Maximum Power Transfer Theorem

Voltage Polarity & Current Direction

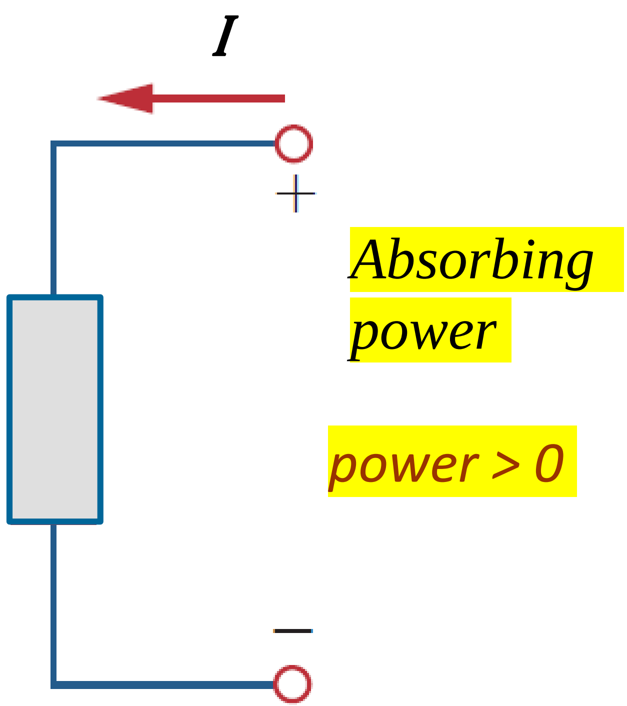
Convention

A convention is a standard way of describing something so that others in the profession can understand what we mean.

By the passive sign convention**, current enters through the positive polarity of the voltage.**

Voltage Polarity

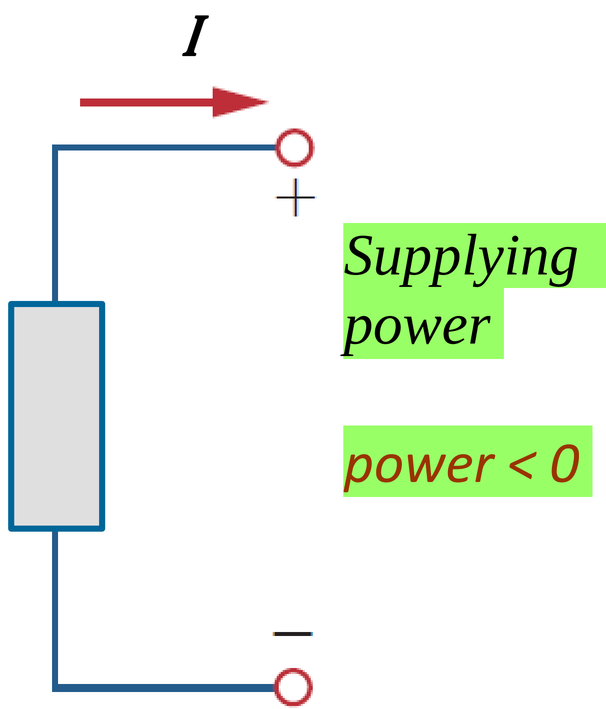
The current direction and voltage polarity play a major role to decide the power sign.



- The *passive sign convention* is satisfied when the current enters the positive terminal of the element.
- Then, P=VI>0 and this means that the element is absorbing this power.

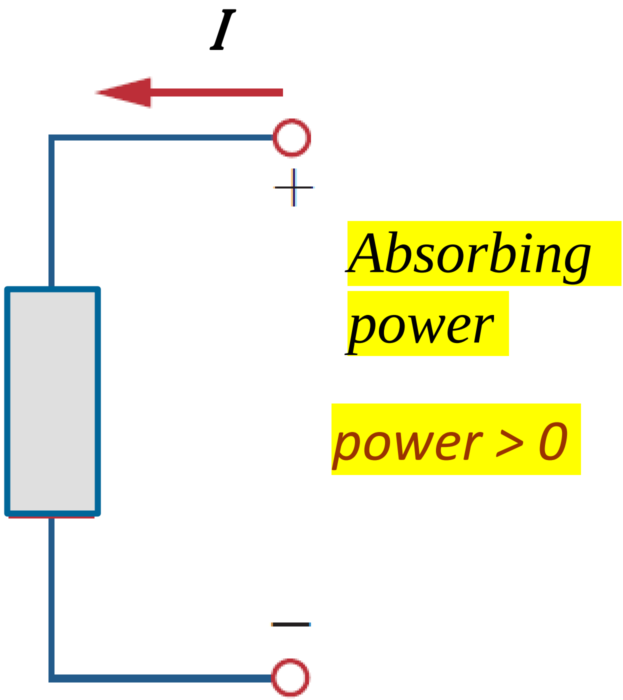
Voltage Polarity

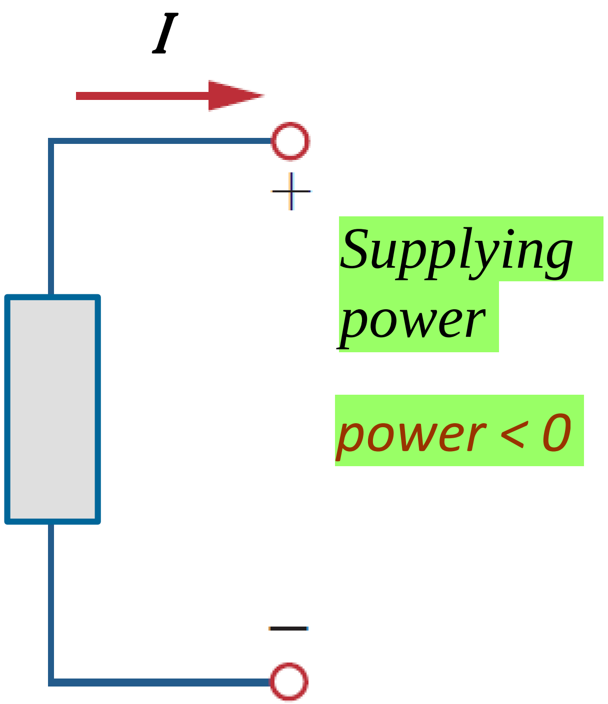
The current direction and voltage polarity play a major role to decide the power sign.



- However, if the current enters the negative terminal of the element, then P<0.
- This means that the element is supplying this power.

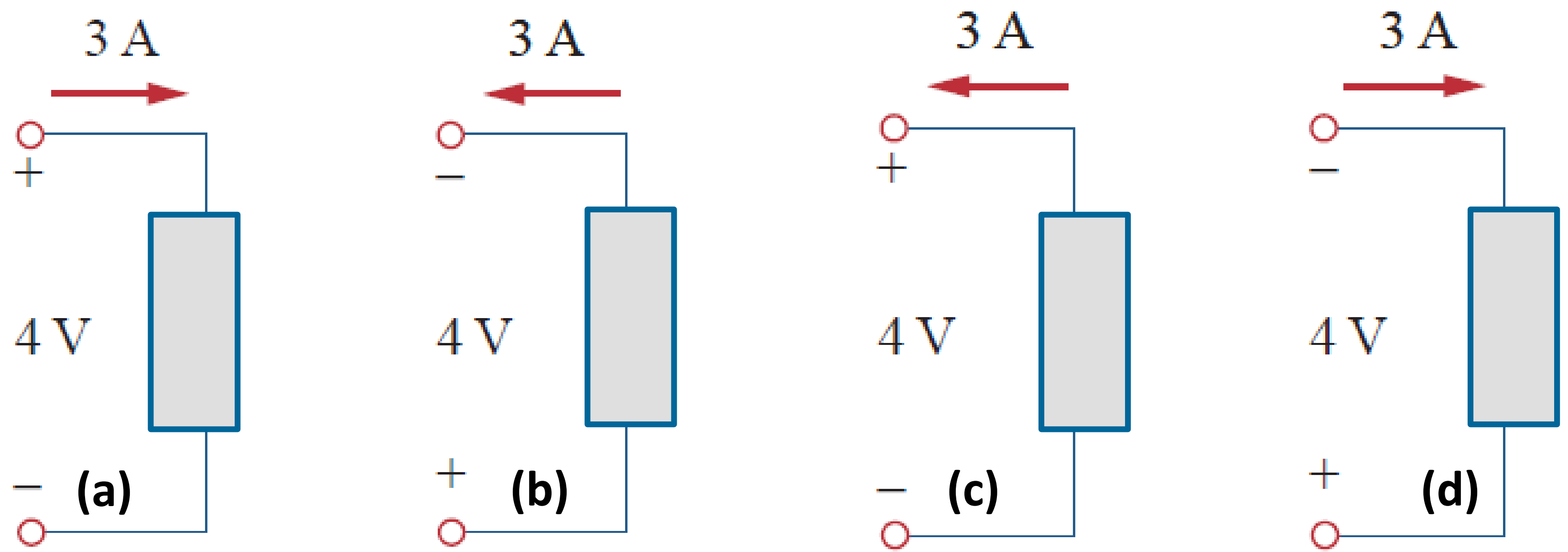
Voltage Polarity





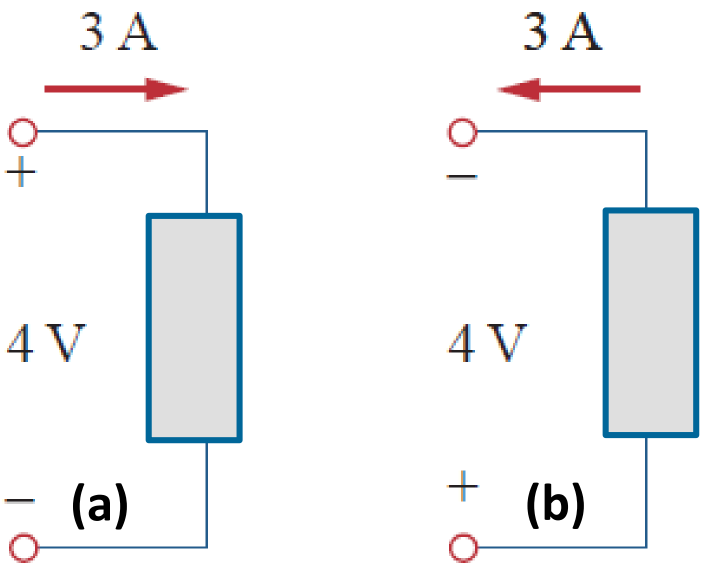
Voltage Polarity - Examples

Are the following components absorbing or supplying power???
Let's calculate the power..

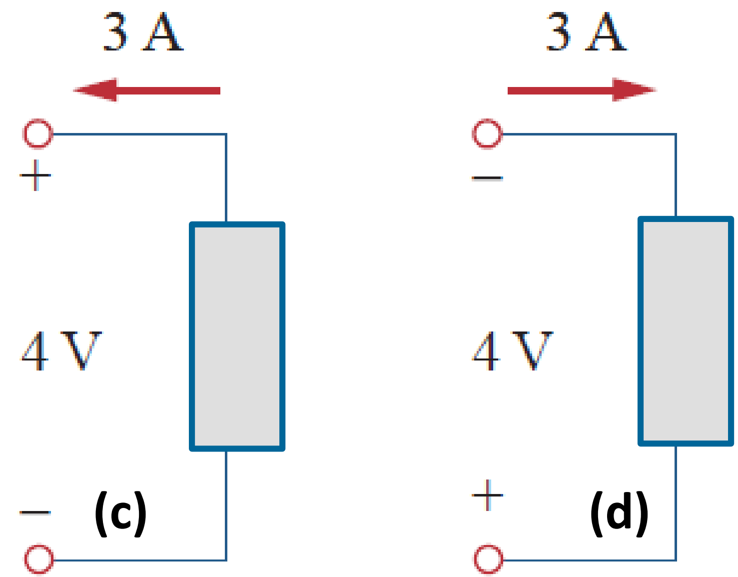


Voltage Polarity – Examples

Absorbing **power of 12 W**

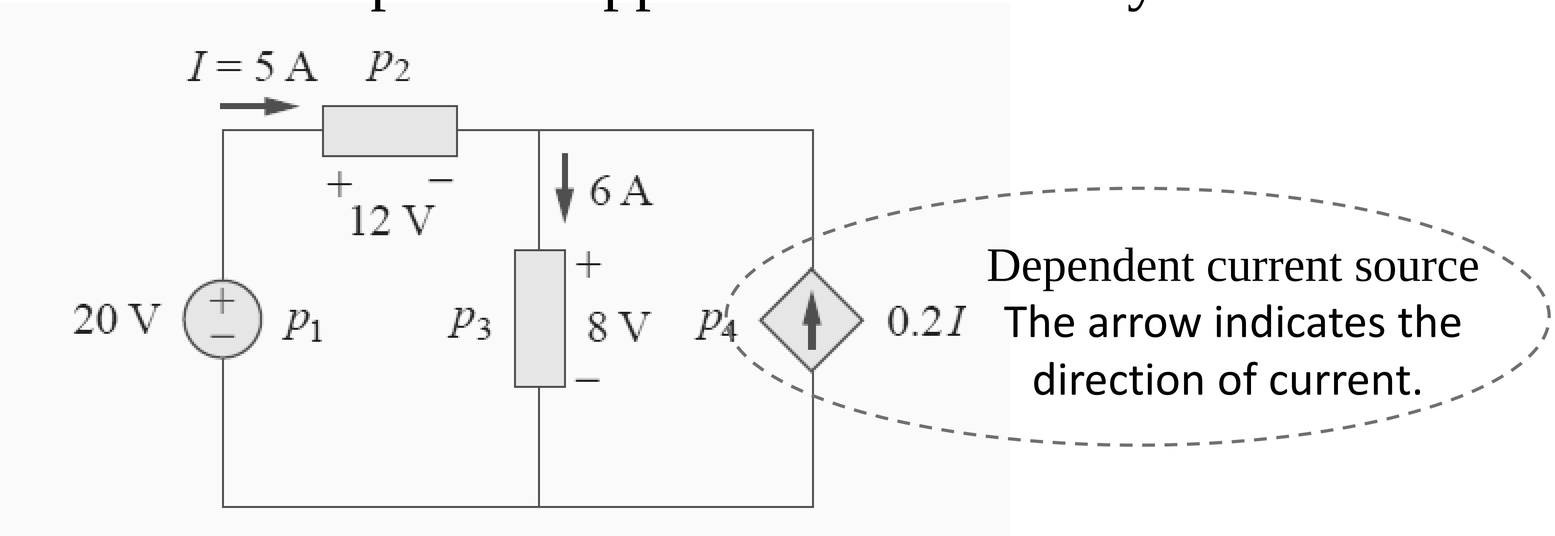


Supplying **power of 12 W**



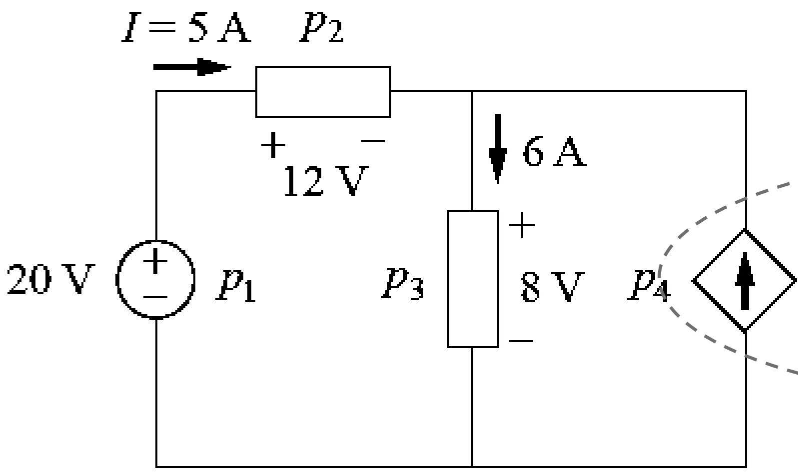
Example

Calculate the power supplied or absorbed by each element.



Example

Calculate the power supplied or absorbed by each element.



Dependent current source The arrow indicates the direction of current.

p1=20*(-5)=-100 W -> *Supplied Power (current into the neg. terminal)*

p2=12*(5)=+60 W -> *Absorbed Power*

p3=8*(6)=+48 W -> *Absorbed Power*

p4= 8*(-0.2*5)= -8 -> *Supplied Power*

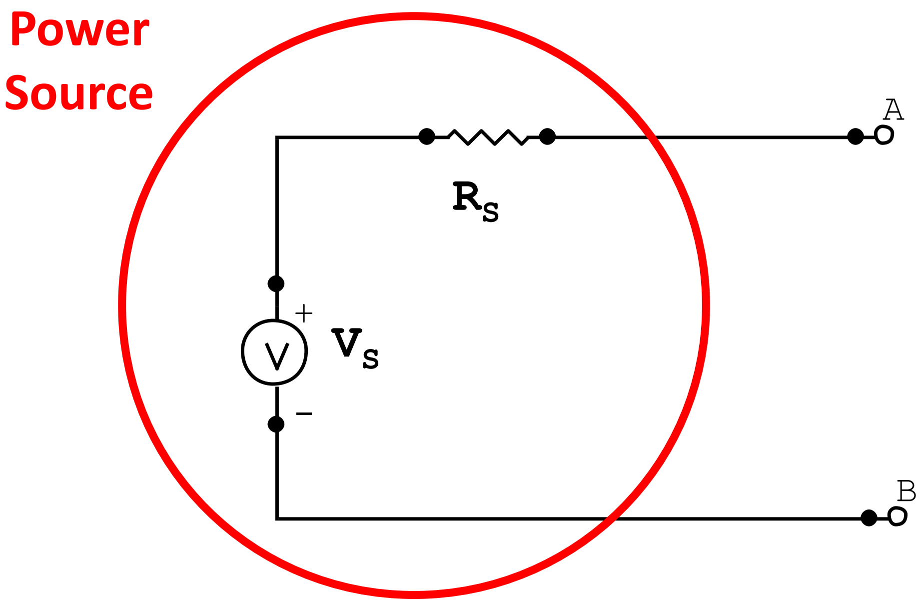
Any observation/comment?

Maximum Power Transfer

Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications where it is desirable to maximize the power delivered to a load.

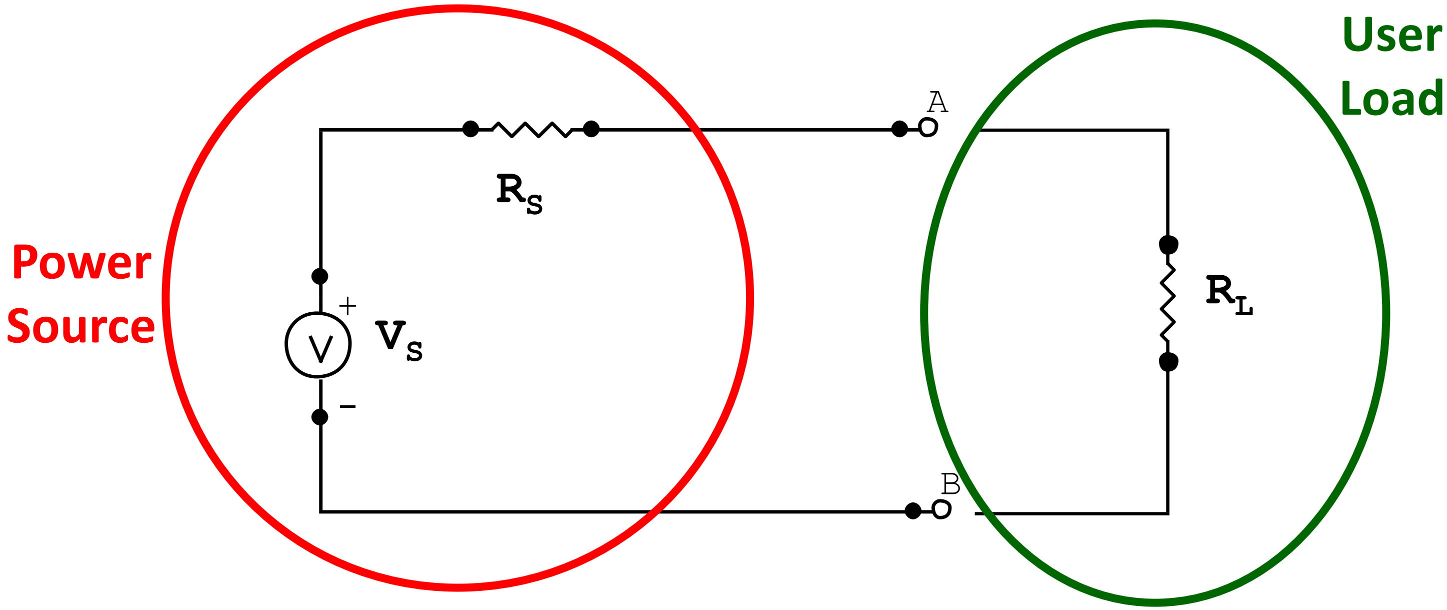
Here, we will address the problem of delivering the maximum power to a load when given a system ***with known internal losses***.



<u>What is the maximum power you can get out of a power source?</u>

We'll only consider voltage sources with a source resistor.

Maximum Power Transfer

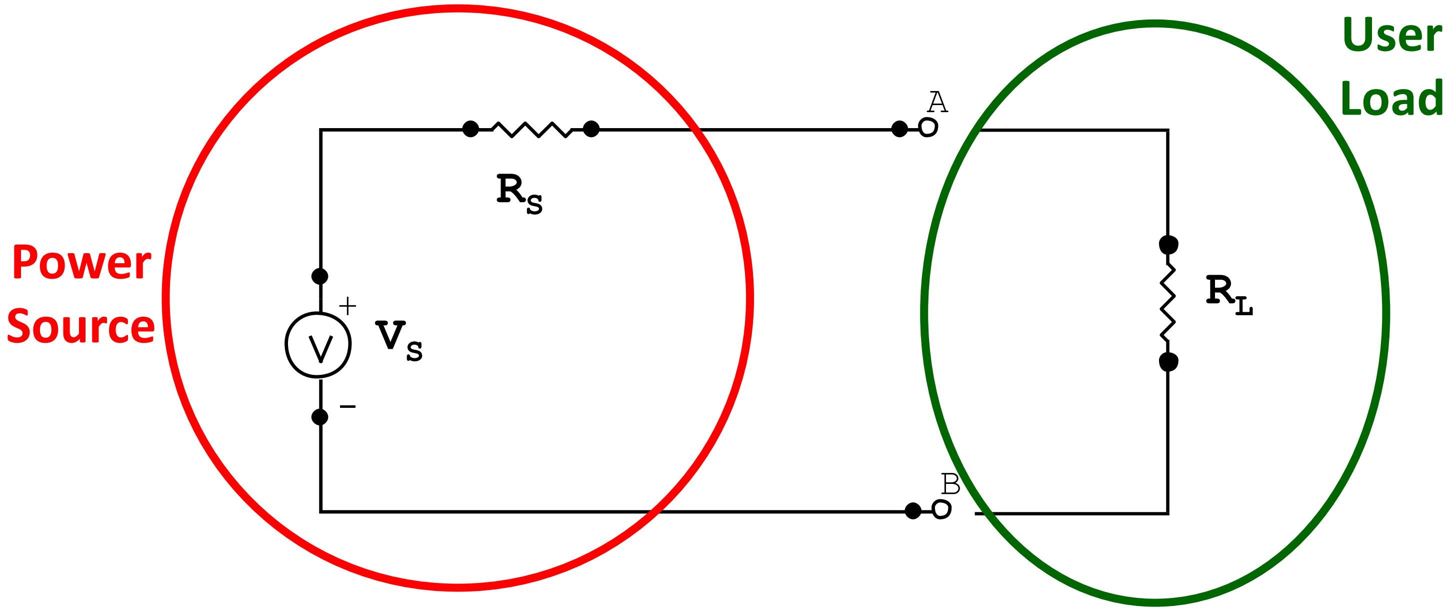


Let us assume that the load can be represented by a resistance R_L. What we need to identify is the value of R_L that maximises the power delivered to the load. The power in the load is given by:

$$P_L = V_L I_L = (I_L)^2 R_L$$

$$I_L = V_S/(R_L + R_S)$$

Maximum Power Transfer



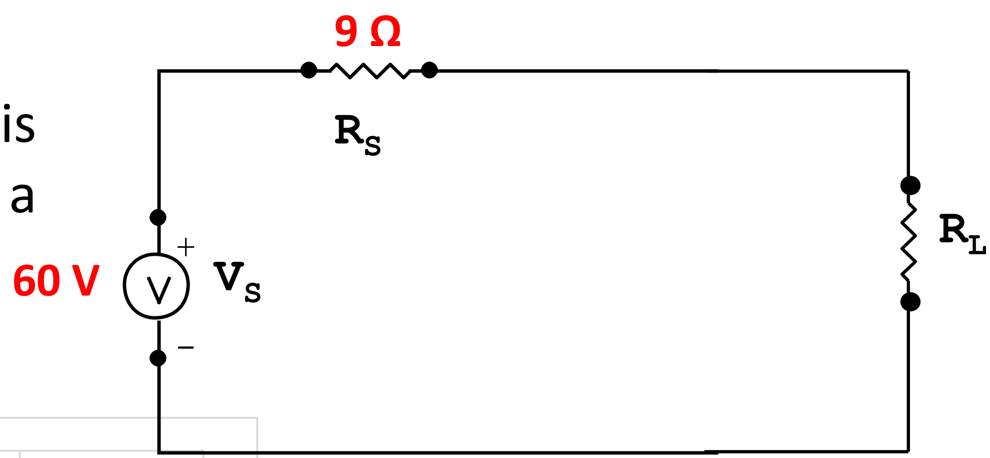
Therefore power is given by

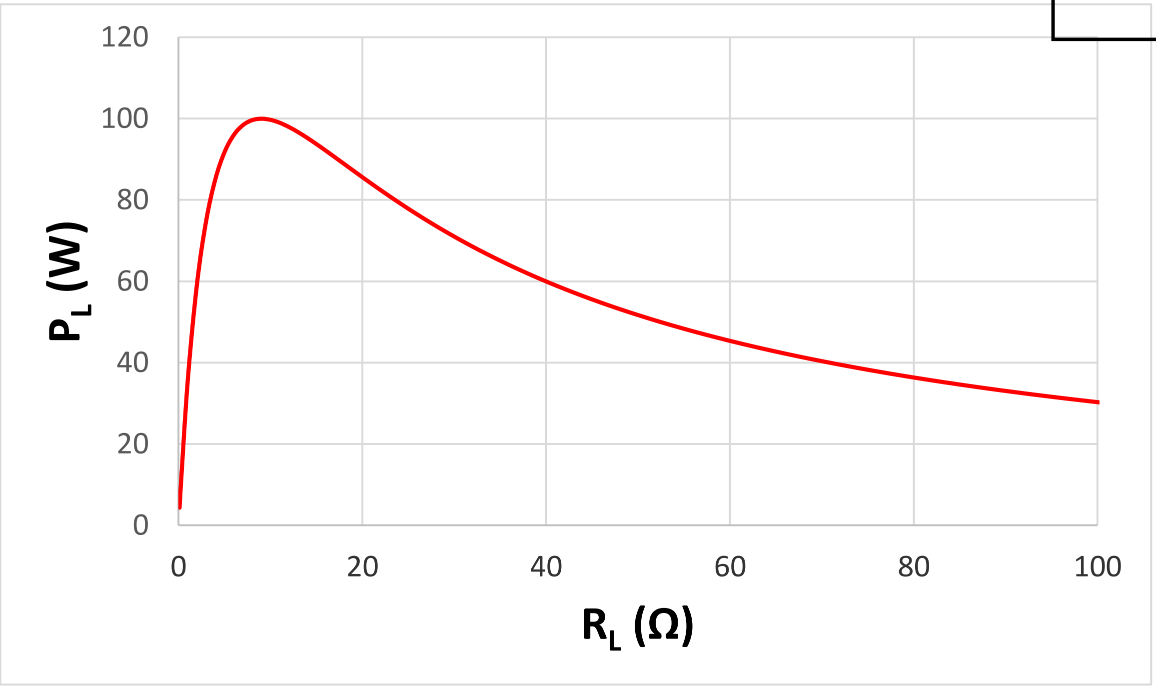
$$P_L = \left(\frac{V_S}{R_L + R_S}\right)^2 R_L = (V_S)^2 \frac{R_L}{(R_L + R_S)^2}$$

Let’s now see an example…

Maximum Power Transfer

Let's calculate the power that is delivered to the load versus a range of resistor load values....

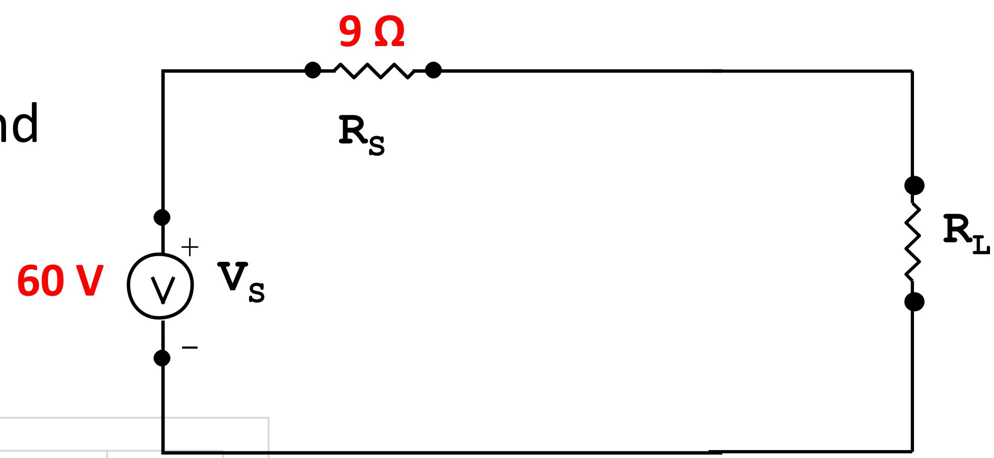




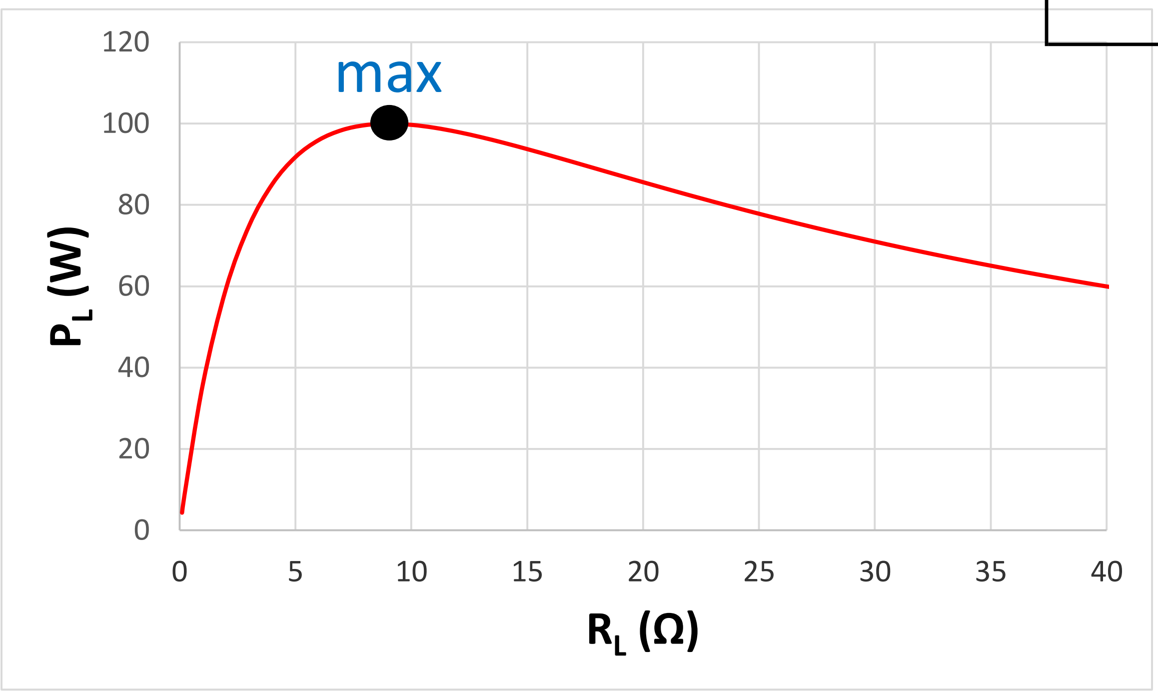
What do you observe from this plot?

$$P_L = \left(\frac{V_S}{R_L + R_S}\right)^2 R_L$$

Maximum Power Transfer



Let's zoom now at the plot and observe when P_L is maximum...



We can see here that the maximum power is transferred to the load when $R_L = R_S = 9\ \Omega$.

Maximum Power Transfer

According to the maximum power transfer theorem, you can extract the maximum power from a battery when the LOAD resistance is matched to the SOURCE resistance ($\mathrm{R_S} = \mathrm{R_L}$).

$$P_{\max\_in\_load} = \left(\frac{V_S}{R_L + R_S}\right)^2 R_L = \frac{R_S (V_S)^2}{(2R_S)^2}$$

You DO need to know this

Maximum Power Transfer

Looking at the power equation:

$$P_{in\_load} = \left(\frac{V_S}{R_L + R_S}\right)^2 \boldsymbol{R_L}$$

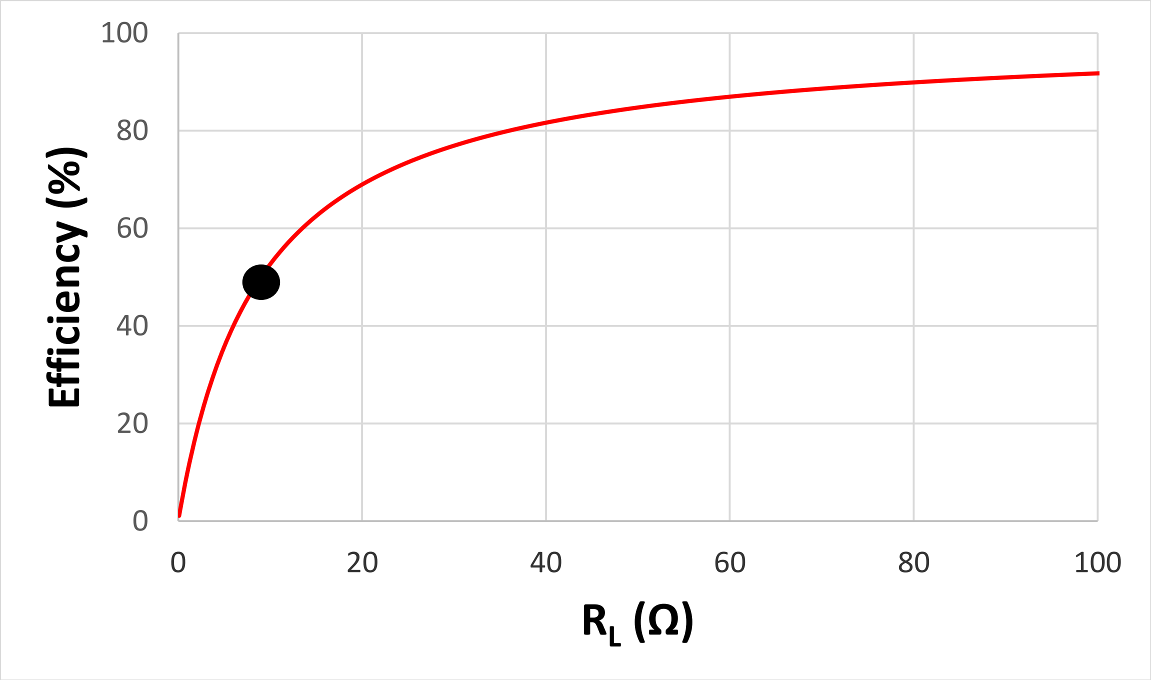
The power wasted is:

$$P_{wasted\_in\_source} = \left(\frac{V_S}{R_L + R_S}\right)^2 \boldsymbol{R_S}$$

Maximum Power Transfer

Let's now calculate the efficiency in the case that $\mathbf{R_L=R_S}$. **Efficiency** can be calculated as the ratio of the power delivered to the load (P_L) to the total power delivered by the source (P_s).

This scenario gives you an efficiency of 50% as half the power will be dissipated in the source resistance.



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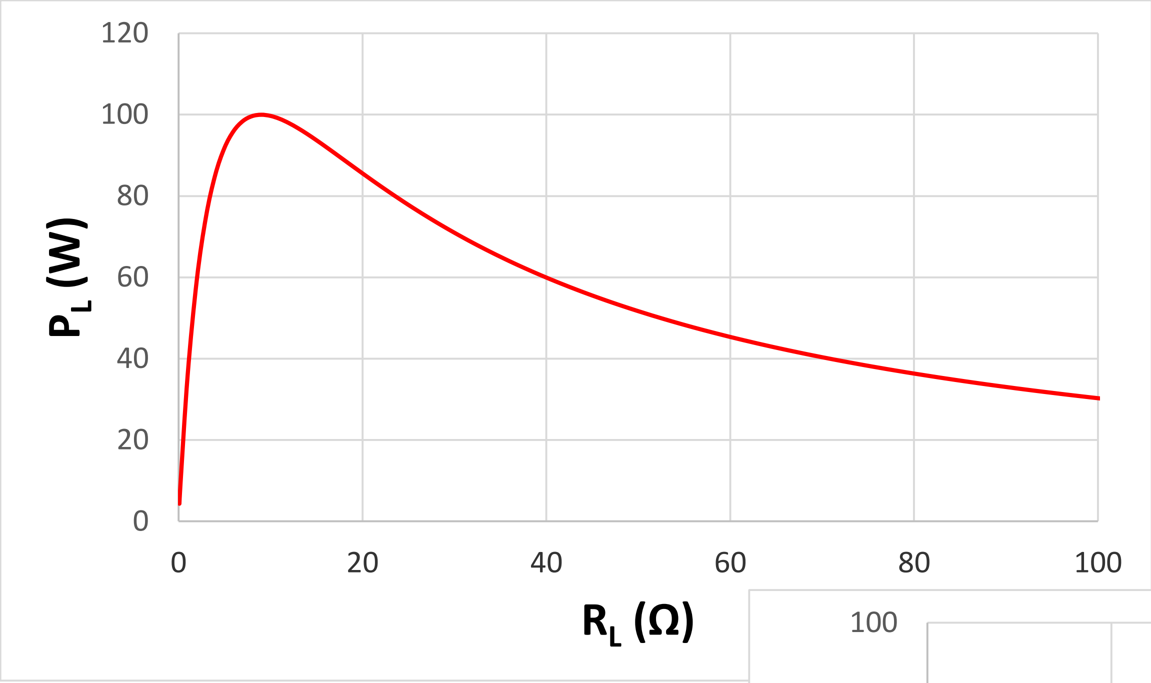
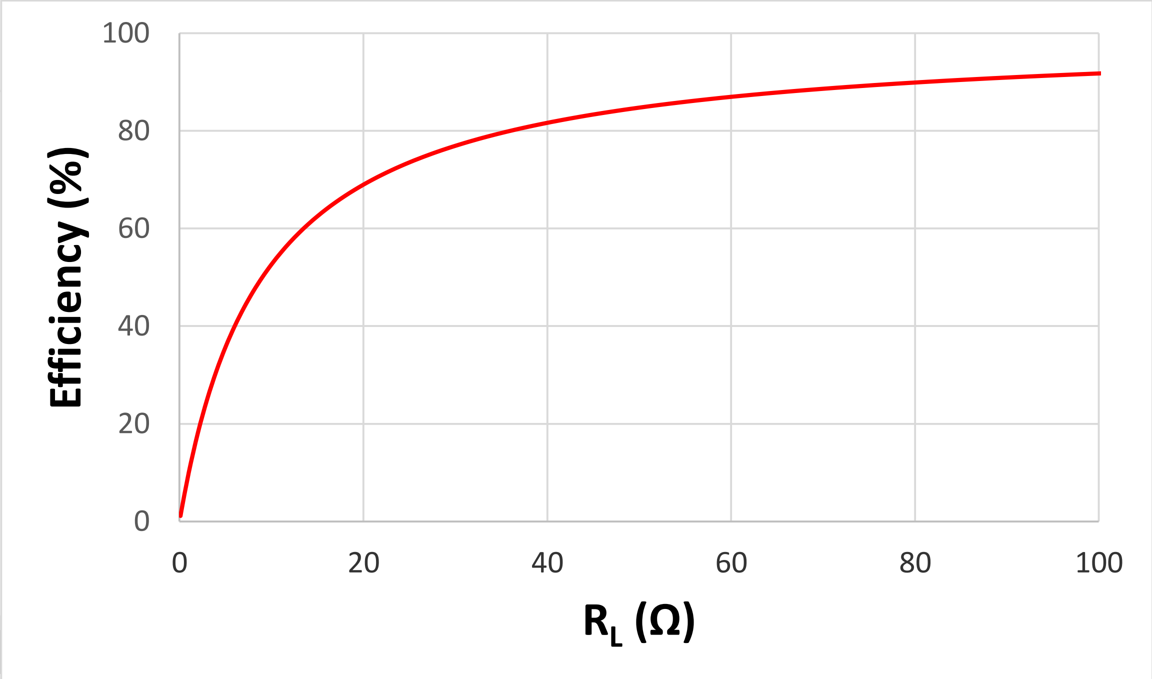
Efficiency vs Max Power

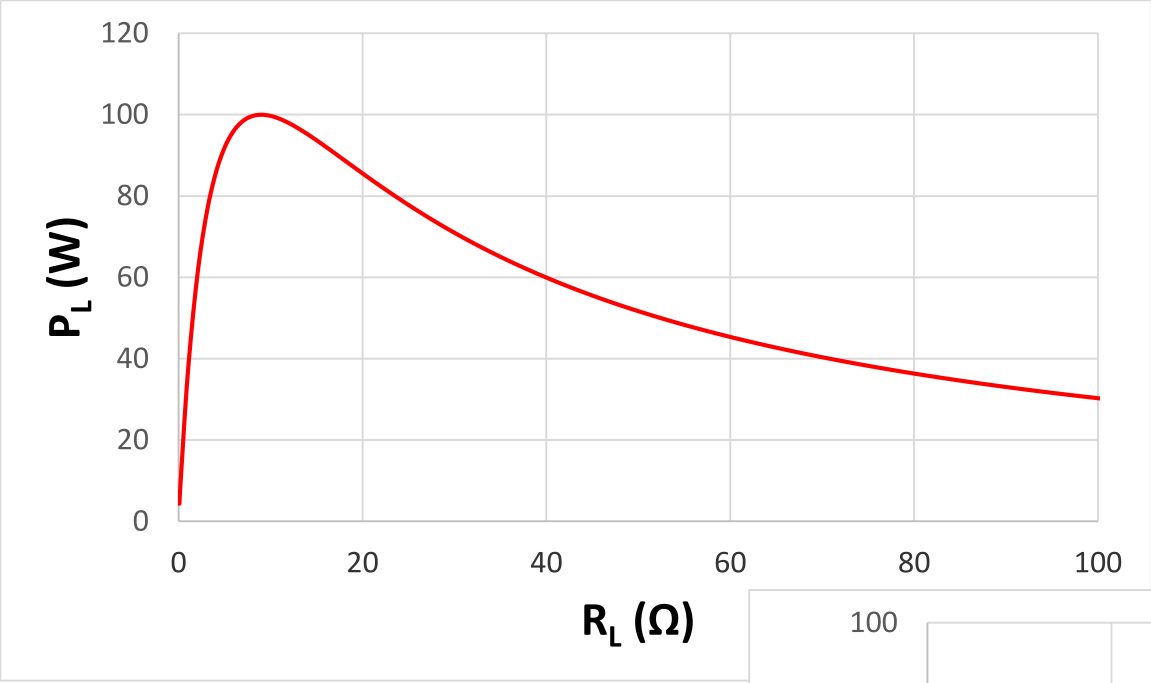


At max power, efficiency is 50%



Max efficiency and max power do not occur at the same location.

Efficiency vs Max Power



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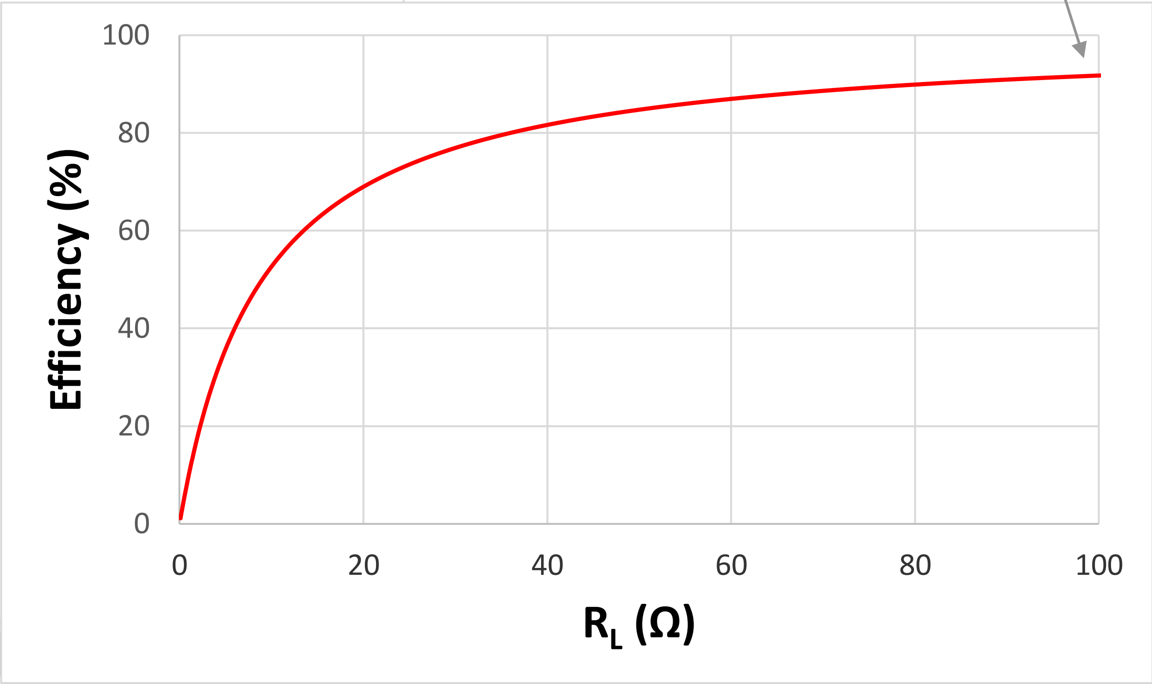
How much power is delivered at maximum efficiency?



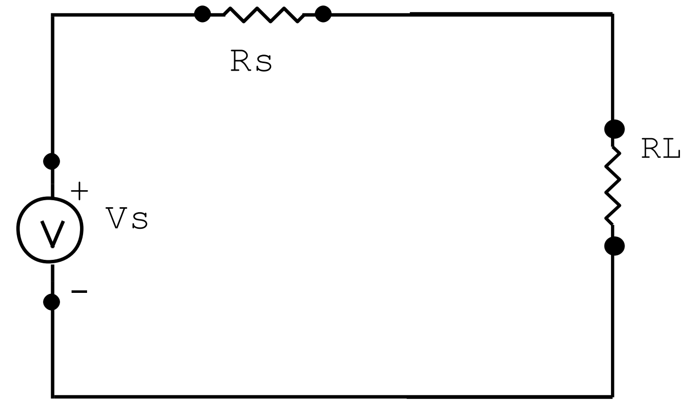
Max efficiency and max power do not occur at the same location.

Efficiency vs Max Power

- If efficiency is the most important factor, then the load should be much larger than the internal resistance of the supply.

- If maximum power transfer is required and efficiency is less important, then the maximum power transfer theorem should be applied.

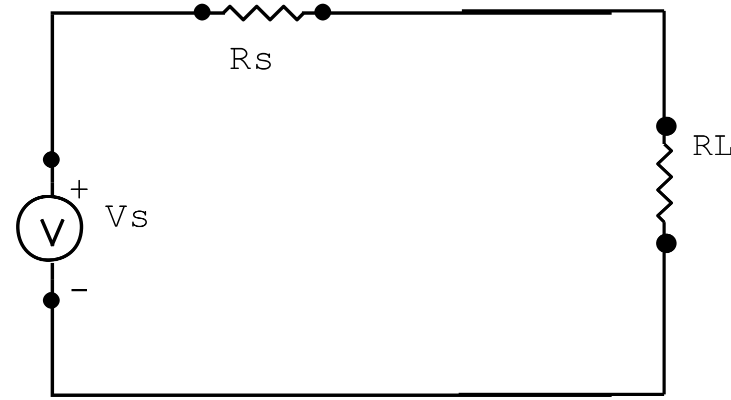
An Example



A fixed load of 16 Ω is applied to a 48 V supply with an internal resistance Rs of 36 Ω.

1. What is the power delivered to the load and lost to the internal resistance of the supply?
2. If the designer has some control over the *internal resistance level of the supply*, what value should he or she make it for maximum power to the load? What is the maximum power to the load?

An Example



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1. Current=923 mA, 30.68 W are lost at the internal resistance of 36 Ohm, 16 W are delivered at the load.
2. Power= 144 W

Exams Papers, Question 5 , January 2015

(d) I have a lightbulb being powered by a battery. It is brightest when it is using the maximum power available. Assuming that the source resistance of by battery (R_S) was 10Ω and the battery voltage was 5V, what equivalent resistance should I design my lamp to have? (4 Marks)

What is the maximum current that can flow when I have achieved my maximum power transfer?

(e) Efficiency is defined as power usefully used over the total power delivered by the battery. What is the power efficiency when I am extracting the maximum power from the battery? Explain your answer. (3 Marks)

(f) If I made my load resistance 10 times larger than the answer you provided in part (d), what would now be the efficiency of your circuit (3 Marks)

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Answer to d:

The same as the source resistance (10 Ohms).

I have two 10 ohm resistors in my circuit and the voltage is 5 volts. So, current = total voltage divided by total current =5/20 = 0.25 Amps.

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Answer to e:

Efficiency = total power used by load divided by total power used
So I could say its 50% from memory and then I'd need to explain, but if we put it simply: two equal valued resistors will consume the same power, so the efficiency must be 50%

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Answer to f: This is a tougher question and is to check if you understand it. A simple way to do this is to calculate the current that flows. The load resistance is 100 Ω (10 times 10), the source resistance is 10 Ω and the voltage is 5 V. So, current is: I= total voltage divided by total resistance = 5/110 = 45 mA

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Answer to f – Continue here:

Power consumed in load = $I^2R = (0.045)^2(100) = 0.206$ Watts

Power consumed in source = $I^2R = (0.045)^2(10) = 0.0206$ Watts

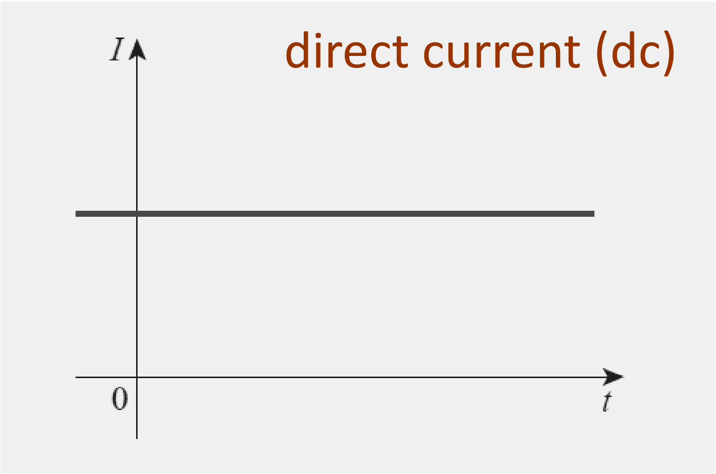
Total power is both added = 0.227 Watts or (5V*45mA=0.227 Watts)

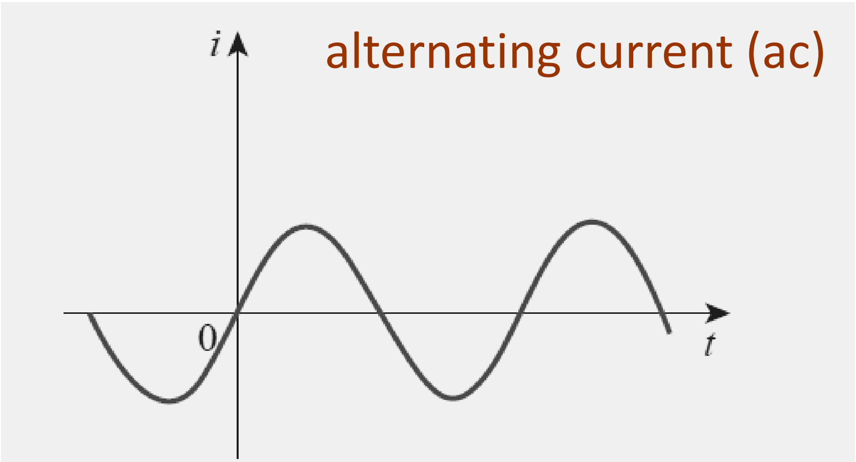
Efficiency= $P_{LOAD}/P_{TOTAL} = 0.206/0.227 = 90.9\%$

DC versus AC current

Direct Current vs Alternative Current

Batteries produce something called direct current (dc). The power that comes from an electric generator (such as power plant) is called alternating current (ac).





It reverses at regular time intervals and can have alternately positive and negative values.

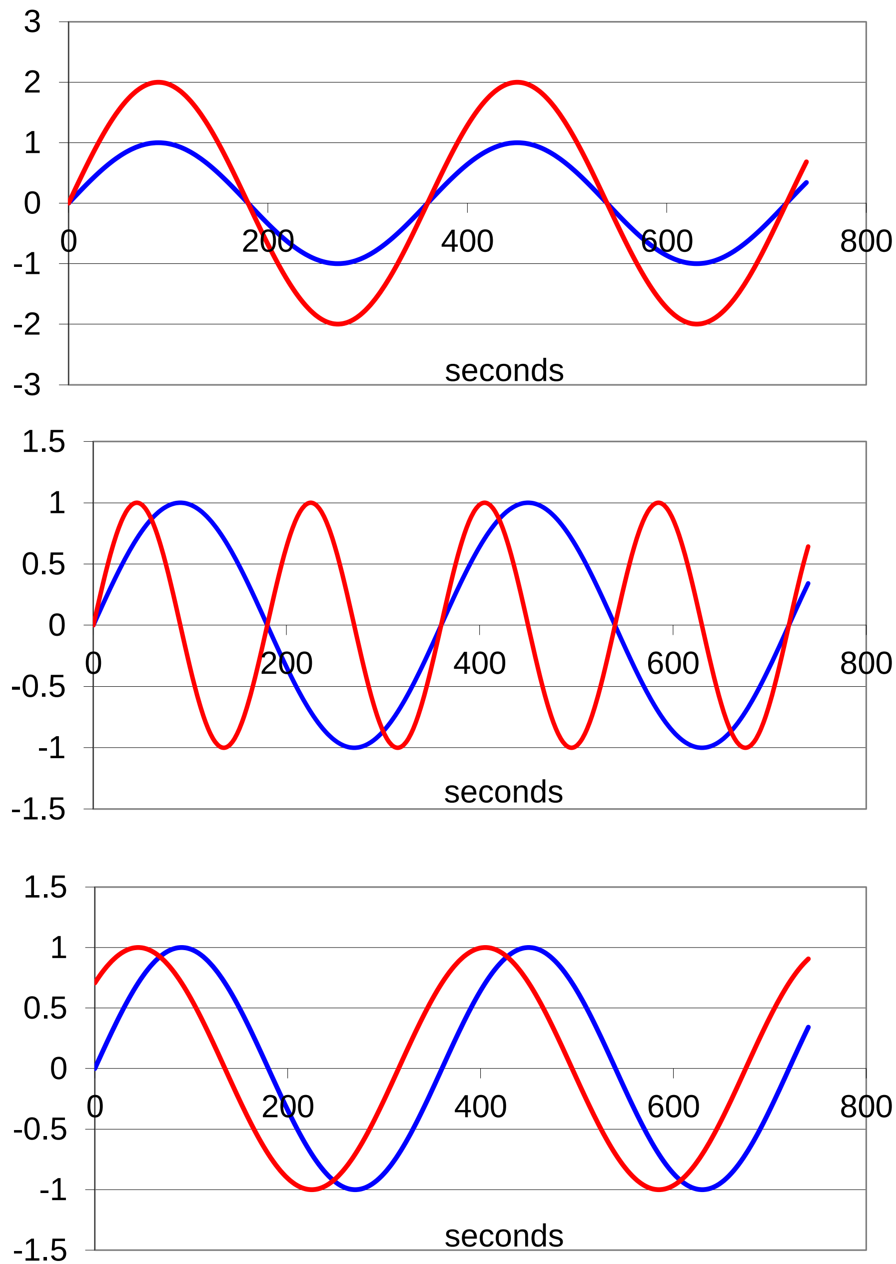
A **direct current (dc)** is a current that remains constant with time.

An **alternative current (ac)** is a current that varies sinusoidally with time.

Sine Wave

A sinusoidal signal (sine or cosine wave) looks as shown. It is periodic and defined by the following key parameters:

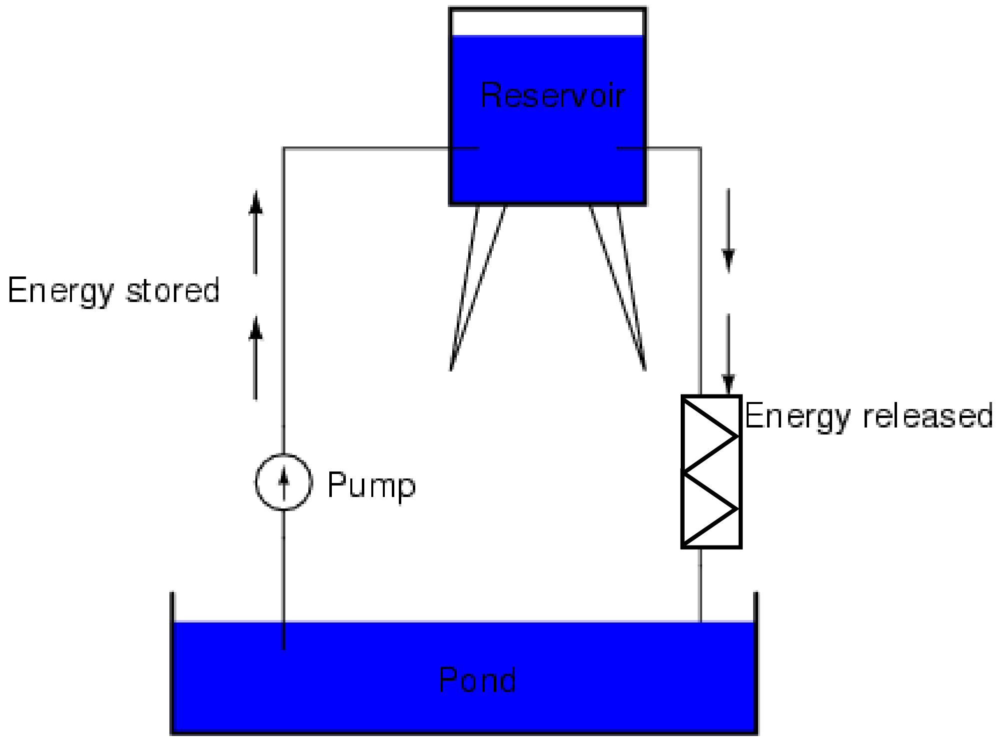
- t   the time instant
- T   its repetition **period**
- F   its **frequency** (f = 1/T)
- A   its **amplitude**
- θ   its **phase** at time zero
- k   any constant (DC) **offset** from zero mean



The equation for this signal is:

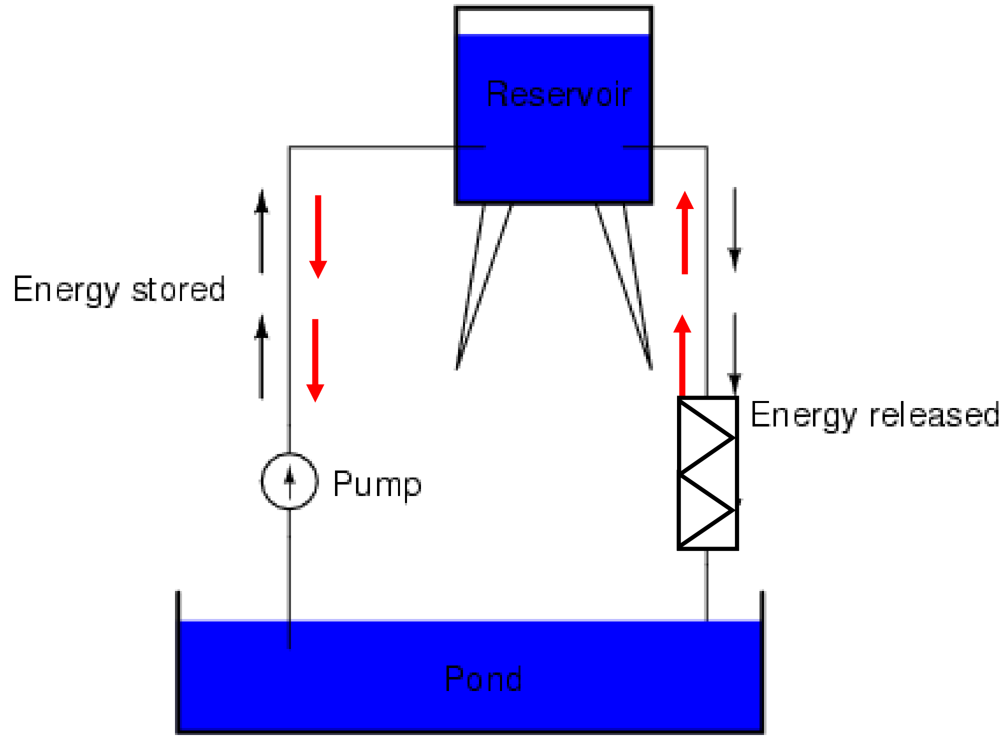
$$s(t) = A\cos\left(2\pi * ft + \theta\right) + k$$

Direct Current (dc)



Direct Current (dc) – the current flows in a single direction (the electron flow will continue to move in the same direction in the circuit). The current magnitude is fixed over time.

Alternating Current (ac)



Alternating Current (Ac) – in terms of the water analogy, the water is pushed then sucked, pushed then sucked by the pump...
Thus, the individual pieces of water/charge do not travel far in any direction, but the movement of water/charge past the machine gives up energy.

A comparison between ac and dc

dc current

- Was invented first.
- It flows in one direction in the circuit.
- Easy to source from batteries.
- Almost all electronic appliances operate on DC.
- Too many losses if going long distances.

ac current

- It reverses its direction while flowing in a circuit.
- Low transmission losses if the voltage is high.
- Easy transfer over long distances and can provide more power.