

Structurale properties of decomposable digraphs

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Part I

Primitive graphs and gabbigubbi

0.1 Introduction

Why we need graphs.

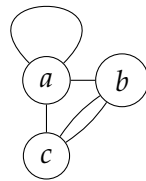
Chapter 1

Graphs and digraphs

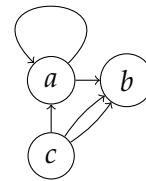
Before going deep into structural properties of decomposable digraphs we first need to establish what a graph is. For some graph $G(V, E)$ where V and E are two sets containing the vertices (also commonly called nodes) and edges of the graph respectively. We define the **size** of the graph to be the number of vertices $|V|$ this is also known as **cardinality** of V . An **edge** $e \in E$ where $e \equiv (a, b)$ and $\{a, b\} \subseteq V$ then e is an edge in G , e is said to be **incident** to a and b . We call $a, b \in V$ **adjacent** if there is an edge (a, b) or (b, a) (an edge between the two given vertices is said to be adjacent). If an edge goes from and to the same vertex (a, a) it is called a **loop**. The set of edges e_1, \dots, e_k is usually describe with the letter E where each edge contains a pair of vertices that are adjacent.

In a graph we have something called a **walk** which is a alternately ordering of vertices and edges in the graph G where the edge in between the two vertices in the ordering is an edge between the vertices in G (for a, e_1, b to be a walk the edge e_1 has to be between a and b). We call a walk closed if the first vertex in the walk is the same as the last.

A **path** in a graph is a walk where each vertex in the ordering can only appear one time. A cycle is a closed walk where the only vertex present more than one time is the first vertex (for the walk to be closed the first vertex has to appear last to also called a closed path).



(a) graph $G(V, E)$ is an example of a graphs, the red edge is a loop, and all pair of vertices in this graph is adjacent.



(b) This is an orientation of the edges in the graph which makes this a digraph

Before delving more specific into graphs and digraphs we must establish some important prerequisite and properties. A graph is called **simple** if there is no loops and no multiple edges. With multiple edges it means multiple edges between the same pair of vertices like in figure Figure ?? between b and c .

A graph is **connected** if there exists a path between all pair of vertices in the graph and **disconnected** otherwise. A graph is called **complete** if there for all pair of vertices in the graph is an edge between them see Figure 1.2.

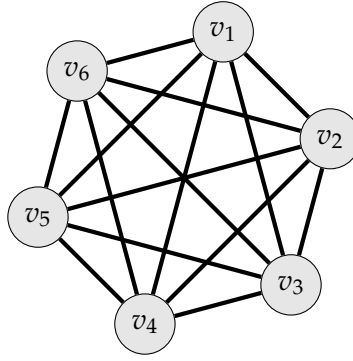


Figure 1.2: Complete graph with 6 vertices.

If we instead of edges have **arcs** between the vertices we call it a **digraph**. An arc is described just like an edge with two adjacent vertices (a, b) the first vertex mentioned in an arc is the vertex **from** where the arc starts, the second vertex is the one the arc is pointing **to**. The set of arcs is normally denoted A like the set of edges is denoted E . So the arc (a, b) goes from a to b , if you wanted it the other way around the arc is (b, a) . These graphs containing only arcs and no edges are called a digraph $G(V, A)$ which is what we in this project are focusing on.

In a digraph a graph we have something called the underlying graph. An underlying graph of a digraph is where all arcs are replaced by edges (edge is used every time we talk about undirected edges between vertices, when direction is mentioned it is called an arc). A digraph is connected if the underlying graph is connected, (also called weakly connected), a digraph can be strongly-connected and semi-connected too. A digraph is called semi-connected if there for each pair u and v exists a path from either u to v or v to u . It is said to be strongly connected if for each pair of vertices u and v there exists a path from both u to v and v to u .

We can use these to describe some specific collection of graphs as the graph tournaments. A tournament is a digraph where the underlying graph is complete. An underlying graph of a digraph is where all arcs are replaced by edges (edge is used every time we talk about undirected edges between vertices, when direction is mentioned it is called an arc). So a complete graph of order 5 any orientation of the edges concludes in a tournament. If instead of replacing the one edge by one arc in either direction, but instead replace it by two arcs the digraph is called semicomplete.

The reason for grouping the digraphs into smaller collections of digraphs (like tournaments is a smaller collection of semicomplete digraphs) is because of problems is easy to describe on specific graphs than general graphs.

We have these graphs called NP-hard problems which sometimes sound easy solvable for graphs but only for some specific graphs we know how to solve it in polynomial time.

Definition 1.0.1. *define NP-hard problems*

In this paper we are focusing on the specific digraphs that are decomposable. A decomposable digraph is a digraph $D = H[G_1, G_2, \dots, G_l H]$ where each G_i is a disconnected graph replacing each vertex of the digraph H