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Task_1

1. sol

$$X_n = \frac{1}{2}\int_0^2 x(t)\, e^{-j\pi n t}\, dt$$

$$= \frac{1}{2}\left(\int_0^{\frac{1}{2}} 2t\, e^{-j\pi n t} + \int_{1/2}^{3/2} (2-2t)\, e^{-j\pi n t}\, dt + \int_{3/2}^{2} 2(t-2)\, e^{-j\pi n t}\, dt\right)$$

$$= \int_0^{1/2} t\, e^{-j\pi n t}\, dt + \int_{1/2}^{3/2} e^{-j\pi n t}\, dt - \int_{1/2}^{3/2} t\, e^{-j\pi n t}\, dt + \int_{3/2}^{2} (-t)\, e^{-j\pi n t} + \int_{3/2}^{2} e^{-j\pi n t}\, dt$$

$$= \left(-2e^{-\frac{1}{2}\pi j n} + 2e^{-\frac{3}{2}\pi j n} - e^{-2\pi j n} + 1\right)\frac{1}{-\pi^2 n^2}.$$

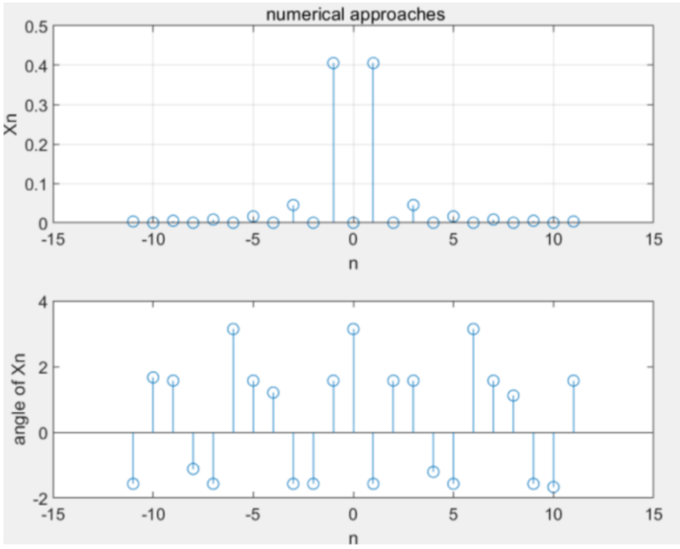
$$\because e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\therefore \text{So } X_n = \begin{cases} 0 & , n \text{ is even} \\ y \times (-1)^{\frac{n+1}{2}} \cdot \frac{4}{\pi^2 n^2} & , n \text{ is odd} \end{cases}$$

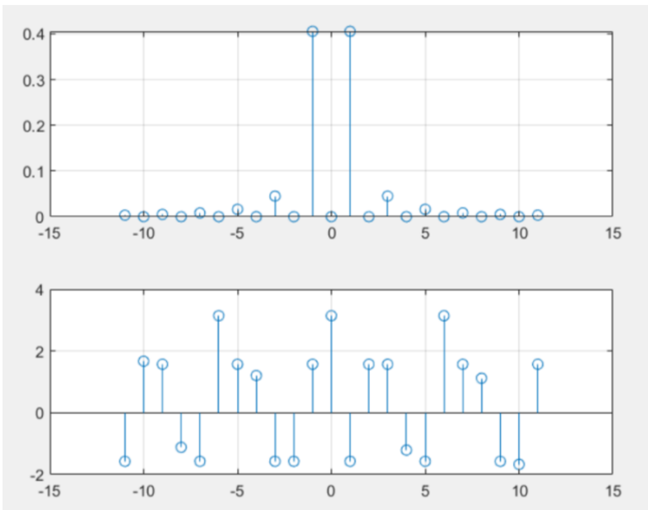
Task_2

Code:

```
N = 11;
n = -N:N;
for k = 1:length(n)
a = (-1)*i*pi*n(k);
   f1 = (@(t)(2*t).*exp(a*t));
   f2 = (@(t)(-2*(t-1)).*exp(a*t));
   f3 = (@(t)(2*(t-2)).*exp(a*t));
Xn(k) = 0.5*quadgk(f1,0,0.5) + 0.5*quadgk(f2,0.5,1.5) + 0.5*quadgk(f3,1.5,2);
x1(k) = conj (x(k));
end
subplot(211)
stem(n,abs(Xn))
title('numerical approaches')
xlabel('n')
ylabel('Xn')
grid on
subplot(212)
stem(n,angle(x))
xlabel('n')
ylabel('angle of Xn')
```




```
subplot(211)
stem(n,abs(x1))
grid on
subplot(212)
stem(n,angle(x1))
grid on
```

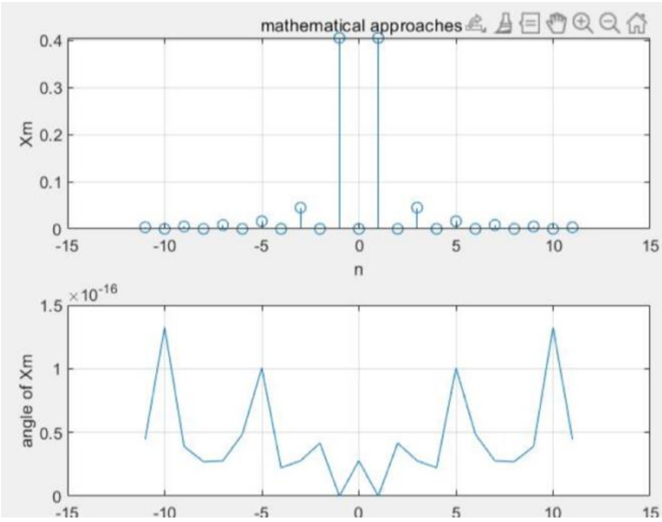


Comment: There is no difference between the two graphs above

<u>Another codes:</u>

```
N = 11;
n = -N:N;
for k = 1:length(n)
    if (mod(n(k),2)==1)
    Xm(k) = i*(-1)^((n(k)+1)/2)*(4/((pi*n(k))^2));
```

```
end
subplot(211)
stem(n,abs(x))
subplot(211)
stem(n,abs(x))
title(' mathematical approaches')
xlabel('n')
ylabel('Xm')
grid on
xlabel('n')
ylabel('angle of Xm')
plot(n,abd(abs(Xm)-abs(Xn)));
```

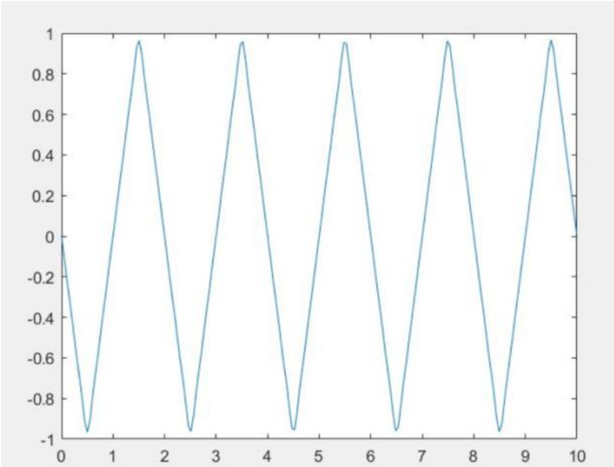


Comment: We can see that the magnitude of the difference between two approaches is 10-16 which is so small that we could ignore it. So, we conclude that the two approaches are same

Task_3

Code:

```
t = linspace(0,10,200);
N = 11;
n = -N:N;
x = 0;
for k=1:length(n)
    a=-i*pi*n(k);
    if (mod(n(k),2)==1) temp = i*(-1)^((n(k)+1)/2)*(4/((pi*n(k))^2))*exp(a*t);
    x = x + temp;
    end
end
plot(t,x)
```

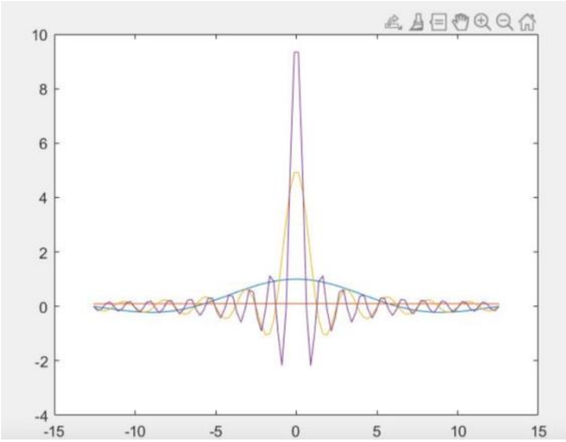


Task 4

Code:

We could conclude from mathematical approach X=T*sin(T/2*w)

```
freqs = linspace(-4*pi,4*pi,100);
T = 1;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
T = 0.1;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
T = 5;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
T = 10;
FT_rect = T*sin(freqs*T/2)./ (freqs*T /2);
plot(freqs, FT_rect)
hold on
```

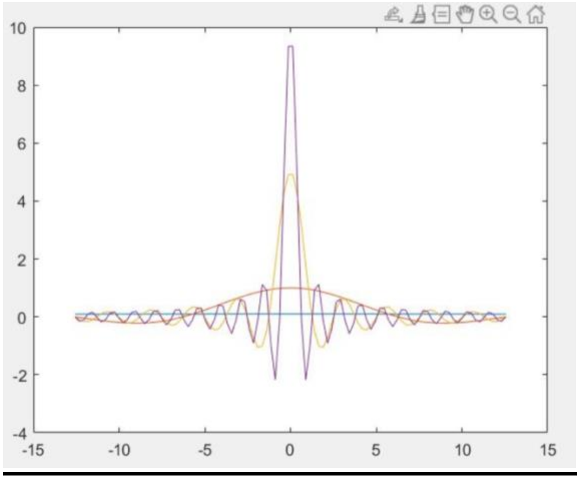


Comment: As T gets bigger and bigger,
the function oscillates more and more,
and the highest point gets higher and higher.

Task_5

Code:

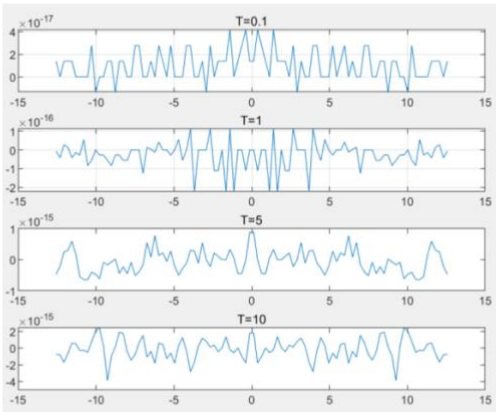
```
freqs = linspace(-4*pi,4*pi,100);
T =0.1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect); hold on T =1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on T =5;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on T =10;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
end
plot(freqs, FT_rect);
hold on
```



Then freqs = linspace(-4*pi,4*pi,100);

```
T =0.1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(411)
plot(freqs,diff)
title('T=0.1')
grid on
freqs = linspace(-4*pi,4*pi,100);
T =1;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(412)
plot(freqs,diff)
title('T=1')
grid on
freqs = linspace(-4*pi,4*pi,100);
T =5;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
```

```
end
subplot(413)
plot(freqs,diff)
title('T=5')
freqs = linspace(-4*pi,4*pi,100);
T =10;
for k = 1:length(freqs)
f = (@(t) exp(-i*freqs (k)*t));
FT_rect(k) = quadgk(f,-T/2,T/2);
FT = T*sin(freqs*T/2)./ (freqs*T /2);
diff=FT_rect-FT;
end
subplot(414)
plot(freqs,diff)
title('T=10')
```



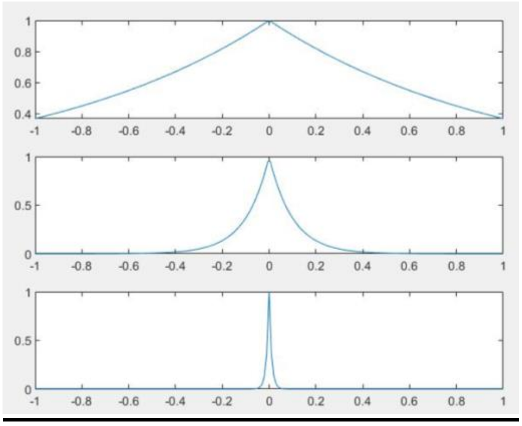
We can see that the difference between two ways is very small, the magnitude is 10 -17 10- 15 . So small that we could ignore it and conclude that the two approaches have the same effects.

<u>Task_6</u>

Code:

```
a = 1;
t = linspace(-1,1,200);
xa = exp(-a*abs(t));
subplot(311)
plot(t, xa)
a = 10;
t = linspace(-1, 1,200);
xa = exp(-a*abs(t));
subplot(312)
plot(t, xa)
a = 100;
t = linspace(-1, 1,199);
```

```
xa = exp(-a*abs(t));
subplot(313)
plot(t, xa)
```



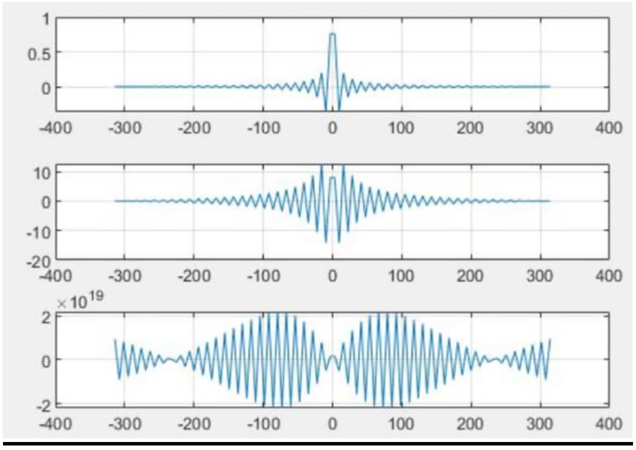
As a become bigger, the x decreases faster, but when t=0, x has the same number 1.

Task_7

Code:

```
fre = linspace(-100*pi,100*pi,100);
a = 1;
T = 1;
for k = 1:length(fre)
    f = (@(t)exp(a*abs(t)).*exp(-i*fre(k)*t));
    FT_rect(k) = quadgk(f,-T/2,T/2);
end
subplot(311)
plot(fre, FT_rect)
grid on
a = 10;
for k = 1:length(fre)
    f = (@(t)exp(a*abs(t)).*exp(-i*fre(k)*t));
    FT_rect(k) = quadgk(f,-T/2,T/2);
end
subplot(312)
plot(fre, FT_rect)
grid on
a = 100;
for k = 1:length(fre)
    f = (@(t)exp(a*abs(t)).*exp(-i*fre(k)*t));
    FT_rect(k) = quadgk(f,-T/2,T/2);
end
subplot(313)
plot(fre, FT_rect)
```

grid on

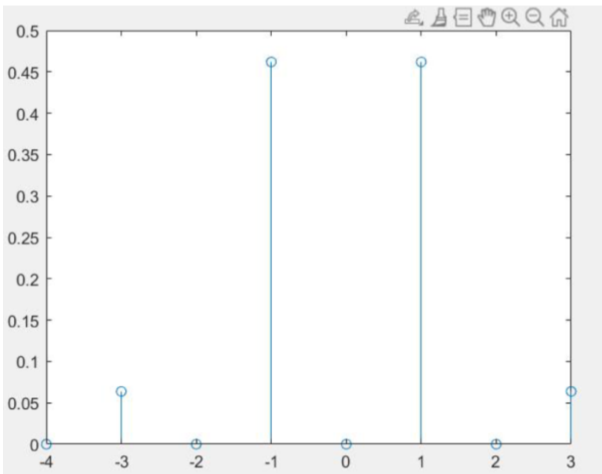


As A increases the graph of X(t) spreads out to the sides,
the peak value of X(t) increases and X(t) becomes more stable.

<u>Task_8</u>

Code:

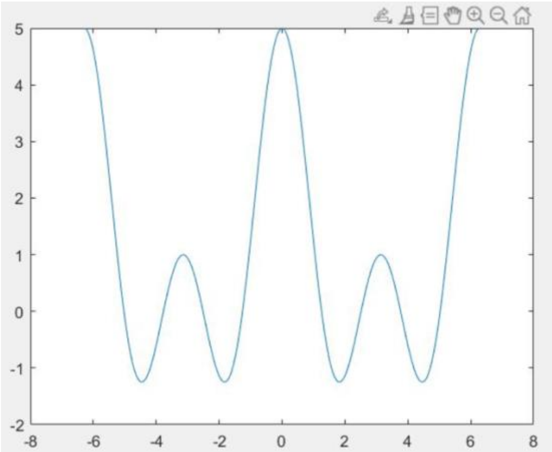
```
N = 8;
n = -4:3;
for k = 1:length(n)
w = pi*n(k)*2;
a = -i*w /N;
f1 = (@(t)(-pi/4).*exp(-a*t));
f2 = (@(t)(pi/4).*exp(-a*t));
x(k) = 1/N*quadgk(f1,-4,-1) + 1/N*quadgk(f2,0,3);
end
stem(n,abs(x))
```



<u>Task_9</u>

Code:

```
fre=linspace(-2*pi,2*pi,200);
n=-2:2;
x=0;
for k=1:length(n)
    x=x+exp(-1*i*fre*n(k));
end
plot(fre,x)
```



I get the same graph.

<u>That's all, thank you!</u>

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