Tutorial Sheet 6 - Stability

Q1 Determine the stability of each of the following systems and comment on the nature of the expected transient responses (i.e. oscillatory or non-oscillatory):

(i) $\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(t)$ (ii) $\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 2 \\ -2 & -3 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}u(t)$

(iii) $\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}u(t)$ (iv) $\boldsymbol{x}(k+1) = \begin{bmatrix} 0.4 & 0.8 \\ -0.4 & 0.2 \end{bmatrix}\boldsymbol{x}(k) + \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}u(k)$

Q2 Determine the conditions on real scalars α and β under which each of the following systems are (a) asymptotically stable, (b) stable and (c) unstable:

(i) $\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 \\ -2 & \alpha \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} \beta \\ 1 \end{bmatrix}u(t)$ (ii) $\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & \beta \\ -2 & -3 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}u(t)$

(iii) $\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -\beta & \alpha \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}u(t)$ (iv) $\boldsymbol{x}(k+1) = \begin{bmatrix} \alpha-1 & 0 \\ -2 & \beta \end{bmatrix}\boldsymbol{x}(k) + \begin{bmatrix} 3 \\ 1 \end{bmatrix}u(k)$

Q3 Consider the mechanical system in figure 1 below, with M = 2 kg, K = 10 N/m and B is the variable damper coefficient.

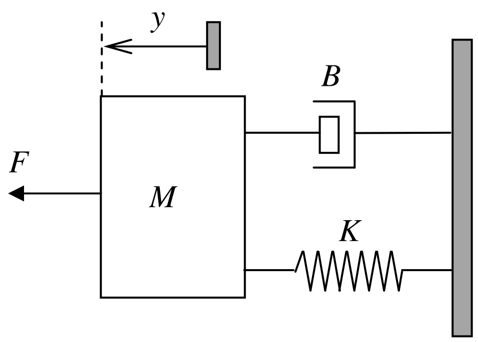

Figure 1: Mechanical system

(i) Determine the state-space model for this system.

(ii) Hence, deduce the range of B for which the system is (a) marginally stable, (b) unstable and (c) asymptotically stable.

Q4 (i) Determine the transfer function for the following systems:

(a) $$\boldsymbol{x}_{k+1} = \begin{bmatrix} 0 & 1 \\ 2 & -2 \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k, \quad y_k = \begin{bmatrix} 2 & 1 \end{bmatrix} \boldsymbol{x}_k + u_k$$

(b) $$\dot{\boldsymbol{x}} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t), \quad y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{x}$$

(ii) Determine the stability of each system directly from the state-space model, and confirm your answer by determining the stability from the equivalent transfer function model.