multiparticle system

Center of Mass

$$\vec{r_c} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + \cdots + m_n\vec{r_n}}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r_i}}{M}$$

$$M = \sum_{i=1}^{n} m_i$$ tatal mass of the system

Motion of Center of Mass

$$\vec{v_c} = \frac{d\vec{r_c}}{dt} = \frac{\sum_{i=1}^{n} m_i \frac{d\vec{r_i}}{dt}}{M} = \frac{\sum_{i=1}^{n} m_i\vec{v_i}}{M}$$

Velocity of Center of Mass

$$M\frac{d\vec{v_c}}{dt} = \frac{\cancel{M}\sum_{i=1}^{n} m_i \frac{d\vec{v_i}}{dt}}{\cancel{M}} = \frac{\cancel{M}\sum_{i=1}^{n} m_i\vec{a_i}}{\cancel{M}} = \sum_{i=1}^{n} m_i\vec{a_i} = \sum_{i=1}^{n}\vec{F_i} = \vec{F}_{net}$$

$$\parallel$$

$M\vec{a_c}$ → Acceleration of Center of Mass

$\vec{F}_{net}$: net external forces

$$\boxed{\vec{F}_{net} = M\vec{a_c}}$$

Newton's 2nd Law for a System of Particles

Center of Mass

Rigid Body

① $m_i \longrightarrow dm$

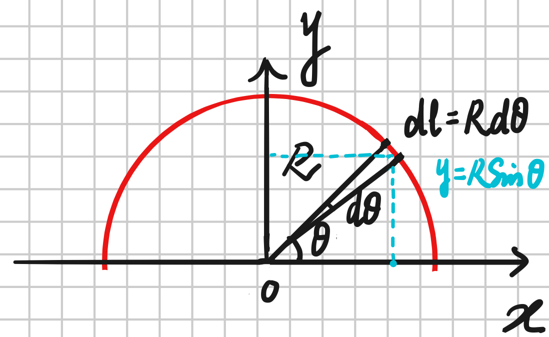
② $\Sigma \longrightarrow \int$

M — total mass of the rigid body

$$x_c = \frac{\int x\,dm}{M} \qquad y_c = \frac{\int y\,dm}{M} \qquad z_c = \frac{\int z\,dm}{M}$$

e.g.

M, R, $\frac{1}{2}$ a thin ring



$$\rho_l = \frac{M}{\pi R}$$ — linear density

$$\begin{cases} dm = \rho_l\,dl = \dfrac{M}{\pi R} R\,d\theta = \dfrac{M}{\pi} d\theta \\ y = R\sin\theta \end{cases}$$

$$y_c = \frac{\int y\,dm}{M} = \frac{\int_0^{\pi} R\sin\theta \frac{\cancel{M}}{\pi} d\theta}{\cancel{M}} = \frac{R}{\pi}\int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{R}{\pi}(-\cos\theta)\Big|_0^{\pi}$$

$$= \frac{R}{\pi}(-(-1-1))$$

$$= \frac{2R}{\pi}$$