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Some symbolic tools for the Fox H-function

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In this note, we explain the code for checking the conditions of the Fox H-function [Fox61]. Here we follow the notation from Kilbas and Saigo [KS].

Let m, n, p, q be configure integers such that

$$0 \leq m \leq q \quad \text{and} \quad 0 \leq n \leq p.$$

Let $a_i, b_j \in \mathbb{C}$ and $\alpha_i, \beta_j \in \mathbb{R}_+$ be the parameters given below:

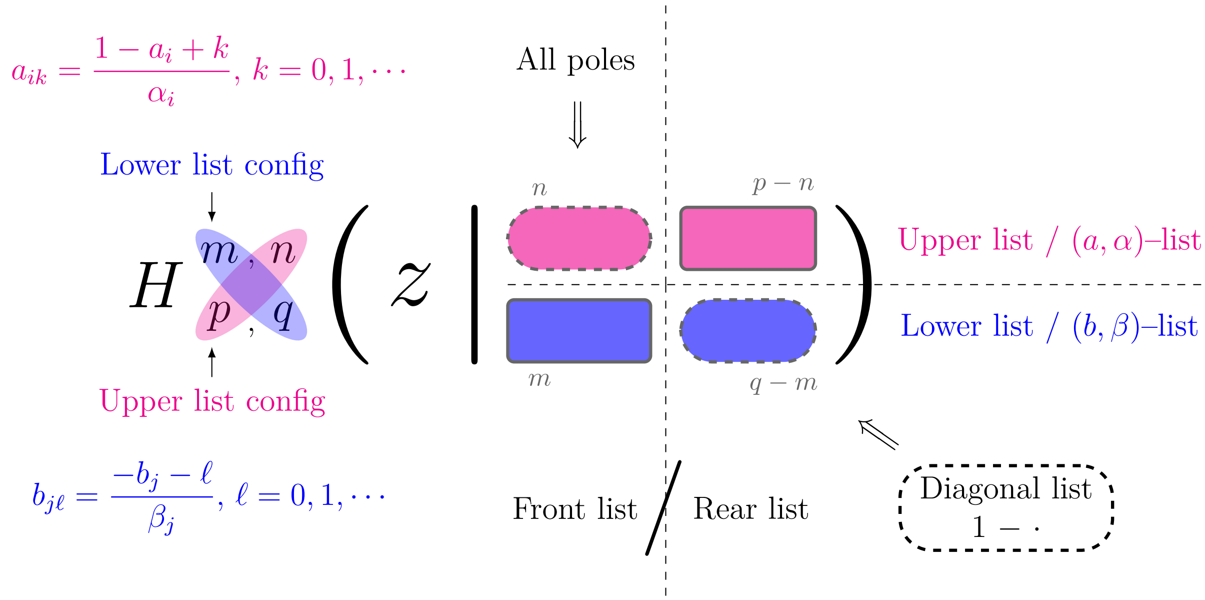
$\in (\mathbb{C}, \mathbb{R}_+)$	Front list	Rear list	
p	$(a_1, \alpha_1), \cdots, (a_n, \alpha_n)$	$(a_{n+1}, \alpha_{n+1}), \cdots, (a_p, \alpha_p)$	Upper list
q	$(b_1, \beta_1), \cdots, (b_m, \beta_m)$	$(b_{m+1}, \beta_{m+1}), \cdots, (b_q, \beta_q)$	Lower list

and denote

$$H_{p,q}^{m,n}(s) := \frac{\displaystyle\prod_{i=1}^{n} \Gamma\left(1 - a_i - \alpha_i s\right)}{\displaystyle\prod_{i=n+1}^{p} \Gamma\left(a_j + \alpha_i s\right)} \times \frac{\displaystyle\prod_{j=1}^{m} \Gamma\left(b_j + \beta_j s\right)}{\displaystyle\prod_{j=m+1}^{q} \Gamma\left(1 - b_j - \alpha_j s\right)}.$$

Then the Fox H-function $H_{2,3}^{2,1}\left(z \,\middle|\, \substack{\cdots \\ \cdots}\right)$ is defined by a Mellin-Barnes type integral of the form

$$H_{m,n}^{p,q}\left(z \,\middle|\, \substack{(a_1,\alpha_1),\cdots,(a_p,\alpha_p) \\ (b_1,\beta_1),\cdots,(b_q,\beta_q)}\right) := \frac{1}{2\pi i} \int_{\mathcal{L}} H_{p,q}^{m,n}(s) z^{-s} s.$$



References

[Fox61] Charles Fox. The G and H functions as symmetrical Fourier kernels. *Trans. Amer. Math. Soc.*, 98:395–429, 1961.

[KS] Anatoly A. Kilbas and Megumi Saigo. *H-transforms*, volume 9 of *Analytical Methods and Special Functions*. Chapman & Hall/CRC, Boca Raton, FL. Theory and applications.