$$H_{4,2}^{4,5}\left(\cdot\,\middle|\,\begin{matrix}(1,\frac{1}{\alpha}),(1,1),(\mathrm{Ceil}(\beta),\beta),(1,1)\\(\frac{1}{2},\frac{\alpha}{2}),(1,1),(3,3),(2,2),(1,\frac{\alpha}{2})\end{matrix}\right)$$

Summary

$$\begin{aligned}a^* &= \frac{1}{\alpha} - \beta + 6\\ \Delta &= \alpha - \frac{1}{\alpha} - \beta + 4\\ \delta &= \frac{2^{-\alpha}\left(2^{\frac{\alpha}{2}+5}\alpha^{\alpha/2} + \alpha^\alpha\right)}{\left(\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}} + 1\right)\left(\beta^\beta + 1\right)}\\ \mu &= 4 - \mathrm{Ceil}(\beta)\\ a_1^* &= \frac{\alpha}{2} - \beta + 5\\ a_2^* &= -\frac{\alpha}{2} + \frac{1}{\alpha} + 1\\ \xi &= \frac{13}{2} - \mathrm{Ceil}(\beta)\\ c^* &= \frac{3}{2}\end{aligned}$$

Poles

1. First ten poles from upper front list

$$a_{i,k} = \begin{pmatrix} 0 & 0\\ \alpha & 1\\ 2\alpha & 2\\ 3\alpha & 3\\ 4\alpha & 4\\ 5\alpha & 5\\ 6\alpha & 6\\ 7\alpha & 7\\ 8\alpha & 8\\ 9\alpha & 9\\ 10\alpha & 10\end{pmatrix}$$

2. First ten poles from lower front list

$$
b_{j,\ell} = \begin{pmatrix}
-\frac{1}{\alpha} & -1 & -1 & -1 \\
-\frac{3}{\alpha} & -2 & -\frac{4}{3} & -\frac{3}{2} \\
-\frac{5}{\alpha} & -3 & -\frac{5}{3} & -2 \\
-\frac{7}{\alpha} & -4 & -2 & -\frac{5}{2} \\
-\frac{9}{\alpha} & -5 & -\frac{7}{3} & -3 \\
-\frac{11}{\alpha} & -6 & -\frac{8}{3} & -\frac{7}{2} \\
-\frac{13}{\alpha} & -7 & -3 & -4 \\
-\frac{15}{\alpha} & -8 & -\frac{10}{3} & -\frac{9}{2} \\
-\frac{17}{\alpha} & -9 & -\frac{11}{3} & -5 \\
-\frac{19}{\alpha} & -10 & -4 & -\frac{11}{2} \\
-\frac{21}{\alpha} & -11 & -\frac{13}{3} & -6
\end{pmatrix}
$$