Test 1

EE106FZ, Semester 1, 2021 Autumn (NOV 21)

Student Number: _____ Student Name: _____

1. [10 marks] Solve $\dfrac{3+2i}{1+5i} - (-2+i)(7-i) = a + b\,i$, and simplify $2a + 4b$ as a value.

2. [10 marks] By DeMoivre's theorem, find all cube roots of $z = -8i$.

3. [10 marks] Evaluate the sum of the series $\displaystyle\sum_{n=1}^{20} \Big(\frac{n^3}{1000} - 3n^2 + 5n - 1 \Big)$.

4. [10 marks] Find the solution to $f(n+2) - 8f(n+1) + 16f(n) = 0$, where $f(1) = 2, f(2) = 8$.

5. [10 marks] Use the ratio test to show convergence or divergence $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}}{5+n}$.

6. [10 marks] Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x^3 + y^3 - 3xy^2 = 8$.

7. [10 marks] If $x = 2(1 - \cos\theta)$ and $y = 2(\theta - \sin\theta)$, find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in the simplest form.

8. [10 marks] If $y = \sinh^{-1} x$, find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in the simplest form.

9. [10 marks] Find the points of inflexion on the following curve: $y = x + \cos x$, $x \in (-\dfrac{\pi}{3}, \dfrac{4\pi}{3})$.

10. [10 marks] Find the values of x at which maximum and minimum of y occur on the curve $y = (x-2)^2(x-7)$.