The Australian National University
School of Computing

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Algebratic effect examples

Example : return 0

$$\frac{\dfrac{}{\Gamma \vdash 0 : \tau}\,[\text{LIT}]}{\Gamma \vdash \text{return } 0 : \tau \mid \emptyset}\,[\text{RET}]$$

Replace τ with Int:

$$\frac{\dfrac{}{\Gamma \vdash 0 : Int}\,[\text{LIT}]}{\Gamma \vdash \text{return } 0 : Int \mid \emptyset}\,[\text{RET}]$$

Example: val x = 1; return x;

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash 1 : \tau_0}\,[\text{LIT}]}{\Gamma \vdash \text{return } 1 : \tau_0 \mid \emptyset}\,[\text{RET}] \qquad \dfrac{\dfrac{\text{x} : \tau_1 \in \Gamma \text{ , x} : \tau_0}{\Gamma \vdash x : \tau_0 \vdash x : \tau_1}\,[\text{VAR}]}{\Gamma, x : \tau_0 \vdash \text{return x} : \tau_1 \mid C_1}\,[\text{RET}]}{\Gamma \vdash \text{val x} = \text{return 1; return x} : \tau_1 \mid \emptyset \cup C_1}\,[\text{VAL}]$$

Replace τ_0 and τ_1 with Int, C_1 with $\emptyset$

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash 1 : Int}\,[\text{LIT}]}{\Gamma \vdash \text{return } 1 : Int \mid \emptyset}\,[\text{RET}] \qquad \dfrac{\dfrac{\text{x} : Int \in \Gamma \text{ , x} : Int}{\Gamma \vdash x : Int \vdash x : Int}\,[\text{VAR}]}{\Gamma, x : Int \vdash \text{return x} : Int \mid \emptyset}\,[\text{RET}]}{\Gamma \vdash \text{val x} = \text{return 1; return x} : Int \mid \emptyset \cup \emptyset}\,[\text{VAL}]$$

Example: def Identity = {(x : Int, ∅) ⇒ return x}; Identity(1,∅);

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{x} : \tau_1 \in \Gamma, (x : Int, \emptyset)}{\Gamma, (x : Int, \emptyset) \vdash x : \tau_1 \mid C}\,[\mathrm{VAR}]}{\Gamma, (x : Int, \emptyset) \vdash \text{return x} : \tau_1 \mid C \cup \overrightarrow{g_j}}\,[\mathrm{RET}]}{\Gamma \vdash \{(\mathrm{x} : Int, \emptyset) \Rightarrow \text{return x}\} : \sigma \mid C}\,[\mathrm{BLOCK}] \qquad \dfrac{\dfrac{Identity :^{C} (\tau'', \emptyset) \to \tau \in \Gamma, Identity :^{C'}}{\Gamma, Identity :^{C'} \vdash Identity \ : \ (\tau'', \emptyset) \to \tau \mid C}\,[\mathrm{LIT}] \qquad \dfrac{}{\Gamma, Identity :^{C'} \vdash 1 : \tau \mid C}\,[\mathrm{LIT}] \qquad \Gamma, Identity :^{C'} \vdash \emptyset : \sigma \mid C}{\Gamma, Identity :^{C'} \ \sigma \vdash Identity(1, \emptyset) : \tau \mid C}\,[\mathrm{APP}]$$

$$\Gamma \vdash \text{def Identity} = \{(\mathrm{x} : \mathrm{Int}, \emptyset) \Rightarrow \text{return x}\}; \text{Identity}(1,\emptyset)\,[\mathrm{DEF}]$$

Replace τ_0 and τ_1 with Int, C_1 with $\emptyset$