$$H_{2,1}^{2,3}\left(\cdot\,\middle|\,\begin{smallmatrix}(1,\frac{1}{\alpha}),(\mathrm{Ceil}(\beta),\beta)\\ (\frac{1}{2},\frac{\alpha}{2}),(1,1),(1,\frac{\alpha}{2})\end{smallmatrix}\right)$$

Summary

$$\begin{aligned}
a^* &= \frac{1}{\alpha} - \beta + 1\\
\Delta &= \alpha - \frac{1}{\alpha} - \beta + 1\\
\delta &= 2^{-\alpha}\left(\frac{1}{\alpha}\right)^{-1/\alpha}\left(2^{\alpha/2}\alpha^{\alpha/2} + \alpha^{\alpha}\right)\beta^{-\beta}\\
\mu &= 1 - \mathrm{Ceil}(\beta)\\
a_1^* &= \frac{1}{2}(\alpha - 2\beta + 2)\\
a_2^* &= \frac{1}{\alpha} - \frac{\alpha}{2}\\
\xi &= \frac{3}{2} - \mathrm{Ceil}(\beta)\\
c^* &= \frac{1}{2}
\end{aligned}$$

Poles

1. First ten poles from upper front list

$$a_{i,k} = \begin{pmatrix} 0\\ \alpha\\ 2\alpha\\ 3\alpha\\ 4\alpha\\ 5\alpha\\ 6\alpha\\ 7\alpha\\ 8\alpha\\ 9\alpha\\ 10\alpha \end{pmatrix}$$

2. First ten poles from lower front list

$$b_{j,\ell} = \begin{pmatrix} -\frac{1}{\alpha} & -1 \\ -\frac{3}{\alpha} & -2 \\ -\frac{5}{\alpha} & -3 \\ -\frac{7}{\alpha} & -4 \\ -\frac{9}{\alpha} & -5 \\ -\frac{11}{\alpha} & -6 \\ -\frac{13}{\alpha} & -7 \\ -\frac{15}{\alpha} & -8 \\ -\frac{17}{\alpha} & -9 \\ -\frac{19}{\alpha} & -10 \\ -\frac{21}{\alpha} & -11 \end{pmatrix}$$