EE206 Assignment 5 *

Due 10th Nov.

1. Find the Laplace Transform of the following functions, **using the definition, NOT the tables.**

   (a) $f(t) = t^n$, where n is a natural number, i.e. $n = 0, 1, 2, \ldots$
   (Hint: write $\mathcal{L}\{t^n\}$) in terms of $\mathcal{L}\{t^{n-1}\}$ using integration by parts. Then use the result to write $\mathcal{L}\{t^n\}$ in terms of $\mathcal{L}\{1\}$

   (b) $f(t) = 2\sinh 3t + \cos 2t$

2. Find the inverse Laplace transform of the following

   (a) $\mathcal{L}^{-1}\left\{\frac{6}{s^2+36s}\right\}$

   (b) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-5)(s-7)}\right\}$

   (c) $\mathcal{L}^{-1}\left\{\frac{(s-1)^3}{s^4}\right\}$

3. Use the Laplace transform to solve the given initial-value problems

   (a) $y'' + 5y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$

   (b) $2\frac{dy}{dt} - y = 0, \quad y(0) = 5$

   (c) $y' - y = 2\cos 6t, \quad y(0) = 0$

   (d) $y'' - 10y' + 25y = 3e^{3t}, \quad y(0) = 0, \quad y'(0) = -1$

4. Use the First Translation (Shift) Theorem to find either $F(s)$ or $f(t)$, as indicated. State in each case how the translation theorem applies.

   (a) $\mathcal{L}\{\cosh(t)\cos(t)\}$

   (b) $\mathcal{L}^{-1}\left\{\frac{(s-1)^2}{(s+2)^4}\right\}$

5. Use the Second Translation (Shift) Theorem to find either $F(s)$ or $f(t)$, as indicated. State in each case how the translation theorem applies.

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(a) $\mathcal{L}\{(3t+1)\mathcal{U}(t-1)\}$

(b) $\mathcal{L}\{\cos(4t-8)\mathcal{U}(t-2)\}$

(c) $\mathcal{L}^{-1}\left\{\dfrac{(1+e^{-s})^2}{s+3}\right\}$