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THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

Year 3

Semester I
2022 - 2023

Exam Paper

EE311FZ
Control Systems Design

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Time allowed: 2 hours

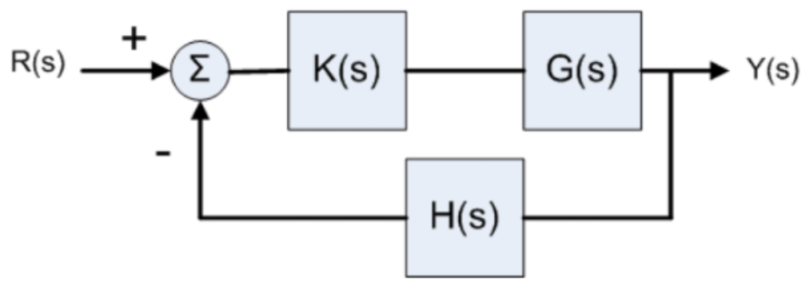
Answer all questions

Instructions: You have 2 hours to complete this examination paper. You then have a further 15 minutes on top of that, to scan and upload your work to the dropbox provided on Moodle for EE311FZ. This additional time should allow for any technical difficulties that may arise. This paper caters for a closed-book examination format. You may use a non-programmable calculator.

Certification of Authenticity: Please be aware on submission of your exam work, you are committing to your lecturer that all the information on your exam script is your own work with no assistance. The Lecturer reserves the right to interview the student if deemed appropriate.

Question 1 [20 marks]

Given the following system:



where

$$G(s) = \frac{1}{s(s+1)}, \qquad H(s) = \frac{1}{s+2}, \qquad K(s) = k$$

(a) Calculate the angles and the intersection points of the asymptotes of the root loci. [4 marks]

(b) Find the breakaway point and the corresponding value of k. [3 marks]

(c) Find the points and the value of corresponding k, where the the root loci may cross the imaginary axis. [3 marks]

(d) Draw the root locus plot on graph paper and annotate important points on the plot. [5 marks]

(e) Use the root locus plot to determine the value of k such that the closed-loop system has a damping ratio ζ of approximately 0.5. Calculations based on graphical approximations are acceptable. [Hint: $\zeta = \cos\phi$] [5 marks]

Question 2 [40 marks]

A continuous-time system is described by the following state-space matrices

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 0 \end{bmatrix}$$

(a) In the system controllable? Comment on your answer. [5 marks]

(b) In the system observable? Comment on your answer. [5 marks]

(c) Determine the input-output transfer function of the system, in the Laplace domain, and comment on the system stability. [5 marks]

(d) Determine, in terms of the response to a unit step input:

(i) the steady-state system response (Hint: Laplace final value Theorem) [3 marks]

(ii) the settling time for the system [2 marks]

(iii) the % overshoot for the system [2 marks]

(iv) the frequency of oscillation of the response [2 marks]

(e) It is desired that the closed-loop system will have poles of $-2 \pm 2\sqrt{3}j$. Design a state feedback controller $u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} x$ to achieve these specifications (determine the value of k_1 and k_2).

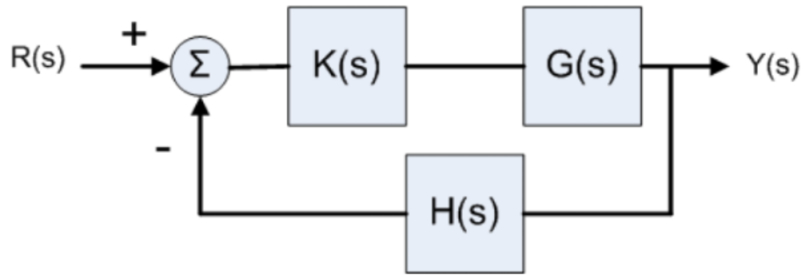
[8 marks]

(f) Due to an inability to measure the states, a state estimator is required. Design a state observer to place the poles of the observer at $-8 \pm 8j$. (You should write down the observer dynamics and then calculate the observer gain $K_e = \begin{bmatrix} k_{e,1} & k_{e,2} \end{bmatrix}^T$)

[8 marks]

Question 3 [20 marks]

Given the following system:



where

$$G(s) = \frac{40}{(s+2)(s+4)}, \qquad H(s) = \frac{1}{s+1}, \qquad K(s) = 1$$

(a) Determine the stability of the feedback system using the Nyquist stability criterion. Draw the Nyquist plot to scale on graph paper and annotate important points on the plot to justify stability/instability.

[15 marks]

Frequency ω	Amplitude $\lvert G(j\omega)H(j\omega)K(j\omega)\rvert$	Phase $\angle G(j\omega)H(j\omega)K(j\omega)$

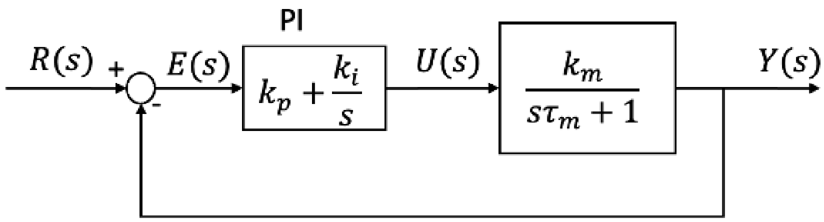
(b) Determine the Gain Margin of the feedback system.

[5 marks]

Question 4 [20 marks]

(a) Show that, for the following PI-controlled system, the steady-state error to a step input is zero (Hint: Laplace final value theorem)

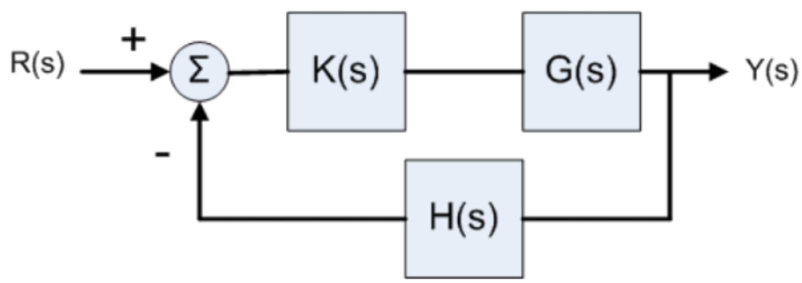
[5 marks]



(b) If the system is instead controlled by a proportional-only controller, i.e. $k_i = 0$ in the above diagram, what is the steady-state error to a unit step?

[5 marks]

(c) Calculate the parameters of a PID controller for the following system using the closed-loop Ziegler-Nicholls method. (Calculate the values of k_p, k_d and k_i)



where

$$G(s) = \frac{100}{(s+1)(s+2)(s+3)}, \qquad K(s) = k_p + k_d s + \frac{k_i}{s}, \qquad H(s) = 1$$

[10 marks]

For your convenience, the closed-loop Ziegler-Nichols rules are given in the following table ($k_d = k_p t_d$, $k_i = k_p / t_i$)

	k_p	t_i	t_d
P	$0.5k_c$		
PI	$0.45k_c$	$t_c/1.2$	
PID	$0.6k_c$	$t_c/2$	$t_c/8$

End of exam.