KF, EKF and Complimentary Filter for Fusing IMU Data

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I. Extended Kalman Filter - type I

A. Process Model

In this assignment, we choose the position ($p = [p_x, p_y, p_z]$) and the rotation ($q = [q_x, q_y, q_z, q_w]$) of the robot in the local map frame as states, i.e.,

$$\mathbf{x} = [p_x, p_y, p_z, q_x, q_y, q_z, q_w]^T .$$

Input vector is chosen as linear acceleration and angular velocity in the body frame, i.e.,

$$\mathbf{u} = [a_x, a_y, a_z, \omega_x, \omega_y, \omega_z].$$

Motion model is obtained by using first-order approximation to the integral:

$$p_{k+1} = p_k + \frac{1}{2}\Delta t^2 R(q_k) a_k$$

$$q_{k+1} = q_k + \Delta t \dot{q}_k = q_k + \Delta t \frac{1}{2} q_k \bigotimes (0, \omega_k),$$

where

$$R(q) = \begin{bmatrix} 2q_w^2 + 2q_x^2 - 1 & 2q_xq_y - 2q_wq_z & 2q_xq_z + 2q_wq_y \\ 2q_xq_y + 2q_wq_z & 2q_w^2 + 2q_y^2 - 1 & 2q_yq_z - 2q_wq_x \\ 2q_xq_z - 2q_wq_y & 2q_yq_z + 2q_wq_x & 2q_w^2 + 2q_z^2 - 1 \end{bmatrix}$$

is the rotation matrix corresponding to quaternion q; When we use the motion model to do predictions, a_k and ω_k is set to be the average mean values returned from the PX4 onboard IMU.

B. Measurement Model

Readings from accelerometers and the magetometer are chosen as observations. We assume that there is no external force and therefore the acceleration measured is from the gravity. As a result,

$$h(\mathbf{x}_{k+1}) = \begin{bmatrix} R(q_t)^T \mathbf{g} \\ R(q_t)^T m_0 \end{bmatrix},$$

where $\mathbf{g} = [0, 0, -g]$ is the gravity acceleration vector in the initial frame (here we assume the body aligns well with the world frame in the initial position) and m_0 is the magnetic vector measured at the the initial position.

C. EKF-based estimator

We follow the standard EKF steps to estimate the position and the rotation.

Predict

Predicted state estimate

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k)$$

Predicted covariance estimate

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k$$

Here Q_k is the covariance matrix for the disturbances in the motion model. In the motion model, we use the mean as control input and the variance of the input is then treated as noise. As a result,

$$\mathbf{Q}_k = \begin{bmatrix} \frac{1}{4}\Delta t^4 R(q_k)^T \Sigma_{a_k} R(q_k) & \mathbf{0} \\ \mathbf{0} & \frac{1}{4}\Delta t^2 Q(\omega_k) \end{bmatrix}$$

Update

Innovation or measurement residual

$$\tilde{\mathbf{y}}_k = z_k - h(\hat{\mathbf{x}}_{k|k-1})$$

Innovation covariance

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

Here R_k is the covariance matrix for observation model. For the acceleration part, we set it to be covariance matrix returned from IMU. For the magnetometer part, because the variance provided by ROS is applicable, we use one typical value (about 10^{-8}) for each axis. As a result,

$$\mathbf{R_k} = \begin{bmatrix} \Sigma_{a_k} & \mathbf{0} \\ \mathbf{0} & \Sigma_{\bar{m}} \end{bmatrix}$$

Near-optimal Kalman gain

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

Updated state estimate

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$$

Updated covariance estimate

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$$

where the state transition and observation matrices are defined to be the following Jacobians

$$\mathbf{F}_k = \frac{\partial f}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k}$$

$$\mathbf{H}_k = \frac{\partial h}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k-1|k-1}}.$$

D. Discussion

We implement this algorithm in EKF_quaternion.py. But the estimator cannot return meaningful results compared to that from PX4 and vicon. As a result, we implement a simpler version EKF as explained below.

II. Simplified Extended Kalman Filter-type II

Here we only use EKF to estimate roll ϕ and pitch θ and Follow the definition in [1].

A. Process Model

$$\begin{bmatrix}\phi_{k+1}\\ \theta_{k+1}\end{bmatrix} = \begin{bmatrix}\phi_k + \Delta t\dot{\phi}_k\\ \theta_k + \Delta t\dot{\theta}_k\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} \tag{2}$$

where ω_x, ω_y, and ω_z are measured by gyro in the body frame.

B. Measurement Model

$$\mathbf{Z}_k = R(\phi,\theta,\psi)\begin{bmatrix}0\\ 0\\ -g\end{bmatrix} = g\begin{bmatrix}\sin\theta\\ -\cos\theta\sin\phi\\ -\cos\theta\cos\phi\end{bmatrix} \tag{3}$$

C. Discussion

We implement this version of EKF in EKF.m. In the model, since we don't the covariance for the process noise, we set it to be $Diag(10^{-6}, 10^{-6})$. Results are given in Fig. 1 and Fig. 2. We use data from imu0.

III. Kalman Filter

We assume that the quadrotor works around $[\phi,\theta,\psi] = [0,0,0]$ and we use Kalman filter to estimate ϕ and θ.

A. Process Model

Since roll, pitch, and yaw are very close to zero, we can first simplify the equation for the angular rate as

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} \approx \begin{bmatrix}\omega_x\\ \omega_y\end{bmatrix}.$$

As a result, the process equation is

$$\begin{bmatrix}\phi_{k+1}\\ \theta_{k+1}\end{bmatrix} = \begin{bmatrix}\phi_k + \Delta t\omega_x\\ \theta_k + \Delta t\omega_y\end{bmatrix}.$$

B. Measurement Model

By linearizing observation model (3), we have

$$\mathbf{Z}_k = R(\phi,\theta,\psi)\begin{bmatrix}0\\ 0\\ -g\end{bmatrix} = g\begin{bmatrix}\theta_k\\ -\phi_k,\end{bmatrix}. \tag{4}$$

C. Discussion

Results are given in Fig. 3 and Fig. 4. Compare to EKF, we can see from Fig. 4, the KF method cannot follow the sharp change.

IV. Complimentary Filter

Let ϕ, θ, ψ be roll, pitch, and yaw respectively. We assume that at the origin of the map frame the z axis is aligned with the gravity direction. As a result, in the FLU coordinate system, at the beginning, the gravity acceleration vector is

$$\mathbf{g} = [0,0,-g]^T$$

.

We also assume on average the external acceleration is zero except the gravity acceleration. By simple geometry knowledge, we have

$$\hat{\phi}_{\mathrm{acc}} = \tan^{-1}(\frac{\mathrm{a_y}}{-\mathrm{a_z}})$$
$$\hat{\theta}_{\mathrm{acc}} = \sin^{-1}(\frac{\mathrm{a_y}}{\mathrm{g}})$$

The rotation rates obtained from the gyroscope in the body frame are related to Euler angles by the relation

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}.$$

Using the above relation, we have

$$\hat{\phi}_{\mathrm{gyro}}(k+1) = \phi_{\mathrm{gyro}}(k) + \Delta t\dot{\phi}(k)$$
$$\hat{\theta}_{\mathrm{gyro}}(k+1) = \theta_{\mathrm{gyro}}(k) + \Delta t\dot{\theta}(k).$$

We use the following formula to fuse the estimation from accelerometer and the gyroscope

$$\hat{\phi} = \alpha\hat{\phi}_{\mathrm{acc}} + (1-\alpha)\hat{\phi}_{\mathrm{gyro}}$$
$$\hat{\theta} = \alpha\hat{\theta}_{\mathrm{acc}} + (1-\alpha)\hat{\theta}_{\mathrm{gyro}}$$

A. Discussion

Results are given in Fig. 5 and Fig. 6. In general, CF estimation agrees well with that from the onboard estimator although there seems to be some constant bias. I also implemented a quaternion based complimentary filter, which is in complimentary_filter_quaternion. But unfortunately, it doesn't work and can only return some random results.

V. PX4 versus Vicon

The results of onboard estimator and those from vicon are given in Fig. 7 and Fig. 8. For position estimation, there is a bias between results from PX4 and those from Vicon. I think it is because the definition of the origin of the map is different for two systems. Similarly, estimates are opposite in qx and qy, which is also due to coordinate definition.

References

[1] P. Narkhede, A. N. Joseph Raj, V. Kumar, V. Karar, and S. Poddar, "Least square estimation-based adaptive complimentary filter for attitude estimation," *Transactions of the Institute of Measurement and Control*, vol. 41, no. 1, pp. 235–245, 2019.

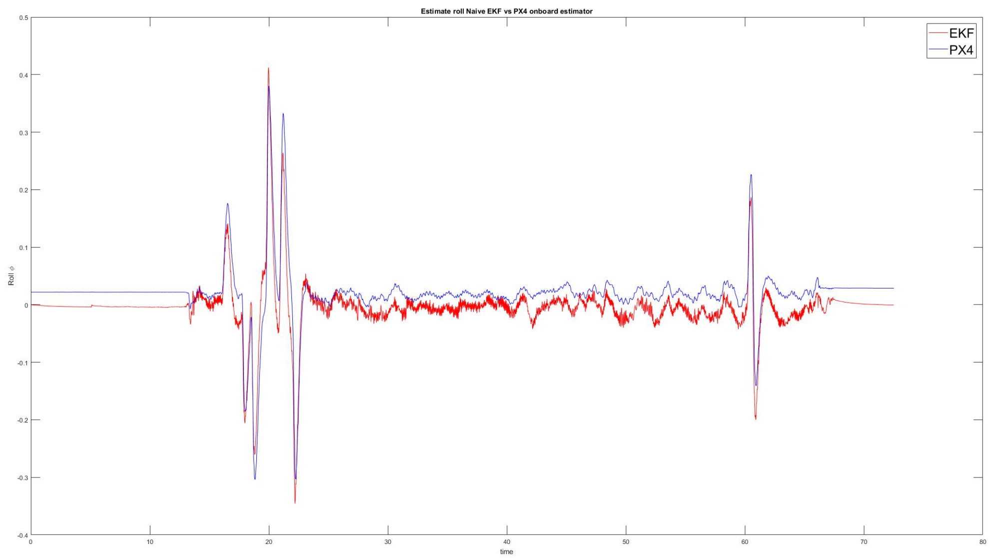

Fig. 1. Roll estimation with EKF.

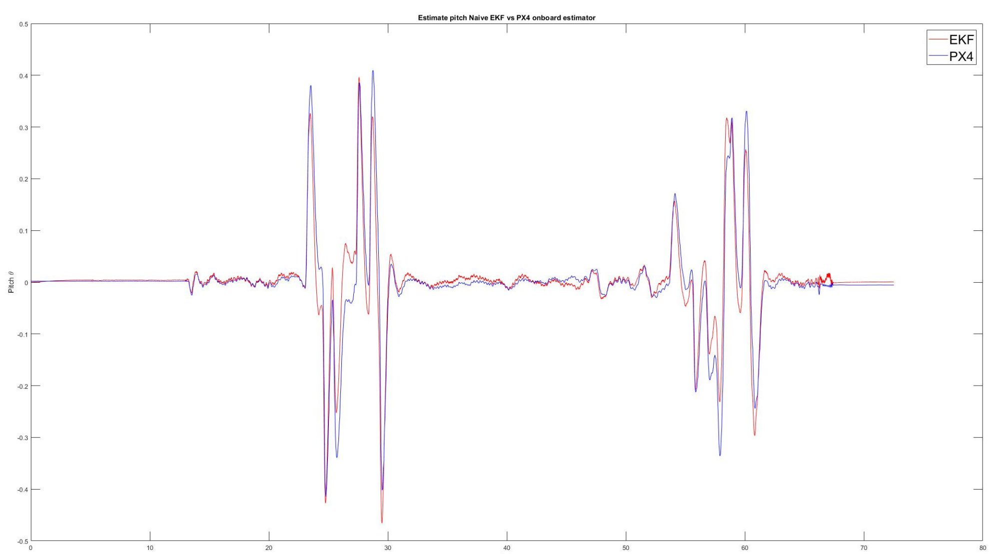

Fig. 2. Pitch estimation with EKF.

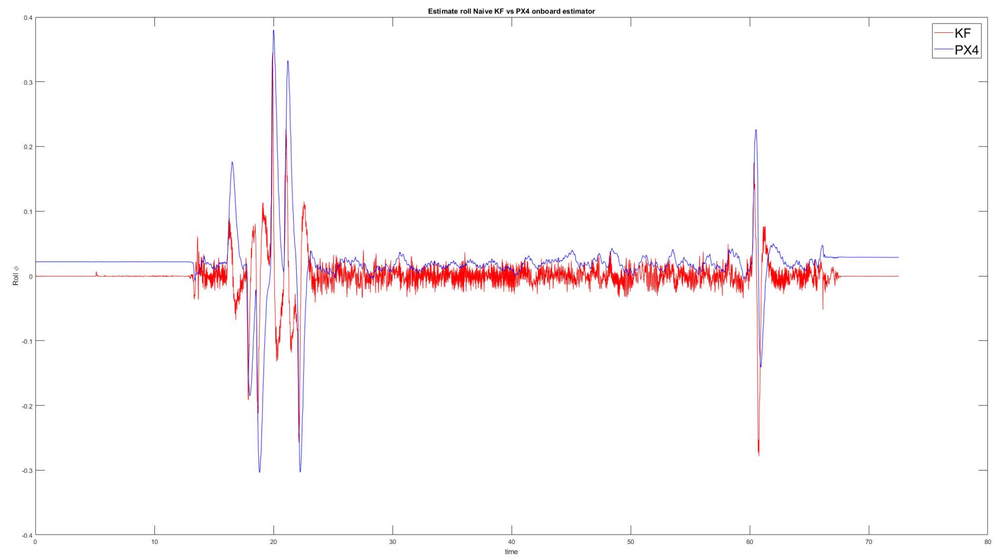

Fig. 3. Roll estimation with KF.

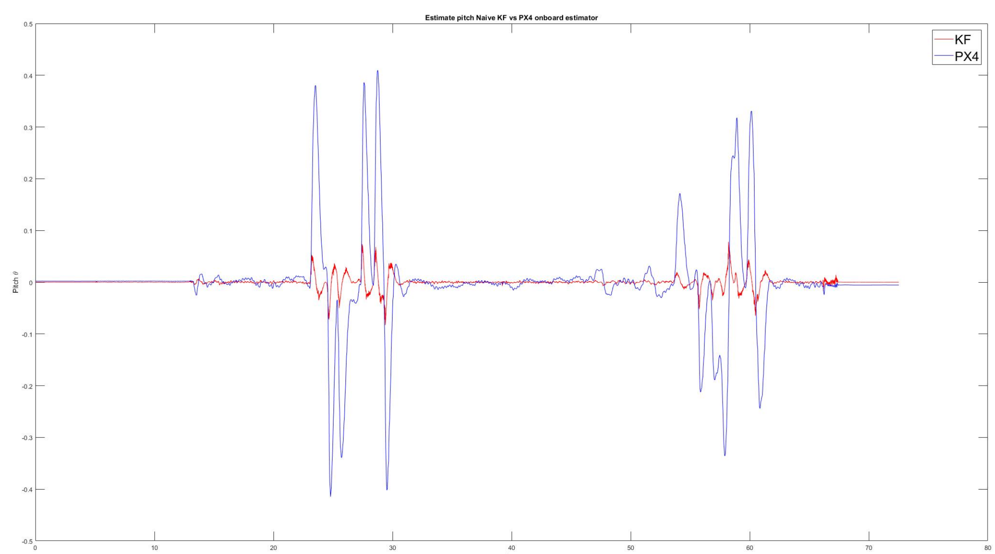

Fig. 4. Pitch estimation with KF.

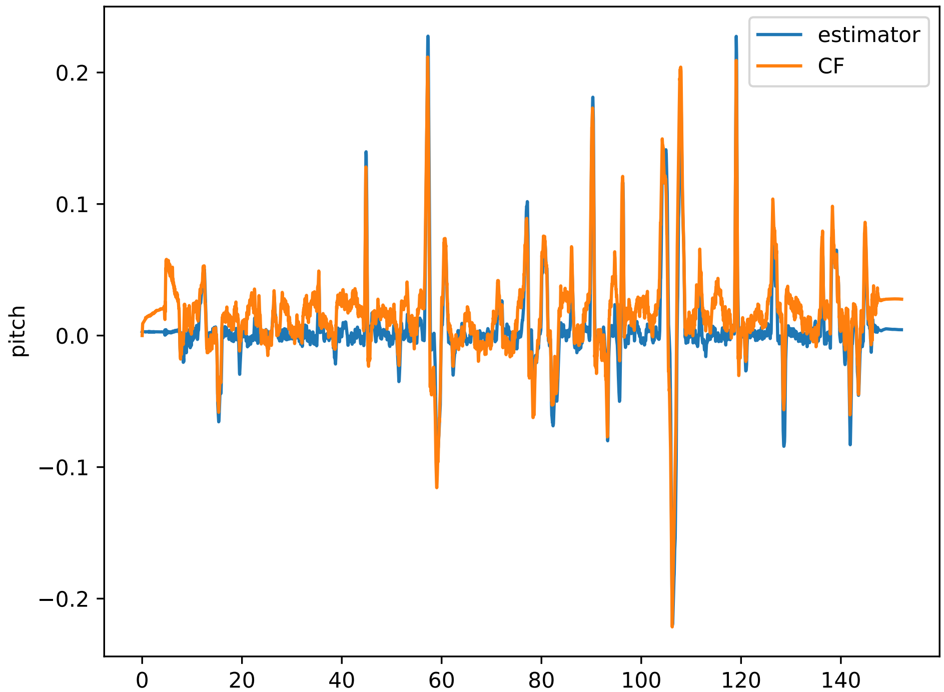

Fig. 5. Pitch estimation with CF.

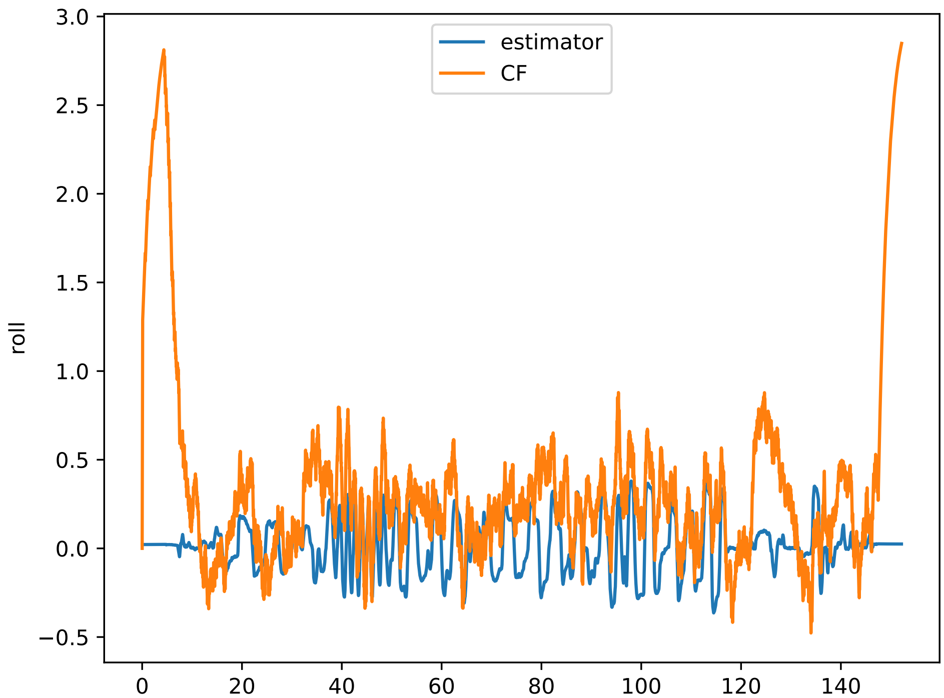

Fig. 6. Roll estimation with CF.

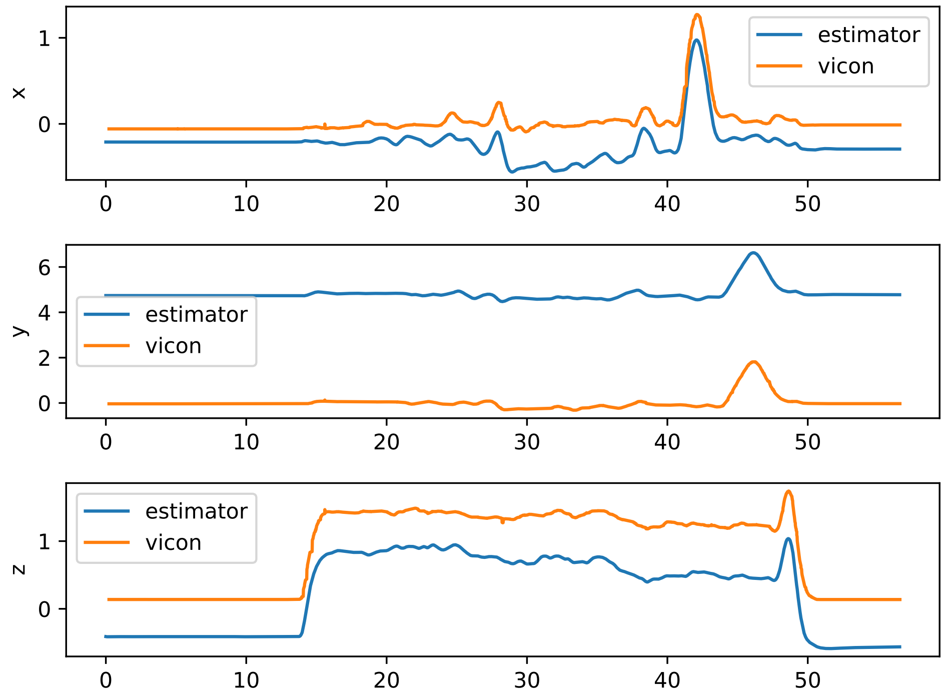

Fig. 7. Position estimate from PX4 and Vicon.

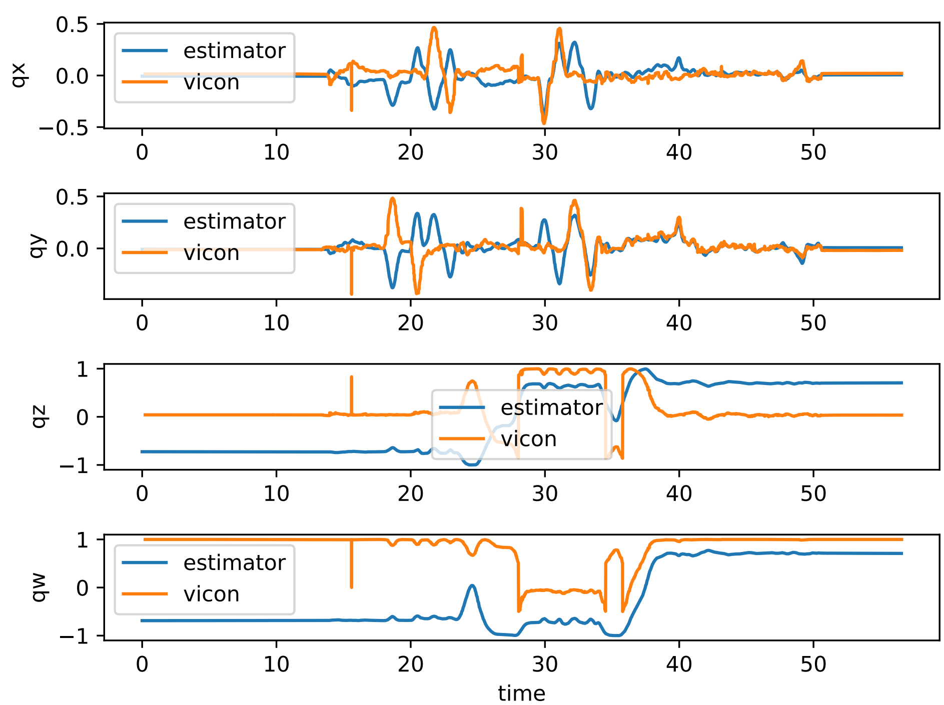

Fig. 8. Rotation estimate from PX4 and Vicon.