EE211 Tutor 1 ①

Q1

Sol.

(1) $\frac{dy(t)}{dt} = u(t) - y(t)$

(2) $\frac{dy(t)}{dt} = 3u(t) - 2\sqrt{y(t)}$

(3) $y(t) = 3u(t)$

(4) $y(t) = 10\sqrt{u(t)}$

(5) $\frac{dy(t)}{dt} = u(t) - a(t)y(t)$, where $a(t)$ is a constant that varies with time!

PS. for all example above, the dependent variable is y, the independent variable is t and the parameters are the constants used.

Q2.

Sol.

(i) This is a system that obey the principle of superposition and homogeneity ie: $Af(x_1) + Bf(x_2) = f(Ax_1 + Bx_2)$ for any A.B

(ii) $y = 2u \Rightarrow Af(u_1) + Bf(u_2) = A(2u_1) + B(2u_1) = 2Au_1 + 2Bu_2$

$y = 2u \Rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) = 2Au_1 + 2Bu_2$

So $Af(u_1) + Bf(u_2) = f(Au_1 + Bu_2) \Rightarrow$ linear

(iii) $y = 2\sqrt{u} \rightarrow Af(u_1) + Bf(u_2) = A \cdot 2\sqrt{u_1} + B2\sqrt{u_2}$

$y = 2\sqrt{u} \rightarrow f(Au_1 + Bu_2) = 2\sqrt{Au_1 + Bu_2}$

Take $A = 1$ $B = 1$ for example.

$2\sqrt{u_1} + 2\sqrt{u_2} \neq 2\sqrt{u_1 + u_2}$ for all of the conditions

So Nolinear.

No.

Data

Q2.

(iv) sol

$y = 2u+1 \Rightarrow Af(u_1) + Bf(u_2) = A(2u_1+1) + B(2u_2+1)$

$= 2(Au_1 + Bu_2) + (A+B)$

$y = 2u+1 \Rightarrow f(Au_1 + Bu_2) = 2(Au_1 + Bu_2) + 1$

Hence $Af(u_1) + Bf(u_2) = f(Au_1 + Bu_2) \Rightarrow$ Nolinear

Q3. $F(s) = \frac{s}{(s+2)(s+5)} = -\frac{2}{3}\cdot\frac{1}{s+2} + \frac{5}{3}\cdot\frac{1}{s+5}$

Sol

So. $f(t) = -\frac{2}{3} e^{-2t} + \frac{5}{3} e^{-5t}$.

Q4. $\frac{dx(t)}{dt} + 3x(t) - 4 = 0 \Leftrightarrow \dot{x}(t) + 3x(t) = 4$

(i)(ii)(iii)

$sX(s) - 0x(0) + 3X(s) = 4$

$\therefore (s+3)X(s) = 5$ Hence $X(s) = \frac{5}{s+3}$

$x(t) = 5 \cdot e^{(-3t)}$

And transfer function model $G(s) = \frac{Y(s)}{X(s)} = \frac{5}{s+3}$

Q5.

(i) Differential equations in the t-domain can be coverted into s-domain, which is easier to resolve.

(ii) Only linear-time-invariant system have transfer function.

EE211 Tutor 1 ②

0306 P③

Q6. ① $G_5 + G_3 \cdot G_4$

② $G_2 (G_5 + G_3 \cdot G_4)$.

③ $\dfrac{G_2 (G_5 + G_3 \cdot G_4)}{1 - G_2 (G_5 + G_3 \cdot G_4) H_1}$.

④ $G_1 \cdot \dfrac{G_2 (\quad)}{1 - G_2 (\quad) H_1}$

⑤ $$G_s = \left[\frac{G_1 G_2 (G_5 + G_3 \cdot G_4)}{1 - G_2 (G_5 + G_3 \cdot G_4) H_1} \right] \Big/ \left[1 + \frac{G_1 G_2 (G_5 + G_3 \cdot G_4)}{1 - G_2 (G_5 + G_3 G_4) H_1} H_2 \right]$$

$$= \frac{G_1 G_2 (G_5 + G_3 G_4)}{1 - G_2 (G_5 + G_3 \cdot G_4) H_1 + \left[G_1 G_2 (G_5 + G_3 \cdot G_4) \right] H_2}$$

Q7. $$G_s = \frac{\dfrac{30 s^2 + 15 s + 10}{3 s^2 + 7 s + 2}}{1 + \dfrac{30 s^2 + 15 s + 10}{3 s^2 + 7 s + 2}} = \frac{30 s^2 + 15 s + 10}{33 s^2 + 22 s + 12}$$

EE211 Tutor 2.

0306

P④

Q1. sol

KVL : $v_i = v_R + v_L + v_C$ ①

$v_R = iR$ ② $\quad v_L = L\frac{di}{dt}$ ③

$v_C = \frac{1}{C}\int i\,dt$ ④

So $v_i = iR + L\frac{di}{dt} + \cancel{\frac{1}{C}\int i\,dt}\, v_C$

$i = C\frac{dv_C}{dt}$ ⑤

$\therefore v_i = R\cdot C\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C$

Hence

$$LC\frac{d^2v_C}{dt} + RC\frac{dv_C}{dt} + v_C = v_i$$

Q2. sol

① $\frac{v_1 - v_i}{R_1} + \frac{v_1 - v_0}{sL} + \frac{v_1 - 0}{\frac{1}{sC}} = 0$

$\frac{v_0 - 0}{R_2} + \frac{v_0 - v_1}{sL} = 0$

Rearranging gives :

$[(R_1LC)s^2 + (R_1R_2C + L)s + (R_1+R_2)]\,v_0 = R_2 v_i$

So $\frac{v_0(s)}{v_i(s)} = \frac{R_2}{(R_1LC)s^2 + (R_1R_2C + L)s + R_1 + R_2}$ ①

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② $\frac{v_1 - v_i}{R_1} + \frac{(v_1 - v_0)}{R_2} + \frac{v_1 - 0}{\frac{1}{sC}} = 0$

$\frac{v_0 - 0}{R_3} + \frac{v_0 - v_1}{R_2} = 0$

$v_0\left[(R_1+R_2+R_3 - sCR_1R_2)\left(\frac{R_2+R_3}{R_3}\right) - (R_1+R_2+R_3)\right] = R_2 v_i$

$\therefore \frac{v_0(s)}{v_i(s)} = \frac{R_2}{[(R_1+R_2+R_3 - sCR_1R_2)\left(\frac{R_2+R_3}{R_3}\right) - (R_1+R_2+R_3)]}$ ②

Q3. sol

Force equilibrium : $F_m + F_{d_1} + F_{d_2} + F_{s1} + F_{s2} = f(t)$ ①

$M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + K_1x + B_2\frac{dx}{dt} + K_2x = f(t)$ ②

P5

Q4 sol.

For M_2 : $F(t) = f_{d2} + f_{s2} + F_M$ ①

For M_1 : $f_{d2} + f_{s2} = f_{d1} + f_{s1} + F_M$ ②

$$\begin{cases} F(t) = B_2(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) + M_2\ddot{x}_2 & ③ \\ B_1\dot{x}_1 + K_1 x_1 + M_1\ddot{x}_1 = B_2(\dot{x}_2 - \dot{x}_1) + K_2(x_2 - x_1) & ④ \end{cases}$$

PS:

$$\begin{cases} \alpha = M_2(B_1 + B_2) \\ \beta = M_2(K_1 + K_2) + B_1B_2 + K_2M \end{cases}$$

Hence $$\frac{X_1(s)}{F(s)} = \frac{M_1 s^2 + (B_1+B_2)s + (K_1+K_2)}{M_1M_2 s^4 + \alpha s^3 + \beta s^2 + (B_1K_2 + B_2K_1)s + K_1K_2}$$ ⑤

Q5. sol

① 1st order

$$y_k = a y_{k-1} + b u_{k-1}$$

$$\begin{cases} 0.109 = 0.098a + 0.065b & ① \\ -0.1 = -0.128a - 0.1b & ② \end{cases}$$

$$\therefore \begin{bmatrix} 0.109 \\ -0.100 \end{bmatrix} = \begin{bmatrix} 0.098 & 0.065 \\ -0.128 & -0.1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \equiv y$$

$y \equiv X\theta \qquad \theta = \begin{bmatrix} a \\ b \end{bmatrix}$

So $$\begin{cases} a = \frac{110}{37} \\ b = -\frac{519}{185} \end{cases}$$

$$\therefore y_k = \frac{110}{37} y_{k-1} - \frac{519}{185} u_{k-1}$$

(ii) Hence $$G(z) = \frac{519/185}{1-(110/37)(z^{-1})}(z^{-1})$$

② 2nd order

$$y_k = a_1 y_{k-1} + a_2 y_{k-2} + b_1 u_{k-1} + b_2 u_{k-2}$$

$$\begin{cases} -0.1 = -0.128a_1 - 0.019a_2 - 0.1b_1 - 0.15b_2 & ① \\ -0.128 = -0.019a_1 + 0.117a_2 - 0.15b_1 - 0.15b_2 & ② \\ -0.019 = 0.117a_1 + 0.109a_2 - 0.15b_1 + 0.065b_2 & ③ \\ 0.117 = 0.109a_1 + 0.098a_2 + 0.065b_1 + 0.065b_2 & ④ \end{cases}$$

$$\therefore \begin{cases} a_1 = -3.09 \\ a_2 = 2.51 \\ b_1 = 0.65 \\ b_2 = 2.55 \end{cases}$$

$$\therefore y_k = -3.09 y_{k-1} + 2.51 y_{k-2} + 0.65 u_{k-1} + 2.55 u_{k-2}$$

(ii) $$G(z) = \frac{Y(z)}{U(z)} = \frac{0.65z^{-1} + 2.55z^{-2}}{1 + 3.09z^{-1} - 2.51z^{-2}}$$

Q6 (i) $$\frac{dV}{dt} = F_{in} - F_{out}$$ ①

$V = Ah$ ② $\rightarrow \frac{dV}{dh} = A\frac{dh}{dt} = F_{in} - F_{out}$ & $F_{out} = \frac{h}{R}$ ③

So $$A\frac{dh}{dt} = F_{in} - \frac{h}{R}$$

(ii) $$F_{out} = \frac{\sqrt{h}}{R}$$ ④

So $$A\frac{dh}{dt} = F_{in} - \frac{\sqrt{h}}{R}$$ ⑤

Q7

Tank I : $A_1 \frac{dh_1}{dt} + \frac{h_1}{R_1} = F_{in}$ ①

Tank II : $A_2 \frac{dh_2}{dt} + \frac{h_2}{R_2} = F_{12}$ ②

$F_{12} = \frac{h_1}{R_1}$ ③

② $\Rightarrow$ $A_2 \frac{dh_2}{dt} + \frac{h_2}{R_2} = \frac{h_1}{R_1}$ ④

Hence $\frac{dh_1}{dt} = A_2 R_1 \frac{d^2 h_2}{dt} + \frac{R_1}{R_2} \cdot \frac{dh_2}{dt}$ ⑤.

Subbing into ①

So $\frac{d^2 h}{dt^2}(A_1 R_1 A_2 R_2) + \frac{dh_2}{dt}(A_1 R_1 + A_2 R_2) + h_2 = R_2 F_{in}$ ⑥

Q8. $F_{12} = \frac{h_1 - h_2}{R_1}$ ① & $F_{out} = \frac{h_2}{R_2}$ ②

$A_1 \frac{dh_1}{dt} = F_{in} - (\frac{h_1 - h_2}{R_1})$ ③

So
$$\begin{cases} A_1 R_1 \frac{dh_1}{dt} = R_1 F_{in} - h_1 + h_2 \\ A_2 R_1 R_2 \frac{dh_2}{dt} = R_2 h_1 - h_2 (R_1 + R_2). \end{cases}$$