EKF and Complimentary Filter for Fusing IMU Data

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I. Extended Kalman Filter

A. Motion Model

In this assignment, we choose the position ($p = [p_x, p_y, p_z]$) and the rotation ($q = [q_x, q_y, q_z, q_w]$) of the robot in the local map frame as states, i.e.,

$$\mathbf{x} = [p_x, p_y, p_z, q_x, q_y, q_z, q_w]^T .$$

Input vector is chosen as linear acceleration and angular velocity in the body frame, i.e.,

$$\mathbf{u} = [a_x, a_y, a_z, \omega_x, \omega_y, \omega_z].$$

Motion model is obtained by using first-order approximation to the integral:

$$p_{k+1} = p_k + \frac{1}{2}\Delta t^2 R^T(q_k) a_k$$

$$q_{k+1} = q_k + \Delta t \dot{q}_k = q_k + \Delta t \frac{1}{2} Q(q_k)\omega_k,$$

where

$$R(q) = \begin{bmatrix} 2q_w^2 + 2q_x^2 - 1 & 2q_xq_y - 2q_wq_z & 2q_xq_z + 2q_wq_y \\ 2q_xq_y + 2q_wq_z & 2q_w^2 + 2q_y^2 - 1 & 2q_yq_z - 2q_wq_x \\ 2q_xq_z - 2q_wq_y & 2q_yq_z + 2q_wq_x & 2q_w^2 + 2q_z^2 - 1 \end{bmatrix}$$

is the rotation matrix corresponding to quaternion q; $Q(q)$ (different from that in Quaternion Primier) is defined as

$$Q(q) = \begin{bmatrix} q_w & -q_z & q_y \\ q_z & q_w & -q_x \\ -q_y & q_x & q_w \\ -q_x & -q_y & -q_z \end{bmatrix}.$$

When we use the motion model to do predictions, a_k and ω_k is set to be the average mean value returned from the IMUs.

B. Measurement Model

Readings from accelerometers and the magetometer are chosen as observations. We assume that there is no external force and therefore the acceleration measured is from the gravity. As a result,

$$h(\mathbf{x}_{k+1}) = \begin{bmatrix} R(q_t)\mathbf{g} \\ R(q_t)m_0 \end{bmatrix},$$

where $\mathbf{g} = [0, 0, -g]$ is the gravity acceleration vector in the initial frame (here we assume the body aligns well with the world frame in the initial position) and m_0 is the magnetic vector measured at the the initial position.

C. EKF-based estimator

We follow the standard EKF steps to estimate the position and the rotation.

Predict

Predicted state estimate

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k)$$

Predicted covariance estimate

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k$$

Here Q_k is the covariance matrix for the disturbances in the motion model. In the motion model, we use the mean as control input and the variance of the input is then treated as noise. As a result,

$$\mathbf{Q}_k = \begin{bmatrix} \frac{1}{4}\Delta t^4 R(q_k)^T \Sigma_{a_k} R(q_k) & \mathbf{0} \\ \mathbf{0} & \frac{1}{4}\Delta t^2 Q(q_k)\Sigma_{\omega_k} Q(q_k)^T \end{bmatrix}$$

Update

Innovation or measurement residual

$$\tilde{\mathbf{y}}_k = z_k - h(\hat{\mathbf{x}}_{k|k-1})$$

Innovation covariance

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

Here R_k is the covariance matrix for observation model. For the acceleration part, we first compute the average value from the three IMUs and compute one covariance corresponding to that average value. For the magnetometer part, because the variance provided by ROS is applicable, we use one typical value (about 10^{-8}) for each axis. As a result,

$$\mathbf{R_k} = \begin{bmatrix} \Sigma_{\bar{a}_k} & \mathbf{0} \\ \mathbf{0} & \Sigma_{\bar{m}} \end{bmatrix}$$

Near-optimal Kalman gain

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

Updated state estimate

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$$

Updated covariance estimate

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$$

where the state transition and observation matrices are defined to be the following Jacobians

$$\mathbf{F}_k = \frac{\partial f}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k}$$

$$\mathbf{H}_k = \frac{\partial h}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k-1|k-1}}$$

II. Complimentary Filter

Let ϕ, θ, ψ be roll, pitch, and yaw respectively. We assume that at the origin of the map frame the z axis is aligned with the gravity direction. As a result, in the FLU coordinate system, at the beginning, the gravity acceleration vector is

$$\mathbf{g} = [0, 0, -g]^T$$

.

We also assume on average the external acceleration is zero except the gravity acceleration. By simple geometry knowledge, we have

$$\hat{\phi}_{\mathrm{acc}} = \tan^{-1}(\frac{\mathrm{a_y}}{-\mathrm{a_z}})$$

$$\hat{\theta}_{\mathrm{acc}} = \sin^{-1}(\frac{\mathrm{a_y}}{\mathrm{g}})$$

The rotation rates obtained from the gyroscope in the body frame are related to Euler angles by the relation

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}.$$

Using the above relation, we have

$$\hat{\phi}_{\mathrm{gyro}}(k+1) = \phi_{\mathrm{gyro}}(k) + \Delta t\dot{\phi}(k)$$
$$\hat{\theta}_{\mathrm{gyro}}(k+1) = \theta_{\mathrm{gyro}}(k) + \Delta t\dot{\theta}(k).$$

We use the following formula to fuse the estimation from accelerometer and the gyroscope

$$\hat{\phi} = \alpha\hat{\phi}_{\mathrm{acc}} + (1-\alpha)\hat{\phi}_{\mathrm{gyro}}$$

$$\hat{\theta} = \alpha\hat{\theta}_{\mathrm{acc}} + (1-\alpha)\hat{\theta}_{\mathrm{gyro}}$$

Assume that the magnetometer has been calibrated at the local inertial frame. The yaw angle estimate is given as

$$\psi = \tan^{-1}(\frac{\mathrm{m_z}\sin\phi - \mathrm{m_y}\cos\phi}{\mathrm{m_x}\cos\theta + \mathrm{m_y}\sin\theta\sin\phi + \mathrm{m_z}\sin\theta\cos\phi})$$

III. Experiment

A. Data alignment

We notice that the data from the magnetometer has the lowest frequency and therefore we use time stamp from the magnetometer the align other data. Specifically, for each time step of the data from the magnetometer, we find data from other sensors that has a time stamp closest but earlier to the current magnetometer time stamp and then choose that data as one sample.

B. Discussion

Our complimentary filter works in the position rosbag provided by the instructor. EKF seems to cannot follow the change of the signal.