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Problem Chosen

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2017 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Abstract

With the quality of life improving, the number of the private cars growing rapidly. Then the question comes that the limited capacity of the route segments could not cover the traffic demand. Luckily, a new type of cars named self-driving cars can extend the capacity of the traffic and reduce the effect of congestion. And the self-driving car can be classified as cooperating and non-cooperating. We analyze the effects of allowing self-driving on the roads and discuss the differences between the cases where the self-driving cars are cooperating and non-cooperating.

Firstly, we come up with the concept of value of time (VOT) and recognize that VOT of self-driving cars differs from that of normal cars. We determine the indicator named the generalized total price to measure the cost of the congestion on certain route segments. We present the way to calculate the generalized total price and take it as the objective function with the only constraint condition $0 \leq \theta \leq 1$ to construct an optimism model based on the pure bottleneck model in literature [1].

Then we analyze the differences between cooperating and non-cooperating cars in the micro. We use the simple physical dynamic model to calculate the average safe following distances. Then we obtain the ideal capacity to judge the route segments which have severe congestions in peak hours. We analyze the principle that the self-driving cars affect the capacity of the route segments. As the ratio of the self-driving cars is increasing, the maximum capacity will grow at the same time.

Next, we determine which route segments are of severe congestions in peak hours. We determine the parameters according to literatures as well and then come out the result. We find out the equilibrium of each route segment when self-driving cars are non-cooperating but discover that the generalized total price keeps decreasing when self-driving cars are cooperating. For instance, we find the equilibrium of the self-driving car ratio of the route segment 184 is 80.4%. Other results are presented in the text.

Finally, we discuss about the best ratio of self-driving cars when dedicating certain lanes to self-driving cars with a minimax procedure to reduce the congestion time as much as possible.

The effect of self-driving cars on the traffic flow in the bottleneck model

Team # 62895

May 16, 2017

Abstract

With the quality of life improving, the number of the private cars growing rapidly. Then the question comes that the limited capacity of the route segments could not cover the traffic demand. Luckily, a new type of cars named self-driving cars can extend the capacity of the traffic and reduce the effect of congestion. And the self-driving car can be classified as cooperating and non-cooperating. We analyze the effects of allowing self-driving on the roads and discuss the differences between the cases where the self-driving cars are cooperating and non-cooperating.

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Finally, we discuss about the best ratio of self-driving cars when dedicating certain lanes to self-driving cars with a minimax procedure to reduce the congestion time as much as possible.

Key Words: Bottleneck Model; Self-driving Cars; Cooperating; Capacity; VOT

A Letter to The Governor of the state of Washington

Dear Governor:

Our team has proposed an analysis of the effects of allowing self-driving, cooperating cars on the roads given by the dataset, and use the index of generalized total price developed by Vickery[1969] in the literature[1] which reflects costs on citizens to measure the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%. A mathematical model is built to help predict the total generalized price cost after identifying the different percentage of self-driving cars on road. The optimal strategy is determined by minimizing the total generalized price.

More specifically, the index is developed after taking the value of time wasted on queuing or traveling, the capacity of road and cost of earlier or later arrivals than expected resulted from different queuing time into consideration. It is apparent that applying self-driving cars can significantly affect the three indicators compared with normal cars. What's more, we also regard the automobile cost that a self-driving car differs from a normal car as another important indicator. From this point of view, our definition manages to depict the difference brought by the introduction of self-driving cars.

Then we divided the cars on the roads into three parts as normal cars, self-driving, non-cooperating cars and self-driving cars. We analysize the effect of self-driving ,non-cooperating cars and self-driving cars in the microscopic model and how they influence the traffic capacity. In the both situations, we solve the optimization model of minimizing the index of total generalized price.

Moreover, we think over what if a lane or several lanes is dedicated to these cars in the two situations above. We determine the best ratio of the self-driving cars by a minimax process.

In conclusion, taking the road segments which we believe heavily overloaded during peak hours infered in our model as examples, our model successfully find equilibria for reference when self-cars are non-cooperating, shown as follows:

No. of road segments	Equilibrium	Optimal Value	Optimized/Real
70 *163.48-164.22, IS 5*)	0.812	819375.424	0.9934463
69 (*163.36-164.22, IS 5*)	0.81	792000.59	1.009281
171 (*3.3-3.69, IS 405*)	0.804	417320.352	1.114399
182 (*9.59-9.96, IS 405*)	0.802	366380.9	1.189525
184 (*10.56-10.93, IS 405*)	0.804	441540.918	1.082918

We shoud note that when the road segment no. 70 reaches equilibrium, the optimization solution is unable to solve the problem of heavy traffic completely. When self-driving cars are cooperating, the generalized total price will decrease with respect to f. So, it will always be a good idea to promote self-driving, cooperating cars.

Besides, if few price increases, the generalized total price will increase generally and the equilibrium will reduce when self-driving cars are non-cooperating.

We have a strong belief that our model can effectively account for the inner relationship between self-driving cars and normal cars, and we wish that the model could help you find an approriate solution to traffic jam.

Yours Sincerely,

Team #62895

Contents

1 Introduction

1.1 Backgroud

With the quality of people's life increasing rapidly, the vehicles become more and more prevailing, which results in horrible traffic jams especially during peak hours.

Increasing the number of lanes of the roads is not enough to cover the need to enhance the traffic capacity in many regions of the United States. In order to solve the problem, the government intends to promote the self-driving car to the public. The Governor of the state of Washington has asked for analysis of the effects of allowing self-driving cars on the road.

So why could self-driving cars improve the traffic condition? In fact, self-driving cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. Besides, if those self-driving cars are cooperating, the capacity will be increased substantially, which we will analyze in detail in the text.

1.2 Process of Our Solution

- We first set up the optimization model in accordance with the bottleneck model referred by Vickrey (1969) in the literature [1]. The objective function is the total generalized cost of the travel of certain sections of the roads, measured by the value of travel time losses (VOT) and the value of schedule delay early or late while the only constraint condition is the ratio of the self-driving cars range from 0 to 1.
- Then we divided the cars on the roads into three parts such as normal cars, self-driving non-cooperating cars and self-driving cars. We analyze the effect of self-driving non-cooperating cars and self-driving cars in the microscopic model and how they influence the traffic capacity. In the both situations, we solve the optimization model mentioned in (a) respectively.
- Moreover, we think over what if a lane or several lanes is dedicated to these cars in the two situations above. We determine the best ratio of the self-driving cars by a minimax process.
- Afterwards, we think of the average traffic volume hours.
- At last, we analyze our results and come up with our suggestions.

After such steps, we can set out to construct our model and finally solve the problem(show our approach in detail).

1.3 Assumptions

Since the real situations are so complex, we simplify the model by applying to some brief models. Below are the assumptions we take about the models.

- Regard the whole section of the road as the bottleneck in peak hours.
- In the bottleneck model, it is important to keep the travel cost constant over time before t^* (the time that a car is willing to pass the bottleneck) as well as after t^*.

- The distances between any self-driving car and any normal car are the same. The distances between a self-driving car and any self-driving car are the same. So are the distances between a normal car and any normal car.
- The value of time remains a constant. Meanwhile, the cost of schedule delay early or late is a linear function of the time that you are early or late, respectively.
- When a car pass the end of the bottleneck, the speed of it is the fastest allowing speed.
- Ignore the traffic jam caused by traffic accident.
- Every lanes has equal cars.
- The route segments list don't cross.
- Select some the original lanes as a special lane for self-driving cars.
- Cannot build additional lanes for self-driving cars.
- For human-driving cars should run on the ordinary lanes, not on the special lanes.
- For self-driving cars should run on the special lanes, not on the ordinary lanes

2 The Bottleneck Model: Macroscopic Evaluation of the Effects of Self-Driving Cars

2.1 Initial Analysis

As Vickrey (1969) said in [1], the pure bottleneck situation is one where a route segment has a fixed capacity substantially smaller relative to traffic demand than that of preceding or succeeding segments. We regard the capacity that guarantees the traffic flowing freely as the capacity of traffic demand. Then we determine some route segments that have severe congestion with their average capacity in peak hours much less than the demanding capacity. Then we can regard those route segments as bottlenecks.

With the consideration above, we apply to the Vickrey's bottleneck model (1969) in literature [1] to analyze the influence of the self-driving car. By the literature [2] and [8], we recognize that the self-driving cars improve the condition of the traffic by increasing the traffic capacity of the roads. The specific explanation is in the section 3.

2.2 Nomenclatures

α	Value of time (VOT) for normal car users the cost of an hour of travel time.(dollars/hour)
β	the cost of an hour earlier arrival than the preferred arrival time t^*.(dollars/hour)
γ	the cost of an hour later arrival than the preferred arrival time t^*.(dollars/hour)
δ	Compound preference parameter $\delta \equiv \beta\gamma/(\beta+\gamma)$. (dollars/hour)
θ	VOT reduction parameter for an robot car: the VOT for an robot car is $\theta\alpha$ with,$\beta/\alpha < \theta < 1$.
C_a	Travel cost for a self-driving car.(dollars)
C_n	Travel cost for a normal car.(dollars)
f	The ratio of self-driving cars on road.
N	The sum of self-driving cars and normal cars.
N_i	i in {a,n}, N_i is the number of type i cars. $N = N_i + N_n$
Q	Queuing time for cars.(hours)
r[f]	Function determining the effective bottleneck capacity for robot cars of s/r[f].
s	Bottleneck capacity for a normal car.(per hour)
t	Arrival time.(hours)
t^*	Preferred arrival time.(hour)
t_{ae}	Moment that the last self-driving car arrives and thus the peak ends.(hour)
t_{as}	Moment that the first self-diving car arrives and thus the peak starts.(hour)
t_e	Moment that the last car arrives and thus the peak ends.(hour)
t_s	Moment that the first car arrives and thus the peak starts.(hour)
TT_{ff}	Free-flow travel time.(hours)
TTC	Total travel cost of $N \times (f \times Ca + (1-f) \times Cn)$.(dollars)

2.3 Explanation of the Parameters

As is known to all, time is precious. We measure the time wasted on queuing or traveling of a normal car by the value of time (VOT), denoted by α. However, when oneâĂŹs car is self-driving, he can use the waiting time to do some other meaningful things rather than pay attention on driving, like reading or doing with some hand-on job. Then we consider that the value of queuing or traveling time of a self-driving car is less than that of a normal car. Meanwhile, we denote the ratio of them by θ, with $0 \leq \theta \leq 1$.

Assume that the time you prefer to pass the end of the bottle neck is t^*, if you arrive earlier or later than t^*, there will be a shadow cost. We denote the shadow cost per hour for arrivals earlier or later than t^* is β or γ, respectively.

According to 3.1, we denote the capacity of bottleneck with respect to the normal cars by s while the capacity of bottleneck with respect to the self-driving cars by $s/r(f)$, where f is the ratio of the self-driving cars and r(f) is the ratio of the capacity of normal cars and that of self-driving cars when the ratio of self-driving cars is f. Apparently, we have $0 \leq f \leq 1$.

2.4 Three reduced models

2.4.1 The classic bottleneck model when there are only normal cars passing the bottleneck

We now briefly reintroduce the classic bottleneck and apply it to the problem we are discussing about. An important assumption of the model is that the travel cost of each car when the cars are all homogeneous in the model should be constant over time as long as arrivals occur. The assumption is reasonable for the reasons that can be referred to [1] or [2].

Then we indicate that the travel cost (C) of a car is the free-flow travel cost and the cost of the bottleneck (CB).

$$C_i = CB_i + \alpha TT_{ff},$$

where the index i is in a, n with a standing for self-driving cars while n for normal cars. Besides, TT_{ff} stands for the time that car spends in passing the section of the road with no congestion. Since the cars are all normal cars, then the TC_n are the same whenever the car passes the end of the bottleneck. Then we can derive the equation below:

$$\begin{cases} -\beta(t_s - t^*) + \alpha TT_{ff} = Q\alpha + \beta(t^* - t) + \alpha TT_{ff},\ if\ \ t < t^* \\ -\gamma(t_e - t^*) + \alpha TT_{ff} = Q\alpha + \gamma(t - t^*) + \alpha TT_{ff},\ if\ \ t \geq t^* \end{cases}$$

where t_s stands for the moment that the first car arrive the end while t_e stands for that of the last car. Notice that the queuing time Q of the first and the last car is zero since they stand for the start and the end of the congestion, respectively. Therefore, the left of the equation are the travel cost of the first car and the last car, respectively. Meanwhile, the right of the equation present the cost of a car arriving at the moment t.

Solve the equation, we get the expression of Q against t:

$$\begin{cases} Q = \dfrac{\beta}{\alpha}(t - t_s),\ if\ \ t < t^* \\ Q = -\dfrac{\gamma}{\alpha}(t - t_e),\ if\ \ t \geq t^* \end{cases}$$

We plot Q against t:

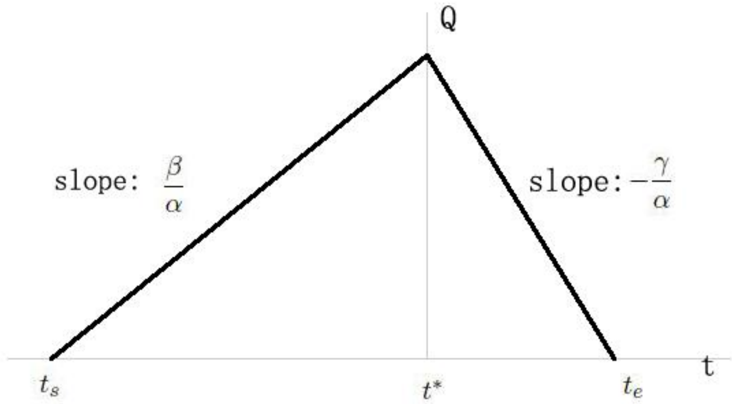


Figure 1: The Plot of Queuing Time against Arrivals

Referring to the literature [6] and citing the equation (4) and (5) in the paper, we get the expression of t_s and t_e:

$$t_s = t^* - \frac{N\gamma}{(\beta+\gamma)s}$$
$$t_e = t^* + \frac{N\beta}{(\beta+\gamma)*s}$$

Where N is the total number of the cars passing the bottleneck during the peak hours.

Then the travel cost of a car is:

$$C_n = \beta(t^* - t_s) + \alpha TT_{ff} = \gamma(t_e - t^*) + \alpha TT_{ff} = \frac{\delta N}{s} + \alpha TT_{ff},$$

where $\delta = \frac{\beta * \gamma}{\beta + \gamma}$.

2.4.2 Assume that self driving cars do not change the capacity of the road but only change VOT

From the analysis above, we get the VOT of the self-driving cars is

$$\alpha_a = \theta\alpha$$

Substitute α_a with α in Model 2.4.1, we get:

$$\begin{cases} Q = \dfrac{\beta}{\alpha\theta}(t - t_{as}),\ if\ \ t < t^* \\ Q = -\dfrac{\gamma}{\alpha\theta}(t - t_{ae}),\ if\ \ t \geq t^* \end{cases}$$

where t_{as} stands for the moment that the first self-driving car arrive the end while t_{ae} stands for that of the last self-driving car.

Still, here we derive that

$$t_{as} = t^* - \frac{N\gamma}{(\beta+\gamma)*s}$$
$$t_{ae} = t^* + \frac{N*\beta}{(\beta+\gamma)*s}$$

Simultaneously, we plot the queuing time of normal cars and self-driving cars against the arrival t on the same coordinate system:

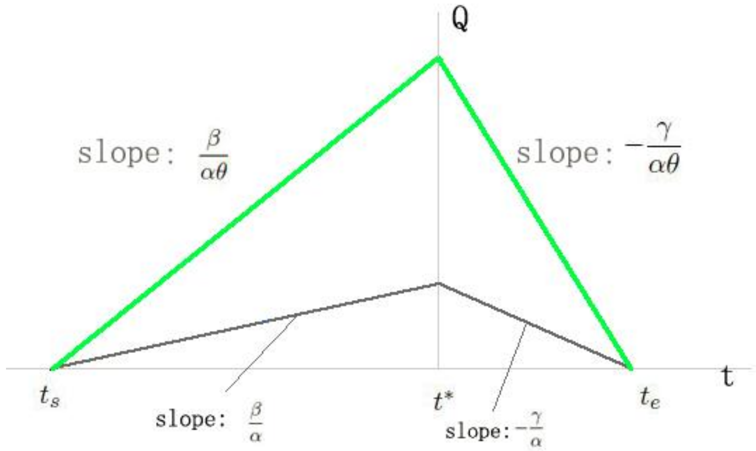


Figure 2: Queuing Time against Arrivals when Self-Driving Cars only Affect VOT

where the black line stands for the situation that all the cars are normal cars while the green line stands for that of self-driving cars. Here we compute the travel cost of a self-driving car:

$$C_a = \delta\frac{N_a}{s} + \alpha\theta TT_{ff}$$

From the plot above, we can see that self-driving cars will extend waiting time although they reduce the value of time wasting on traveling.

When the types of cars are mixed with the ratio of the self-driving cars f, we can derive the travel cost of normal cars and self-driving cars, referring to the literature [7] and [9]:

$$C_a = \delta_a \frac{N_a + \frac{\alpha_a}{\alpha_n} N_n}{s} + \alpha\theta TT_{ff} = \delta \frac{N*f + \theta N(1-f)}{s} + \alpha\theta TT_{ff}$$
$$C_n = \delta_n \frac{N_a + N_n}{s} + \alpha TT_{ff} = \delta \frac{Nf + N(1-f)}{s} + \alpha TT_{ff} = \delta \frac{N}{s} + \alpha TT_{ff}$$

2.4.3 Assume that self driving cars only change the capacity of the road but do not change VOT

From the analysis above, we know that the self-driving cars can increase the capacity of the roads. Especially when the ratio of the self-driving car f, the capacity will be s/r(f).

Firstly, we compare the two circumstances where there are only self-driving cars and where there are only normal cars. When there are only self-driving cars, we can simply substitute

$$s_a = \frac{s}{r(1)}$$

with s in Model 2.4.1.
Then we get the results:

$$\begin{cases} Q = \dfrac{\beta}{\alpha}(t - t_{as}),\ if\ \ t < t^* \\ Q = -\dfrac{\gamma}{\alpha}(t - t_{ae}),\ if\ \ t \geq t^* \end{cases}$$

where

$$t_{as} = t^* - \frac{N\gamma}{(\beta+\gamma)s} r(1);$$
$$t_{ae} = t^* + \frac{N\beta}{(\beta+\gamma)s} r(1).$$

Simultaneously, we plot the queuing time of normal cars and self-driving cars against the arrival t on the same coordinate system in figure 3, where the black line stands for the situation that all the cars are normal cars while the green line stands for that of self-driving cars.We calculate the travel cost:

$$C_a = \delta r(1) \frac{N}{s} + \alpha TT_{ff}.$$

From the expression above we can infer that the cost of N_a self driving cars passing the bottleneck is equivalent to the cost of $N'_a = N_a * r(1)$ self-driving cars with the capacity s passing the bottleneck. This thought is vital for the derivations below.

Then we come to the situation that there are heterogeneity types of the cars with the ratio of the self-driving cars f.

In accord with the thought above, the situation suffices to that there are $N(1-f)$ normal cars and $N'_a = N * f * r(f)$ self-driving cars with the capacity s passing the bottleneck. However, the two types of the normal cars must be regarded to be heterogeneous although the capacity of the road to them are both s.

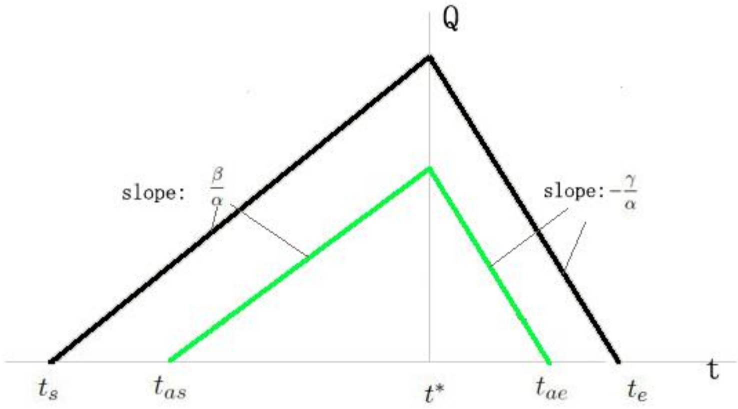


Figure 3: Queuing Time against Arrivals when Self-Driving Cars only Affect Capacity

Then as the literature [9] shows, we can compute the cost of a normal car or a self-driving car:

$$C_a = \delta_a \frac{N_a' + N_n}{s} + \alpha TT_{ff} = \delta \frac{Nfr(f) + N(1-f)}{s} + \alpha TT_{ff}$$
$$C_n = \delta_n \frac{N_a' + N_n}{s} + \alpha TT_{ff} = \delta \frac{Nfr(f) + N(1-f)}{s} + \alpha TT_{ff} = C_a$$

2.5 The full optimization model

Now we consider what if self driving cars not only change the capacity of the road but also change VOT. Based on the three reduced models, the analyses and derivations of the full model is relatively brief. For instance, we directly consider of the situation that there are heterogeneity types of the cars with the ratio of the self-driving cars f. So it can easily suffices to that there are $N(1-f)$ normal cars and $N_a = Nfr(f)$ self-driving cars with the capacity s passing the bottleneck. Now the difference between the normal cars and the $N_a = Nfr(f)$ so-called self-driving cars is only up to VOTs of them.

Then as the literature [9] shows, we can compute the cost of a normal car or a self-driving car:

$$C_a = \delta_a \frac{N_a' + \frac{\alpha_a}{\alpha_n} N_n}{s} + \alpha\theta TT_{ff} = \delta \frac{Nfr(f) + N\theta(1-f)}{s} + \alpha\theta TT_{ff}$$
$$C_n = \delta_n \frac{N_a' + N_n}{s} + \alpha TT_{ff} = \delta \frac{Nfr(f) + N(1-f)}{s} + \alpha TT_{ff}$$

Then we calculate the total travel cost, which is the cost of all the cars (TTC): $TTC = N_a C_a + N_(n) C_n$Simplify the expression we get:

$$TTC = N(\delta \frac{N}{s}(r(f)f + (1-\theta)f^2 - 2f + \theta f + 1) + \alpha TT_{ff}((1-f) + f\theta))$$

2.6 The total generalized price optimism model

In the literature [2] points out that we considerations about the total travel cost is not enough, we ought to think of the automobile cost that a self-driving car differs from a normal car, denoted by MC_a. Generally, MC_a is positive according to [2]. Then the total generalized price(p) can be denoted as the total travel cost (TTC) adding to the total automobile cost. $P = TTC + MC_a Nf$ Finally, we take P as the objective function and would like to minimize it with the only constraint condition $0 \le f \le 1$.

Finally, we take TTC as the objective function and would like to minimize it with the constraint condition $0 \le f \le 1$.

3 Microscopic Model of Cars On Road: Ideal capacity S and $r[f]$ calculation

3.1 Assumptions of the model

Assuming that:

- the cars are evenly distributed on the road moving at the same speed(Vkm/h)
- Self-driving,cooperating cars:which have the sensors.
- Non self-drivingcars: which don't have sensors and cooperating devices.

Consider self-driving , cooperating cars into two models.

Model1(M1): Each self- driving car doesn't have cooperating devices.The self-driving cars modulates their own deceleration and don't communicate with other self-driving cars.

Model2(M2): All self-driving cars have cooperating devices can only communicate with their neighboring vehicles.

3.2 Nomenclatures

Ts	the delay until a car with sensors detects an emergency situation
Tc	the perception-reaction time of all the cooperating cars until its brake is automatically applied.
a_s	the deceleration rate which each self-driving car owns in model 1.
a_c	the deceleration rate which each self-driving , cooperating car owns in model 2.
a_{min}	the minimum deceleration rates for all the cars on the road.
a_{max}	the maximum deceleration rates for all the cars on the road.
D_m	the average safe following distance that Non self-driving cars maintain to their preceding cars.
D_s	the average safe following distance that self-driving cars maintain to their preceding cars in M1.
D_c	the average safe following distance that cooperating cars maintain to their preceding cars in M2.
V	the velocity of the cars on the road.
N	Average daily traffic counts Year 2015
N_D	Number of Lanes DECR MP direction
N_I	Number of Lanes INCR MP direction

3.3 The description of the Model

Due to the complexity of the mixed traffic system, we introduce the average safe distance($\bar{D}$) to simplify the model.

3.3.1 Model 1(M1): Each self- driving car doesn't have cooperating devices

Each self- driving car doesn't have cooperating devices. The average safe following distance is calculated as:

$$\bar{D} = (1-f)D_m + fD_s$$

Where f is the proportion of the self-driving cars of the total number of cars. The average safe following distance D_m,D_s are as follows.

- D_m
  As to [11] , the time gaps that human drivers maintain with their proceding cars are assumed to be distributed according to Gaussian distribution with a mean of 1.1s and standard deviation of 0.15s. And these values are taken from a statistical analysis of the UMTRI ACC FOT baseline case human driving data. Thus, we adapt them as the human driver reaction time until they apply the brakes. And D_m is calculated as follows:

  $$D_m = \frac{1.1V}{3.6}$$

  The constant 3.6 is to convert car velocity V from km/h to m/s

- D_s
  Assuming that the deceleration rate a_s of self-driving cars is distributed according to uniform distribution[a_{max},a_{min}] according to [8]. The average safe following distance of self-driving cars is calculated as followed:

  $$\begin{aligned} D_m &= \int_{|a_{min}|}^{|a_{max}|} \left(\frac{T_s V}{3.6} + \frac{V^2}{25.96 a_s} - \frac{V^2}{25.96|a_{max}|}\right) f(a_s) da_s \\ &= \frac{T_s V}{3.6} + \frac{V^2 ln(\frac{|a_{max}|}{|a_{min}|})}{25.92(|a_{max}|-|a_{min}|)} - \frac{V^2}{25.92|a_{max}|} \end{aligned}$$

  The term $25.92|a_s|$ are from $2|a_s|$ times 12.96, which is the constant to convert V^2 from $(km/h)^2$ to $(m/s)^2$. $f(a_s)$is the probability density function of a_s, which denotes as

  $$f(a_s) = \frac{1}{|a_{max}|-|a_{min}|}$$

3.3.2 Model 2(M2): All self-driving cars have cooperating devices

If all self-driving cars have cooperating devices, the average safe following distance is calculated as:

$$\bar{D} = (1-f)D_m + fD_c$$

f is the proportion of the self-driving cars of the total number of cars. The average safe following distance D_m, D_c are as follows.

- D_m
  D_m is the same as the D_m in model 1.

- D_c
All self-driving carswithcooperating devices can only cooperate their neighbors. So there are 3 cases we must consider. And the average safe following distance that self-driving, cooperating cars maintain :

$$D_c = P_1 D_{c1} + P_2 D_{c2} + P_3 D_{c3}$$

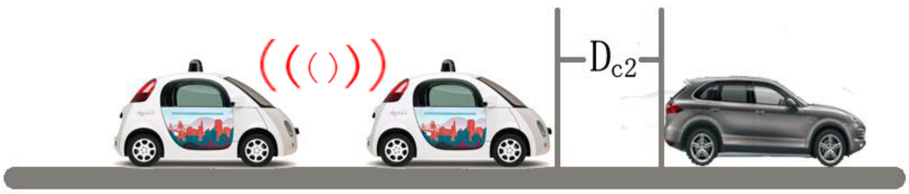
Where P_1,p_2 and p_3 are the probabilityof that case1, case2 and case3 respectively. D_{c1}, D_{c2} and D_{c3} are the average safe following distance that self-driving, cooperating cars maintain in case1, case2 and case3.

Case 1: **Neither the preceding nor following car can cooperate**

In this case, P_1 is $(1-f)^2$. And $D_{c1}$is same as D_s in model1 as follows:

$$D_{c1} = D_s = \frac{T_s V}{3.6} + \frac{V^2 ln(\frac{|a_{max}|}{|a_{min}|})}{25.92(|a_{max}|-|a_{min}|)} - \frac{V^2}{25.92|a_{max}|}$$

Case 2: **The preceding car cannot cooperate, but the following car can**



In this case,P_2 is $f*(1-f)$. D_{c2} is calculated as follows:

$$D_{c2} = \frac{TsV}{3.6} + \int_{|a_{min}|}^{|a_{max}|} f(a_c)(\frac{V^2}{25.92a_c})da_c - \frac{V^2}{25.92|a_{max}|}$$
$$= \frac{T_s V}{3.6} - \frac{nV^2}{25.92(|a_{max}|-|a_{min}|)^n} \int_{|a_{min}|}^{|a_{max}|} \frac{(|a_{max}|-a_c)^{n-1}}{a_c} da_c$$

Where $f(a_c)$ is the probability density function of the absolute value of the negotiated deceleration rate a_c related to the average number of self-driving, cooperating cars n, which is shown as

$$f(a_c) = (\frac{n}{|a_{max}|-|a_{amin}|}) * (\frac{|a_max|-a_c}{a_{max}|-|a_{amin}|})$$

According to[8], if f<1, the n=(2-f)/(1-f), while if f=1, then n=$\inf$ and $a_c$is equal to a_{min}. So, $D_{c2} = (T_s V/3.6) + [V^2/(25.92|a_{min}|)] - [V^2/(25.92|a_{max}|)]$

Case 3: **The preceding car can communicate**

In this case, P_3 is f. D_{c3} is calculated as follows:

$$D_{c3} = \frac{T_c V}{3.6}$$

3.4 Capacity Calculation

According [12], The capacity is defined as the maximum hourly rate at which persons or cars reasonably can be expected to traverse a point or a uniform section of a lane or roadway during a given time period under prevailing roadway, traffic, and control conditions. From [8], the capacity (cars/hour/lanes)can be estimated as follow:

$$S = 3600V/[3.6(l + \bar{D})]$$

Where $l(m)$ is the average vehicle length and $\bar{D}$ is the average safe following distance.

3.5 r(f) Calculation

According to the process of $\bar{D}$ calculation in section 4.3, $\bar{D}(f)$ is the function with respect to f, so does $S(f)$. $r(f)$ is calculated as follow:

$$r(f) = \frac{S(0)}{S(f)}$$

Where $S(f) = 3600V/3.6(l + \bar{D}(f))$.

3.6 Result

Here are the values of the parameters we assume. a_{min} and a_{max} are $-5m/s^2$ and $-8.5m/s^2$ referring to [10]. the time gaps that human drivers maintain is 1.1s according [11], T_s is 0.245s as in[16], T_c is 0.181s as in [18]. The average car length l is 4.3m referring to [17].

We use the value we obtain above to calculate the ideal capacities in different velocities without self-driving cars and plot the results by R as follows:

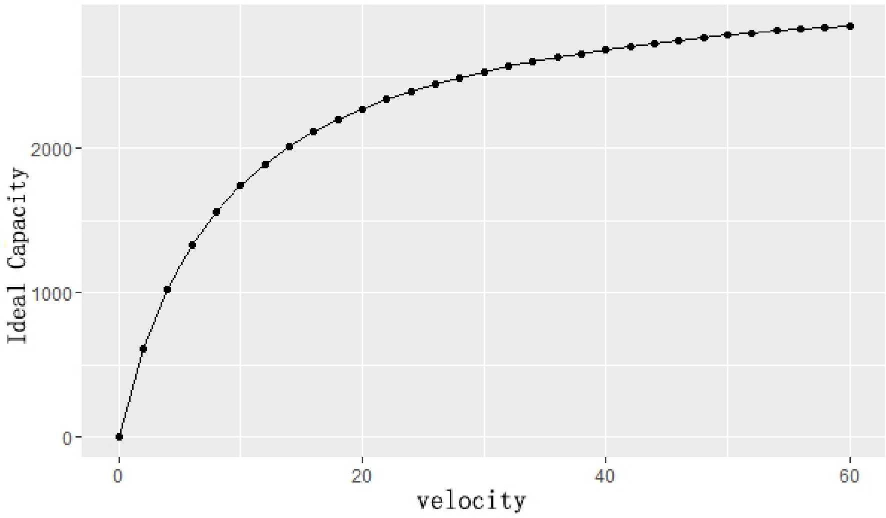


Figure 4: Ideal Capacity Without Self-Driving Cars

As we can see, the higher velocity is, the larger capacity is. So we define the capacity in 60 miles/h as the maximum capacity of the road. From the results, the maximum capacity of the road is 2856.4315 with 100% human driving cars.

Now we change the proportion of self-driving cars to plot the capacity against the velocity in M1 and M2 respectively as follow:

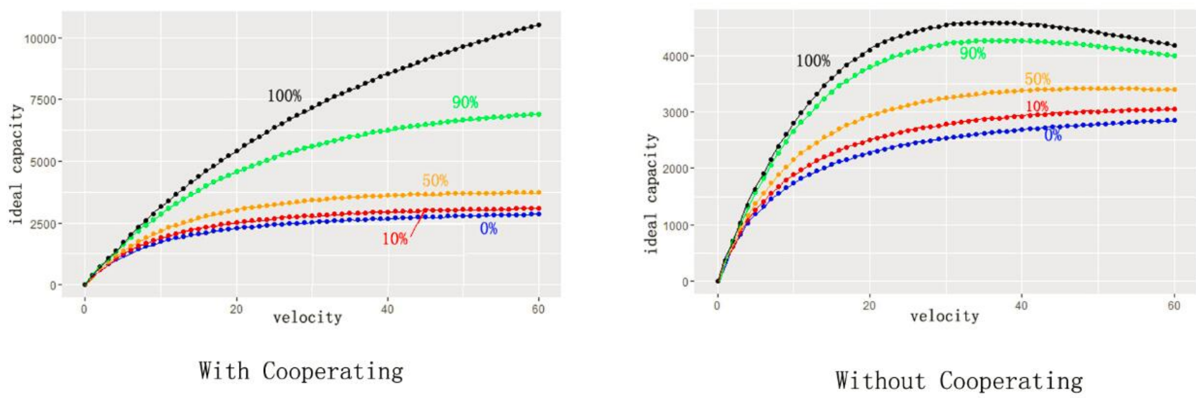


Figure 5: Capacity of 10%, 50%, 90% Self-Driving Cars

As we can see, at the same level velocity, the capacities of 10% , 50%, 90% self-driving cars are increasing whether it's in M1 or M2 or not. When with cooperating (M2) , different proportions of self-driving cars roughly have the same tendency while velocity changing. When without cooperating (M1), different proportions of self-driving cars roughly have the same tendency except 90% and 100% self-driving cars.

4 Analysis of Result

4.1 Determination of the severe congestion

We use the maximum capacity of the road to judge which road exceeds the the maximum capacity at peak and we regard them as the overloaded road. Now we calculate the real capcity on rush hour as follow:

$$S_r = N0.08/(N_D + N_I)$$
$$\Delta S = 2856.4315 - S_r$$

When ΔS is larger than 0, the route segment is overloaded. We plot ΔS against all the route segmen as follow:

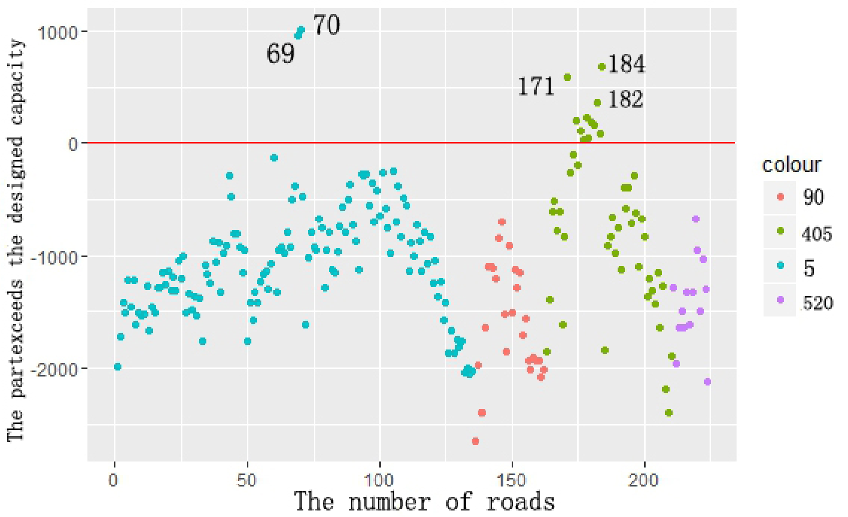


Figure 6: Comparison between real capcity and maximum capacity without self-driving car

According to the plot, we can obersve overloaded roads. We choose the most severe five segments for analysis, which are 69, 70, 171, 182 and 184.

4.2 Determination of the parameters

For the USA, we determine VOT as 18.82 dollars/h, recommended by [13] and [14]. Then we take the ratios β/α and γ/α as the values 39/64 and 1521/640, respectively, according to [15], suggested by [2]. In the peak hours, we use the given background information and let the total number of the cars passing the bottleneck be 8% the daily traffic volume of the route segments. The value of MC_a is 1.13 dollars, derived by [2].

4.3 Results Analysis

4.3.1 The results of the model

For instance, as for the routes segment 184, we minimize the generalized total price with the accuracy in f being 0.002. We plot the generalized total price (P) against (100f), then we get the diagram below. If non-cooperating:

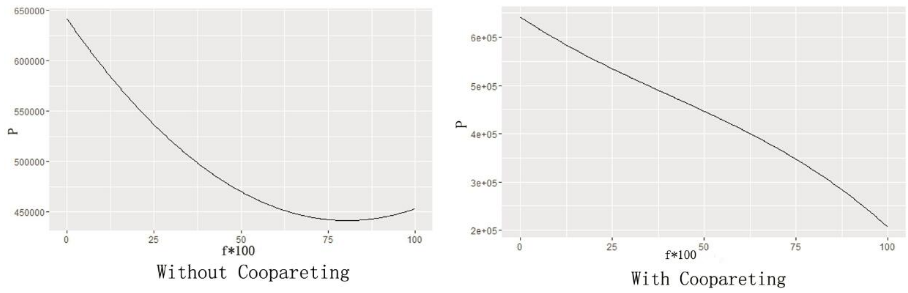


Figure 7: The generalized total price against 100f

The case where self-driving cars are non-cooperating: The output shows when the ratio $f = 0.804$ the generalized total price goes to the minimum value $P = 441540.92$ dollars. If $\theta < 0.388$, the function will be a valley function. Else, itâĂŹs decreasing with f range from 0 to 1. We denote the critical point as θ^* The case where self-driving cars are cooperating: No matter how θ changes, the $P_\theta(f)$ is concave and decreasing. Hence, we determine the equilibrium of the ratio of self-driving cars with respect to making the generalized total price least. In the same way, we get the best values of the other four route segments, respectively. The results are shown in the table below.

As we can see, the last column donates that the capacities corresponse to crital points is larger than their real capacities except the NO. 70 route segment. For the NO. 70 route segment, we can use larger or smaller f value to guarantee to enlarge the capacity larger than real one, although sacrifice for a little cost.

4.4 Sensitivity with the parameter θ

We still take the route segment 184 for instance, letting θ vary from 0 to 1 by the step 0.1. Then we get the plots of P against t when θ valued 0.1, 0.5 and 0.9: when cooperating 5.4.3

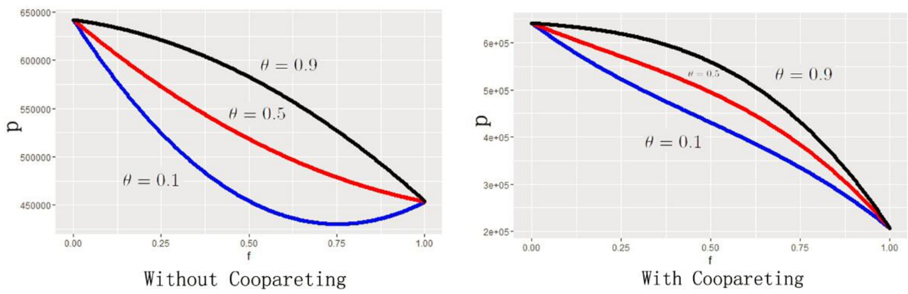


Figure 8: Plots of P against t when θ valued 0.1, 0.5 and 0.9

The explanation of the result of sensitivity with the parameter θ We can see from the plot that

Table 1: Analysis of Non-cooperating

No. of road segments	θ^*	Critical Point	Optimal Value	Optimized/Real
70	0.381	0.812	819375.424	0.9934463
69	0.383	0.81	792000.59	1.009281
171	0.389	0.804	417320.352	1.114399
182	0.391	0.802	366380.9	1.189525
184	0.388	0.804	441540.918	1.082918

when the parameter θ growing, the function $P_\theta(f)$ become more convex and monotonous. The reason of the phenomenon is that the queuing time decreases with θ increasing, which has been pointed out in section 2.4.2. Then when θ is small, the effect of the queuing time delaying weighs a lot while the extension of the capacity counts less. So with f increasing over certain level, P will not decrease but increase. While if θ is relatively large, then the effect that self-driving cars delay the queuing time is impacted while the effects of the extension of the capacity becomes significance, especially cooperating case where $r(f)$ is concave and decreasing. Then $P_\theta(f)$ decreases faster and faster with f growing large.

4.5 The sensitivity of the parameter MC_a

Taking route segments 184 for instance, we plot the generalized total price against f with $\theta = 2$ fixed and MC_a ranging from 1 to 4, we get the plot below:

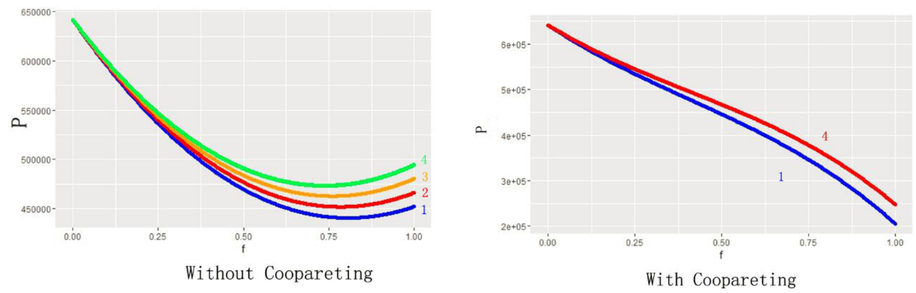


Figure 9: Plots of P against t when θ valued 0.1, 0.5 and 0.9

From the plot, if the self-driving car is non-cooperating, then we can infer that with MC_a increasing, 5he equilibrium of f decreases while the minimum cost increasing. Notice that MC_a mainly stands for the fuel cost and the price of the self-driving cars, according to [2]. The real meaning of the phenomenon is that with price of the fuel or the self-driving car growing, the minimum general total cost will be necessary much more while the demand of the ratio of the self-driving car decreases. Besides, if cooperating, the general cost increases with no equilibrium.

4.6 Discussion about designing the special lane for self-driving lane

We select some the original lanes as a speciallane for self-driving cars and cannot build additional lanes for self-driving cars. Human-driving cars should run on the ordinary lanes, not on the special lanes. While self-driving cars should run on the special lanes, not on the ordinary lanes. As in [6], we center the arriving time t^* to zero, Thus t_a and t_b as follow:

$$\begin{gathered} t_a = -\gamma \frac{N}{(\gamma+\beta)S_0} \\ t_b = -\beta \frac{N}{(\gamma+\beta)S_0} \\ \Delta t = t_b - t_a = \frac{N}{S_0} \\ \Delta t = \frac{N(1-f)/(n-n_1)}{S_0} \\ \Delta t' = \frac{Nfr(1)/n_1}{S_0} \end{gathered}$$

Where S_0 is 2856.4315, N is the capacity of one direction road , n is the number of lanes in one direction road. n_1 is the number of lanes that are changed into special lanes. $r(1)$ is 0.683 in

M1 and 0.271 in M2(Section 3.5).Δt is the whole congestion time about human-driving cars per lane. $\Delta t'$ is the whole congestion time about self-driving cars per lane.

When we separate the two types of cars on the road. We want to the longest congestion time among these of all the lanes be a smallest value. Now we should optimize the objective function as follow :

$$\min \max \{\Delta t, \Delta t'\}$$

Then we obtain f against n and n_1:

$$f = \frac{n_1}{nr(1) - n_1 r(1) + 1}$$

According to the f function, we use it to calculate the best f of different numbers of special lane. And we applied the designing of special lane for the top five sever congestion route segment to compute their own congestion times in two case.

Table 2: f-value (percentage of self-driving car)

f-value	One-special-lane	Two-special-lane	Three-special-lane	Four-special-lane
Two-lane-road				
Non-cooperating	0.5941834			
cooperating	0.7868957			
Three-lane-road				
Non-cooperating	0.4226607	0.7454392		
cooperating	0.6486628	0.8807405		
Four-lane-road				
Non-cooperating	0.3279822	0.5941834	0.8145573	
cooperating	0.5517395	0.7868957	0.9172022	
Five-lane-road				
Non-cooperating	0.267958	0.4939557	0.6871335	0.8541566
cooperating	0.480015	0.7111239	0.8470669	0.9365891

Table 3: Congestion Time(h)

DECR.MP.direction	One-special-lane	INCR MP direction	One-special-lane	Two-special-lane
70		70		
Non-cooperating	1.1001992	Non-cooperating	0.51760008	2.34781528
cooperating	0.57774184	cooperating	0.24249112	1.42875216
69		69		
Non-cooperating	1.0820144	Non-cooperating	0.50904472	2.3090084
cooperating	0.5681924	cooperating	0.23848304	1.4051364
171		171		
Non-cooperating	0.9774496	Non-cooperating	0.9774496	
cooperating	0.51328304	cooperating	0.51328304	
182		182		
Non-cooperating	0.9149384	Non-cooperating	0.9149384	
cooperating	0.4804568	cooperating	0.4804568	
184		184		
Non-cooperating	1.005864	Non-cooperating	1.005864	
cooperating	0.528204	cooperating	0.528204	

5 Conclusion

5.1 Strengths

- The model carefully analysizes how the proportion of self-driving cars affects road capacity.
- We apply the pure Bottleneck model, which is classic and universal.
- We thoroughly analysize the difference between cooperating car and uncooperating in principle.

5.2 Drawbacks

- The result can be very complex if applied to all road segments.
- Some parameters have great sensitivity while some are subjective and hard to estimate.

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Appendices

Appendix A First appendix

some text...

Here are simulation programmes we used in our model as follow.

Input R source:

```
Dbar<-function(f, Mv, amax = 8.5, amin = 5,Ts = 0.245, Tc = 0.181){
  Mv = Mv*1.609344
  if(f<1){
    n = (2-f)/(1-f)
    integrand = function(x) {(amax-x)^(n-1)/x}
    q = integrate(integrand, lower = amin, upper = amax)
    Dm = 1.1*Mv/3.6
    Ds = (Ts*Mv/3.6)+(Mv^2*log(amax/amin))/(25.92*(amax-amin))-Mv^2/(25.92*amax)
    Ds1 = Ds
    Ds2 = Ts*Mv/3.6-Mv^2/(25.92*amax)+n*Mv^2/(25.92*(amax-amin)^n)*q$value
  }else{
    Dm = 1.1*Mv/3.6
    Ds = (Ts*Mv/3.6)+(Mv^2*log(amax/amin))/(25.92*(amax-amin))-Mv^2/(25.92*amax)
    Ds1 = Ds
    Ds2=Ts*Mv/3.6+Mv^2/(25.92*amin)-Mv^2/(25.92*amax)
  }
  Ds3 = Tc*Mv/3.6
  D2 = (1-f)^2*Ds1+(1-f)*f*Ds2+f*Ds3
  return(f*D2+(1-f)*Dm)
}


CDbar<-function(f, Mv, amax = 8.5, amin = 5,Ts = 0.245, Tc = 0.181){
  Mv = Mv*1.609344
  Dm = 1.1*Mv/3.6
  Dc = Tc*Mv/3.6
  return(f*Dc+(1-f)*Dm)
}


NDbar<-function(f, Mv, amax = 8.5, amin = 5,Ts = 0.245, Tc = 0.181){
  Mv = Mv*1.609344
  Dm = 1.1*Mv/3.6
  Ds = (Ts*Mv/3.6)+(Mv^2*log(amax/amin))/(25.92*(amax-amin))-Mv^2/(25.92*amax)
  return(f*Ds+(1-f)*Dm)
}


S <-function(f,Mv, state, amax = 8.5, amin = 5,Ts = 0.245, Tc = 0.181,l = 4.3){
  source("Dbar.R")
  source("NDbar.R")
  source("CDbar.R")
  if (state == 1)
  {s = 3600*Mv*1.609344/(3.6*(l+Dbar(f,Mv,amax, amin,Ts, Tc)))
  }
  if (state == 0){
    s = 3600*Mv*1.609344/(3.6*(l+NDbar(f,Mv,amax, amin,Ts, Tc)))
  }
  if (state == 2){
    s = 3600*Mv*1.609344/(3.6*(l+CDbar(f,Mv,amax, amin,Ts, Tc)))
```

```
  }
  return(s)
}

library(ggplot2)
dat=read.csv("2017_MCM_Problem_D_Data.csv")


###########
theta=0.2
t=184##źńÂůţÄśàžÅ
Mv=1.609344*60###ËÙűÈ
x=1###ÓĐÎÞ¡żżě
N=dat$Average.daily.traffic.counts.Year_2015*0.08
y=4####automobile cost

#############
Ts=0.245;Tc=0.181
D=29.50464
alpha=18.82;l=4.3;r=1521/640*alpha;beta=39/64*alpha;L=dat$endMilepost-dat$startMilepost
S0=3600*Mv/(3.6*(l+D))
delta=beta*r/(beta+r);TTff=L/Mv
lanes=dat$Number.of.Lanes.DECR.MP.direction+dat$Number.of.Lanes.INCR.MP.direction
n=N/lanes

#####################

f=seq(0,1,0.002)
SS=rep(0,length(seq(0,1,0.002)))
i=1
for (f in seq(0,1,0.002)) {
  SS[i]=S(f,60,x)
  i=i+1
}
f=seq(0,1,0.002);f2=f^2
rr=S(0,60,x)/SS
p<-lm(rr~f+f2)

rr=p$coefficients[1]+p$coefficients[2]*f+p$coefficients[3]*f2
Cn=delta*N[t]/S0*(1-f)+delta*N[t]*(1/S0)*rr*f+alpha*TTff[t]
Ca=delta*N[t]/S0*theta*(1-f)+delta*N[t]*(1/S0)*rr*f+theta*alpha*TTff[t]

##############
W=((1-f)*Cn+f*Ca)*N[t]+N[t]*f*y
qplot(f*100,W,geom = c("line"))

############
result2<-min(W)
result1<-(order(W)[1]-1)/500
c(result1,result2)
```