EE114
Intro to Systems & Control

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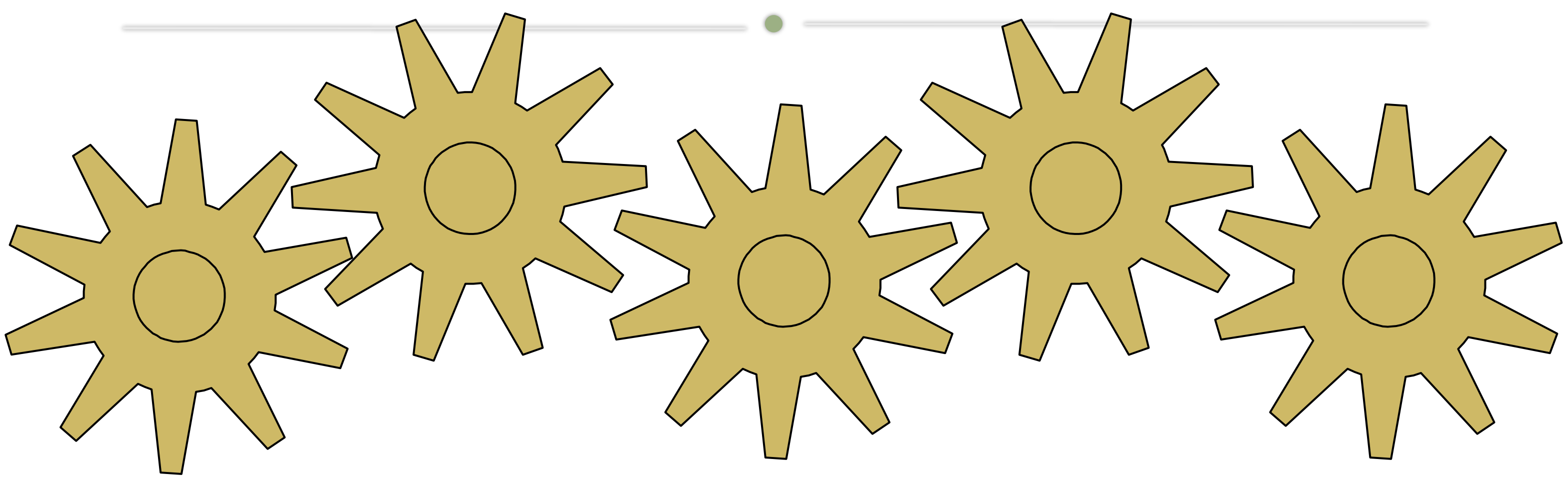
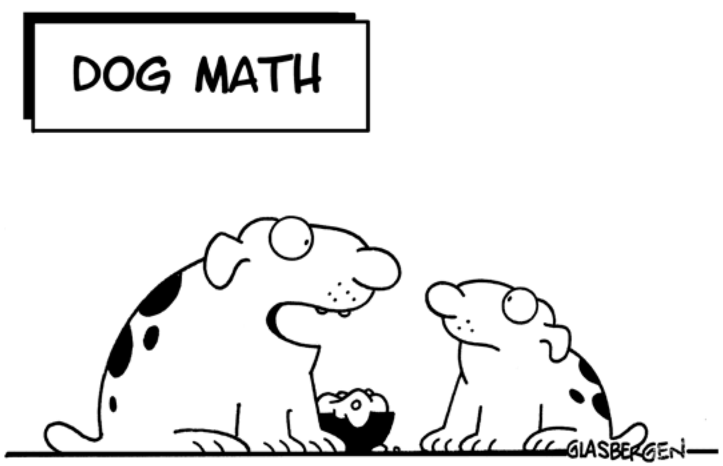


So far …

- We’ve introduced the concept of control & feedback control …
- We’ve talked about systems …
- We’ve started to model simple dynamical systems - an RC circuit, thus far …



“If I have 3 bones and Mr. Jones takes away 2, how many fingers will he have left?”

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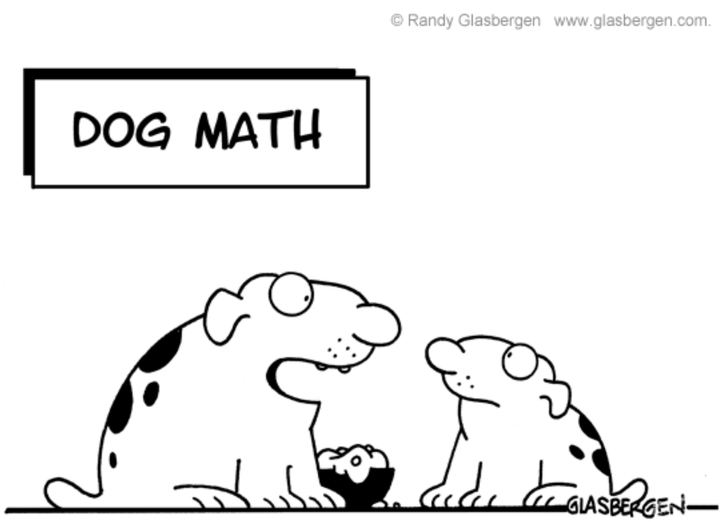

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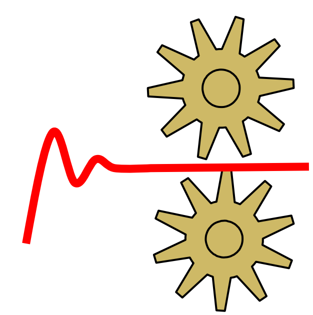
- **Today, we shall model some more dynamical systems …**



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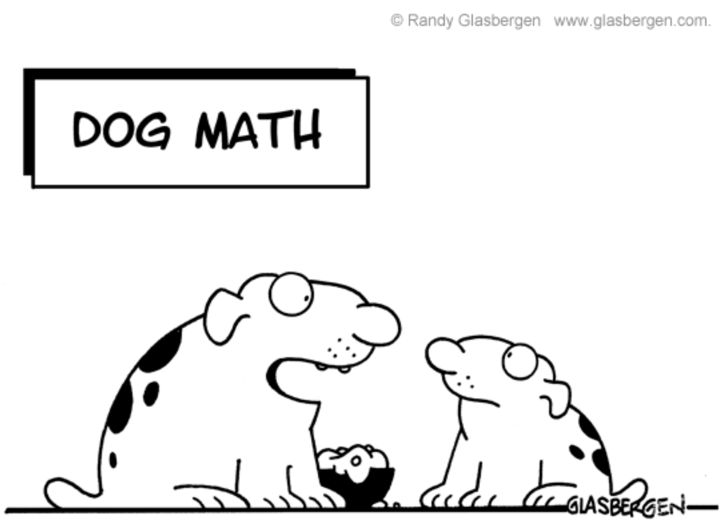

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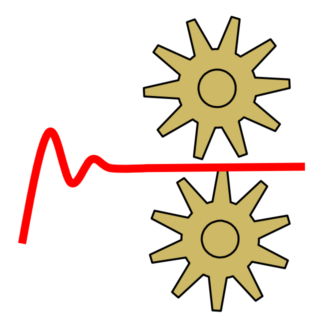
Fantastic !

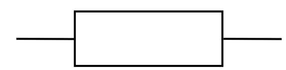
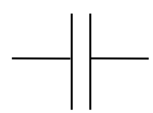
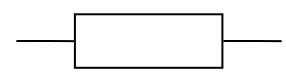


A Quick Recap …

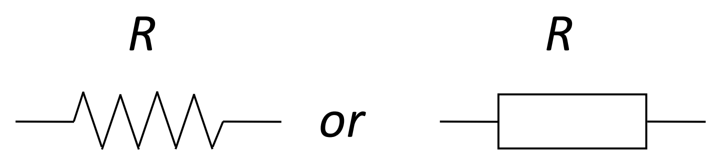
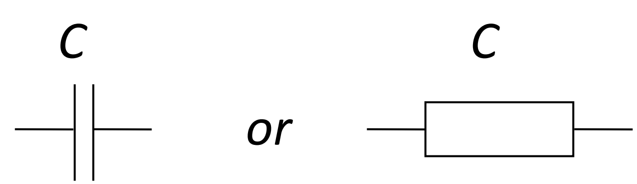
Modelling Electrical Systems

- Analytic procedure:
  - Physical model – circuit diagram.
  - Variables – voltages, currents.
  - Equilibrium relation – Kirchoff's Current Law (KCL).
  - Compatibility relation – Kirchoff's Voltage Law (KVL).
  - Physical relations are summarised in the following table:

Modelling Electrical Systems

Component	**Physical Law**	**Symbol**
Resistance (R)	$v = iR$	*R*  *or* *R* 
Inductance (L)	$$v = L\frac{di}{dt}$$	*L*  *or* *L* 
Capacitance (C)	$$v = \frac{1}{C}\int i\,dt$$ or $$i = C\frac{dv}{dt}$$	*C*  *or* *C* 

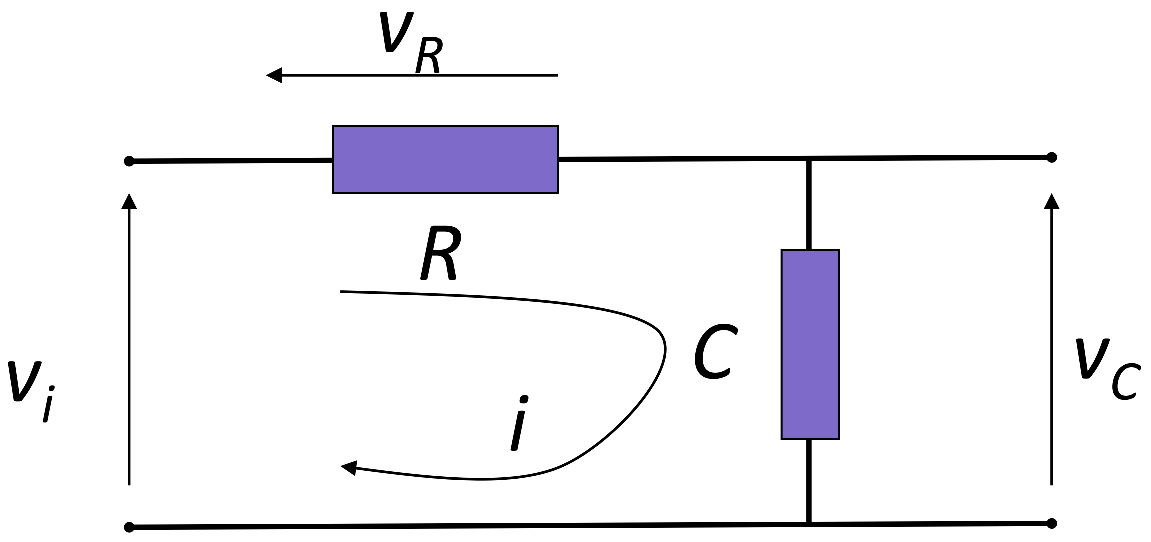
Modelling Electrical Systems

Component	Physical Law	Symbol
Resistance (R)	$v = iR$	
Inductance (L)	$v = L\frac{di}{dt}$	
Capacitance (C)	$v = \frac{1}{C}\int i dt$ or $i = C\frac{dv}{dt}$	

Note that we are assuming lumped parameters and ideal components.

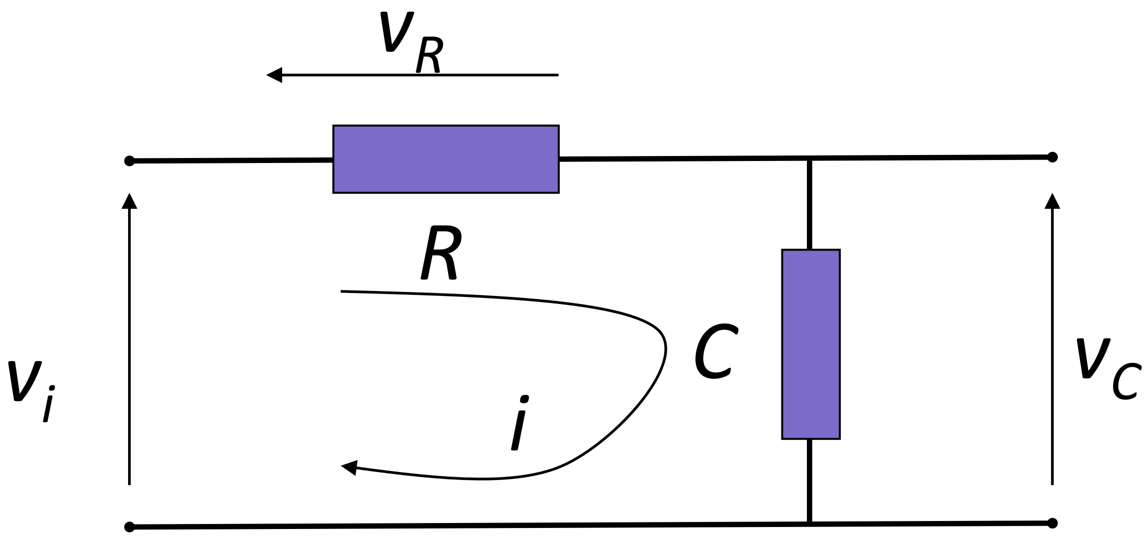
Modelling Electrical Systems

- ***Ex 3.3 Determine a mathematical model for the resistor/ capacitor filter circuit shown below:***



Modelling Electrical Systems

- ***Ex 3.3 Determine a mathematical model for the resistor/ capacitor filter circuit shown below:***



$$v_i = RC\frac{dv_C}{dt} + v_C$$

Continuing …

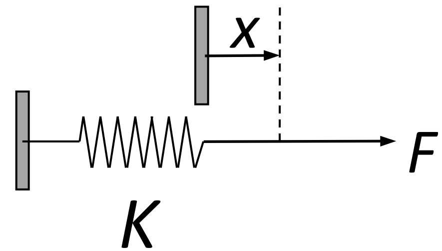
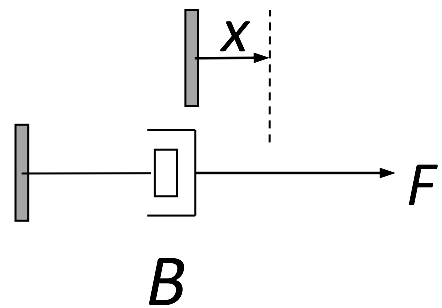
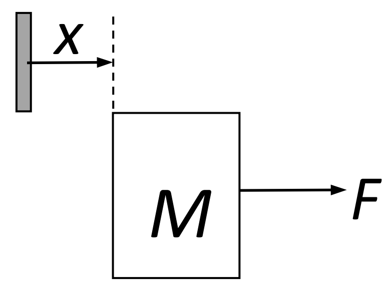
Modelling Mechanical Systems

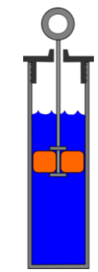
Modelling Mechanical Systems

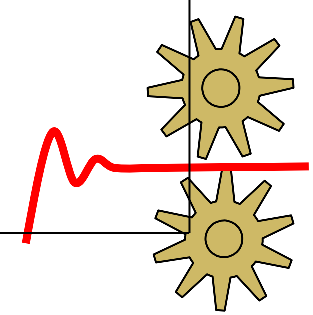
- Analytic procedure:
  - Physical model – schematic showing geometry of the system with respect to an arbitrary configuration and reference co-ordinate frame.
  - Variables – force and position.
  - Equilibrium/ Compatibility relations – Energy conservation or force equilibrium.
  - Physical relations (assuming ideal components):



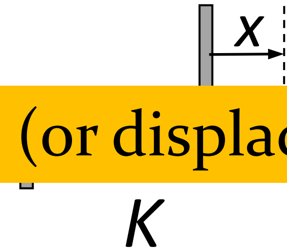
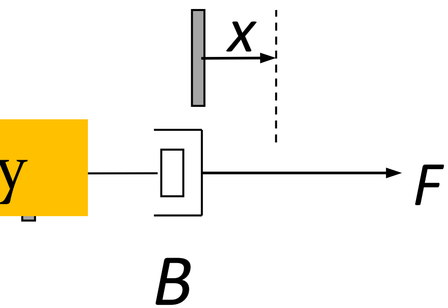
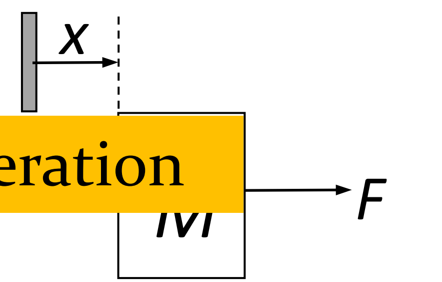
Modelling Mechanical Systems

Component	Physical Law	Symbol
Spring (K)	$F = Kx$	
Damper (or Dashpot) (B)	$F = B\dot{x}$	
Mass (M)	$F = M\ddot{x}$	





Modelling Mechanical Systems

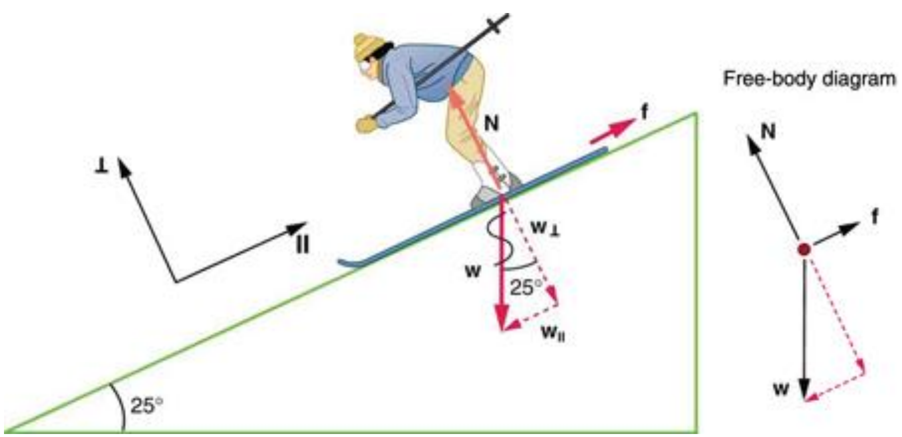
Component	Physical Law	Symbol
Spring (K)	$F = Kx$ Position (or displacement)	
Damper (or Dashpot) (B)	$F = B\dot{x}$ Velocity	
Mass (M)	$F = M\ddot{x}$ Acceleration	

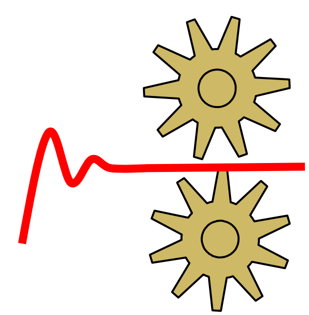
Modelling Mechanical Systems

- **Free body diagram** – In mechanics the concept of a free body diagram (FBD) (or force diagram) is used to analyse mechanical systems.

- Each mass is viewed as a free body isolated from the rest of the system with only the forces acting on it shown.

- Force balance equations are then written for each mass.





Modelling Mechanical Systems

- ***Ex 3.6(a) Determine a mathematical model for the spring-mass-damper system, whose physical model is shown in step 1 below:***

Modelling Mechanical Systems

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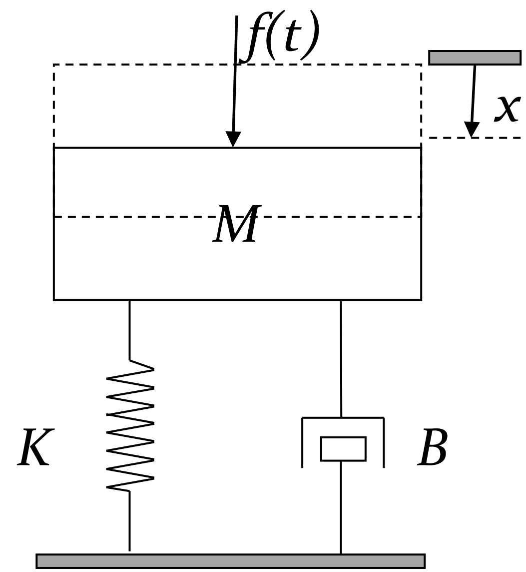
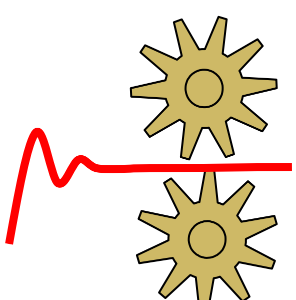
Step 1: Physical model assuming ideal components:

Actual system – ‘Suspension’ within a shoe or a bicycle for example



Physical model



Modelling Mechanical Systems

Solution …

Step 2: Model variables defined on the physical model

Modelling Mechanical Systems

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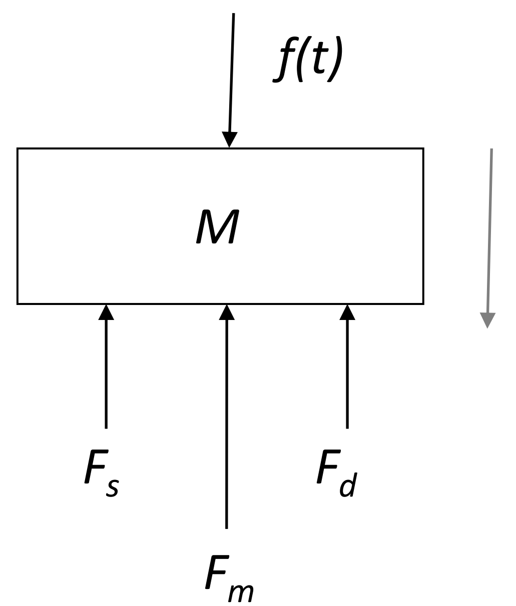
Step 3: From the free body diagram showing only the forces acting on *M* we obtain the Force equilibrium equation:



Free body diagram

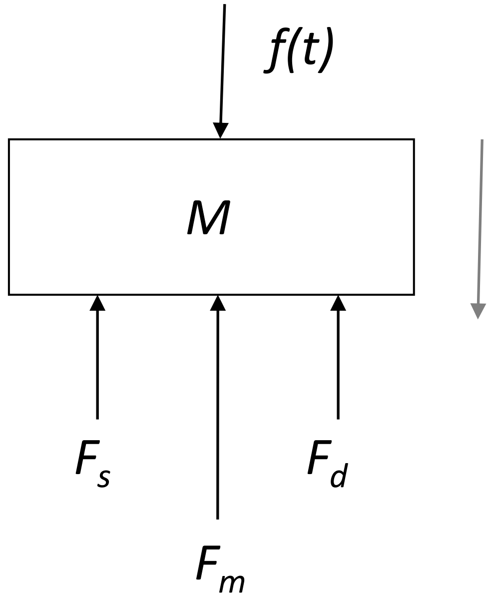
Modelling Mechanical Systems

Solution …

Step 2: Model variables defined on the physical model

Step 3: From the free body diagram showing only the forces acting on *M* we obtain the Force equilibrium equation:

$$F_m + F_d + F_s = f(t)$$

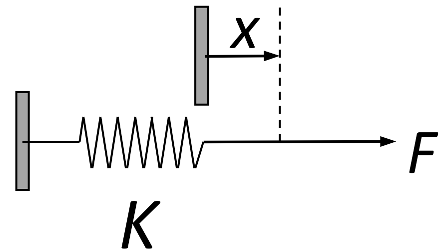
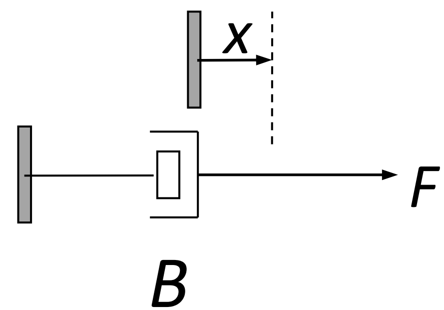
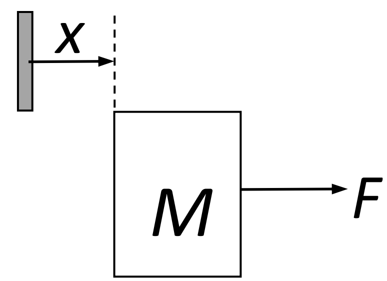


Free body diagram

Modelling Mechanical Systems

Solution … $$F_m + F_d + F_s = f(t)$$

Step 4: Using the physical force-geometry relations this becomes:

Component	Physical Law	Symbol
Spring (K)	$F = Kx$	
Damper (or Dashpot) (B)	$F = B\dot{x}$	
Mass (M)	$F = M\ddot{x}$	

Modelling Mechanical Systems

Solution … $$F_m + F_d + F_s = f(t)$$

Step 4: Using the physical force-geometry relations this becomes:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

Modelling Mechanical Systems

Solution …

$$F_m + F_d + F_s = f(t)$$

Step 4: Using the physical force-geometry relations this becomes:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

$$M\ddot{x} + B\dot{x} + Kx = f(t)$$

Modelling Mechanical Systems

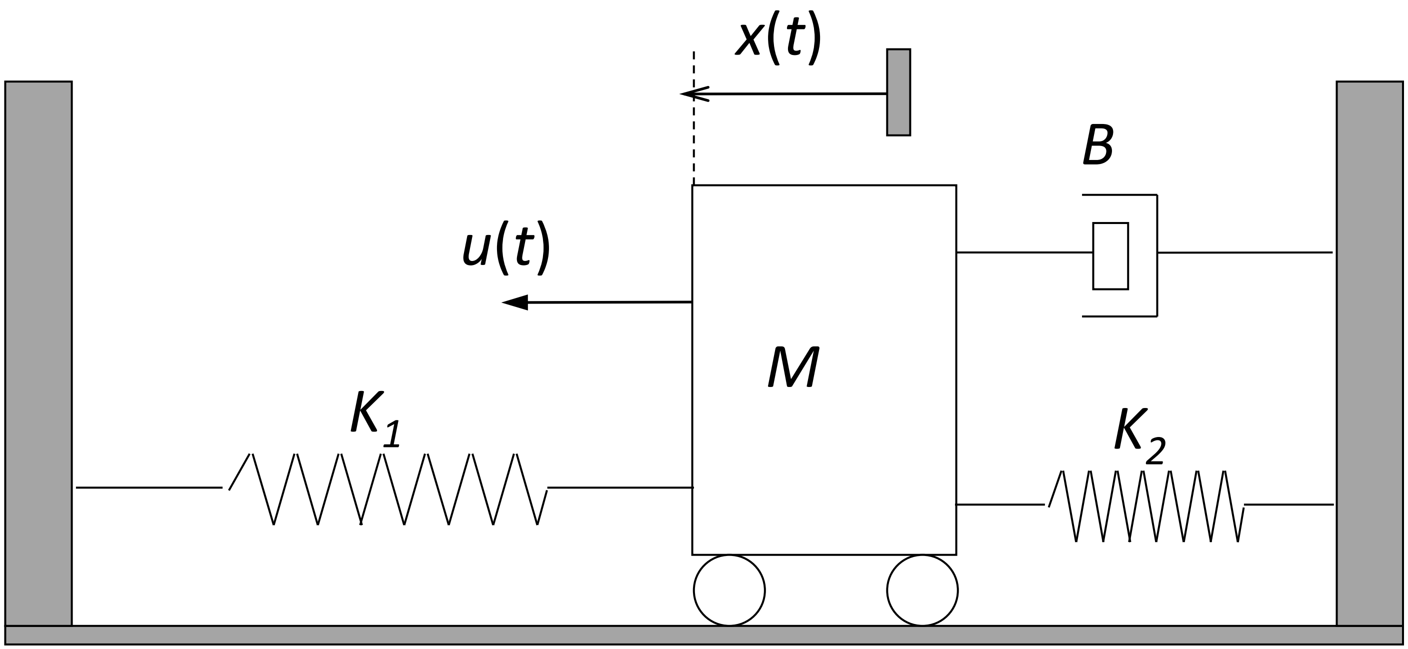
- ***Ex 3.6(b) Determine a mathematical model for a pogo stick (when in contact with the ground).***

Carry out this exercise yourself.

Hint: The pogo stick is effectively a mass-spring system!

Modelling Mechanical Systems

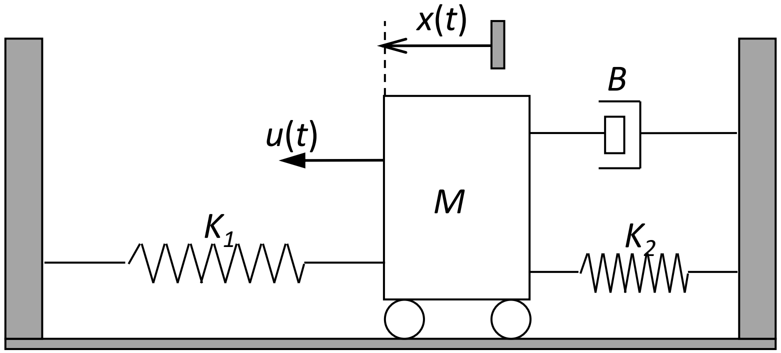
- ***Ex 3.7 Determine a mathematical model for the system, whose physical model is shown below:***



Physical Model

Modelling Mechanical Systems

Solution ...

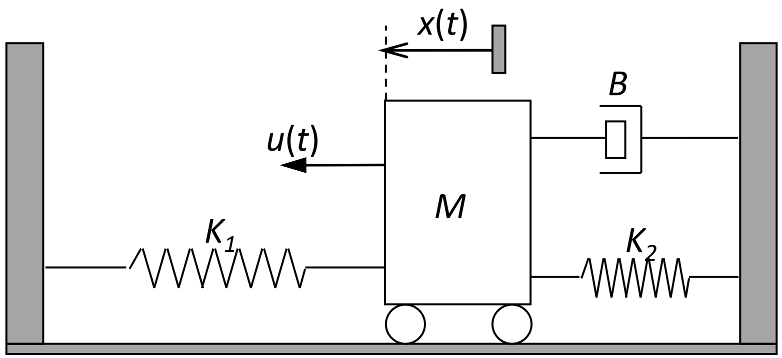


Physical Model

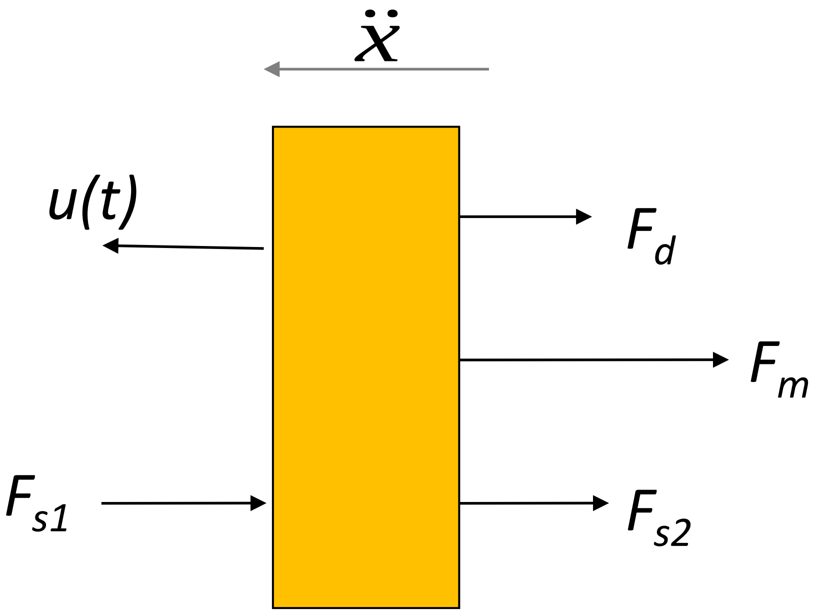
Modelling Mechanical Systems

Solution ...

Free Body Diagram for Mass *M* is given by:



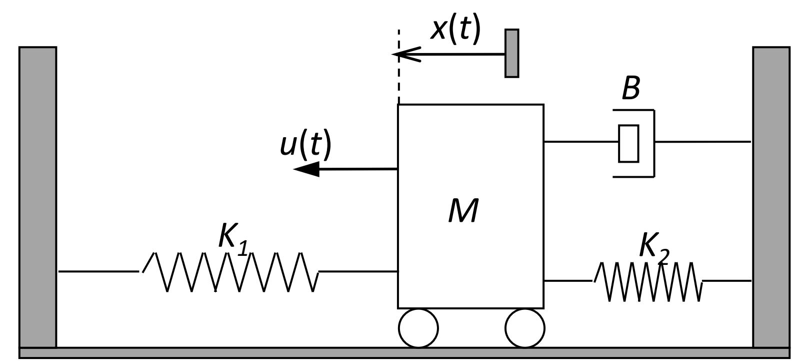
Physical Model



Free body diagram

Modelling Mechanical Systems

Solution …



Physical Model

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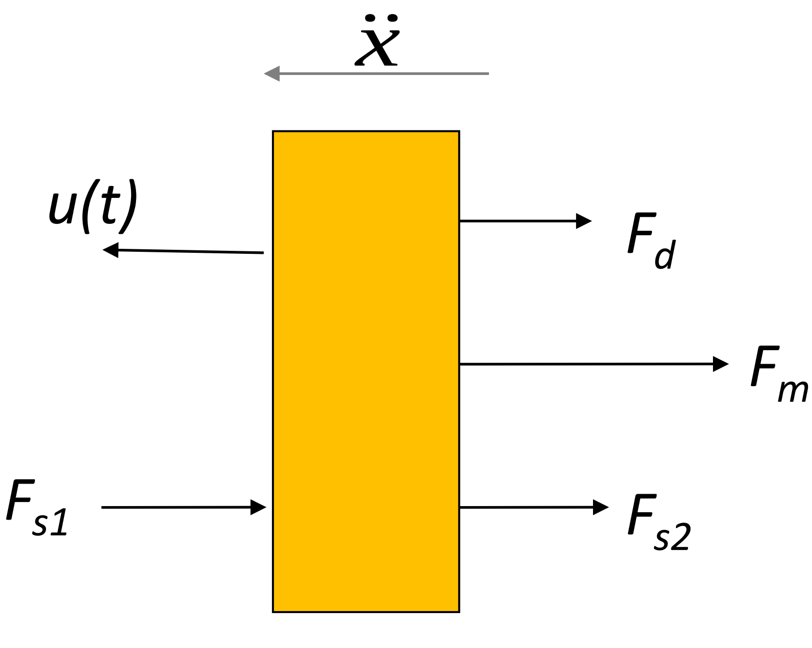


Free body diagram

Thus, the Force equilibrium equation is:

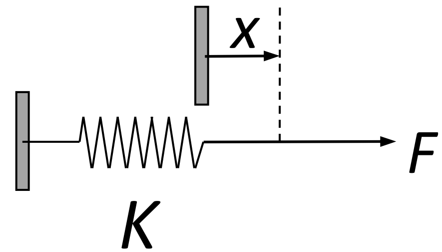
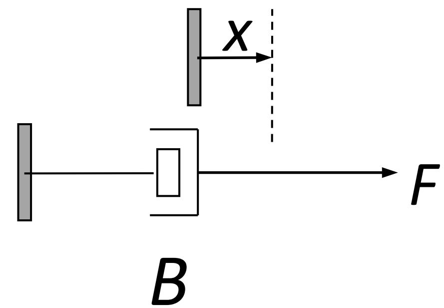
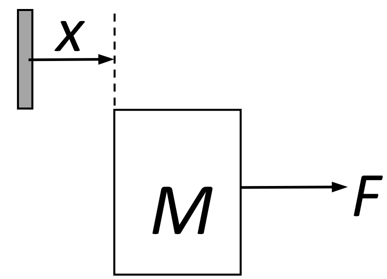
$$F_m + F_d + F_{s1} + F_{s2} = u(t)$$

Modelling Mechanical Systems

Solution …

$$F_m + F_d + F_{s1} + F_{s2} = u(t)$$

Using the physical force-geometry relations this becomes:

Component	Physical Law	Symbol
Spring (K)	$F = Kx$	
Damper (or Dashpot) (B)	$F = B\dot{x}$	
Mass (M)	$F = M\ddot{x}$	

Modelling Mechanical Systems

Solution …

$$F_m + F_d + F_{s1} + F_{s2} = u(t)$$

Using the physical force-geometry relations this becomes:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + K_1 x + K_2 x = u(t)$$

Modelling Mechanical Systems

Solution ...

$$F_m + F_d + F_{s1} + F_{s2} = u(t)$$

Using the physical force-geometry relations this becomes:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + K_1x + K_2x = u(t)$$

$$M\ddot{x} + B\dot{x} + (K_1 + K_2)x = u(t)$$

Modelling Mechanical Systems

Solution …

$$F_m + F_d + F_{s1} + F_{s2} = u(t)$$

Using the physical force-geometry relations thi

Note, by way of assumptions in the previous examples, we assume that we are using ideal components (springs, dampers) and that there is no friction between the mass and the ground.

$$M\ddot{x} + B\dot{x} + (K_1 + K_2)x = u(t)$$