Contents

6 Additional Topics in Trigonometry

6.1 Law of sines

Used to solve oblique (no right angels) triangles.

To solve any triange, you need one side then two other parts of information This sets up a common ration between a side length and the sine of the opposite side.

Formula:

$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

6.2 Law of cosines

Also used to solve trianges Used to solve when you have Side Side Side

$a^2 = b^2 + c^2 - 2bc \cos A$

OR

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

Swap this around to match which ever you are using–just put the two that aren't negative on the top on the bottom

6.3 Vectors in the plane

6.4 Vectors and dot products

Properties of dot products

1. $u \cdot v = v \cdot u$
2. $u \cdot (v + w) = u \cdot v + u \cdot w$
3. $c(u \cdot v) = cu \cdot v = u \cdot cv -$ c is a constant

Angle between two vectors:

$\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$

If $u \cdot v = 0$ then they are orthagonal.

6.5 Trigonometric form of a complex number

6.5.1 Absolute value of a complex number

$|a + bi| = \sqrt{a^2 + b^2}$

Graph complex numbers on a regular plot, except Y axis is imaginary.

6.5.2 Trig form of a Complex number

The trig form of $z = a + bi$ is

$z = r(\cos\theta + i\sin\theta)$

6.5.3 Product and quotient of complex numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Product: $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

Quotient: $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

6.5.4 Powers of complex numbers

if $z = r(\cos\theta + i\sin\theta)$

then: $z^n = r^n(\cos n\theta + i\sin n\theta)$

6.5.5 Roots of a complex number

The nth root of the complex number $z = r(\cos\theta + i\sin\theta)$ has n distinct roots:

$\sqrt[n]{r}(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n})$

Where k is 0, 1, 2, 3, ... n-1

9 Topics in Anaylitic Geometry

9.1 1

9.1.1 Circles

The locus of points all points in a plane that are equidistant to a center point

Equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$ where (h, k) is the center of the circle and r is the radius

Eccentricity: 0

9.1.2 Parabolas

Set of all points equidistant from a fixed line (directrix) and a fixed point (focus).

The midpoint between the directrix and the foucs is the vertex (which is actully on the line)

The axis is orthagonal to the directrix.

Equation for a parabola:

$(x-h)^2 = 4p(y-k), p \neq 0$

Vertex: (h, k)

Directrix: $y = k - p$

Vertical Axis

Focus: $(h, k+p)$ Eccentricity: 1

Focus is on the axis p units from the vertex

When the vertex is on the origin, then $x^2 = 4py$

9.1.3 Reflective property

Reflective property: from the focus if light is emitten the rays will end up being parallel if they hit the parabola.

A triangle between the focus, a point on the parabola, and the intersection of the tangent line at the point on the parabola and the Axis is an isocelies triangle.

9.2 Ellipses

Definition: all points in a plane that the sum of the distances from two fixed points are constant. These two points are the foci.

Major axis: the longer one, minor is the shorter one

Equation:

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Center: (h, k)

Major axis length: $2a$

Minor axis length: $2b$

Foci are on the major axis, c units from the center; $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$; e is always $0 < e < 1$ for an eclipse.

9.3 Hyperbolas and Rotation of Conics

Definition: all points in a plane who the difference of the distances between two points are equal.

Terms:

Transverse Axis: The axis that crosses through both curves

Equation: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Center: (h, k)

Eccentricity: $\frac{\sqrt{a^2+b^2}}{a}$

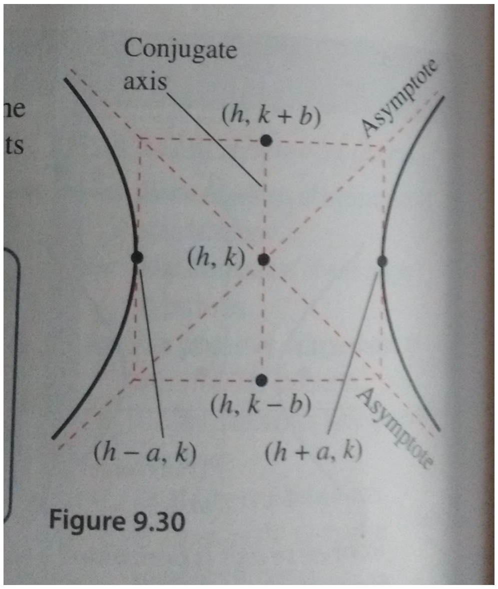


Figure 9.30

9.3.1 General Equation of Conics

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Classifying: use the discriminate: $h = B^2 - 4AC$

Parabola: $h = 0$ and $A = 0$ or $B = 0$

Circle: $h < 0$ and $A = C$

Ellipse: $h < 0$ and $A \neq C$

Hyperbola: $h > 0$

9.4 Parametric Equations

9.4.1 Eliminating the Parameter/Paramtric $\rightarrow$ Rectangular Equation

1. Solve for t
2. Subsutite t into the other equation

9.4.2 Parametric $\rightarrow$ Symmetric equation

1. Solve for t in all of the equations
2. Set all the ts equal to each other. Example: $\frac{x-2}{4} = \frac{y-1}{-1} = \frac{z-8}{-5}$

9.5 Polar Coordinates

9.5.1 Rectangular to Polar Conversion

$x = r\cos\theta$

$y = r\sin\theta$

9.5.2 Rectangular to Polar Conversion

$\tan\theta = \frac{y}{x}$

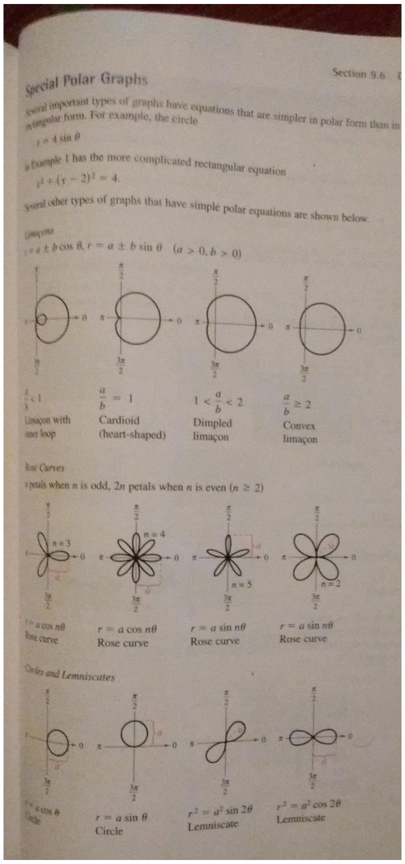
$r^2 = x^2 = y^2$

9.6 Graphs of Polar Equations

Common Polar Equations:

Line: $\theta =< Angle >$

Circle with center at origin: $r = c$



Section 9.6

Special Polar Graphs

Several important types of graphs have equations that are simpler in polar form than in rectangular form. For example, the circle

$r = 4 \sin \theta$

in Example 1 has the more complicated rectangular equation

$x^2 + (y - 2)^2 = 4.$

Several other types of graphs that have simple polar equations are shown below.

Limaçons

$r = a \pm b \cos \theta, r = a \pm b \sin \theta \quad (a > 0, b > 0)$

$\frac{a}{b} < 1$ Limaçon with inner loop

$\frac{a}{b} = 1$ Cardioid (heart-shaped)

$1 < \frac{a}{b} < 2$ Dimpled limaçon

$\frac{a}{b} \geq 2$ Convex limaçon

Rose Curves

n petals when n is odd, $2n$ petals when n is even ($n \geq 2$)

$r = a \cos n\theta$ Rose curve

$r = a \cos n\theta$ Rose curve

$r = a \sin n\theta$ Rose curve

$r = a \sin n\theta$ Rose curve

Circles and Lemniscates

$r = a \cos \theta$ Circle

$r = a \sin \theta$ Circle

$r^2 = a^2 \sin 2\theta$ Lemniscate

$r^2 = a^2 \cos 2\theta$ Lemniscate

9.7 Polar Equations of Conics

Equations:

$r = \frac{ep}{1 \pm e \cos \theta}$: vertical directrix

$r = \frac{ep}{1 \pm e \sin \theta}$: horizonital directrix

$e > 0$ is the eccentricity

$|p|$ is the distance between the focus and the directirx

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10.1 Three Dimensional Coordinate System

10.1.1 Equation of a sphere

$$(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2$$

Where the center is at (h, j, k)

10.2 Vectors in Space

10.3 The Cross Product of Two Vectors

Gets vector that is orthagonal to two other ones, and the magnitude is the area of the paralleogram that the other vectors make

Finding the cross product: $\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ (find the determinant)

10.3.1 Properties of Cross Products

1. $u \times v = -(v \times u)$
2. $u \times (v + w) = (u \times v) + (u \times w)$
3. $c(u \times v) = (cu) \times v = u \times (cv) - c$ is a constant
4. $u \times 0 = 0 \times u = 0$
5. $u \times u = 0$
6. $u \cdot (v \times w) = (u \times v) \cdot w$
7. $u \times v$ is orthagonal to both u and v
8. $\|u \times v\| = \|u\| \|v\| \sin \theta$
9. $u \times v = 0$ if and only if u and v are scalar multiples of each other
10. $\|u \times v\| =$ area of parallelogram having u and v as adjacent sides

10.3.2 Triple Scalar Product

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

10.3.3 Geometric Property of the Triple Scalar Product

Volume V of a parallelpiped with vectors u, v, and w as adjacent edges is given by

$$V = |u \cdot (v \times w)|$$

10.4 Lines and Planes in space

10.4.1 Parametric Equations of a Line in Space

A line L parallel to the vector $v = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ is:

$$x = x_1 + at_1, y = y_1 + bt, z = z_1 + ct$$

10.4.2 Standard Equation of a plane in space

The plane containing the point (x_1, y_1, z_1) and having normal vector $\langle a, b, c \rangle$ is:

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

10.4.3 Finding Intersection of two planes

Solve the system of equations of the equations of the two planes to get one simpler equation.

10.4.4 Distance between a Point and a Plane

$$D = \|proj_n \vec{PQ}\| = \frac{|\vec{PQ} \cdot n|}{\|n\|} \tag{1}$$

Where Q is the point off the plane, n is the normal vector of the plane, and P is some point on the plane.

8

8.1 Sequences and Series

8.2 Arithmatic Sequences and Partial Sums

8.3 Geometric Sequences and Series

Sum of a finite geometric series:

$$a_1(\frac{1-r^n}{1-r})$$

Sum of an infinite geomtric series (if $0 \leq r \leq 1$)

$$\frac{a_1}{1-r}$$

8.4 The Bionomial Theorem

expansion of $(x+y)^n$:

$$x^n + nx^n - 1y...$$

The coefficient of $x^{n-r}y^r$ is

$${}_nC_r$$

You can also use pascal's triangle