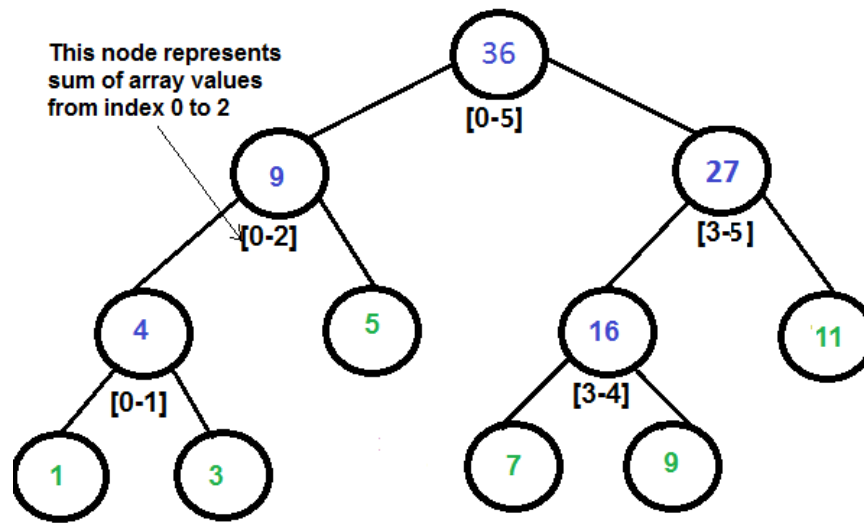


Lazy Propagation in Segment Tree

Segment tree is introduced in [previous post](#) with an example of range sum problem. We have used the same “Sum of given Range” problem to explain Lazy propagation



Segment Tree for input array {1, 3, 5, 7, 9, 11}

How does update work in Simple Segment Tree?

In the [previous post](#), update function was called to update only a single value in array. Please note that a single value update in array may cause multiple updates in Segment Tree as there may be many segment tree nodes that have a single array element in their ranges.

Below is simple logic used in previous post.

- 1) Start with root of segment tree.
- 2) If array index to be updated is not in current node's range, then return
- 3) Else update current node and recur for children.

Below is code taken from previous post.

```

/* A recursive function to update the nodes which have the given
index in their range. The following are parameters
tree[] --> segment tree
si      --> index of current node in segment tree.
          Initial value is passed as 0.
ss and se --> Starting and ending indexes of array elements
              covered under this node of segment tree.
              Initial values passed as 0 and n-1.
i       --> index of the element to be updated. This index
              is in input array.
diff --> Value to be added to all nodes which have array
          index i in range */
void updateValueUtil(int tree[], int ss, int se, int i,
                    int diff, int si)
{
    // Base Case: If the input index lies outside the range
    // of this segment
    if (i < ss || i > se)
        return;

    // If the input index is in range of this node, then
    // update the value of the node and its children
    st[si] = st[si] + diff;
    if (se != ss)
    {
        int mid = getMid(ss, se);
        updateValueUtil(st, ss, mid, i, diff, 2*si + 1);
        updateValueUtil(st, mid+1, se, i, diff, 2*si + 2);
    }
}

```

Run on IDE

What if there are updates on a range of array indexes?

For example add 10 to all values at indexes from 2 to 7 in array. The above update has to be called for every index from 2 to 7. We can avoid multiple calls by writing a function updateRange() that updates nodes accordingly.

```

/* Function to update segment tree for range update in input
array.
si -> index of current node in segment tree
ss and se -> Starting and ending indexes of elements for
              which current nodes stores sum.
us and ue -> starting and ending indexes of update query
diff -> which we need to add in the range us to ue */
void updateRangeUtil(int si, int ss, int se, int us,
                    int ue, int diff)
{
    // out of range
    if (ss > se || ss > ue || se < us)
        return ;

    // Current node is a leaf node
    if (ss == se)
    {
        // Add the difference to current node
        tree[si] += diff;
        return;
    }

    // If not a leaf node, recur for children.
    int mid = (ss+se)/2;

```

```

updateRangeUtil(si*2+1, ss, mid, us, ue, diff);
updateRangeUtil(si*2+2, mid+1, se, us, ue, diff);

// Use the result of children calls to update this
// node
tree[si] = tree[si*2+1] + tree[si*2+2];
}

```

Run on IDE

Lazy Propagation – An optimization to make range updates faster

When there are many updates and updates are done on a range, we can postpone some updates (avoid recursive calls in update) and do those updates only when required.

Please remember that a node in segment tree stores or represents result of a query for a range of indexes. And if this node's range lies within the update operation range, then all descendants of the node must also be updated. For example consider the node with value 27 in above diagram, this node stores sum of values at indexes from 3 to 5. If our update query is for range 2 to 5, then we need to update this node and all descendants of this node. With Lazy propagation, we update only node with value 27 and postpone updates to its children by storing this update information in separate nodes called lazy nodes or values. We create an array lazy[] which represents lazy node. Size of lazy[] is same as array that represents segment tree, which is tree[] in below code.

The idea is to initialize all elements of lazy[] as 0. A value 0 in lazy[i] indicates that there are no pending updates on node i in segment tree. A non-zero value of lazy[i] means that this amount needs to be added to node i in segment tree before making any query to the node.

Below is modified update method.

```

// To update segment tree for change in array
// values at array indexes from us to ue.
updateRange(us, ue)
1) If current segment tree node has any pending
   update, then first add that pending update to
   current node.
2) If current node's range lies completely in
   update query range.
   ....a) Update current node
   ....b) Postpone updates to children by setting
        lazy value for children nodes.
3) If current node's range overlaps with update
   range, follow the same approach as above simple
   update.
   ...a) Recur for left and right children.
   ...b) Update current node using results of left
        and right calls.

```

Is there any change in Query Function also?

Since we have changed update to postpone its operations, there may be problems if a query is made to a

node that is yet to be updated. So we need to update our query method also which is `getSumUtil` in [previous post](#). The `getSumUtil()` now first checks if there is a pending update and if there is, then updates the node. Once it makes sure that pending update is done, it works same as the previous `getSumUtil()`.

Below are programs to demonstrate working of Lazy Propagation.

C/C++

```
// Program to show segment tree to demonstrate lazy
// propagation
#include <stdio.h>
#include <math.h>
#define MAX 1000

// Ideally, we should not use global variables and large
// constant-sized arrays, we have done it here for simplicity.
int tree[MAX] = {0}; // To store segment tree
int lazy[MAX] = {0}; // To store pending updates

/* si -> index of current node in segment tree
   ss and se -> Starting and ending indexes of elements for
               which current nodes stores sum.
   us and ue -> starting and ending indexes of update query
   diff -> which we need to add in the range us to ue */
void updateRangeUtil(int si, int ss, int se, int us,
                    int ue, int diff)
{
    // If lazy value is non-zero for current node of segment
    // tree, then there are some pending updates. So we need
    // to make sure that the pending updates are done before
    // making new updates. Because this value may be used by
    // parent after recursive calls (See last line of this
    // function)
    if (lazy[si] != 0)
    {
        // Make pending updates using value stored in lazy
        // nodes
        tree[si] += (se-ss+1)*lazy[si];

        // checking if it is not leaf node because if
        // it is leaf node then we cannot go further
        if (ss != se)
        {
            // We can postpone updating children we don't
            // need their new values now.
            // Since we are not yet updating children of si,
            // we need to set lazy flags for the children
            lazy[si*2 + 1] += lazy[si];
            lazy[si*2 + 2] += lazy[si];
        }

        // Set the lazy value for current node as 0 as it
        // has been updated
        lazy[si] = 0;
    }

    // out of range
    if (ss>se || ss>ue || se<us)
        return ;

    // Current segment is fully in range
    if (ss>=us && se<=ue)
    {
        // Add the difference to current node
        tree[si] += (se-ss+1)*diff;
    }
}
```

```

    // same logic for checking leaf node or not
    if (ss != se)
    {
        // This is where we store values in lazy nodes,
        // rather than updating the segment tree itself
        // Since we don't need these updated values now
        // we postpone updates by storing values in lazy[]
        lazy[si*2 + 1] += diff;
        lazy[si*2 + 2] += diff;
    }
    return;
}

// If not completely in rang, but overlaps, recur for
// children,
int mid = (ss+se)/2;
updateRangeUtil(si*2+1, ss, mid, us, ue, diff);
updateRangeUtil(si*2+2, mid+1, se, us, ue, diff);

// And use the result of children calls to update this
// node
tree[si] = tree[si*2+1] + tree[si*2+2];
}

// Function to update a range of values in segment
// tree
/* us and eu -> starting and ending indexes of update query
   ue -> ending index of update query
   diff -> which we need to add in the range us to ue */
void updateRange(int n, int us, int ue, int diff)
{
    updateRangeUtil(0, 0, n-1, us, ue, diff);
}

/* A recursive function to get the sum of values in given
   range of the array. The following are parameters for
   this function.
   si --> Index of current node in the segment tree.
           Initially 0 is passed as root is always at'
           index 0
   ss & se --> Starting and ending indexes of the
               segment represented by current node,
               i.e., tree[si]
   qs & qe --> Starting and ending indexes of query
               range */
int getSumUtil(int ss, int se, int qs, int qe, int si)
{
    // If lazy flag is set for current node of segment tree,
    // then there are some pending updates. So we need to
    // make sure that the pending updates are done before
    // processing the sub sum query
    if (lazy[si] != 0)
    {
        // Make pending updates to this node. Note that this
        // node represents sum of elements in arr[ss..se] and
        // all these elements must be increased by lazy[si]
        tree[si] += (se-ss+1)*lazy[si];

        // checking if it is not leaf node because if
        // it is leaf node then we cannot go further
        if (ss != se)
        {
            // Since we are not yet updating children of si,
            // we need to set lazy values for the children
            lazy[si*2+1] += lazy[si];
            lazy[si*2+2] += lazy[si];
        }
    }
}

```

```

        // unset the lazy value for current node as it has
        // been updated
        lazy[si] = 0;
    }

    // Out of range
    if (ss>se || ss>qe || se<qs)
        return 0;

    // At this point we are sure that pending lazy updates
    // are done for current node. So we can return value
    // (same as it was for query in our previous post)

    // If this segment lies in range
    if (ss>=qs && se<=qe)
        return tree[si];

    // If a part of this segment overlaps with the given
    // range
    int mid = (ss + se)/2;
    return getSumUtil(ss, mid, qs, qe, 2*si+1) +
        getSumUtil(mid+1, se, qs, qe, 2*si+2);
}

// Return sum of elements in range from index qs (query
// start) to qe (query end). It mainly uses getSumUtil()
int getSum(int n, int qs, int qe)
{
    // Check for erroneous input values
    if (qs < 0 || qe > n-1 || qs > qe)
    {
        printf("Invalid Input");
        return -1;
    }

    return getSumUtil(0, n-1, qs, qe, 0);
}

// A recursive function that constructs Segment Tree for
// array[ss..se]. si is index of current node in segment
// tree st.
void constructSTUtil(int arr[], int ss, int se, int si)
{
    // out of range as ss can never be greater than se
    if (ss > se)
        return ;

    // If there is one element in array, store it in
    // current node of segment tree and return
    if (ss == se)
    {
        tree[si] = arr[ss];
        return;
    }

    // If there are more than one elements, then recur
    // for left and right subtrees and store the sum
    // of values in this node
    int mid = (ss + se)/2;
    constructSTUtil(arr, ss, mid, si*2+1);
    constructSTUtil(arr, mid+1, se, si*2+2);

    tree[si] = tree[si*2 + 1] + tree[si*2 + 2];
}

/* Function to construct segment tree from given array.
This function allocates memory for segment tree and
calls constructSTUtil() to fill the allocated memory */
void constructST(int arr[], int n)
{

```

```

    // Fill the allocated memory st
    constructSTUtil(arr, 0, n-1, 0);
}

// Driver program to test above functions
int main()
{
    int arr[] = {1, 3, 5, 7, 9, 11};
    int n = sizeof(arr)/sizeof(arr[0]);

    // Build segment tree from given array
    constructST(arr, n);

    // Print sum of values in array from index 1 to 3
    printf("Sum of values in given range = %d\n",
        getSum(n, 1, 3));

    // Add 10 to all nodes at indexes from 1 to 5.
    updateRange(n, 1, 5, 10);

    // Find sum after the value is updated
    printf("Updated sum of values in given range = %d\n",
        getSum(n, 1, 3));

    return 0;
}

```

Run on IDE

Java

```

// Java program to demonstrate lazy propagation in segment tree
class LazySegmentTree
{
    final int MAX = 1000; // Max tree size
    int tree[] = new int[MAX]; // To store segment tree
    int lazy[] = new int[MAX]; // To store pending updates

    /* si -> index of current node in segment tree
       ss and se -> Starting and ending indexes of elements for
       which current nodes stores sum.
       us and eu -> starting and ending indexes of update query
       ue -> ending index of update query
       diff -> which we need to add in the range us to ue */
    void updateRangeUtil(int si, int ss, int se, int us,
        int ue, int diff)
    {
        // If lazy value is non-zero for current node of segment
        // tree, then there are some pending updates. So we need
        // to make sure that the pending updates are done before
        // making new updates. Because this value may be used by
        // parent after recursive calls (See last line of this
        // function)
        if (lazy[si] != 0)
        {
            // Make pending updates using value stored in lazy
            // nodes
            tree[si] += (se - ss + 1) * lazy[si];

            // checking if it is not leaf node because if
            // it is leaf node then we cannot go further
            if (ss != se)
            {
                // We can postpone updating children we don't
                // need their new values now.
                // Since we are not yet updating children of si,

```

```

        // we need to set lazy flags for the children
        lazy[si * 2 + 1] += lazy[si];
        lazy[si * 2 + 2] += lazy[si];
    }

    // Set the lazy value for current node as 0 as it
    // has been updated
    lazy[si] = 0;
}

// out of range
if (ss > se || ss > ue || se < us)
    return;

// Current segment is fully in range
if (ss >= us && se <= ue)
{
    // Add the difference to current node
    tree[si] += (se - ss + 1) * diff;

    // same logic for checking leaf node or not
    if (ss != se)
    {
        // This is where we store values in lazy nodes,
        // rather than updating the segment tree itself
        // Since we don't need these updated values now
        // we postpone updates by storing values in lazy[]
        lazy[si * 2 + 1] += diff;
        lazy[si * 2 + 2] += diff;
    }
    return;
}

// If not completely in rang, but overlaps, recur for
// children,
int mid = (ss + se) / 2;
updateRangeUtil(si * 2 + 1, ss, mid, us, ue, diff);
updateRangeUtil(si * 2 + 2, mid + 1, se, us, ue, diff);

// And use the result of children calls to update this
// node
tree[si] = tree[si * 2 + 1] + tree[si * 2 + 2];
}

// Function to update a range of values in segment
// tree
/* us and eu -> starting and ending indexes of update query
   ue -> ending index of update query
   diff -> which we need to add in the range us to ue */
void updateRange(int n, int us, int ue, int diff) {
    updateRangeUtil(0, 0, n - 1, us, ue, diff);
}

/* A recursive function to get the sum of values in given
range of the array. The following are parameters for
this function.
si --> Index of current node in the segment tree.
Initially 0 is passed as root is always at'
index 0
ss & se --> Starting and ending indexes of the
segment represented by current node,
i.e., tree[si]
qs & qe --> Starting and ending indexes of query
range */
int getSumUtil(int ss, int se, int qs, int qe, int si)
{
    // If lazy flag is set for current node of segment tree,
    // then there are some pending updates. So we need to
    // make sure that the pending updates are done before
    // processing the sub sum query

```



```

if (lazy[si] != 0)
{
    // Make pending updates to this node. Note that this
    // node represents sum of elements in arr[ss..se] and
    // all these elements must be increased by lazy[si]
    tree[si] += (se - ss + 1) * lazy[si];

    // checking if it is not leaf node because if
    // it is leaf node then we cannot go further
    if (ss != se)
    {
        // Since we are not yet updating children of si,
        // we need to set lazy values for the children
        lazy[si * 2 + 1] += lazy[si];
        lazy[si * 2 + 2] += lazy[si];
    }

    // unset the lazy value for current node as it has
    // been updated
    lazy[si] = 0;
}

// Out of range
if (ss > se || ss > qe || se < qs)
    return 0;

// At this point sure, pending lazy updates are done
// for current node. So we can return value (same as
// was for query in our previous post)

// If this segment lies in range
if (ss >= qs && se <= qe)
    return tree[si];

// If a part of this segment overlaps with the given
// range
int mid = (ss + se) / 2;
return getSumUtil(ss, mid, qs, qe, 2 * si + 1) +
       getSumUtil(mid + 1, se, qs, qe, 2 * si + 2);
}

// Return sum of elements in range from index qs (query
// start) to qe (query end). It mainly uses getSumUtil()
int getSum(int n, int qs, int qe)
{
    // Check for erroneous input values
    if (qs < 0 || qe > n - 1 || qs > qe)
    {
        System.out.println("Invalid Input");
        return -1;
    }

    return getSumUtil(0, n - 1, qs, qe, 0);
}

/* A recursive function that constructs Segment Tree for
array[ss..se]. si is index of current node in segment
tree st. */
void constructSTUtil(int arr[], int ss, int se, int si)
{
    // out of range as ss can never be greater than se
    if (ss > se)
        return;

    /* If there is one element in array, store it in
    current node of segment tree and return */
    if (ss == se)
    {
        tree[si] = arr[ss];
        return;
    }

```

```

    }

    /* If there are more than one elements, then recur
       for left and right subtrees and store the sum
       of values in this node */
    int mid = (ss + se) / 2;
    constructSTUtil(arr, ss, mid, si * 2 + 1);
    constructSTUtil(arr, mid + 1, se, si * 2 + 2);

    tree[si] = tree[si * 2 + 1] + tree[si * 2 + 2];
}

/* Function to construct segment tree from given array.
   This function allocates memory for segment tree and
   calls constructSTUtil() to fill the allocated memory */
void constructST(int arr[], int n)
{
    // Fill the allocated memory st
    constructSTUtil(arr, 0, n - 1, 0);
}

// Driver program to test above functions
public static void main(String args[])
{
    int arr[] = {1, 3, 5, 7, 9, 11};
    int n = arr.length;
    LazySegmentTree tree = new LazySegmentTree();

    // Build segment tree from given array
    tree.constructST(arr, n);

    // Print sum of values in array from index 1 to 3
    System.out.println("Sum of values in given range = " +
        tree.getSum(n, 1, 3));

    // Add 10 to all nodes at indexes from 1 to 5.
    tree.updateRange(n, 1, 5, 10);

    // Find sum after the value is updated
    System.out.println("Updated sum of values in given range = " +
        tree.getSum(n, 1, 3));
}
}
// This Code is contributed by Ankur Narain Verma

```

Run on IDE

Output:

```

Sum of values in given range = 15
Updated sum of values in given range = 45

```