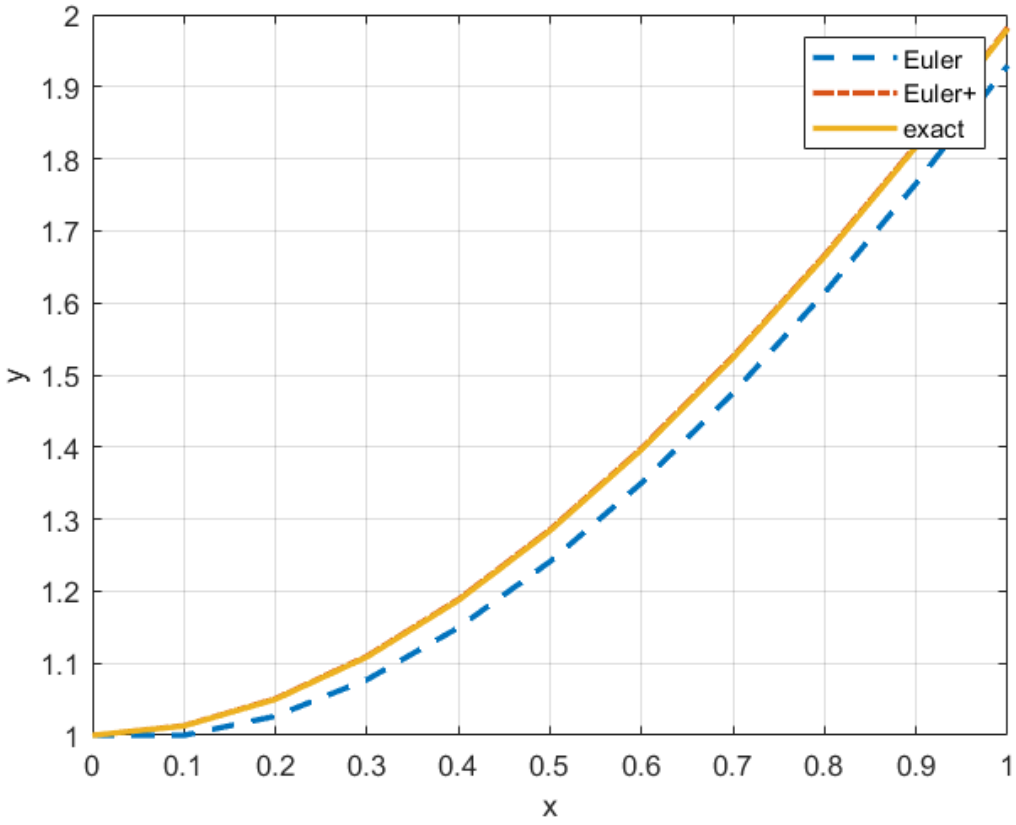
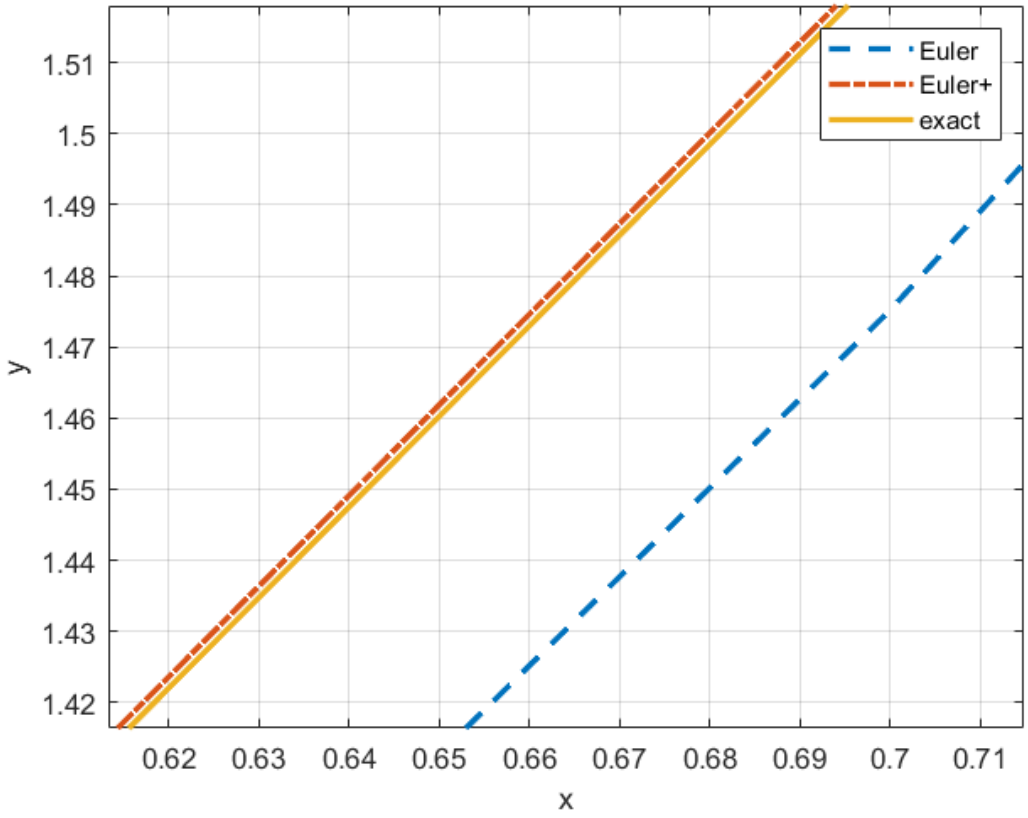
# 1. 实验一

用欧拉法和改进欧拉法求 $y' = -y + ax + 1$ （其中 $a = 1 + 2rand(1)$），$y(0) = 1$。并作图比较。

## 1.1. 实验结果






通过比较，不难发现经过改进后欧拉法得到的结果优于初始结果，其原因在于欧拉法用差商[y(xi+1)-y(xi)]/h 近似代替y(xi)的导数，局部截断误差较大；改进欧拉法先用欧拉法求出预报值，再利用梯形公式求出校正值，局部截断误差比欧拉法低了一阶，较大程度地提高了计算精度。

## 1.2. 实验参数

工作区

| 名称 | 值 |
| --- | --- |
| a | 2.6650 |
| f | @(x,y)a*x-y+1 |
| h | 0.1000 |
| k | 10 |
| n | 10 |
| x | *1x11 double* |
| y1 | *1x11 double* |
| y2 | *1x11 double* |
| y3 | *1x11 sym* |

```
>> y1'          >> y2'

ans =          ans =

    1.0000         1.0000
    1.0000         1.0133
    1.0266         1.0507
    1.0773         1.1098
    1.1495         1.1887
    1.2412         1.2854
    1.3503         1.3982
    1.4752         1.5256
    1.6142         1.6662
    1.7660         1.8188
    1.9292         1.9822
```

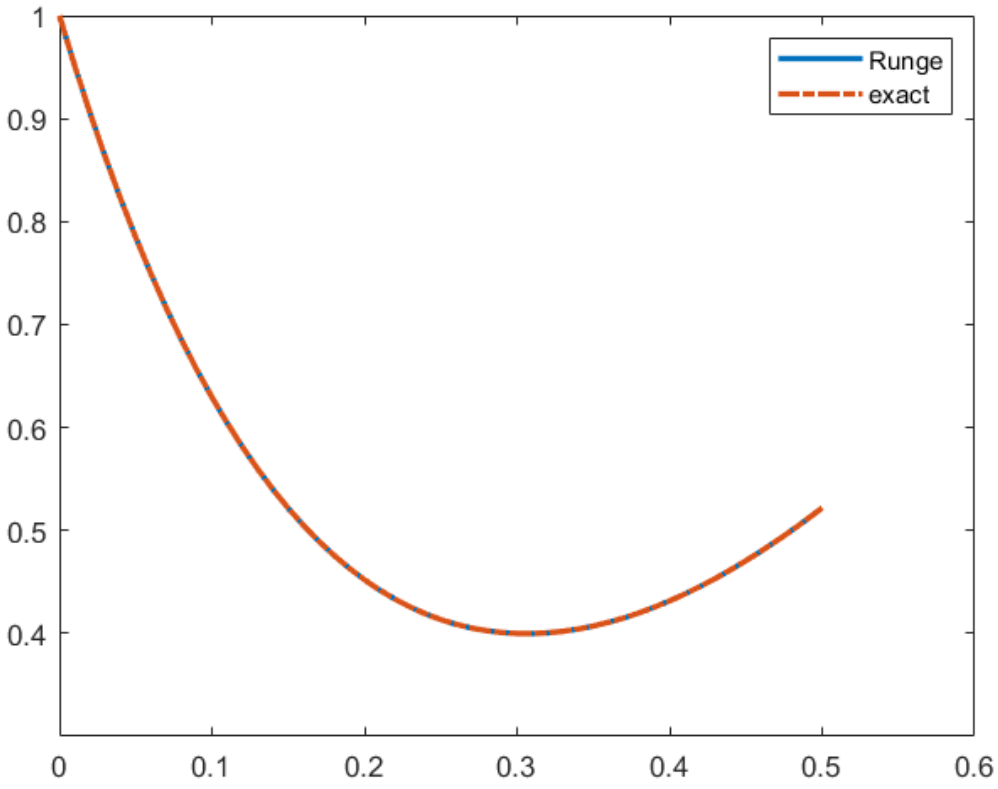
## 1.3. 实验代码

```
clear;
rand('seed',1851960);
a = 1+2*rand(1);

f = @(x,y)a*x-y+1;
h = 0.1;
n = 1/h;
x = zeros(1,n+1);
y1 = zeros(1,n+1);              % 欧拉法
y2 = zeros(1,n+1);              % 改进欧拉法
x(1) = 0;
y1(1) = 1;
y2(1) = 1;
for k =1:n
    x(k+1) = x(1)+h*k;
    y1(k+1) = y1(k)+h*f(x(k),y1(k));
    y2(k+1) = y2(k)+h*f(x(k),y2(k));
    y2(k+1) = y2(k)+h/2*(f(x(k),y2(k))+f(x(k+1),y2(k+1)));
end
y3 = dsolve('Dy=b*x-y+1','y(0)=1','x');            % 精确解
y3 = subs(y3,'b',a);
y3 = subs(y3,'x',x);
figure(1);

plot(x,y1,'--','linewidth',2)
hold on
plot(x,y2,'-.','linewidth',2)
hold on
plot(x,y3,'-','linewidth',2)
legend('Euler','Euler+','exact')

xlabel('x');
ylabel('y');
grid on;
```

# 2. 实验二

用龙格库塔方法求解 $y' = -ay + ax^2 + ax$（其中 $a = randi(5,1,1)$），$0 \le x \le 0.5$，$y(0) = 1$。

## 2.1. 实验结果




可见在图示范围内，用龙格库塔方法求解的精度较高。

## 2.2. 实验参数

工作区

| 名称 | 值 |
| --- | --- |
| a | 5 |
| h | 0.0100 |
| i | 50 |
| n | 50 |
| x | *1x51 double* |
| y | *1x51 double* |
| y2 | *1x51 sym* |

```
>> test2
>> y'

ans =

    1.0000
    0.9515
    0.9058
    0.8629
    0.8226
    0.7848
    0.7493
    0.7162
    0.6852
    0.6562
    0.6293
    0.6043
    0.5811
    0.5596
    0.5398
    0.5216
    0.5048
    0.4896
    0.4758
    0.4632
    0.4520
    0.4420
    0.4332
    0.4255
    0.4189
    0.4134
    0.4088
    0.4052
    0.4026
    0.4008
    0.3999
    0.3998
    0.4005
    0.4020
    0.4042
    0.4071
    0.4107
    0.4150
    0.4199
    0.4254
    0.4316
    0.4383
    0.4456
    0.4534
    0.4617
    0.4705
    0.4799
    0.4897
    0.5000
    0.5107
    0.5219
```

## 2.3. 实验代码

```
clear;
rand('seed',1851960);
n=50;
x=zeros(1,n+1);
y=zeros(1,n+1);
x(1)=0;
y(1)=1;
h=0.01;
a=randi(5,1,1);

for i=1:n
    x(i+1)=x(i)+h;
    y(i+1)=runge_kutta(@f,x(i),y(i),h);
end

y2 = dsolve('Dy=-a*y+a*x*x+a*x','y(0)=1','x');            % 精确解
```

```
y2 = subs(y2,'a',a);
y2 = subs(y2,'x',x);

plot(x,y,'linewidth',2)
hold on
plot(x,y2,'-.','linewidth',2)
legend('Runge','exact')
```

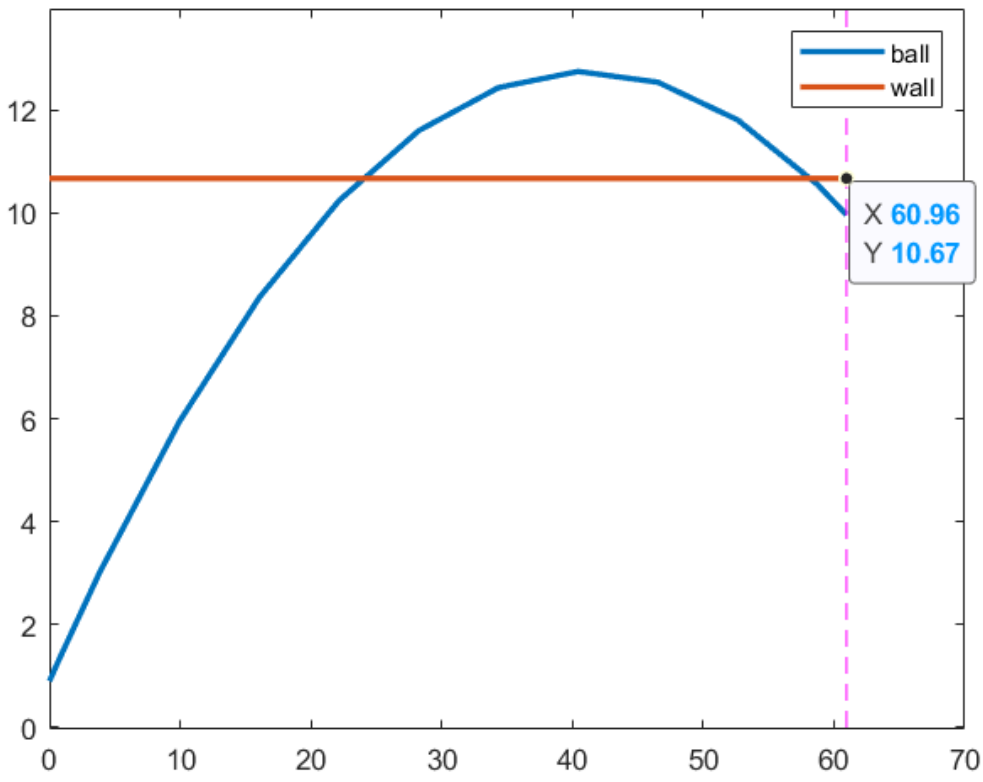
```
function y=runge_kutta(f,x,y,h)
k1=f(x,y);
k2=f(x+0.5*h,y+0.5*h*k1);
k3=f(x+0.5*h,y+0.5*h*k2);
k4=f(x+h,y+h*k3);
y=y+(1/6)*(k1+k2*2+2*k3+k4)*h;
```

```
function f=f(x,y)
rand('seed',1851960)
a=randi(5,1,1);
f=-a*y+a*x*x+a*x;
```

# 3. 实验三

棒球击出的速度为每秒30.48米，与水平线夹角为30度，球拍离地面0.9米，忽略空气和风力的阻力，球能否飞过离本垒60.96米远，10.67米高的围墙？需要多少时间？

## 3.1. 实验结果




由图可知，球无法飞过围墙，此过程经历的时间为：$t = \frac{60.96}{30.48 \times 0.5\sqrt{3}} s = 2.31s$

## 3.2. 实验参数

x

14x1 double

| | 1 | 2 |
|---|---|---|
| 1 | 0 | |
| 2 | 0.1247 | |
| 3 | 0.7482 | |
| 4 | 3.8659 | |
| 5 | 9.9619 | |
| 6 | 16.0579 | |
| 7 | 22.1539 | |
| 8 | 28.2499 | |
| 9 | 34.3459 | |
| 10 | 40.4419 | |
| 11 | 46.5379 | |
| 12 | 52.6339 | |
| 13 | 58.7299 | |
| 14 | 60.9600 | |

x y

14x1 double

| | 1 |
|---|---|
| 1 | 0.9000 |
| 2 | 0.9719 |
| 3 | 1.3281 |
| 4 | 3.0269 |
| 5 | 5.9536 |
| 6 | 8.3577 |
| 7 | 10.2391 |
| 8 | 11.5978 |
| 9 | 12.4339 |
| 10 | 12.7473 |
| 11 | 12.5380 |
| 12 | 11.8061 |
| 13 | 10.5515 |
| 14 | 9.9619 |

## 3.3. 实验代码

```
clear;
[x,y]=ode23('f2',[0,60.96],0.9);
y1=0*x+10.67;
plot(x,y,'linewidth',2)
hold on
plot(x,y1,'linewidth',2)
hold on
plot([60.96,60.96],ylim,'m--')
legend('ball','wall')
```

```
function f=f2(x,y)
v0=30.48;g=9.8;
a=sin(pi/6)*v0;b=cos(pi/6)*v0;
f=a/b-g*x/b^2+0*y;
```