Project 4 Car Tracking

Cheng Zhong 16307110259

January 1, 2020

Problem 1: Bayesian Network Basics

(a) We have:

$$\begin{aligned}
P(C_2=1|D_2=0) &= \frac{P(C_2=1,D_2=0)}{P(D_2=0)} \\
&= \frac{P(C_2=1,D_2=0)}{\sum_{C_1}\sum_{C_2} P(D_2=0,C_1,C_2)} \\
&= \frac{0.5*\eta}{0.5*\epsilon*\eta+0.5*(1-\epsilon)*(1-\eta)+0.5*(1-\epsilon)*\eta+0.5*\epsilon*(1-\eta)} \\
&= \eta
\end{aligned}$$

(b) Since

$$\begin{aligned}
P(C_2|D_2=0,D_3=1) &\propto P(C_2,D_2=0,D_3=1) \\
&\propto \sum_{c_1} P(C_2|D_2=0)\,P(C_3=c_3|C_2)\,P(D_3=1|C_3=c_3) \\
&= \alpha < 1-\eta, \eta >< \epsilon+\eta-2\epsilon\eta, 1-\epsilon-\eta+2\epsilon\eta > \\
&= \alpha < \epsilon+\eta-3\epsilon\eta-\eta^2+2\epsilon\eta^2, \eta-\epsilon\eta-\eta^2+2\epsilon\eta^2 >
\end{aligned}$$

After normalization, we can get

$$\alpha = \frac{1}{\epsilon+2\eta-4\epsilon\eta-2\eta^2+4\epsilon\eta^2}$$

So,

$$P(C_2=1|D_2=0,D_3=1) = \frac{\eta-\epsilon\eta-\eta^2+2\epsilon\eta^2}{\epsilon+2\eta-4\epsilon\eta-2\eta^2+4\epsilon\eta^2}$$

(c) Suppose $\epsilon = 0.1$ and $\eta = 0.2$

i. After put the value into the formula above, we can get

$$\begin{aligned}
P(C_2=1|D_2=0) &= 0.2000 \\
P(C_2=1|D_2=0,D_3=1) &= 0.4157
\end{aligned}$$

ii. After adding the second sensor reading $D_3 = 1$, the posterior probability of P($C_2 = 1$) increase. That's because the value of C_2 is related to the values of C_3 and D_3, when $D_3 = 1$ is more likely to occur under $C_3 = 1$, the posterior probability of $C_3 = 1$ rises when $D_3 = 1$ is observed.

iii. While we set $\eta = 0.2$, in order to keep $P(C_2=1|D_2=0,D_3=1) = P(C_2=1|D_2=0,D_3=1)$, So,

$$\begin{aligned}
\eta &= \frac{\eta-\epsilon\eta-\eta^2+2\epsilon\eta^2}{\epsilon+2\eta-4\epsilon\eta-2\eta^2+4\epsilon\eta^2} \\
\epsilon &= 0.5
\end{aligned}$$