Tutorial Sheet 6 - Solutions

Q1/Q2 Refer to Notes.

Q3 $G(s)=\dfrac{3K_p}{s-2}, H(s)=1$ Hence: CLTF $\dfrac{\frac{3K_p}{s-2}}{1+\frac{3K_p}{s-2}} \Rightarrow \dfrac{3K_p}{s-2+3K_p}$

Pole given by: $s-2+3K_p=0 \Rightarrow s=2-3K_p$

For stability, Re(s) < 0: $2-3K_p<0 \Rightarrow K_p>0.67$

Q4 $G(s)=\dfrac{K_p}{s+1}, H(s)=1$. Hence:

CLTF $\dfrac{\frac{K_p}{s+1}}{1+\frac{K_p}{s+1}} \Rightarrow \dfrac{K_p}{s+1+K_p} \Rightarrow \dfrac{\frac{K_p}{1+K_p}}{1+s\left(\frac{1}{1+K_p}\right)} \equiv \dfrac{K}{1+s\tau}$ (*standard first order system*)

Hence the time constant is given by: $\tau=\dfrac{1}{1+K_p}$

We know that the 2% settling time is 4τ, which has to be equal to 1s.

Hence: $4\tau=\dfrac{4}{1+K_p}=1 \Rightarrow 4=1+K_p \Rightarrow K_p=3$

Q5 (i) CLTF $=\dfrac{K_p}{s+1+K_p}$. Setting s = 0 gives the gain of the system: $\dfrac{K_p}{1+K_p}$

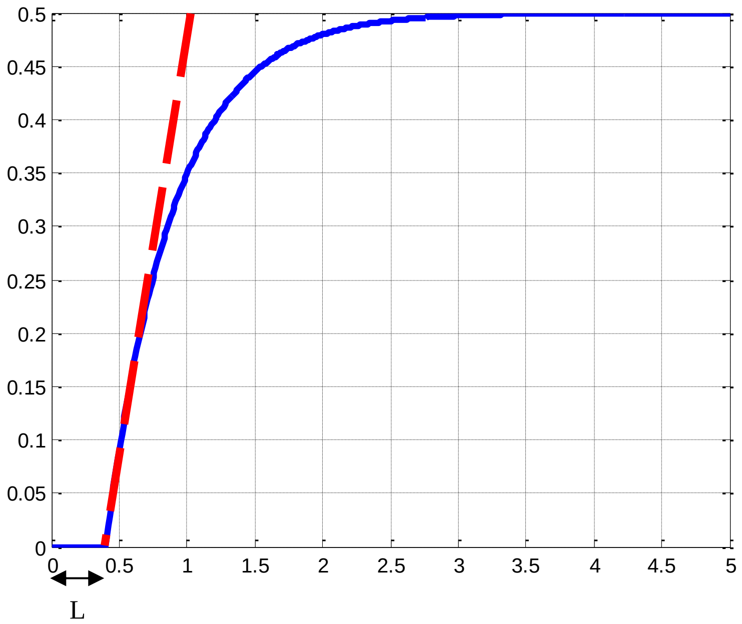
Thus for a **unit** step input the steady-state output will be: $\dfrac{K_p}{1+K_p}$

Hence, the steady-state error is: $1-\dfrac{K_p}{1+K_p}=\dfrac{1+K_p-K_p}{1+K_p}=\dfrac{1}{1+K_p}$

(ii) No, as in order to get 0 error, we would need $K_p=\infty$, which is not practical!

(iii) PID solves this problem by the inclusion of integral action. The integral action eliminates the error completely.

Q6



From the graph, we obtain:

$$L = 0.4 \qquad \text{(the dead time)}$$

and $$R = \frac{0.5}{0.6} = \frac{5}{6} \qquad \text{(the steepest slope)}$$

Hence, from the Ziegler-Nichols table (refer to notes) we obtain the following PID parameters:

$$K_p = \frac{1.2}{RL} = 3.6, \quad t_i = 2L = 0.8 \quad \text{and} \quad t_d = 0.5L = 0.2$$

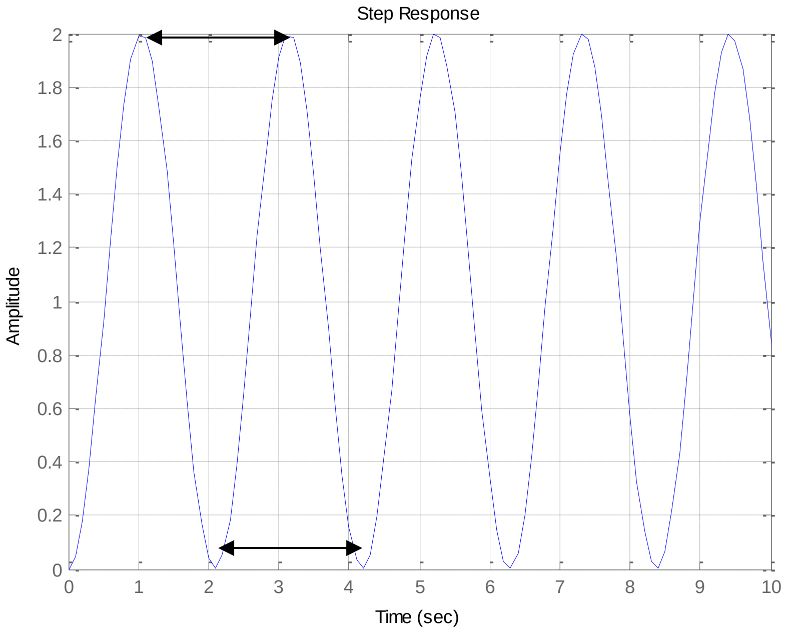
Hence: $$K_i = \frac{K_p}{t_i} = \frac{3.6}{0.8} = 4.5 \quad \text{and} \quad K_d = K_p t_d = (3.6)(0.2) = 0.72$$

Thus, the PID controller is given by:

$$G_c(s) = \frac{K_d s^2 + K_p s + K_i}{s} = \frac{0.72s^2 + 3.6s + 4.5}{s}$$

or: $$G_c(s) = K_p\left(1 + \frac{1}{t_i s} + t_d s\right) = 3.6\left(1 + \frac{1}{0.8s} + 0.2s\right)$$

Q7



$K_c = 3$ (given)

t_c = period of oscillations (as indicated in figure above) ≈ 2.1s

Hence, from the Ziegler-Nichols table (refer to notes) we obtain the following PID parameters:

$$K_p = 0.6K_c = 1.8, \quad t_i = \frac{t_c}{2} = 1.05 \quad \text{and} \quad t_d = \frac{t_c}{8} = 0.2625$$

Hence: $K_i = \dfrac{K_p}{t_i} = \dfrac{1.8}{1.05} = 1.71$ and $K_d = K_p t_d = (1.8)(0.2625) = 0.47$

Thus, the PID controller is given by:

$$G_c(s) = \frac{K_d s^2 + K_p s + K_i}{s} = \frac{0.47s^2 + 1.8s + 1.71}{s}$$

or: $$G_c(s) = K_p\left(1 + \frac{1}{t_i s} + t_d s\right) = 1.8\left(1 + \frac{1}{1.05s} + 0.625s\right)$$