Conservative Forces
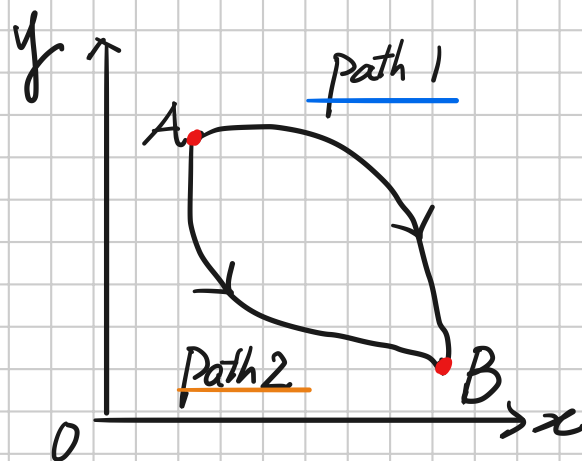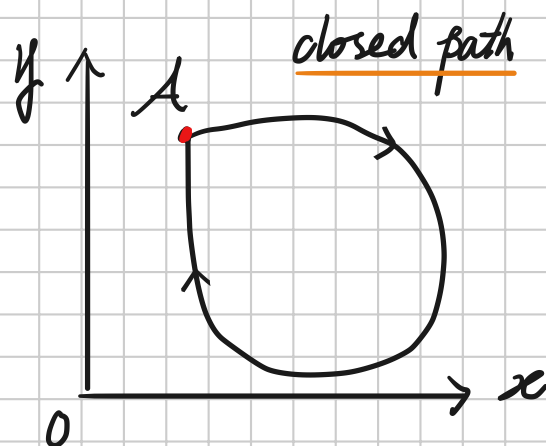
①

$$\int_{A\;(path\,1)}^{B} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{A\;(path\,2)}^{B} \vec{F}(\vec{r}) \cdot d\vec{r}$$

②

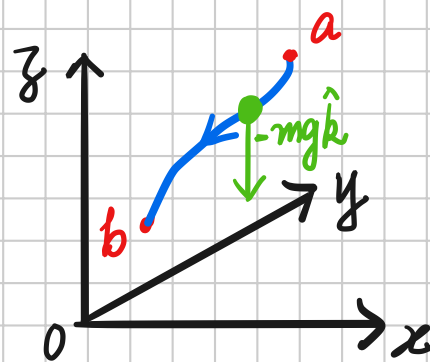$$\oint \vec{F}(\vec{r}) \cdot d\vec{r} = 0$$

Potential Energy

① gravitational potential energy

$$W = \int_{a}^{b} (-mg\,\hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= -mg \int_{a}^{b} dz = -mg(z_b - z_a) = -(mg\,z_b - mg\,z_a)$$

$$= mg\,z_a - mg\,z_b$$

$$= mg(z_a - z_b)$$

② elastic potential energy

$$W = \int_{x_i}^{x_f} f_x \, dx = \int_{x_i}^{x_f} -kx \, dx = -k \int_{x_i}^{x_f} x \, dx$$

$f_x = -kx$

frictionless

massless ideal spring

$$= -\frac{1}{2}kx^2 \Big|_{x_i}^{x_f} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$