Tutorial Sheet 1 - Solutions

Q1 (i) $\dfrac{dy(t)}{dt} = 3u(t) - 2y(t)$

(ii) $\dfrac{dy(t)}{dt} = 3u(t) - 2\sqrt{y(t)}$

(iii) $y(t) = 3u(t)$

(iv) $y(t) = 3\sqrt{u(t)}$

(v) $\dfrac{dy(t)}{dt} = 3u(t) - a(t)y(t)$, where $a(t)$ is a constant that varies with time!

For all examples, the dependent variable is y, the independent variable is t and the parameters are the constants used.

Q2 (i) This is a system that obeys the principle of superposition. In other words:

$$Af(x_1) + Bf(x_2) = f(Ax_1 + Bx_2) \text{ for any constants } A \text{ and } B$$

(ii) $y = 2u \rightarrow Af(u_1) + Bf(u_2) = A(2u_1) + B(2u_2) = 2Au_1 + 2Bu_2$

$y = 2u \rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) = 2Au_1 + 2Bu_2$

Hence: $Af(u_1) + Bf(u_2) = f(Au_1 + Bu_2) \Rightarrow$ Linear

(iii) $y = 2\sqrt{u} \rightarrow Af(u_1) + Bf(u_2) = A(2\sqrt{u_1}) + B(2\sqrt{u_2}) = 2A\sqrt{u_1} + 2B\sqrt{u_2}$

$y = 2\sqrt{u} \rightarrow f(Au_1 + Bu_2) = 2\sqrt{Au_1 + Bu_2}$

Take A = 1, B = 1 for example:

$$Af(u_1) + Bf(u_2) = 2\sqrt{u_1} + 2\sqrt{u_2}$$

$$f(Au_1 + Bu_2) = 2\sqrt{u_1 + u_2}$$

Hence: $Af(u_1) + Bf(u_2) \neq f(Au_1 + Bu_2) \Rightarrow$ Nonlinear

(iv) $y = 2u + 1 \rightarrow Af(u_1) + Bf(u_2) = A(2u_1 + 1) + B(2u_2 + 1) = 2(Au_1 + Bu_2) + A + B$

$y = 2u + 1 \rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) + 1$

Hence: $Af(u_1) + Bf(u_2) \neq f(Au_1 + Bu_2) \Rightarrow$ Nonlinear

Q3 $$F(s) = \frac{s}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5} = \frac{A(s+5) + B(s+2)}{(s+2)(s+5)}$$

$s = -5: \quad -5 = B(-3) \Rightarrow B = \dfrac{5}{3}$ $\qquad s = -2: \quad -2 = A(3) \Rightarrow A = -\dfrac{2}{3}$

$\Rightarrow F(s) = -\dfrac{2}{3}\left(\dfrac{1}{s+2}\right) + \dfrac{5}{3}\left(\dfrac{1}{s+5}\right)$ $\qquad \Rightarrow f(t) = -\dfrac{2}{3}e^{-2t} + \dfrac{5}{3}e^{-5t}$

Q4 (i)

$$\frac{dx(t)}{dt} + 3x(t) - 4 = 0 \rightarrow sX(s) - x(0) + 3X(s) - \frac{4}{s} = 0$$

$$\Rightarrow sX(s) - 1 + 3X(s) = \frac{4}{s} \quad \Rightarrow X(s)(s+3) = \frac{4}{s} + 1 = \frac{4+s}{s}$$

$$\Rightarrow X(s) = \frac{s+4}{s(s+3)}$$

(ii)

$$X(s) = \frac{s+4}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} = \frac{A(s+3)+Bs}{s(s+3)}$$

$$s = 0: \quad 4 = A(3) \Rightarrow A = \frac{4}{3} \qquad s = -3: \quad 1 = B(-3) \Rightarrow B = -\frac{1}{3}$$

$$\Rightarrow X(s) = \frac{4}{3}\left(\frac{1}{s}\right) - \frac{1}{3}\left(\frac{1}{s+3}\right) \quad \Rightarrow x(t) = \frac{4}{3} - \frac{1}{3}e^{-3t} = \frac{1}{3}\left(4 - e^{-3t}\right)$$

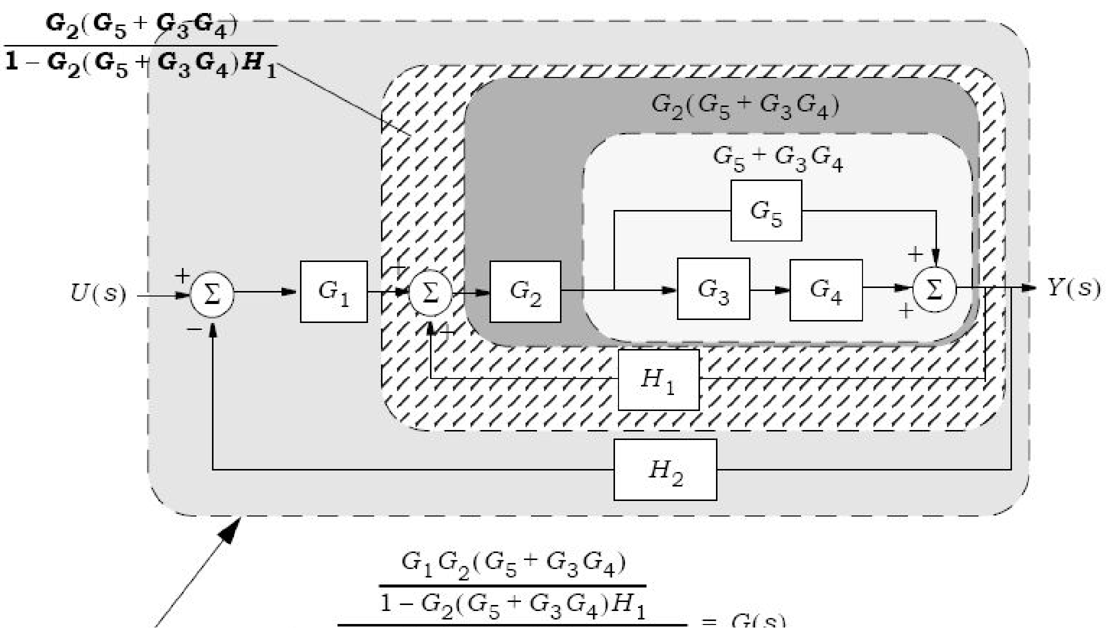
(iii)

$$\frac{dx(t)}{dt} + 3x(t) - 4 = 0 \rightarrow sX(s) + 3X(s) - \frac{4}{s} = 0 \qquad \Rightarrow X(s) = \frac{4}{s(s+3)}$$

Q5 are (i) Transfer functions represent the system so that the input, output and system distinct parts. They can conveniently represent the interconnection of several subsystems. In this form, we no longer work with differentials but rather with an algebraic expression.

(ii) Can't implement initial conditions (hence, we use zero initial conditions). Applicable to linear systems only!

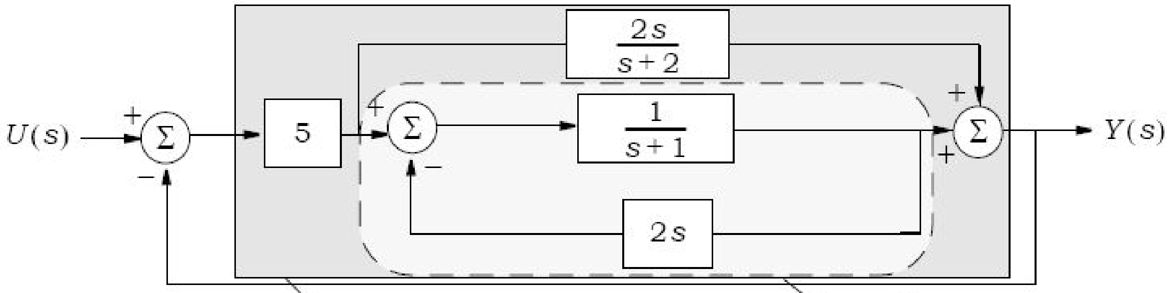
Q6



The transfer function of the block diagram can be obtained in steps, as shown in the diagramabove, giving the overall transfer function as:

$$G(s) = \frac{\dfrac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1}}{1+\dfrac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1}H_2} = \frac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1+(G_1G_2(G_5+G_3G_4))H_2}$$

Q7



$$5\left[\frac{1}{3s+1}+\frac{2s}{s+2}\right] = \frac{30s^2+15s+10}{3s^2+7s+2}$$

$$\frac{\frac{1}{s+1}}{1+\frac{1}{s+1}\cdot 2s} = \frac{1}{3s+1}$$

$$G(s) = \frac{\frac{30s^2+15s+10}{3s^2+7s+2}}{1+\frac{30s^2+15s+10}{3s^2+7s+2}} = \frac{30s^2+15s+10}{3s^2+7s+2+30s^2+15s+10} = \frac{30s^2+15s+10}{33s^2+22s+12}$$