We let $x > 0$,

1. (a) $f(-x) = -x\cdot\cos(-x) = -x\cos x = -f(x)$, And $f(0)=0$

so it's odd

(b) $f(-x) = x^4+4x$ $\quad f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

So it's neither.

(c) $f(-x) = e^{-x} - e^{x} = -f(x)$ $\quad f(0) = 1-1=0$

So it's odd.

(d) $f(-x) = x^5 = f(x)$ so it's even

(e) $f(-x) = -x+5 = f(x)$ so it's even

2. (a) $A(\alpha) = \int_{-2}^{2} x\cdot\cos\alpha x\,dx = 0$

$B(\alpha) = \int_{-2}^{2} x\cdot\sin\alpha x\,dx = 2\int_0^2 x\cdot\sin\alpha x\,dx$

$= 2\left(-\frac{\cos\alpha x}{\alpha}\cdot x\Big|_0^2 + \frac{1}{\alpha}\int_0^2 \cos\alpha x\,dx\right)$

$= 2\left(-\frac{\cos\alpha 2}{\alpha} + \frac{1}{\alpha^2}\sin\alpha x\Big|_0^2\right) = -\frac{2\cos\alpha 2}{\alpha}$ $\qquad f(x) = \sum_{\alpha=1}^{\infty} -\frac{2\cos\alpha 2}{\alpha}\sin\alpha x$

(b) $A(\alpha) = \int_{-1}^{0} -x^2\cdot\cos\alpha x\,dx + \int_0^1 x^2\cdot\cos\alpha x\,dx = 0$

$B(\alpha) = 2\int_0^1 x^2\cdot\sin\alpha x\,dx = 2\left(-x^2\frac{\cos\alpha x}{\alpha} + \frac{2}{\alpha^2}\sin\alpha x\cdot x + \frac{2}{\alpha^3}\cos\alpha x\right)\Big|_0^1$

$= -\frac{2}{\alpha}\cos\alpha + \frac{4}{\alpha^2}\sin\alpha + \frac{4}{\alpha^3}\cos\alpha - \frac{4}{\alpha^3}$

$f(x) = \sum_{\alpha=1}^{\infty}\left(-\frac{2}{\alpha}\cos\alpha + \frac{4}{\alpha^2}\sin\alpha + \frac{4}{\alpha^3}\cos\alpha - \frac{4}{\alpha^3}\right)\sin\alpha x$

(c) $A(\alpha) = 2\int_1^2 \cos\alpha x\,dx = 2\frac{\sin\alpha x}{\alpha}\Big|_1^2 = 2\left(\frac{\sin 2\alpha - \sin\alpha}{\alpha}\right)$

$B(\alpha) = \int_{-2}^{-1}\sin\alpha x\,dx + \int_1^2 \sin\alpha x\,dx = 0$

$f(x) = \sum_{\alpha=1}^{\infty} 2\left(\frac{\sin 2\alpha - \sin\alpha}{\alpha}\right)\cos\alpha x$

3. $A(\alpha) = \int_0^2 (2-x)x\cos\alpha x\,dx = 2\int_0^2 x\cdot\cos\alpha x\,dx - \int_0^2 x^2\cos\alpha x\,dx$

$= 2\left(\frac{\sin\alpha x}{\alpha}x + \frac{\cos\alpha x}{\alpha^2}\right)\Big|_0^2 - \left(x^2\cdot\frac{\sin\alpha x}{\alpha} + \frac{2}{\alpha^2}\cos\alpha x\cdot x - \frac{2}{\alpha^3}\sin\alpha x\right)\Big|_0^2$

$= \frac{2}{\alpha^3}\sin 2\alpha - \frac{2}{\alpha^2}\cos 2\alpha - \frac{2}{\alpha^2}$

$$B(\alpha)=\int_0^2 (2-x)x\sin\alpha x\,dx = 2\int_0^2 x\cdot\sin\alpha x\,dx-\int_0^2 x^2\cdot\sin\alpha x\,dx$$

$$=2\left(-\frac{\cos\alpha x}{\alpha}x+\frac{\sin\alpha x}{\alpha^2}\right)\Big|_0^2-\left(-\frac{\cos\alpha x}{\alpha}x^2+\frac{2}{\alpha^2}\sin\alpha x\cdot x+\frac{2}{\alpha^3}\cos\alpha x\right)\Big|_0^2$$

$$=\frac{2}{\alpha^3}-\frac{2}{\alpha^2}\sin 2\alpha-\frac{2}{\alpha^3}\cos 2\alpha$$

4.(a) $$C(\alpha)=\int_{-\infty}^{\infty} f(x)e^{-j\alpha x}dx=\int_0^2 e^{-j\alpha x}dx$$

$$=-\frac{e^{-j\alpha x}}{\alpha j}\Big|_0^2=-\frac{1}{\alpha j}\left(e^{-2\alpha j}-1\right)$$

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}-\frac{1}{\alpha j}\left(e^{-2\alpha j}-1\right)e^{j\alpha x}d\alpha$$

(b) $$C(\alpha)=\int_0^{\pi} x\cdot e^{-j\alpha x}dx$$

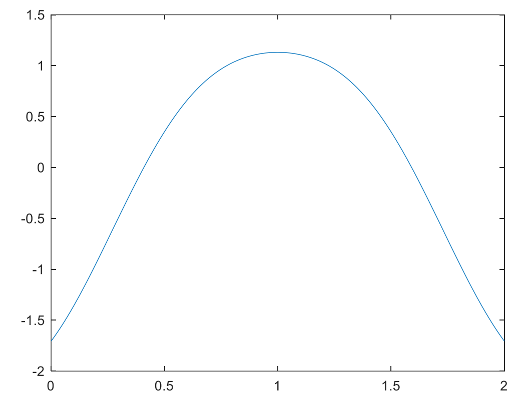
$$=-\frac{e^{-\alpha jx}}{\alpha j}\cdot x\Big|_0^{\pi}-\int_0^{\pi}-\frac{e^{-\alpha jx}}{\alpha j}dx$$

$$=-\frac{\pi\cos\alpha\pi}{\alpha j}+\frac{1}{\alpha j}\cdot\left(-\frac{e^{-\alpha jx}}{\alpha j}\right)\Big|_0^{\pi}=\frac{\cos\alpha\pi-1}{\alpha^2}+\frac{\pi\cos\alpha\pi}{\alpha}j$$

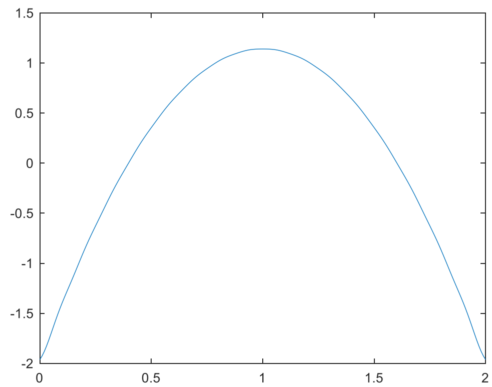
$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\cos\alpha\pi-1}{\alpha^2}+\frac{\pi\cos\alpha\pi}{\alpha}j\right)e^{j\alpha x}d\alpha$$

3 图

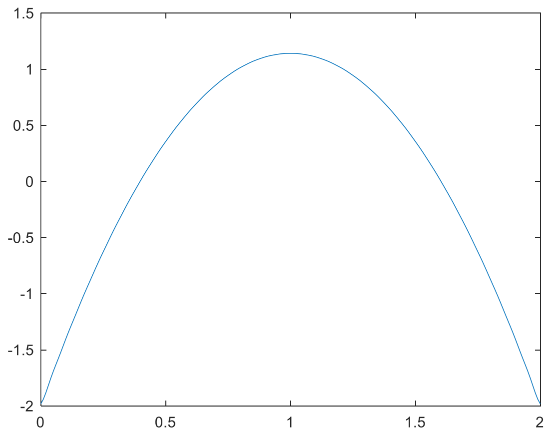
①α=5



②α=50



③α=100



Matlab Code

```
a = 0; %input
w = 0:a;
x = 0:0.01:2;
f1 = 0;
f2 = 0;
for k = 1:length(w)
   a= 2/k^3*sin(2*k)-2/k^2*cos(2*k)-2/k^2;
   b= 2/k^3-2/k^2*sin(2*k)-2/k^3*cos(2*k);
   f1 = f1 + a*cos(k*x);
   f2 = f2 + b*sin(k*x);
   f = f1 + f2 - 4/3;
end
plot(x,f)
```