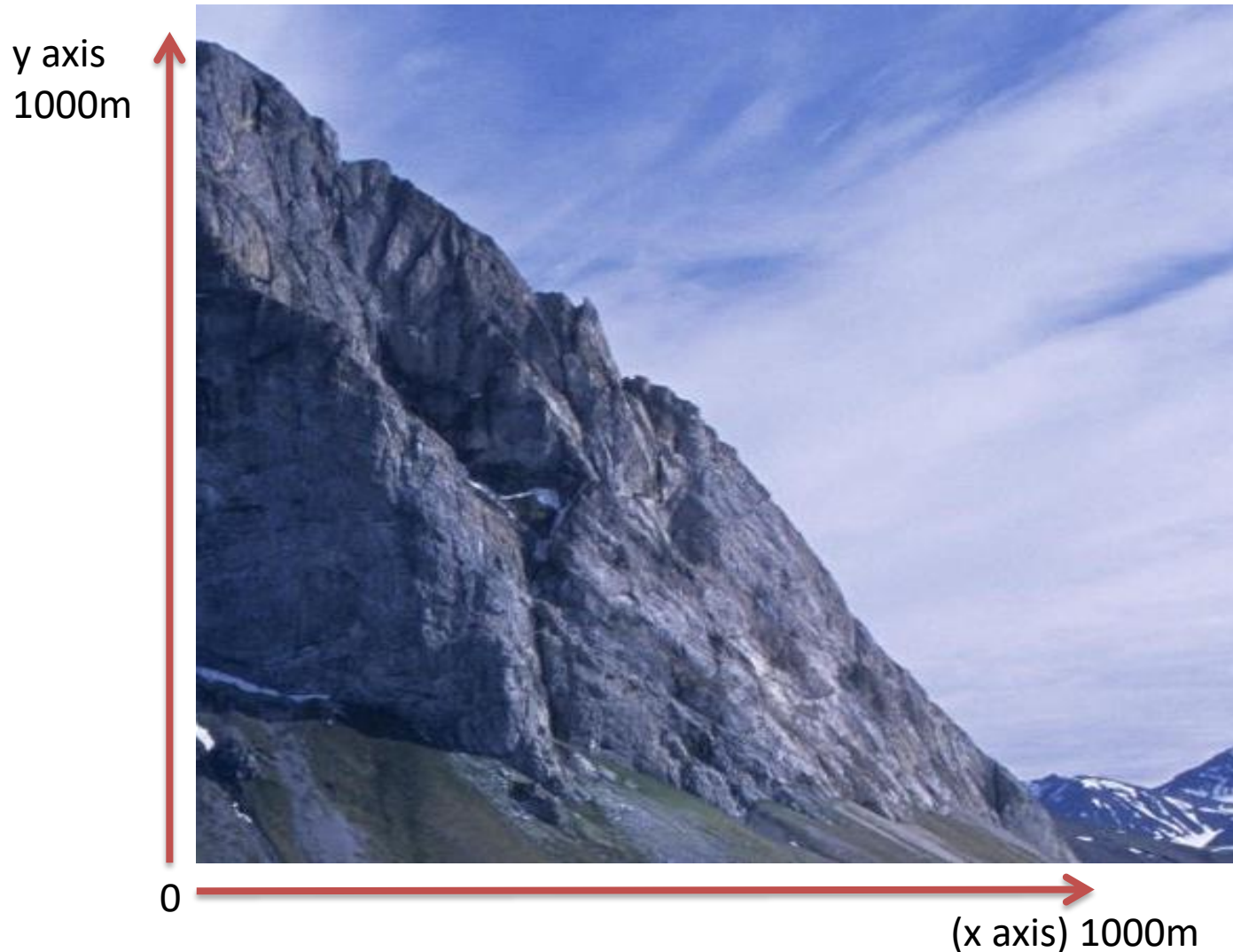
A model tells us the **important information** in an **easy-to-use** format

Models are **simplifications**. The complexity of the model depends on how **precisely** we want to represent the real behaviour

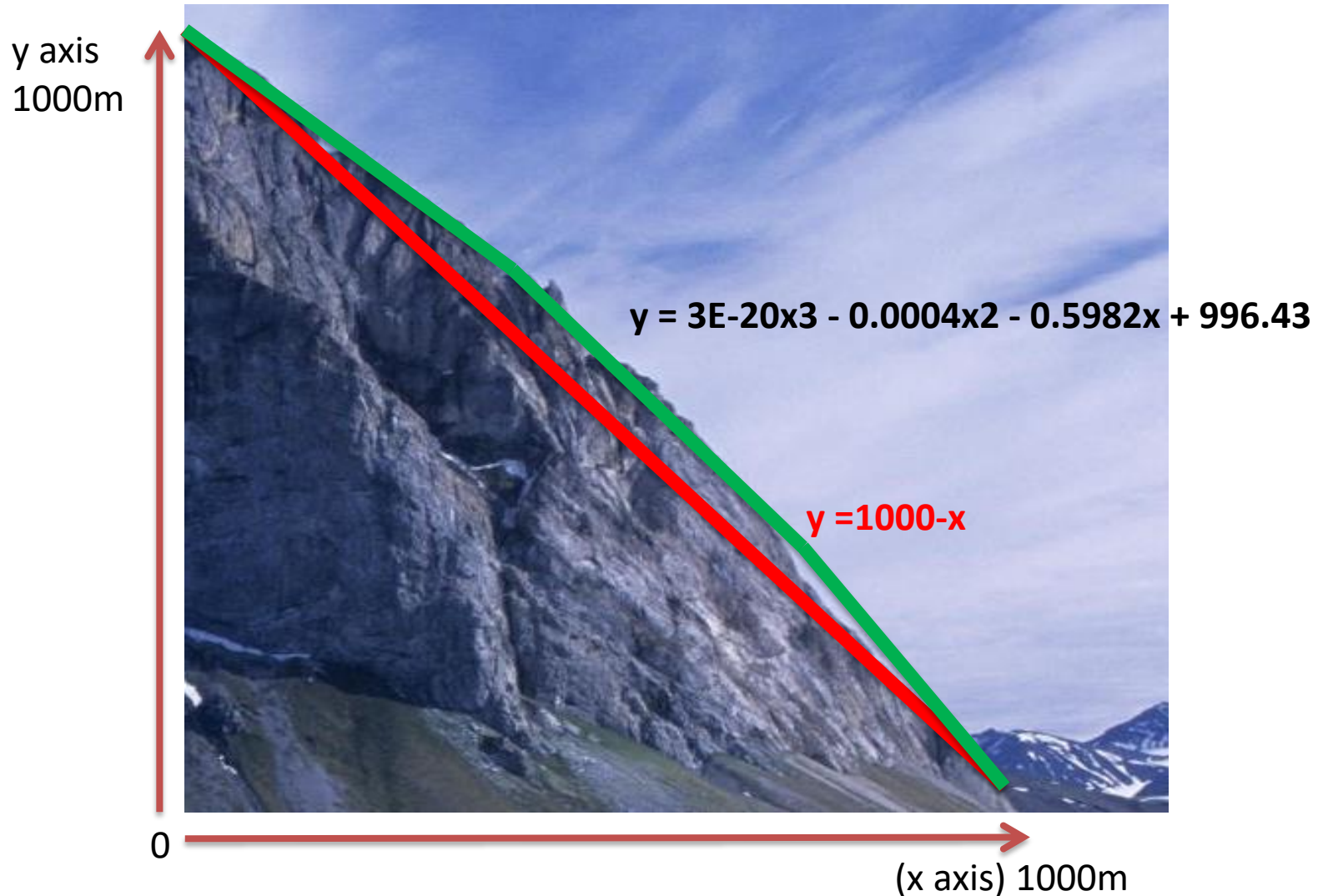
Models can be derived from the **underlying physics** (theoretical), but more commonly are found by **taking measurements** (empirical) and ideally combining the two approaches.

Models are **NOT** perfect, but they can be useful

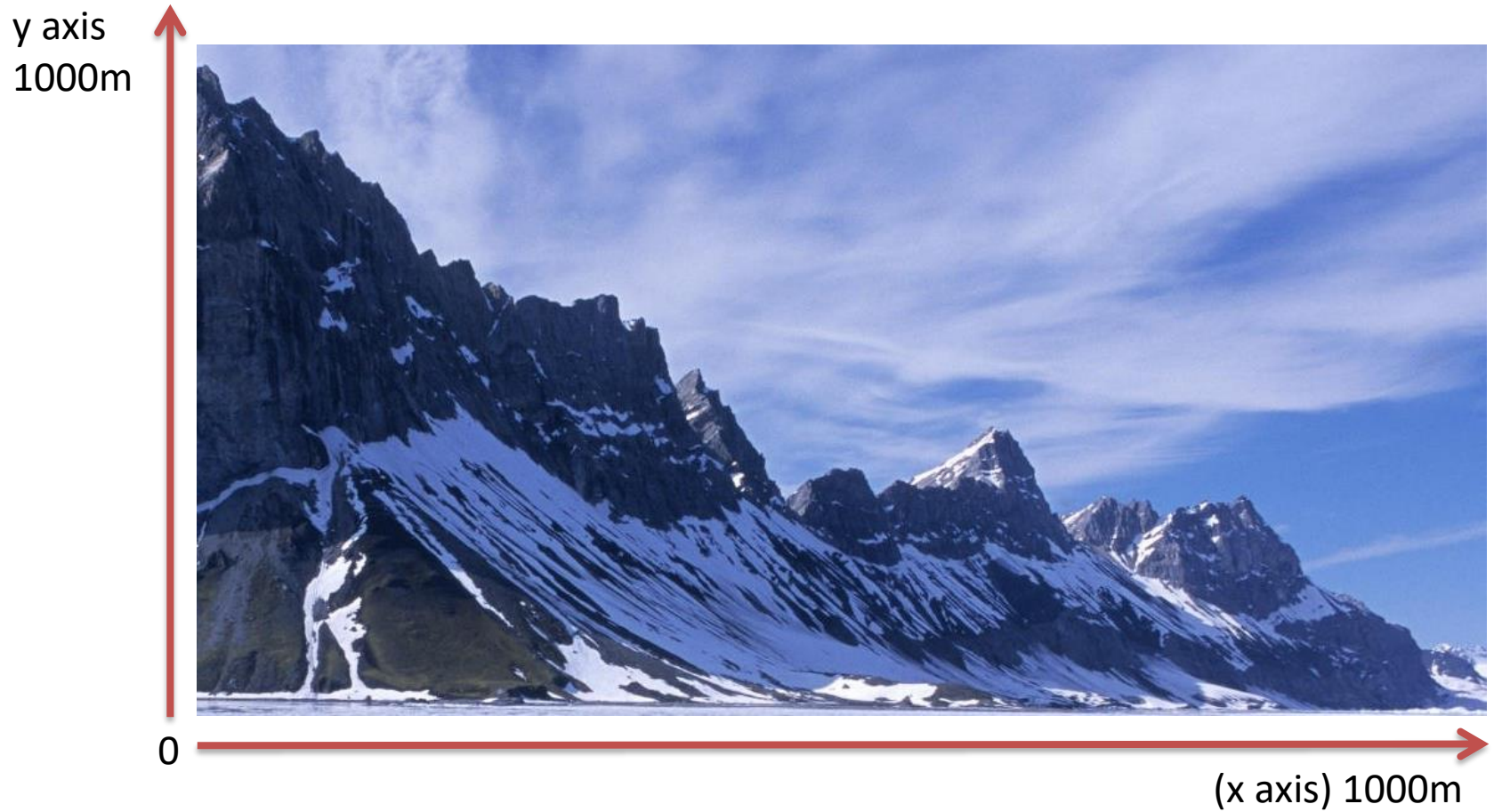
What sort of curve (we call it a model) could we use for this mountain to describe its slope?



We like using **MATHS** as the language to explain things in a repeatable fashion, and it helps solve problems as well



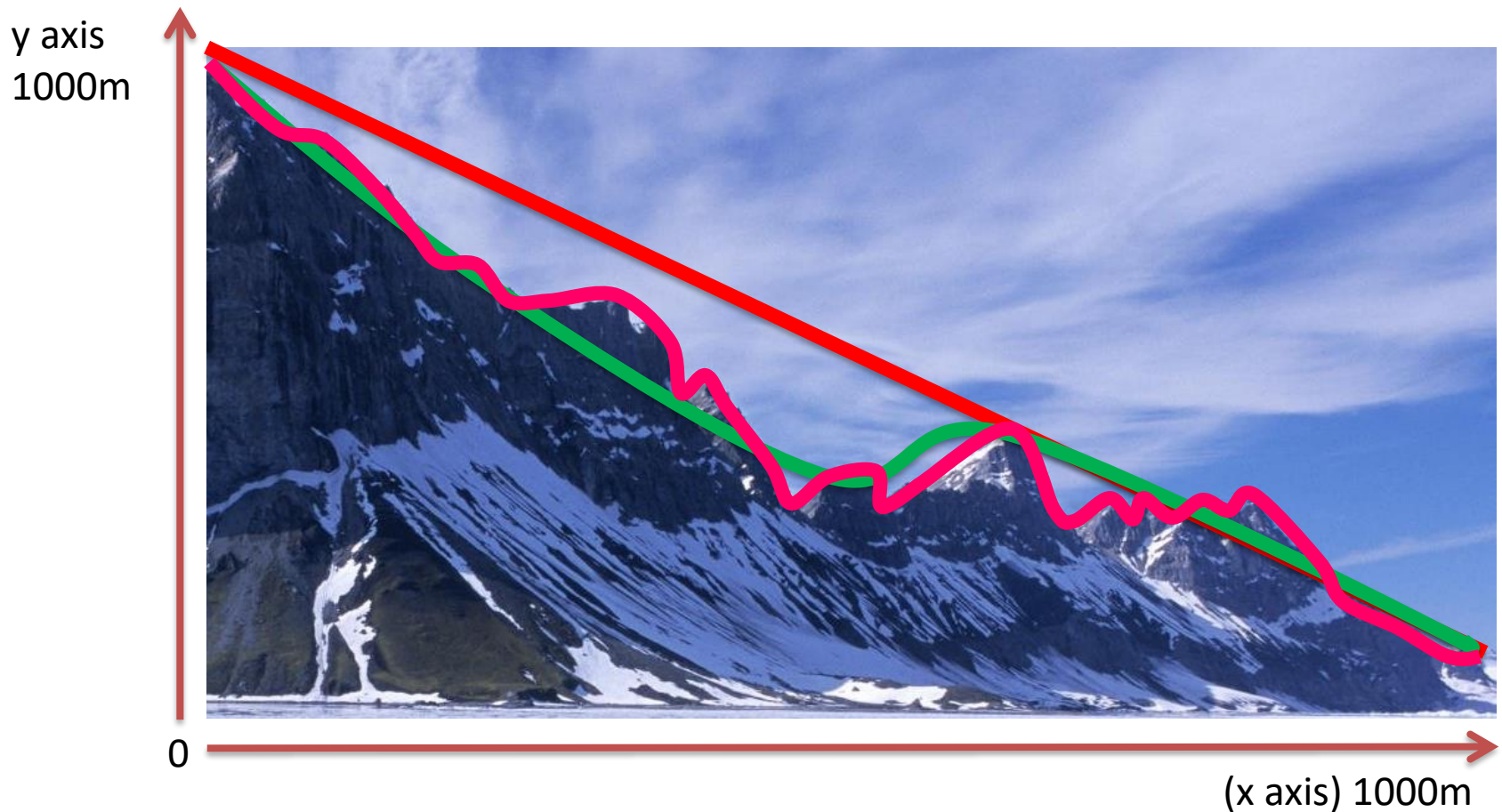
What sort of Models could we use for this mountain?





## A choice: **simplicity versus accuracy**

Reality is messy, our tools only work well with simple models



Question: for your application... Do you really need to know about all the bumps and dips???

# What is a model?

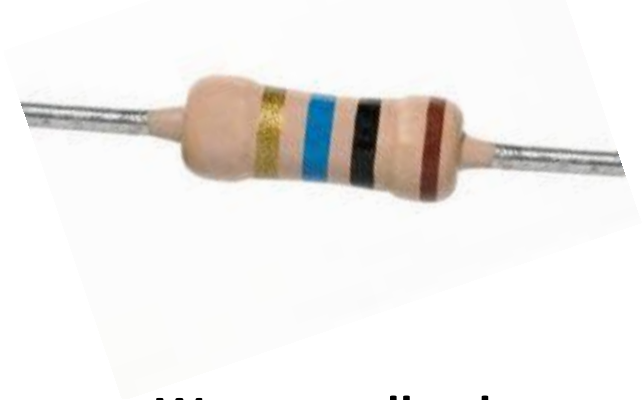
It's a tool to make our **lives easy**... Simple brains

It's a **mathematical representation** of reality that allows us to use maths to extract some useful information.

**It's imperfect**, it has errors. It will never match reality perfectly... But it's a good first guess..

A good model is “**close enough**” but “**simple enough**” to use

# Looking at a Resistor

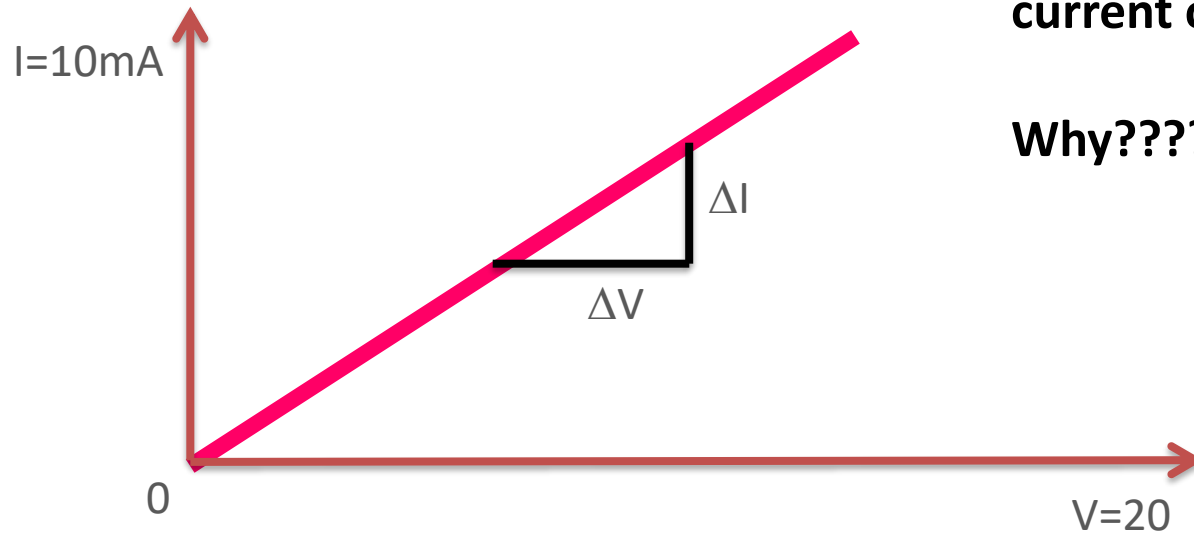


Ohms Law

$$V = I * R$$

We normally plot  
voltage on X axis and  
current on Y.

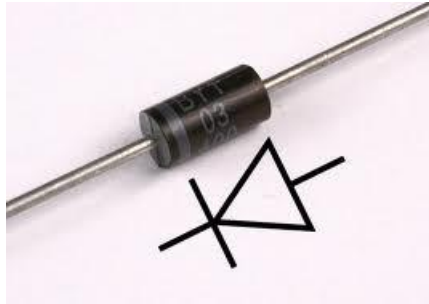
Why????



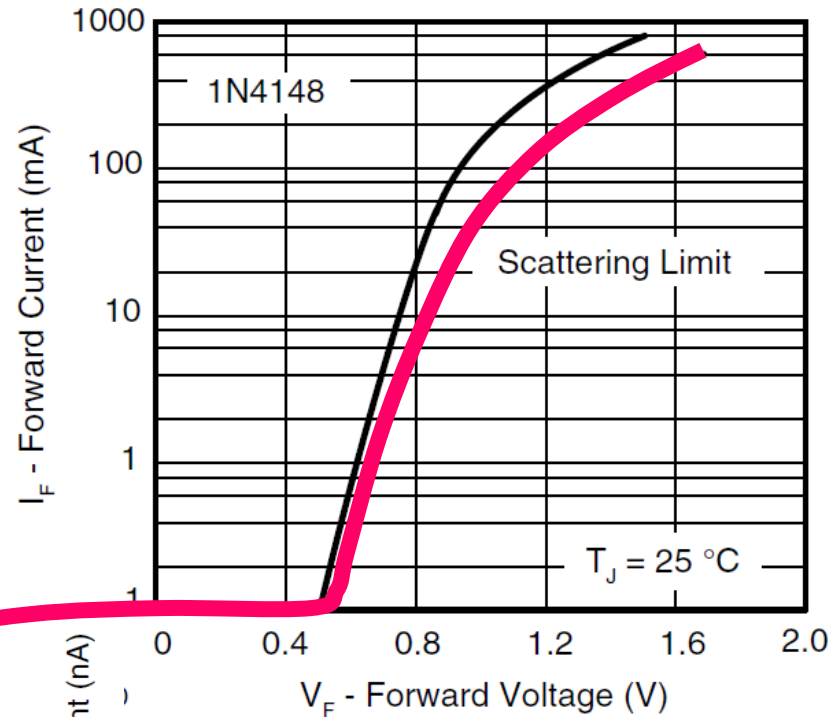
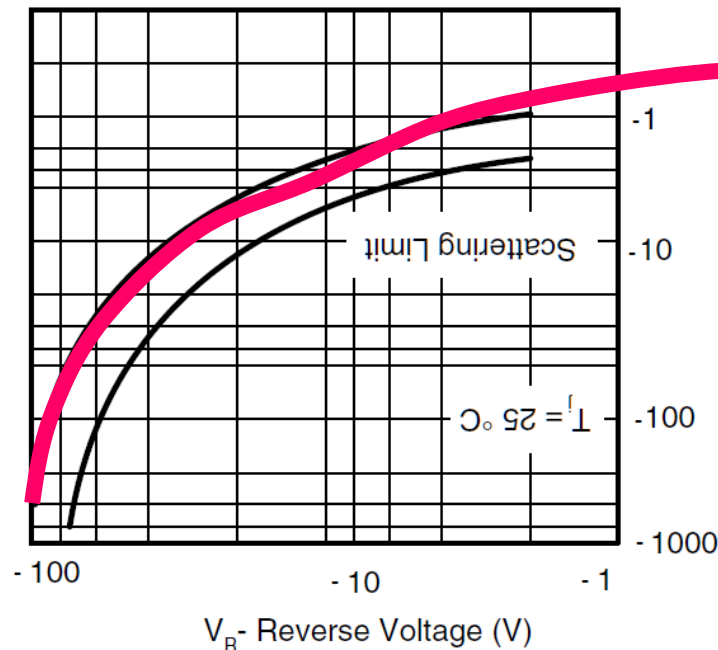
What does the slope of the line correspond to??



# Looking at a Diode



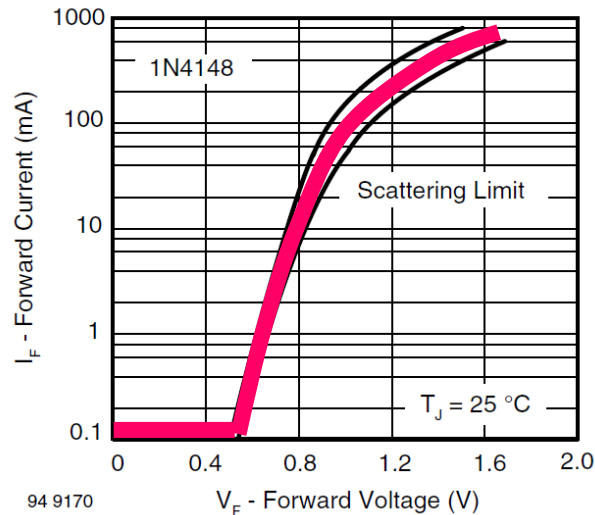
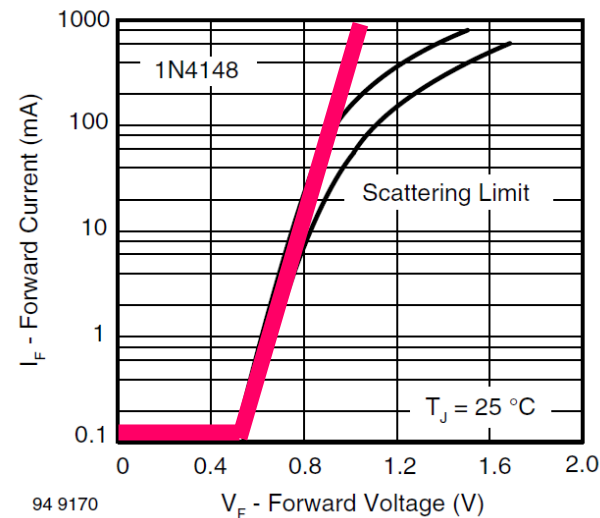
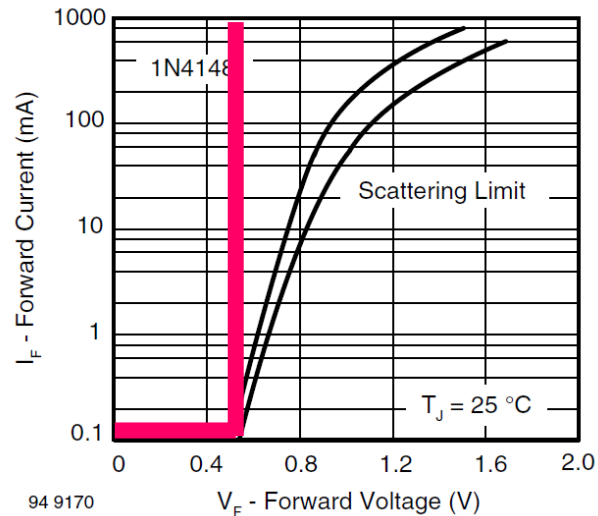
Reverse current is 1000 times smaller than forward current



$$I_D = I_S \left( \exp\left(\frac{V_D}{nV_T}\right) - 1 \right)$$

$n$  = ideality factor

# One device, multiple models



**Which model is most accurate?**

**Which model is the best to use?**

# A circuit-based model

We have tools to solve circuits, and sometimes we can see ways to simplify circuits to help us solve them for a specific purpose.

So can we take a mathematical expression and convert it to a circuit representation.

Lets add some new circuit elements to our collection

# Independent Sources



$$I_{\text{out}} = 0.3 \text{ (or any constant)}$$



$$V_{\text{out}} = 0.3 \text{ (or any constant)}$$

These are constant valued sources. Nothing effects what they do. An independent voltage source is a battery.

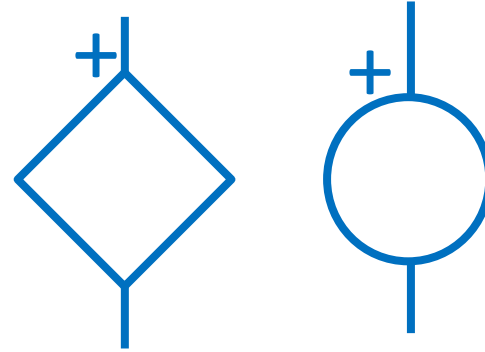
Independent current sources are rarer.

# Dependent Sources



Voltage Controlled Current Source

$$I_{\text{out}} = k V_{\text{control}}$$



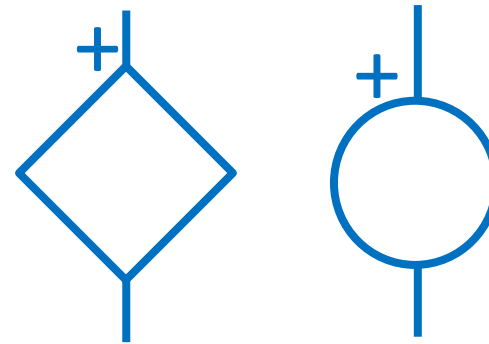
Voltage Controlled Voltage Source

$$V_{\text{out}} = k V_{\text{control}}$$



Current Controlled Current Source

$$I_{\text{out}} = k I_{\text{control}}$$



Current Controlled Voltage Source

$$V_{\text{out}} = k I_{\text{control}}$$

Don't worry about how these are made, they are normally useful quirks of physics. With these, you need to write out what their controlling signal is as symbols don't tell you enough

# Making a circuit model (1)

I am told that for this “device”, the following happens

the current through the device is unknown, but the voltage across the device is a constant value  $k$

$$\Delta V = k$$

$$I = \text{unknown}$$

This is the expression for a constant voltage source, or a battery.

We have no information about what the current through the device is.



# Making a circuit model (2)

I am told that for this “device”, the following happens

the voltage across a device is unknown, but the current through this device is **k** amps

$$\begin{aligned}\Delta V &= \text{unknown} \\ I &= k\end{aligned}$$

This is the expression for a constant current source

We have no information about what the voltage dropped across the device is.

# Making a circuit model (3)

I am told that for this “device”, the following happens

the voltage across this device is proportional (k) to the current flowing through it.

$$\Delta V = k * I$$

This is the expression for a resistor, where resistance  $R=k$

# Making a circuit model (4)

I am told that for this “device”, the following happens

the current through the device is proportional (**k**) to the voltage applied across it.

$$I = k * \Delta V$$

This is again Ohm's law but for a conductance. The conductance of a device is inversely proportional to its resistance ( $R=1/k$ )

# Making a circuit model (5)

I am told that for this “device”, the following happens

the voltage across a device is unknown, but the current through the device is  $k$  times the current  $I_{in}$  that is being measured somehow.

$$\begin{aligned}\Delta V &= \text{unknown} \\ I &= k * I_{in}\end{aligned}$$

This is the expression for a current controlled current source.

**NOTE:**  $I_{in}$  is not the same as the current through the device. It's from somewhere else in the circuit.

# Making a circuit model (6)

I am told that for this “device”, the following happens

the voltage across a device is unknown, but the current through the device is  $k$  times the voltage  $V_{in}$  that is being measured somehow.

$$\Delta V = \text{unknown}$$

$$I = k * V_{in}$$

This is the expression for a voltage controlled current source.

**NOTE:**  $V_{in}$  is not the same as the voltage across the device. It's from somewhere else in the circuit.

# Making a circuit model (7)

I am told that for this “device”, the following happens

the current through the device is unknown but the voltage is **k** times the value of  $V_{in}$ , which is measured somehow.

$$\Delta V = k * V_{in}$$

$$I = \text{unknown}$$

This is the expression for a voltage controlled voltage source

**NOTE:**  $V_{in}$  is not the same as the voltage across the device. It's from somewhere else in the circuit.



# Making a circuit model (8)

I am told that for this “device”, the following happens

the current through the device is unknown but the voltage is  $k$  times the value of  $V_{in}$ , which is measured somehow.

$$\begin{aligned}\Delta V &= k * I_{in} \\ I &= \text{unknown}\end{aligned}$$

This is the expression for a current controlled voltage source

**NOTE:**  $I_{in}$  is not the same as the current through the device. It's from somewhere else in the circuit.

# Making a circuit model (the fun part)

The important thing to understand here is that we have two types of devices – those that produce current and those that produce voltages.

You cannot successfully mix current and voltage output devices. The question “3 amps + 4 volts =” is meaningless.

So imagine that you have two forms of equations that you can use

$$I_{XY} = 3 + V_X(0.2) - 13V_1 + 5V_2 + 0.1I_3$$

$$V_{XY} = 0.2 + 5I_{XY} + 13V_1 - 2V_2 - 1.1I_3$$

# Making a circuit model (the fun part)

$$I_{XY} = 3 + V_{XY}*(0.2) - 13*V_1 + 5*V_2 + 0.1*I_3$$

Something that adds a current that is proportional to the current  $I_3$

Something that adds a current that is proportional to the voltage  $V_1$  and  $V_2$

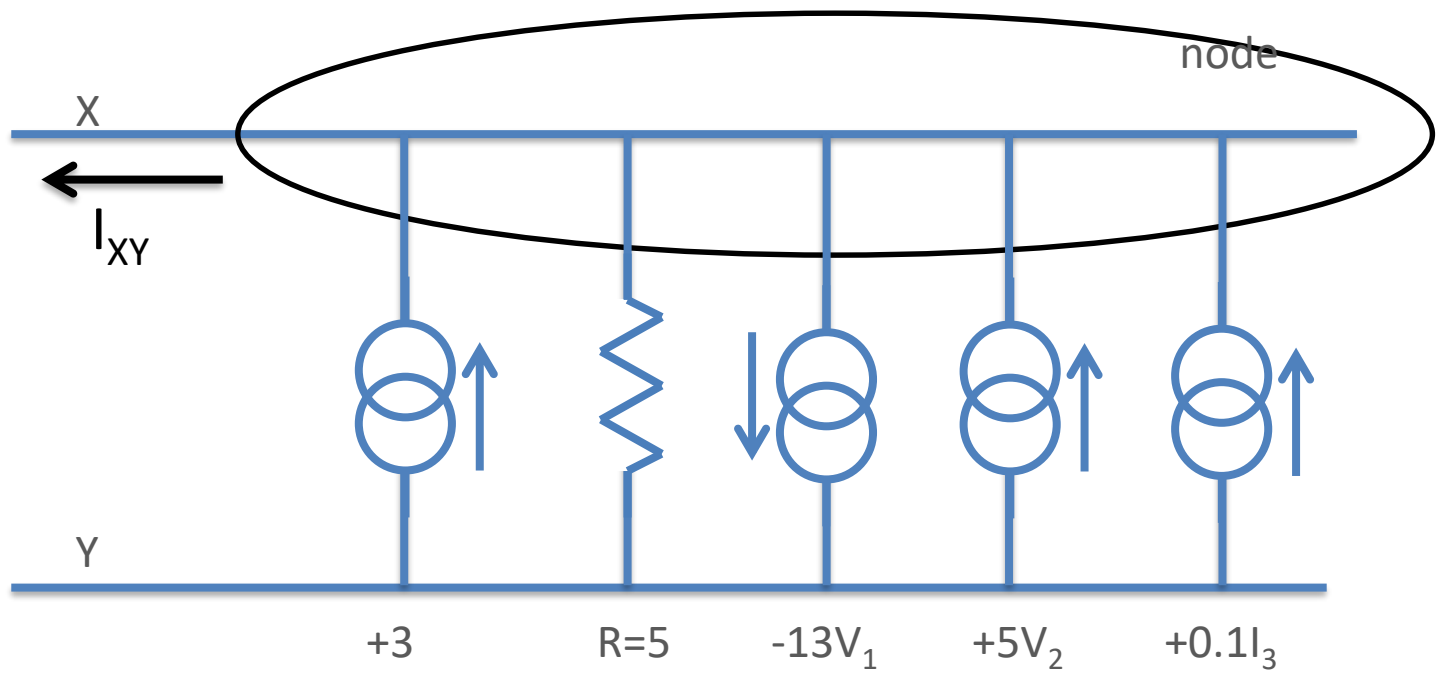
Something that adds a current that is proportional to the voltage across the points XY

Something that adds a constant current... easy

# Making a circuit model (the fun part)

$$I_{XY} = 3 + V_{XY}*(0.2) - 13*V_1 + 5*V_2 + 0.1*I_3$$

Remember, KCL says that currents add at a node. Be careful with the arrows.



# Making a circuit model (the fun part)

$$V_{xy} = 0.2 + 5 I_{xy} + 13 V_1 - 2 V_2 - 1.1 I_3$$

Something that adds a voltage that is proportional to the current  $I_3$

Something that adds a voltage that is proportional to the voltage  $V_1$  and  $V_2$

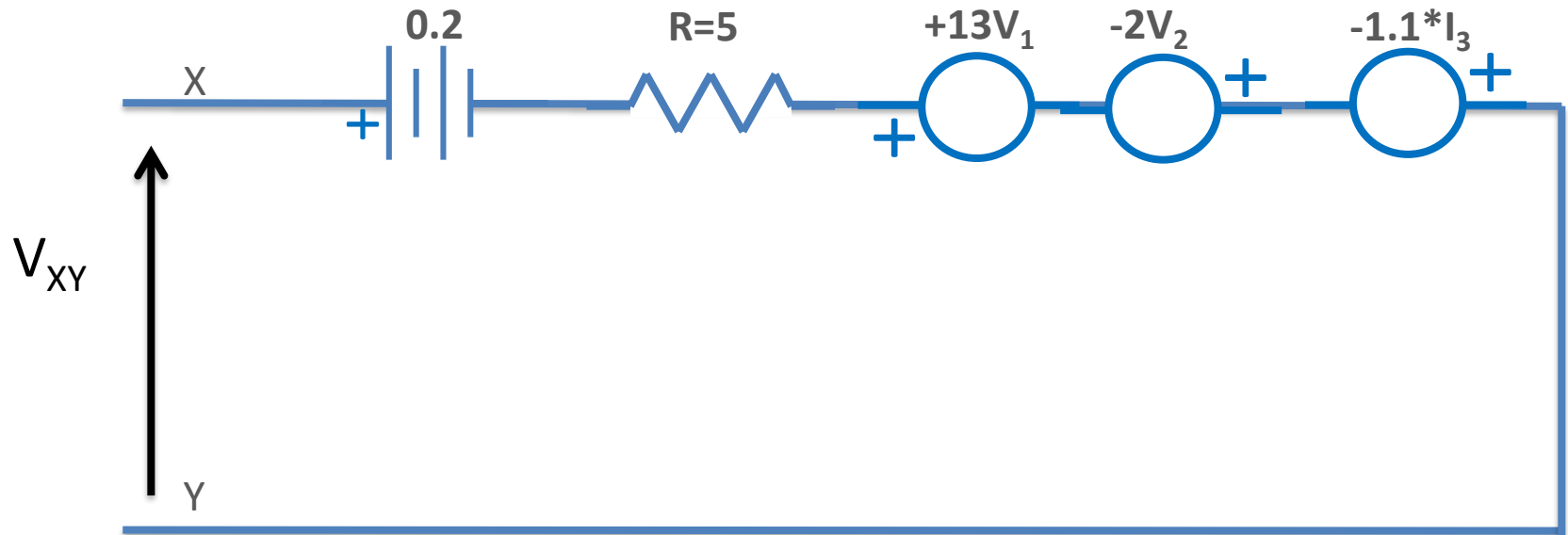
Something that adds a voltage that is proportional to the current flowing through it

Something that adds a constant voltage... easy

# Making a circuit model (the fun part)

$$V_{XY} = 0.2 + 5 I_{XY} + 13*V_1 - 2*V_2 - 1.1*I_3$$

Remember, KVL says that voltages add in a loop.  
Be careful of the signs.





# Making a circuit model

Circuit models have a directly equivalent mathematical model. We use whichever one is easiest.

In circuit models, we will attempt to never use  $()^2$  or anything complicated. Our aim is simplicity

Remember all models are simplifications. Computers are great at crunching equations.

They'll give you the answer to 6 decimal places but they can't tell you if it's the right answer to the problem you wanted to solve.

# Biasing and Small Signal Behaviour

What is a small signal?

It's a change in your signal that is small.

What is small?

Small enough that you don't get any weird behaviours

Seriously... What is small??

I am being serious... There is no good definition

Got an example?

Usually these are AC signals, for example microphone audio output signal (mV)

What slope does the person experience?

How far can they go before they experience a different slope?



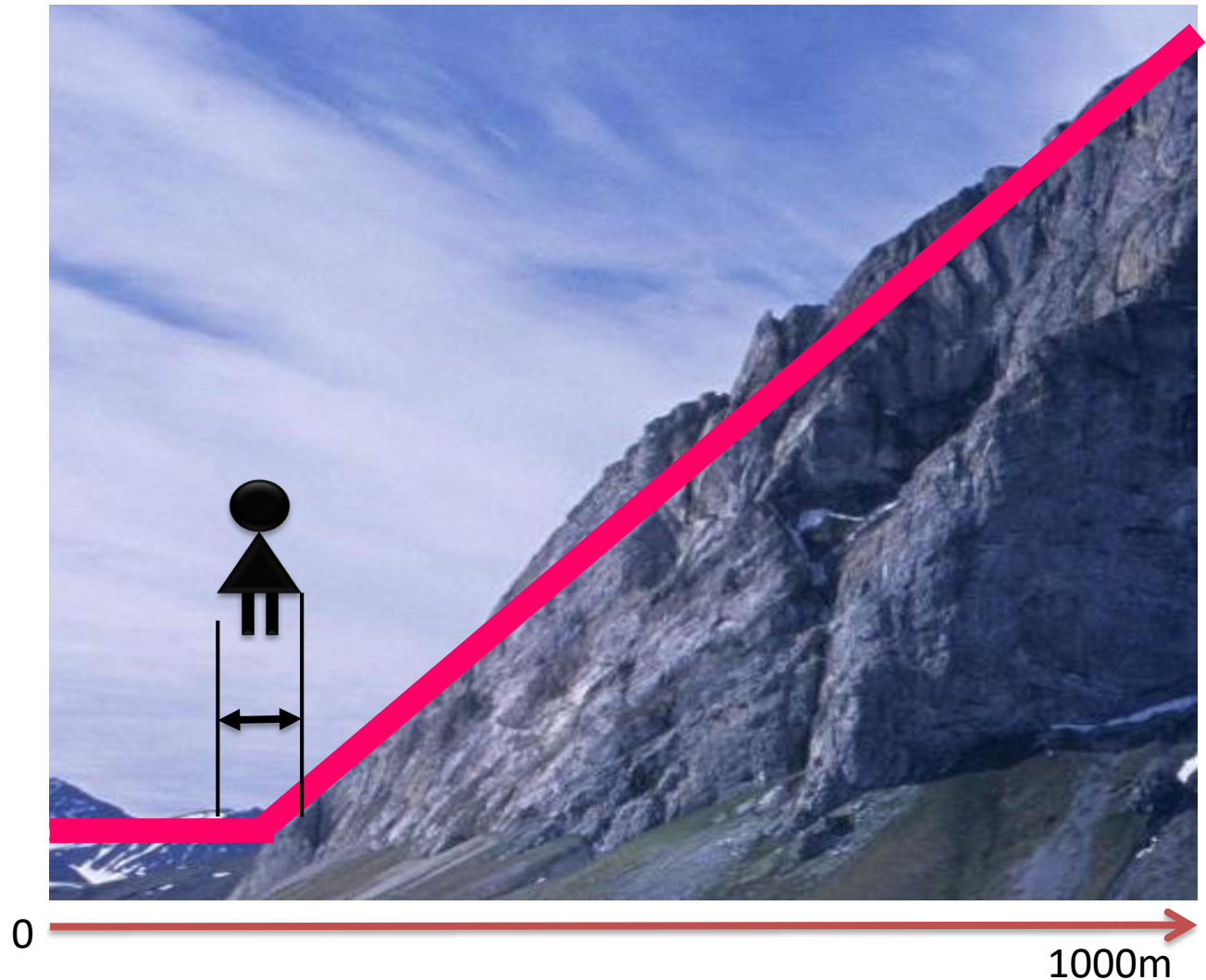
What slope does the person experience?

How far can they go before it changes significantly?



What slope does the person experience?

How far can they go before the slope changes significantly?







As I walk left to right... there will be points where I may end up walking downhill.

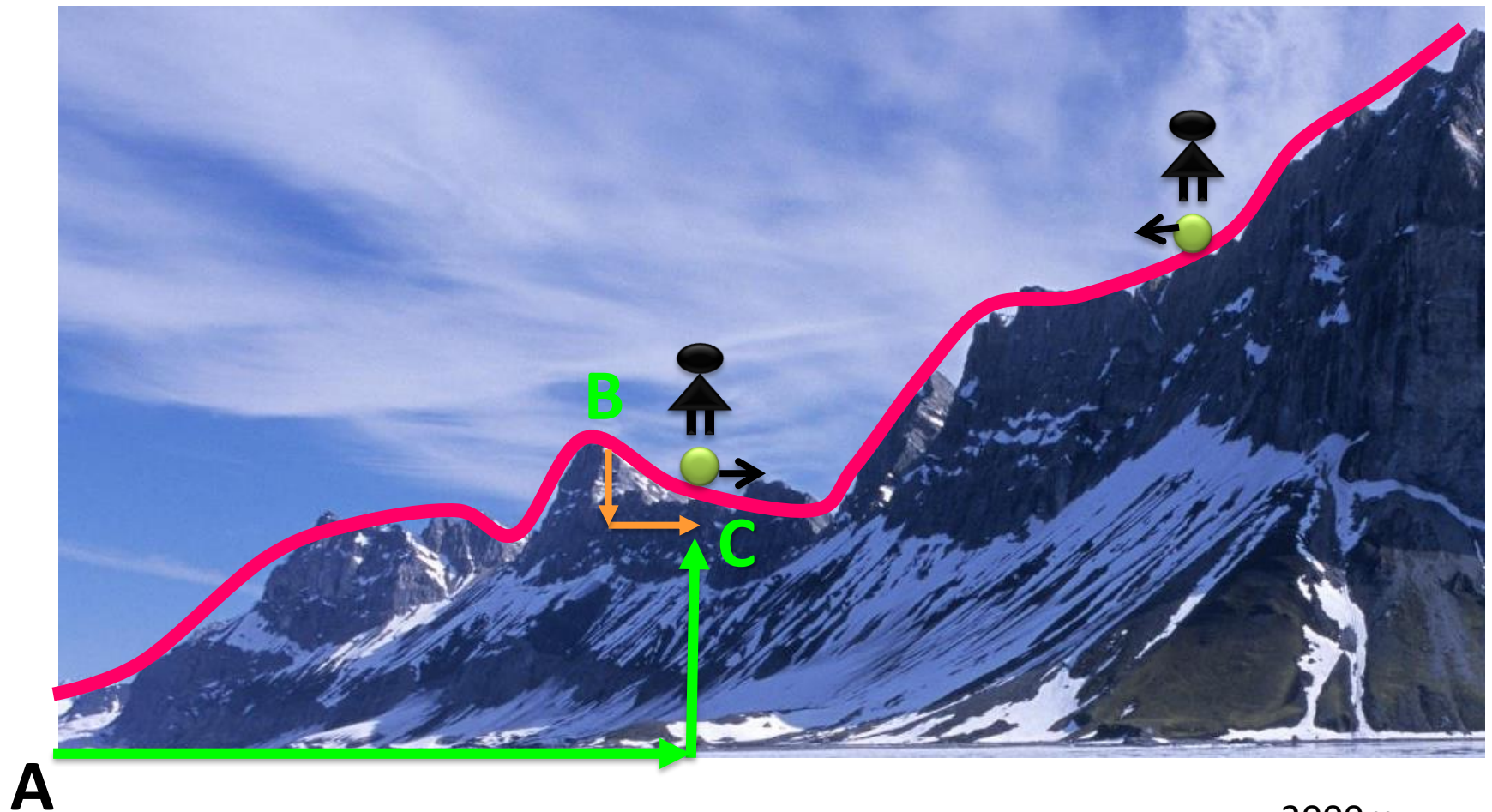
What does this mean??



Over a very small distance, everything looks like a straight line



The slope of the line at each point can of course change.



2000m

So am I walking uphill or not... it depends on your perspective..

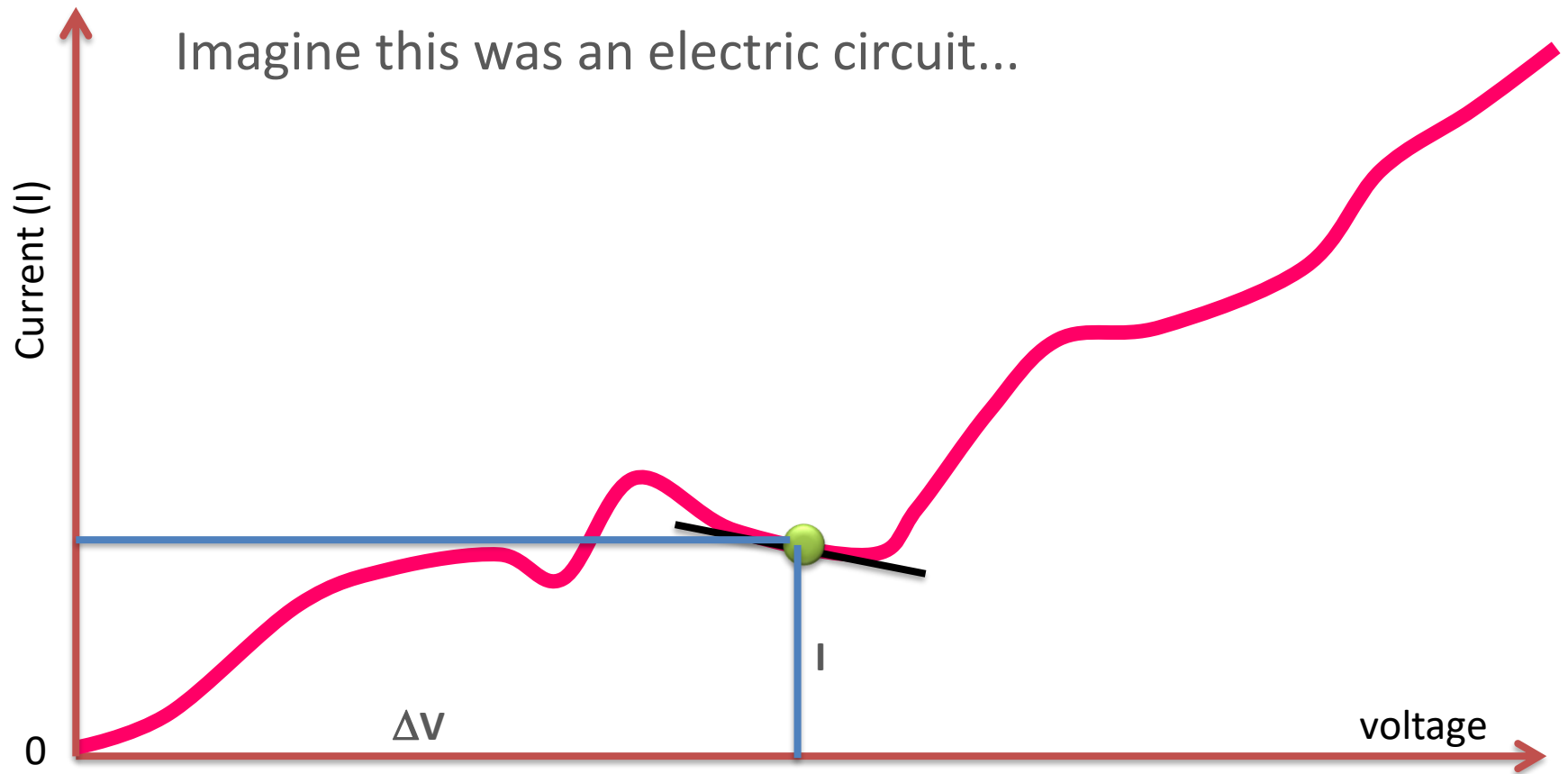
If I am starting at A....

If I am starting at B...



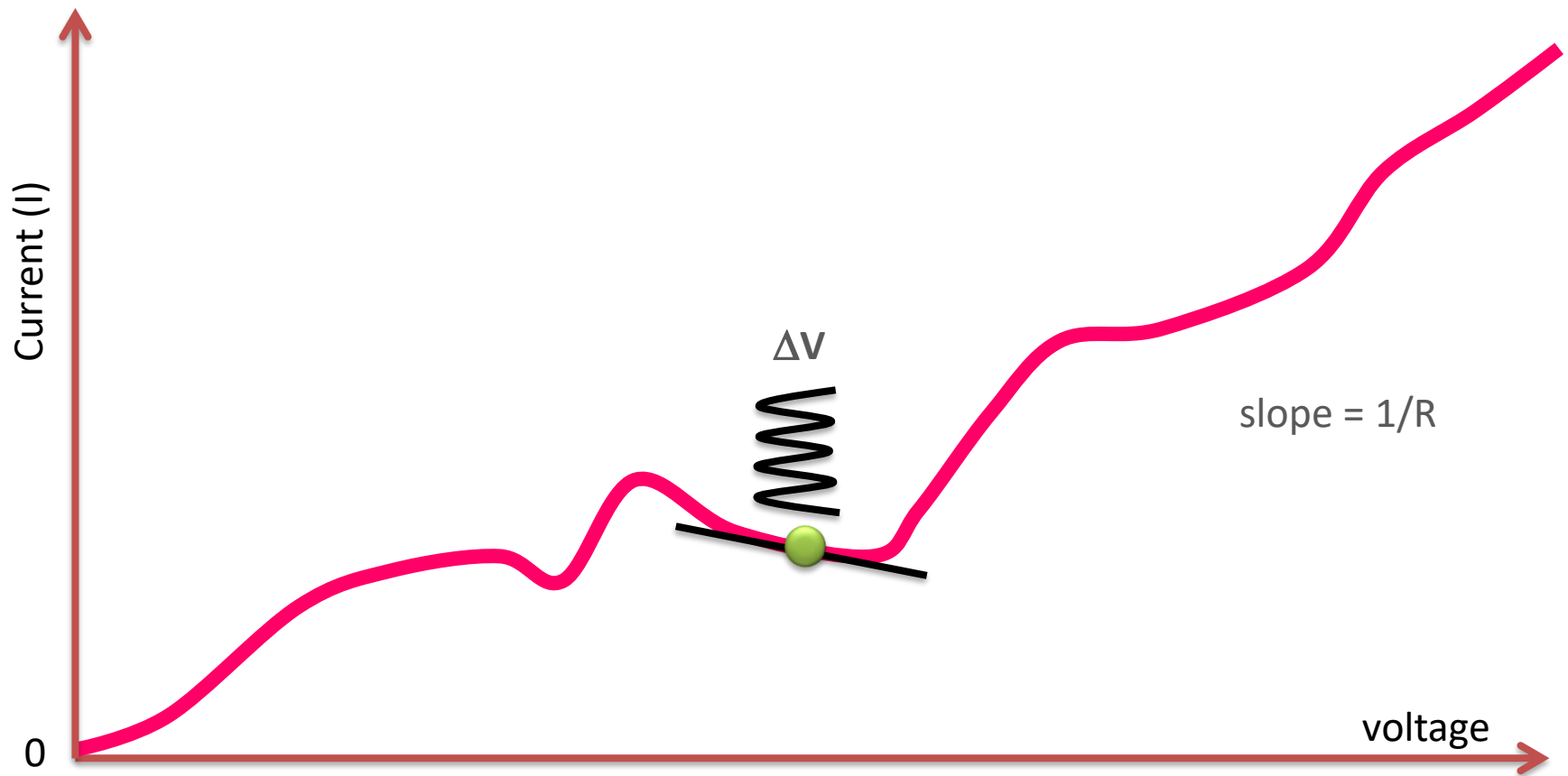
**The same applies to Voltages and Currents**

Imagine this was an electric circuit...



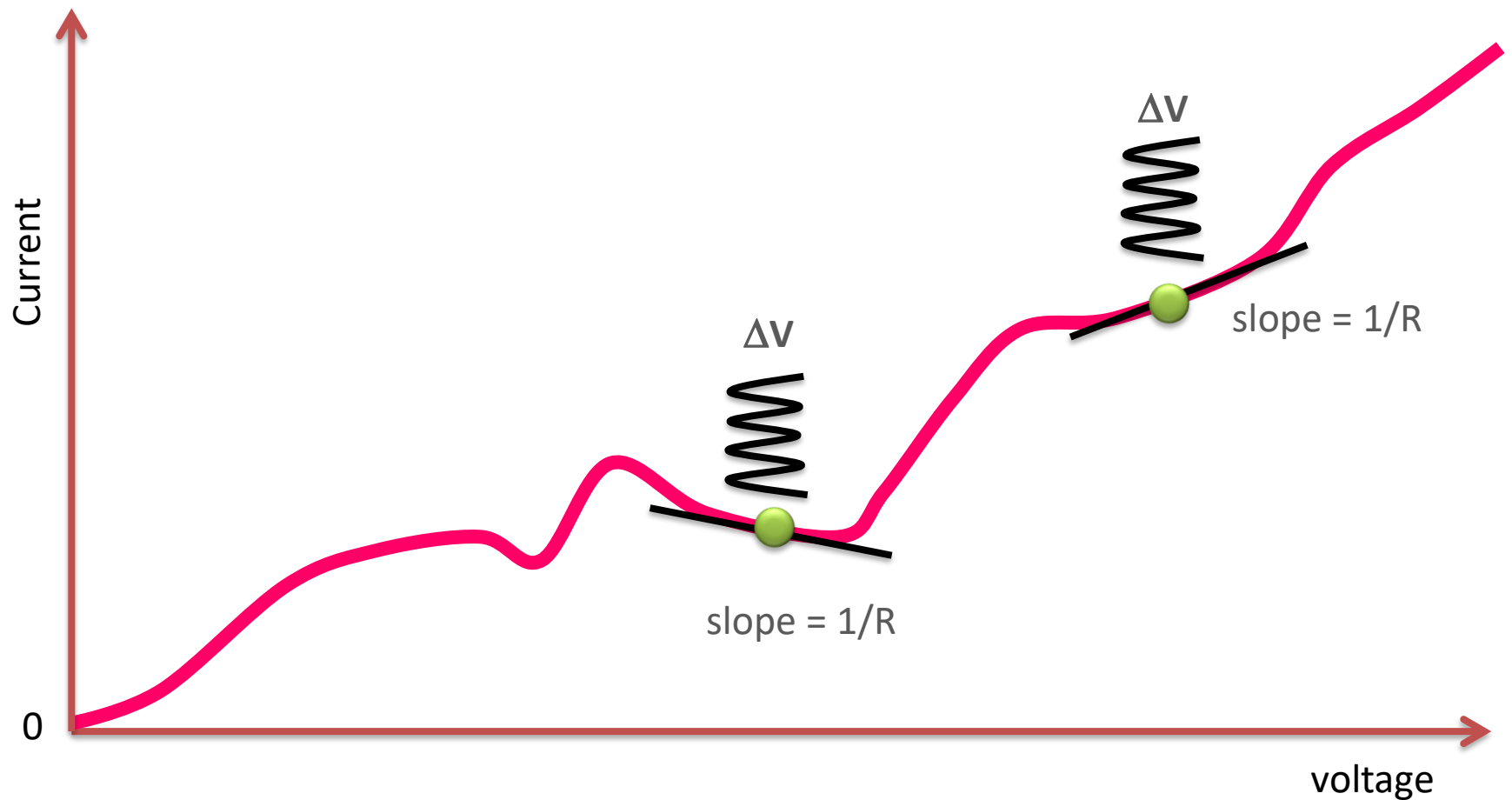
If I take the total voltage dropped, and the total current...

I get a value of resistance given by  $\Delta V = I * R$



Now imagine the numbers where  $R=100$ ,  $V=10$ ,  $I= 0.1$

Looking at this curve, what would happen to the current if I increased the voltage a little... would it go up or down?



A small change in voltage causes a small change in current

$$\Delta V = R_{\text{small}} (\Delta I)$$

This Resistance to small changes is **not always positive** and it's clearly **not always the same** for different parts of the curve and it's not the **same as the resistance for large signals**

# Small Signals and Large Signals

**Small Signals** are small changes that are so small that they stay in a region on the curve where everything looks like a **straight line** (we call straight-line regions **LINEAR**) **(this is the correct exam-paper answer)**

**Small Signals** are always considered to be time-varying signals with zero average values – often picked up from the sensors and need to be amplified.

**Small Signal Resistance** is given by

$$\Delta V = R_{\text{small}} (\Delta I)$$

**Small signals** use lower case letters ( $v, i$ )

**Large signals** use upper case letters ( $V, I$ )

**Large signals** are any and all signals that are not small

**Large signal Resistance** is given by

$$\Delta V = R (I)$$

# What is Biasing

The **Bias Point** is where you **get the behaviour** you want for your small signals. You choose it and then you design your biasing. A good bias point is far away from sharp changes in your curve.

**Biasing** is the process of **getting to the point** in the curve that you want. The curve represents the behaviour of the device

**Biasing** is done through the use **of DC signals**.

**Small Signals** are small changes that are so small that they stay in a linear region on the curve, anything between 10 and 100 times smaller than the big signals is a good definition of small.

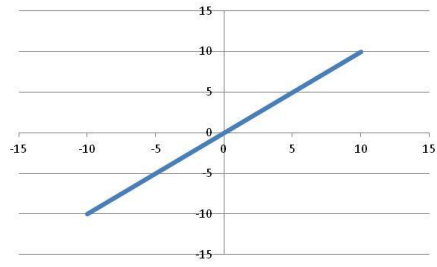
**Your real signal** is your **small signal + biasing**. Or else it's just a very small signal and won't let you do much.



The slide features a decorative element on the left side consisting of several vertical bars of varying heights and shades of green, creating a layered, totem-like effect.

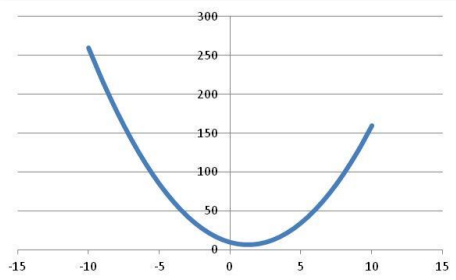
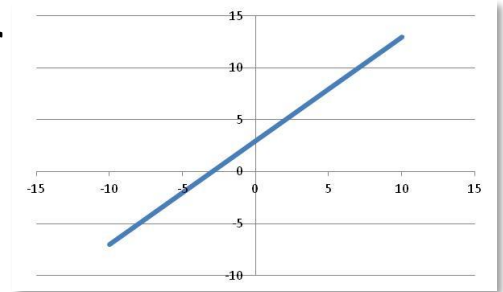
Some more curves or models....

# Linear, Quadratic, Cubic, etc...



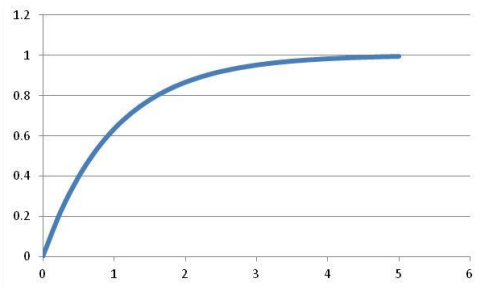
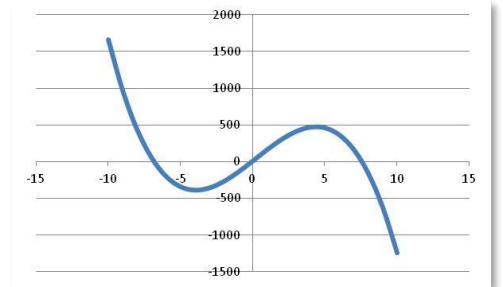
**Linear**, special term  
**AFFINE** as it goes through  
the origin. It does not  
need to be 1:1, just a  
straight line

**Linear**



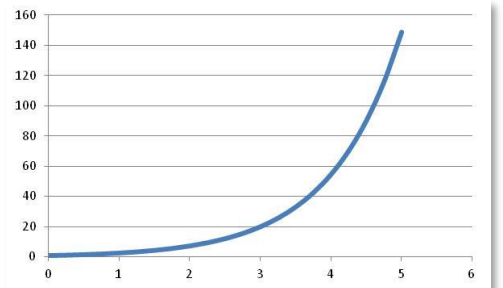
**Quadratic**

**Cubic**

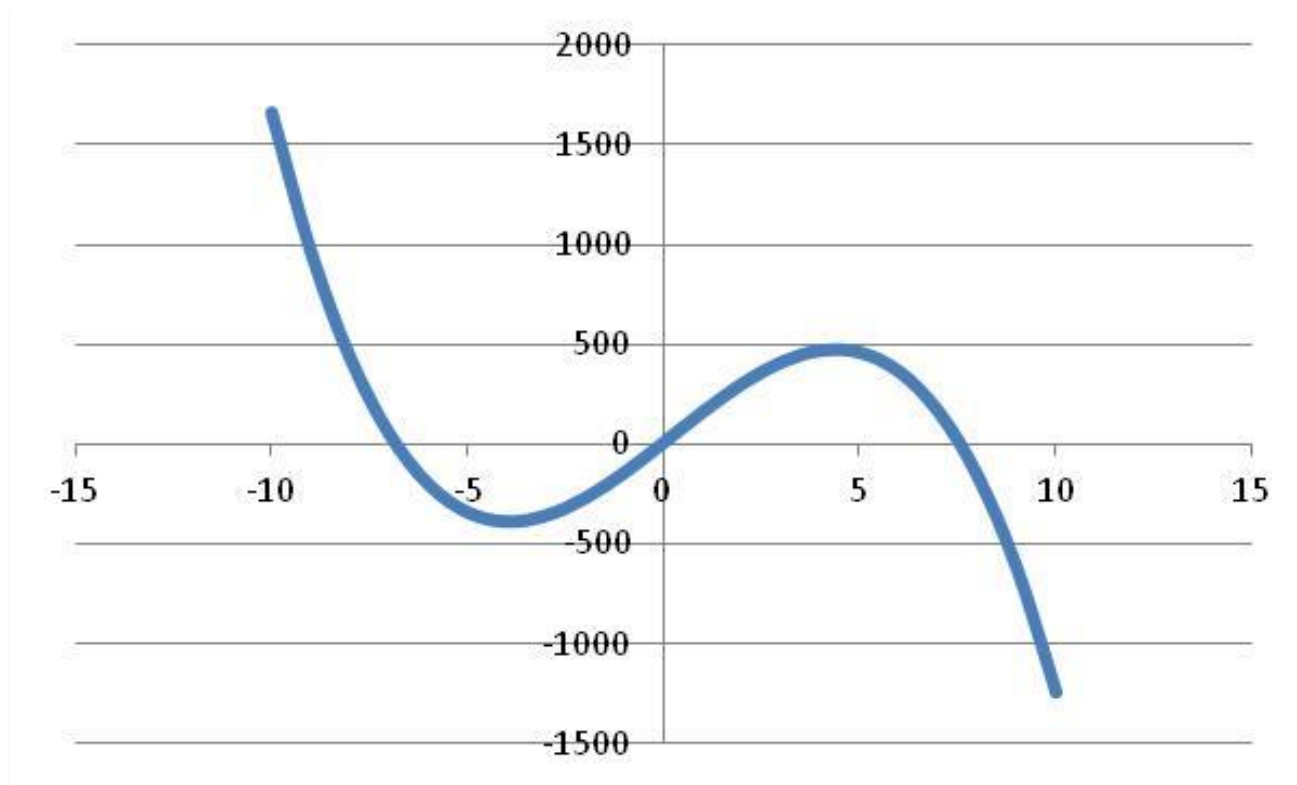


**Asymptotic**  
exponential but  
there are other  
forms that do  
this too

**Exponential**



# Piecewise Linear



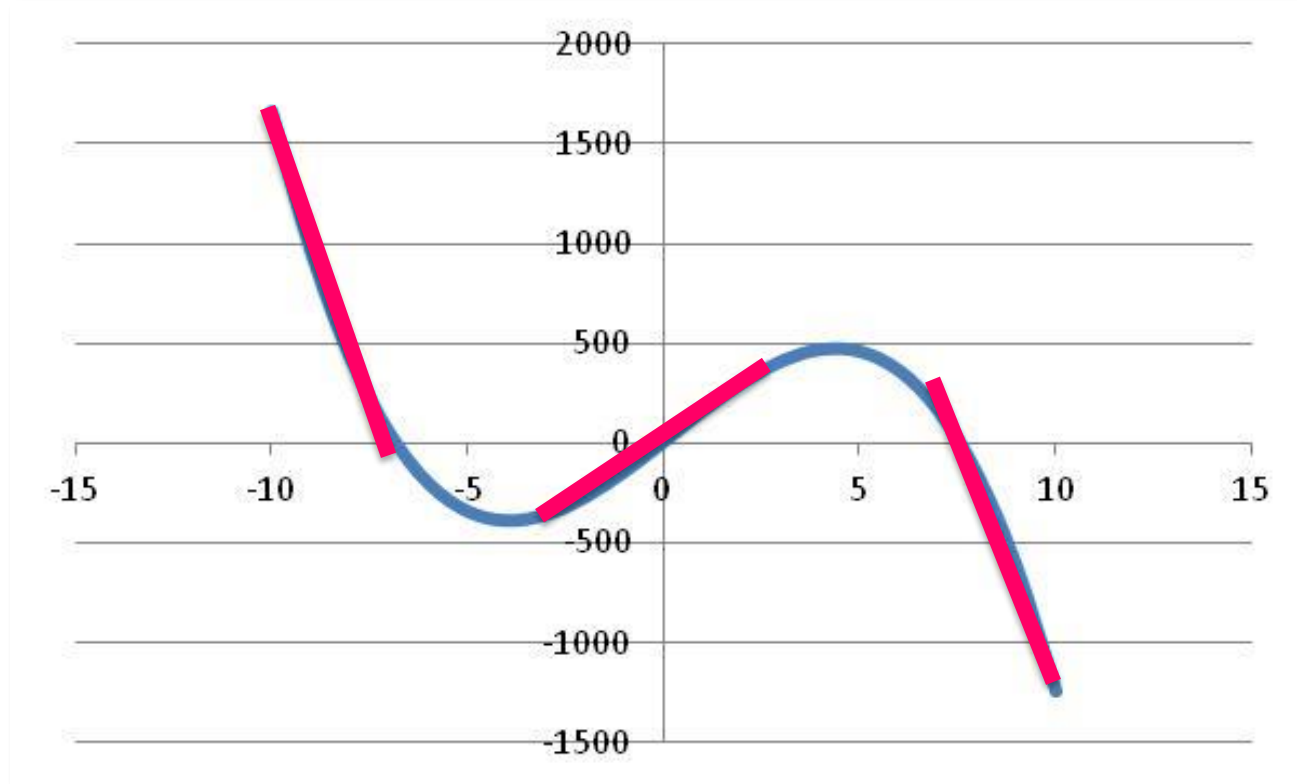
**We say something is piecewise linear if we can remake the curve using a bunch of linear sections. It cannot ever be a perfect fit unless the pieces are infinitely small. The smaller the pieces, the better the fit.**

# Piecewise Linear – small pieces



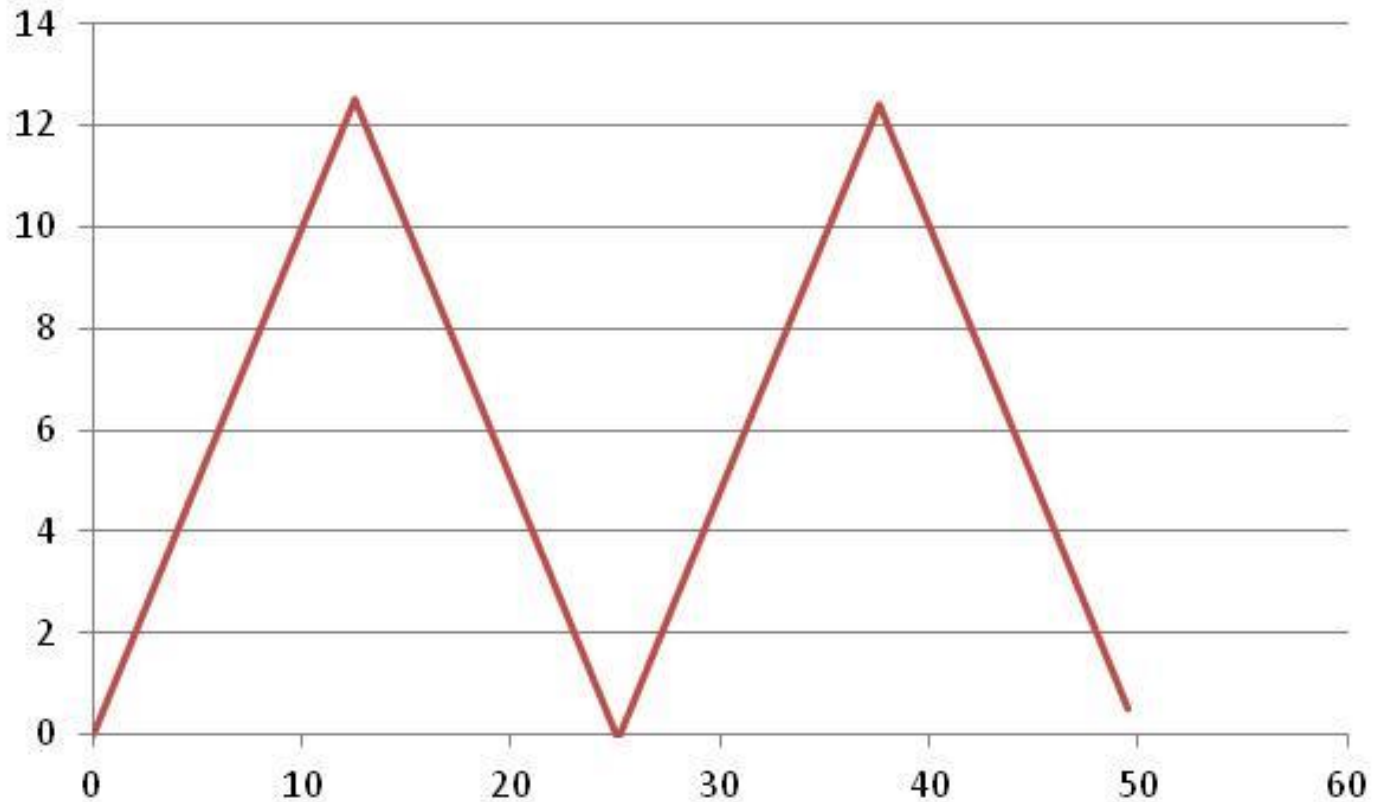
**This should remind you of numerical techniques for doing integration (simpson's rule)**

# Piecewise Linear – small pieces



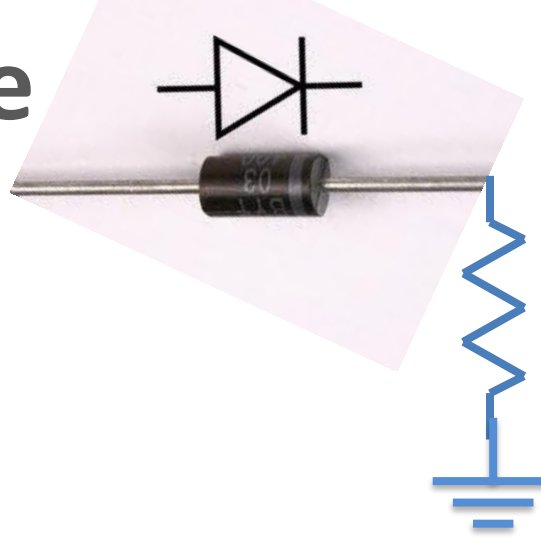
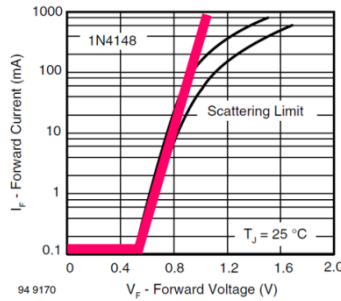
Usually when we are doing piecewise linear approximations... We ignore the messy bits and only look at whether there are large regions that can be well approximated by a straight line... Otherwise it's not that useful to us.

# Piecewise Linear – small pieces



Some shapes are very easy to represent as piece-wise linear, provided you avoid wondering what happens at the junctions 😊

# An example

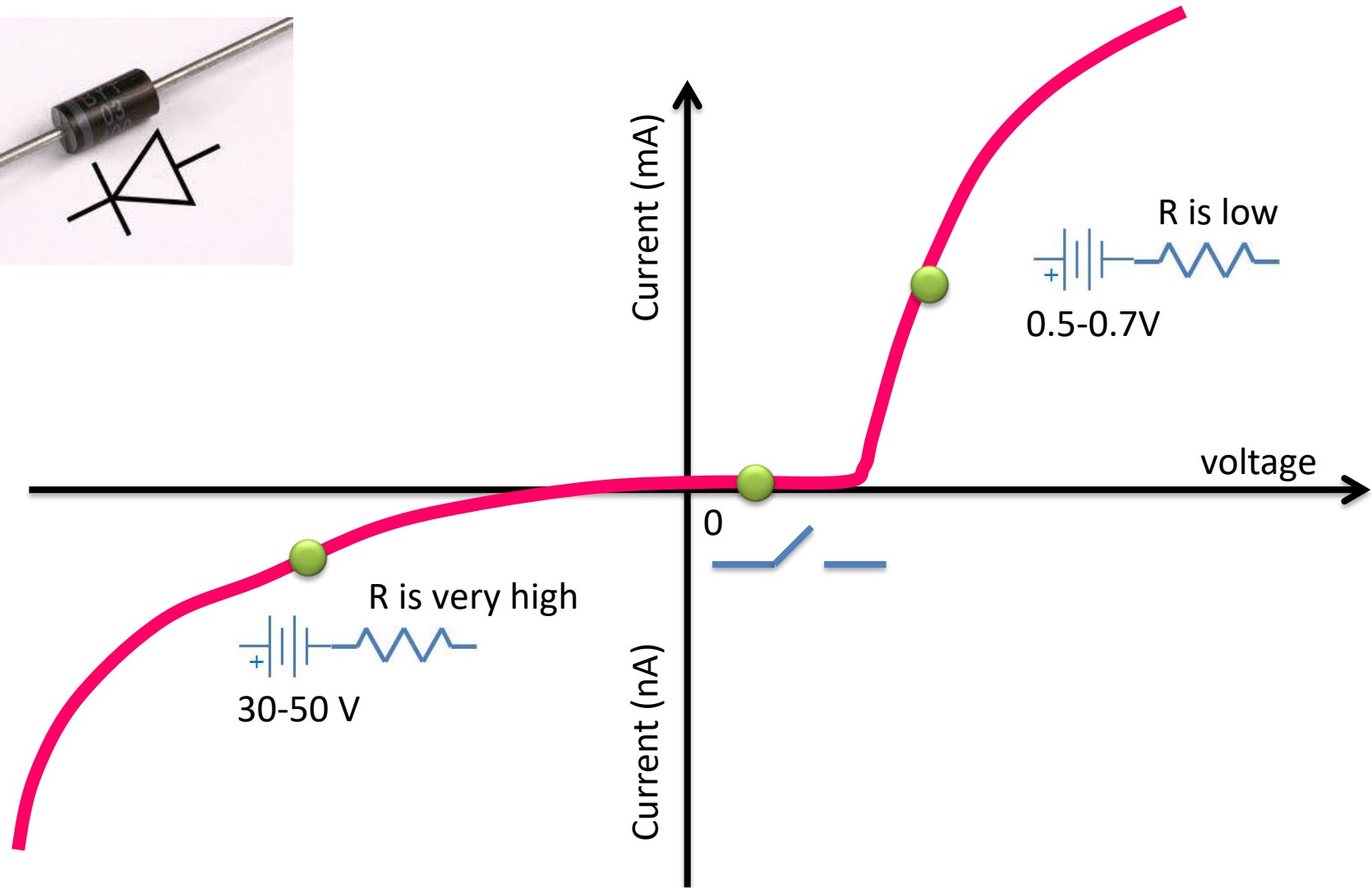


Threshold voltage = 0.7 Volts  
On resistance of 1 ohm  
Load Resistance of 100 ohms

I input a sinusoidal signal of amplitude  $\pm 100\text{ mV}$ ,  
what voltage do I see on the resistor? Sketch!

I input a sinusoidal signal of amplitude  $\pm 100\text{ mV}$  with a  $+1\text{ V}$  constant offset  
what voltage do I see on the resistor? Sketch!

# Looking at a Diode





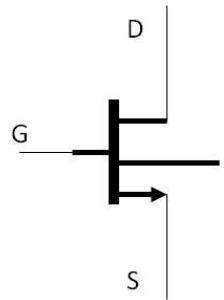
# Equations with more than 1 input

For diodes, capacitors, resistors, inductors, the current is dependent only one factor, the voltage applied across it.

There are some devices where the current that flows through it is dependent on **MORE THAN ONE VALUE**. The most important one of these is a transistor

More on this later, but one type of transistor has the following equation for current flow

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$



# Scary Solid-state equations

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$I_D = I_s \left( \exp\left(\frac{V_D}{nV_T}\right) - 1 \right)$$

You will not be asked to derive them or memorise them

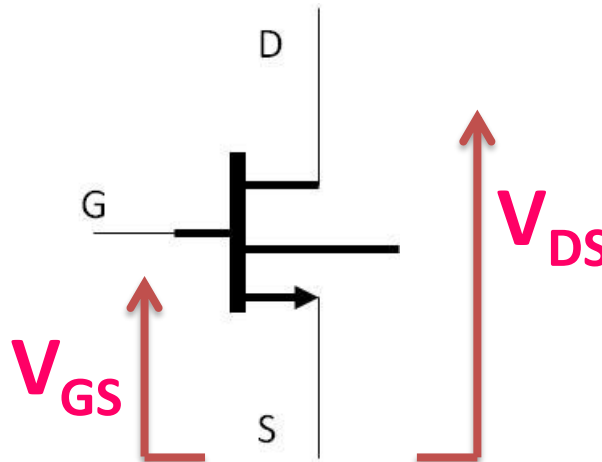
You do need to be able to interpret them, which bits are constant and which bits cause a change

They explain behaviours but they're not perfect either

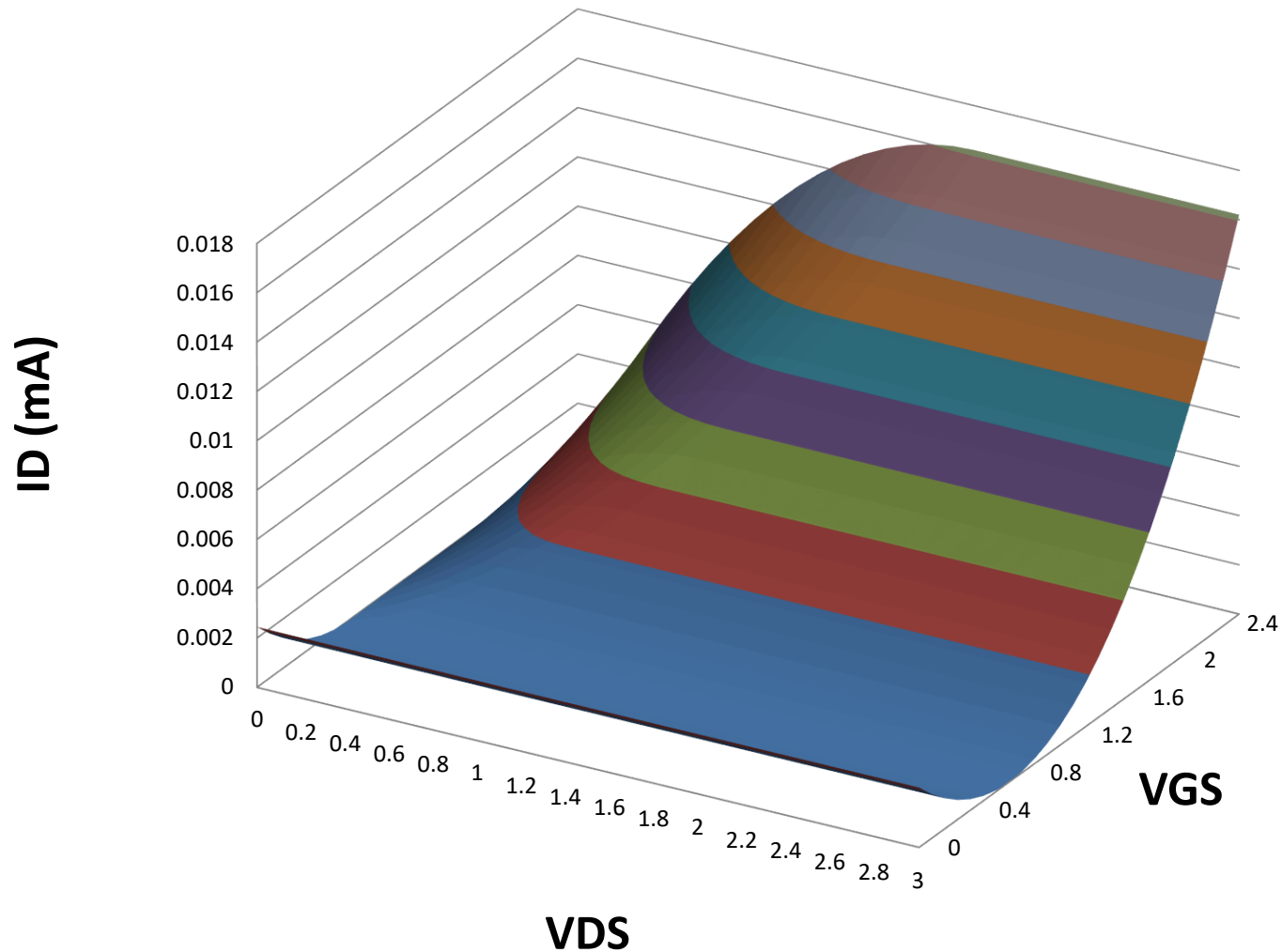
# Explaining this equation – just a little

These are constants that depend on the construction and physical make up of the transistor. They do not change

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

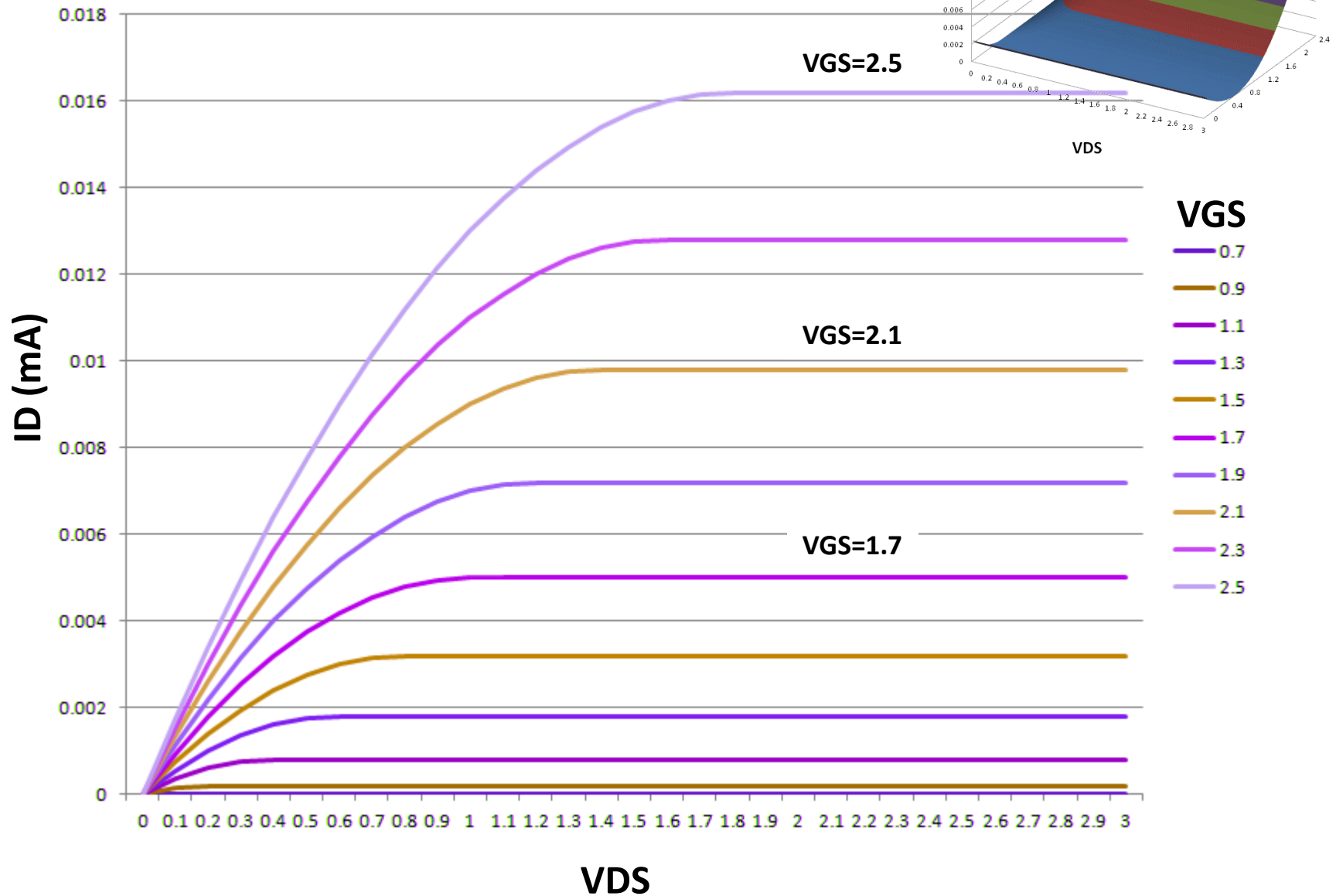


# Plotting this equation



This was drawn from the equations.. Can anyone spot the problem???

# Plotting this equation



# 3D Models

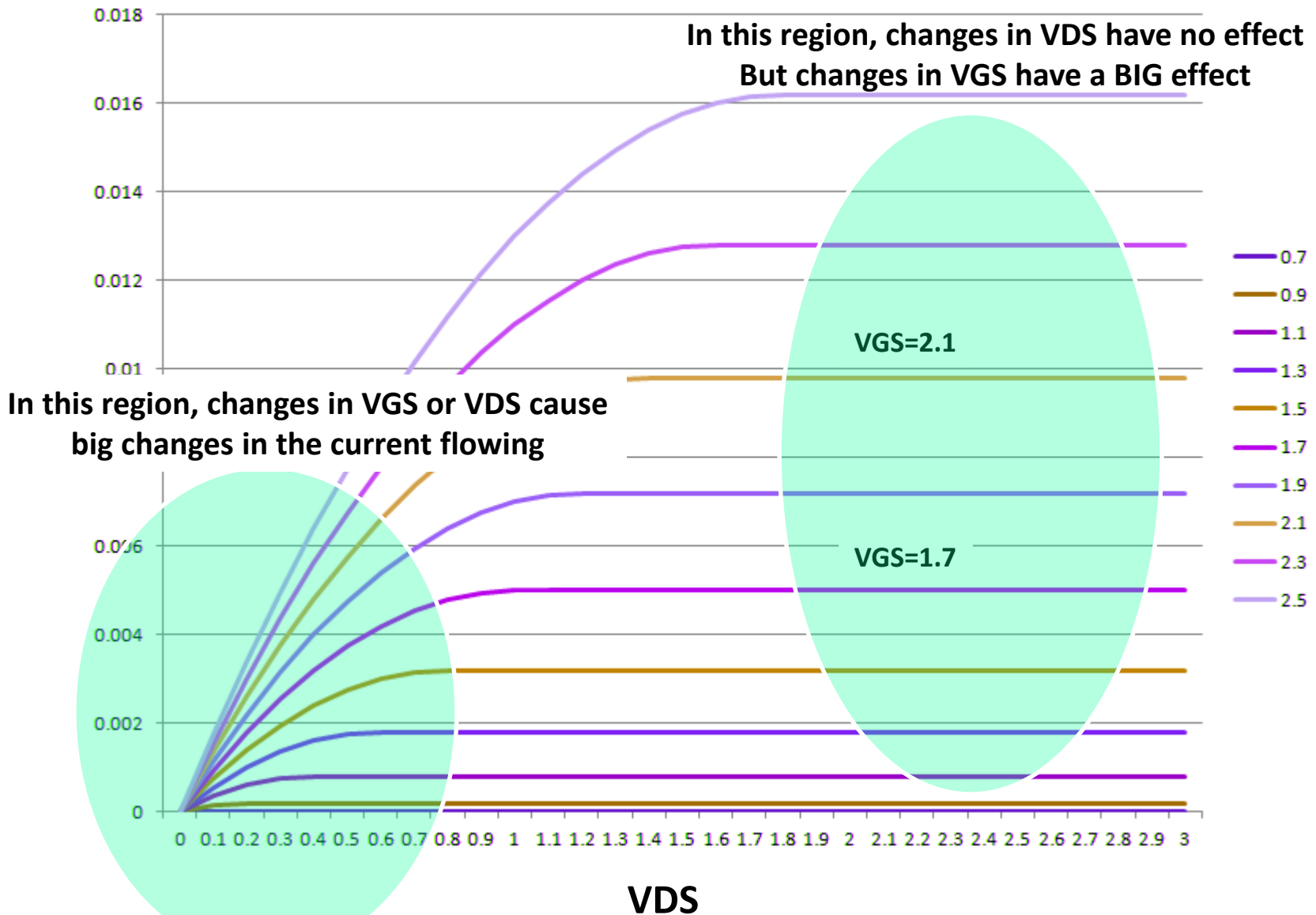
Much harder to visually interpret

These lines are smooth but reality definitely does not provide us with smooth lines

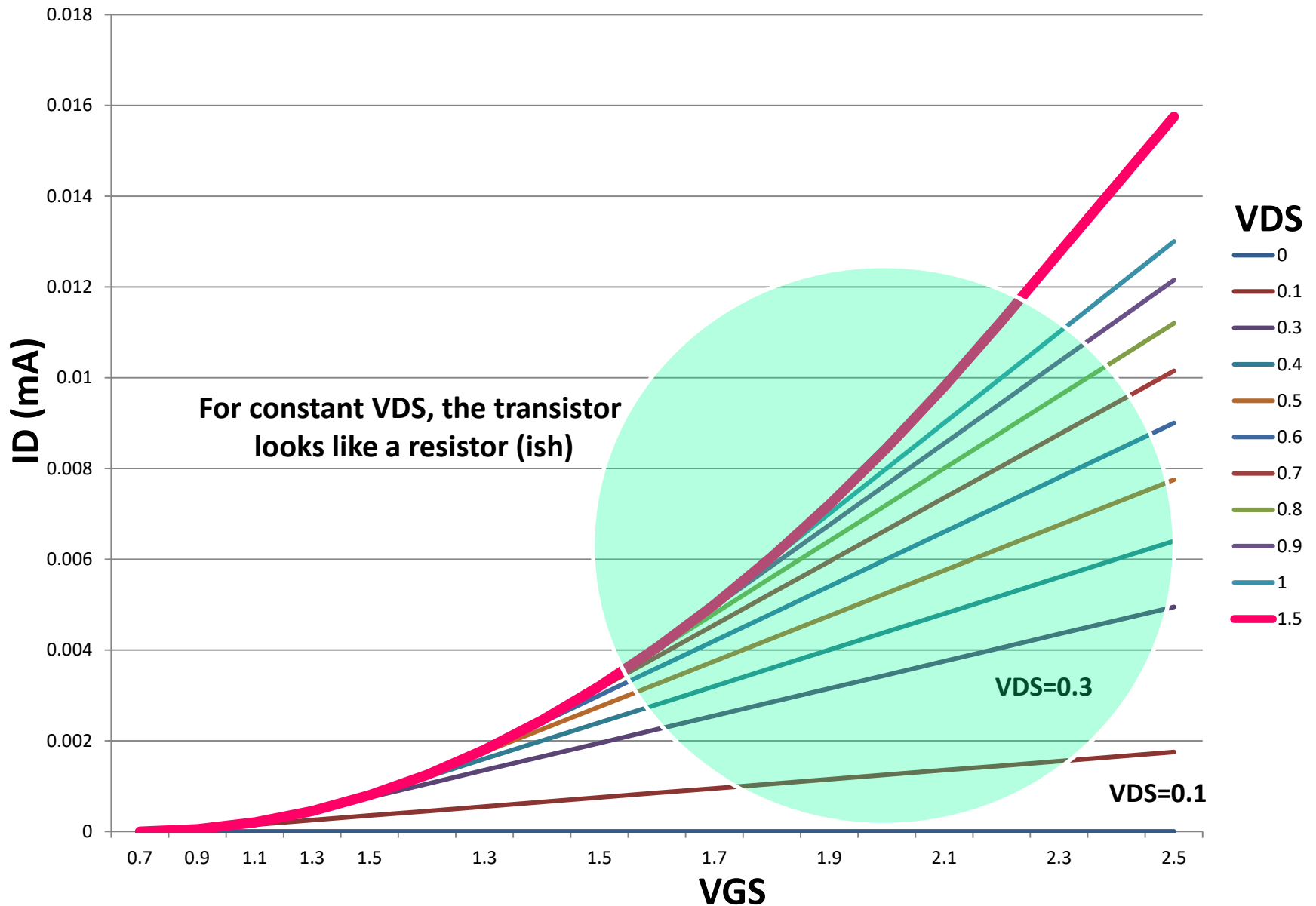
What we are looking for is something that we can use in these models... Can we get them to do something interesting or useful?

So lets look at the plots and see if anything jumps out (straight lines are always useful).


# Plotting this equation



# Plotting this equation (different X Axis)







Now we are going to use the different behaviours for small and large signals to make amplifiers