Adaptive Real-Time Strategies: A Dynamic Framework for Game Theory and Strategic Interactions ⋆

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Abstract. In traditional game theory, the oversimplification of human decision-making and uncertainty in dynamic scenarios often undermines its applicability to real-world interactions. The Iterated Prisoner's Dilemma, for instance, while foundational, assumes static strategies and perfect rationality, which does not accurately represent the evolution of cooperation over time. Our proposal introduces the Strategic Adaptive Interaction Framework (SAIF), a pioneering integration of Deep Reinforcement Learning (DRL) with Agent-Based Modeling (ABM), designed to simulate and understand the complex and dynamic nature of strategic ecosystems more realistically.
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1 Introduction

1.1 Research Objectives

- To develop a framework that allows for real-time strategy adaptation by agents in response to the ongoing dynamics of the game, thereby overcoming the limitations of static strategy assumptions in traditional game theory.
- To create a holistic model of strategic interaction that captures both individual and collective decision-making processes, providing insights into the emergent behaviors that drive real-world ecosystems.

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– To explore non-obvious pathways to cooperation, demonstrating how and why cooperation can prevail even in scenarios traditionally dominated by competitive strategies.

1.2 Methodological Approach

– **Deep Reinforcement Learning:** Utilizing DRL, SAIF enables agents to dynamically adjust their strategies based on immediate outcomes and anticipated future scenarios. This adaptability introduces a level of strategic foresight previously unattainable in game theory, closely mirroring human learning and decision-making processes.
– **Agent-Based Modeling:** By simulating a network of autonomous agents each with distinct behaviors, SAIF captures the complexity and realism of strategic interactions. This approach is particularly adept at illustrating emergent behaviors from the collective dynamics of multiple agents.

1.3 Innovations in SAIF

– **Real-time Strategy Adaptation:** Unlike traditional models that fix strategies from the outset, SAIF's agents continually evolve their tactics based on past interactions and future predictions, enhancing the framework's applicability to fluctuating real-world scenarios.
– **Holistic Strategic Development:** Integrating DRL with ABM, SAIF considers both micro-level individual learning and macro-level group influences, providing a comprehensive view of strategic evolution across a network of interactions.

1.4 Potential Applications

– **Theoretical and Practical Expansion:** Initially focused on theoretical constructs like the Iterated Prisoner's Dilemma, SAIF's application spectrum will extend to real-world challenges, including negotiation tactics, market dynamics, social network behaviors, and ecological conservation strategies.
– **Policy and Strategic Planning:** SAIF's enhanced predictive capabilities can substantially aid policymakers and business leaders in crafting strategies that are more reflective of actual human behavior in strategic contexts.
– **Educational Integration:** As a teaching tool, SAIF offers a hands-on approach to understanding complex strategic interactions, benefiting both students and professionals in grasping the nuanced realities of strategy.

[luyao] **New: provide in formal academic paper writing the research description that extends the Q&A research interview on the GitHub.**

2 Background

As for traditional game theory, the game environment is essential for simplifying complex interactions into analyzable models. It allows theorists and practitioners to predict outcomes based on rational choice theory, making it invaluable in economics, political science, psychology, and beyond.

However, the traditional game theory framework, with its assumption of rationality and fixed payoffs, faces limitations in capturing the full spectrum of human and AI behavior. Human decisions are often influenced by irrational factors such as emotions, biases, and social norms, which are not easily quantified in game theory models. Moreover, AI agents, especially those using machine learning, can adapt their strategies based on new data, leading to a dynamic evolution of strategies not always accounted for in static game models.

Therefore, while the game environment of defined players, strategies, and payoffs remains a cornerstone of game theory, its evolution is inevitable and necessary to revolutionize how humans connect, behave, and interact. Here are some potential situations:

2.1 Enhanced Decision-Making

The achievements of AlphaZero, discussed in *A General Reinforcement Learning Algorithm That Masters Chess, Shogi, and Go through Self-Play* [1], highlight AI's potential to enhance decision-making. By mastering complex games through self-play and deep learning, AlphaZero not only outperformed human champions but also devised strategies beyond human conception. This example points to a future where AI's ability to process and learn from vast data sets can lead to superior decision-making in various sectors, demonstrating a significant shift towards leveraging AI for more informed, rational, and effective problem-solving across diverse fields.

2.2 Enhancing Social Interactions

Virtual Reality (VR) and Augmented Reality (AR), powered by AI, are creating immersive social experiences, allowing people to connect and interact in digital spaces that are increasingly realistic. This could lead to new forms of social interaction, breaking down geographical barriers and creating new communities based on shared interests rather than physical proximity.

By embracing the complexities of human behavior and the capabilities of AI, we can develop richer, more dynamic models that better reflect the intricacies of real-world interactions. This evolution promises not only to enhance our theoretical understanding but also to offer practical insights for navigating the challenges and opportunities of an increasingly interconnected world.

3 An Illustration Example: IPD

3.1 Setting Up the Scenario:

- **Objective:** Demonstrate how dynamic strategy adaptation can lead to different outcomes in the IPD.
- **Method:** Use DRL for strategy adaptation where each agent learns and evolves their strategy based on the game's progression.
- **Tools:** Google Colab for coding, Python for implementation (using libraries like TensorFlow for DRL, and Mesa for ABM).
- **Importance:** This approach not only facilitates the exploration of emergent cooperative behaviors that traditional static models may overlook but also enhances the predictive capabilities of the model, making it more applicable to complex, real-world scenarios. Additionally, the integration of ABM provides a comprehensive view of individual and collective dynamics, offering deeper insights into the systemic behavior and potential outcomes of strategic interactions.

3.2 Implementation Steps

Define the Environment

- The IPD game is set up where agents can choose to either cooperate (C) or defect (D) in each round.
- The payoff matrix remains as in traditional IPD, but agents can adapt their strategy after each round.

3.3 Model Agent Behaviors

- Each agent is modeled using a DRL algorithm, which allows them to learn from past games and adjust their strategies based on what they anticipate the opponent will do in future rounds.
- Agents store and analyze outcomes of past interactions to refine their decision-making process.

3.4 Simulation and Learning

- Initialize a population of agents with random strategies.
- Let them interact in numerous rounds of IPD, observing and learning from each interaction.
- Agents update their strategies continuously using a DRL model, which optimizes their decisions based on a reward structure that values both short-term gains and long-term cooperation.

3.5 Analysis

- Compare the evolution of strategies under SAIF with traditional static approaches.
- Evaluate how often and under what conditions cooperation emerges.
- Analyze the stability and efficiency of the resulting equilibria compared to traditional Bayesian Nash Equilibrium or Subgame Perfect Equilibrium.

3.6 Outcome Visualization

- Use Python's visualization libraries (e.g., Matplotlib, Seaborn) to illustrate the strategic evolution and outcome distribution over time.

A The Pioneers in the History of Game Theory

- 1947: Game theory became a distinct field with the publication of *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern in 1947. This foundational work emphasized strategies among rational players and established game theory as its own discipline.[2]
- 1950: In 1950, John F. Nash Jr introduced the concept of mixed-strategy Nash Equilibrium, expanding the analysis of strategic interactions to include probabilistic strategies. Nash's work distinguished between non-cooperative and cooperative games, enriching our understanding of both independent and collective decision-making.[3]
- 1965: Reinhard Selten in 1965 introduced dynamic games, allowing for the analysis of sequential strategic interactions. This advancement was a significant extension of Nash's equilibrium concepts.[4]
- 1967: In the late 1960s, specifically 1967, John C. Harsanyi shifted the focus towards games with imperfect information, providing a theoretical foundation for analyzing scenarios where players' decisions are influenced by uncertainty regarding other players' intentions.[5]

B Review Classic Games, Nash Equilibrium and the Analytical Tools

B.1 Exploring Inspirational Games in Strategic or Normal Form

The game I have chosen to analyze is the "Resource Management Game." This game simulates a scenario in which three companies, labeled A, B, and C, make strategic decisions regarding investments in research and development (R&D) to compete in the market. Each company has the option to invest at Low (L), Medium (M), or High (H) levels. The strategic choices are presented in a 3x3 matrix format, reflecting the simplified payoff matrix for Company A, assuming Companies B and C choose similarly. This matrix illustrates the net income based on their respective investment levels. The setup effectively captures the essence of strategic interaction in competitive markets, focusing on investment decisions and their implications for market dynamics and company performance.

Inspiration and Significance The game is particularly inspiring due to its reflection of real-world competitive dynamics, showcasing how strategic choices directly impact market success. The decision-making structure goes beyond simple binary choices, emphasizing the complexity and interdependence of corporate strategy.

- Depth of Strategy: The game offers a richer strategic environment compared to simpler 2x2 matrix games, allowing for more detailed planning and forecasting.
- Real-World Relevance: It mimics actual corporate decision-making processes, highlighting the importance of strategic investment in innovation.
- Illustration of Strategic Interdependence: The game demonstrates how a company's payoff is affected by the actions of its competitors, emphasizing the need for anticipation and strategic flexibility.
- you must provide basic discussions to compare the definition, theorem, and proof

As a student interested in finance, I find the significance of game theory, particularly demonstrated through games like the "Resource Management Game," resonates deeply due to its applicability in financial markets and corporate strategy. This game serves as a microcosm of the strategic decisions that companies face in competitive markets, where investment decisions not only affect individual company performance but also shape industry dynamics. In the finance sector, firms often make investment decisions influenced not only by their internal metrics but also by the actions of competitors. The Resource Management Game aids in understanding how companies can strategically position themselves in a market where investment levels significantly impact both their returns and market behavior.

Game Theory and Business Strategy The game's significance lies in demonstrating the intricacies of strategic planning and the competitive advantage gained through investment in innovation. By introducing a range of strategic options (Low, Medium, High), it complicates the analysis, making it a potent example for exploring strategic interactions and the concept of Nash Equilibrium in a more complex setting. Solving the game involves identifying Nash Equilibria to determine the optimal strategy for each company, considering the likely responses of competitors. In essence, the "Resource Management Game" enhances the understanding of strategic interactions by illustrating the balance between risk and reward in competitive investments, offering valuable insights into both game theory and business strategy.

For more information and to try the game, visit the following link: Resource Management Game on Google Colab.

B.2 Delving into Extensive-Form Games

Introduction: One of the most interesting and widely discussed extensive-form games in game theory literature is the "Ultimatum Game." It was introduced in the seminal paper *An experimental analysis of ultimatum bargaining* [6]

Description of the Ultimatum Game The Ultimatum Game involves two players: a proposer and a responder. The proposer is given a sum of money (or any divisible good) and must decide how to split this sum between themselves and the responder. The proposer makes an offer to the responder, who then chooses either to accept or reject it. If the responder accepts the offer, the money is split according to the proposal. If the offer is rejected, both players receive nothing, highlighting the interplay between economic self-interest and fairness [7].

Compelling Aspects of the Ultimatum Game

- Irrational Rejections: What makes the Ultimatum Game particularly compelling is the frequency of irrational rejections. Responders often reject offers that are low but still better than nothing. From a purely rational economic perspective, any positive amount of money should be accepted, as receiving something is better than receiving nothing (refer to Figures 1 and 2). However, in practice, offers perceived as unfair—often those less than 30% of the total—are frequently rejected. This behavior challenges the traditional economic assumption that agents are purely self-interested and money-maximizing.
- Variability Across Cultures: The Ultimatum Game has been instrumental in cross-cultural studies, illustrating how notions of fairness and acceptable behavior vary significantly across different societies. For instance, [8] used the game to show how economic decisions are influenced by cultural contexts, challenging the universality of the "rational economic agent" model. These studies reveal that economic behavior cannot be fully understood without considering the cultural backdrop against which it unfolds.

B.3 Critiquing Nash Equilibrium and Envisioning Innovations:

A key limitation of current analytical tools, such as Nashpy and QuantEcon, and the Nash Equilibrium concept is their reliance on assumptions of rationality and complete information. These assumptions often do not align with real-world scenarios. Moreover, these tools frequently struggle with dynamic interactions and games that evolve, as well as the computational complexity in analyzing large, complex games (refer to Figure 5 for Mindmap).

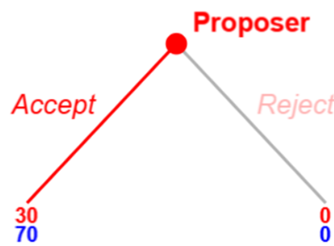

Fig. 1. The SPNE of the game when the proposer gives 30% of the money, the decision is "ACCEPT"

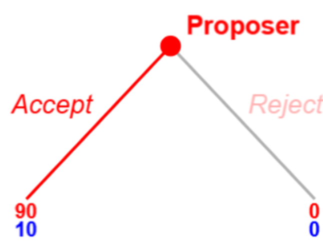

Fig. 2. The SPNE of the game when the proposer gives 90% of the money, the decision is "ACCEPT"

Case Study: The Traveler's Dilemma A specific example illustrating these limitations is the "Traveler's Dilemma". This game involves two players (travelers) who must independently choose a compensation amount from a set range, say between \$2 and \$100. The rules are as follows: If both players choose the same amount, then both are paid that amount. If one player chooses a lower amount than the other, the player who chose the lower amount is considered more honest and is paid their chosen amount plus a small bonus (say \$2), while the other player is penalized by the same amount (\$2) and receives their chosen amount minus \$2 [9].

The Nash Equilibrium of this game, under the assumption of rationality and complete information, suggests that both players will choose the lowest possible number (\$2 in this case). This outcome results from each player undercutting the other to avoid being penalized, despite higher payouts being possible if both cooperated by choosing \$100. However, in real-life implementations of the game, players often do not choose \$2, instead opting for significantly higher amounts. This behavior contradicts the prediction of Nash Equilibrium under strict rationality, suggesting that players consider other factors such as fairness, potential cooperation, or fear of appearing greedy.

Proposed Enhancements to Analytical Tools To address these limitations, analytical tools like Nashpy and QuantEcon could integrate features that:

- **Dynamic Strategy Modeling:** Enhance capabilities to model dynamic games where players' strategies evolve based on historical interactions and changing information landscapes.
- **Information Asymmetry Models:** Develop models that can handle games with incomplete or asymmetric information, providing more accurate predictions in such environments.

Bridging Theoretical and Practical Applications My aspirations lie in bridging the gap between theoretical computer science and practical, real-world applications, specifically through the lens of artificial intelligence and machine learning. The contribution I aim to make is to improve the predictive capabilities of game-theoretic tools by incorporating AI-driven analysis of past player behaviors and outcomes. Additionally, I envision providing tools that can simulate and analyze complex interactions under varied conditions, thereby offering strategic insights applicable in economics, politics, business, and beyond.

B.4 Bayesian (Subgame Perfect) Nash Equilibrium: The definitions

B.4.1. The Economist Perspectives

Refer to Textbook: Osborne, Martin J. and Ariel Rubinstein. 1994. A Course in Game Theory.

Definition 1 (Bayesian Nash Equilibrium). *A **Bayesian Nash Equilibrium** $\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\succsim_i)\rangle$ is a Nash equilibrium of the strategic game defined as follows.*

- *The set of players is the set of all pairs (i, t_i) for $i \in N$ and $t_i \in T_i$.*
- *The set of actions of each player (i, t_i) is A_i.*
- *The preference ordering $\succsim^*_{(i,t_i)}$ of each player (i, t_i) is defined by*

$$a^* \succsim^*_{(i,t_i)} b^* \text{ if and only if } L_i(a^*, t_i) \succsim_i L_i(b^*, t_i),$$

where $L_i(a^*, t_i)$ is the lottery over $A \times \Omega$ that assigns probability $p_i(\omega)/p_i(\tau_i^{-1}(t_i))$ to $((a^*(j, \tau_j(\omega)))_{j\in N}, \omega)$ if $\omega \in \tau_i^{-1}(t_i)$, zero otherwise.

Definition 2 (Subgame Perfect Equilibrium). *Let $\Gamma = \langle N, H, P, (\succsim_i)\rangle$ be an extensive game with perfect information. A strategy profile s^* such that for every player $i \in N$ and every nonterminal history $h \in H \backslash Z$ for which $P(h) = i$ we have*

$$O_h(s^*_{-i}|_h, s^*_i|_h) \succsim_i |_h O_h(s^*_{-i}|_h, s_i)$$

for every strategy s_i of player i in the subgame $\Gamma(h)$.

B.4.2. The Computer Scientist Perspectives

Refer to Textbook: *Shoham, Yoav and Leyton-Brown, Kevin.* 2008. MULTIAGENT SYSTEMS (Chapter 6, Page 170, DEFINITION 6.3.7)

Definition 3 (Bayes–Nash equilibrium). *A **Bayes–Nash equilibrium** is a mixed-strategy profile s that satisfies $\forall i: s_i \in BR_i(s_{-i})$.*

Definition 4 (Subgame-perfect Equilibria). *The **subgame-perfect equilibria** (SPE) of a game G are all strategy profiles s such that for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'.*

B.5 Bayesian (Subgame Perfect) Nash Equilibrium: The Thereom

B.5.1. The Economist Perspectives

Proposition 1. *Every finite extensive game with perfect information has a subgame perfect equilibrium.*

Proof. Let $\Gamma = \langle N, H, P, (\succsim_i)\rangle$ be a finite extensive game with perfect information. We construct a subgame perfect equilibrium of Γ by induction on $\ell(\Gamma(h))$; at the same time we define a function R that associates a terminal history with every history $h \in H$ and show that this history is a subgame perfect equilibrium outcome of the subgame $\Gamma(h)$.

If $\ell(\Gamma(h)) = 0$ (i.e., h is a terminal history of Γ) define $R(h) = h$. Now suppose that $R(h)$ is defined for all $h \in H$ with $\ell(\Gamma(h)) \leq k$ for some $k \geq 0$. Let h^* be a history for which $\ell(\Gamma(h^*)) = k + 1$ and let $P(h^*) = i$. Since $\ell(\Gamma(h^*)) = k + 1$ we have $\ell(\Gamma(h^*, a)) \leq k$ for all $a \in A(h^*)$. Define $s_i(h^*)$ to be a $\succsim_i$-maximizer of $R(h^*, a)$ over $a \in A(h^*)$, and define $R(h^*) = R(h^*, s_i(h^*))$. By induction we have now defined a strategy profile s in Γ; by Lemma 1 this strategy profile is a subgame perfect equilibrium of Γ.

B.5.2. The Computer Scientist Perspectives

Theorem 1. *Given a two-player perfect-information extensive-form game with ℓ leaves, the set of subgame-perfect equilibrium payoffs can be represented as the union of $O(\ell^2)$ axis-aligned rectangles and can be computed in time $O(\ell^3)$.*

Proof. We proceed by induction on the structure of the game tree. For the base case, consider a game tree with a single leaf (i.e., a single decision node). In this case, the set of subgame-perfect equilibrium payoffs is trivially a single point, and hence a rectangle.

For the inductive step, assume the theorem holds for all game trees with fewer than ℓ leaves. Consider a game tree with ℓ leaves. By the inductive hypothesis, each subgame rooted at a child of the root can be solved independently, with the set of equilibrium payoffs for each subgame representable as a union of axis-aligned rectangles.

To show the time complexity is $O(\ell^3)$, we count the operations required to compute the equilibrium for each subgame. The union of rectangles from each subgame can be computed in $O(\ell^2)$ time, and there are ℓ subgames to consider.

Therefore, by induction, the set of subgame-perfect equilibrium payoffs for the entire game is a union of $O(\ell^2)$ rectangles, and the entire solution can be computed in $O(\ell^3)$ time.

C Game Theory Glossary Tables

Table 1. Basic Glossaries in Game Theory

Glossary	Definition	Sources
Zero-sum Game	A situation where one's gain is exactly balanced by another's loss.	[2]
Dominant Strategy	A strategy that is best for a player, regardless of others' strategies.	[3]
Mixed Strategy	A random choice among actions based on set probabilities.	[12]
Pareto Efficiency	An allocation where improving one's outcome would worsen another's.	[4]
Prisoner's Dilemma	A game illustrating why two individuals might not cooperate, even if it appears that it is in their best interest to do so.	[5]

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