[thm]Lemma Definition[section] [thm]Example

# Algorithm for Finding linkage in a Totally $\phi$-decomposable Digraph

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# Decomposable Digraph

A digraph $D = S[H_1, \ldots, H_s]$ is called Decomposable where $|S| = s$ and each $H_i$ are disjoint digraphs replacing every vertex in $S$. The digraphs $H_i$ will in this presentation be called **hauses**. For a set of digraphs $\phi$ we can talk about $\phi$-decomposable digraphs. For a digraph $D = S[H_1, \ldots, H_s]$ to be $\phi$-decomposable either $D \in \phi$ or $S \in \phi$.

For a digraph $D = S[H_1, \ldots, H_s]$ to be totally $\phi$-decomposable it has to be $\phi$-decomposable and if $D \notin \phi$, then $S \in \phi$ and each $H_i$ has to be totally $\phi$-decomposable for $i = 1, \ldots, s$.

# The weak linkage Problem

Given two pair of vertices $s_1, t_1$ and $s_2, t_2$ finding arc-disjoint paths between each pair is 2-weak linkage problem. Then the $k$-weak linkage problem is finding k arc-disjoint paths between k pair of terminals. where a terminal pair is a source and a zink in the paths of the solution of the linkage problem. When talking about linkage problem for decomposable digraph, we can have hauses with terminals in and some without any terminals. The hauses with no terminals in is called **clean hauses**. Then a terminal pair can either be inside the same hauses or in different hauses. If a terminal pair is contained inside the same hause it is called an internal pair other wise we call the pair external. The same with a path if it is fully containd inside a hause it is called an internal path other wise it is external. you can have an external path for a internal piar if the path go out of the hause and in agian.

# Notation for the weak linkage problem

Every thing on this list is denoted like this unless it is specifed noot to be like that.

- a natruel number $k$ is going to repricent the number of terminal pairs there in the linkage problem.
- $\Pi$ denote the set of terminal pairs $(s_1, t_1), \ldots, (s_k, t_k)$.
- $D$ is the totally $\phi$-decomposable digraph where each hause is denoted $H_i$ for $i = 1, \ldots, s$.

## Bombproof sets

### Definition

[?] We say that a class of digraphs $\phi$ is Bombproof is there exsists a polynomial algorithm $\mathcal{A}_\phi$ to find a totally $\phi$-decomposition of every totally $\phi$-decomposable digraph and, for every integer $c$, there exists a polynomial algorithm $\mathcal{B}_\phi$ to decide the weak $k$-linkage problem for the class

$$\phi(c) := \bigcup_{D \in \phi} D(c) \tag{1}$$

for a integer $c$, the class denoted $D(c)$ is the for a digraph $D$ you can first add as many parallel arcs to arcs that already exists in $D$ you can blow up $b$ vertices where $0 \leq b \leq c$ the digraph that is blown up to has a size $\leq c$.

### Lemma

*[?] Let $D$ be a digraph, $\Pi$ a list of $k$ terminal pairs and $H \subset D$ a clean module with respect to $\Pi$. Let $D'$ be the contraction of $H$ into a single vertex $h$. Then $D$ has a waek $\Pi$-linkage if and only if $D'$ has a weak $\Pi$-linkage.*