SIMULATION STUDIES

The treatment effects is measured as the difference between $y_{1,i,t}$ and the predicted $y_{0,i,t}$. Since the true DGP is unknown, the only way to consider which method is more likely to yield more accurate $y_{0,i,t}$ in a wide array of situations is to conduct computer simulations. In the DGPs below, we assume that the common factors $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ are $i.i.d. N(0,1)$; the factor loadings $\gamma_{1,i}$, $\gamma_{2,i}$ and $\gamma_{3,i}$ are also $i.i.d. N(0,1)$, unless they are specified otherwise. The coefficients are set at $\beta_1 = 1$ and $\beta_2 = 4$. The DGPs are designed as follows:

DGP1. Model with exogenous variables and common factors:

$$y_{i,t} = x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

The covariates $x_{k,i,t}(k = 1,2)$ are (positively) correlated with the factors as follows:

$$x_{k,i,t} = 1 + \rho_{k,i}x_{k,i,t-1} + c_{1,i}f_{1,t} + c_{2,i}f_{2,t} + c_{3,i}f_{3,t} + \eta_{k,i,t}, \quad k = 1,2$$

where $\rho_{k,i} \sim i.i.d\ U(0.1,0.9)$, $c_{1,i}, c_{2,i}, c_{3,i} \sim i.i.d\ U(1,2)$ and $\eta_{k,i,t}, u_{i,t} \sim i.i.d\ N(0,1)$.

DGP2. Model with exogenous variables and common factors:

$$y_{i,t} = x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

The covariates $x_{k,i,t}(k = 1,2)$ follow an autoregressive moving average (ARMA) process as

$$x_{k,i,t} = 1 + \rho_{k,i}x_{k,i,t-1} + \eta_{k,i,t} + \varphi_{k,i}\eta_{k,i,t-1}, \quad k = 1,2,$$

where $\rho_{k,i}, \varphi_{k,i} \sim i.i.d\ U(0.1,0.9)$ and $\eta_{k,i,t} \sim i.i.d\ N(0,1)$.

DGP3. Non-linear model with exogenous variables and common factors:

$$y_{i,t} = \Lambda(x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t}) + u_{i,t}$$

$$\Lambda(x) = \frac{1}{1 + e^{-x}}$$

The covariates $x_{k,i,t}(k = 1,2)$ follow an autoregressive moving average (ARMA) process as

$$x_{k,i,t} = 1 + \rho_{k,i}x_{k,i,t-1} + \eta_{k,i,t} + \varphi_{k,i}\eta_{k,i,t-1}, \quad k = 1,2,$$

where $\rho_{k,i}, \varphi_{k,i} \sim i.i.d\ U(0.1,0.9)$ and $\eta_{k,i,t} \sim i.i.d\ N(0,1)$.

DGP4. DGP1 with heteroskedasticity:

$$y_{i,t} = x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

where $u_{i,t} \sim i.i.d\ N(0, \sigma_i^2)$ and $\sigma_i^2 \sim i.i.d\ U(0,1)$.

DGP5. DGP1 with autocorrelation:

$$y_{i,t} = x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

$$u_{i,t} = \sqrt{1 - \varphi_i^2}(\varphi_i\, u_{i,t-1} + \zeta_{i,t})$$

where $\varphi_i \sim i.i.d\ U(0.1,0.9)$ and $\zeta_{i,t} \sim i.i.d\ N(0,1)$

DGP6. DGP1 with heteroscedasticity and autocorrelation:

$$y_{i,t} = x_{1,i,t}\beta_1 + x_{2,i,t}\beta_2 + \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

$$u_{i,t} = \varphi_i\, u_{i,t-1} + \zeta_{i,t}$$

where $\varphi_i \sim i.i.d\ U(0.1,0.9)$ and $\zeta_{i,t} \sim i.i.d\ N(0,1)$

DGP7. Pure factor model:

$$y_{i,t} = \gamma_{1,i}f_{1,t} + \gamma_{2,i}f_{2,t} + \gamma_{3,i}f_{3,t} + u_{i,t}$$

where $y_{i,t}$ not depended on $x_{k,i,t}$. The covariates $x_{k,i,t}(k = 1,2)$ are (positively) correlated with factors as follows:

$$x_{k,i,t} = 1 + \rho_{k,i}x_{k,i,t-1} + c_{1,i}f_{1,t} + c_{2,i}f_{2,t} + c_{3,i}f_{3,t} + \eta_{k,i,t}, \quad k = 1,2$$

where $\rho_{k,i} \sim i.i.d\ U(0.1,0.9)$, $c_{1,i}, c_{2,i}, c_{3,i} \sim i.i.d\ U(1,2)$ and $\eta_{k,i,t}, u_{i,t} \sim i.i.d\ N(0,1)$.

The treatment and control groups consist of 1 and $N - 1$ units, respectively. The treatment for unit 1 starts at time $T_0 + 1$.The other $(N - 1)$ units are not subject to treatment. We let $N = 40, 70$ and the pretreatment time $T_0 = 40,70$ as well. The posttreatment periods are set at $T - T_0 = 10$; that is, $T = 40,60$. The number of replications is set at $R = 2,000$.

We consider four criteria for comparison: the coverage probability of confidence interval for treatment effect (CP) at each post-treatment period, the width of confidence interval for treatment effect (WCI) at each post-treatment period, the mean of the sum of squared error for the actual outcomes and the counterfactuals at each post-treatment period (MSE), the median absolute deviation for the actual outcomes and the counterfactuals at each post-treatment period(MAD). We consider the performances obtained by constructing the counterfactuals of $y_{1,t}(t = T_0 + 1, \ldots, T)$ via QCM and QCMX. The simulation results are summarized in Tables 1 –7. We also plot simulation results for DGPs 1–7 when N = 40 and T = 60 in Figures 1– 7. In general, we find that

(1) When the outcome variable is depended on covariates (DGP 1-6), QCMX method is able to approximately dominate QCM all of the times, adding covariates to help predict that greater confidence probability, narrower confidence interval, lesser MSE and lesser MAD to be obtained.

(2) When the outcome variable is not depended on covariates (DGP 7), the QCMX method adds interference variables unrelated to the outcome variable compared with QCM method, which makes the confidence interval wider and MSE larger, but has no significant impact on CP and WCI.

TABLE 1 Simulation results of QCM and QCMX methods for DGP1

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.917	0.938	4.054	3.236	45.557	31.894	26.829	26.146
	2	0.926	0.949	4.341	3.291	46.967	33.275	27.153	26.692
	3	0.919	0.937	4.361	3.280	51.954	35.811	27.124	26.762
	4	0.893	0.926	4.544	3.411	58.137	41.288	27.330	27.041
	5	0.895	0.924	4.542	3.404	63.075	43.881	27.307	27.007
	6	0.894	0.918	4.583	3.465	64.186	44.776	27.181	27.040
	7	0.896	0.917	4.785	3.532	67.631	48.319	27.537	26.976
	8	0.894	0.918	4.644	3.428	67.342	46.868	27.734	27.426
	9	0.894	0.912	4.577	3.396	64.789	46.424	27.176	27.031
	10	0.886	0.918	4.571	3.532	66.521	47.293	27.622	27.433
70	1	0.936	0.966	4.195	2.810	42.242	22.197	26.358	24.895
	2	0.933	0.965	4.228	2.879	44.926	24.006	26.608	25.225
	3	0.926	0.956	4.435	2.909	50.836	26.935	26.907	25.293
	4	0.919	0.951	4.312	2.890	50.642	27.786	26.878	25.399
	5	0.924	0.951	4.405	2.961	50.785	28.204	26.794	25.240
	6	0.927	0.961	4.457	2.962	50.395	28.682	26.831	25.335
	7	0.928	0.958	4.290	3.034	49.593	29.591	26.733	25.300
	8	0.929	0.962	4.616	3.127	50.901	28.887	26.815	25.472
	9	0.925	0.959	4.436	3.074	50.413	29.090	26.926	25.682
	10	0.919	0.954	4.310	2.908	50.493	29.046	26.820	25.638
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.926	0.943	4.014	3.211	41.726	29.745	26.541	26.473
	2	0.917	0.933	4.294	3.349	45.695	33.415	26.832	26.834
	3	0.909	0.928	4.397	3.536	49.275	36.181	26.979	27.091
	4	0.908	0.931	4.385	3.442	51.485	37.285	26.820	27.097
	5	0.899	0.923	4.691	3.650	53.883	39.856	27.054	27.311
	6	0.902	0.926	4.316	3.485	53.398	39.392	27.174	27.279
	7	0.913	0.934	4.570	3.628	51.085	37.309	27.172	27.389
	8	0.909	0.931	4.578	3.645	53.420	39.434	27.209	27.431
	9	0.908	0.932	4.289	3.502	52.306	38.246	27.230	27.511
	10	0.917	0.929	4.382	3.419	54.872	41.094	27.286	27.501
70	1	0.940	0.965	3.875	2.694	36.639	21.141	25.916	24.564
	2	0.943	0.963	4.228	3.009	43.298	25.391	26.234	24.819
	3	0.931	0.953	4.083	2.901	44.210	26.339	26.191	25.072
	4	0.923	0.954	4.280	3.028	48.234	28.820	26.284	25.110
	5	0.926	0.952	4.295	2.942	47.838	29.166	26.458	25.110
	6	0.925	0.952	4.603	3.032	49.732	30.594	26.226	25.102
	7	0.927	0.949	4.269	2.937	49.914	30.961	26.295	25.316
	8	0.928	0.954	4.228	2.948	48.792	29.101	26.391	25.273
	9	0.926	0.954	4.343	3.043	46.869	28.363	26.470	25.385
	10	0.917	0.949	4.495	3.006	50.386	30.607	26.325	25.350

TABLE 2 Simulation results of QCM and QCMX methods for DGP2

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.894	0.926	5.388	3.271	71.123	37.627	28.707	27.480
	2	0.886	0.928	5.598	3.309	74.441	39.117	29.103	27.992
	3	0.893	0.928	5.873	3.467	79.651	42.014	29.138	28.182
	4	0.886	0.929	5.682	3.499	79.544	42.004	29.261	28.226
	5	0.890	0.934	5.763	3.417	83.560	43.468	29.134	28.532
	6	0.894	0.923	5.464	3.219	81.593	44.013	29.218	28.526
	7	0.888	0.923	5.929	3.546	87.381	47.140	29.189	28.397
	8	0.872	0.912	5.903	3.599	88.462	47.493	29.300	28.452
	9	0.873	0.927	5.732	3.470	88.290	48.357	29.116	28.472
	10	0.875	0.913	5.879	3.578	88.678	48.794	29.251	28.505
70	1	0.910	0.954	5.397	2.855	70.502	28.901	29.315	26.933
	2	0.917	0.960	5.381	2.752	69.157	27.688	29.511	27.126
	3	0.904	0.948	5.465	2.779	77.429	31.974	29.644	27.492
	4	0.899	0.949	5.473	2.833	79.162	33.262	29.612	27.643
	5	0.903	0.947	5.703	2.987	80.916	33.708	29.587	27.635
	6	0.896	0.942	5.488	2.883	80.070	34.247	29.689	27.646
	7	0.891	0.942	5.830	3.045	85.142	36.219	29.673	27.678
	8	0.905	0.946	5.667	2.958	81.117	34.489	29.740	27.734
	9	0.904	0.945	5.703	2.945	81.515	34.491	29.657	27.629
	10	0.916	0.957	5.490	2.889	78.530	33.204	29.879	27.775
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.908	0.942	5.067	3.283	63.763	34.865	28.629	27.564
	2	0.901	0.936	5.424	3.423	72.354	40.101	28.913	28.147
	3	0.902	0.931	5.466	3.421	76.044	43.085	29.259	28.296
	4	0.888	0.924	5.638	3.505	81.476	45.981	29.146	28.308
	5	0.889	0.923	5.690	3.564	84.407	47.789	29.140	28.412
	6	0.886	0.924	5.768	3.515	82.995	46.819	29.278	28.468
	7	0.889	0.921	5.811	3.557	85.442	49.194	29.344	28.637
	8	0.887	0.923	5.494	3.678	83.952	47.907	29.293	28.591
	9	0.894	0.924	5.585	3.634	81.923	46.562	29.318	28.478
	10	0.894	0.927	5.548	3.468	80.847	46.470	29.233	28.511
70	1	0.925	0.960	5.066	2.897	64.207	27.510	29.584	27.268
	2	0.910	0.958	5.249	2.952	74.085	32.522	29.754	27.764
	3	0.919	0.958	5.408	2.843	71.420	30.963	29.844	27.901
	4	0.923	0.960	5.542	2.911	73.641	31.848	29.948	27.993
	5	0.910	0.954	5.912	3.054	81.692	36.224	30.098	28.070
	6	0.911	0.953	5.396	2.920	79.294	34.804	30.129	28.089
	7	0.907	0.951	5.469	2.946	80.400	35.771	29.996	28.066
	8	0.896	0.948	5.620	2.999	82.671	37.161	29.943	28.118
	9	0.899	0.944	5.716	3.032	85.715	39.007	30.040	28.047
	10	0.906	0.950	5.855	3.075	82.494	35.601	29.977	28.107

TABLE 3 Simulation results of QCM and QCMX methods for DGP3

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.907	0.913	1.077	1.072	2.161	2.139	3.808	3.886
	2	0.896	0.906	1.087	1.070	2.153	2.128	3.798	3.906
	3	0.908	0.919	1.065	1.085	2.141	2.118	3.807	3.925
	4	0.915	0.923	1.087	1.084	2.105	2.087	3.828	3.920
	5	0.922	0.930	1.075	1.067	2.114	2.133	3.800	3.920
	6	0.913	0.926	1.066	1.063	2.046	2.013	3.801	3.926
	7	0.908	0.918	1.049	1.048	2.102	2.094	3.793	3.915
	8	0.909	0.927	1.086	1.082	2.110	2.090	3.792	3.938
	9	0.925	0.927	1.072	1.085	2.133	2.130	3.804	3.931
	10	0.926	0.929	1.065	1.064	2.144	2.105	3.832	3.924
70	1	0.924	0.930	1.049	1.046	2.053	2.064	3.851	3.913
	2	0.911	0.924	1.053	1.070	2.171	2.130	3.848	3.919
	3	0.923	0.930	1.079	1.073	2.108	2.085	3.863	3.926
	4	0.924	0.930	1.054	1.040	2.071	2.053	3.867	3.937
	5	0.923	0.929	1.072	1.059	2.141	2.116	3.871	3.932
	6	0.914	0.924	1.108	1.092	2.199	2.177	3.867	3.950
	7	0.906	0.917	1.077	1.089	2.126	2.114	3.850	3.932
	8	0.931	0.938	1.069	1.037	2.078	2.050	3.872	3.969
	9	0.917	0.929	1.095	1.091	2.110	2.082	3.860	3.965
	10	0.919	0.922	1.088	1.078	2.139	2.111	3.846	3.939
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.903	0.899	1.076	1.075	2.135	2.136	3.854	3.893
	2	0.916	0.923	1.134	1.142	2.211	2.194	3.865	3.936
	3	0.915	0.921	1.058	1.047	2.147	2.128	3.862	3.942
	4	0.916	0.925	1.107	1.106	2.096	2.080	3.850	3.941
	5	0.906	0.907	1.061	1.064	2.182	2.161	3.861	3.939
	6	0.905	0.919	1.076	1.066	2.110	2.097	3.845	3.938
	7	0.909	0.912	1.091	1.094	2.214	2.192	3.863	3.949
	8	0.912	0.918	1.123	1.103	2.225	2.212	3.853	3.951
	9	0.920	0.930	1.062	1.079	2.114	2.111	3.847	3.951
	10	0.914	0.927	1.080	1.074	2.107	2.099	3.845	3.946
70	1	0.929	0.935	1.037	1.055	2.069	2.051	3.898	3.933
	2	0.923	0.924	1.068	1.058	2.118	2.101	3.902	3.959
	3	0.913	0.925	1.089	1.093	2.144	2.113	3.886	3.948
	4	0.928	0.940	1.053	1.047	2.094	2.092	3.900	3.960
	5	0.934	0.939	1.078	1.073	2.018	2.002	3.893	3.959
	6	0.919	0.926	1.084	1.091	2.158	2.140	3.890	3.960
	7	0.927	0.935	1.040	1.047	2.055	2.055	3.886	3.950
	8	0.922	0.930	1.066	1.042	2.058	2.020	3.913	3.962
	9	0.917	0.918	1.037	1.019	2.155	2.142	3.902	3.980
	10	0.924	0.933	1.051	1.051	2.024	2.004	3.918	3.971

TABLE 4 Simulation results of QCM and QCMX methods for DGP4

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.907	0.925	4.174	3.098	47.463	31.392	26.562	26.020
	2	0.906	0.926	4.465	3.287	51.440	34.043	27.052	26.568
	3	0.893	0.917	4.482	3.167	55.365	36.354	27.232	26.551
	4	0.898	0.921	4.492	3.399	54.067	36.171	27.436	27.065
	5	0.903	0.924	4.483	3.427	55.775	37.982	27.309	27.193
	6	0.902	0.927	4.529	3.232	57.706	38.211	27.311	27.006
	7	0.897	0.923	4.424	3.323	56.909	38.796	27.602	27.169
	8	0.887	0.918	4.688	3.354	60.962	41.426	27.266	27.028
	9	0.904	0.924	4.367	3.361	58.210	40.236	27.426	27.285
	10	0.898	0.921	4.382	3.377	59.662	40.963	27.608	27.286
70	1	0.941	0.965	4.122	2.735	40.134	21.030	26.003	24.381
	2	0.930	0.954	4.030	2.721	41.431	22.722	26.110	24.442
	3	0.933	0.960	4.280	2.748	44.575	24.143	26.059	24.671
	4	0.924	0.949	4.497	2.879	46.611	26.086	26.206	24.675
	5	0.927	0.953	4.063	2.696	44.725	24.667	26.312	24.691
	6	0.924	0.957	4.070	2.702	46.855	25.369	26.544	24.926
	7	0.935	0.963	4.305	2.776	49.126	26.358	26.688	25.241
	8	0.924	0.960	4.198	2.725	51.107	28.423	26.500	25.185
	9	0.918	0.955	4.267	2.873	51.332	29.068	26.601	25.154
	10	0.917	0.945	4.238	2.811	52.869	29.956	26.682	25.102
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.922	0.941	3.867	3.127	39.363	28.640	26.234	26.143
	2	0.916	0.932	4.214	3.261	46.483	33.809	26.733	26.799
	3	0.917	0.940	4.294	3.432	49.309	35.226	26.838	27.061
	4	0.909	0.933	4.371	3.555	50.494	36.600	26.832	26.974
	5	0.896	0.919	4.513	3.529	51.879	38.500	26.817	27.057
	6	0.911	0.929	4.467	3.531	49.682	35.370	26.942	26.973
	7	0.892	0.915	4.544	3.558	54.463	41.232	26.868	26.966
	8	0.893	0.920	4.485	3.587	56.764	40.658	27.041	27.075
	9	0.898	0.919	4.540	3.456	57.924	42.960	27.037	27.064
	10	0.905	0.927	4.600	3.526	56.635	41.988	27.042	27.173
70	1	0.945	0.966	4.010	2.722	38.071	22.444	25.804	24.637
	2	0.928	0.957	4.362	2.882	43.293	25.919	26.207	24.949
	3	0.928	0.954	4.279	2.919	42.966	25.422	26.342	25.011
	4	0.928	0.956	4.100	2.810	44.803	26.127	26.460	25.156
	5	0.925	0.947	4.169	2.866	47.995	28.927	26.536	25.321
	6	0.923	0.950	4.297	2.978	46.635	28.470	26.263	25.114
	7	0.932	0.958	4.211	2.899	47.580	28.458	26.380	25.304
	8	0.930	0.958	4.350	2.889	47.757	27.912	26.414	25.420
	9	0.930	0.950	4.222	3.003	51.344	30.188	26.642	25.551
	10	0.927	0.951	4.326	2.954	45.996	27.664	26.450	25.322

TABLE 5 Simulation results of QCM and QCMX methods for DGP5

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.915	0.930	4.178	3.159	48.053	31.428	26.630	26.192
	2	0.908	0.929	4.321	3.212	51.950	34.486	27.087	26.773
	3	0.903	0.924	4.367	3.292	54.460	36.871	27.275	26.767
	4	0.902	0.923	4.633	3.451	54.016	36.157	27.312	26.979
	5	0.900	0.923	4.336	3.380	56.189	38.091	27.295	27.095
	6	0.897	0.918	4.697	3.556	58.146	39.033	27.241	27.082
	7	0.901	0.923	4.413	3.297	57.104	39.151	27.631	27.069
	8	0.897	0.920	4.639	3.496	60.760	42.135	27.369	27.130
	9	0.902	0.924	4.286	3.309	58.863	41.076	27.539	27.194
	10	0.900	0.921	4.583	3.334	59.872	41.401	27.703	27.307
70	1	0.946	0.968	4.055	2.695	39.998	21.278	26.217	24.423
	2	0.928	0.952	4.087	2.687	42.543	23.634	26.192	24.611
	3	0.930	0.960	4.264	2.897	45.127	24.637	26.304	24.724
	4	0.924	0.948	4.345	2.954	47.871	26.757	26.439	24.748
	5	0.931	0.954	4.062	2.754	44.309	24.882	26.479	24.870
	6	0.929	0.957	4.160	2.811	47.772	26.302	26.612	25.042
	7	0.927	0.963	4.367	2.919	49.809	27.195	26.671	25.438
	8	0.930	0.958	4.437	2.851	52.129	29.591	26.648	25.282
	9	0.917	0.954	4.281	2.877	51.858	29.390	26.568	25.405
	10	0.911	0.946	4.191	2.956	53.633	30.583	26.597	25.152
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.919	0.933	4.007	3.273	39.687	29.101	26.333	26.356
	2	0.920	0.934	4.320	3.440	46.831	34.831	26.755	26.840
	3	0.921	0.935	4.489	3.432	49.217	35.595	26.994	27.025
	4	0.909	0.931	4.436	3.586	51.745	37.498	26.824	26.968
	5	0.896	0.918	4.557	3.595	53.440	39.742	27.043	27.305
	6	0.906	0.931	4.530	3.501	51.753	36.923	27.046	27.052
	7	0.897	0.916	4.533	3.594	55.396	42.224	26.839	27.215
	8	0.898	0.922	4.389	3.488	57.152	41.015	27.055	27.107
	9	0.899	0.925	4.508	3.605	58.868	43.640	27.030	27.291
	10	0.907	0.928	4.665	3.652	57.782	42.144	27.068	27.383
70	1	0.951	0.970	3.913	2.761	37.536	22.349	25.955	24.546
	2	0.932	0.956	4.270	2.889	43.223	25.814	26.044	25.086
	3	0.920	0.955	4.229	3.014	43.793	25.981	26.382	25.164
	4	0.928	0.958	4.115	2.747	45.050	26.660	26.447	25.281
	5	0.927	0.948	4.121	2.921	48.762	29.443	26.661	25.505
	6	0.918	0.948	4.382	2.981	47.510	28.939	26.300	25.269
	7	0.929	0.954	4.332	2.990	47.974	29.080	26.509	25.471
	8	0.932	0.957	4.388	3.008	48.551	28.526	26.465	25.444
	9	0.925	0.949	4.378	2.881	50.079	29.806	26.620	25.629
	10	0.927	0.955	4.334	2.966	46.801	27.991	26.521	25.478

TABLE 6 Simulation results of QCM and QCMX methods for DGP6

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.908	0.928	4.234	3.252	48.677	32.633	26.956	26.386
	2	0.907	0.927	4.486	3.289	53.364	35.512	27.351	26.931
	3	0.903	0.925	4.554	3.405	56.260	37.764	27.552	27.003
	4	0.900	0.923	4.616	3.494	55.106	37.493	27.548	27.226
	5	0.901	0.921	4.458	3.374	56.910	39.276	27.651	27.370
	6	0.897	0.918	4.812	3.655	58.206	40.382	27.662	27.234
	7	0.899	0.920	4.552	3.420	57.520	40.049	27.688	27.359
	8	0.899	0.919	4.720	3.673	62.161	43.971	27.645	27.345
	9	0.902	0.921	4.446	3.502	59.887	42.558	27.853	27.460
	10	0.898	0.918	4.562	3.481	61.051	43.196	27.969	27.529
70	1	0.941	0.966	4.154	2.889	40.883	22.175	26.480	24.707
	2	0.926	0.956	4.151	2.844	44.088	24.905	26.362	24.810
	3	0.926	0.956	4.371	3.032	46.196	26.050	26.603	25.083
	4	0.917	0.949	4.381	3.057	48.914	28.229	26.557	25.009
	5	0.932	0.952	4.051	2.885	45.592	26.112	26.772	25.087
	6	0.925	0.957	4.286	2.987	48.352	27.275	26.879	25.284
	7	0.925	0.960	4.515	3.066	51.061	28.546	27.131	25.691
	8	0.928	0.960	4.428	3.010	52.578	30.277	26.899	25.493
	9	0.921	0.952	4.290	3.079	51.769	30.176	26.844	25.473
	10	0.915	0.943	4.337	2.966	53.448	31.306	26.913	25.551
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.922	0.936	4.023	3.303	40.926	29.772	26.588	26.547
	2	0.918	0.930	4.392	3.529	48.422	35.795	26.946	27.222
	3	0.918	0.935	4.337	3.624	50.631	37.363	27.259	27.461
	4	0.908	0.931	4.634	3.711	53.595	39.082	27.274	27.387
	5	0.896	0.915	4.679	3.626	55.339	41.119	27.418	27.499
	6	0.907	0.931	4.568	3.598	53.070	38.778	27.245	27.446
	7	0.895	0.913	4.668	3.671	56.840	43.569	27.214	27.539
	8	0.894	0.916	4.440	3.599	58.170	42.109	27.242	27.413
	9	0.896	0.923	4.715	3.635	60.452	44.910	27.404	27.533
	10	0.910	0.931	4.765	3.659	58.720	43.676	27.370	27.788
70	1	0.946	0.966	3.978	2.912	38.261	23.498	26.247	24.964
	2	0.930	0.955	4.169	2.976	44.027	26.865	26.442	25.255
	3	0.924	0.954	4.409	3.122	44.516	26.949	26.624	25.463
	4	0.927	0.955	4.207	2.936	45.801	27.696	26.661	25.560
	5	0.927	0.945	4.198	3.000	50.042	30.651	26.875	25.728
	6	0.924	0.944	4.484	3.128	48.466	30.628	26.588	25.405
	7	0.933	0.957	4.315	3.111	48.940	29.884	26.824	25.692
	8	0.928	0.954	4.430	3.068	49.727	29.367	26.838	25.733
	9	0.918	0.951	4.376	2.924	50.456	30.666	26.830	26.092
	10	0.919	0.948	4.357	3.135	47.524	29.263	26.875	25.687

TABLE 7 Simulation results of QCM and QCMX methods for DGP7

T0	Time	CP		MAD		MSE		WCI	
N=40		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.936	0.932	1.215	1.212	2.735	2.820	5.658	5.898
	2	0.925	0.931	1.135	1.151	2.720	2.838	5.690	5.952
	3	0.926	0.932	1.171	1.213	2.652	2.793	5.598	5.922
	4	0.936	0.939	1.149	1.127	2.723	2.809	5.651	5.933
	5	0.929	0.936	1.208	1.200	2.855	2.879	5.667	5.923
	6	0.927	0.930	1.132	1.140	2.649	2.685	5.689	5.913
	7	0.923	0.932	1.105	1.122	2.758	2.840	5.654	5.910
	8	0.932	0.936	1.165	1.180	2.627	2.743	5.656	5.934
	9	0.929	0.938	1.158	1.181	2.797	2.890	5.692	5.951
	10	0.926	0.932	1.221	1.207	2.840	2.927	5.661	5.939
70	1	0.935	0.938	1.133	1.166	2.717	2.794	5.532	5.775
	2	0.945	0.952	1.126	1.147	2.525	2.593	5.593	5.833
	3	0.943	0.951	1.138	1.153	2.520	2.596	5.567	5.824
	4	0.940	0.948	1.151	1.164	2.568	2.660	5.572	5.853
	5	0.949	0.950	1.090	1.111	2.507	2.570	5.573	5.826
	6	0.935	0.943	1.145	1.158	2.561	2.625	5.598	5.852
	7	0.945	0.947	1.130	1.173	2.658	2.781	5.584	5.818
	8	0.948	0.955	1.211	1.219	2.684	2.730	5.628	5.862
	9	0.948	0.951	1.103	1.078	2.475	2.536	5.559	5.804
	10	0.946	0.950	1.127	1.129	2.481	2.514	5.570	5.821
N=70		QCM	QCMX	QCM	QCMX	QCM	QCMX	QCM	QCMX
40	1	0.934	0.941	1.167	1.172	2.684	2.787	5.717	5.972
	2	0.935	0.938	1.203	1.190	2.680	2.820	5.729	5.961
	3	0.944	0.948	1.178	1.190	2.626	2.718	5.701	5.938
	4	0.926	0.933	1.144	1.128	2.554	2.694	5.673	5.938
	5	0.931	0.931	1.179	1.200	2.739	2.866	5.713	5.987
	6	0.949	0.943	1.145	1.168	2.662	2.799	5.706	5.989
	7	0.936	0.938	1.148	1.174	2.657	2.757	5.727	5.987
	8	0.929	0.936	1.127	1.145	2.595	2.692	5.677	5.979
	9	0.932	0.936	1.237	1.233	2.700	2.769	5.665	5.969
	10	0.928	0.929	1.145	1.147	2.733	2.845	5.697	5.952
70	1	0.948	0.948	1.176	1.175	2.618	2.652	5.444	5.662
	2	0.953	0.955	1.211	1.224	2.521	2.639	5.452	5.665
	3	0.953	0.949	1.118	1.121	2.467	2.543	5.464	5.681
	4	0.948	0.946	1.156	1.167	2.603	2.664	5.466	5.670
	5	0.949	0.945	1.130	1.131	2.511	2.555	5.502	5.728
	6	0.945	0.953	1.132	1.111	2.474	2.555	5.468	5.709
	7	0.944	0.949	1.164	1.152	2.474	2.569	5.447	5.678
	8	0.946	0.950	1.096	1.102	2.395	2.469	5.452	5.689
	9	0.951	0.957	1.131	1.135	2.370	2.447	5.432	5.669
	10	0.952	0.955	1.126	1.140	2.435	2.491	5.448	5.716

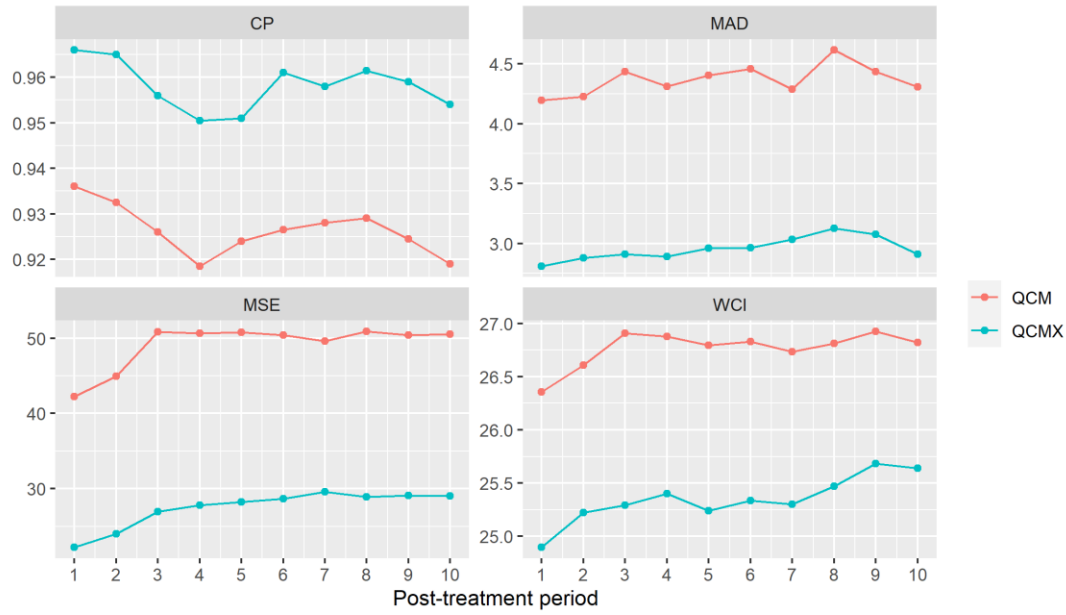

FIGURE 1 Plot of simulation result for DGP1 when N = 40 and T = 70

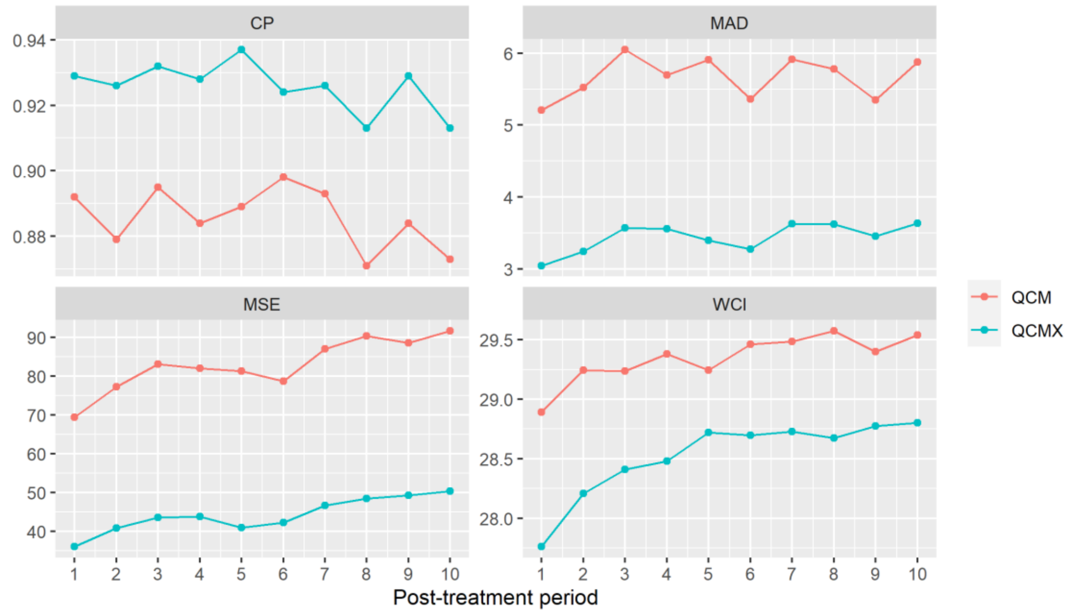

FIGURE 2 Plot of simulation result for DGP2 when N = 40 and T = 70

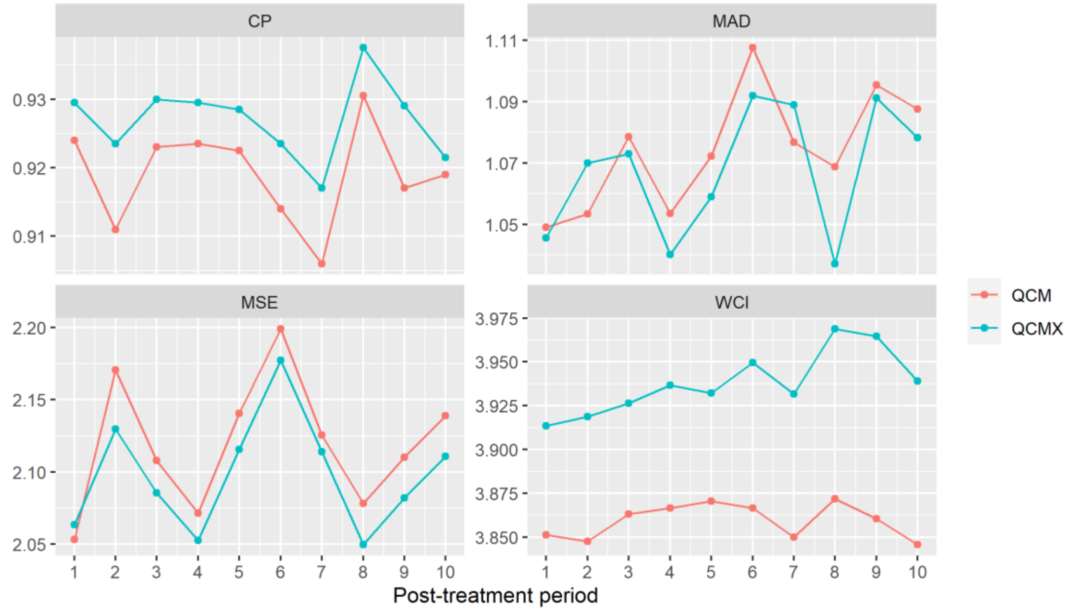

FIGURE 3 Plot of simulation result for DGP3 when N = 40 and T = 70

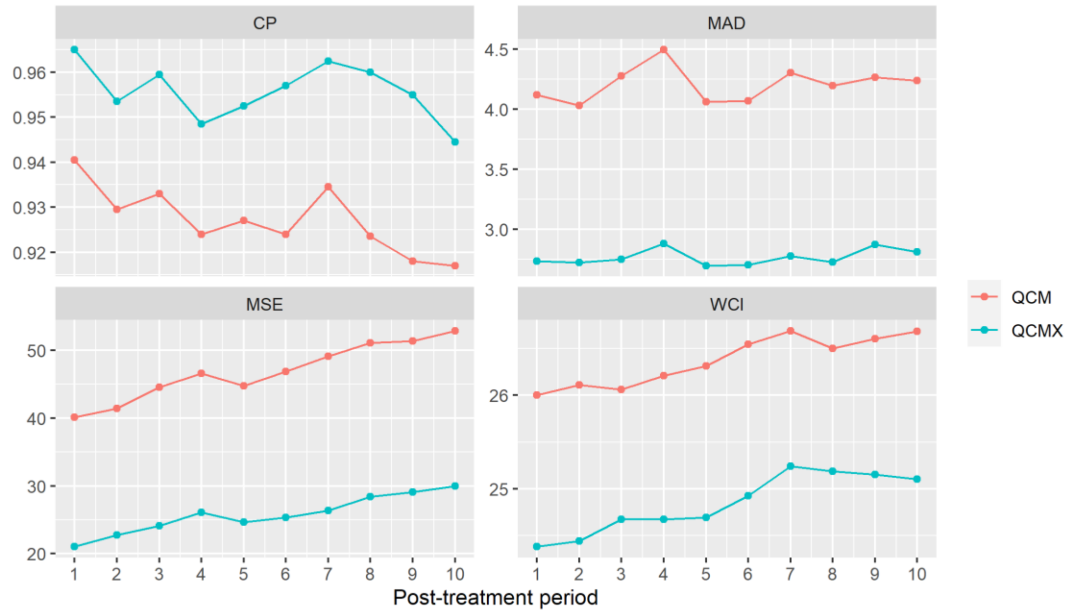

FIGURE 4 Plot of simulation result for DGP4 when N = 40 and T = 70

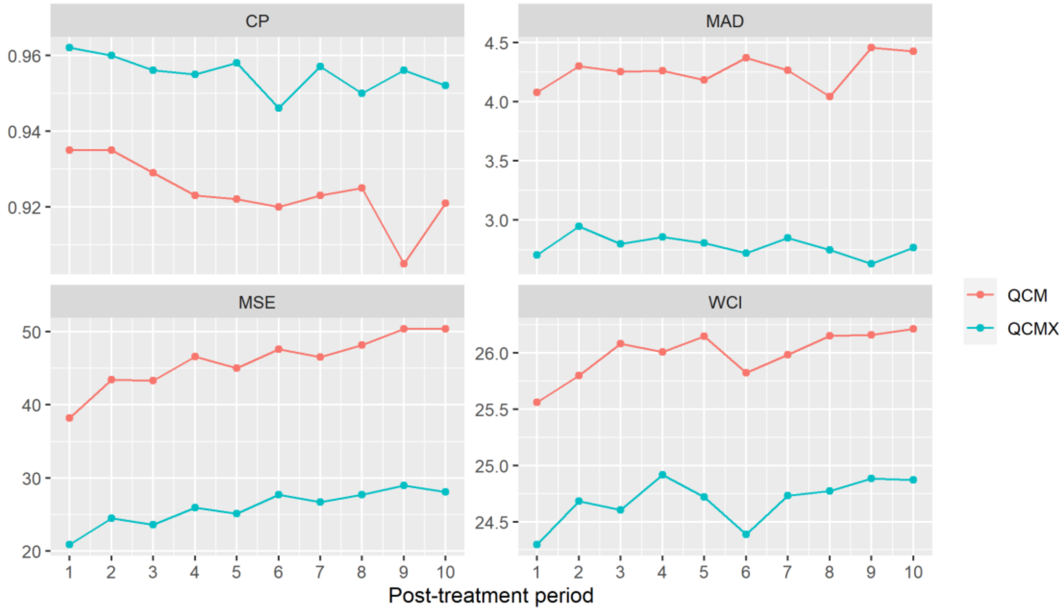

FIGURE 5 Plot of simulation result for DGP5 when N = 40 and T = 70

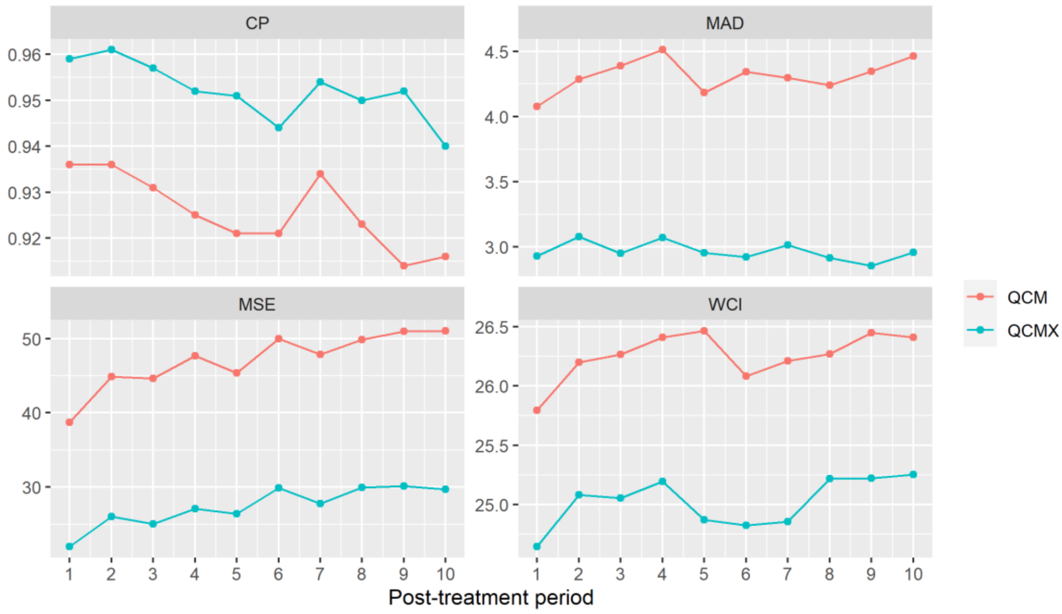

FIGURE 6 Plot of simulation result for DGP6 when N = 40 and T = 70

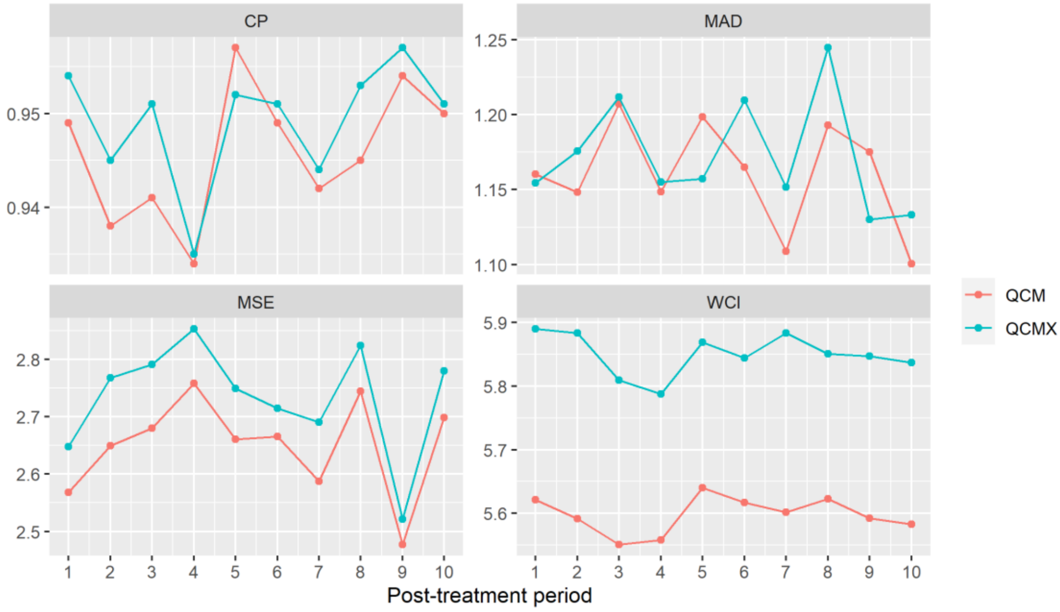

FIGURE 7 Plot of simulation result for DGP7 when N = 40 and T = 70