EE204FZ
Lecture 8
Operational Amplifiers

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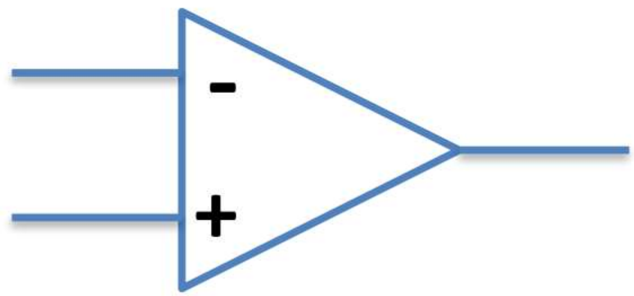
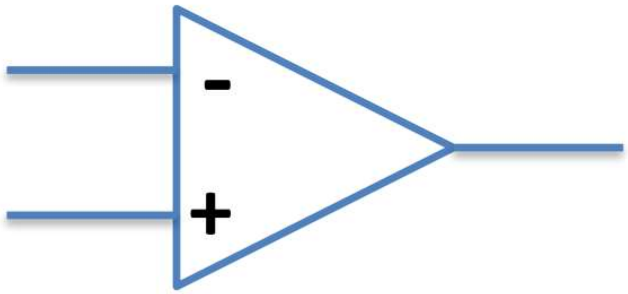
What is an Ideal Op-Amp?



- They amplify the DIFFERENCE of two inputs;
- They have INFINITE input impedance;
- They have ZERO output impedance;
- They have INFINITE gain.

Some other characteristics also include zero offset voltage, infinite bandwidth, etc.

What is a "Practical" Op-Amp?



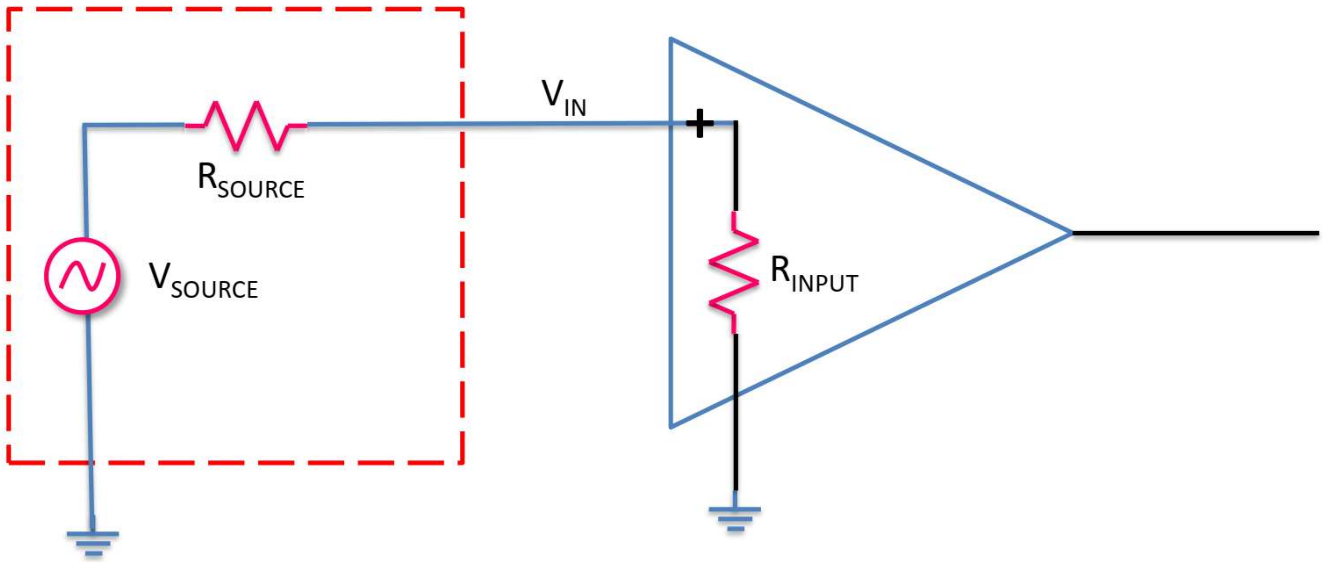
- They amplify the DIFFERENCE of two inputs;
- They have very high input impedance

  (typically allowing 1 – 2 pA of input current, or $R > 1\ \mathrm{T\Omega}$);
- They have very low output impedance

  (good ones can change output voltage at 500 V/μs);
- They have nearly INFINITE gain

  (up to 200k for low-speed op-amps, > 1k for high-speed devices).

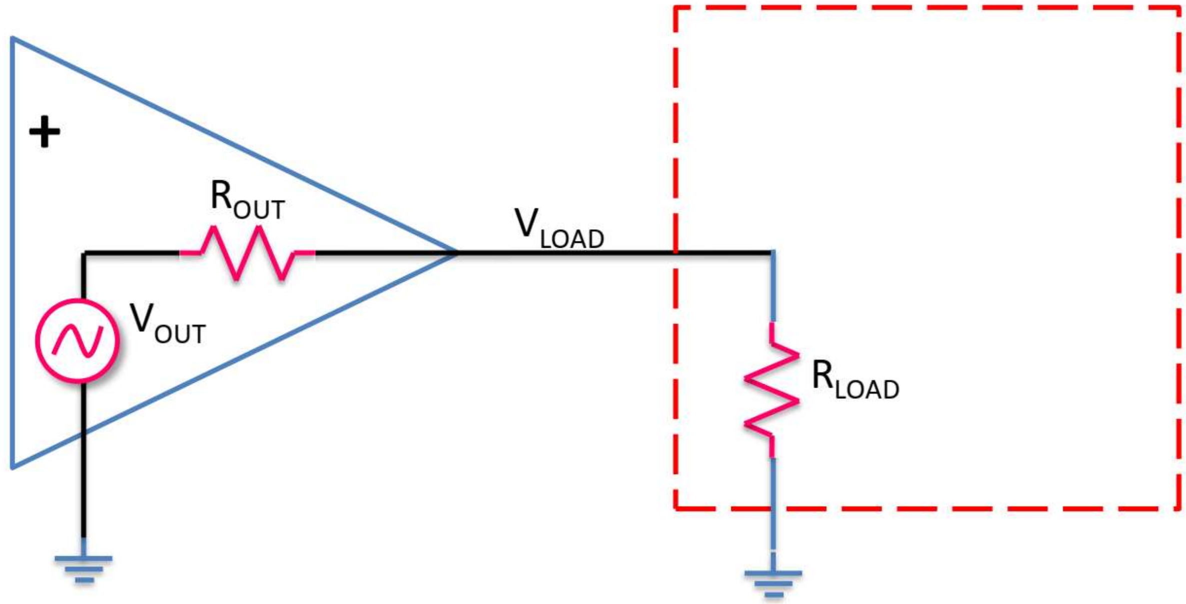
Why is High Input Impedance Important?



$$V_{\text{IN}} = V_{\text{SOURCE}} - R_{\text{SOURCE}} I_{\text{IN}}$$
$$= V_{\text{SOURCE}} - R_{\text{SOURCE}} \frac{V_{\text{SOURCE}}}{R_{\text{SOURCE}} + R_{\text{IN}}}$$
$$= V_{\text{SOURCE}} \left(1 - \frac{R_{\text{SOURCE}}}{R_{\text{SOURCE}} + R_{\text{IN}}}\right)$$

- To make $V_{\text{IN}} = V_{\text{SOURCE}}$, we need to make I_{IN} zero, or make R_{IN} infinitely large.
- Pulling current from a high output impedance source drops voltage.

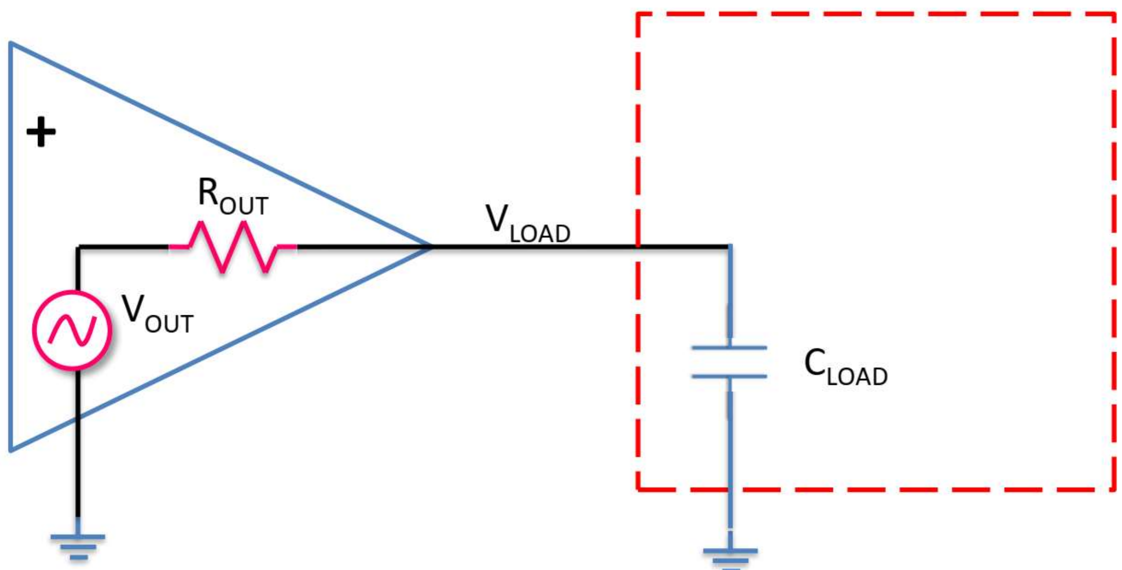
Why is Low Output Impedance Important?



$$V_{\text{LOAD}} = V_{\text{OUT}} - R_{\text{OUT}} I_{\text{OUT}} = R_{\text{LOAD}} I_{\text{OUT}}$$

- Like last time, we can make $V_{\text{LOAD}} = V_{\text{OUT}}$ if we make I_{OUT} equal to zero. This occurs if R_{LOAD} is very large. This can occur if we are connecting to a high impedance input (e.g., a FET). If R_{OUT} was very large, then by the resistor divider rule, almost no signal would be seen at R_{LOAD}.

Why is Low Output Impedance Important?

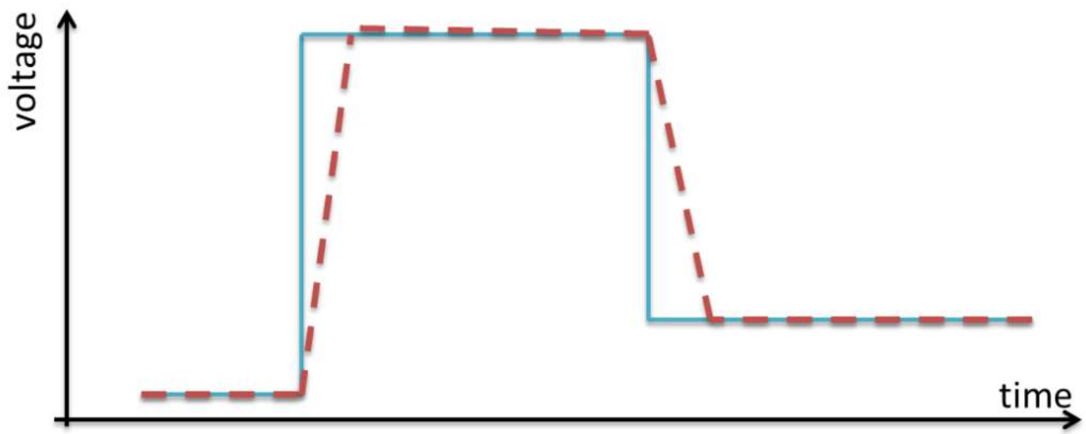


- If we have a capacitor as the output, then the capacitor will need to charge/discharge. The time it takes to get to 97% is approximately 6 time constants. Of course it depends on where you start and end. However, as a time constant is in this case

$$\tau = R_{\text{OUT}} C_{\text{LOAD}}$$

- Reducing R_{OUT} will mean that we get more current and thus more rapid charging/discharging of the capacitors. $R_{\text{OUT}} = 0$ would mean infinitely fast changes in the output voltage.

What is the Slew Rate?

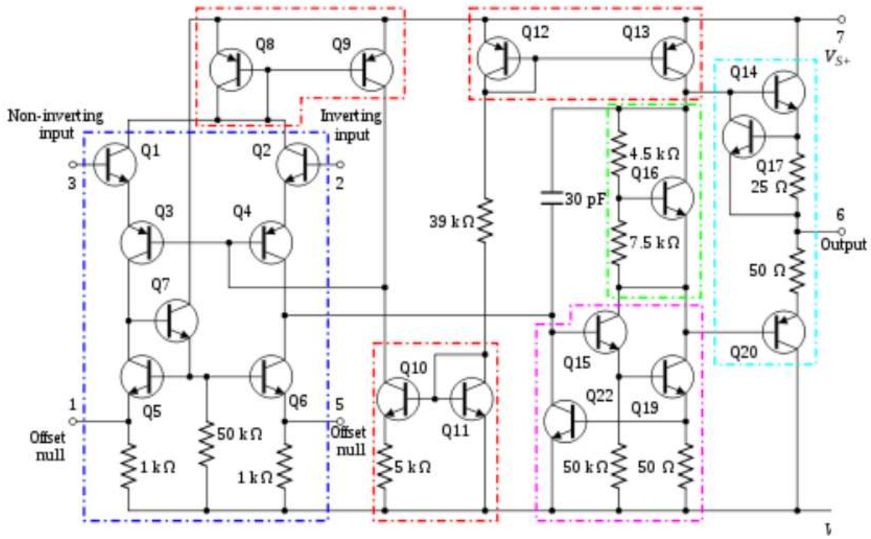


- When the output is about to change, the presence of capacitors and inductors will slow down the change – capacitors are usually the problem.
- The slew rate is the fastest speed at which an output can change. It is of course dependent on the amount of capacitance connected to the output.
- There is no such thing as infinitely fast slew rates as that would mean zero output impedance (infinite current delivery) or zero capacitance.

LM741 slew rate = 5 V/µs at 100 pF

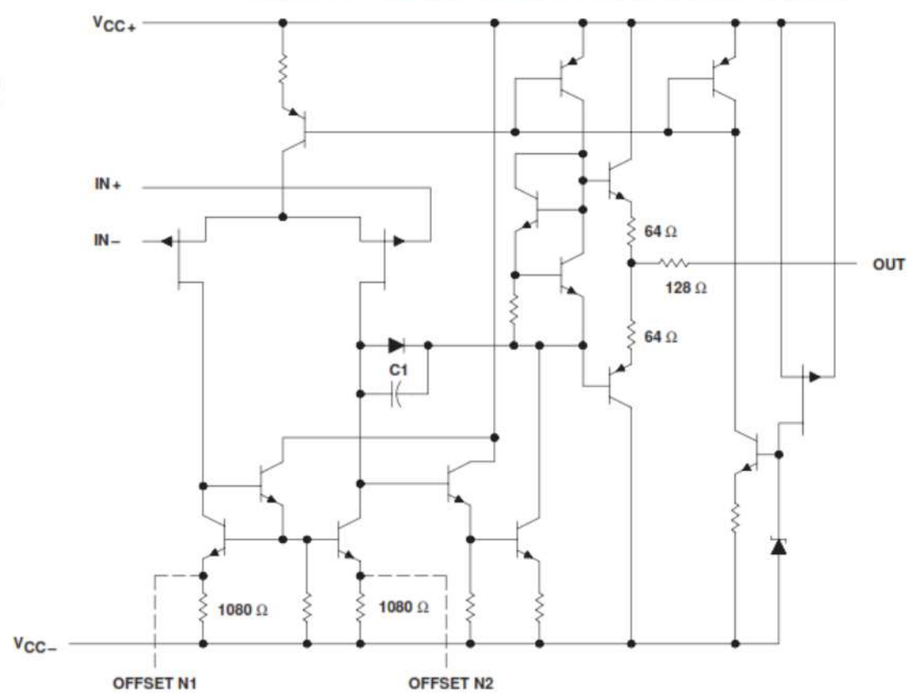
- **Note**: a 1 MHz signal with 5 V_{pk} goes from 0 to 5 V in a quarter period or 0.25 µs, so this op-amp is at least 4 times too slow for a 1 MHz signal of this amplitude.

What is Inside an Op-Amp?

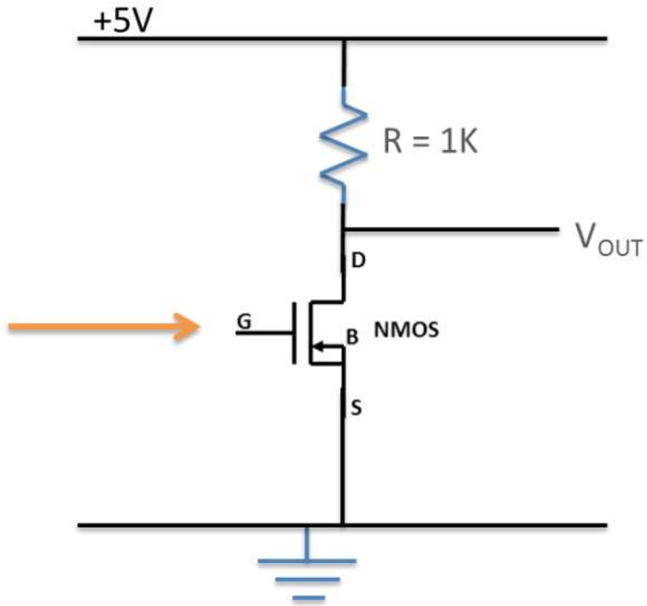


BJT implementation (LM741)

CMOS implementation (TL081)



The Key Design Philosophy for Op-Amps



- The gain of this amplifier is $-g_m R_L$.
- Imagine I want a gain of 10.0, how do I get it?
- Each of these is dependent on temperature and other characteristics.
- Resistors are 5% accurate if you are lucky.
- Manufacturing effects could add another few percent error.
- In silicon technology, ±33% is considered good.

What is a Typical Op-Amp Gain?

- We don't know… nor do we really care.
- We ask for it to be LARGE, and ideally as large as possible. How large depends on the application. The first place to look is the datasheet. It will depend on frequency, but more about that later.

Gain=100K
at low freq.

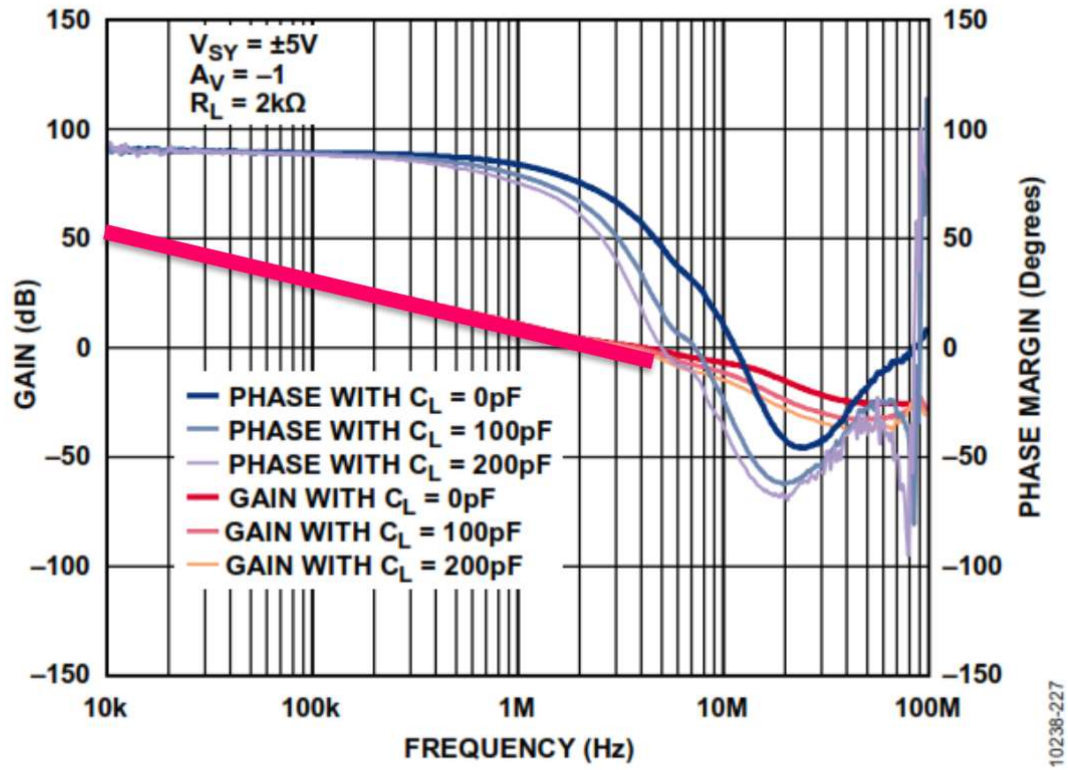
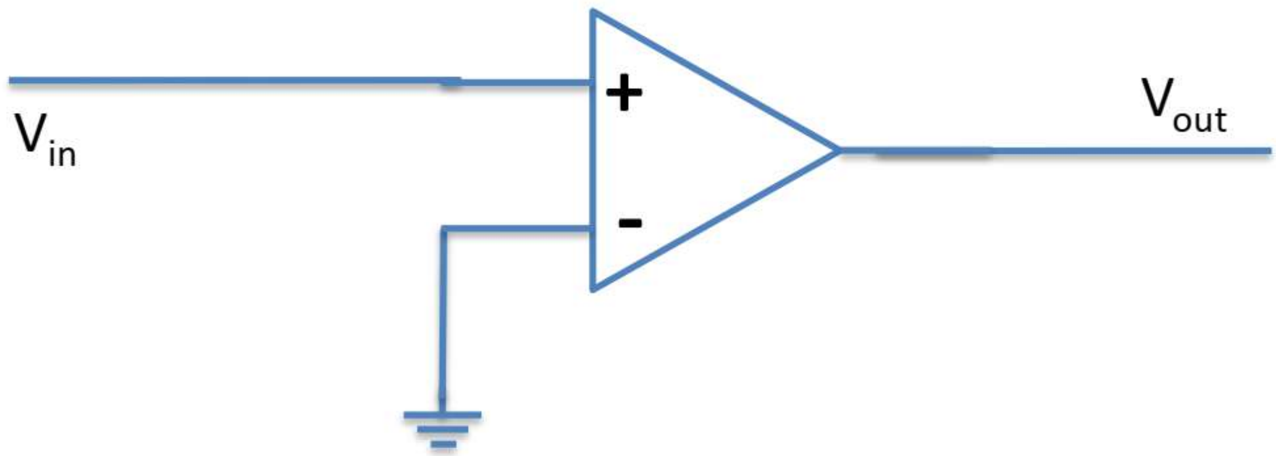


Figure 28. Open-Loop Gain and Phase vs. Frequency, V_{SY} = ±5 V

The Key Design Philosophy for Op-Amps

- We do not care what the values of our parameters are, provided that
  - THE GAIN IS BIG ENOUGH;
  - THE INPUT IMPEDANCE IS HIGH ENOUGH;
  - THE OUTPUT IMPEDANCE IS LOW ENOUGH.
- With all of these, the aim is to design/use an op-amp that goes so far beyond our requirements, that we don’t mind if it is 50% out.

Using an Op-Amp

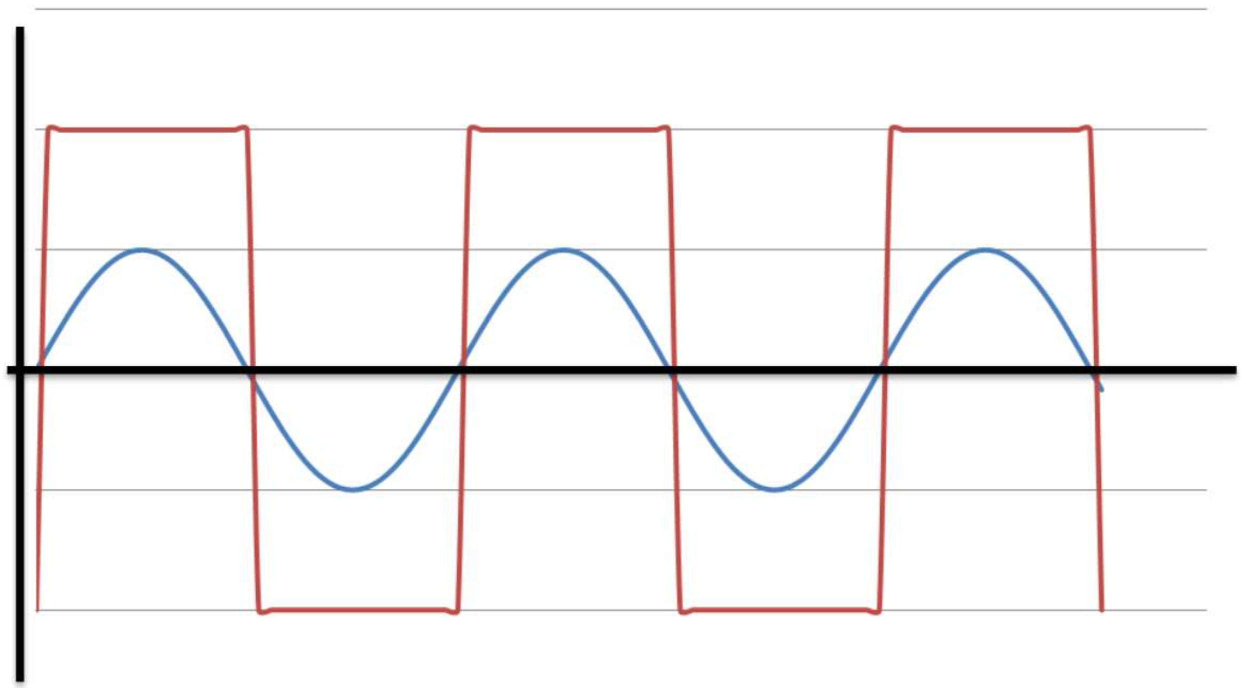


- Consider a zero-mean sinusoidal signal (V_{IN}). The output of the op-amp is given by the expression

$$V_{OUT} = A(V_+ - V_-)$$

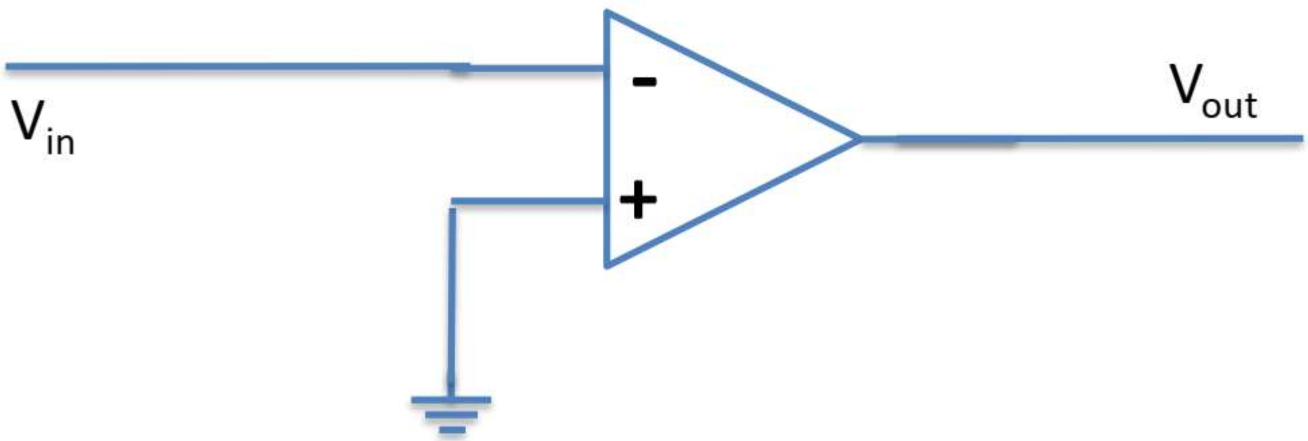
- If A is equal to 100k, and the maximum output voltage is (say) ±10 V, then the output will look like this.

Using an Op-Amp



- If V_{CC} = 10 V and the gain is 100k, then even 0.1 mV of a difference between the inputs and the outputs would be sufficient to make the output hit the +/- voltage rails.
- This use of an op-amp works very well as a comparator. Is A>B?

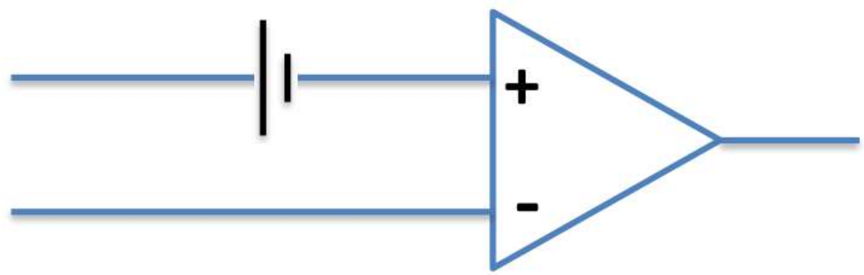
Using an Op-Amp



- If I swap the inputs so that V_{IN} goes into the MINUS pin, what do you think the output would look like now?

Other Characteristics: Input Offset Voltage

- Due to manufacturing tolerances and component variations, op-amps will never have perfectly matched inputs. One input will be higher in electric potential than the other so that even if both are equal externally, internally there would be a difference. This is called the input offset voltage and it looks like a small voltage added to the PLUS input.



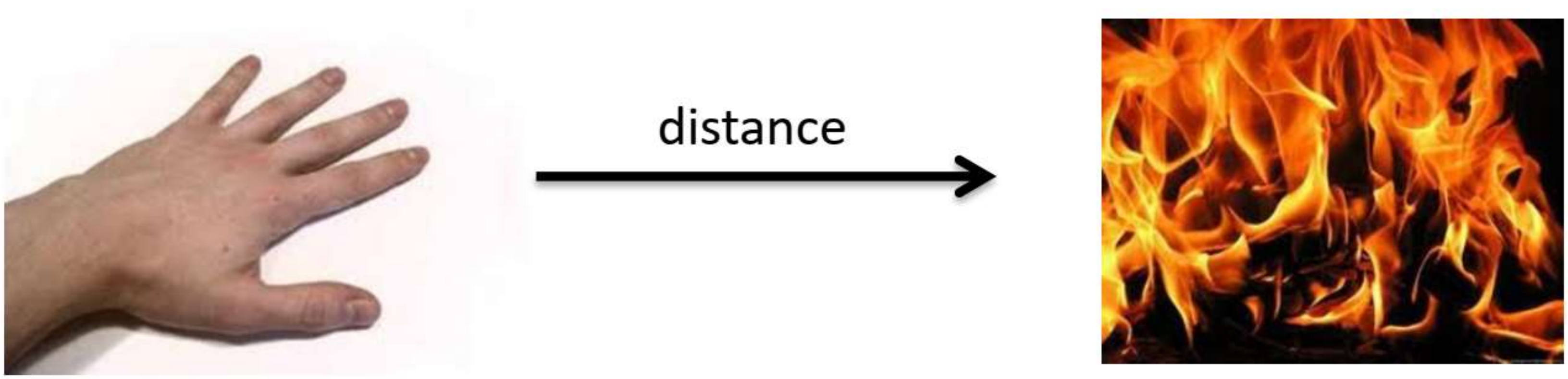
- This limits how small a difference you can detect reliably, or if you wish to take very sensitive measurements. There are circuit tricks to make it smaller, but you need to pick special op-amps.

Chopper stabilized op-amps: < 1 µV
Best bipolar op-amps: 10 – 25 µV
High-speed op-amps: 100 – 2000 µV
Trimmed CMOS op-amps: < 1000 µV

General purpose op-amps: 50 – 500 µV
Best JFET input op-amps: 100 – 1000 µV
Cheap CMOS op-amps: 5000 – 50,000 µV

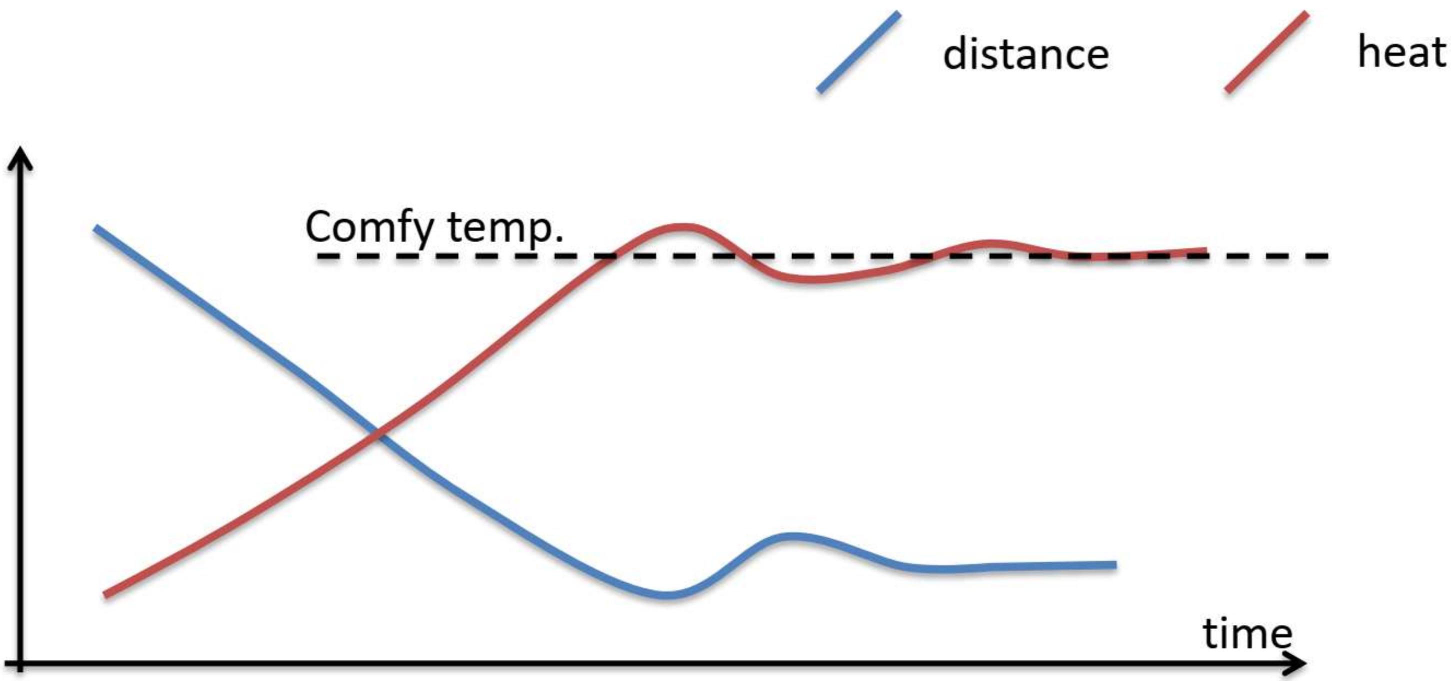
Feedback

- This will be covered in dedicated modules later. To make it simple:
- Feedback is a process in which information about the past and/or current values of the output of a system influences the current or future values of that output.



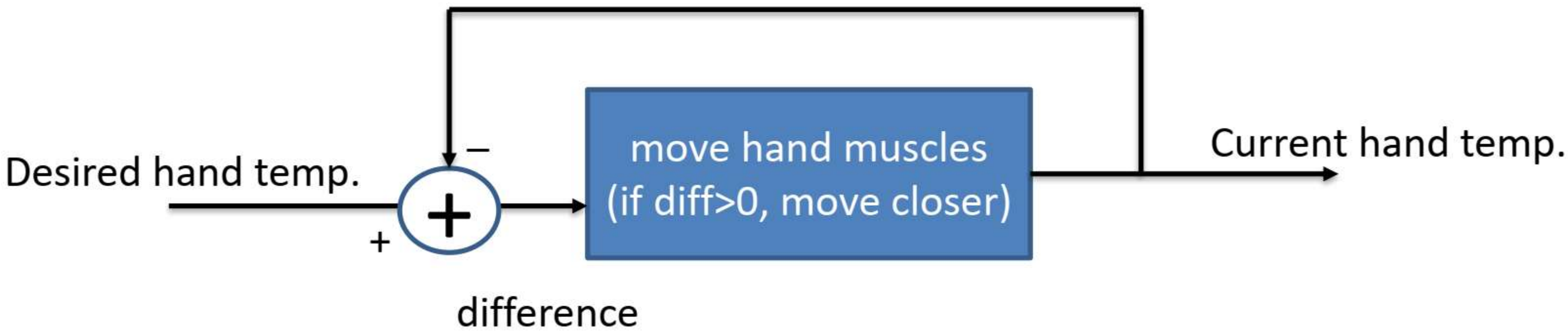
- Consider you have a hand near a fire. Your hand is cold and you want to warm it up to a comfortable temperature. You decide to move your hand closer to the fire (variable is distance). How do you choose when to stop?

Feedback



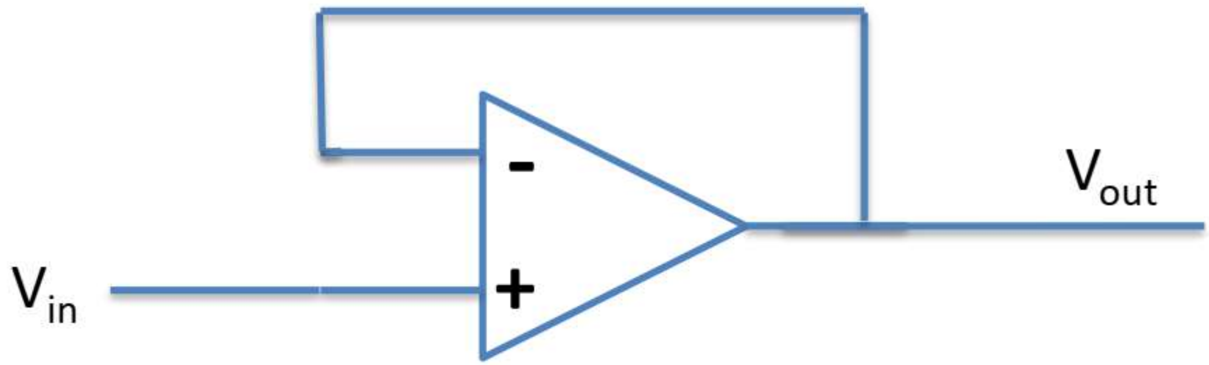
- You move your hand forward and it gets warm;
- You keep moving it. It gets warmer;
- If you move too far and it gets too warm, you pull your hand back;
- Eventually you'll move back and forward a little until it is perfect.

Feedback



- Where the feedback signal works to limit the increase in the output, we call this negative feedback.
- The other type of feedback is called positive feedback and instead of the difference we get the sum. What effect would this have in our example?

In an Op-Amp



Let's explore this scenario:

$$V_{\mathrm{OUT}} = A(V_+ - V_-) = A(V_{\mathrm{IN}} - V_{\mathrm{OUT}})$$

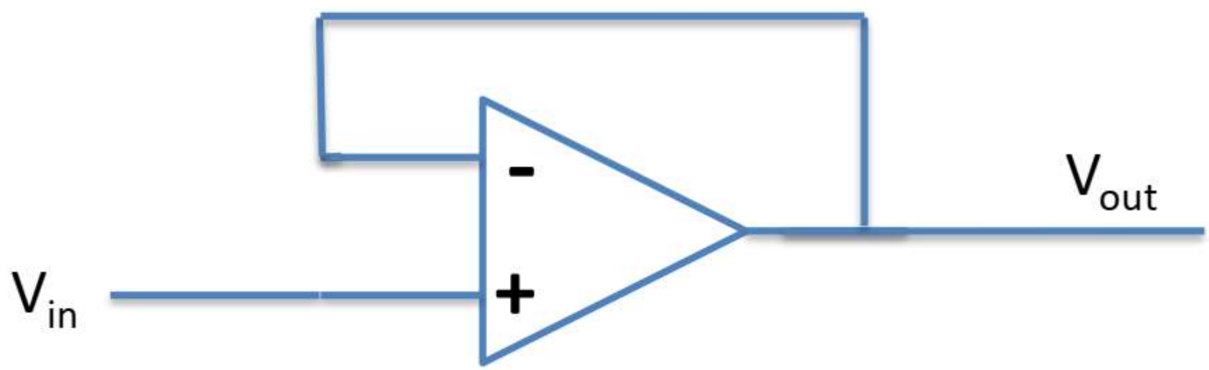
We have

$$V_{\mathrm{OUT}} = \frac{A}{1+A} V_{\mathrm{IN}}$$

If A is very large (100k), then pretty much this says $V_{OUT} = V_{IN}$.

- If V_{OUT} increases above V_{IN}, then the difference is negative and V_{OUT} falls.
- As V_{OUT} falls, it drops below V_{IN}, and V_{OUT} increases. The balance point is when $V_{OUT} = V_{IN}$.

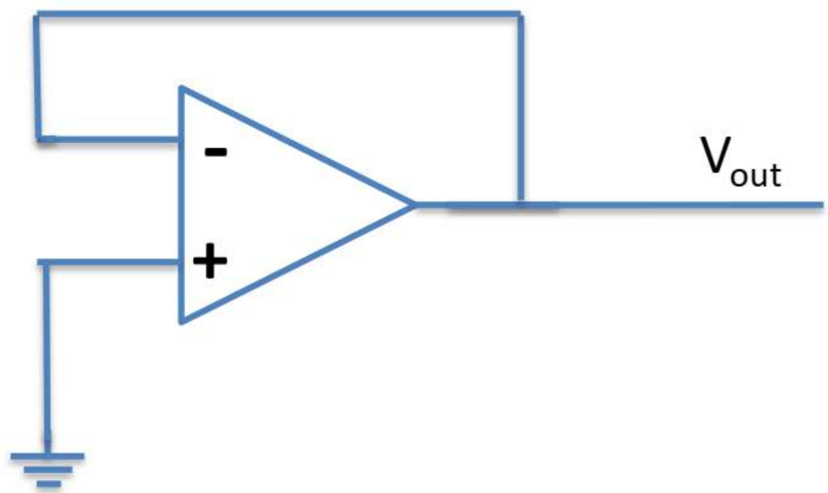
Using Feedback to Simplify Designs



- In any scenario where there is negative feedback and the output voltage is not maxed out, we can assume that the difference in the two inputs (PLUS and MINUS) is very very small.

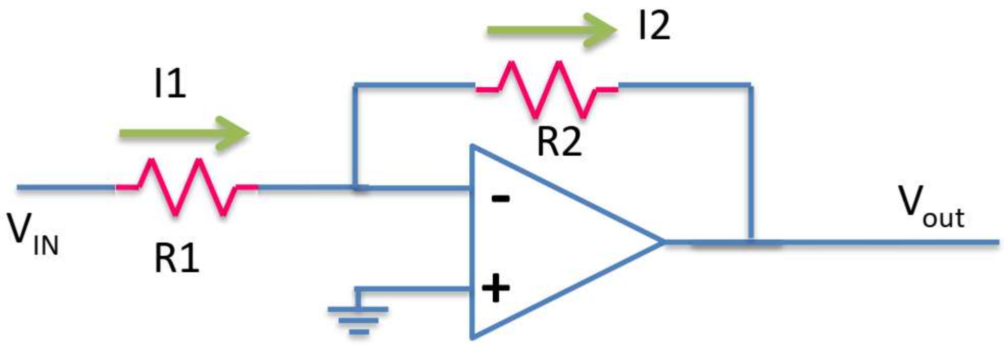
Assumption: The PLUS and MINUS inputs have the same voltage when there is a negative feedback loop.

Virtual Earth (Ground)



- If the assumption is true that PLUS and MINUS have the same voltage, then if I connect one to ground, then the other must also have the same voltage – i.e., zero. We call the MINUS pin in this scenario a virtual earth (ground).
- To get a negative feedback to work we commonly use the MINUS pin of the op-amp, thus we must then connect the earth to the PLUS pin.

An Amplifier



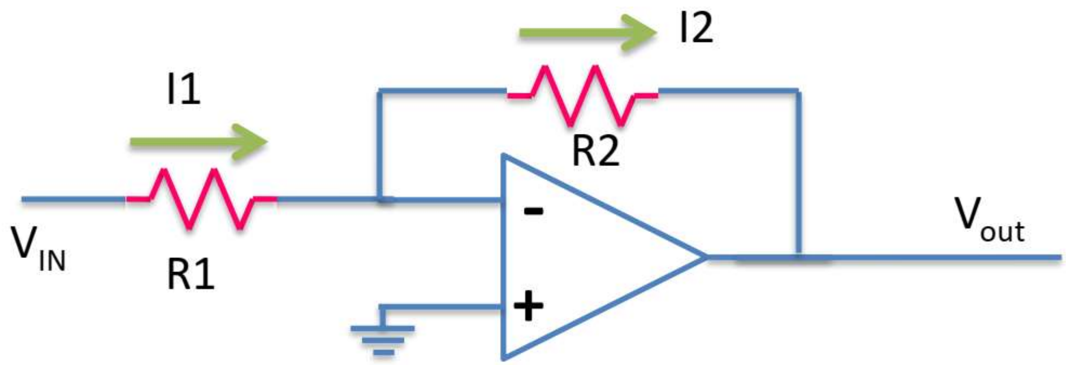
- In this circuit, the PLUS pin is connected to EARTH.
- There is a feedback path, so that means the MINUS pin has ZERO voltage.
- V_{IN} is not zero, so through R_1 flows current I_1, $I_1 = V_{\mathrm{IN}}/R_1$.
- But no current can flow into an ideal op-amp (infinite input impedance), therefore the same current must flow through R_2, $I_2 = I_1$.
- A current through a resistor drops voltage I_2R_2, so

$$V_{\mathrm{OUT}} = V_{-} - I_2R_2 = V_{-} - I_1R_2 = -I_1R_2$$

- Insert the expression for I_1:

$$V_{\mathrm{OUT}} = -\left(\frac{V_{\mathrm{IN}}}{R_1}\right)R_2 = -V_{\mathrm{IN}}\left(\frac{R_2}{R_1}\right)$$

An Amplifier



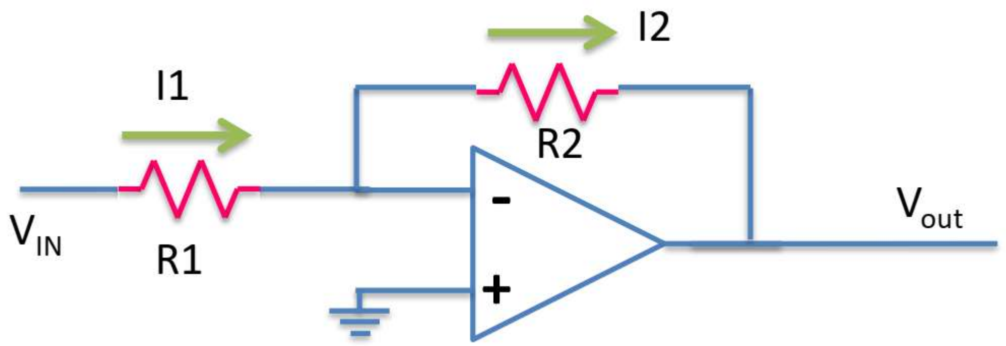
$$V_{\mathrm{OUT}} = -\left(\frac{R_2}{R_1}\right)V_{\mathrm{IN}}$$

Therefore, gain is

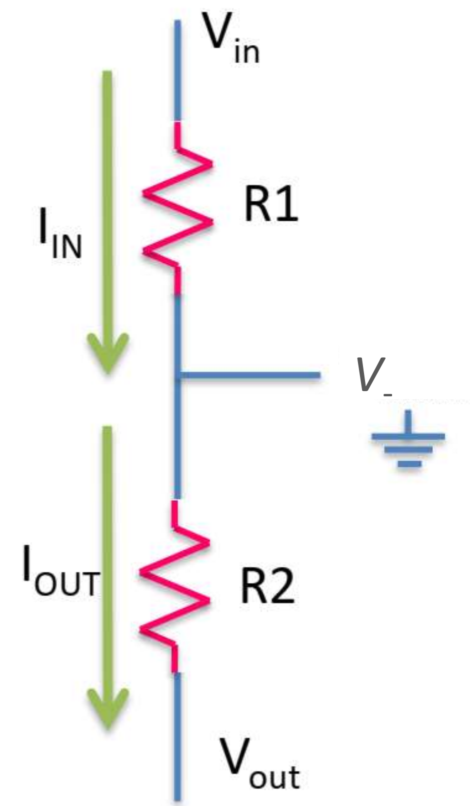
$$A = -\frac{R_2}{R_1}$$

- We call this the closed-loop gain of the system – closed because there is feedback. The gain without feedback is called open-loop gain.
- Also remember we have some assumptions here, e.g., $V_+ = V_-$, virtual earth, ideal op-amp, etc.

An Amplifiers: In Plain English

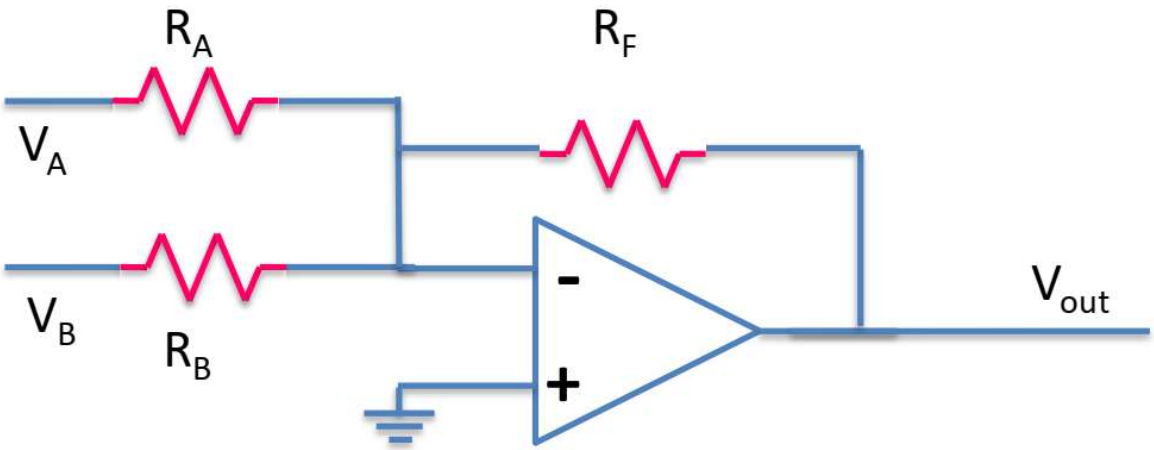


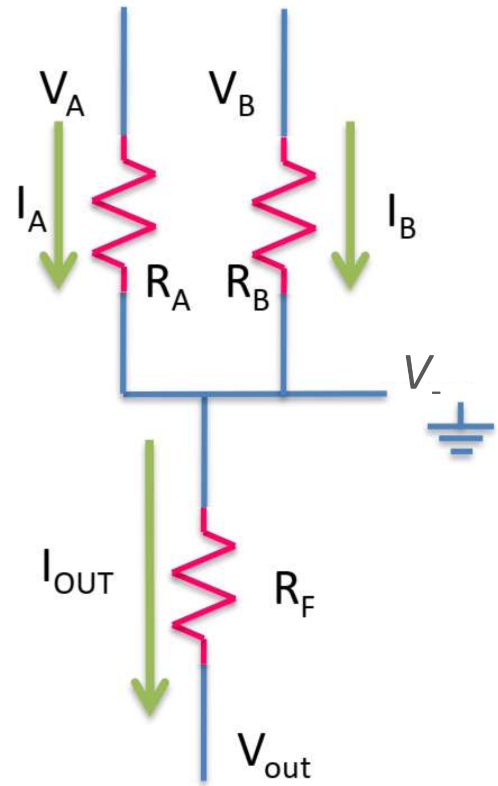
- If V_{OUT} gets too negative, V_- will be negative. Then $A(V_+ - V_-)$ becomes a positive number and V_{OUT} will try to rise, and we get this bigger/smaller oscillation until they balance ($V_+ = V_- = 0$).
- So as the PLUS pin is at zero volts, the feedback will always work to force the MINUS pin to zero volts. It will do this by changing the value of V_{OUT} to whatever value is needed to achieve this.
- Consider them vertically stacked instead. The middle point is zero, but no current can escape. In this scenario, the current is V_{IN}/R_1, but it must then flow through the second resistor. V_{OUT} will keep changing until it is the right value to get the voltage divider right.



$$I_{\mathrm{IN}} = I_{\mathrm{OUT}} \text{ and } \frac{V_{\mathrm{IN}}-0}{R_1} = \frac{0-V_{\mathrm{OUT}}}{R_2}$$

Another Amplifier: Two Inputs





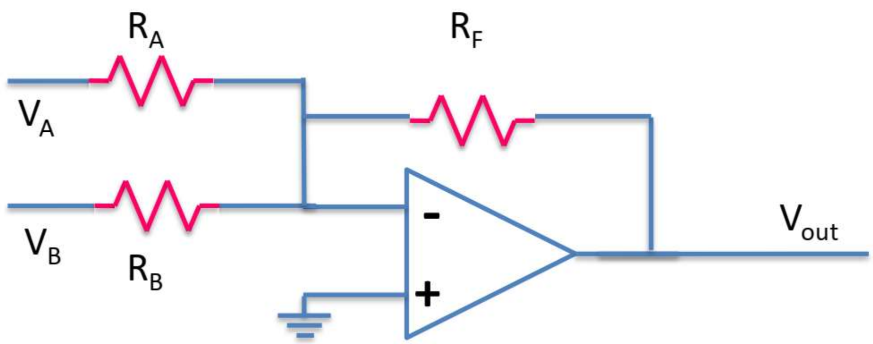
- What we have here are two inputs, but the theory is the same. V_- must stay at zero, and V_{OUT} moves.
- It is harder to do a divider for voltage here, so let's do nodal analysis with currents. What goes in must come out:

$$I_{\mathrm{A}} = V_{\mathrm{A}}/R_{\mathrm{A}} \text{ and } I_{\mathrm{B}} = V_{\mathrm{B}}/R_{\mathrm{B}}$$

$$I_{\mathrm{OUT}} = I_{\mathrm{A}} + I_{\mathrm{B}}$$

$$V_{\mathrm{OUT}} = -I_{\mathrm{OUT}} R_{\mathrm{F}}$$

Another Amplifier: Two Inputs



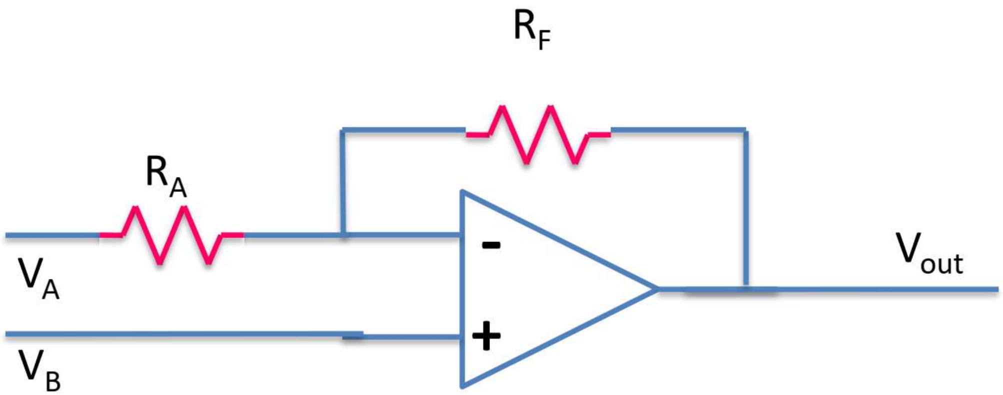
$$V_{\mathrm{OUT}} = -R_{\mathrm{F}}\left(\frac{V_{\mathrm{A}}}{R_{\mathrm{A}}} + \frac{V_{\mathrm{B}}}{R_{\mathrm{B}}}\right) = -[V_{\mathrm{A}}\left(\frac{R_{\mathrm{F}}}{R_{\mathrm{A}}}\right) + V_{\mathrm{B}}\left(\frac{R_{\mathrm{F}}}{R_{\mathrm{B}}}\right)]$$

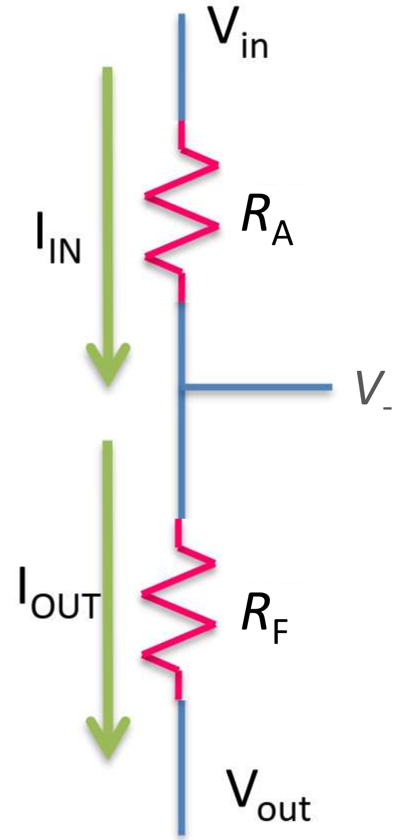
- Let us imagine that all the resistors are equal, then what we get is:

$$V_{\mathrm{OUT}} = -(V_{\mathrm{A}} + V_{\mathrm{B}})$$

- This gives us the function of summation, and the design is called an inverting summer (summing amplifier). You can use multiple branches to add more voltages together.

Another Amplifier: Two Inputs





- In this case, we do not have V_+ set to the ground. What will happen here?
- Same analysis: But this time V_- will try to match V_+ which is equal to V_B.

$$I_{\mathrm{IN}} = \frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{R_{\mathrm{A}}} \text{ and } I_{\mathrm{OUT}} = \frac{V_{\mathrm{B}} - V_{\mathrm{OUT}}}{R_{\mathrm{F}}}$$

$$I_{\mathrm{IN}} = I_{\mathrm{OUT}}$$

Another Amplifier: Two Inputs

$$I_{\mathrm{IN}} = \frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{R_{\mathrm{A}}} \text{ and } I_{\mathrm{OUT}} = \frac{V_{\mathrm{B}} - V_{\mathrm{OUT}}}{R_{\mathrm{F}}}$$

$$I_{\mathrm{IN}} = I_{\mathrm{OUT}}$$

$$\frac{V_{\mathrm{A}} - V_{\mathrm{B}}}{R_{\mathrm{A}}} = \frac{V_{\mathrm{B}} - V_{\mathrm{OUT}}}{R_{\mathrm{F}}}$$

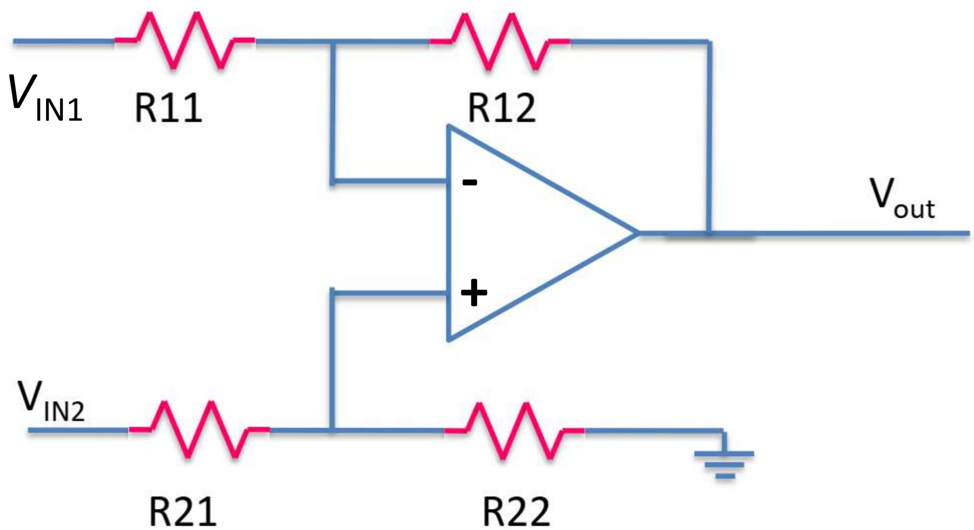
$$V_{\mathrm{OUT}} = -V_{\mathrm{A}}\left(\frac{R_{\mathrm{F}}}{R_{\mathrm{A}}}\right) + V_{\mathrm{B}}\left(1 + \frac{R_{\mathrm{F}}}{R_{\mathrm{A}}}\right)$$

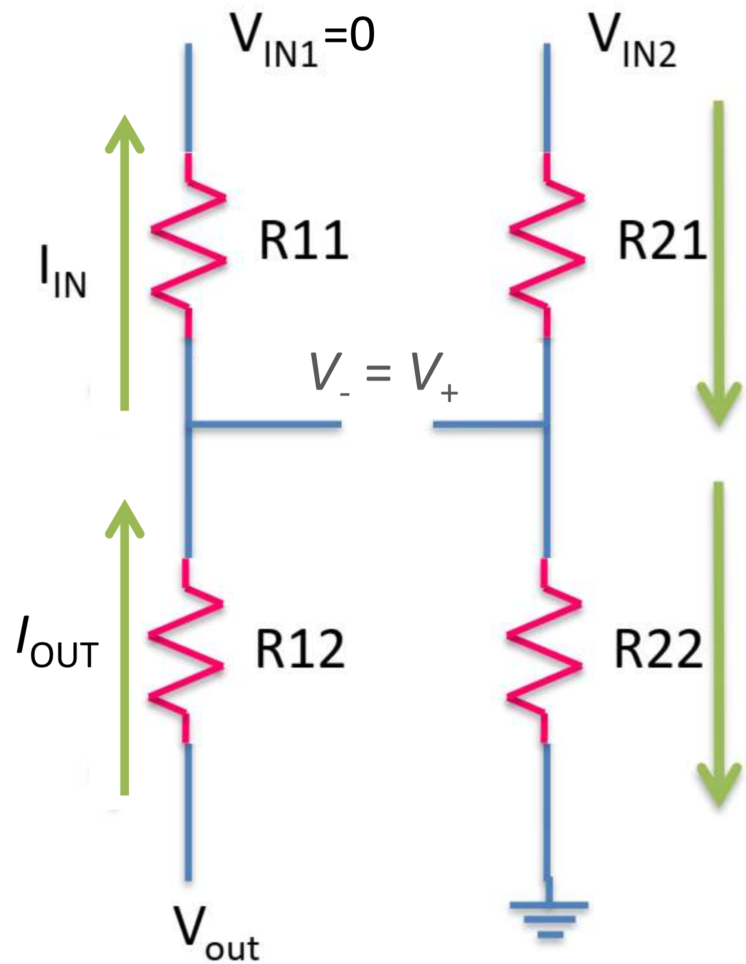
$$= (V_{\mathrm{B}} - V_{\mathrm{A}})\left(\frac{R_{\mathrm{F}}}{R_{\mathrm{A}}}\right) + V_{\mathrm{B}}$$

$$V_{\mathrm{OUT}} \approx (V_{\mathrm{B}} - V_{\mathrm{A}})\left(\frac{R_{\mathrm{F}}}{R_{\mathrm{A}}}\right)$$

- This makes a subtraction function which is accurate if R_F/R_A is very big. We should make R_F big and R_A small.

Another Configuration





- This circuit is a bit tricky. We will use the superposition theorem to solve it.
- If V_{IN2} = 0, $V_{\mathrm{OUT}} = -\left(\frac{R_{12}}{R_{11}}\right) V_{\mathrm{IN1}}$.
- If V_{IN1} = 0, we can write down two equations:

$V_+ = V_{\mathrm{IN}} \;\frac{R_{22}}{R_{21}+R_{22}}$ and $V_- = V_{\mathrm{OUT}}(\frac{R_{11}}{R_{11}+R_{12}})$

Another Configuration

$$V_+ = V_-$$

$$V_{\mathrm{IN}}\ \frac{R_{22}}{R_{21}+R_{22}} = V_{\mathrm{OUT}}\frac{R_{11}}{R_{11}+R_{12}}$$

$$V_{\mathrm{OUT}} = \ (\frac{R_{22}}{R_{11}})(\frac{R_{11}+R_{12}}{R_{21}+R_{22}})V_{\mathrm{IN2}}\ \text{(when } V_{\mathrm{IN1}} = 0\text{)}$$

- When V_{IN1} and V_{IN2} are both present, the V_{OUT} would be the sum of the two cases:

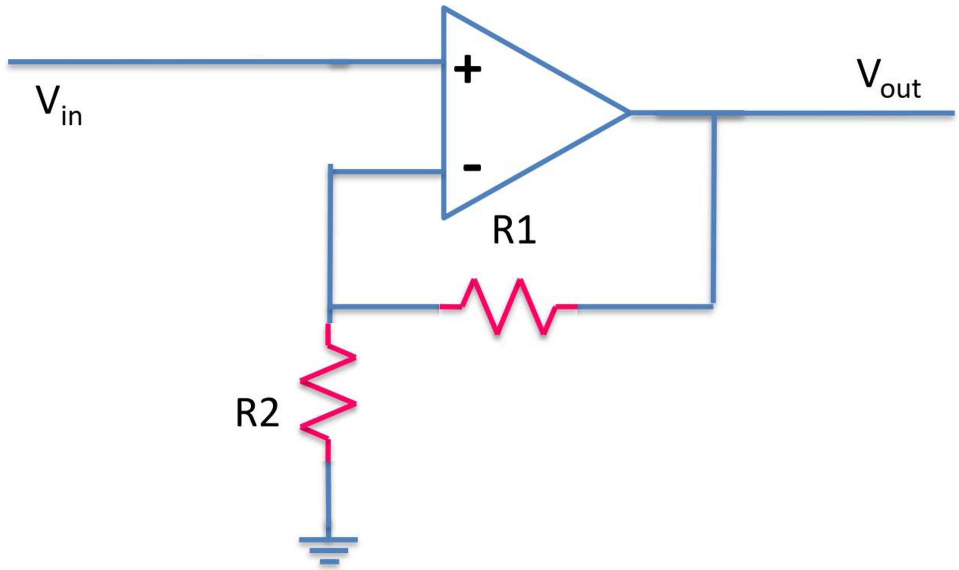
$$V_{\mathrm{OUT}} = -\left(\frac{R_{12}}{R_{11}}\right)V_{\mathrm{IN}}\ + (\frac{R_{22}}{R_{11}})(\frac{R_{11}+R_{12}}{R_{21}+R_{22}})V_{\mathrm{IN2}}$$

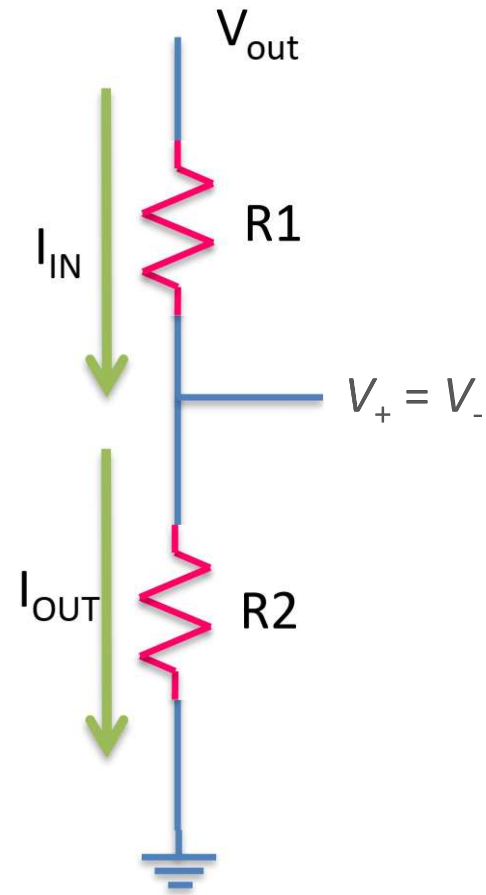
- If we assume that $R_{11} = R_{21}$ and $R_{12} = R_{22}$,

$$V_{\mathrm{OUT}} = (\frac{R_{12}}{R_{11}})(V_{\mathrm{IN2}} - V_{\mathrm{IN1}})$$

- We amplify the difference, and the exact ratio and scaling depend on the resistors.

A Different Configuration





- This is a different configuration for the resistors but the principle remains the same.
- V_+ and V_- need to stay very close to each other if there is working negative feedback.

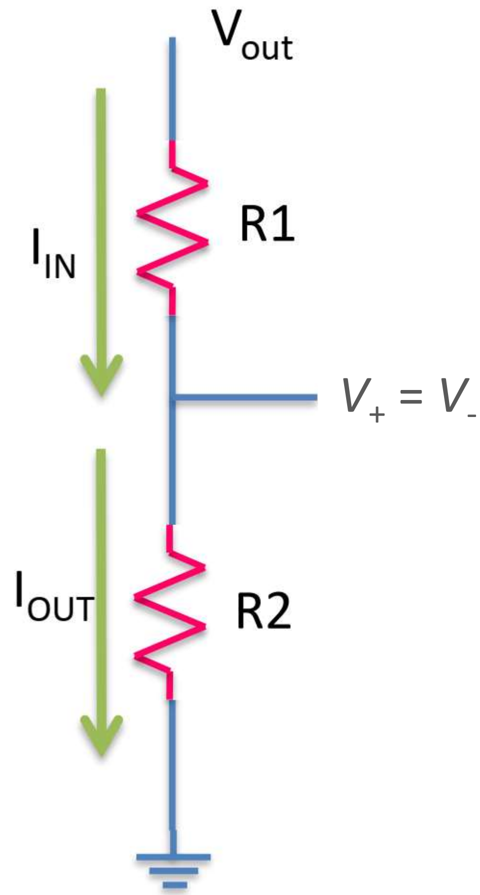
Non-Inverting Amplifier

$$I_{\mathrm{IN}} = \frac{V_{\mathrm{OUT}} - V_{-}}{R_1} \text{ and } I_{\mathrm{OUT}} = \frac{V_{-} - 0}{R_2}$$

$$I_{\mathrm{IN}} = I_{\mathrm{OUT}}$$

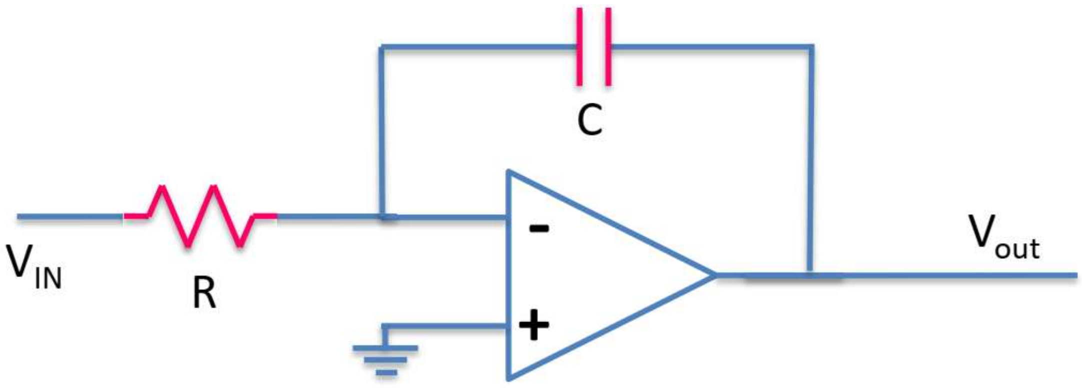
$$\frac{V_{\mathrm{OUT}} - V_{-}}{R_1} = \frac{V_{-}}{R_2} \text{ and } V_{-} = V_{+} = V_{\mathrm{IN}}$$

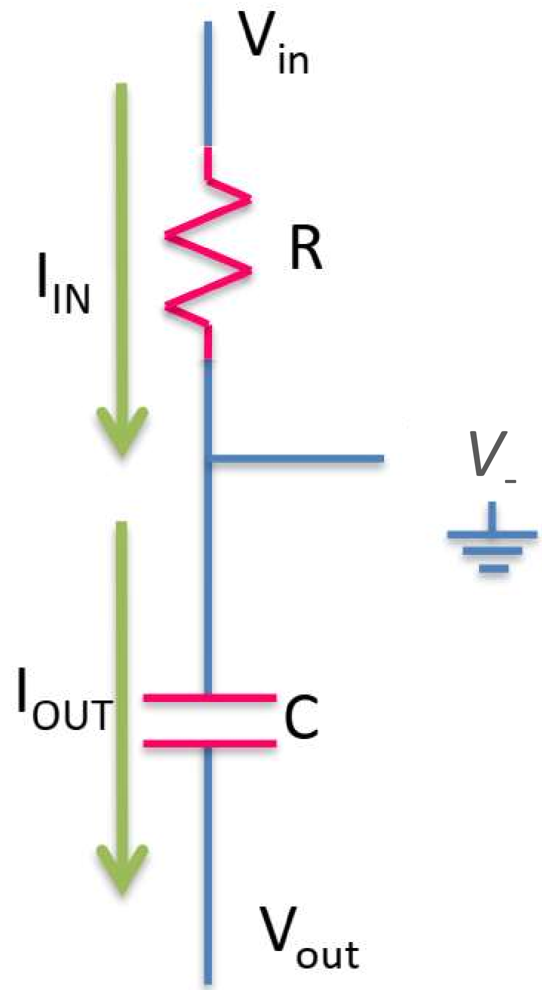
$$V_{\mathrm{OUT}} = (1 + \frac{R_1}{R_2})V_{\mathrm{IN}}$$



- The important thing to note here is that there is NO MINUS SIGN.
- This is a non-inverting amplifier and the gain is given as shown. If you want a gain of 1.00, make R_1 = 0 or R_2 = infinity.

An Amplifier: With a Capacitor



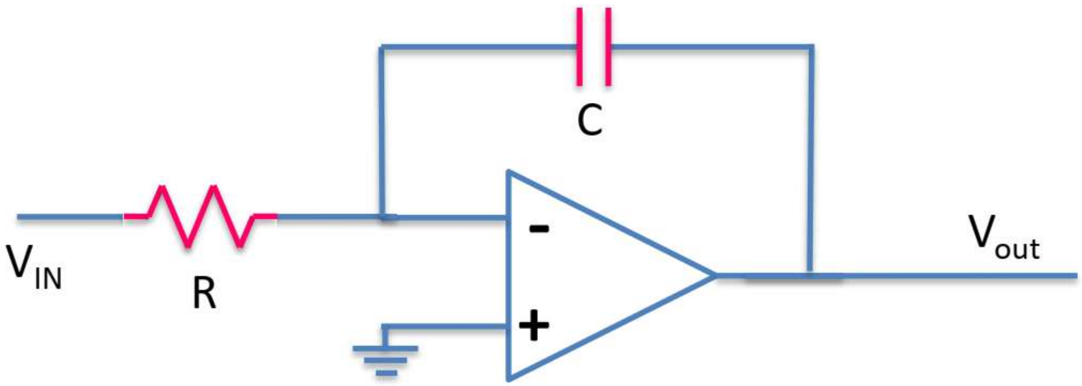


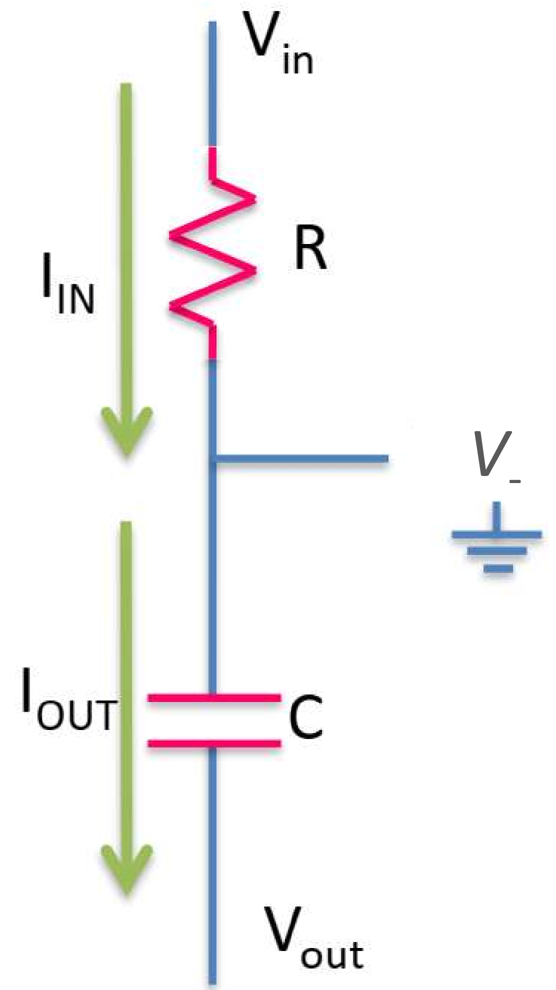
- There are two ways to look at this circuit. The first is to treat it as a normal amplifier.

$$A = -\frac{Z_C}{R} = -\frac{1}{R}\frac{1}{j\omega C} = -\frac{1}{j\omega RC}$$

- We have a frequency dependent gain. When frequency is zero, the gain is infinite. When frequency is high, the gain is low.

An Amplifier: With a Capacitor





- A second way is to look at the current:

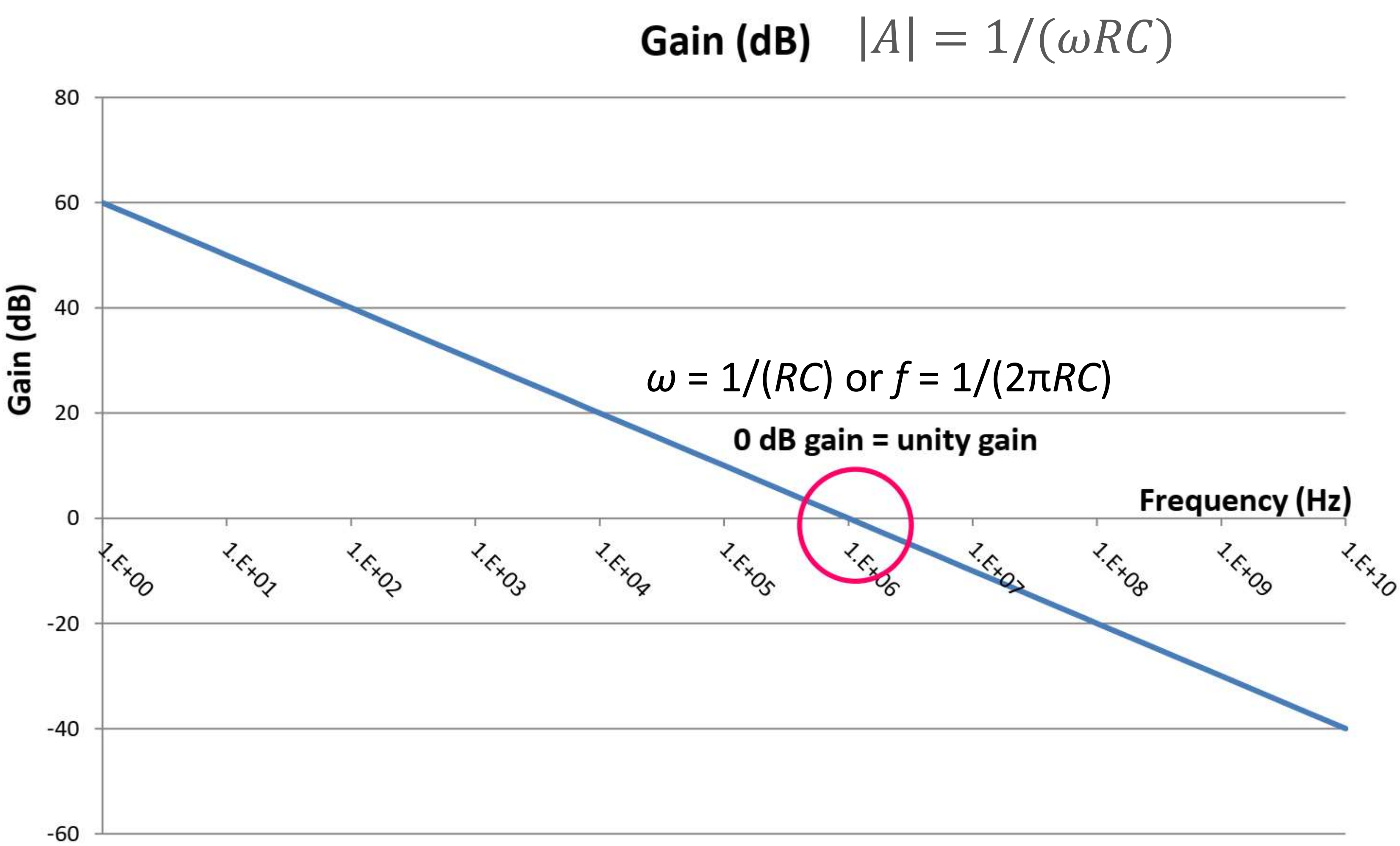
$$I_{\mathrm{IN}} = \frac{V_{\mathrm{IN}} - V_{-}}{R} \text{ and } I_{\mathrm{OUT}} = C\,\frac{d(V_{-} - V_{\mathrm{OUT}})}{dt}$$

$$\frac{V_{\mathrm{IN}}}{R} = -C\,\frac{dV_{\mathrm{OUT}}}{dt}$$

$$V_{\mathrm{OUT}} = -\left(\frac{1}{RC}\right)\int V_{\mathrm{IN}}\,dt$$

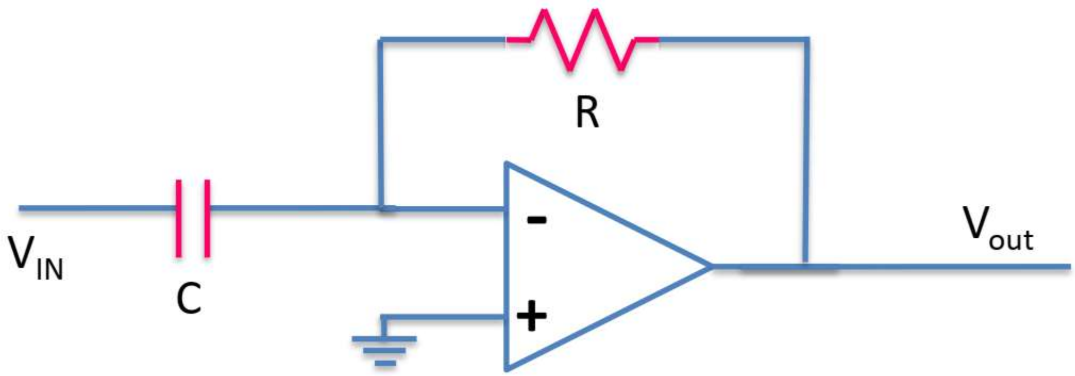
- V_{OUT} is the INTEGRAL of V_{IN}, this gives us an INTEGRATOR function.

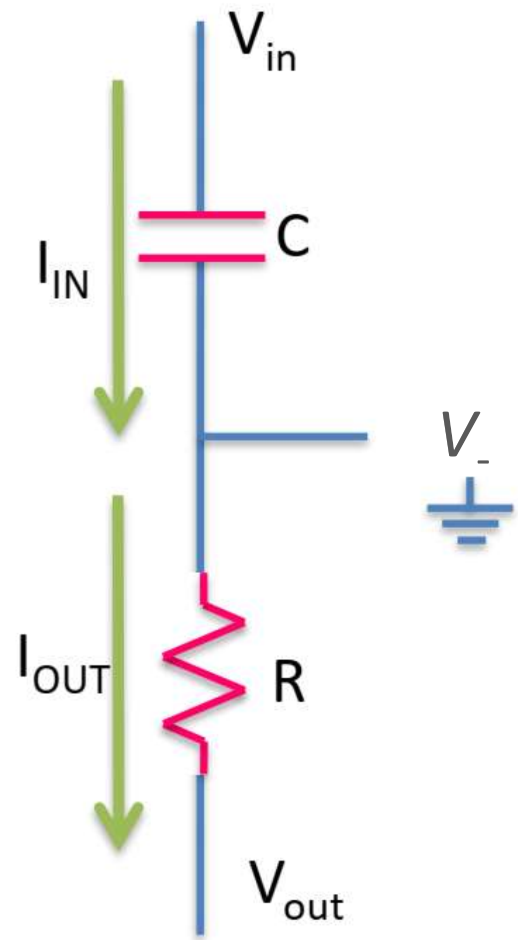
An Integrator



Attention! The plot is in the log-log scale.

Another Amplifier: With a Capacitor





- Swap the capacitor and the resistor. Let's redo the calculation:

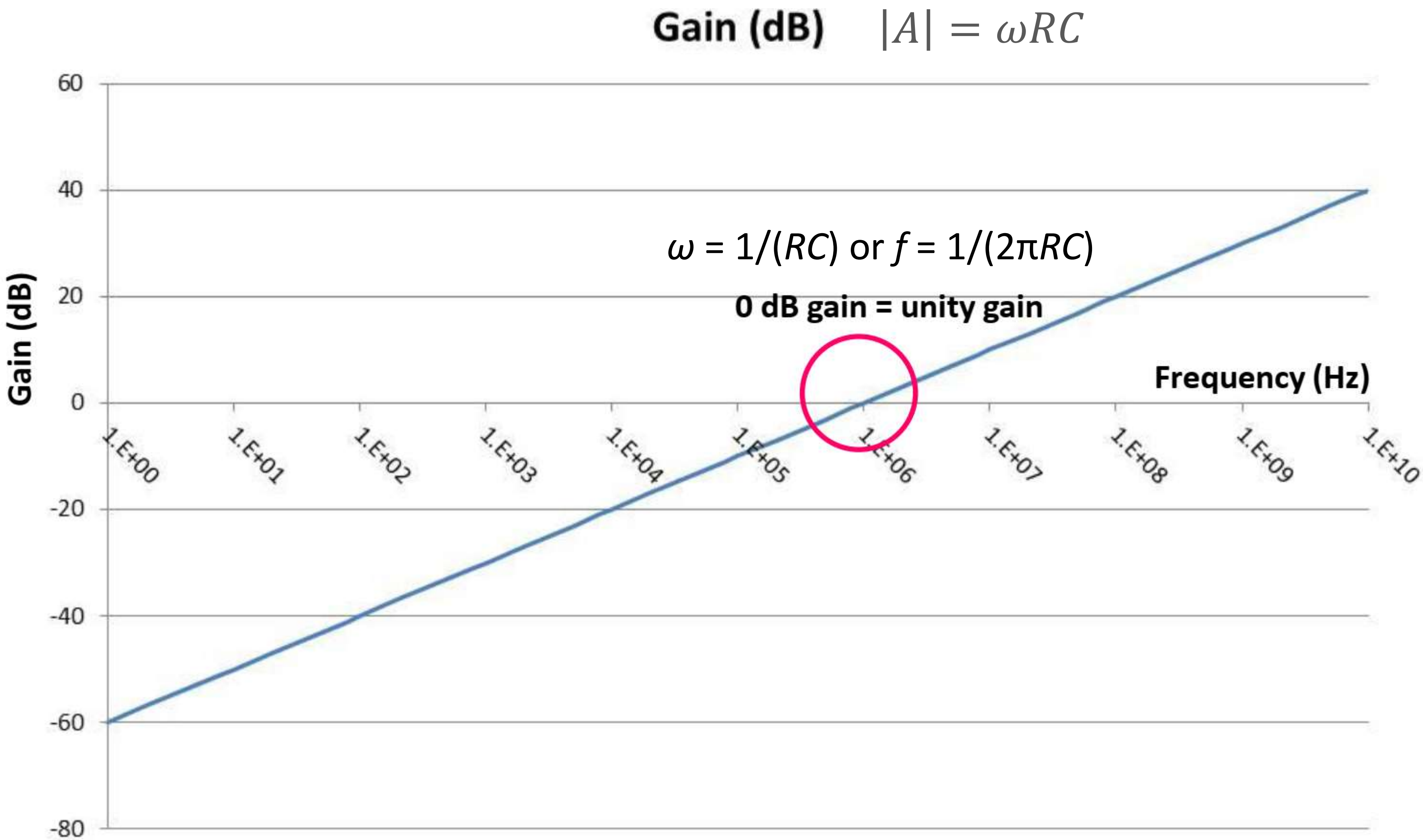
$$I_{\mathrm{IN}} = C\frac{d(V_{\mathrm{IN}} - V_{-})}{dt} \text{ and } I_{\mathrm{OUT}} = \frac{V_{-} - V_{\mathrm{OUT}}}{R}$$

$$C\frac{dV_{\mathrm{IN}}}{dt} = \frac{-V_{\mathrm{OUT}}}{R}$$

$$V_{\mathrm{OUT}} = -(RC)\frac{dV_{\mathrm{IN}}}{dt}$$

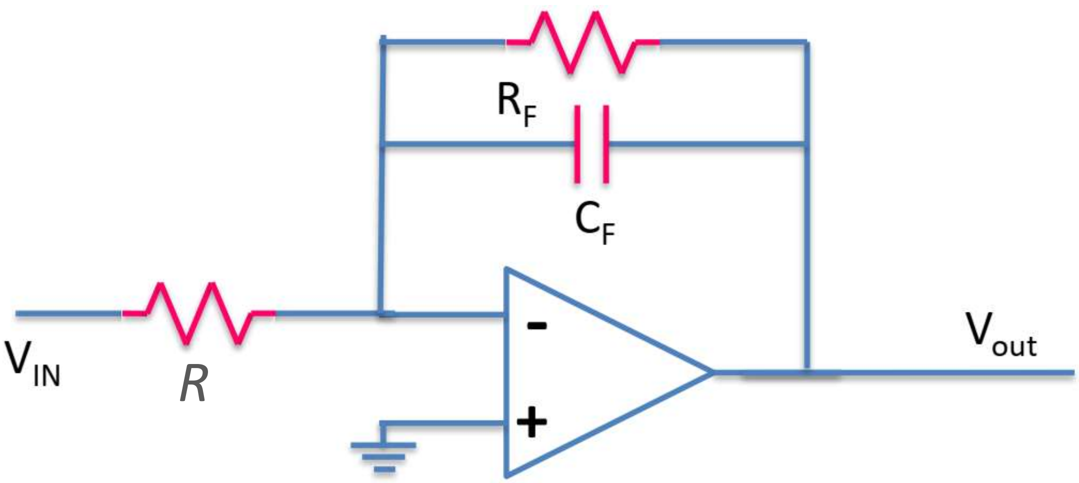
- V_{OUT} is the DIFFERENTIAL of V_{IN}, this gives us a DIFFERENTIATOR function.

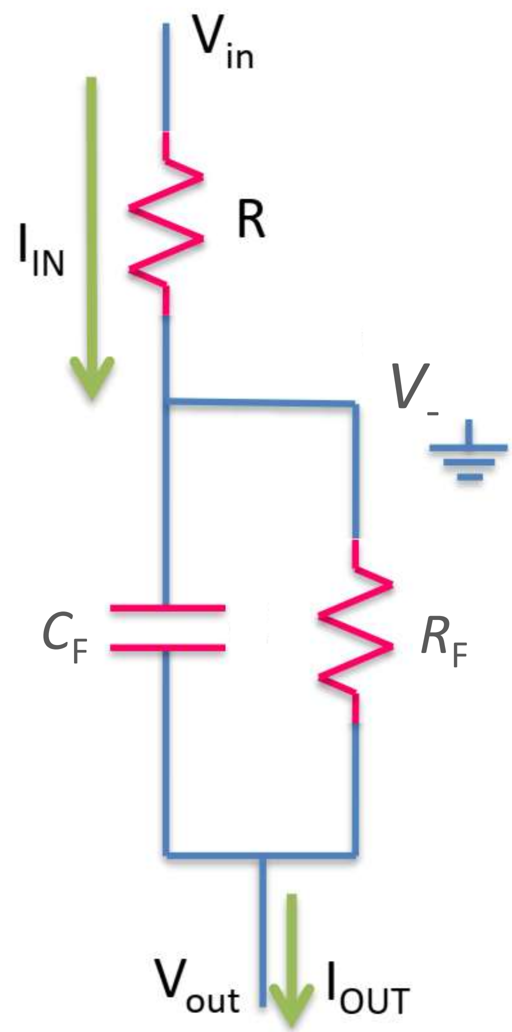
A Differentiator



Attention! The plot is in the log-log scale.

More Complex Structures



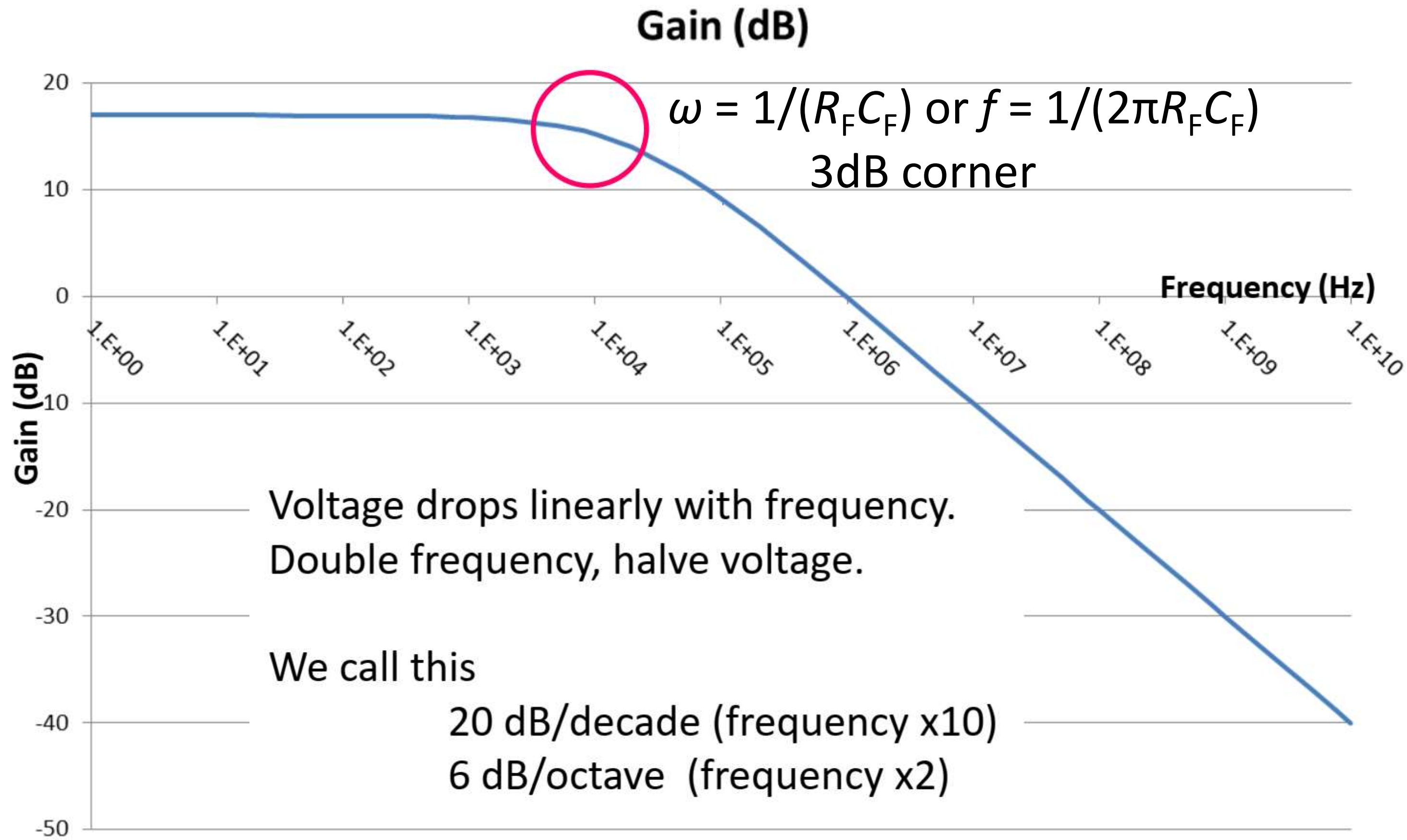


- Let's look at the gain:

$$A = -\frac{Z_{\mathrm{F}} \parallel R_{\mathrm{F}}}{R} = -\frac{1}{R}\left(\frac{1}{\frac{1}{R_{\mathrm{F}}} + j\omega C_{\mathrm{F}}}\right)$$

$$A = -\left(\frac{R_{\mathrm{F}}}{R}\right)\left(\frac{1}{1 + j\omega R_{\mathrm{F}} C_{\mathrm{F}}}\right)$$

Low-Pass Filter With Corner at $1/(2\pi RC)$



- We call filters with a single "corner" a single-pole filter. You can get multiple pole filters (higher order filters). You get a pole every time you hit a frequency point that halves the amplitude. Normally it's one pole per capacitor.

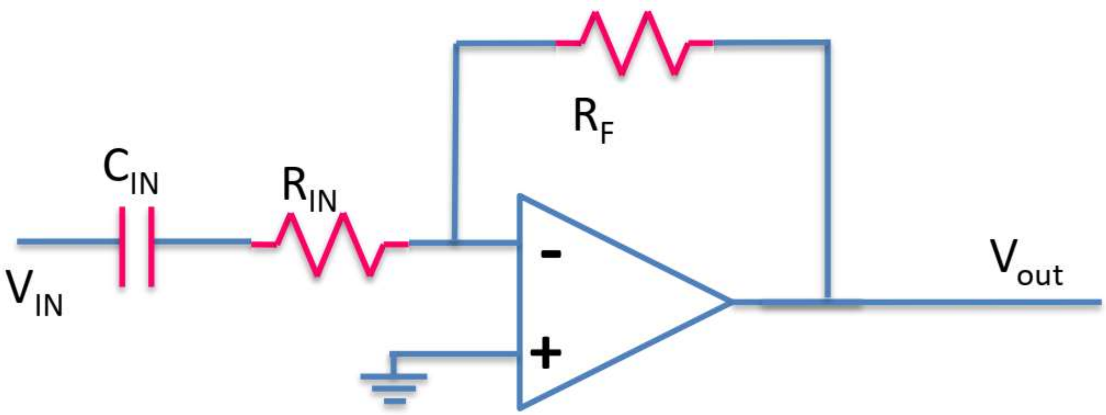
Other Characteristics: Unity Gain Bandwidth

- One characteristic often quoted for op-amps is unity gain bandwidth.
- This works on the assumption that even with the best of intentions, there is parasitic capacitances in the circuit, if nothing else arising from the transistors. This will cause the roll-off in gain that you see.
- Unity gain bandwidth assumes that we have first-order roll-off (a single aggregate capacitor effect) and it is the bandwidth for which gain is > 1. Or more simply, the frequency at which gain = 1.
- MORE IMPORTANTLY. If we have a single capacitor effect (first-order filter effect), then the gain will change by a factor of 10 if the frequency changes by a factor of 10. The product (so-called gain-bandwidth product or GBW) is a constant. So

If	$G = 1$	at BW = 100 MHz	(GBW = 100M)
then	$G = 10$	at BW = 10 MHz	(GBW = 100M)
then	$G = 100$	at BW = 1 MHz	(GBW = 100M)

- This is not perfectly true in all cases, but it is a useful guide.

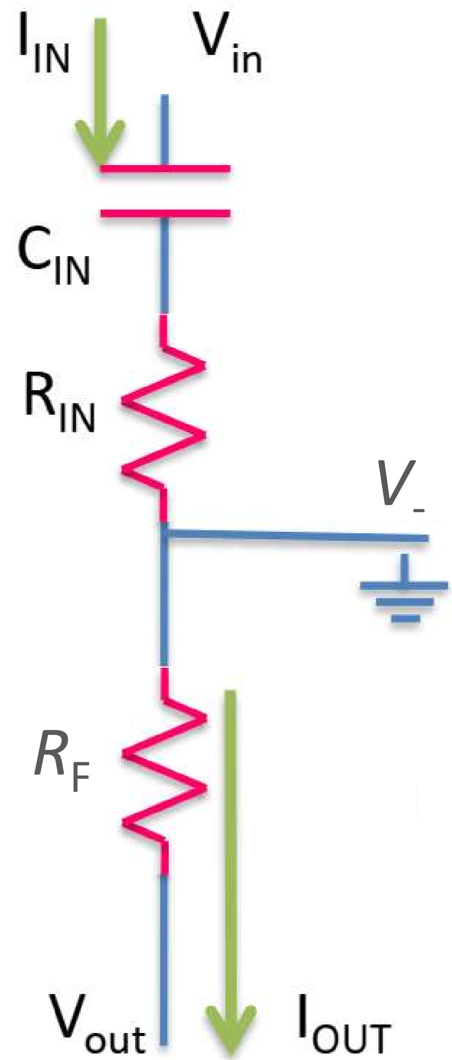
More Complex Structures



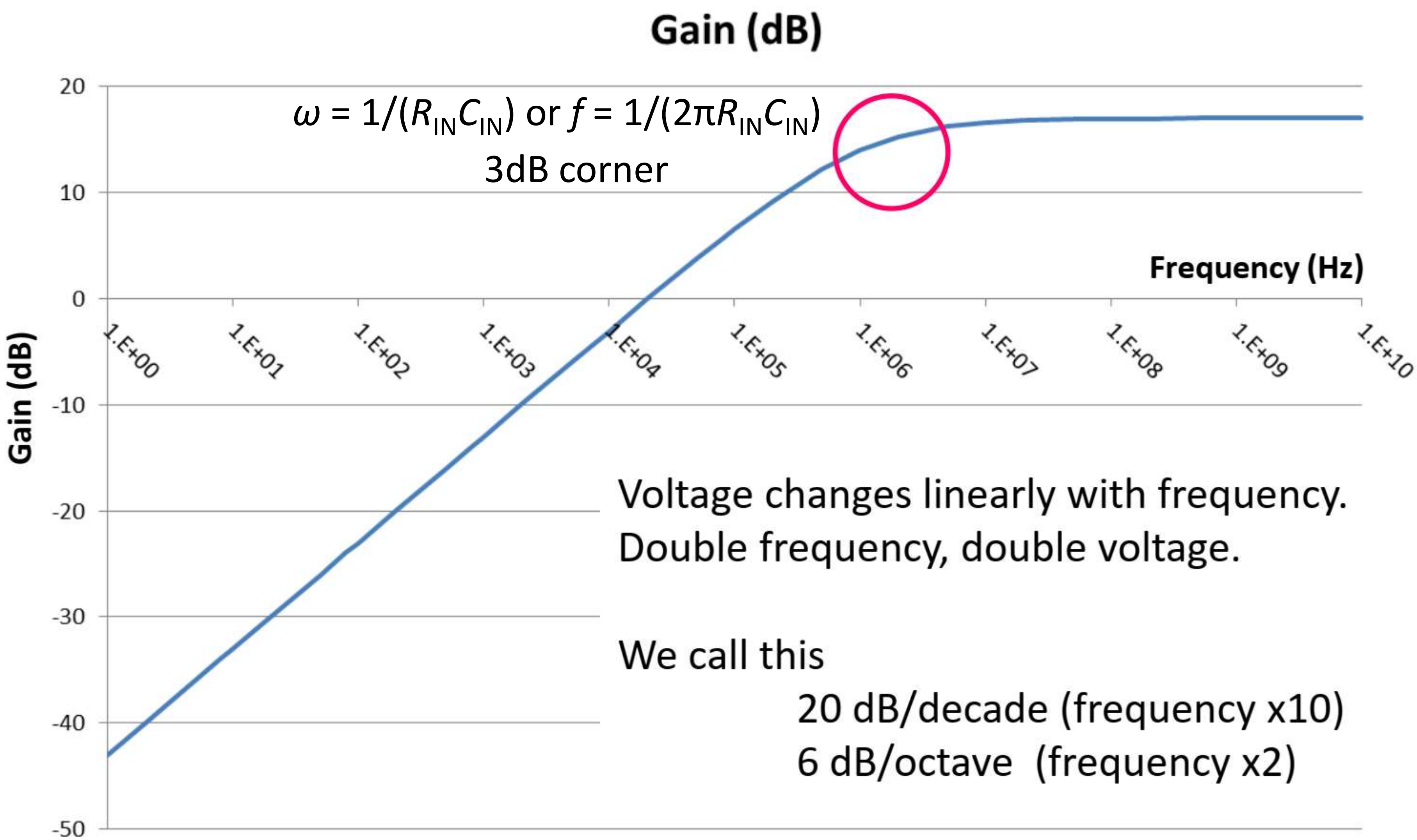
- Let's look at the gain:

$$A = -\frac{R_F}{R_{IN} + Z_{IN}} = -R_F\left(\frac{1}{R_{IN} + \frac{1}{j\omega C_{IN}}}\right)$$

$$A = -\left(\frac{j\omega R_F C_{IN}}{1 + j\omega R_{IN} C_{IN}}\right)$$



High-Pass Filter With Corner at $1/(2\pi RC)$

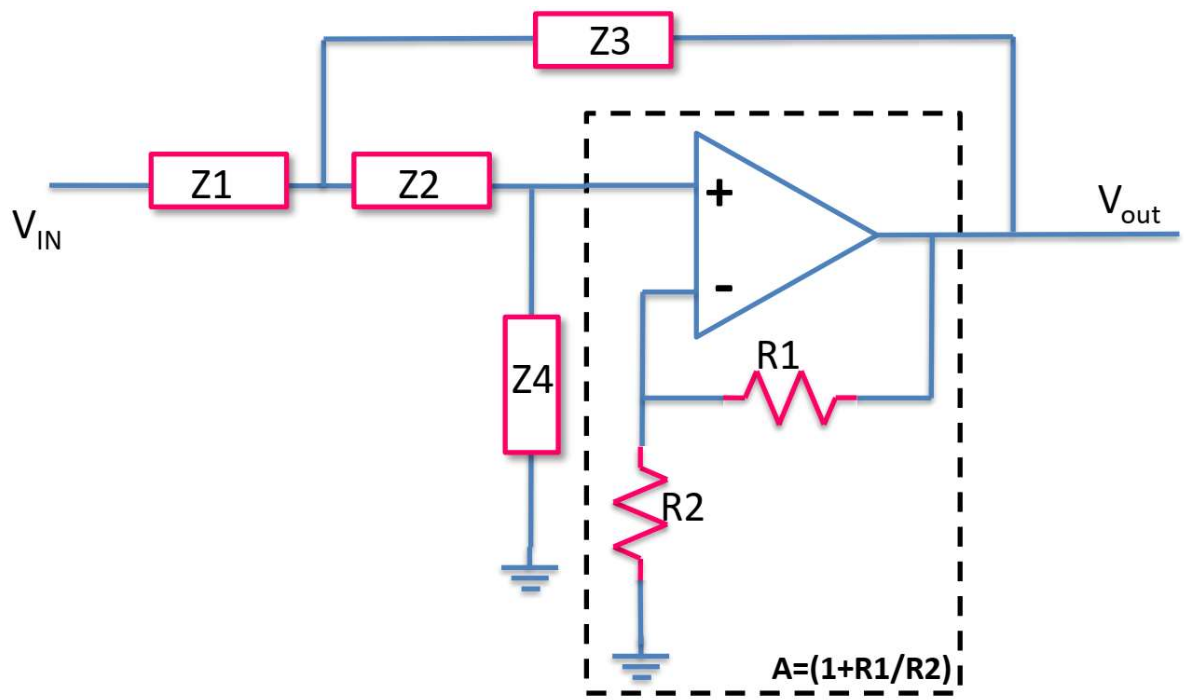


Voltage changes linearly with frequency.
Double frequency, double voltage.

We call this

20 dB/decade (frequency x10)
6 dB/octave (frequency x2)

The Sallen-Key Filter Structure

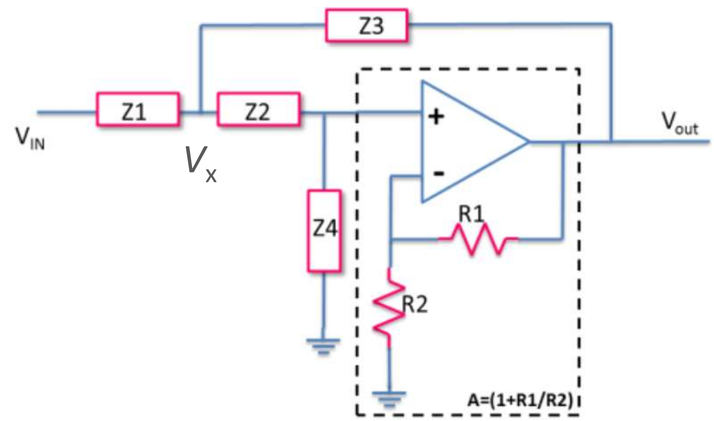


- This structure is a general-purpose structure for constructing filters. It is well understood. We have to look at what the gain equation is. The important first step is that the dashed-box is a standard non-inverting amplifier where we control gain. We're going to set R_1 = 0 and assume unity gain for the moment.
- What we need to figure out is what the value of V_+ is.

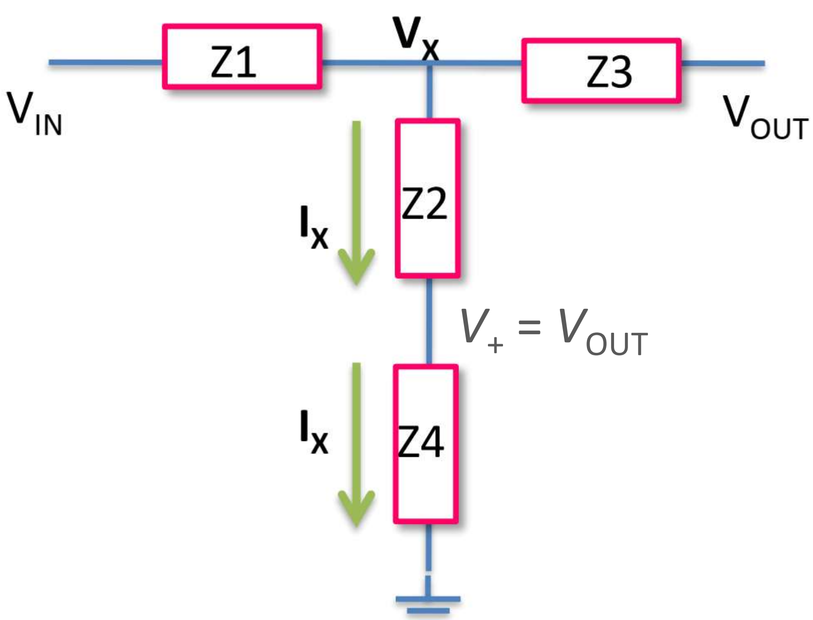
The Sallen-Key Filter Structure

- We're only after V_+. Let's re-draw the circuit and accept the following criteria. There is non-inverting amplification of A = 1.0 (We can change the value of gain by changing R_1 and R_2 if we want). This means

$$V_{\mathrm{OUT}} = A \cdot V_+ = V_+$$



- We're going to find V_x, get the current I_x, and then work out $V_+ = I_x Z_4$.



Kirchhoff's current law at point x:

$$\frac{V_{\mathrm{x}} - V_+}{Z_2} = \frac{V_{\mathrm{IN}} - V_{\mathrm{x}}}{Z_1} + \frac{V_{\mathrm{OUT}} - V_{\mathrm{x}}}{Z_3}$$

$$\frac{V_{\mathrm{x}} - V_{\mathrm{OUT}}}{Z_2} = \frac{V_{\mathrm{IN}} - V_{\mathrm{x}}}{Z_1} + \frac{V_{\mathrm{OUT}} - V_{\mathrm{x}}}{Z_3}$$

Kirchhoff's current law at V_+:

$$I_{\mathrm{x}} = \frac{V_{\mathrm{x}} - V_{\mathrm{OUT}}}{Z_2} = \frac{V_{\mathrm{OUT}} - 0}{Z_4}$$

$$V_{\mathrm{x}} = (1 + \frac{Z_2}{Z_4}) V_{\mathrm{OUT}}$$

The Sallen-Key Filter Structure

- We insert $V_\mathrm{X} = (1+\frac{Z_2}{Z_4})V_\mathrm{OUT}$ into equation $\frac{V_\mathrm{X}-V_\mathrm{OUT}}{Z_2} = \frac{V_\mathrm{IN}-V_\mathrm{X}}{Z_1} + \frac{V_\mathrm{OUT}-V_\mathrm{X}}{Z_3}$:

$$\frac{(1+\frac{Z_2}{Z_4})V_\mathrm{OUT} - V_\mathrm{OUT}}{Z_2} = \frac{V_\mathrm{IN} - (1+\frac{Z_2}{Z_4})V_\mathrm{OUT}}{Z_1} + \frac{V_\mathrm{OUT} - (1+\frac{Z_2}{Z_4})V_\mathrm{OUT}}{Z_3}$$

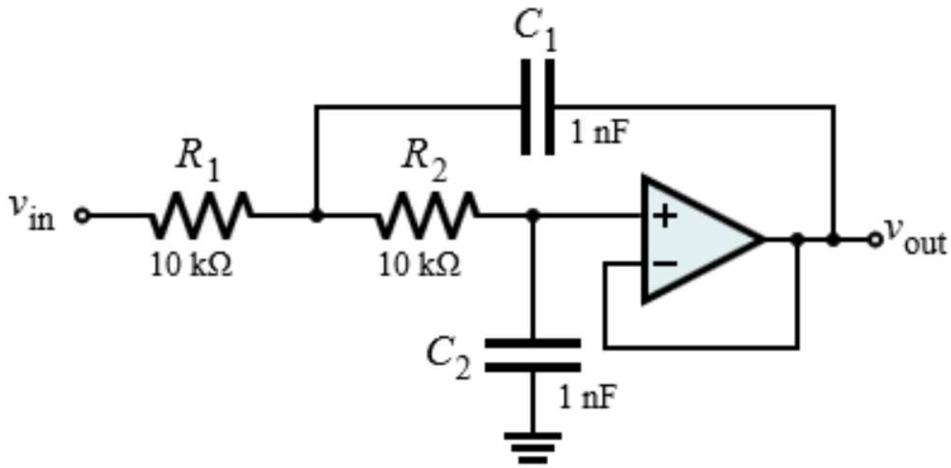
- This equation now contains only V_{IN} and V_{OUT}. Solving this equation gives the "gain" of the circuit:

$$\frac{V_\mathrm{OUT}}{V_\mathrm{IN}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4}$$

- If all Z's are complex, then this is a second order filter (In fact, this requirement is usually too strong. We only need either Z_1 and Z_2 or Z_3 and Z_4 to be imaginary).

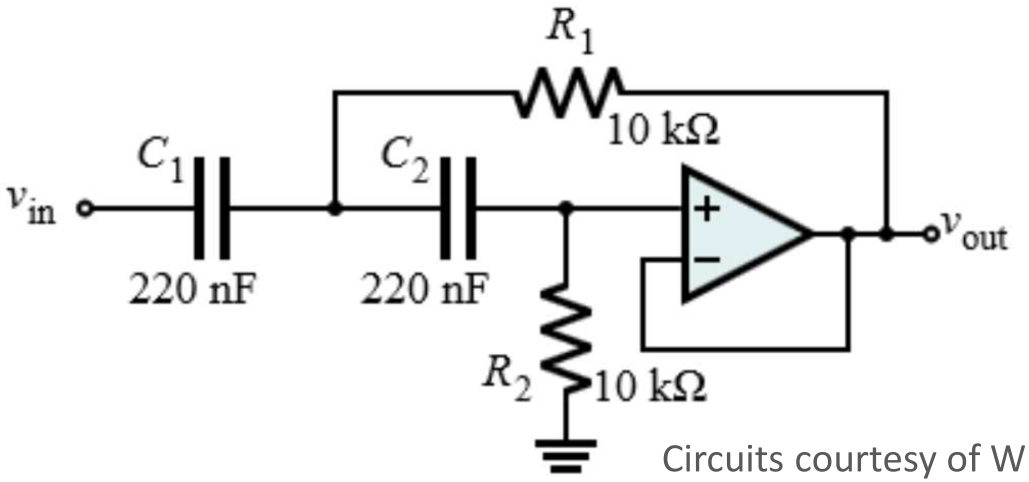
Sallen-Key Filter Examples

- Low-Pass Filter (LPF):



$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

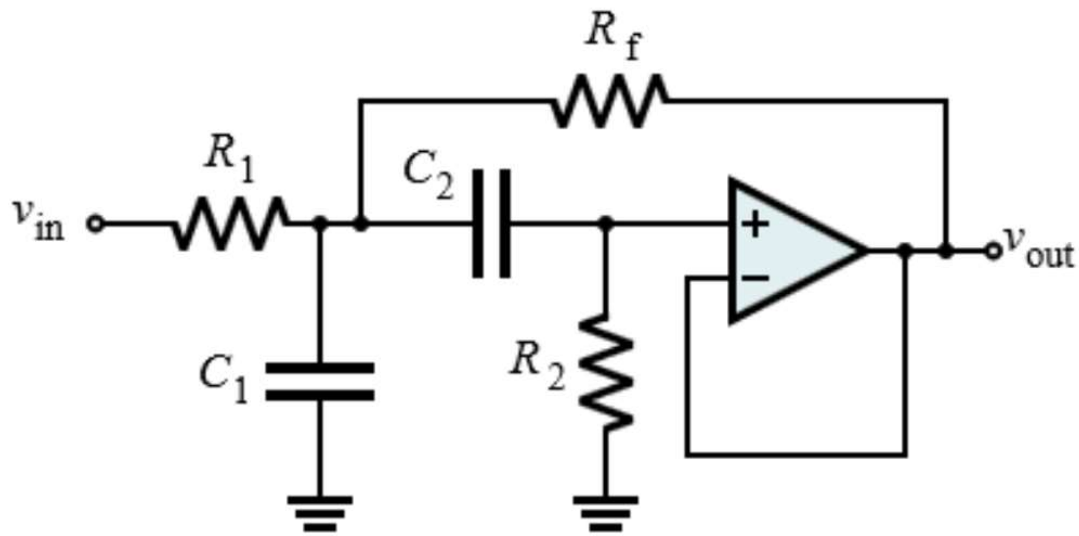
- High-Pass Filter (HPF):



$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Sallen-Key Filter Examples

- Band-Pass Filter (BPF):



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{R_f + R_1}{C_1 C_2 R_1 R_2 R_f}}$$

- In these three circuits, and in the analysis, we assumed gain = 1 by setting $R_1 = 0$ and $R_2 = \infty$. However, we can add gain by the equation $A = 1 + (R_1/R_2)$. This is possible for all circuits and just adds amplification to the filter function.
- More gain tends to widen the bandwidth of our filter as the passband starts higher and needs longer to decay before it is a small number (< 1).