Lance

Assign 7   1130

$\boxed{\text{Q1. (a)}}$ $(f_1, f_2) = \int_0^{\lambda} \cos x \cdot \sin^3 x \, dx$

$= \int_0^{\lambda} \sin^3 x \, d(\sin x)$

$= \frac{1}{4} \sin^4 x \Big|_0^{\lambda}$

$= 0$ So they are orthogonal

$\boxed{\text{(b)}}$ $(f_1, f_2) = \int_{\frac{1}{4}\pi}^{\frac{5}{4}\pi} e^x \cdot \sin x \, dx$

$= \frac{(\sin x - \cos x) e^x}{2} \Big|_{\cancel{0}}^{\cancel{0}}$

$= \cancel{2} - 0 = 0$

So they're orthogonal

$\boxed{\text{(c) Sol}}$.

① $\int_0^P \sin\left(\frac{n\lambda x}{P}\right) \sin\left(\frac{m\lambda x}{P}\right) dx$

$= \frac{-P}{n\lambda} \cos\left(\frac{n\lambda x}{P}\right) \sin\left(\frac{m\lambda x}{P}\right)\Big|_0^P + \frac{m}{n}\int_0^P \cos\left(\frac{n\lambda x}{P}\right) \cos\left(\frac{m\lambda x}{P}\right) dx$ ①

② $\int_0^P \cos\left(\frac{n\lambda x}{P}\right) \cos\left(\frac{m\lambda x}{P}\right) dx$

$= \frac{P}{n\lambda} \sin\left(\frac{n\lambda x}{P}\right) \cos\left(\frac{m\lambda x}{P}\right)\Big|_0^P + \frac{m}{n}\int_0^P \sin\left(\frac{n\lambda x}{P}\right) \sin\left(\frac{m\lambda x}{P}\right) dx$ ②

By ① and ②,

So $\int_0^P \sin\left(\frac{n\lambda x}{P}\right) \sin\left(\frac{m\lambda x}{P}\right) dx$

$= \frac{m}{n} \cdot \frac{1}{1 - \frac{m^2}{n^2}} \cdot \frac{P}{n\lambda} \sin\left(\frac{n\lambda x}{P}\right) \cos\left(\frac{m\lambda x}{P}\right)\Big|_0^P$

$= 0$

That's all.

$\boxed{\text{Q2. (a)}}$

$\int_0^{\infty} f_1(x) f_2(x) \cdot \cancel{x0} \cdot w(x) \, dx$

$= \int_0^{\infty} \left(\frac{1}{2}x^2 - 2x + 1\right) e^{-x} dx$

$= \frac{1}{2}\left(-e^{-x} \cdot x^2\right)\Big|_0^{\infty} - \left(-e^{-x} \cdot x\right)\Big|_0^{\infty}$

$= \left[-e^{-x}\left(\frac{1}{2}x^2 - x\right)\right]_0^{\infty}$

$= -\lim_{x\to\infty} \frac{\frac{1}{2}x^2 - x}{e^x}$, $= -\lim_{x\to\infty} \frac{x-1}{e^x}$,

$= -\lim_{x\to\infty} \frac{1}{e^x} = 0$

$\boxed{\text{Q2 (b)}}$

$I = \int_0^{\infty} (-x+1) e^{-x} dx$

$= -\left(-e^{-x} \cdot x\right)\Big|_0^{\infty}$

$= \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$

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2. (a) sol.

$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx$
$= \frac{1}{2}\left(\int_{-2}^{0}(-1-\frac{1}{2}x)dx + \int_0^2 (1-\frac{1}{2}x)dx\right)$
$= 0$

$a_n = \frac{1}{2}\left[\int_{-2}^{0}(1-\frac{x}{2})\cos\frac{n\pi}{2}x\,dx + \int_0^2 (1-\frac{x}{2})\cos\frac{n\pi}{2}dx\right].$
$= \frac{1}{2}\left[(-1-\frac{x}{2})\frac{\sin\frac{n\pi}{2}x}{n\pi/2}\Big|_{-2}^{0} + \frac{2}{n\pi}\int_{-2}^{0}\frac{1}{2}\sin\frac{n\pi}{2}x\,dx\right] + \frac{1}{2}\left[(1-\frac{x}{2})\cdots\cdots\right]$
$= \frac{1}{2n\lambda}\left(-\frac{2}{n\lambda}\cos\frac{n\lambda}{2}x\right)\Big|_{-2}^{0} + \frac{1}{2n\lambda}\left(-\frac{2}{n\lambda}\cos\frac{n\lambda}{2}x\right)\Big|_{0}^{2}$
$= 0.$

$b_n = \frac{1}{2}\left[\int_{-2}^{0}(1-\frac{x}{2})\sin\frac{n\lambda}{2}x\,dx + \int_0^2 (1-x)\sin\frac{n\lambda}{2}x\,dx\right]$
$= \frac{1}{2}\cdot\frac{2}{\frac{n\lambda}{2}} = \frac{2}{n\lambda}$

3(b)

$a_0 = \int_{-1}^{0} 0\,dx + \int_0^1 \frac{e^{-10x}-e^{-10}}{1-e^{-10}}dx$
$= \frac{(1-11e^{-10})}{10-10e^{-10}}.$

$a_n = \int_0^1 \frac{e^{-10x}-e^{-10}}{1-e^{-10}}\cos n\lambda x\,dx$
$= \frac{n\lambda}{100+n^2\lambda^2}(1-e^{-10}\cos n\pi).$

$b_n = \int_0^1 \frac{e^{-10x}-e^{-10}}{1-e^{-10}}\sin n\lambda x\,dx$
$= \frac{1}{n\lambda} + \frac{10-10e^{-10}\cos n\pi}{100+n^2\lambda^2}.$

3(c)

$a_0 = \int_{-1}^{0}(x+1)^2dx + \int_0^1 (x-1)^2 dx$
$= (\frac{1}{3}-0) + (0-(-\frac{1}{3})) = \frac{2}{3}$

$a_n = \int_{-1}^{0}(x+1)^2\cos(n\lambda x)\,dx$
$= -\frac{2}{n\lambda}(-\frac{1}{n\lambda}+0)$
$= \frac{2}{n^2\lambda^2}$

$b_n = \int_0^1 (x-1)^2\sin n\lambda x\,dx$
$= -(x-1)^2\frac{\cos n\lambda x}{n\lambda}\Big|_0^1 + \frac{2}{n\lambda}\int_0^1 (x-1)\cos n\lambda x\,dx$
$= \frac{1}{n\lambda} + \frac{2\cos n\lambda - 2}{n^3\pi^3}.$

3(d)

$a_0 = \frac{1}{2}\ \frac{3}{4}$

$a_1 = \frac{1}{2}\left(\int_0^1 x\cos\frac{n\lambda}{2}x\,dx + \int_1^2 \cos\frac{n\lambda}{2}x\,dx\right)$

So $a_1 = \left(\frac{2}{n^2\lambda^2}\cos\frac{n\lambda}{2}\right) - \left(\frac{1}{n\lambda}\sin\frac{n\lambda}{2}\right) \frac{2}{n^2\lambda^2}$

$b_n = \frac{1}{2}\left(\int_0^1 x\sin\frac{n\lambda}{2}x\,dx + \int_1^2 \sin\frac{n\lambda}{2}x\,dx\right)$
$= \frac{2}{n^2\lambda^2}\sin\frac{n\lambda}{2} - \frac{1}{n\lambda}\cos n\lambda$

Campus