Tutorial Sheet 1 - Solutions

Q1 & Q2 Refer to notes.

Q3 (i) $\frac{dy(t)}{dt} = 3u(t) - 2y(t)$

(ii) $\frac{dy(t)}{dt} = 3u(t) - 2\sqrt{y(t)}$

(iii) $y(t) = 3u(t)$

(iv) $y(t) = 3\sqrt{u(t)}$

(v) $\frac{dy(t)}{dt} = 3u(t) - a(t)y(t)$, where $a(t)$ is a constant that varies with time!

For all examples, the dependent variable is *y*, the independent variable is *t* and the parameters are the constants used.

Q4 (i) This is a system that obeys the principle of superposition and homogeneity, i.e.:

$$Af(x_1) + Bf(x_2) = f(Ax_1 + Bx_2) \text{ for any constants } A \text{ and } B$$

(ii) $y = 2u \rightarrow Af(u_1) + Bf(u_2) = A(2u_1) + B(2u_2) = 2Au_1 + 2Bu_2$

$y = 2u \rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) = 2Au_1 + 2Bu_2$

Hence: $Af(u_1) + Bf(u_2) = f(Au_1 + Bu_2) \Rightarrow$ Linear

(iii) $y = 2\sqrt{u} \rightarrow Af(u_1) + Bf(u_2) = A(2\sqrt{u_1}) + B(2\sqrt{u_2}) = 2A\sqrt{u_1} + 2B\sqrt{u_2}$

$y = 2\sqrt{u} \rightarrow f(Au_1 + Bu_2) = 2\sqrt{Au_1 + Bu_2}$

Take A = 1, B = 1 for example:

$$Af(u_1) + Bf(u_2) = 2\sqrt{u_1} + 2\sqrt{u_2}$$

$$f(Au_1 + Bu_2) = 2\sqrt{u_1 + u_2}$$

Hence: $Af(u_1) + Bf(u_2) \neq f(Au_1 + Bu_2) \Rightarrow$ Nonlinear

(iv) $y = 2u + 1 \rightarrow Af(u_1) + Bf(u_2) = A(2u_1 + 1) + B(2u_2 + 1) = 2(Au_1 + Bu_2) + A + B$

$y = 2u + 1 \rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) + 1$

Hence: $Af(u_1) + Bf(u_2) \neq f(Au_1 + Bu_2) \Rightarrow$ Nonlinear

Q5 (i) KVL: $v_i = v_R + v_L$

Now, $v_R = iR$ and $v_L = L\frac{di}{dt}$

Hence the first equation becomes: $v_i = iR + L\frac{di}{dt}$ (*relating v_i to i*)

Q5 (ii) KVL: $v_i = v_R + v_L$

Here, we want out the relationship between v_L and v_i.

Now, $v_R = iR$ and hence the equation becomes: $v_i = iR + v_L$

We need to eliminate i. We know that: $v_L = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int v_L$

Hence: $v_i = iR + v_L \rightarrow v_i = \frac{R}{L}\int v_L + v_L$ (*relating v_i to v_L*)

Differentiating once to give a differential equation model:

$$\frac{dv_i}{dt} = \frac{R}{L}v_L + \frac{dv_L}{dt} \qquad \text{(relating } v_i \text{ to } v_L\text{)}$$

Q6 KVL: $v_i = v_R + v_L + v_C$

Here, we want out the relationship between v_L and v_C.

We know that: $v_R = iR$ and $v_L = L\frac{di}{dt}$

Hence the equation becomes: $v_i = iR + L\frac{di}{dt} + v_C$

We need to eliminate i. We know that: $i = C\frac{dv_C}{dt}$

Hence: $v_i = RC\frac{dv_C}{dt} + L\frac{d}{dt}\left(C\frac{dv_C}{dt}\right) + v_C$

$$\Rightarrow v_i = RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C$$

We normally express this as:

$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = v_i$$

or

$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{1}{LC}v_i$$