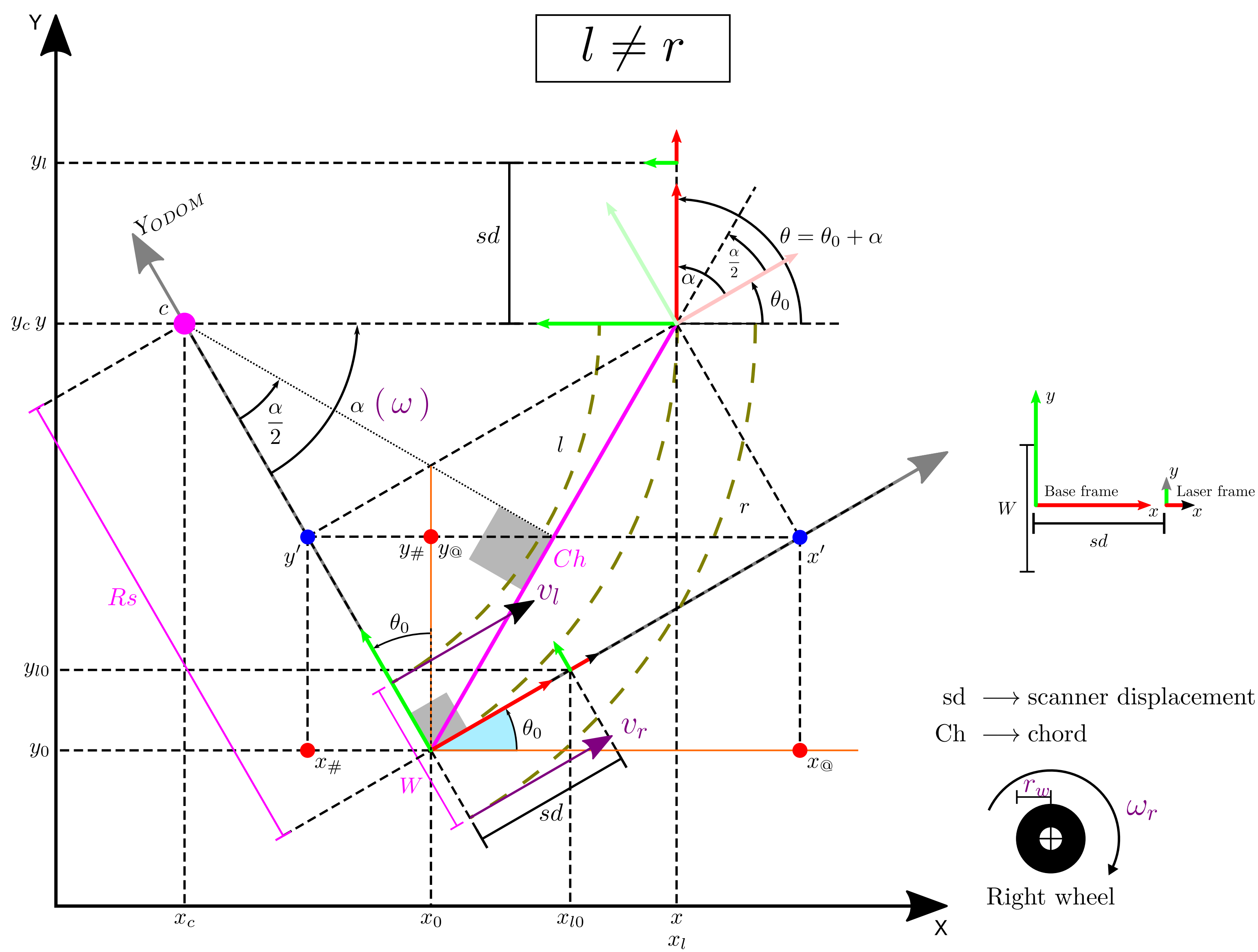
Robot's coordinates in the global reference frame



Given:

the robot's initial pose, (x_0, y_0, θ_0), and the motion commands, (l, r).

$$\begin{aligned}
\alpha &= \omega\, t \\
r &= \left(Rs + \frac{W}{2}\right) \alpha \\
l &= \left(Rs - \frac{W}{2}\right) \alpha \\
\dot{r} &= \left(Rs + \frac{W}{2}\right) \dot{\alpha} \longrightarrow v_r = \left(Rs + \frac{W}{2}\right) \omega \\
\dot{l} &= \left(Rs - \frac{W}{2}\right) \dot{\alpha} \longrightarrow v_l = \left(Rs - \frac{W}{2}\right) \omega
\end{aligned}$$

$$\begin{aligned}
\omega &= \frac{v_r - v_l}{W} \\
Rs &= \frac{v_r + v_l}{2\,\omega} = \frac{W}{2}\,\frac{v_r + v_l}{v_r - v_l}
\end{aligned}$$

Distance travelled by each wheel:

$$\begin{aligned}
D_r &= r_w\, \phi_r \\
D_l &= r_w\, \phi_l
\end{aligned}$$

$$\begin{aligned}
\dot{D}_r &= r_w\, \dot{\phi}_r \longrightarrow v_r = r_w\, \omega_r \\
\dot{D}_l &= r_w\, \dot{\phi}_l \longrightarrow v_l = r_w\, \omega_l
\end{aligned}$$

$$\begin{aligned}
\omega &= \frac{r_w}{W}\,(\omega_r - \omega_l) \\
Rs &= \frac{W}{2}\left(\frac{\omega_r + \omega_l}{\omega_r - \omega_l}\right)
\end{aligned}$$

$$\begin{aligned}
x_l &= x + sd\, \cos(\theta) \\
y_l &= y + sd\, \sin(\theta)
\end{aligned}$$

$$
\begin{aligned}
\cos(u \pm v) &= \cos u \, \cos v \mp \sin u \, \sin v \\
\sin(u \pm v) &= \sin u \, \cos v \pm \cos u \, \sin v
\end{aligned}
$$

$$
\begin{aligned}
\cos(u - v) - \cos(u + v) &= 2 \sin(u) \sin(v) \\
\sin(u + v) - \sin(u - v) &= 2 \cos(u) \sin(v)
\end{aligned}
$$

$$
Ch = 2 \left(Rs + \frac{W}{2} \right) sin \left(\frac{\alpha}{2} \right)
$$

$$
\begin{aligned}
x &= x_0 + x_{@} + x_{\#} \\
&= x_0 + x' \cos(\theta_0) + y' \cos\left(\theta + 90^\circ\right) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos\left(\theta + 90^\circ\right) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) - Ch \sin\left(\frac{\alpha}{2}\right) \sin(\theta) \\
&= x_0 + Ch \cos\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= x_0 + 2 \left(Rs + \frac{W}{2} \right) sin \left(\frac{\alpha}{2} \right) \cos\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}
$$

$$
v_x = \dot{x} = \left(Rs + \frac{W}{2} \right) \omega \cos(\theta_0 + \alpha)
$$

$$
\begin{aligned}
y &= y_0 + y_{@} + y_{\#} \\
&= y_0 + x' \sin(\theta_0) + y' \sin\left(\theta + 90^\circ\right) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \sin\left(\theta + 90^\circ\right) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos(\theta_0) \\
&= y_0 + Ch \sin\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= y_0 + 2 \left(Rs + \frac{W}{2} \right) sin \left(\frac{\alpha}{2} \right) \sin\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}
$$

$$v_y = \dot{y} = \left(Rs + \frac{W}{2}\right) \omega \sin(\theta_0 + \alpha)$$

Another way to calculate the robot's coordinates in the global reference frame:

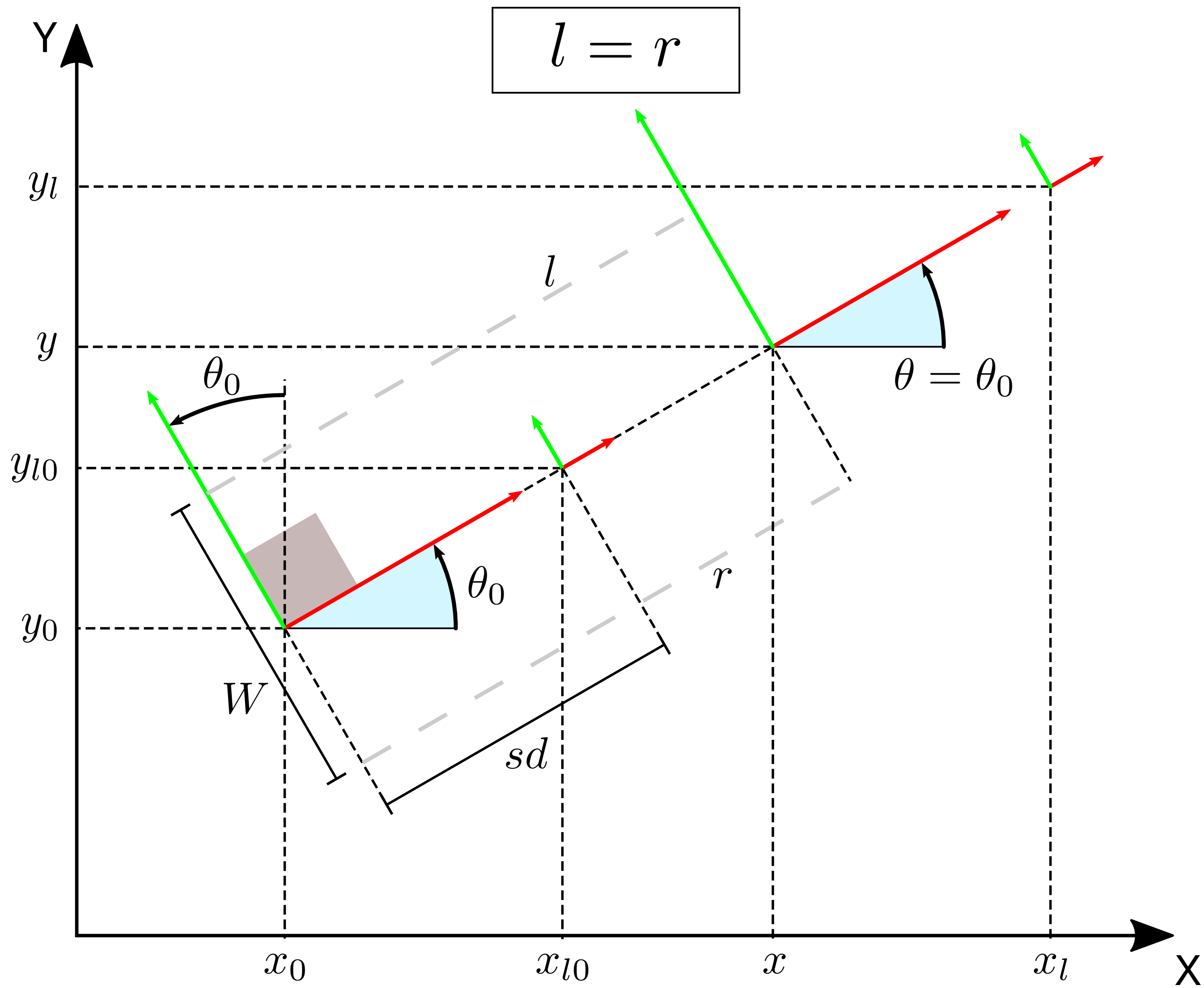
$$\begin{aligned}
x_c &= x_0 + \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0 + 90°\right) \\
&= x_0 - \left(Rs + \frac{W}{2}\right) sin(\theta_0) \\
y_c &= y_0 + \left(Rs + \frac{W}{2}\right) \sin\left(\theta_0 + 90°\right) \\
&= y_0 + \left(Rs + \frac{W}{2}\right) \cos(\theta_0)
\end{aligned}$$

$$\begin{aligned}
x_c &= x + \left(Rs + \frac{W}{2}\right) \cos\left(\theta + 90°\right) \\
&= x - \left(Rs + \frac{W}{2}\right) sin(\theta) \\
y_c &= y + \left(Rs + \frac{W}{2}\right) \sin\left(\theta + 90°\right) \\
&= y + \left(Rs + \frac{W}{2}\right) \cos(\theta_0)
\end{aligned}$$

$$\begin{aligned}
x &= x_c + \left(Rs + \frac{W}{2}\right) \sin(\theta) \\
&= x_0 - \left(Rs + \frac{W}{2}\right) \sin(\theta_0) + \left(Rs + \frac{W}{2}\right) \sin(\theta) \\
&= x_0 + \left(Rs + \frac{W}{2}\right) \left(\sin(\theta) - \sin(\theta_0)\right) \\
&= x_0 + 2\left(Rs + \frac{W}{2}\right) \cos\left(\theta_0 + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)
\end{aligned}$$

$$
\begin{aligned}
y &= y_c - \left(Rs + \frac{W}{2}\right)\cos(\theta) \\
&= y_0 + \left(Rs + \frac{W}{2}\right)\cos(\theta_0) - \left(Rs + \frac{W}{2}\right)\cos(\theta) \\
&= y_0 + \left(Rs + \frac{W}{2}\right)\big(\cos(\theta_0) - \cos(\theta)\big) \\
&= y_0 + 2\left(Rs + \frac{W}{2}\right)\sin\left(\theta_0 + \frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right)
\end{aligned}
$$

Robot's coordinates in the global reference frame



Given the robot's initial pose, (x_0, y_0, θ_0), and the motion commands, (l, r):

$$
\begin{aligned}
x &= x_0 + l \cos(\theta_0) \\
y &= y_0 + l \sin(\theta_0) \\
x_l &= x + sd \cos(\theta) \\
y_l &= y + sd \sin(\theta)
\end{aligned}
$$