Tutorial Sheet 3 - Solutions

Q1 (i)

$$F(s)=\frac{s}{(s+2)(s+5)}=\frac{A}{s+2}+\frac{B}{s+5}=\frac{A(s+5)+B(s+2)}{(s+2)(s+5)}$$

$$s=-5:\quad -5=B(-3)\Rightarrow B=\frac{5}{3} \qquad\qquad s=-2:\quad -2=A(3)\Rightarrow A=-\frac{2}{3}$$

$$\Rightarrow F(s)=-\frac{2}{3}\left(\frac{1}{s+2}\right)+\frac{5}{3}\left(\frac{1}{s+5}\right) \qquad\qquad \Rightarrow f(t)=-\frac{2}{3}e^{-2t}+\frac{5}{3}e^{-5t}$$

(ii)

$$F(s)=\frac{s^2+8}{s(s^2+2s-8)}=\frac{A}{s}+\frac{B}{s-2}+\frac{C}{s+4}=\frac{A(s-2)(s+4)+Bs(s+4)+Cs(s-2)}{s(s-2)(s+4)}$$

$$s=0:\quad 8=A(-8)\Rightarrow A=-1$$
$$s=2:\quad 12=B(12)\Rightarrow B=1$$
$$s=-4:\quad 24=C(24)\Rightarrow C=1$$

$$\Rightarrow F(s)=\frac{-1}{s}+\frac{1}{s-2}+\frac{1}{s+4} \qquad\qquad \Rightarrow f(t)=-1+e^{2t}+e^{-4t}$$

Q2 (a) (i)

$$\frac{dx(t)}{dt}+3x(t)-4=0\rightarrow sX(s)-x(0)+3X(s)-\frac{4}{s}=0$$

$$\Rightarrow sX(s)-1+3X(s)=\frac{4}{s} \qquad \Rightarrow X(s)(s+3)=\frac{4}{s}+1=\frac{4+s}{s}$$

$$\Rightarrow X(s)=\frac{s+4}{s(s+3)}$$

(ii)

$$X(s)=\frac{s+4}{s(s+3)}=\frac{A}{s}+\frac{B}{s+3}=\frac{A(s+3)+Bs}{s(s+3)}$$

$$s=0:\quad 4=A(3)\Rightarrow A=\frac{4}{3} \qquad\qquad s=-3:\quad 1=B(-3)\Rightarrow B=-\frac{1}{3}$$

$$\Rightarrow X(s)=\frac{4}{3}\left(\frac{1}{s}\right)-\frac{1}{3}\left(\frac{1}{s+3}\right) \qquad \Rightarrow x(t)=\frac{4}{3}-\frac{1}{3}e^{-3t}=\frac{1}{3}\left(4-e^{-3t}\right)$$

(iii)

$$\frac{dx(t)}{dt}+3x(t)-4=0\rightarrow sX(s)+3X(s)-\frac{4}{s}=0 \qquad\qquad \Rightarrow X(s)=\frac{4}{s(s+3)}$$

Q2 (b) (i) $\dfrac{d^2x(t)}{dt} - 4x(t) = 4 \rightarrow s^2X(s) - sx(0) - \dot{x}(0) - 4X(s) = \dfrac{4}{s}$

$$\Rightarrow s^2X(s) - 2s - 1 - 4X(s) = \frac{4}{s}$$

$$\Rightarrow X(s)(s^2 - 4) = \frac{4}{s} + 2s + 1 = \frac{4 + 2s^2 + s}{s}$$

$$\Rightarrow X(s) = \frac{2s^2 + s + 4}{s(s^2 - 4)}$$

(ii) $X(s) = \dfrac{2s^2 + s + 4}{s(s^2 - 4)} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s-2}$

$$= \frac{A(s+2)(s-2) + Bs(s-2) + Cs(s+2)}{s(s+2)(s-2)}$$

$s = 0: \quad 4 = A(-4) \Rightarrow A = -1$

$s = -2: \quad 10 = B(8) \Rightarrow B = \dfrac{5}{4}$

$s = 2: \quad 14 = C(8) \Rightarrow C = \dfrac{7}{4}$

$$\Rightarrow X(s) = -\frac{1}{s} + \frac{5}{4}\left(\frac{1}{s+2}\right) + \frac{7}{4}\left(\frac{1}{s-2}\right) \quad \Rightarrow x(t) = -1 + \frac{5}{4}e^{-2t} + \frac{7}{4}e^{2t}$$

(iii) $\dfrac{d^2x(t)}{dt} - 4x(t) = 4 \rightarrow s^2X(s) - 4X(s) = \dfrac{4}{s} \quad \Rightarrow X(s) = \dfrac{4}{s(s^2 - 4)}$

Q3 Tut1, Q5(i) $v_i = iR + L\dfrac{di}{dt} \rightarrow V_i(s) = RI(s) + sLI(s) \quad \Rightarrow \dfrac{I(s)}{V_i(s)} = \dfrac{1}{sL + R}$

Tut1, Q5(ii) $\dfrac{dv_i}{dt} = \dfrac{R}{L}v_L + \dfrac{dv_L}{dt} \rightarrow sV_i(s) = \dfrac{R}{L}V_L(s) + sV_L(s) \quad \Rightarrow \dfrac{V_L(s)}{V_i(s)} = \dfrac{s}{s + \frac{R}{L}}$

Tut1, Q6 $LC\dfrac{d^2v_C}{dt^2} + RC\dfrac{dv_C}{dt} + v_C = v_i \rightarrow LCs^2V_C(s) + RCsV_C(s) + V_C(s) = V_i(s)$

$$\Rightarrow \frac{V_C(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Tut1, Q8 $$M\frac{d^2x}{dt^2}+(B_1+B_2)\frac{dx}{dt}+(K_1+K_2)x=f(t)$$

$$\rightarrow s^2MX(s)+s(B_1+B_2)X(s)+(K_1+K_2)X(s)=F(s)$$

$$\Rightarrow \frac{X(s)}{F(s)}=\frac{1}{Ms^2+(B_1+B_2)s+(K_1+K_2)}$$

Tut1, Q9 $$A\frac{dh}{dt}+\frac{h}{R}=F_{in}\rightarrow AsH(s)+\frac{1}{R}H(s)=F_{in}(s)\ \Rightarrow \frac{H(s)}{F_{in}(s)}=\frac{1}{sA+\frac{1}{R}}$$

Tut1, Q10 $$\frac{d^2h_2}{dt^2}(A_1A_2R_1R_2)+\frac{dh_2}{dt}(A_1R_1+A_2R_2)+h_2=R_2F_{in}$$

$$\rightarrow s^2(A_1A_2R_1R_2)H_2(s)+s(A_1R_1+A_2R_2)H_2(s)+H_2(s)=R_2F_{in}(s)$$

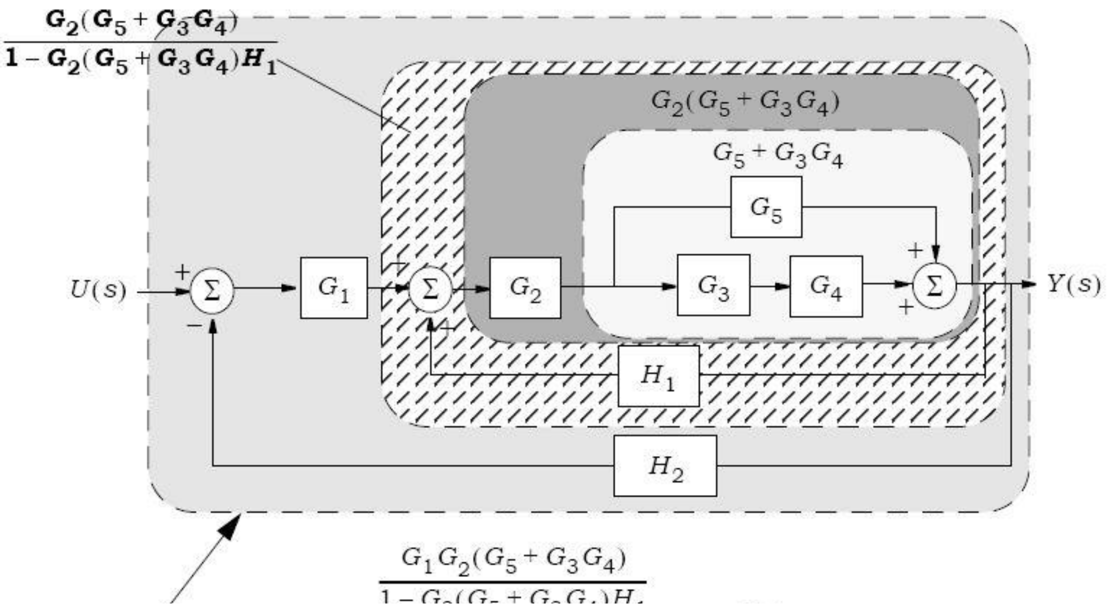
$$\Rightarrow \frac{H_2(s)}{F_{in}(s)}=\frac{R_2}{A_1A_2R_1R_2s^2+(A_1R_1+A_2R_2)s+1}$$

Q4 (i) Refer to Notes

(ii) Can't implement initial conditions (hence, we use zero initial conditions)

Q5 Refer to Notes

Q6

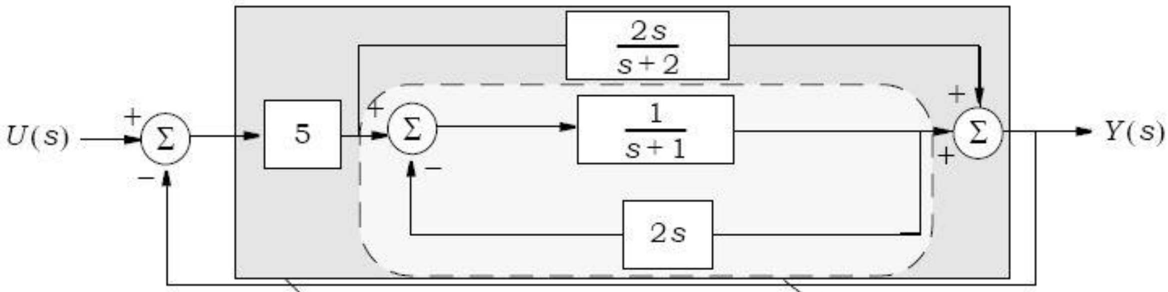


The transfer function of the block diagram can be obtained in steps, as shown in the diagramabove, giving the overall transfer function as:

$$G(s)=\frac{\dfrac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1}}{1+\dfrac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1}H_2}=\frac{G_1G_2(G_5+G_3G_4)}{1-G_2(G_5+G_3G_4)H_1+(G_1G_2(G_5+G_3G_4))H_2}$$

$G_1G_2G_5+G_1G_2G_3G_4$

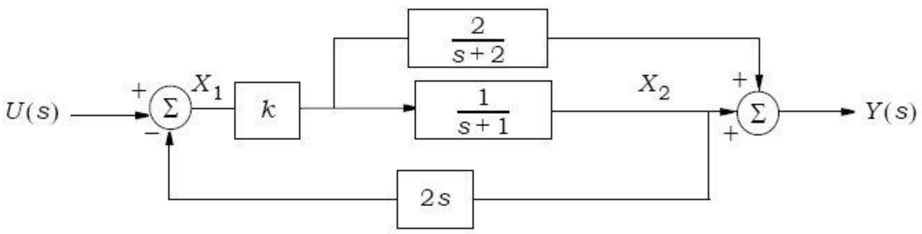
Q7



$$5\left[\frac{1}{3s+1}+\frac{2s}{s+2}\right] = \frac{30s^2+15s+10}{3s^2+7s+2} \qquad \frac{\frac{1}{s+1}}{1+\frac{1}{s+1}\cdot 2s} = \frac{1}{3s+1}$$

$$G(s) = \frac{\frac{30s^2+15s+10}{3s^2+7s+2}}{1+\frac{30s^2+15s+10}{3s^2+7s+2}} = \frac{30s^2+15s+10}{3s^2+7s+2+30s^2+15s+10} = \frac{30s^2+15s+10}{33s^2+22s+12}$$

Q8



Cannot easily simplify this configuration using the standard block diagram reduction rules. Instead we can write equations for the different points in the block diagram and then eliminate the intermediate variables, as follows:

$$X_1(s) = U(s) - 2sX_2(s) \qquad (1)$$

$$X_2(s) = \frac{k}{s+1}X_1(s) \qquad (2)$$

$$Y(s) = X_2(s) + \frac{2k}{s+2}X_1(s) \qquad (3)$$

We can elimate $X_2(s)$ from equation (1) and (3) using equation (2) to give:

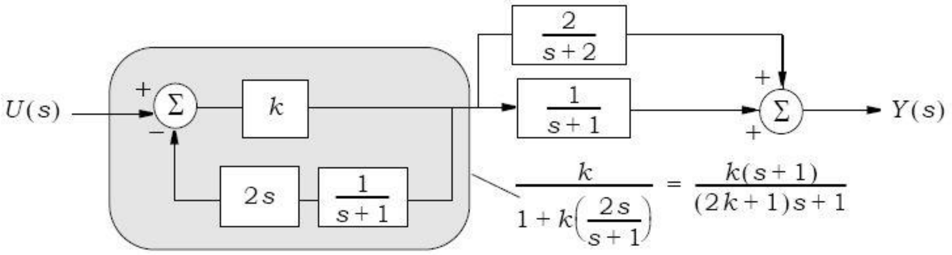
$$X_1(s) = U(s) - 2s\frac{k}{s+1}X_1(s) \qquad (4)$$

$$Y(s) = \frac{k}{s+1}X_1(s) + \frac{2k}{s+2}X_1(s) = \frac{3ks+4k}{(s+1)(s+2)}X_1(s) \qquad (5)$$

Rewriting (4) as:

$$\left[1+2s\frac{k}{s+1}\right]X_1(s) = U(s) \Rightarrow X_1(s) = \frac{U(s)}{1+\frac{2sk}{s+1}} = \frac{(s+1)}{s(2k+1)+1}U(s)$$

Note - It is in fact possible to use block diagram reduction methods to solve this problem by first spotting that it can be redrawn as follows (i.e the feedback loop can be moved to the left of the 1/(s+1) block as follows:



Thus from the block diagram:

$$\frac{Y(s)}{U(s)} = \frac{k(s+1)}{(2k+1)s+1}\left[\frac{2}{s+2}+\frac{1}{s+1}\right] = \frac{k(s+1)}{(2k+1)s+1}\left[\frac{3s+4}{(s+2)(s+1)}\right]$$

$$= \frac{3ks+4k}{(s+2)((2k+1)s+1)} = \frac{3ks+4k}{(2k+1)s^2+(4k+3)s+2}$$

The gain of a system is given by:

$$\lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{3ks+4k}{(2k+1)s^2+(4k+3)s+2} = \frac{4k}{2} = 2k$$

Therefore for unity gain $2k = 1 \Rightarrow k = \frac{1}{2}$

$$\Rightarrow G(s) = \frac{1.5s+2}{2s^2+5s+2} = \frac{1.5s+2}{(2s+1)(s+2)}$$