EE114
Intro to Systems & Control

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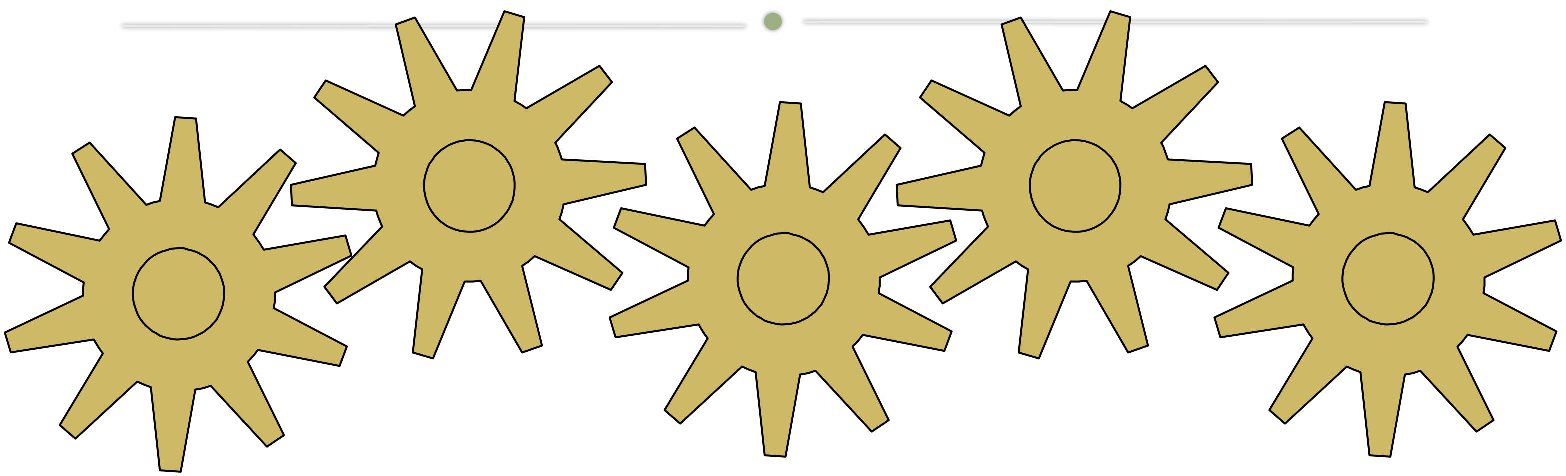

So far …

- We've introduced the concept of control and, in particular, feedback control …

- We've looked at what a system is, how systems can be categorized and illustrated the need for mathematical modelling …

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- We’ve looked at what a system is, how systems can be categorized and illustrated the need for mathematical modelling …

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- **Now, we are going to look at the mathematical modelling of a few simple systems.**

Modelling of Basic Systems

- As mentioned already, obtaining a mathematical representation of a system allows us to:
  - Understand the characteristics of the system (useful for design purposes).
  - Simulate the system (useful for scenario testing, forecasting, etc.).
  - Provide a basis for control system design (for stability, optimising performance, etc.).

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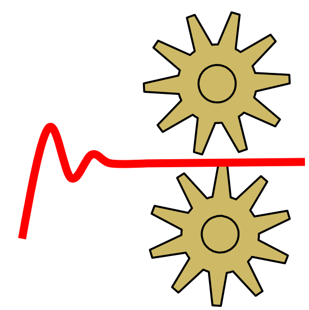
- In essence, a mathematical model of a system will increase our understanding of it.



Modelling of Basic Systems

- The basic modelling procedure is as follows:
  - 1 – Draw a schematic diagram of the system and define the variables.
  - 2 – Using physical laws, write equations for each component.
  - 3 – Parameterise the model (using experiment design and/or system identification techniques).
  - 4 – Validate the model.

Modelling of Basic Systems

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  - **Objective of modelling** – what is the purpose of the model? This dictates the level of accuracy (and hence complexity) required. Different modelling objectives include design/synthesis, analysis and control.

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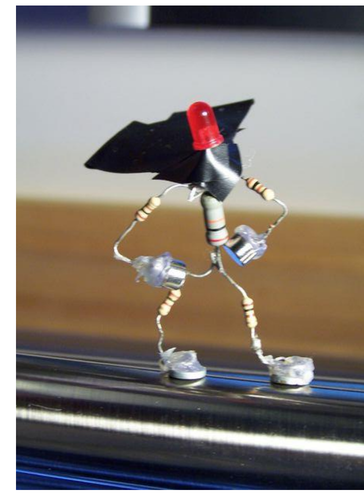

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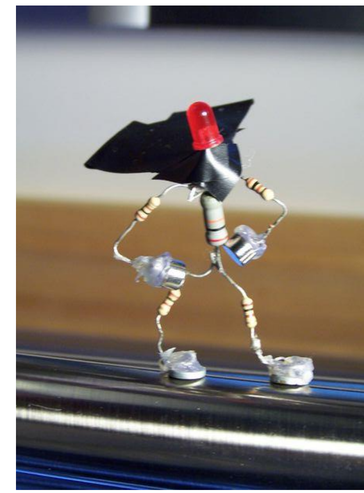


- For static electrical systems, such as resistor networks, we use Ohm's law, which states that the voltage across a resistor is directly proportional to the current flowing through it:

$$v = iR$$

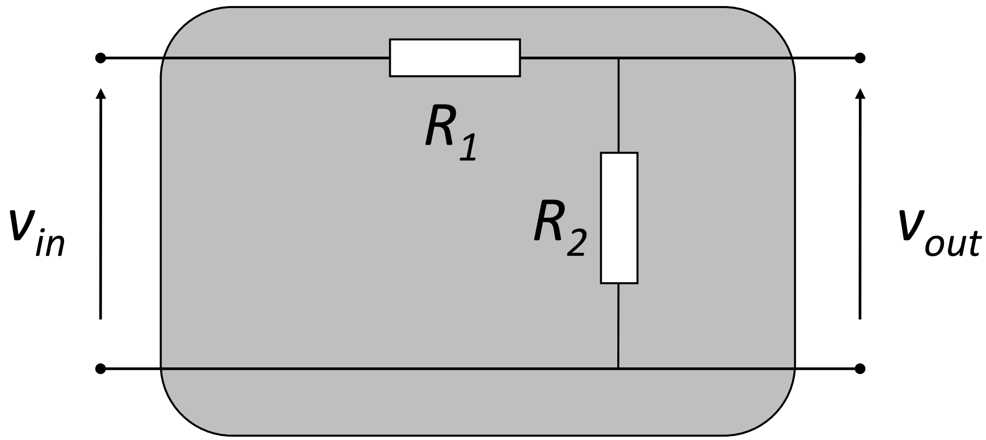
v is the voltage across the resistor in volts (system output)
i is the current through the resistor in amps (system input)
R is the resistance in ohms (system model)

Modelling of Static Systems

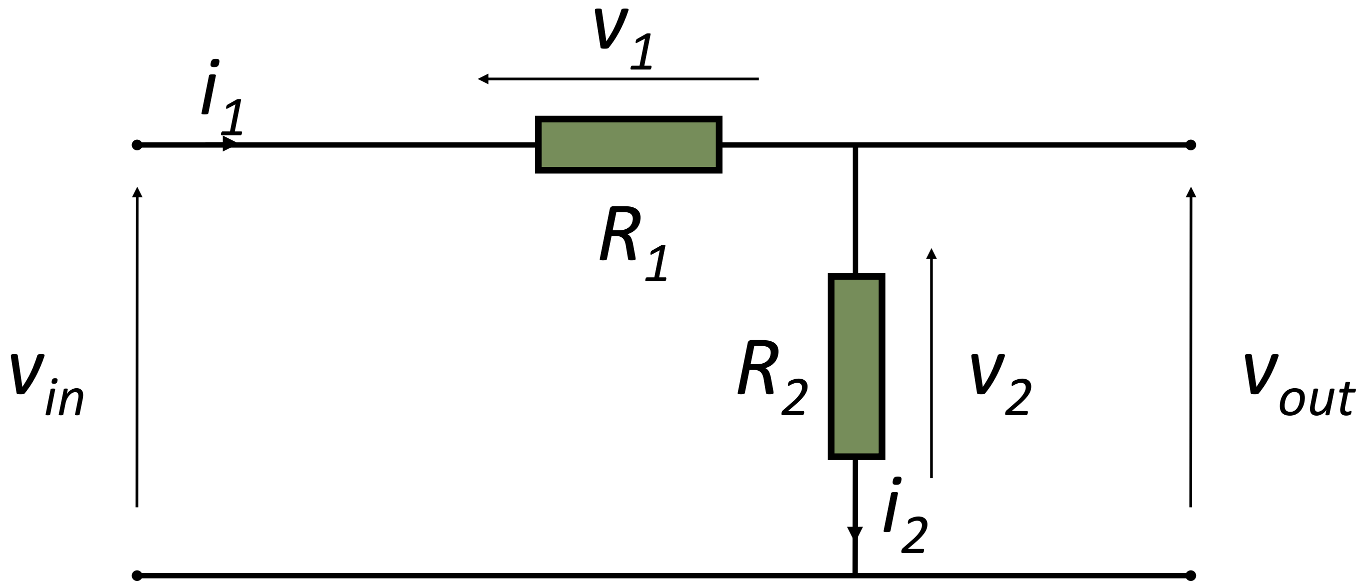
- Note that there are no dynamics in this system. Here, a step change in the input will result in a step change in the output.
- The model is given by a resistor value with a tolerance specification.
- Different resistor networks will involve parallel and series configurations.
- Furthermore, you may need to apply Kirchoff's current and voltage laws (KCL and KVL) in order to obtain a suitable model representation.

Modelling of Static Systems

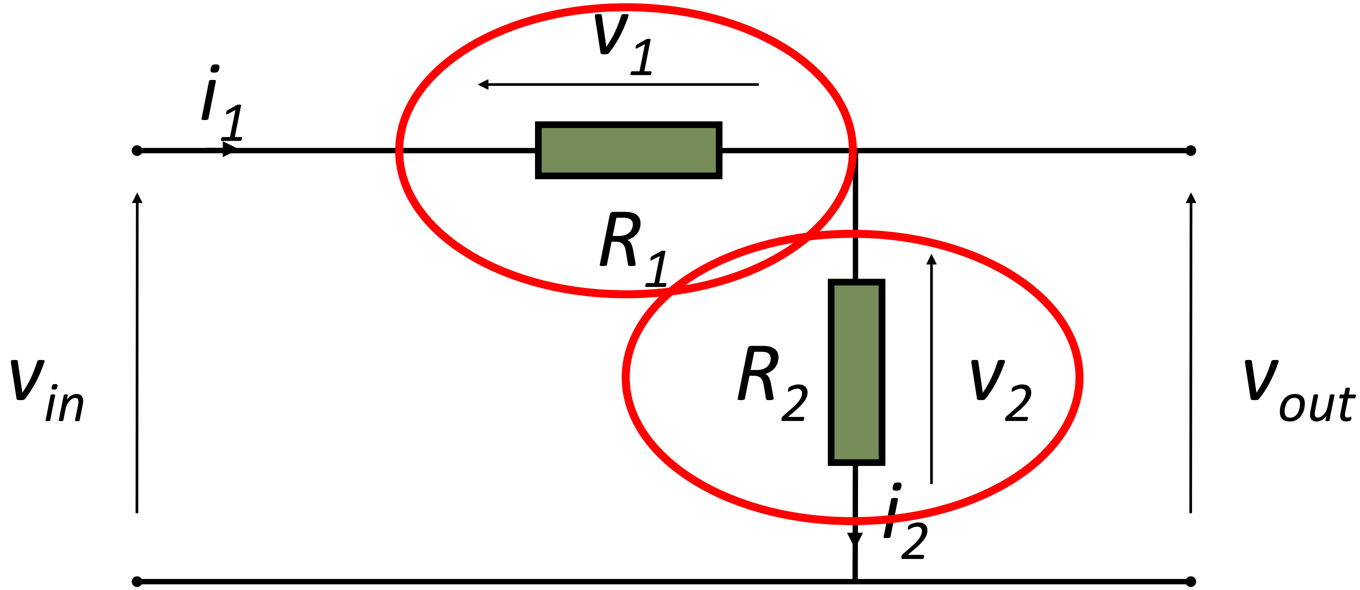
- ***Ex. 3.1 Derive, from first principles, the input-output relationship for the voltage divider circuit below:***



Solution:

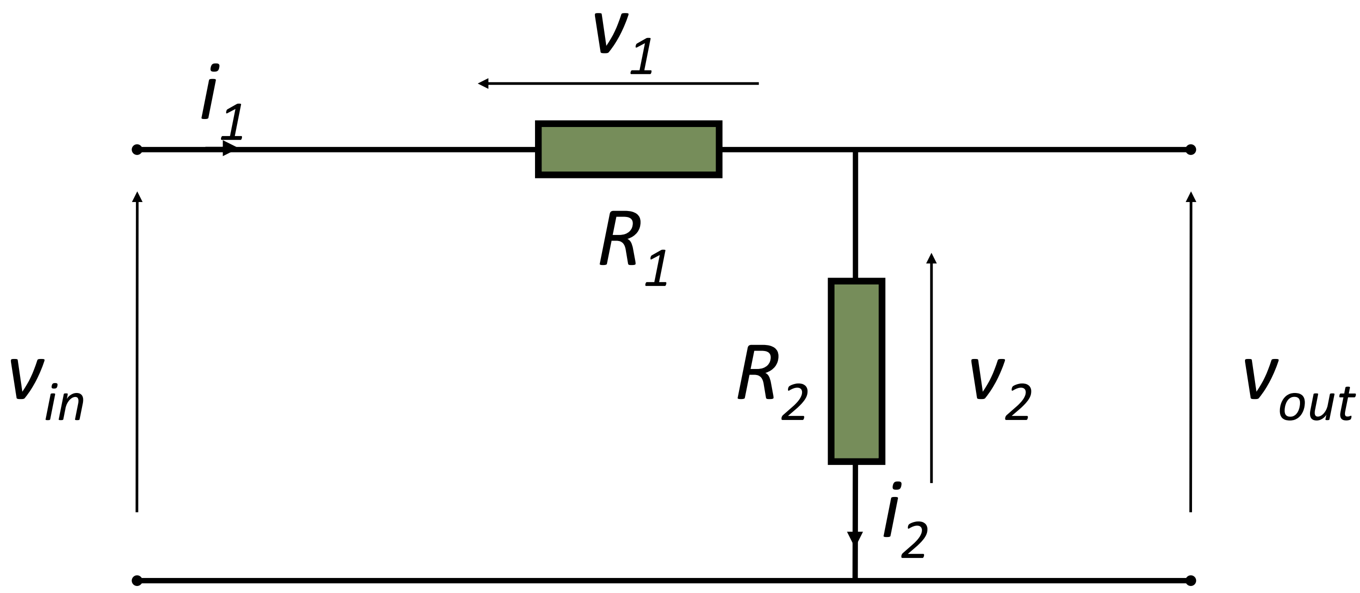


Solution:



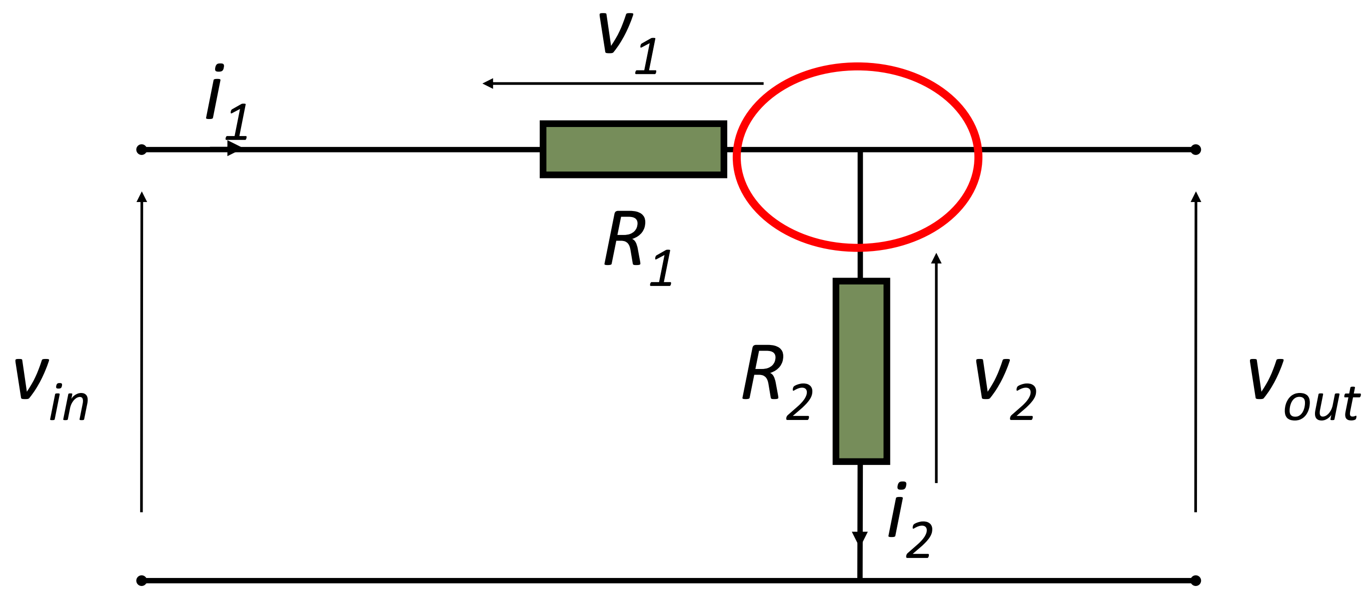
Ohm's Law …

Solution:



Ohm's Law … $v_1 = i_1 R_1$ and $v_2 = i_2 R_2$

Solution:



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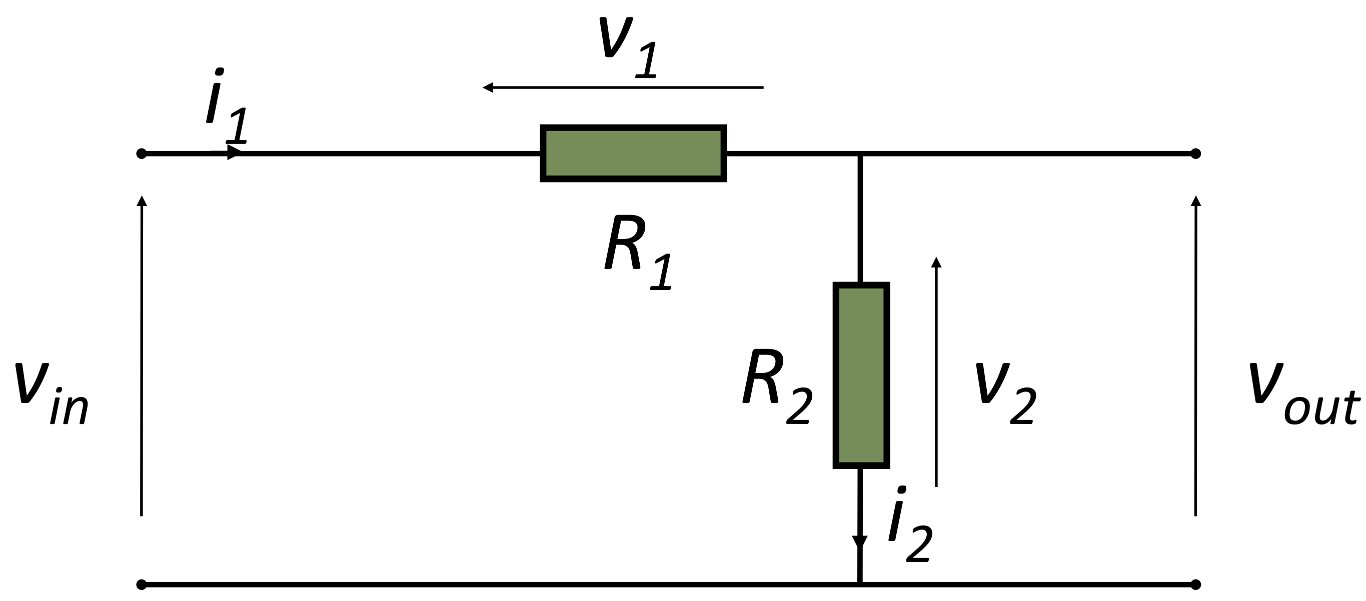
KCL …

Solution:



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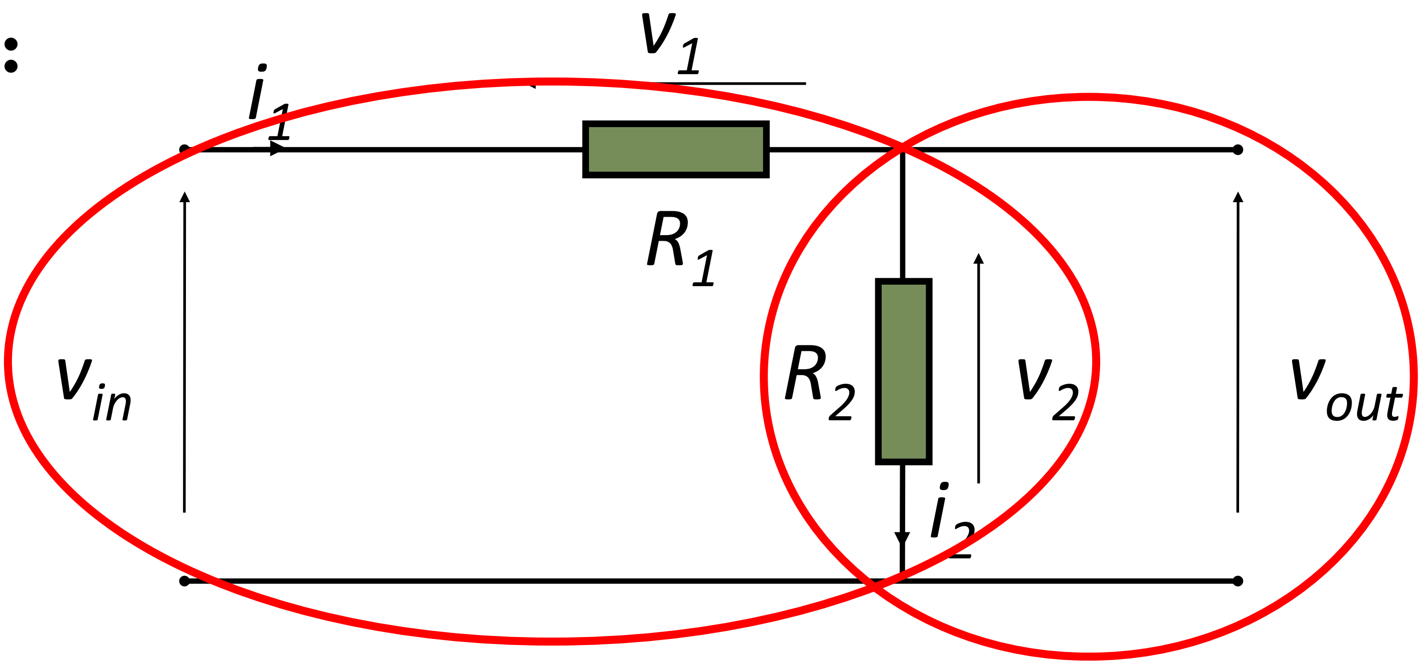

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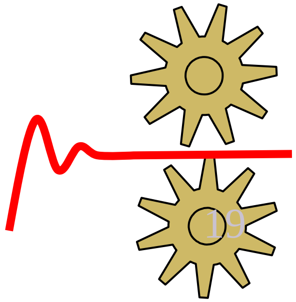
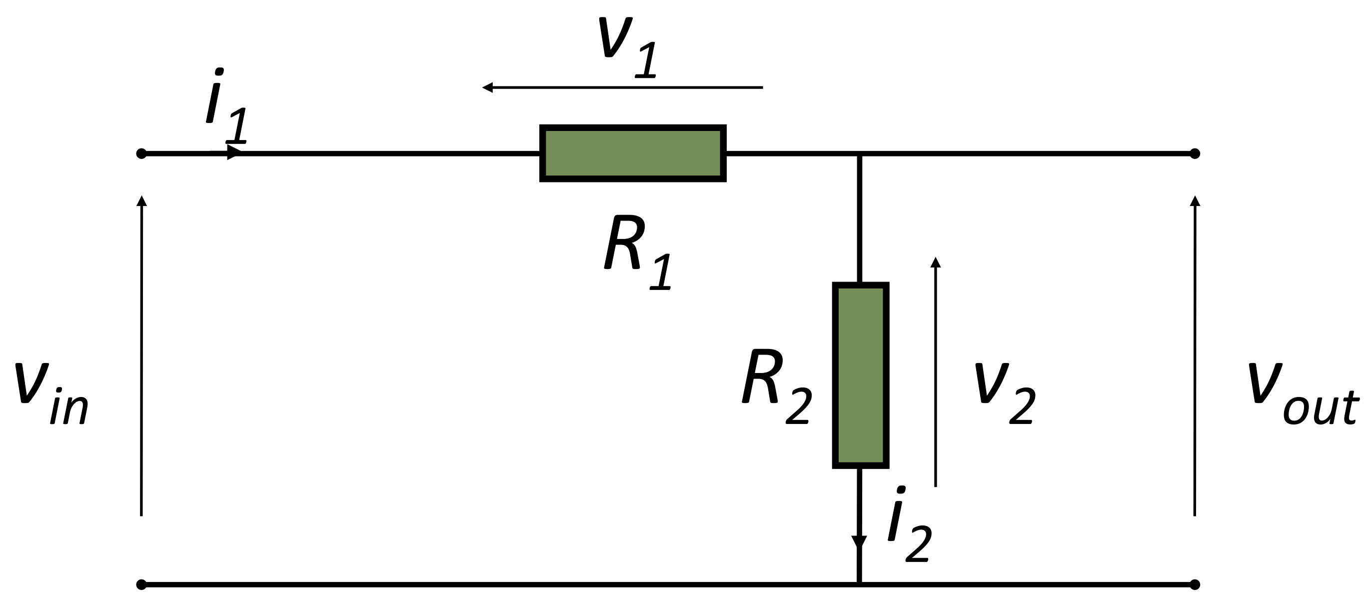
KVL …



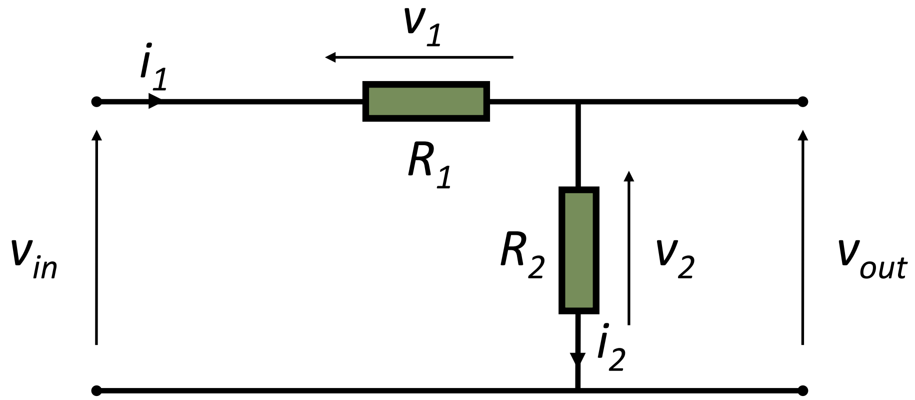
Solution:



Ohm's Law … $v_1 = i_1 R_1$ and $v_2 = i_2 R_2$

KCL … $i_1 = i_2$

KVL … $v_{in} = v_1 + v_2$ and $v_{out} = v_2$



$$v_1 = i_1 R_1 \text{ and } v_2 = i_2 R_2$$

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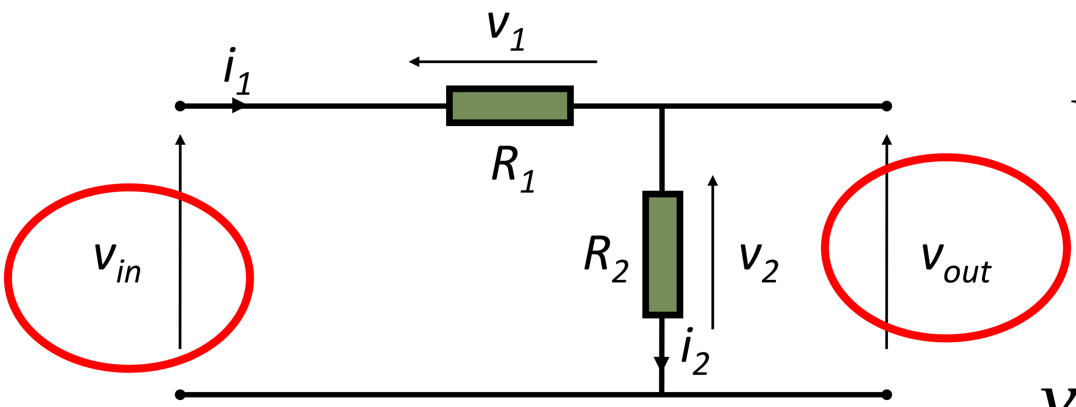
Hence …



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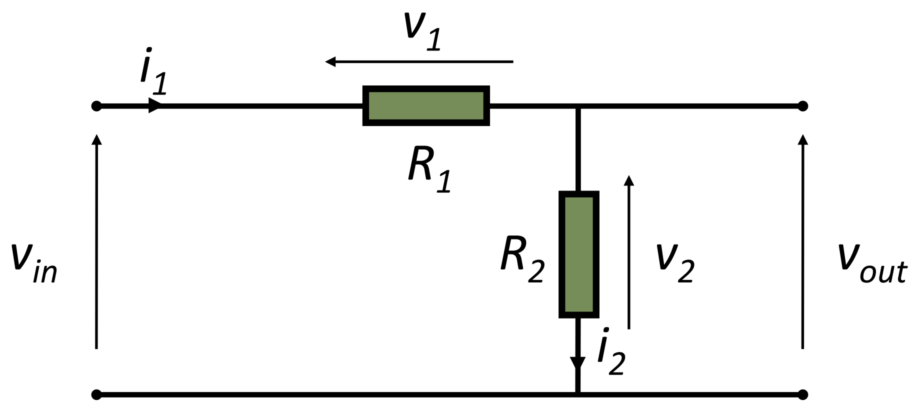
$$\frac{v_{out}}{v_{in}} = \frac{v_2}{v_1 + v_2}$$



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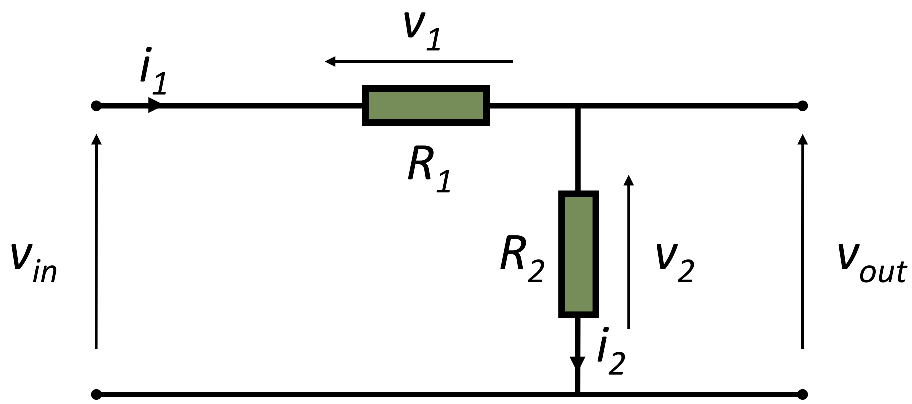

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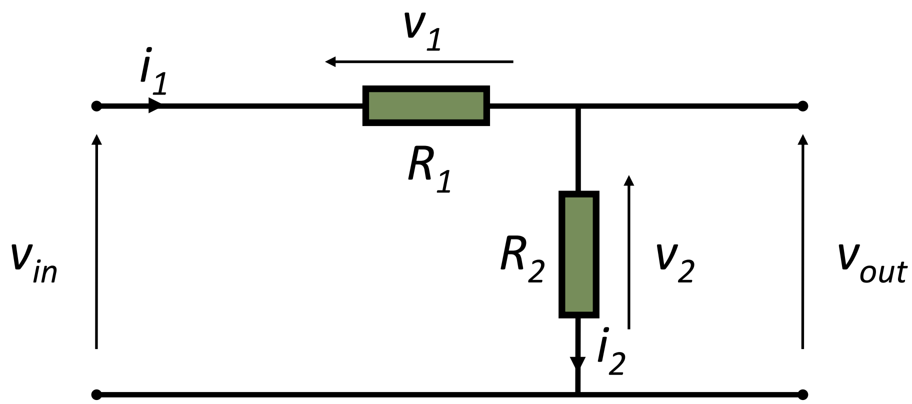

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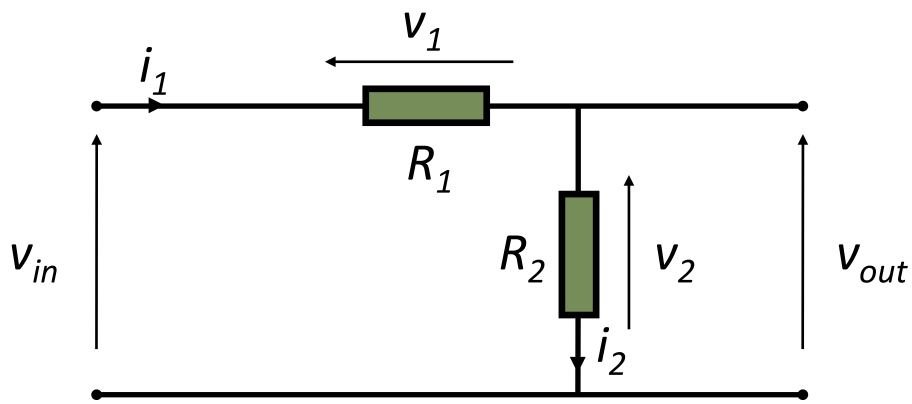
$$= \frac{R_2}{R_1 + R_2}$$



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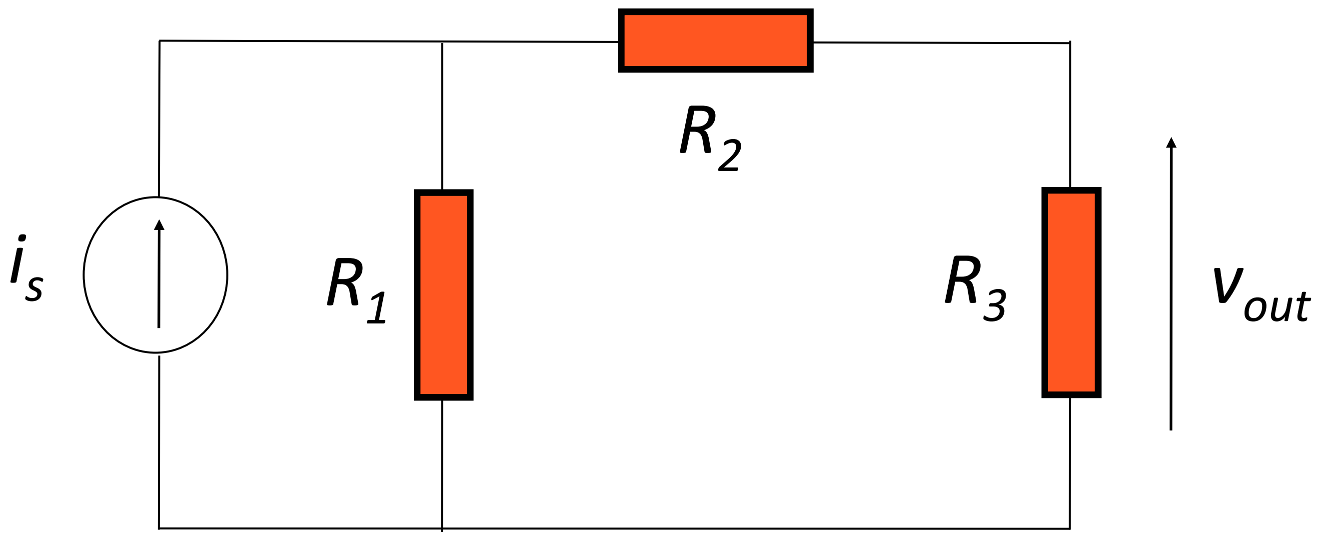
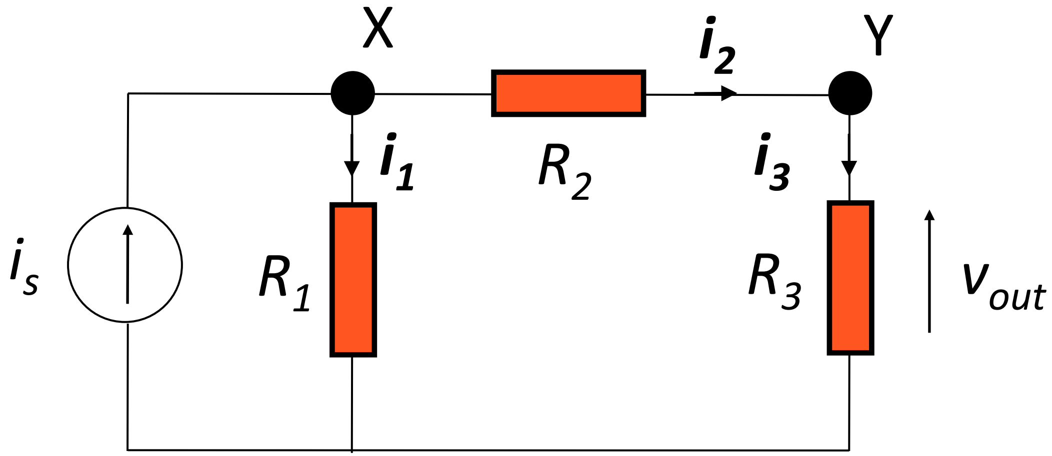
$$= \frac{R_2}{R_1 + R_2}$$ **... voltage divider rule**

Modelling of Static Systems

- ***Ex. 3.2(a) Derive the input-output relationship for the following circuit given that the input is the current source i_s and the output is the voltage v_{out} (across resistor R_3 as shown):***

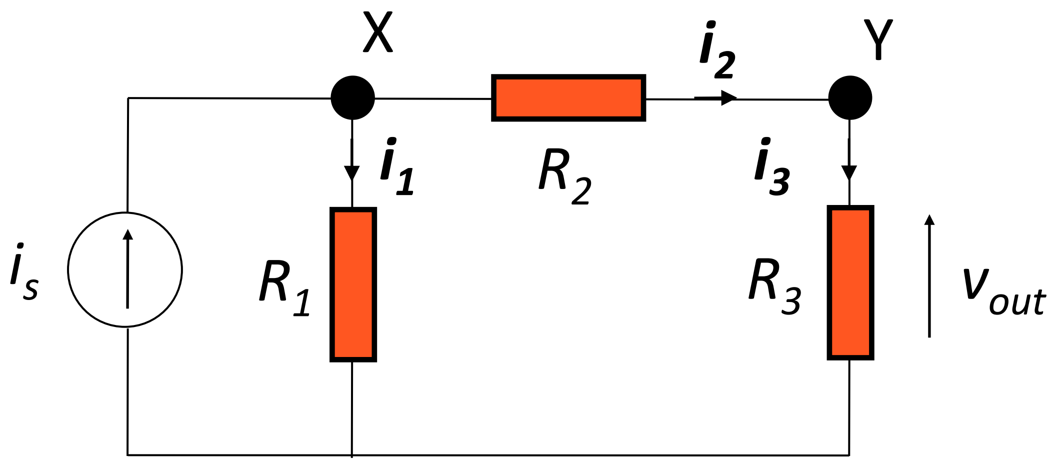


Solution:


X
Y
i_2
i_1
R_2
i_3
i_s
R_1
R_3
v_{out}


JOIN THE RESISTANCE!
OHMMMMM...
WWW.MANDOR.SK

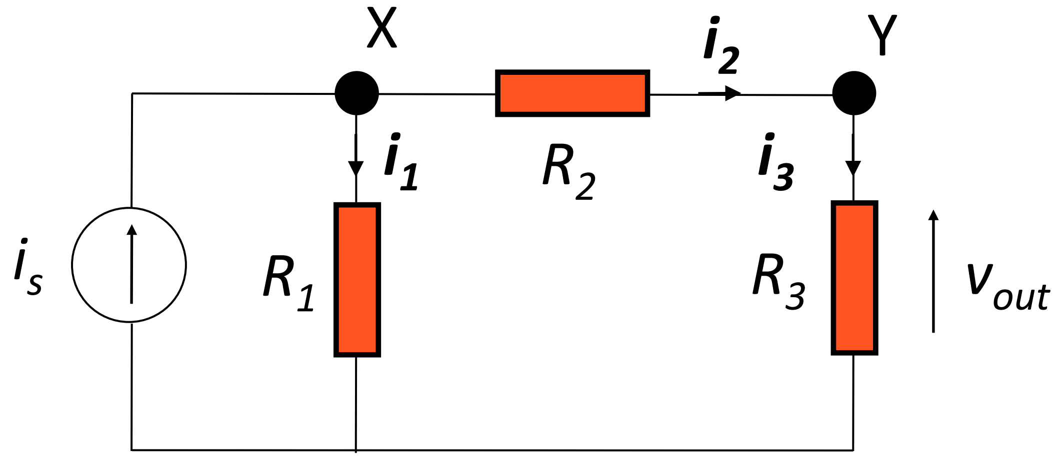
Solution:



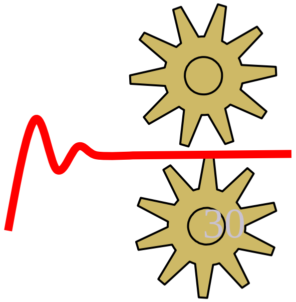
Using nodal analysis, we apply KCL to each of the nodes X and Y, as shown.

The voltage at node X is given by v_X while the voltage at node Y is v_{out} in this case.

Solution:



KCL @ node X: $$i_s = i_1 + i_2 \Rightarrow i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2}$$



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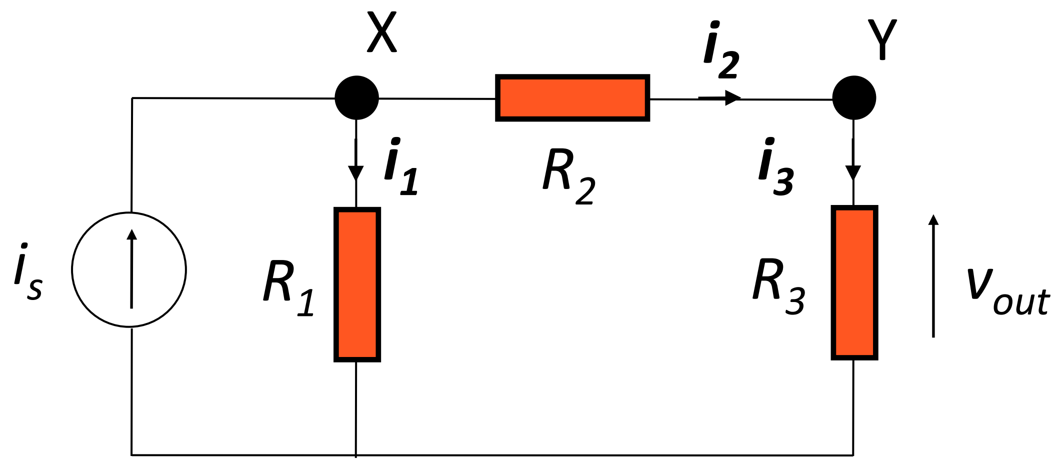
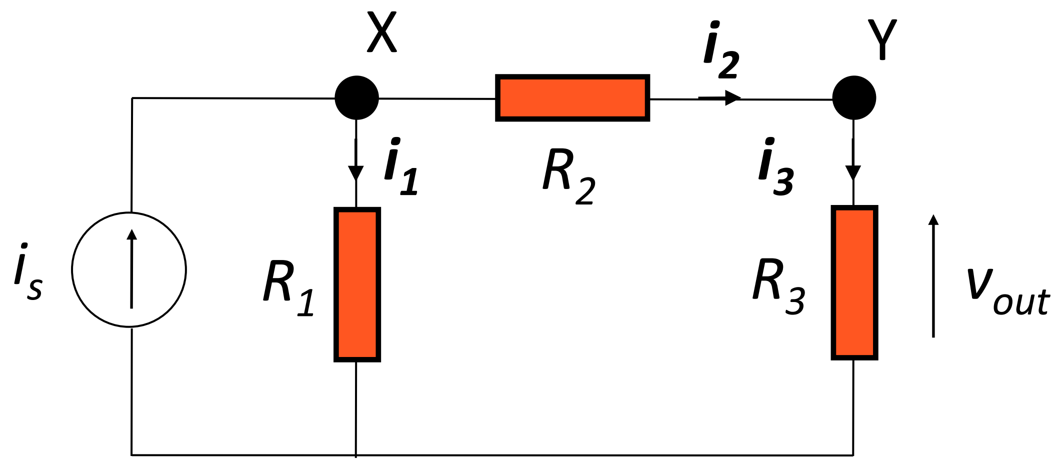


KCL @ node X: $$i_s = i_1 + i_2 \Rightarrow i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2}$$

KCL @ node Y: $$i_2 = i_3 \Rightarrow \frac{v_X - v_{out}}{R_2} = \frac{v_{out}}{R_3}$$

Solution:

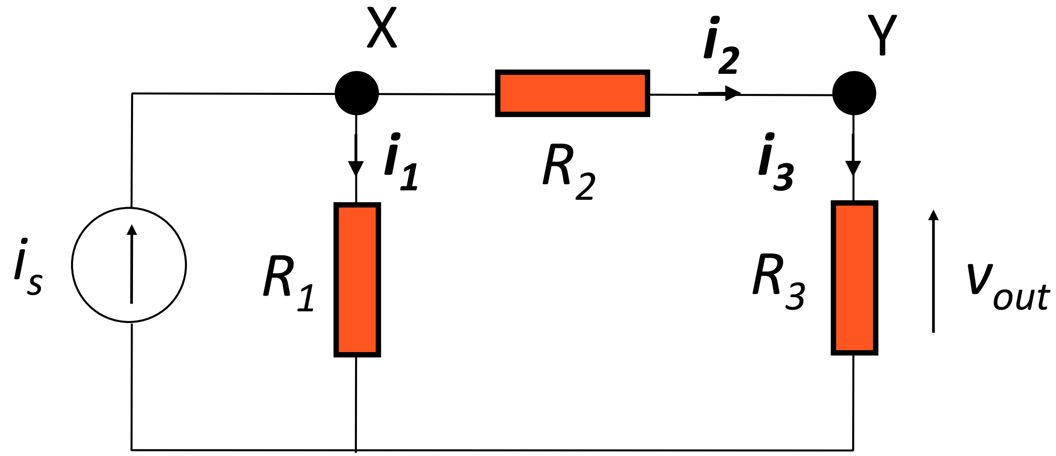


$$i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2}$$

$$\frac{v_X - v_{out}}{R_2} = \frac{v_{out}}{R_3}$$

Rearranging gives:

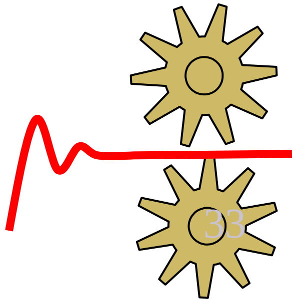
Solution:



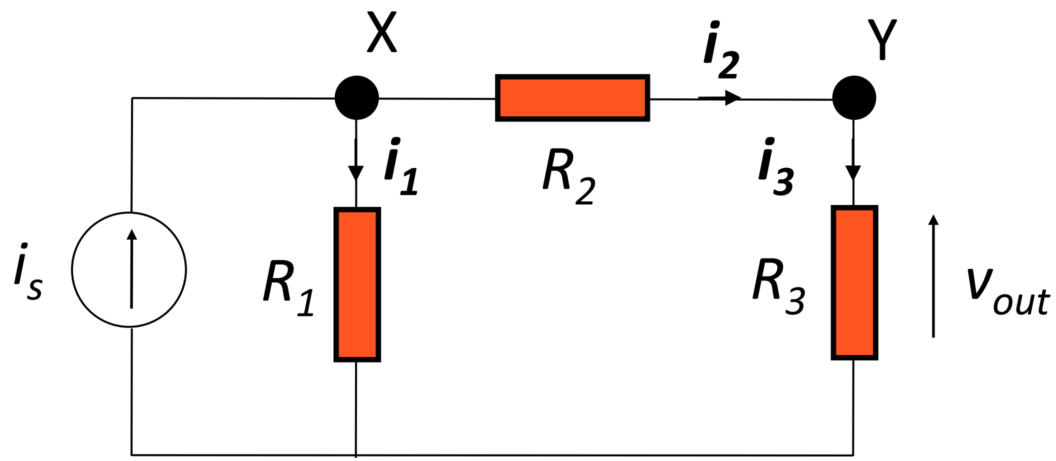
$$i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2} \qquad \frac{v_X - v_{out}}{R_2} = \frac{v_{out}}{R_3}$$

Rearranging gives:

$$v_X - v_{out} = \frac{R_2}{R_3} v_{out}$$



Solution:

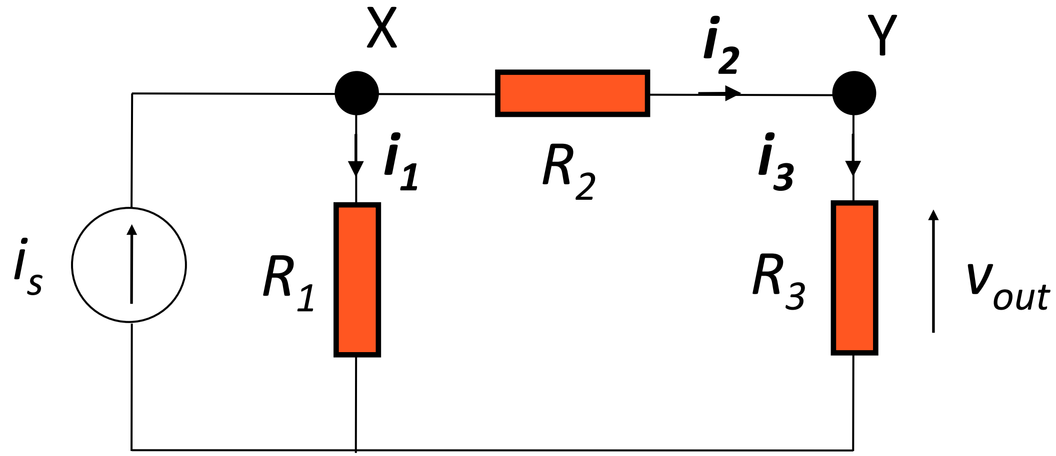


$$i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2} \qquad \frac{v_X - v_{out}}{R_2} = \frac{v_{out}}{R_3}$$

Rearranging gives:

$$v_X - v_{out} = \frac{R_2}{R_3} v_{out} \qquad \Rightarrow v_X = v_{out}\left(1 + \frac{R_2}{R_3}\right)$$

Solution:



$$i_s = \frac{v_X}{R_1} + \frac{v_X - v_{out}}{R_2}$$

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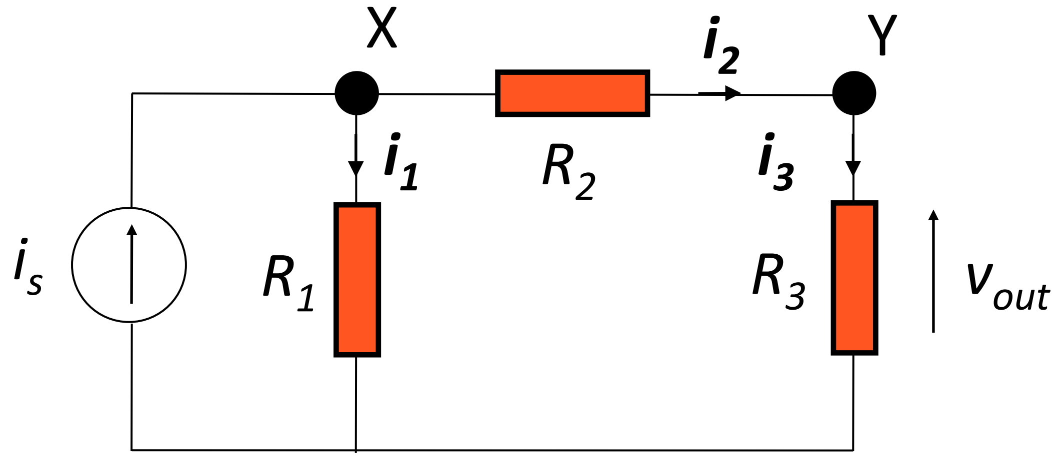
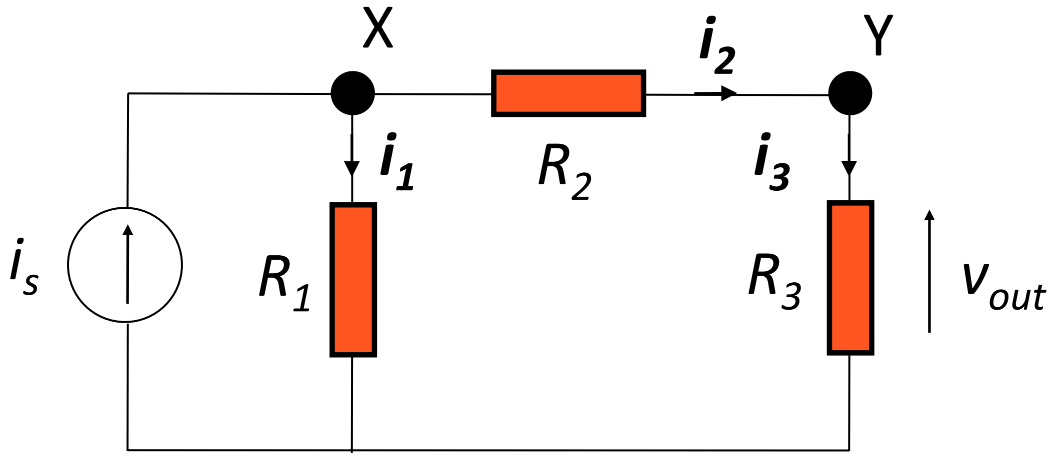


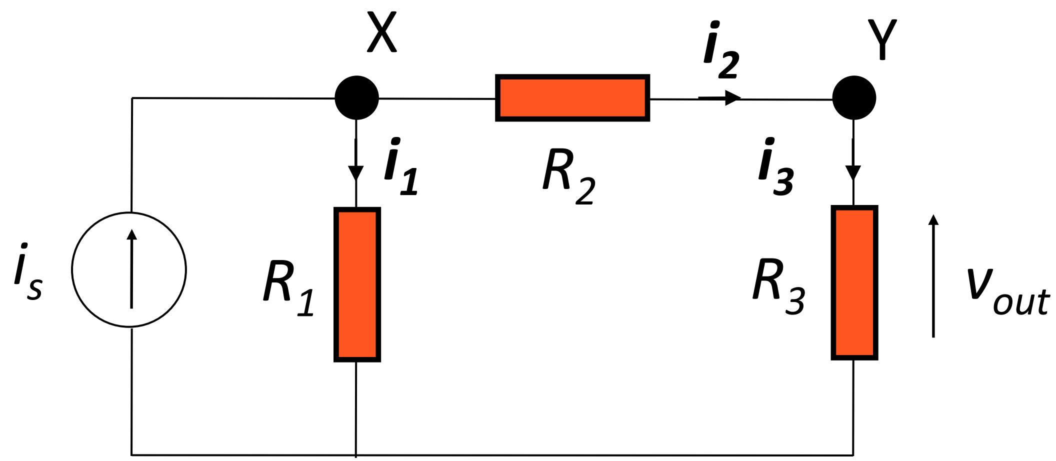
$$i_s = \frac{v_{out}\left(1+\dfrac{R_2}{R_3}\right)}{R_1} + \frac{v_{out}}{R_3}$$

Solution:



$$i_s = \frac{v_{out}\left(1+\dfrac{R_2}{R_3}\right)}{R_1} + \frac{v_{out}}{R_3} \qquad \Rightarrow i_s = \left(\frac{\left(1+\dfrac{R_2}{R_3}\right)}{R_1} + \frac{1}{R_3}\right) v_{out}$$

Solution:

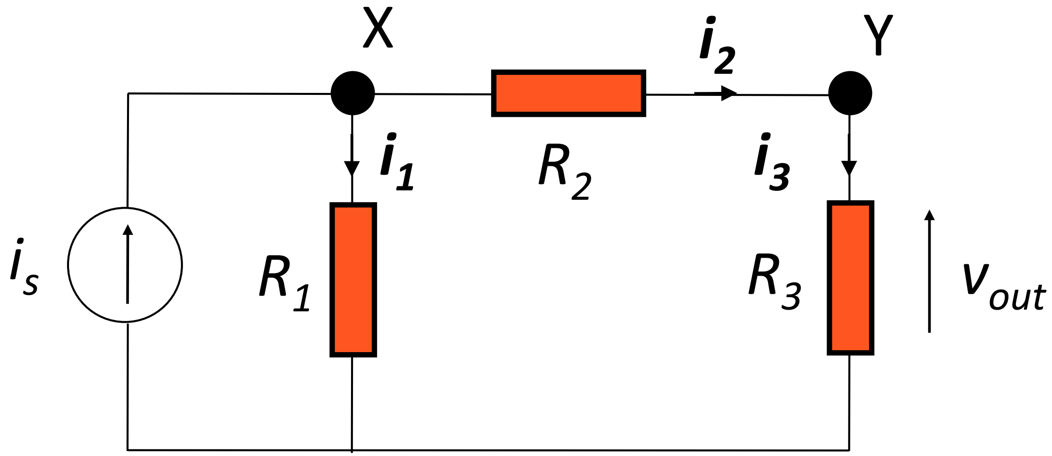


$$i_s = \frac{v_{out}\left(1+\frac{R_2}{R_3}\right)}{R_1} + \frac{v_{out}}{R_3}$$

$$\Rightarrow i_s = \left(\frac{\left(1+\frac{R_2}{R_3}\right)}{R_1}+\frac{1}{R_3}\right)v_{out}$$

$$\Rightarrow \frac{i_s}{v_{out}} = \left(\frac{(R_3+R_2)+R_1}{R_1R_3}\right)$$

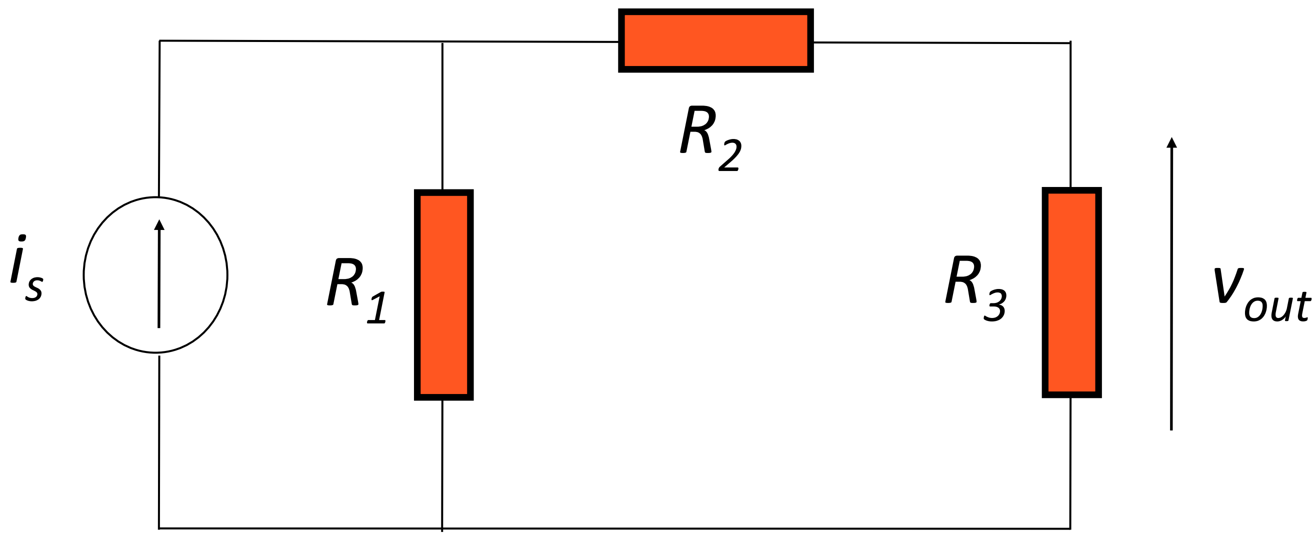
Solution:

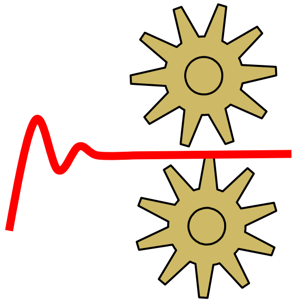


$$\Rightarrow \frac{v_{out}}{i_s} = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

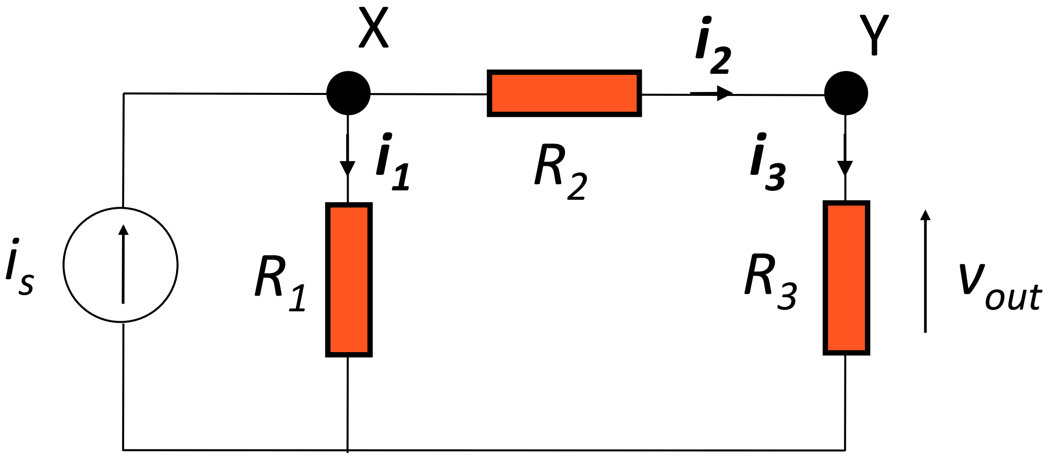
Modelling of Static Systems

- ***Ex. 3.2(b) Given that $R_1 = R_3 = 0.5\Omega$, $R_2 = 1\Omega$ and $i_s = 8A$, determine the voltage v_{out} for the circuit.***



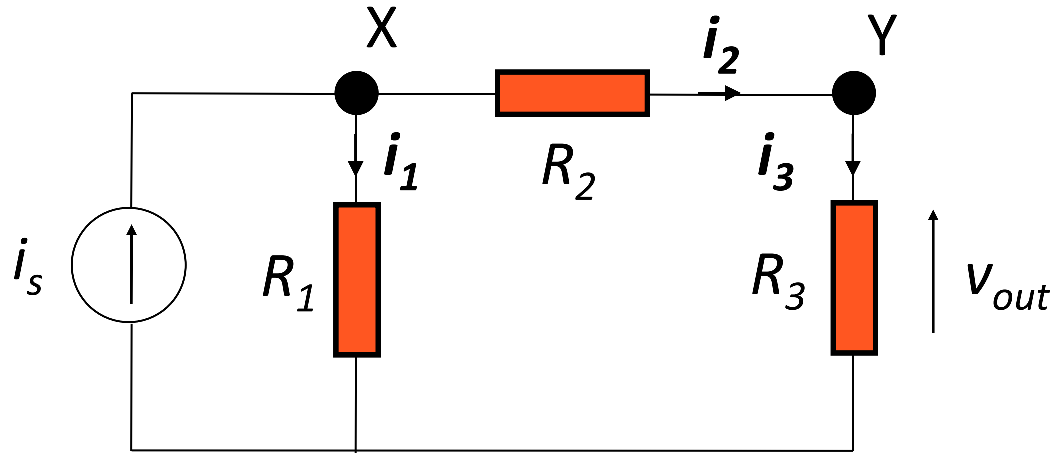


Solution:



$$v_{out} = \frac{R_1 R_3}{R_1 + R_2 + R_3} i_s \quad = \frac{(0.5)(0.5)}{0.5 + 1 + 0.5}(8)$$

Solution:



$$v_{out} = \frac{R_1 R_3}{R_1 + R_2 + R_3} i_s \quad = \frac{(0.5)(0.5)}{0.5 + 1 + 0.5}(8)$$

$$= \frac{0.25}{2}(8) \quad = 1V$$

Modelling of Static Systems

- Overall, in the case of modelling of static systems, we can say:
  - A static relationship exists between input and output.
  - Static = memoryless = instantaneous.
  - We start with some basic laws for given system type - for example, Ohm's law for an electrical circuit.
  - Model parameters may be subject to error (e.g. resistance tolerance values).
  - We should also state any assumptions made in deriving model.

Modelling of Static Systems

- In general, all models are subject to assumptions and all models have errors.
- In the case of the previous examples, the wire in the circuit has resistance along its entire length – we can argue that this is negligible in relation to the value of the resistor itself.
- We also assume that the components are ideal.

Modelling of Static Systems

- Ideally, the complete model description should give:
  - the model structure,
  - the parameters of the model,
  - modelling assumptions, including range of model validity, and
  - some measure of the error in the model.

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  - the parameters of the model,
  - modelling assumptions, including range of model validity, and
  - some measure of the error in the model.

- In practice, most real systems have dynamics associated with them – even resistors!

- Sometimes these dynamics have a negligible effect on the system behaviour that we are interested in studying allowing us to assume steady state (or static) conditions.