1. 实验1

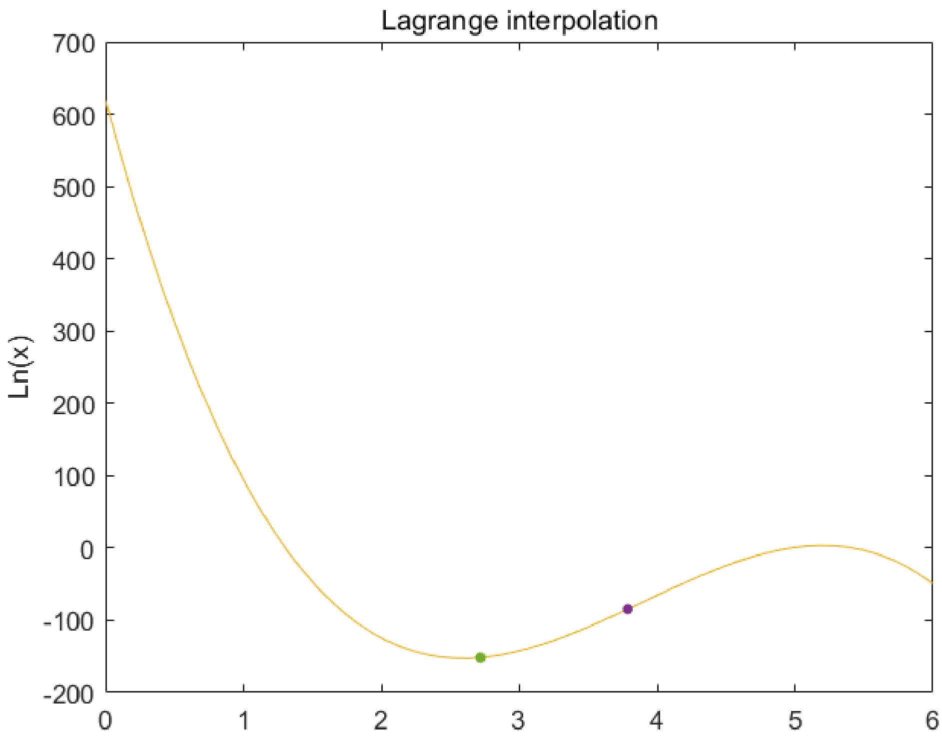
1.1. Lagrange插值

1. Lagrange 插值公式：

   $Ln(x) = 17.612 * x^3 + 205.97 * x^2 - 713.08 * x + 618.77;$

2. 绘制曲线及计算坐标：

   $x_i = [3.79, 2.72];\ \ y_i = [-84.39, -151.44];$



3. 参数截图：

名称	值
a	[5.1625,5.0180,1.2966,1.2875]
ans	*1x2 sym*
b	[3.5451,1.4296,2.0536,4.4987]
i	4
k	4
l	*4x1 sym*
L	*1x1 sym*
L1	*4x4 double*
m	4
V	[-7.6120,87.3642,-297.6731,255.6836]
x	*1x60 double*
X	[5.1625,5.0180,1.2966,1.2875]
x0	[3.7868,2.7190]
Y	[3.5451,1.4296,2.0536,4.4987]
y0	[-84.3891,-151.4350]

4. 代码附录:

```
%exp1.1 2020/11/7 zgz
rand('seed',1851960);
%points
a = 1+5*rand(1,4);
b = 1+5*rand(1,4);

%construct coordinates
x = linspace(0,6,60);
y = lagrange(a,b,x);

%draw
plot(x,y);
ylabel('Ln(x)');
title('Lagrange interpolation');
hold on

%draw additional points
x0 = 2 + 3*rand(1,2);
y0 = lagrange(a,b,x0);
for i = 1:2
    plot(x0(1,i),y0(1,i),'.','MarkerSize',14);
end
```

```
function y=lagrange(x0,y0,x)
%此函数用于形成拉格朗日插值公式;

n = length(x0);
m = length(x);
y = zeros(1,m);
for i=1:m
   z=x(i);s=0;
   for k=1:n
      L=1;
      for j=1:n
         if j~=k
            L=L*(z-x0(j))/(x0(k)-x0(j));
         end
      end
      s=s+L*y0(k);
   end
   y(i)=s;
end
```

```
%%%% 求拉格朗日多项式及基函数   %%%%
%%%%                Liu Deping               %%%%
%%%%              2020.06.14               %%%%
%输入的量:n+1个节点(x_i,y_i)(i = 1,2, ... , n+1)横坐标向量X，纵坐标向量Y
%输出的量：n次拉格朗日插值多项式L和基函数l
X=input('请输入横坐标向量X:\nX=');  %输入的数据为一维数组，例如：[1,3,4,5]（下
同）；
Y=input('请输入纵坐标向量Y:\nY=');
m = length(X);
L = ones(m,m);
for k = 1 : m
```

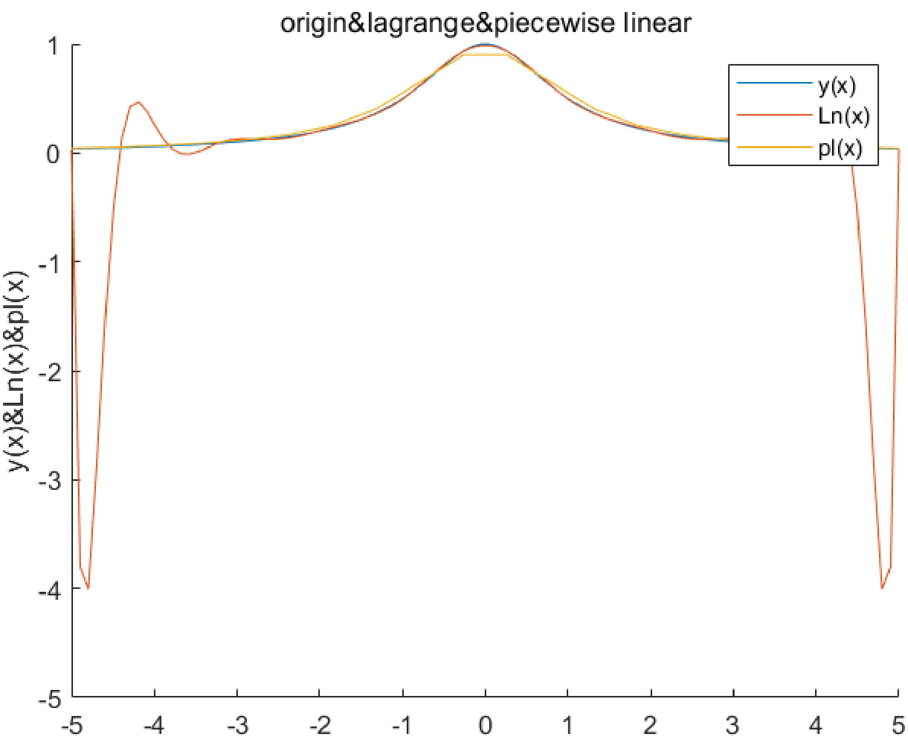
```
    V = 1;
    for i = 1 : m
        if k ~= i
            V = conv(V,poly(X(i))) / (X(k) - X(i));
        end
    end
    L1(k, :) = V;
    l(k, :) = poly2sym(V);
end
fprintf('基函数为：\n');
for k=1:m
    fprintf('q%d(x)=%s\n',k,l(k));
end
L = Y * l;
fprintf('拉格朗日多项式为:\nP(x)=%s\n',L);
```

1.2. 观察Runge现象及分段线性插值

1. Lagrange 插值公式：

$$Ln(x) = -3.202e^{-23} * x^{17} + 4.093e^{-8} * x^{16} + 1.437e^{-19} * x^{15} - 3.472e^{-6} * x^{14}$$
$$+4.289e^{-19} * x^{13} + 0.0001186 * x^{12} - 3.003e^{-16} * x^{11} - 0.002114 * x^{10} - 3.541e^{-15} * x^{9}$$
$$+0.02131 * x^{8} - 1.227e^{-14} * x^{7} - 0.1241 * x^{6} - 1.573e^{-14} * x^{5} + 0.4157 * x^{4}$$
$$-4.154e^{-16} * x^{3} - 0.803 * x^{2} - 9.729e^{-16} * x + 0.9868$$

2. 绘制曲线，发现在横坐标两端出现龙格现象，即发生过拟合；分段线性插值（pl）绘图如下：



3. 分析：

　一般情况下，多项式的次数越多，需要的数据就越多，而预测也就越准确；插值次数越高，插值结果越偏离原函数。为了解决 Runge 现象，可采取分段线性插值，使得用分段线性函数进行拟合，避免了过拟合，但是函数失去了平滑性，即出现了不可导点。

4. 参数截图：

名称	值
a	[27.0389,15.8748,15.8476,22.6203,16.2739]
ans	*1x1 sym*
b	[59.9833,51.6518,23.1771,9.7016,6.8645]
i	18
k	18
l	*18x1 sym*
L	*1x1 sym*
L1	*18x18 double*
m	18
n	18
p	[1.3795,-56.3655,576.4023]
V	*1x18 double*
x	*1x100 double*
X	*1x18 double*
x0	*1x18 double*
x3	*1x20 double*
y	*1x25 double*
Y	*1x18 double*
y0	*1x18 double*
y1	*1x100 double*
y2	*1x100 double*
y3	*1x20 double*

5. 代码附录:

```
%exp1.2 2020/11/7 zgz
rand('seed',1851960);
%construct coordinates
n  = randi([8,20],1,1);
x0 = linspace(-5,5,18);
y0 = 1./(1+x0.^2);
x  = linspace(-5,5,100);

%original
y1 = 1./(1+x.^2);

%lagrange
y2 = lagrange(x0,y0,x);

%pl
x3  = linspace(-5,5,20);
y3 = pl(20,-5,5);

%draw
plot(x,y1);
hold on
plot(x,y2);
hold on
plot(x3,y3);
ylabel('y(x)&Ln(x)&pl(x)');
legend('y(x)','Ln(x)','pl(x)');
title('origin&lagrange&piecewise linear');
```

```
%draw additional points
hold on
```

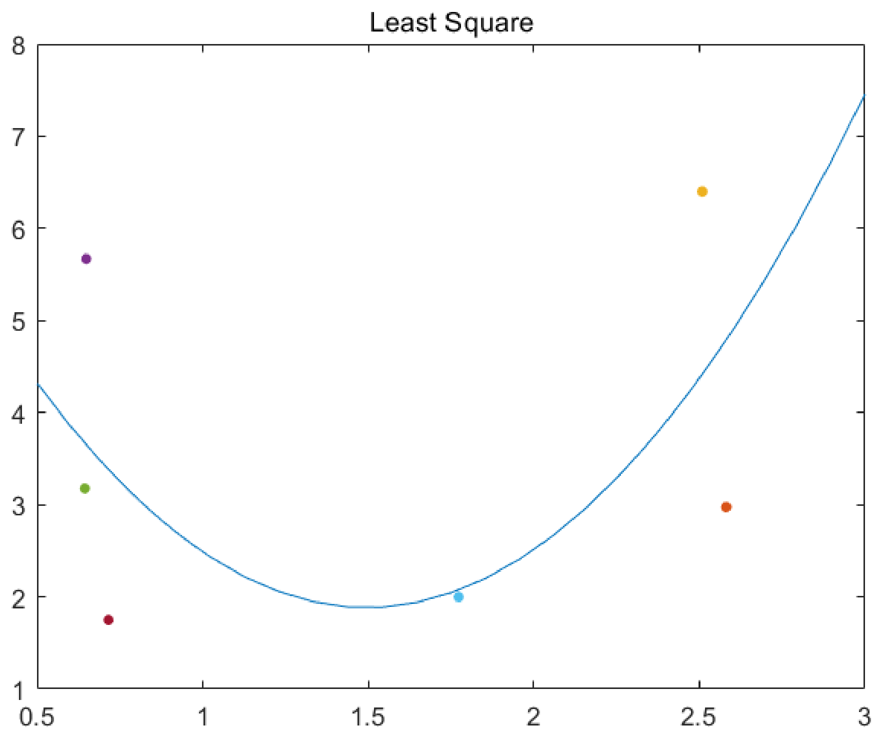
```
function Sn=pl(L,b1,b2)
%当在区间内取i个等距节点时对应的小区间的中点值Si并绘制出图形
%b1代表左边界，b2代表右边界
%L可以是一个数组，也可以是一个数字

 %插值绘图
 n=length(L);
 for i=1:n
     s=L(i);
     L1=linspace(b1,b2,s+1);
     for j=2:s+1
             X(j-1)=(L1(j-1)+L1(j))/2; %寻找两端点中点值
             Sn(j-1) = (((X(j-1)-L1(j))/(L1(j-1)-L1(j)))/(1+L1(j-1)^2))+
(((X(j-1)-L1(j-1))/(L1(j)-L1(j-1)))/(1+L1(j)^2)) %中点值函数值
     end
     hold on
 end
```

```
%%%% 求拉格朗日多项式及基函数  %%%%
%%%%                Liu Deping                %%%%
%%%%              2020.06.14                %%%%
%输入的量:n+1个节点(x_i,y_i)(i = 1,2, ... , n+1)横坐标向量X，纵坐标向量Y
%输出的量：n次拉格朗日插值多项式L和基函数l
X=input('请输入横坐标向量X:\nX=');  %输入的数据为一维数组，例如：[1,3,4,5]（下
同）；
Y=input('请输入纵坐标向量Y:\nY=');
m = length(X);
L = ones(m,m);
for k = 1 : m
    V = 1;
    for i = 1 : m
        if k ~= i
            V = conv(V,poly(X(i))) / (X(k) - X(i));
        end
    end
    L1(k, :) = V;
    l(k, :) = poly2sym(V);
end
fprintf('基函数为：\n');
for k=1:m
    fprintf('q%d(x)=%s\n',k,l(k));
end
L = Y * l;
fprintf('拉格朗日多项式为:\nP(x)=%s\n',L);
```

2. 实验2

2.1. 最小二乘法拟合（用polyfit函数）

1. 绘制曲线和数据点：



2. 参数截图：

名称	值
a	[2.5812,2.5090,0.6483,0.6438,1.7725,0.7148]
b	[2.9751,6.3982,5.6692,3.1777,1.9986,1.7503]
i	6
p	[2.4593,-7.3530,7.3806]
x	*1x25 double*
y	*1x25 double*

3. 代码附录：

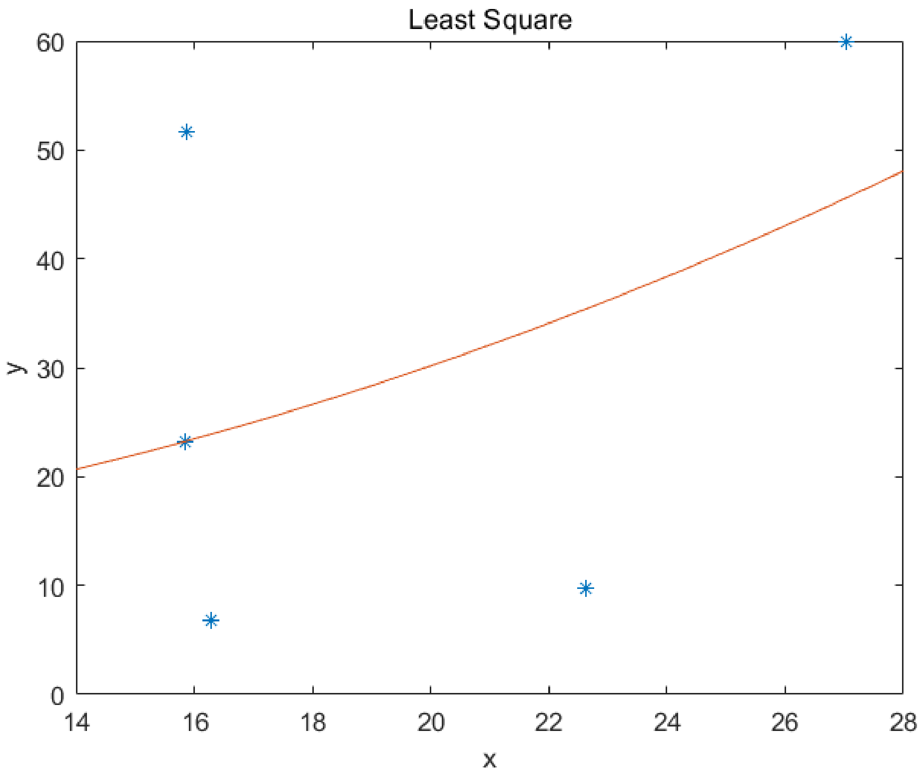
```
%exp2.1 2020/11/7 zgz
rand('seed',1851960);
%points
a = 0.5+2.5*rand(1,6);
b = 1.5+7*rand(1,6);

%fit
p = polyfit(a,b,2);

%construct coordinates
x = linspace(0.5,3,25);
y = polyval(p,x);

%draw
plot(x,y);
ylabel('');
title('Least Square');
hold on

%draw additional points
for i = 1:6
    plot(a(i),b(i),'.','MarkerSize',14);
```

```
    hold on
end
```

2.2. 最小二乘法拟合（用矩形方法求解）

1. 拟合结果：

$y = 11.53 + 0.047x^2$;



2. 参数截图：

名称	值
a	[27.0389,15.8748,15.8476,22.6203,16.2739]
ans	0.0466
b	[59.9833,51.6518,23.1771,9.7016,6.8645]
fun	*1x1 inline*
i	21
n	[11.5290,0.0466]
n0	[1,1]
s	*1x1 sym*
t	*1x1 sym*
x	*1x100 double*
y	*22x1 double*
z	*22x1 double*

3. 代码附录：

```
%exp2_2 2020/11/7 zgz
rand('seed',1851960);
%init
a = 18*rand(1)+15*rand(1,5);
b = 19*rand(1)+80*rand(1,5);
x = linspace(14,28,100);
```

```
fun = inline('n(1)+n(2)*x.^2','n','x');
n0  = [1,1];
n=nlinfit(a,b,fun,n0);
plot(a,b,'*');
hold on
plot(x,n(1)+n(2)*x.^2);
```

2.3. Newton-Cotes系列数值求积公式

1. 比较精度（小区间的宽度取 $w = 0.01$）：

   矩形求积法误差：$loss\_rec = 0.0012$;

   梯形求积法误差：$loss\_tra = 7.4162e^{-6}$;

   因此梯形求积法误差精度较高;

2. 参数截图：

名称	值
a	9
b	13
f	@(t)exp(-0.5*t).*(t+pi/b)
i	6
loss_rec	0.0012
loss_tra	7.4162e-06
p	[2.4593,-7.3530,7.3806]
s	4.4833
s_rec	4.4845
s_tra	4.4833
x	1x25 double
y	1x25 double

3. 代码附录：

```
%exp2_3 2020/11/7 zgz
rand('seed',1851960);
%init
a = randi(10,1,1);
b = randi([5 15],1,1);

%calculate sum
% y = exp(-0.5*t)*(t+pi/b);
s_rec = rec_sum(0,a*pi,b);%rec
s_tra = tra_sum(0,a*pi,b);%tra

%calculate loss
f = @(t) exp(-0.5*t).*(t+pi/b);
s = integral(f,0,a*pi);
loss_rec = abs(s-s_rec);
loss_tra = abs(s-s_tra);
```

```
function rec_sum = rec_sum(floor,ceiling,b)
%矩形法求定积分

%init
s = 0;
i = 1;
w = 0.01; %矩形的宽度为0.01
n = (ceiling-floor)/w; %份数
t = floor;
y = exp(-0.5*t)*(t+pi/b);

while i < n
    s = s+w*y;
    t = t+w;
    y = exp(-0.5*t)*(t+pi/b);
    i = i+1;
end

rec_sum = s;

end
```

```
function tra_sum = tra_sum(floor,ceiling,b)
%梯形法求定积分

%init
s = 0;
i = 1;
w = 0.01; %梯形的宽度为0.01
n = (ceiling-floor)/w; %份数
t = floor;
y1 = exp(-0.5*t).*(t+pi/b);
y2 = exp(-0.5*(t+w)).*((t+w)+pi/b);

while i < n
    s = s+w*(y1+y2)/2;
    t = t+w;
    y1 = y2;
    y2 = exp(-0.5*(t+w)).*((t+w)+pi/b);
    i = i+1;
end

tra_sum = s;

end
```

2.4. Romberg求积公式

1. 积分值：3.57174
2. 参数截图：

名称 ▲	值
a	2.0812
b	2.4108
fun	@(x)x^b
s	3.5717

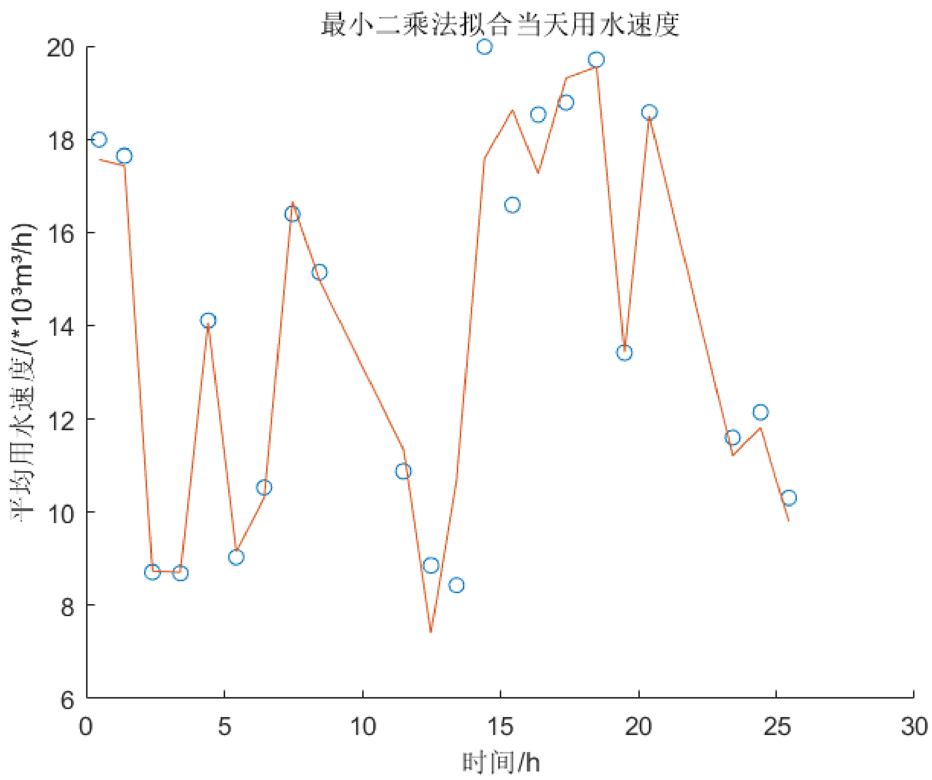
3. 代码附录：

```
%exp2_4 2020/11/7 zgz
rand('seed',1851960);
%init
a = 2.5*rand(1,1);
b = 3*rand(1,1);

%calculate sum
fun = @(x) x^b;
[s,~,~] = romberg(fun,0,a,1e-5);
```

```
function [R,k,T]=romberg(fun,a,b,tol)
% 龙贝格(Romberg数值求解公式)
% author:
%   -gongwanlu
% inputs:
%   -fun：积分函数句柄
%   -a/b：积分上下限romberg
%   -tol：积分误差
% Outputs:
%   -R：7阶精度Romberg积分值
%   -k：迭代次数
%   -T：整个迭代过程
%
% Example
% fun=@(x)4./(1+x^2);
% [R,k,T]=romberg(fun,0,1,1e-6)
%
k=0; % 迭代次数
n=1; % 区间划分个数
h=b-a;
T=h/2*(fun(a)+fun(b));
err=1;
while err>=tol
    k=k+1;
    h=h/2;
    tmp=0;
    for i=1:n
        tmp=tmp+fun(a+(2*i-1)*h);
    end
    T(k+1,1)=T(k)/2+h*tmp;
    for j=1:k
        T(k+1,j+1)=T(k+1,j)+(T(k+1,j)-T(k,j))/(4^j-1);
    end
    n=n*2;
    err=abs(T(k+1,k+1)-T(k,k));
end
R=T(k+1,4);
```

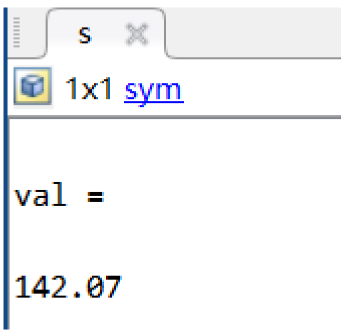
2.5. 估计某地居民的用水速度和每天用水量

1. 拟合该小区当天用水速度的图像如下：



总用水量估计：$142.07\times10^{3}m^{3}$。

2. 参数设置：



	x: 22x1 double	y: 22x1 double	a: 21x1 double
	1	1	1
1	0.4600	17.9900	-8.3607e-...
2	1.3800	17.6432	2.1003e-13
3	2.4000	8.7119	-2.4519e-...
4	3.4100	8.6901	1.7659e-09
5	4.4200	14.1082	-8.7849e-...
6	5.4400	9.0311	3.2029e-06
7	6.4500	10.5287	-8.8612e-...
8	7.4700	16.3969	0.0019
9	8.4500	15.1472	-0.0320
10	11.4900	10.8760	0.4261
11	12.4900	8.8547	-4.4933
12	13.4200	8.4291	37.4582
13	14.4300	19.9841	-245.2985
14	15.4400	16.5906	1.2485e+...
15	16.3700	18.5284	-4.8607e...
16	17.3800	18.7862	1.4148e+...
17	18.4800	19.7069	-2.9789e...
18	19.5000	13.4205	4.3194e+...
19	20.4000	18.5751	-3.9843e...
20	23.4200	11.5966	2.0215e+...
21	24.4300	12.1434	-3.9715e...

名称 ▲	值
a	21x1 double
ans	1x1 sym
b	2.4108
fun	@(x)x^b
i	21
s	1x1 sym
t	1x1 sym
x	22x1 double
y	22x1 double
z	22x1 double

3. 代码附录：

```
%exp2_5 2020/11/7 zgz
rand('seed',1851960);
%init
x =
[0.46,1.38,2.40,3.41,4.42,5.44,6.45,7.47,8.45,11.49,12.49,13.42,14.43,15
.44,16.37,17.38,18.48,19.50,20.40,23.42,24.43,25.45]';
y = 8+12*rand(22,1);
a = polyfit(x,y,20);
z = polyval(a,x);
```

```
syms t

%cal int
s = 0;
for i = 1:21
   s = s+int(a(1,i)*t^(21-i),t,0,24);
end
s = vpa(s,5);

scatter(x, y);
hold on;
plot(x, z);
hold on;
```