Mathematic Presentation of a Vector

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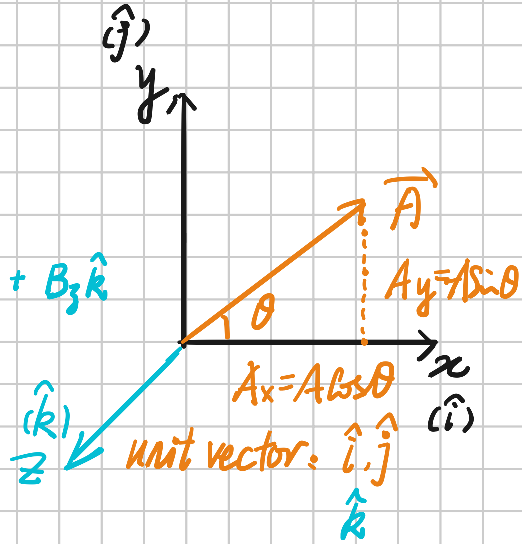
① unit-vector notation

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$



② magnitude - angle notation

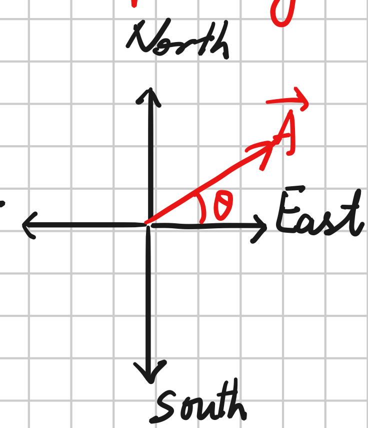
magnitude $A = \sqrt{A_x^2 + A_y^2}$

direction $\vec{A}$ is in the direction θ above +x.

: "of" : "by"

Vector $\vec{A}$ is in the direction θ North of East.

(East by North)



Addition of Vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} = (\underbrace{A_x + B_x}_{C_x})\hat{i} + (\underbrace{A_y + B_y}_{C_y})\hat{j} = C_x\hat{i} + C_y\hat{j}$$

unit-vector notation

$$|\vec{C}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} = \sqrt{C_x^2 + C_y^2}$$

$$\theta = tg^{-1}\left(\frac{C_y}{C_x}\right)$$

magnitude-angle notation

Multiplying of Vectors

① multiplying a vector by a scalar

e.g. $m, \vec{a}$

$$\vec{F} = m\vec{a}$$

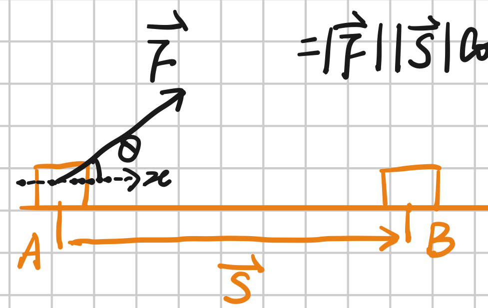
② multiplying a vector by a vector

scalar product of vector (dot product)

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

e.g. $W = \vec{F} \cdot \vec{S}$

$= |\vec{F}|\,|\vec{S}| \cos\theta$



$$\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = \begin{cases} 1 \\ -1 \end{cases} \text{ if } |\vec{A}| = |\vec{B}| = 1$$

$$\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$$

vector product of vector (cross product)

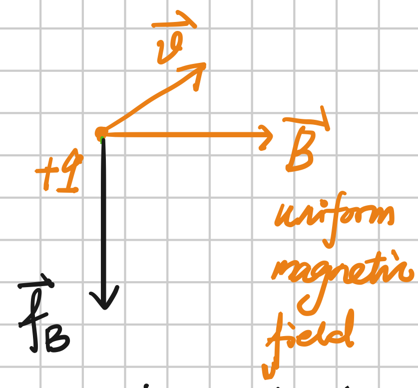
$$\vec{a} \times \vec{b} = \vec{c}$$

$$|\vec{c}| = |\vec{a}|\,|\vec{b}| \sin\theta$$

$$\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0$$

$$\vec{A} \perp \vec{B} \Rightarrow |\vec{A} \times \vec{B}| = 1 \text{ if } |\vec{A}| = |\vec{B}| = 1$$

e.g



$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\boxed{\begin{aligned} &\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \\ &\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0 \end{aligned}}$$

$$\hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$$
$$\hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k} = 0$$

$$\vec{A}\times\vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$
$$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{i} = -\hat{k}$$
$$\hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{j} = -\hat{i}$$
$$\hat{k}\times\hat{i} = \hat{j}, \quad \hat{i}\times\hat{k} = -\hat{j}$$

e.g.

$$\vec{a} = 3.0\hat{i} - 4.0\hat{j}$$
$$\vec{b} = -2.0\hat{i} + 3.0\hat{k}$$

① Determine the angle between $\vec{a}$ and $\vec{b}$.

$$\vec{a}\cdot\vec{b} = (3.0)(-2.0) = -6.0$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$= (5.0)(3.6)\cos\theta$$

$$|\vec{a}| = \sqrt{3.0^2 + 4.0^2} = 5.0$$
$$|\vec{b}| = \sqrt{2.0^2 + 3.0^2} = 3.6$$

$$\theta = \cos^{-1}\left(\frac{-6.0}{(5.0)(3.6)}\right) = 110^\circ$$

② $\vec{C} = \vec{a}\times\vec{b} = ?$

$$\vec{C} = (3.0\hat{i} - 4.0\hat{j})\times(2.0\hat{i} + 3.0\hat{k})$$

$$= (3.0)(3.0)\underset{-\hat{j}}{\underset{\parallel}{(\hat{i}\times\hat{k})}} + (-4.0)(-2.0)\underset{-\hat{k}}{\underset{\parallel}{(\hat{j}\times\hat{i})}} + (-4.0)(3.0)\underset{\hat{i}}{\underset{\parallel}{(\hat{j}\times\hat{k})}}$$

$$= -12\hat{i} - 9.0\hat{j} - 8.0\hat{k}$$