The Australian National University
School of Computing

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Algebratic effect examples

Example : return 0

$$\frac{\dfrac{}{\Gamma \vdash 0 : \tau}\,[\text{LIT}]}{\Gamma \vdash \text{return } 0 : \tau \mid \emptyset}\,[\text{RET}]$$

Replace τ with Int:

$$\frac{\dfrac{}{\Gamma \vdash 0 : Int}\,[\text{LIT}]}{\Gamma \vdash \text{return } 0 : Int \mid \emptyset}\,[\text{RET}]$$

Example: val x = 1; return x;

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash 1 : \tau_0}\,[\text{LIT}]}{\Gamma \vdash \text{return } 1 : \tau_0 \mid \emptyset}\,[\text{RET}] \qquad \dfrac{\dfrac{\text{x} : \tau_1 \in \Gamma \text{ , x} : \tau_0}{\Gamma \vdash x : \tau_0 \vdash x : \tau_1}\,[\text{VAR}]}{\Gamma, x : \tau_0 \vdash \text{return x} : \tau_1 \mid C_1}\,[\text{RET}]}{\Gamma \vdash \text{val x} = \text{return 1; return x} : \tau_1 \mid \emptyset \cup C_1}\,[\text{VAL}]$$

Replace τ_0 and τ_1 with Int, C_1 with $\emptyset$

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash 1 : Int}\,[\text{LIT}]}{\Gamma \vdash \text{return } 1 : Int \mid \emptyset}\,[\text{RET}] \qquad \dfrac{\dfrac{\text{x} : Int \in \Gamma \text{ , x} : Int}{\Gamma \vdash x : Int \vdash x : Int}\,[\text{VAR}]}{\Gamma, x : Int \vdash \text{return x} : Int \mid \emptyset}\,[\text{RET}]}{\Gamma \vdash \text{val x} = \text{return 1; return x} : Int \mid \emptyset \cup \emptyset}\,[\text{VAL}]$$

Example: def Identity = {(x : Int, $\emptyset$) $\Rightarrow$ return x}; Identity(1,$\emptyset$);

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{x} : \tau_1 \in \Gamma, \Gamma, x : Int, \emptyset : \sigma}{\Gamma, x : Int, \emptyset : \sigma \vdash x : \tau_1 | \ C}\,[\mathrm{VAR}]}{\Gamma, x : Int, \emptyset : \sigma \vdash \mathrm{return\ x} : \tau_1 \ | \ C \cup \overrightarrow{g_j}}\,[\mathrm{RET}]}{\Gamma \vdash \{(\mathrm{x} : Int, \emptyset) \Rightarrow \mathrm{return\ x}\} : \sigma \mid C}\,[\mathrm{BLOCK}] \quad \dfrac{\dfrac{Identity :^{C} (\tau_2, \emptyset) \to \tau_0 \in \Gamma, Identity :^{C'}}{\Gamma, Identity :^{C} \vdash Identity \ : \ (\tau_2, \emptyset) \to \tau_0 \mid C}\,[\mathrm{TRANS}] \quad \dfrac{}{\Gamma, Identity :^{C'} \vdash 1 : \tau_0 \mid C}\,[\mathrm{LIT}] \quad \Gamma, Identity :^{C'} \vdash \emptyset : \sigma | C}{\Gamma, Identity :^{C} \sigma \vdash Identity(1, \emptyset) : \tau_0 \mid C}\,[\mathrm{APP}]}{\Gamma \vdash \mathrm{def\ Identity} = \{(\mathrm{x} : \mathrm{Int}, \emptyset) \Rightarrow \mathrm{return\ x}\}; \mathrm{Identity}(1,\emptyset) : \tau_0;}\,[\mathrm{DEF}]$$

Replace C with $\emptyset$ (for [Ret] to apply at leftmost proof tree), $\tau_{0,1,2}$ with Int, $\overrightarrow{g_j}$ with $\emptyset$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{x} : Int \in \Gamma, \Gamma, x : Int, \emptyset : \emptyset}{\Gamma, x : Int, \emptyset : \emptyset \vdash x : Int | \ \emptyset}\,[\mathrm{VAR}]}{\Gamma, x : Int, \emptyset : \emptyset \vdash \mathrm{return\ x} : Int \ | \ \emptyset}\,[\mathrm{RET}]}{\Gamma \vdash \{(\mathrm{x} : Int, \emptyset) \Rightarrow \mathrm{return\ x}\} : \emptyset \mid \emptyset}\,[\mathrm{BLOCK}] \quad \dfrac{\dfrac{Identity :^{\emptyset} (Int, \emptyset) \to Int \in \Gamma, Identity :^{\emptyset}}{\Gamma, Identity :^{\emptyset} \vdash Identity \ : \ (Int, \emptyset) \to Int \mid \emptyset}\,[\mathrm{TRANS}] \quad \dfrac{}{\Gamma, Identity :^{\emptyset} \vdash 1 : Int \mid \emptyset}\,[\mathrm{LIT}] \quad \Gamma, Identity :^{\emptyset} \vdash \emptyset : \emptyset | \emptyset}{\Gamma, Identity :^{\emptyset} \emptyset \vdash Identity(1, \emptyset) : Int \mid \emptyset}\,[\mathrm{APP}]}{\Gamma \vdash \mathrm{def\ Identity} = \{(\mathrm{x} : \mathrm{Int}, \emptyset) \Rightarrow \mathrm{return\ x}\}; \mathrm{Identity}(1,\emptyset) : Int;}\,[\mathrm{DEF}]$$