Some Open Problems I am Interested in

Guanyu Li

October 31, 2018

Problem 1 (Kervaire Invariant Problem in 126, Geometrical Topology)**.** In 1960, Kervaire proved that there is a manifold homotopic to a sphere S^2 without any smooth structure [4]. What he did is actually constructing a "Kervaire invariant" that lives in $\pi_2^s(S^0)$. In 1969, Browder proved that the Kervaire invariant can only live in $\pi_{2^n-2}^s(S^0)$ [1] and soon Barratt, Jones, Mahowald proved for $n = 2, 3, 4, 5, 6$ the fundamental group is not trivial and the element exists [2]. Since then 126 has become an open problem for the existence. In 2016, Hill, Hopkins and Ravenel proved $\pi_{256k-2}^s(S^0) = 0$ so that the invariant does not exist for $n \geq 8$. Thus the only left one is $n = 7$ [3].

References

[1] Browder, William., *The Kervaire invariant of framed manifolds and its generalization*. Annals of Mathematics. **90** (1) (1969): 157186. JSTOR 1970686.

[2] Barratt, Michael G.; Jones, J. D. S.; Mahowald, Mark E., *Relations amongst Toda brackets and the Kervaire invariant in dimension 62*. Journal of the London Mathematical Society. 2. 30 (3) (1984): 533550. MR 0810962.

[3] Hill, Michael A.; Hopkins, Michael J.; Ravenel, Douglas C., *On the nonexistence of elements of Kervaire invariant one*. Annals of Mathematics. **184** (1) (2016).: 1262. arXiv:0908.3724.

[4] Kervaire, Michel A., *A manifold which does not admit any differentiable structure*, Commentarii Mathematici Helvetici. **34** (1960): 257270. MR 0139172.