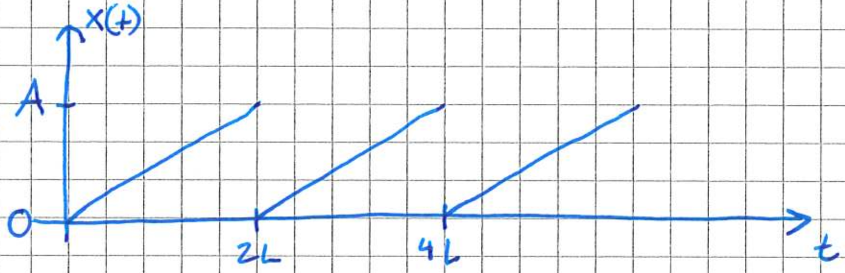
Fourier række for savtakket signal

Vi betragter signalet $x(t)$ defineret som

$$x(t) = A\frac{t}{2L} \qquad \text{for} \quad t \in [0, 2L]$$

og $\quad x(t+2L) = x(t)$. Signalet er vist herunder (Periode: $2L$)



Følgende formler benyttes til udregning af Fourier koefficienter

$$a_n = \frac{1}{L}\int_{-L}^{L} x(t)\cos\left(\frac{n\pi t}{L}\right)dt \qquad n \geq 0 \qquad (1)$$

$$b_n = \frac{1}{L}\int_{-L}^{L} x(t)\sin\left(\frac{n\pi t}{L}\right)dt \qquad n > 0 \qquad (2)$$

Koefficienten a_0 udregnes ved brug af (1)

$$\begin{aligned} a_0 &= \frac{1}{L}\int_{0}^{2L} \frac{A}{2L}\, t \cos\left(\frac{0\pi t}{L}\right)dt \\ &= \frac{A}{2L^2}\int_0^{2L} t\, dt = \frac{A}{2L^2}\left[\frac{t^2}{2}\right]_0^{2L} \\ &= A \end{aligned}$$

Til udregning af de øvrige koefficienter benyttes partiel integration

$$\int_a^b u(x)v'(x)\,dx = \left[u(x)v(x)\right]_a^b - \int_a^b u'(x)v(x)\,dx \qquad (3)$$

Først udregnes a_n-koefficienterne

$$a_n = \frac{A}{2L^2}\int_0^{2L} t\cdot\cos\left(\frac{n\pi t}{L}\right)dt$$

Vi anvender (3) med $u(t) = t$ og $v(t) = \dfrac{\sin\left(\frac{n\pi t}{L}\right)}{\frac{n\pi}{L}}$

$$\begin{aligned} a_n &= \frac{A}{2L^2}\left(\left[t\cdot\frac{\sin\left(\frac{n\pi t}{L}\right)}{\frac{n\pi}{L}}\right]_0^{2L} - \underbrace{\int_0^{2L} 1\cdot\frac{\sin\left(\frac{n\pi t}{L}\right)}{n\pi/L}\,dt}_{0}\right) \\ &= \frac{A}{2L^2}\, 2L\, \frac{\sin(n2\pi)}{\frac{n\pi}{L}} \\ &= 0 \end{aligned}$$

Vi anvender ③ med $u(t) = t$ og $v(t) = \frac{-\cos\left(\frac{n\pi t}{L}\right)}{\frac{n\pi}{L}}$ til udregning af b_n-koefficienterne i ②

$$b_n = \frac{A}{2L^2}\int_0^{2L} t \sin\left(\frac{n\pi t}{L}\right) dt$$

$$= \frac{A}{2L^2}\left(\left[t \cdot \frac{-\cos\left(\frac{n\pi t}{L}\right)}{\frac{n\pi}{L}}\right]_0^{2L} - \underbrace{\int_0^{2L} 1 \cdot \frac{-\cos\left(\frac{n\pi t}{L}\right)}{\frac{n\pi}{L}}}_{=0}\right)$$

$$= \frac{-A}{2L^2}\, 2L \cdot \frac{1}{\frac{n\pi}{L}}$$

$$= \frac{-A}{n\pi}$$

Samlet fås

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{A}{n\pi} \sin\left(\frac{n\pi t}{L}\right)$$