Kirchhoff's Law

Voltage & Current Divider

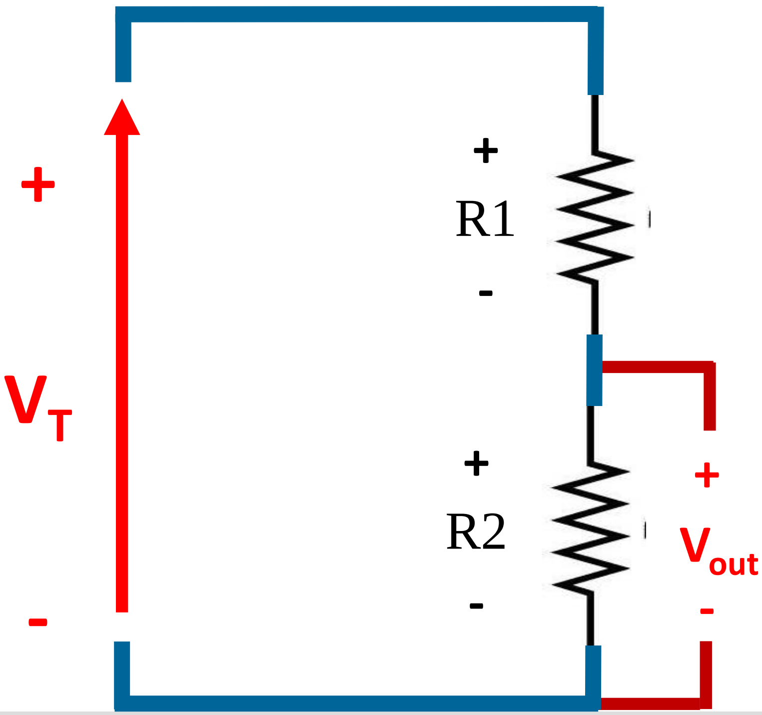
Voltage Divider Theorem

A *voltage divider* is a circuit that produces an output voltage that is a fraction of its input voltage.

A simple example of a voltage divider is two resistors connected in series, with the input voltage applied across the resistor pair (V_T) and the output voltage (V_{out}) emerging from the connection between them.

In the simple resistive circuit shown here, the relationship between the input voltage, V_T, and the output voltage, V_{out} is:

$$V_{out} = V_T\left(\frac{R_2}{R_1+R_2}\right)$$



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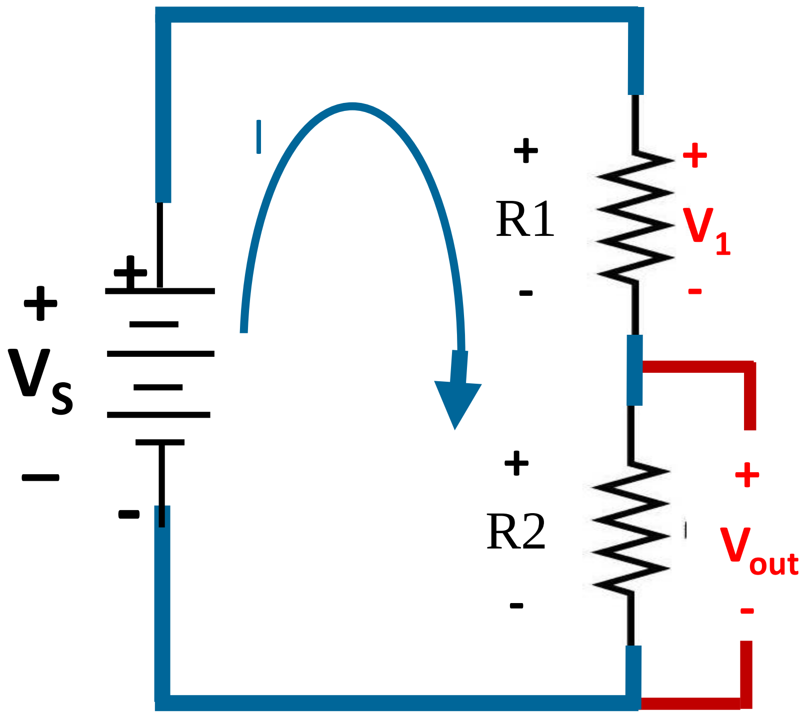


Let's see how we can derive the formula:

$$V_{out} = IR_2$$

$$I = \frac{V_S}{R_1 + R_2}$$

$$V_{out} = V_S\left(\frac{R_2}{R_1 + R_2}\right)$$

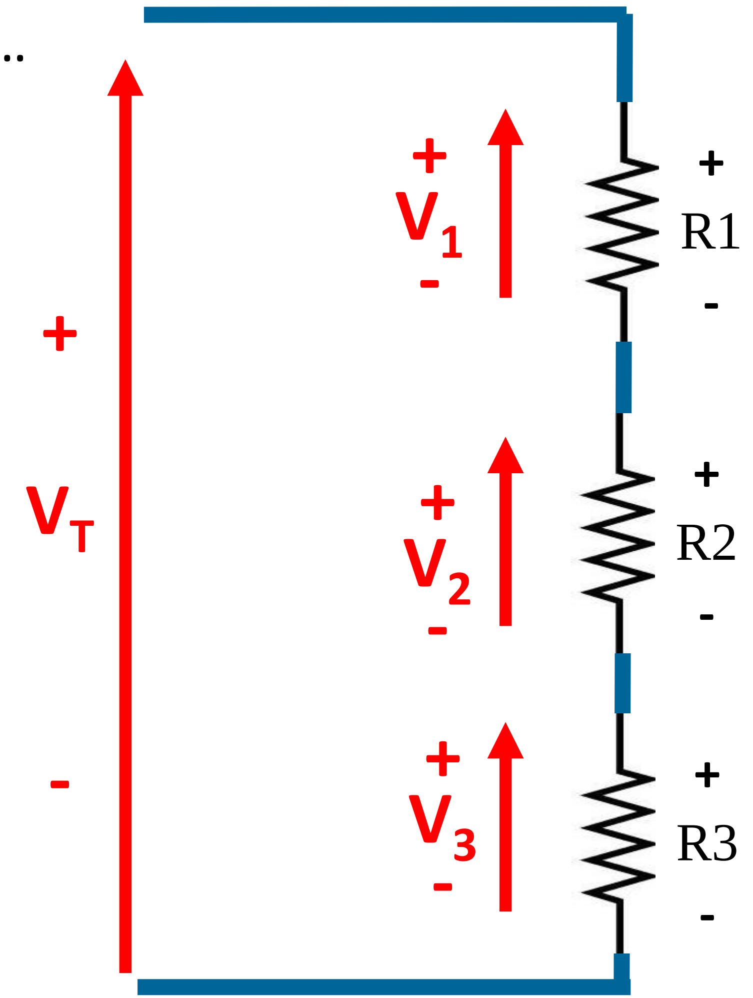
Voltage Divider Theorem

Below you can find the formula for 3 resistors…

$$V_{\#} = V_T\left(\frac{R_{\#}}{R_{TOTAL}}\right)$$

$$V_1 = V_T\left(\frac{R_1}{R_1+R_2+R_3}\right)$$

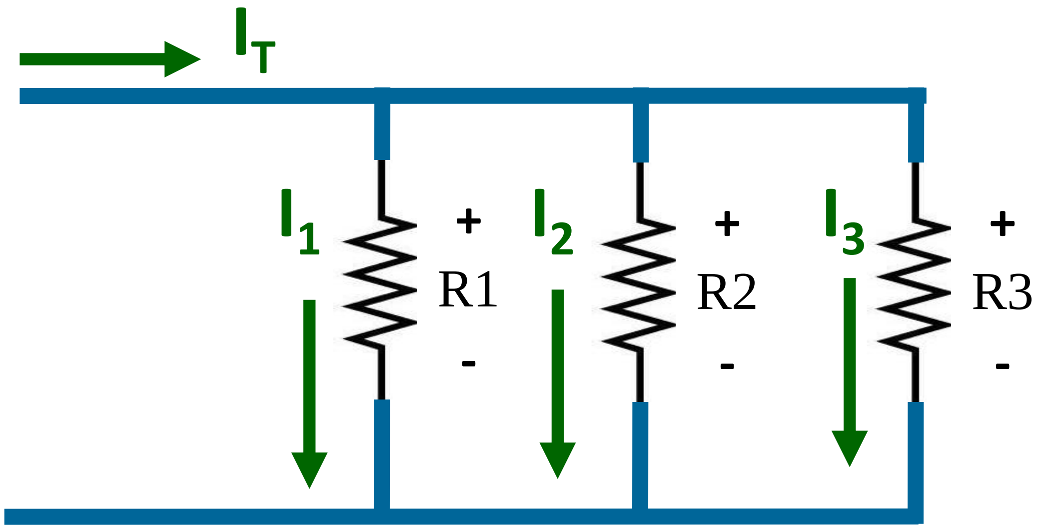
$$V_T = V_1 + V_2 + V_3$$



Current Divider Theorem

A *current divider* is a simple circuit that produces an output current that is a fraction of its input current.

If two or more resistances are in parallel, the total current will be split between them in *inverse proportion* to their resistances.



$$I_T = I_1 + I_2 + I_3$$

$$I_{\#} = I_T \left(\frac{R_{TOTAL}}{R_X} \right)$$

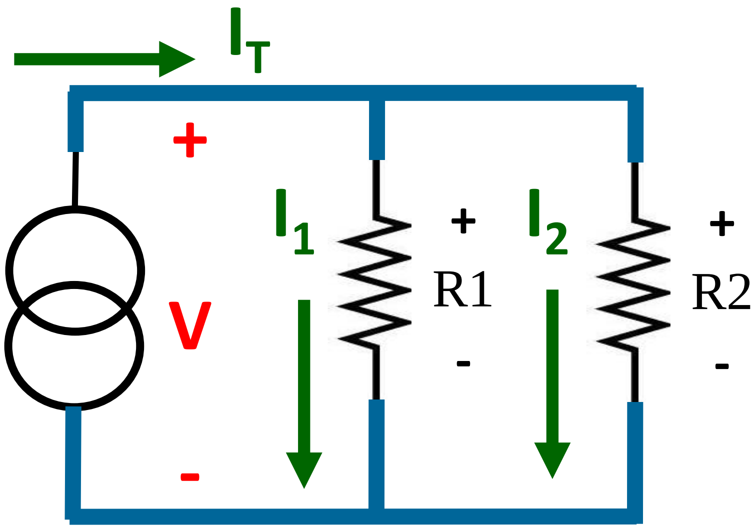
$$I_1 = I_T \left(\frac{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}}{R_1} \right)$$

If the resistance have the same value the current is split equally.

Current Divider Theorem

A *current divider* is a simple circuit that produces an output current that is a fraction of its input current.

Let's see how we can derive the formula:



I$= \frac{V}{R} = GV$, where G is the conductance

$$I_T = I_1 + I_2$$

$$I_T = G_1V + G_2V$$

$$I_T = (G_1 + G_2)V \qquad \textbf{(1)}$$

$$I_1 = G_1V \qquad \textbf{(2)}$$

From (1) & (2), we calculate:

$$I_1 = I_T\left(\frac{G_1}{G_1 + G_2}\right)$$

$$I_1 = I_T\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}}\right) = I_T\left(\frac{R_2}{R_1+R_2}\right)$$

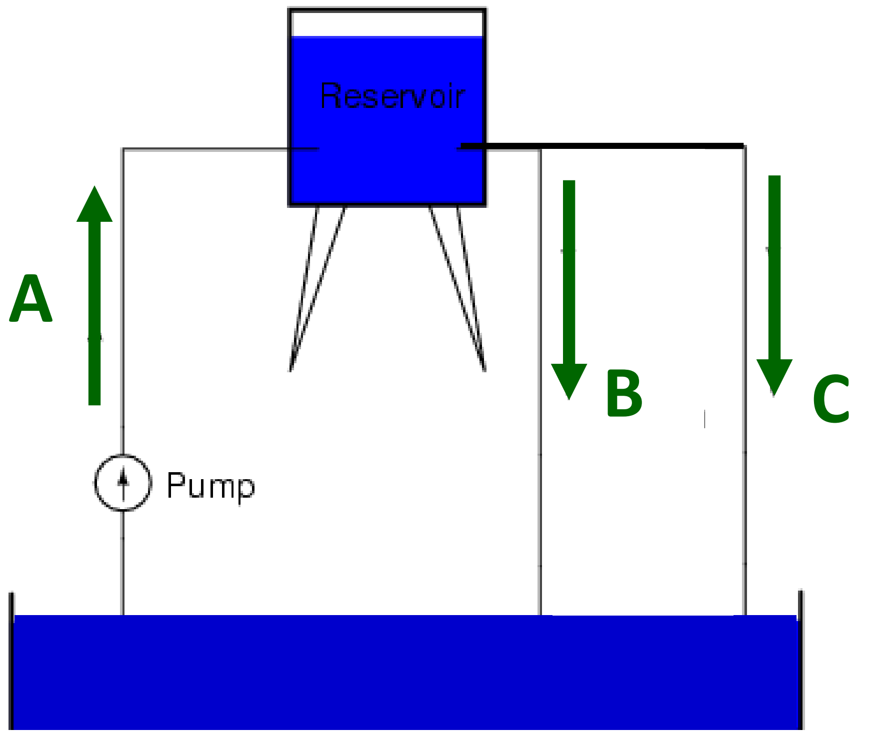
Kirchhoff's Laws

Ohm's law is an important tool that help us analyze circuits. However, we can have a powerful tool to analyze electric circuits, if we combine Ohm's law with the following laws:

- **Kirchhoff's Current Law**
- **Kirchhoff's Voltage Law**

Kirchhoff's *Current* Law (KCL)

For a node … what is the relationship between the current flowing in and the current flowing out (for a stable state)??



Kirchhoff's *Current* Law (KCL)

Kirchhoff's Current Law (KCL) has three equivalent versions:

KCL states that the sum of currents entering a node is equal to the sum of the currents leaving the node.

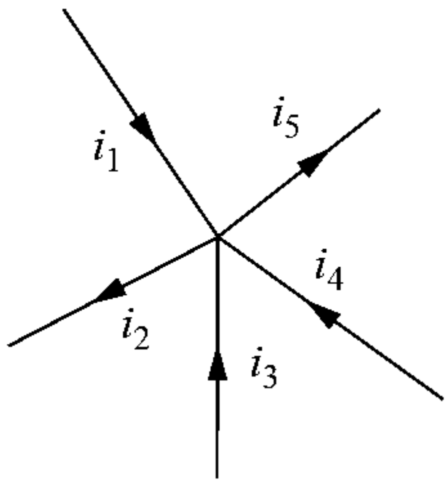
KCL states that the **algebraic** sum of currents entering a node is zero.

KCL states that the **algebraic** sum of currents leaving a node is zero.

We take into account the signs of the currents

Currents entering a node may be regarded as *positive*, while currents leaving the node may be taken as *negative* <u>or vice versa</u>.

Kirchhoff's *Current* Law (KCL)



In this example, the current that enters the node is considered *positive*, and the current leaving the node is considered *negative*, as shown below:

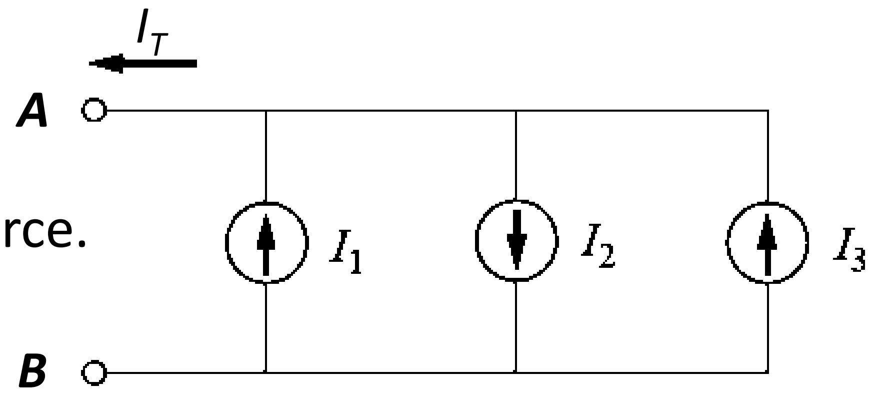
$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

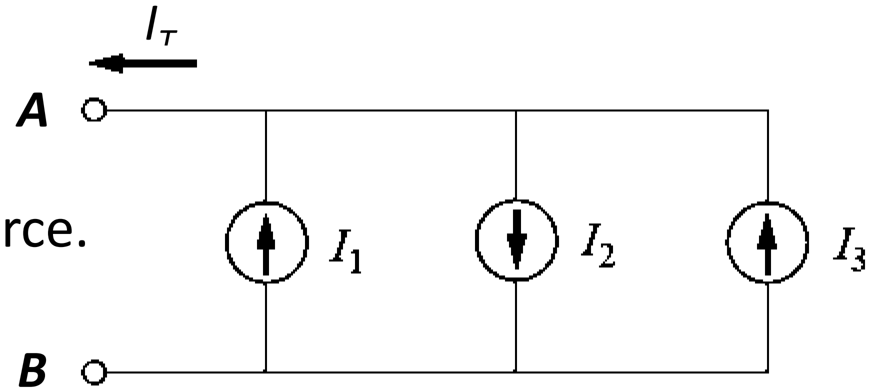
Kirchhoff's *Current* Law (KCL)

Let's see another example.

Let's calculate the equivalent current source.



Kirchhoff's *Current* Law (KCL)

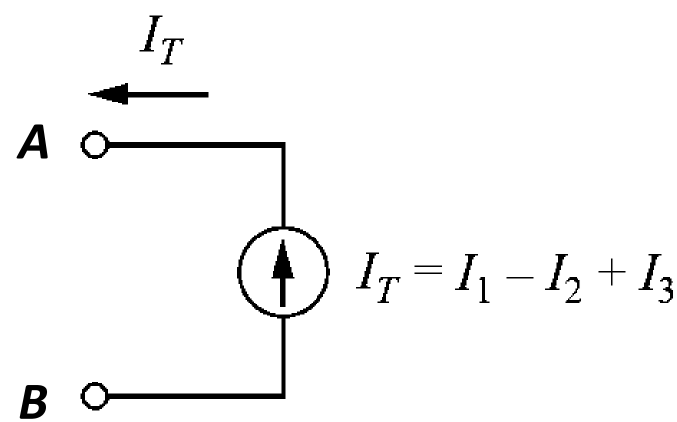


Let's see another example.

Let's calculate the equivalent current source.

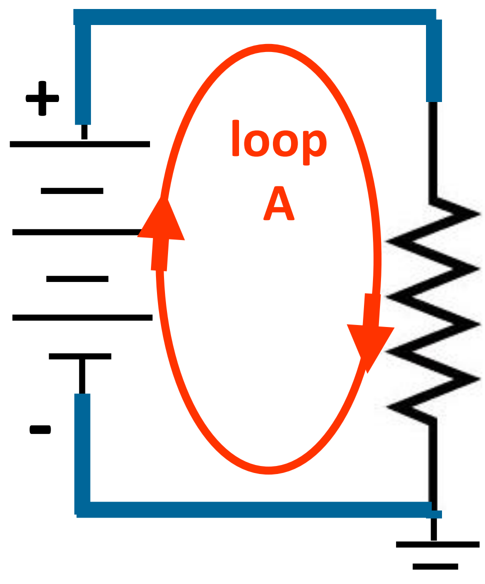
Applying KCL, we find that: $I_T + I_2 = I_1 + I_3$

$$I_T = I_1 - I_2 + I_3$$



Kirchhoff's *Voltage* Law (KVL)

Kirchhoff's Voltage Law (KVL) states that the **algebraic** sum of all voltages around a closed path (or loop) is zero. In other words, the sum of voltage rises equals the sum of voltage drops.



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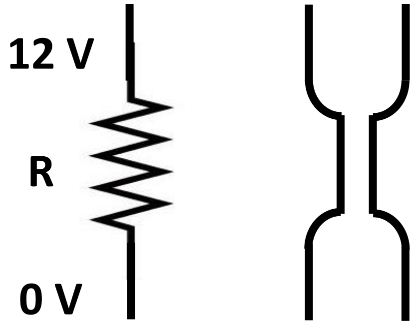
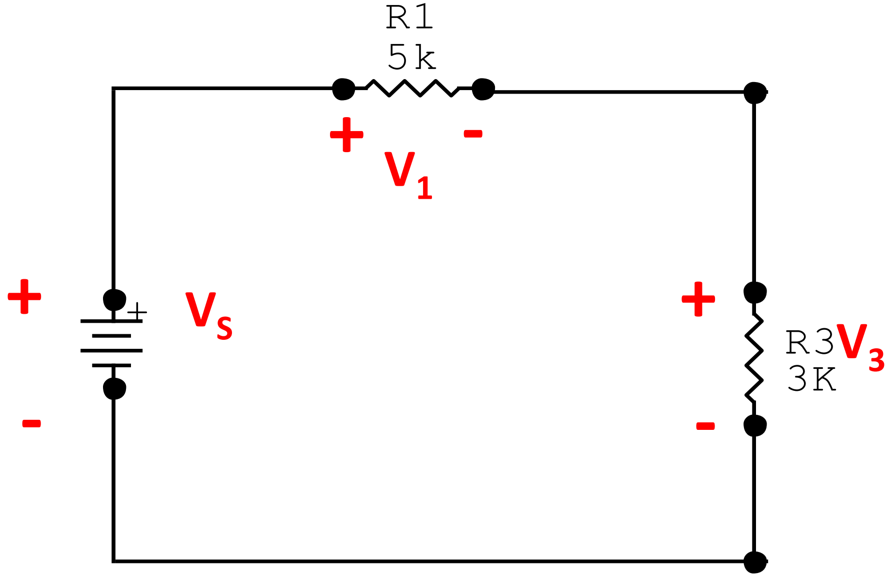
In terms of the water analogy model, the water pressure can increase or fall in some places (for example due to constriction). In the same way, a resistor will create voltage drop. See examples:



Kirchhoff's *Voltage* Law (KVL)

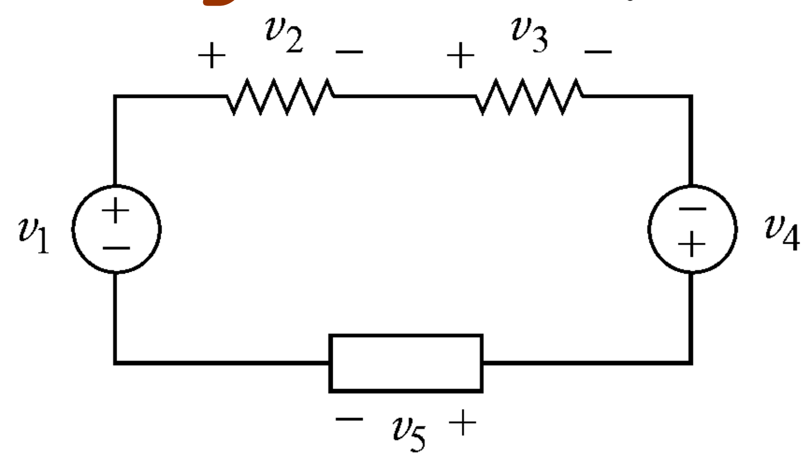
Let's see a very simple example of Kirchoff's Voltage Law.
As we said already, according to KVL:
Sum of voltage drops = Sum of voltage rises



$$V_s = V_1 + V_3$$

Kirchhoff's *Voltage* Law (KVL)

Let's see another example:



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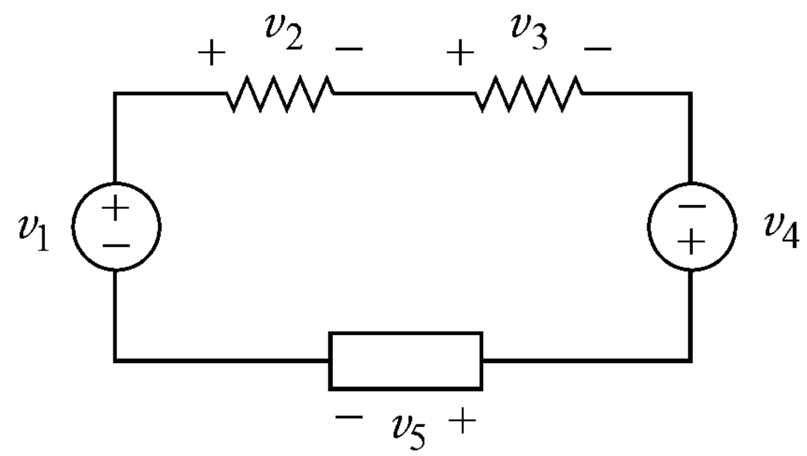
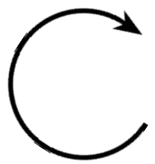
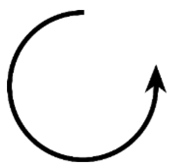




If we go *clockwise around the loop*, the voltages are -v1, +v2, +v3, -v4, +v5.

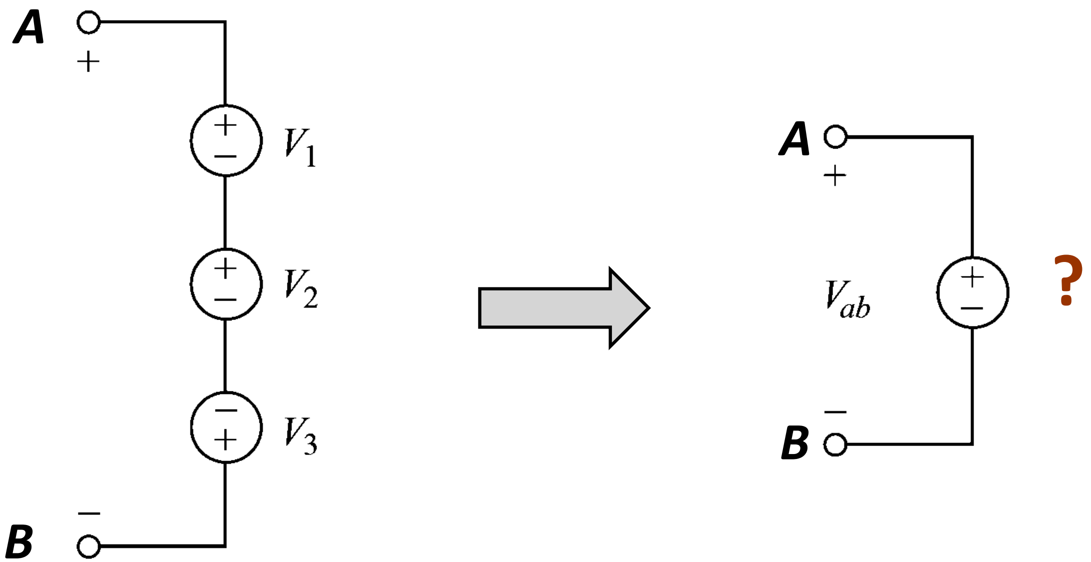
If we go *counterclockwise around the loop*, the voltages are +v1, -v5, +v4, -v3, -v2



Either way, ***v*2 + *v*3 + *v*5 = *v*1 + *v*4**

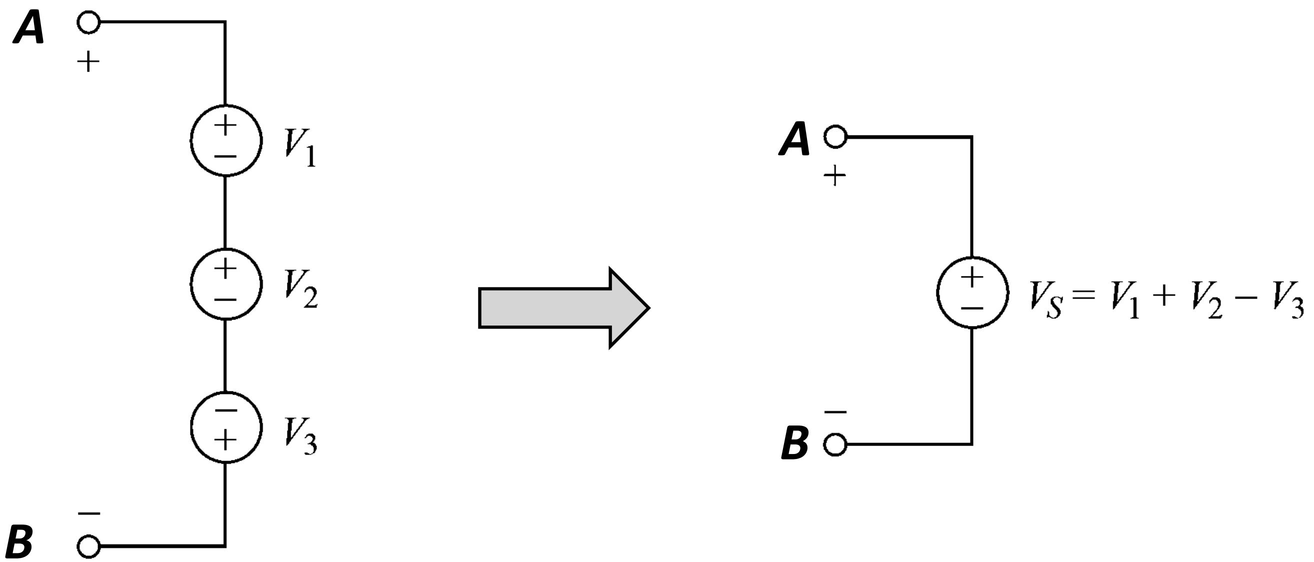
Kirchhoff's *Voltage* Law (KVL)

Let's see another example:



Kirchhoff's *Voltage* Law (KVL)

Let's see another example:



The combined voltage is the algebraic sum of the voltages of the individual sources.

Kirchhoff's Laws - Summary

Kirchhoff's Current Law (KCL)

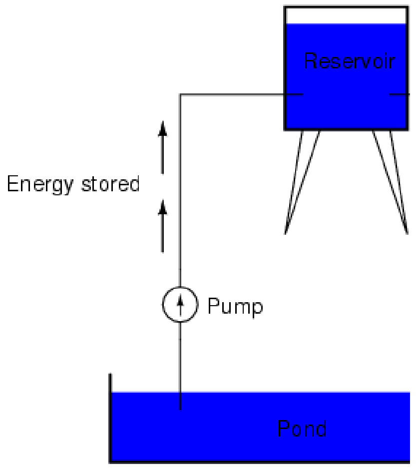
The algebraic sum of currents flowing into (or out of) a node equals zero. ***Or what goes in must come out.***

Kirchhoff's Voltage Law (KVL)

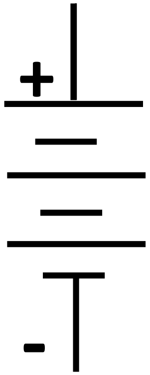
The algebraic sum of voltage drops around a loop must equal zero. ***Or what goes up must come down.***

Real Batteries

Real Batteries

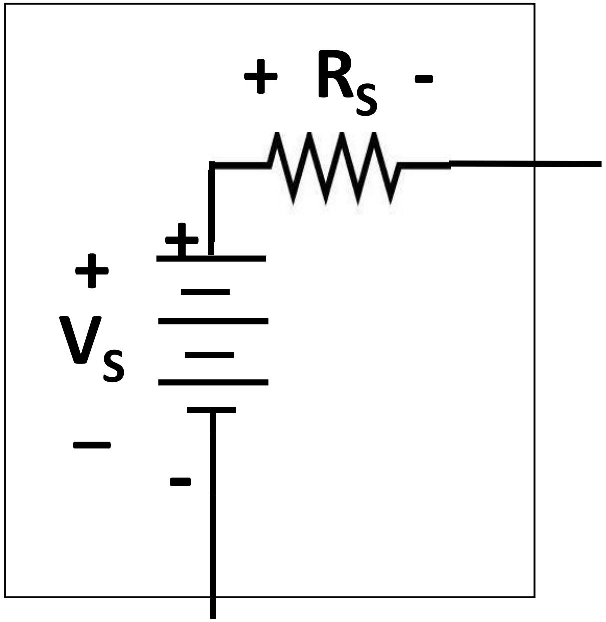


Even the best pump has an input and output pipe *diameter*. *There is a* ***limit to the amount of water*** *that can be sucked/pushed through the pump*.



Similarly, in batteries, even if there is a lot of potential charge energy, ***there are limits on the amount of current*** *that can be produced*.

Real Batteries



We can model a real battery by saying that it produces a voltage of value $\mathbf{V_S}$ and that it has an internal resistance of $\mathbf{R_S}$ Ohms. ***All batteries will have some internal resistance to limit the current.***

Applications that demand high currents will have problems with batteries with high internal resistance.

Real Batteries

The most important information for any battery is its *voltage rating* and its *ampere-hour (Ah) rating*.

The ampere-hour (Ah) rating gives information on the duration that a battery of fixed voltage can supply a current.

As an example, theoretically a battery with an ampere-hour rating of 100 will theoretically provide:

- 1 A for 100 hours
- 10 A for 10 hours
- 100 A for 1 hour
- etc

How much power is in a battery?

Batteries have a finite amount of energy (usually quoted as **mAh** or **Ah**). This is how much charge they can deliver (current * time) at their rated voltage.

Example: A 1.5V alkaline AA battery typically has an ampere-hour rating of 2800 mAh. What is the total energy capacity?

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Example: A 1.5V alkaline AA battery typically has an ampere-hour rating of 2800 mAh. What is the total energy capacity?

$$Power = V * I$$

$$\begin{aligned} Energy &= Power * Time \\ &= V * I * T \\ &= (1.5)(I * T) \\ &= (1.5V)(2.8A * 3600sec) \\ &= 15120\ Joules \end{aligned}$$

How much power is wasted in a battery?

Example: A 1.8 V battery has internal resistance of 1 Ω. It is connected to a 100 Ω load resistance. What is the power dissipated in the load and also in the battery?

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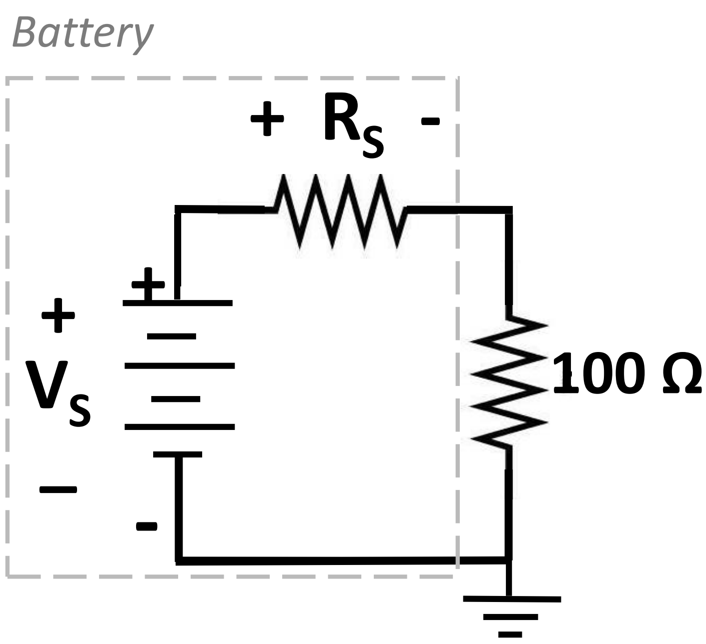
Example: A 1.8 V battery has internal resistance of 1 Ω. It is connected to a 100 Ω load resistance. What is the power dissipated in the load and also in the battery?

The total resistance and total current is:

$$R_T = 100 + 1 = 101\ \Omega$$
$$I_T = 1.8/101 \approx 18mA$$



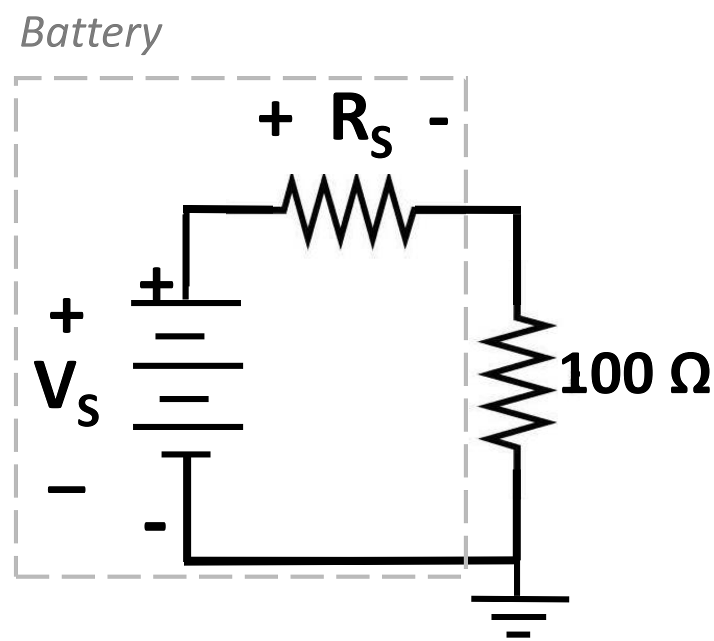
How much power is wasted in a battery?

Example: A 1.8 V battery has internal resistance of 1 Ω. It is connected to a 100 Ω load resistance. What is the power dissipated in the load and also in the battery?

$I_T \approx 18mA \qquad R_T = 101\ \Omega$

The dissipated power at the battery and the resistor is:

$$P_{batt} = I^2 \mathrm{R}_S = (0.018)^2(1) = 0.3mW$$
$$P_{res} = I^2 R = (0.018)^2(100) = 32.4mW$$



It depends on the batteries, but AA batteries start with 1Ω source resistance but get bigger/worse as the battery discharges.

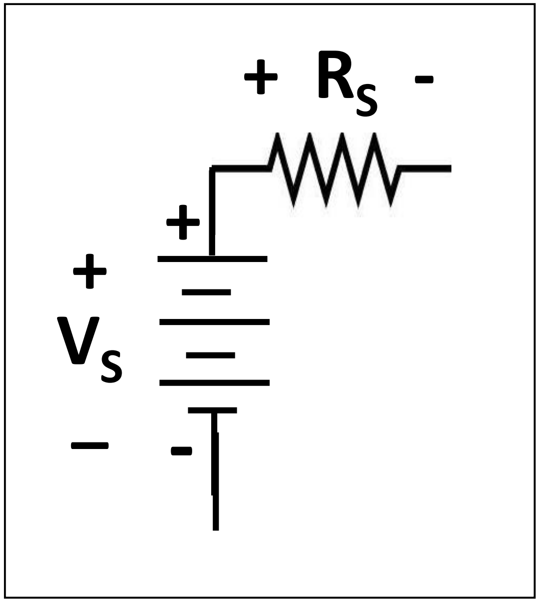
More Examples

Example 1: How long will a 9 V battery with a rating of 520 mAh will provide a current of 20 mA?

Example 2: How long can a 1.5 V battery provide a current of 250 mA to light the bulb if its rating is 16 Ah?

Measuring the internal resistance

It's important to know how to measure the internal resistance of a battery in the laboratory.
Do NOT use an ohmmeter.



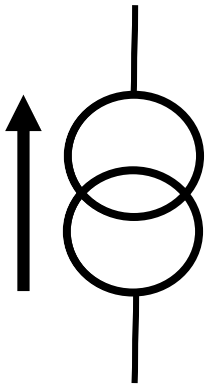
Steps:
1. First, measure the voltage across the battery. If no current flows, then the voltage dropped across R_S is zero. This gives $\mathbf{V_S}$.
2. Then, place a known resistor (say 1kΩ) is series and measure the voltage dropped across your resistor ($\mathbf{V_R}$) and the current flowing ($\mathbf{I_R}$).

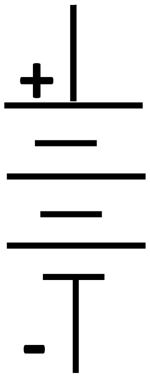
The voltage should be V_S but it will not be. The difference will be the (I_R* R_S). So this gives that:

$$R_S = \frac{V_S - V_R}{I_R}$$

Ideal Current and Voltage Sources

The symbol of the current source always shows the direction of current flow





Ideal sources do not occur in nature, nor they can be manufactured. Things that look almost like an "**ideal voltage sources**" are possible. Things that look like "**ideal current sources**" are rare.

Both sources in principle require an **infinite energy source**.