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CS422 (323)

Q1.

(a)
$$\begin{cases} \phi & \text{world - x - axis} & f1. \\ \theta & \text{world - z - axis} & f2. \\ \psi & \text{current - x - axis} & f3 \\ \alpha & \text{world - z - axis} & f4 \end{cases}$$

$$\cancel{H = Rot_{x,\phi}} \qquad H = \overset{(f4)}{Rot_{z,\alpha}} \overset{(f2)}{Rot_{z,\theta}} \overset{(f1)}{Rot_{x,\phi}} \cdot \overset{(f3)}{Rot_{x,\psi}} \quad ①$$

$$\therefore H = Rot_{z,\alpha} \cdot Rot_{z,\theta} Rot_{x,\phi} Rot_{x,\psi} \longleftarrow$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\phi & -s\phi & 0 \\ 0 & s\phi & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\psi & -s\psi & 0 \\ 0 & s\psi & c\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $x_1 = \left(\frac{1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}}\right)^T \quad y_1 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)^T \quad z_1 = \left(0 \quad 1 \quad 0\right)^T$

$$\therefore H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad R_1^0 = (H_1)^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

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P2/P5

Q1.

$w = J \cdot q$ $\quad$ w_2^0

(c) $w_1^0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad w_2^1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $\quad R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.

$$w_2^0 = R_1^0 w_1^0 \cdot w_2^{21} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

(d) $H = Rot_{z,\theta} Trans_{z,d} Trans_{x,a} Rot_{x,\alpha}$.

The following pairs can be commuted:

$$\left\{\begin{aligned} H &= Rot_{z,\theta} \cdot Trans_{z,d} Trans_{x,a} Rot_{x,\alpha} && (0) \\ &= Trans_{z,d} Rot_{z,\theta} \cdot Trans_{x,a} Rot_{x,\alpha} && (1) \\ &= Rot_{z,\theta} Trans_{z,d} Rot_{x,\alpha} Trans_{x,a} && (2) \\ &= Rot_{z,\theta} Trans_{x,a} Trans_{z,d} Rot_{x,\alpha} && (3) \\ &= Trans_{z,d} Rot_{z,\theta} Rot_{x,\alpha} Trans_{x,a} && (4) \end{aligned}\right\} \text{5 totally !}$$

$$Rot_{z,\theta} = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Trans_{z,d} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Rot_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha & -s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

{ Because all the 4 pairs are orthogonal matrix, they can be commuted and will not cause any influence of the homogeneous transformation H.

Q1.

(e) sol.

$$\begin{cases} \text{position} & q_0 \to q_f \\ \text{times} & t_0 = 0 \to t_f = 1. \\ \text{velocities} & v_0 = v_f = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} q_0 \\ 0 \\ q_f \\ 0 \end{bmatrix}$$

cubic polynomial $q(t)$:

$$\begin{cases} q(t_0) = q_0 & \dot{q}(t_0) = v_0 = 0 \\ q(t_f) = q_f & \dot{q}(t_f) = v_f = 0 \end{cases}$$

$$\therefore \begin{cases} q_0 = a_0 \\ 0 = a_1 \\ q_f - q_0 = a_2 + a_3 \\ 0 = 2a_2 + 3a_3 \end{cases}$$

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad ①$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\begin{cases} q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 \\ v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2 \\ q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{cases}$$

$$\therefore q(t) = q_0 + 3(q_f - q_0)t^2 - 2(q_f - q_0)t^3$$

(f). Jocabian

$$X = \begin{bmatrix} d_3 c_1 s_2 - d_2 s_1 \\ d_3 s_1 s_2 + d_2 c_1 \\ d_3 c_2 \end{bmatrix}$$

derivative :

$$J = \begin{bmatrix} -d_3 s_1 s_2 - d_2 c_1 & d_3 c_1 c_2 & c_1 s_2 \\ d_3 c_1 s_2 - d_2 s_1 & d_3 s_1 c_2 & s_1 s_2 \\ 0 & -d_3 s_2 & c_2 \end{bmatrix} \quad ①$$

$$\det(J) = 0 \longrightarrow d_3^2 \sin\theta_2 = 0 \ (\text{singular}). \quad ②$$

Hence $d_3 = 0$ or $\theta_2 = 0, \pi$. ③

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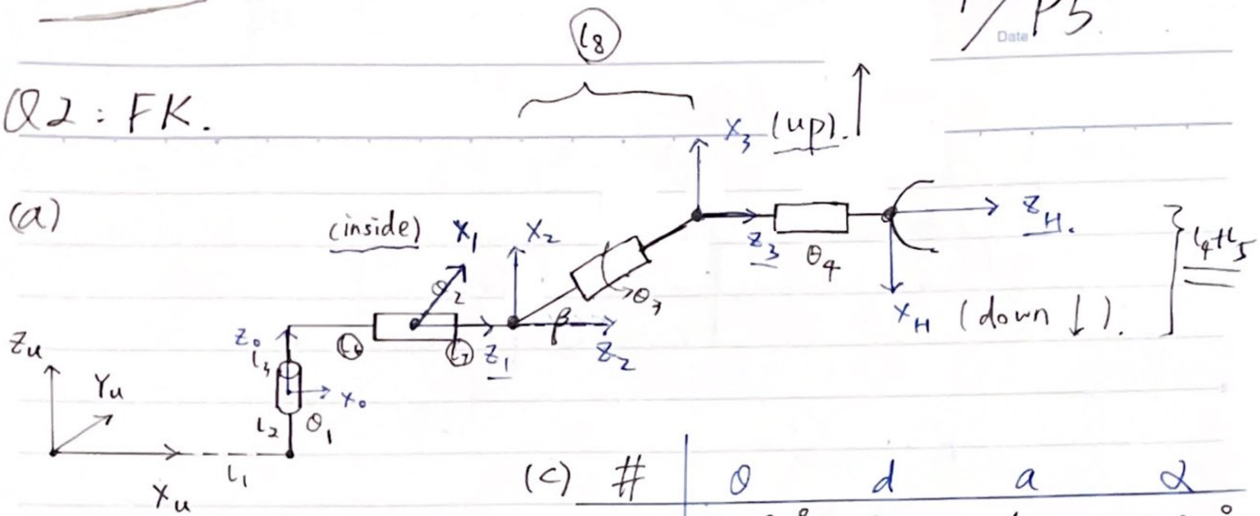
Q2 : FK.

(a)



(b) $T^u_0 = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c)

#	θ	d	a	α
0-1	$\theta_1+90°$	l_3	l_6	$+90°$
1-2	$\theta_2+90°$	l_7	0	0
2-3	θ_3	l_8	l_4+l_5	0
3-H	$\theta_4+180°$	l_9	0	0

(d) $A^0_1 = \begin{bmatrix} c(\theta_1+\frac{\pi}{2}) & 0 & s(\theta_1+\frac{\pi}{2}) & l_6 c(\theta_1+\frac{\pi}{2}) \\ s(\theta_1+\frac{\pi}{2}) & 0 & -c(\theta_1+\frac{\pi}{2}) & l_6 s(\theta_1+\frac{\pi}{2}) \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 & 0 & c\theta_1 & -l_6 s\theta_1 \\ c\theta_1 & 0 & s\theta_1 & l_6 c\theta_1 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A^1_2 = \begin{bmatrix} -s\theta_2 & -c\theta_2 & 0 & 0 \\ c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A^2_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_8 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & l_8 s\theta_3 \\ 0 & 0 & 1 & l_8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A^3_H = \begin{bmatrix} c(\theta_4+\pi) & -s(\theta_4+\pi) & 0 & 0 \\ s(\theta_4+\pi) & c(\theta_4+\pi) & 0 & 0 \\ 0 & 0 & 1 & l_9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -c\theta_4 & s\theta_4 & 0 & 0 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & l_9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) $T^u_H = T^u_0 \cdot \left(A^0_1 A^1_2 A^2_3 A^3_H \right)$

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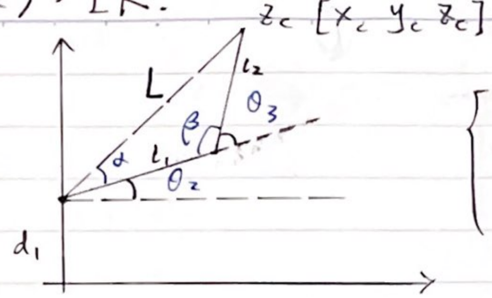
P_5 / P_5

Q3 : IK.



As the figure defined :

$$\begin{cases} L = \sqrt{r^2 + s^2} \\ s = z_c - d_1 \\ r = \sqrt{x_c^2 + y_c^2} . \end{cases}$$

$$\begin{cases} \cos\alpha = \dfrac{L^2 + l_1^2 - l_2^2}{2 \cdot L \cdot l_1} \\ \sin\alpha = \pm \sqrt{1 - c^2\alpha} . \end{cases} \qquad \alpha = \mathrm{atan2}(\cos\alpha, \sin\alpha) \quad ③$$

$$\cancel{\tan}\ \theta_2 = \mathrm{atan2}(r, z_c - d) - \alpha \quad ②$$

easy to get: $\theta_1 = \mathrm{atan2}(x_c, y_c)$. ①

$$\begin{cases} \cos\beta = \dfrac{l_1^2 + l_2^2 + L}{2 l_1 l_2} \\ \sin\beta = \pm \sqrt{1 - c^2\beta} . \end{cases} \qquad \beta = \mathrm{atan2}(\cos\beta, \sin\beta) \quad ④$$

$$\theta_3 = 180^\circ - \beta \quad ⑤$$

Overall,

$$\begin{cases} \theta_1 = \mathrm{atan2}(x_c, y_c) \\ \theta_2 = \mathrm{atan2}(r, z_c - d) - \mathrm{atan2}(\cos\alpha, \sin\alpha) \\ \theta_3 = \cancel{\mathrm{atan}}\ \pi - \mathrm{atan2}(\cos\beta, \sin\beta) . \end{cases}$$

(OVER).