



SUN YAT-SEN UNIVERSITY

中山大學數學學院

SCHOOL OF MATHEMATICS



# A Puzzle of Triangle



Can you solve the challenging problem?

**LI GUANYU**

**Mathematics and Applied Mathematics**

# CONTENTS

**01**

## **Construction**

Triangles and quadrangle

**02**

## **Invariant**

To count the number

**03**

## **Proof**

Green's theorem

**04**

## **Analogies**

What is a space?

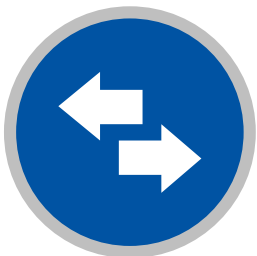
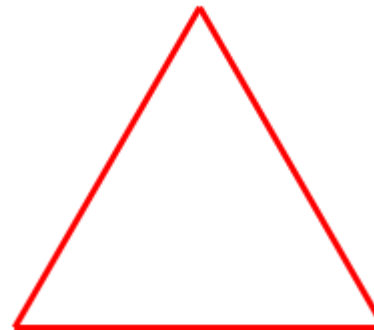
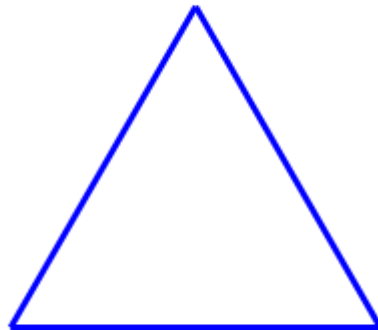
# PART ONE

## The Construction

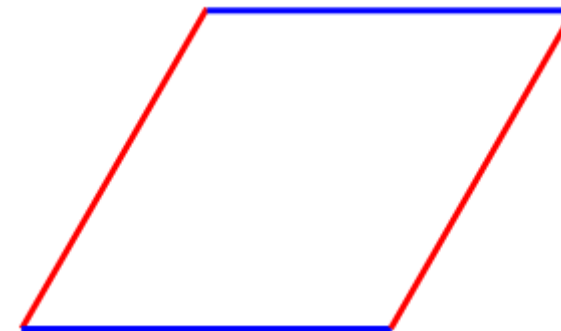
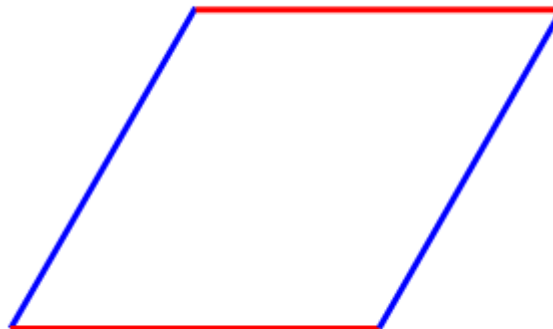
# Where do we start?



## Triangles



## Parallelogram

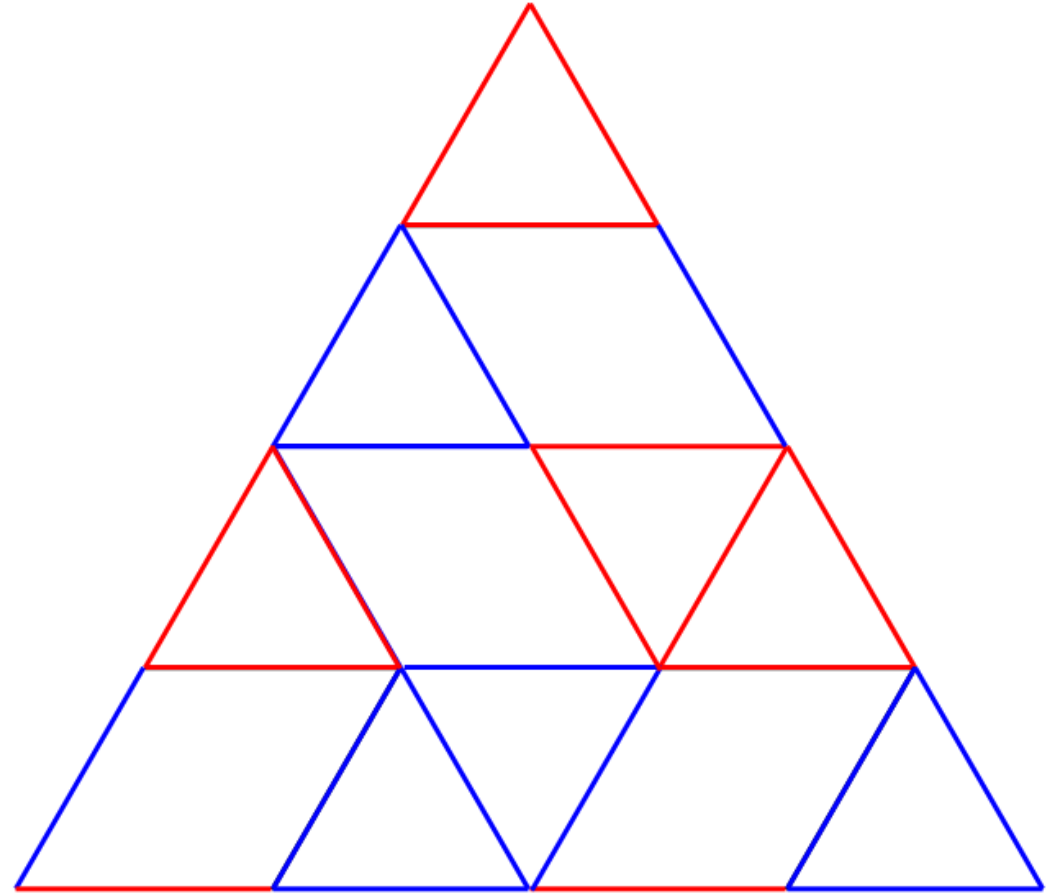
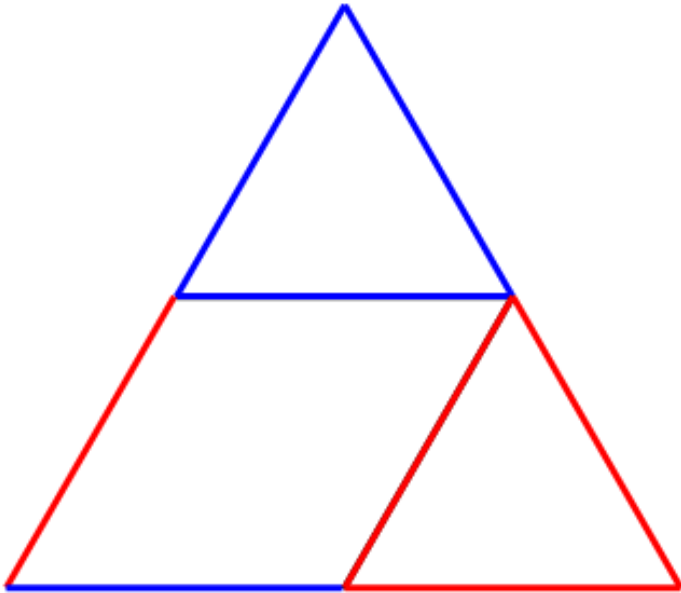


## Rule of Gluing

Blue side can just be glued with another blue side, and so are the red sides.

# Examples

Use these pieces of graphs to form a larger regular triangle.



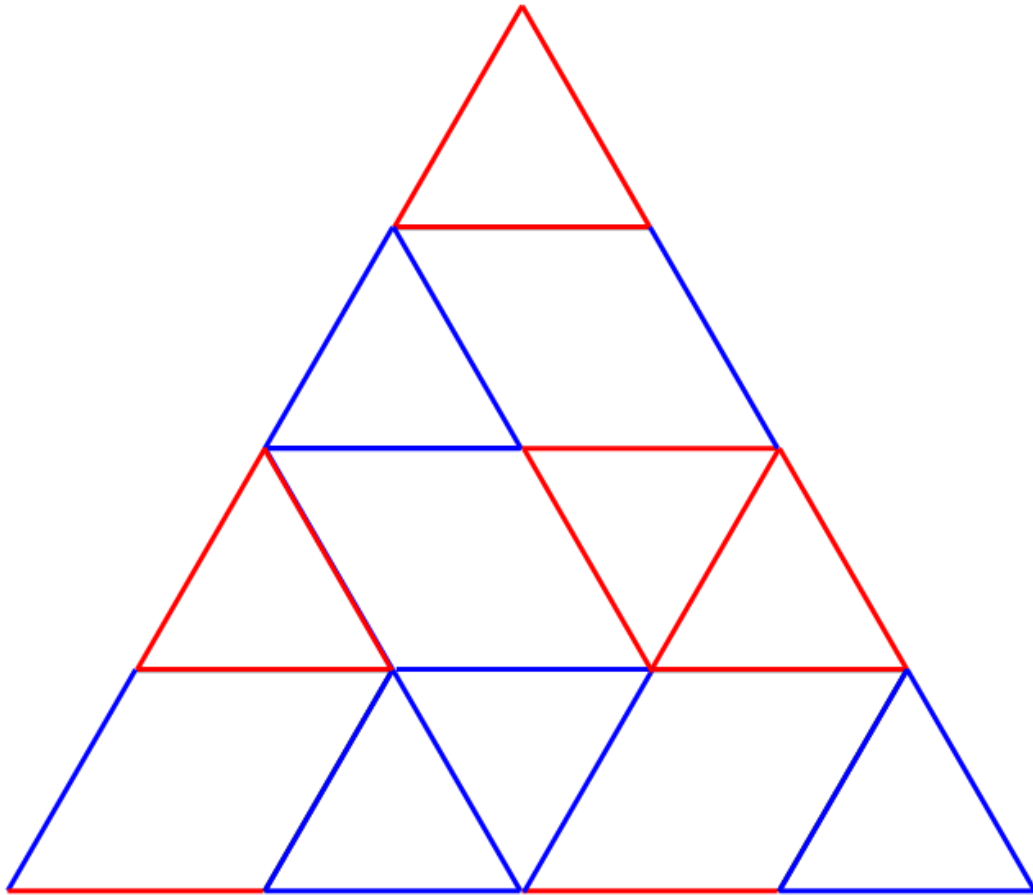
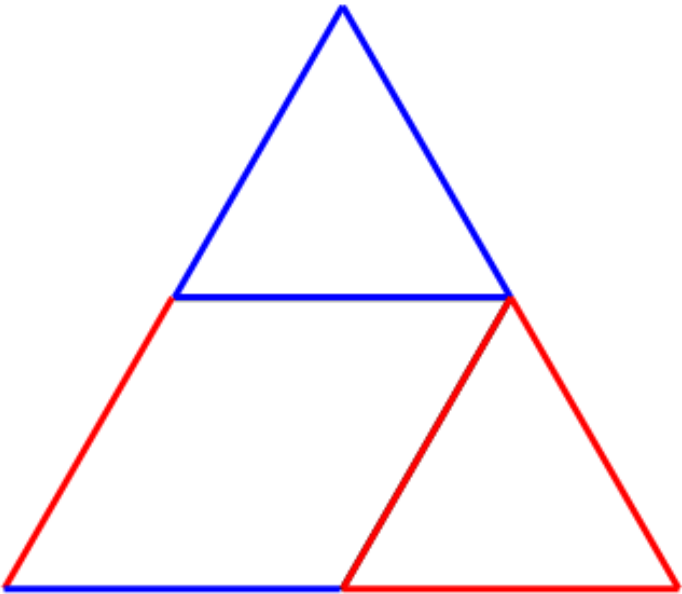
# PART TWO

## **The Invariant**



# Counting

The number of red/blue pieces of each side of the larger triangle





## Conclusion

**SUM**

The sum of numbers of blue and red pieces of each side equals.

The number of blue pieces of each side equals.

**Blue**



# **The Proof**

## A simple try

**What causes  
the  
difficulties**

**How can we  
eliminate the  
barriers**

**The Core**

**The quadrangle  
makes the color  
changes.**

**Discussing  
the situations  
of quadrangle**

**From “local”  
to “global”**



**So what is the proof?**

**MAYBE we need something else to determine the color**

## **Vectors**

---

- Suppose we can attach a unit vector pointing out for each blue side.

**Use better description**

## **Observation**

---

- All the small pieces have the sum of vectors 0 and the sum remains after gluing.

**Translate the rule of gluing**

**01**

The color can be regarded as another mathematical objects.

**02**

The shapes and rules can be translated as equations.

**03**

The sum remains, so is the invariant.

# **The Analogy**

# **Analogy**



**Counter-  
example?**



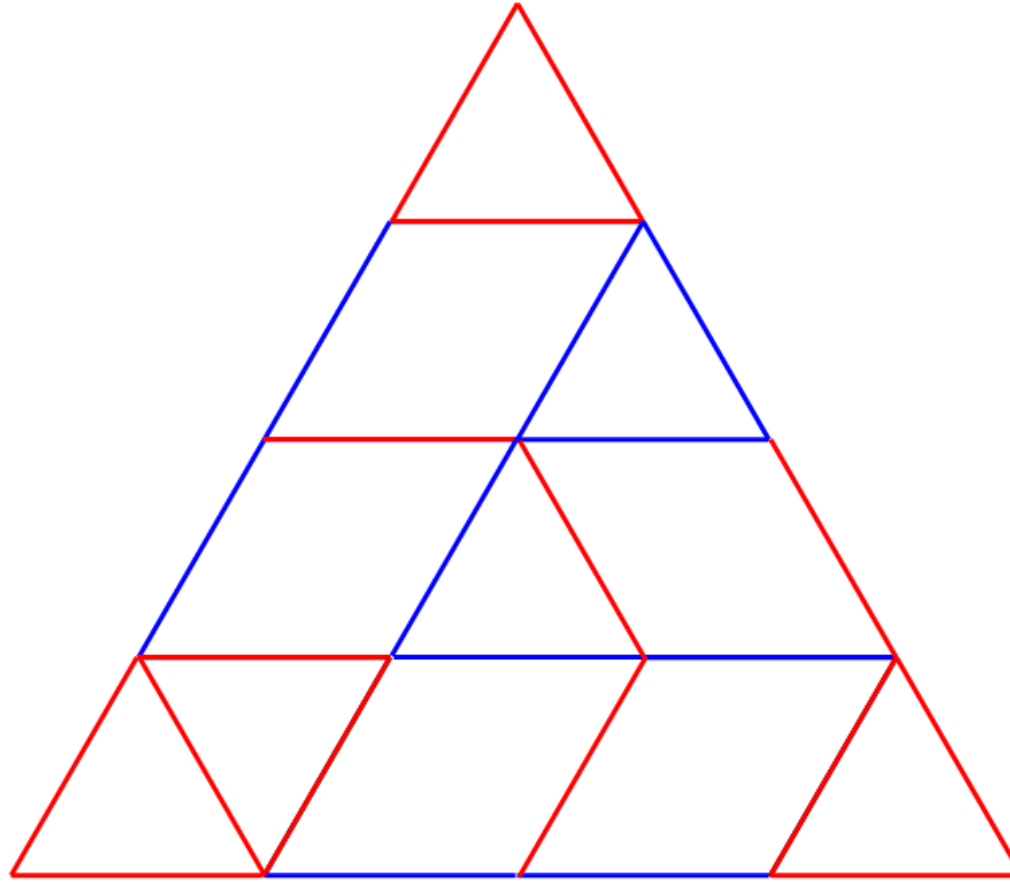
**Cut off part  
of the  
triangle**



**Is there  
any  
similarity  
of this  
proof?**

# Counterexample

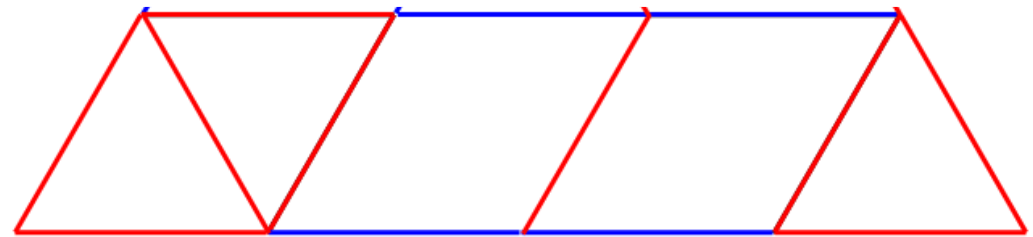
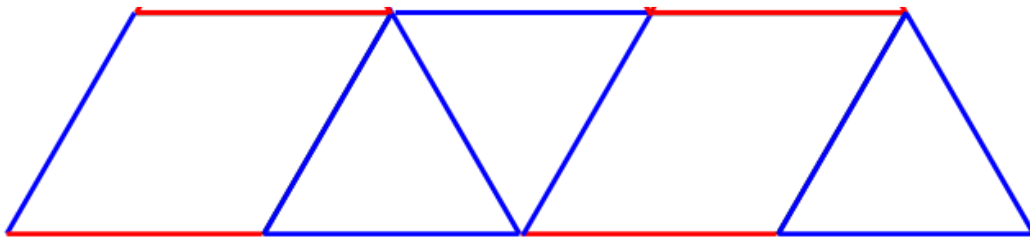
Why the statement fails to explain this regular triangle?





# Cut off

How about we erase part of the triangle?





# Emmmmm

Analogy of the proof? Construct the complement?

**THANKS**