SIMULATION STUDIES

The treatment effects is measured as the difference between 𝑦 1 it and the predicted 𝑦 0 it . Since the true DGP is unknown, the only way to consider which method is more likely to yield more accurate 𝑦 0 it in a wide array of situations is to conduct computer simulations. In the DGPs below, we assume that the common factors , , and are i.i.d.N(0,1); the factor loadings , and are also i.i.d.N(0,1), unless they are specified otherwise. The DGPs are designed as follows:

**DGP1.** Model with exogenous variables and common factors:

The covariates are (positively) correlated with the factors and extra factors as follows:

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where , , , , ,

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**DGP2.** Model with exogenous variables and common factors:

The covariates follow an autoregressive moving average (ARMA) process as

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where ，，，，.

**DGP3.** Non-linearmodel with exogenous variables and common factors:

The covariates are (positively) correlated with the factors and extra factors as follows:

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where , , , ,

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**DGP4.** DGP1 with heteroscedasticity:

where and

**DGP5.** DGP1 with autocorrelation:

where and

**DGP6.** DGP1 with heteroscedasticity and autocorrelation:

where and

**DGP7.** Pure factor model:

The covariates are (positively) correlated with the factors and extra factors as follows:

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where , , , , ,

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The treatment and control groups consist of 1 and units, respectively. The treatment for unit 1 starts at time .The other units are not subject to treatment. We let and the pretreatment time as well. The posttreatment periods are set at ; that is, . The number of replications is set at .

with:

**DGP6:**

with:

**DGP7:**

with:，

**DGP1 non-linear:**

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with:，，，，，

We consider three criteria for comparison: the mean of the absolute bias for the true observation and the counterfactuals at each posttreatment period (MAB), the mean of the sum of squared error for the true observation and the counterfactuals at each posttreatment period (MSE), and the mean of the ratio of absolute counterfactuals and absolute true outcomes at each posttreatment period (MAP). We consider the performances obtained by constructing the counterfactuals of

y 1t (t = T 0 + 1, …,T) via approaches E1–E7. For N = 50, when y it is stationary, we use the stepwise method to select a

subset of available control units. When y it is nonstationary, we use the random split method by first splitting the sample

randomly into two groups (G = 2). The simulation results are summarized in Tables 1 –7. We also plot the root mean

square prediction errors of different methods for DGPs 1–7 when N = 30 and T = 60 in Figures 1– 7. In general, we find

that