

# Sorting Algorithms

1. Selection
2. Bubble
3. Insertion
4. Merge
5. Quick
6. Shell

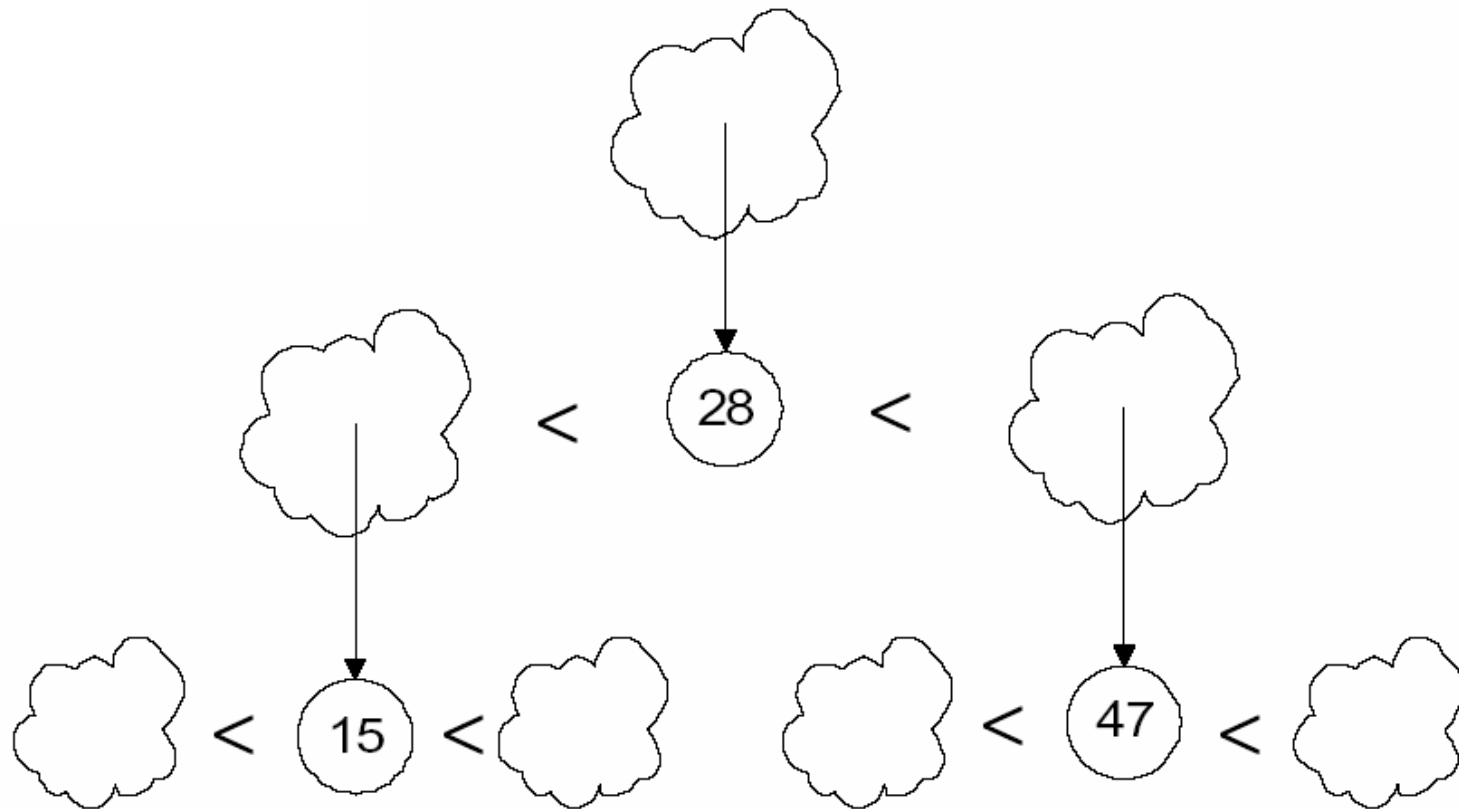
# Sorting algorithms

- Metode Insertion, selection dan bubble sort memiliki worst-case performance yang bernilai kuadratik
- Apakah algoritma berbasis comparison yang tercepat ?

$O(n \log n)$

- Mergesort dan Quicksort

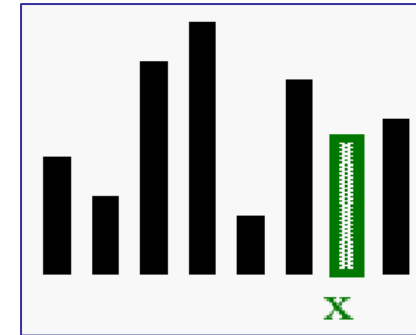
# Idea of Quicksort



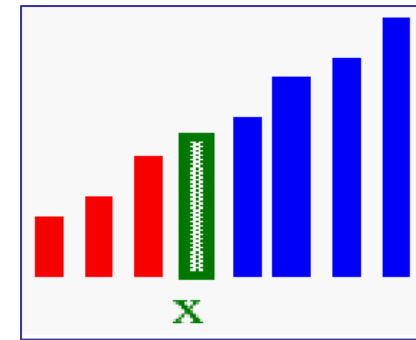
- Ambil sebuah “pivot”.
- Bagi menjadi 2 : bagian yang kurang dari dan bagian yang lebih dari pivot
- Urutkan masing-masing bagian secara rekursif

# Idea of Quicksort

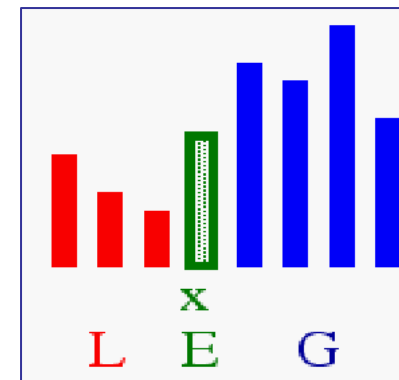
1. **Select:** pick an element



2. **Divide:** rearrange elements so that **x goes to its final position E**



3. **Recur and Conquer:** recursively sort



# Quicksort Algorithm

Misal diberikan sebuah array  $A$  memiliki  $n$  elemen (integer)  $\rightarrow p = 0; r = n-1$

- Array  $A[p..r]$  dipartisi menjadi dua non-empty subarray :  $A[p..q]$  and  $A[q+1..r]$ 
  - Seluruh elemen dalam array  $A[p..q]$  lebih kecil dari seluruh elemen dalam array  $A[q+1..r]$
- Seluruh sub array diurutkan secara rekursif dengan cara memanggil fungsi `quicksort ( )`

# Quicksort Code

```
Quicksort_rekursif(A, p, r)
{
    if (p < r)
    {
        q = Partition(A, p, r);
        Quicksort_rekursif(A, p, q);
        Quicksort_rekursif(A, q+1, r);
    }
}
```

# Partition

- Terlihat bahwa, seluruh aksi terjadi dalam fungsi `partition()`
  - Rearranges subarray secara in place
  - Hasil akhir:
    - Dua subarray
    - Seluruh elemen pada subarray pertama  $\leq$  seluruh elemen pada subarray kedua
  - Return value berupa index dari elemen “pivot” – yang memisahkan kedua subarray tsb
- *How do you suppose we implement this?*

# Partition In Words

- Partition(A, p, r):
  - Pilih sebuah elemen yang bertindak sebagai “pivot” (*which?*)
  - Pecah array menjadi dua bagian, A[p..i] and A[j..r]
    - Seluruh element dalam A[p..i]  $\leq$  pivot
    - Seluruh element dalam A[j..r]  $\geq$  pivot
  - (**HOW ?**)
  - Increment i until A[i]  $\geq$  pivot
  - Decrement j until A[j]  $\leq$  pivot
  - Jika  $i < j$ , maka Swap A[i] and A[j]
  - Jika tidak, return j
  - Repeat until  $i \geq j$
  - return j



# Partition Code

```
Partition(A, p, r)
    x = A[p]; //pivot=elemen posisi pertama
    i = p ;   //inisialisasi
    j = r ;
    repeat
        while(A[j] > x)
            j--;
        while(A[i] < x)
            i++;
        if (i < j){
            Swap(A, i, j);
            j--;
            i++;
        }
        else
            return j;
    until i >= j
    return j
```

12	35	9	11	3	17	23	15	31	20
----	----	---	----	---	----	----	----	----	----

QuickSort(0,9)

- $X = \text{PIVOT}$  merupakan indeks ke -0
- $\text{PIVOT} = 12$
- terdapat variabel  $i$  dan  $j$  ,  $i=0$  ,  $j=9$
- variabel  $i$  untuk mencari bilangan yang lebih dari atau sama dengan PIVOT. Cara kerjanya : selama  $\text{Data}[i] < \text{PIVOT}$  maka nilai  $i$  ditambah.
- variabel  $j$  untuk mencari bilangan yang lebih kecil dari atau sama dengan PIVOT. Cara kerjanya : selama  $\text{Data}[j] > \text{PIVOT}$  maka nilai  $j$  dikurangi

$q = \text{Partition}(0, 9)$

12	35	9	11	3	17	23	15	31	20
----	----	---	----	---	----	----	----	----	----

SWAP

PIVOT = 12

$i = 0 \ j = 4$

$i < j$  maka SWAP

3	35	9	11	12	17	23	15	31	20
---	----	---	----	----	----	----	----	----	----

SWAP

PIVOT = 12

$i = 1 \ j = 3$

$i < j$  maka SWAP

3	11	9	35	12	17	23	15	31	20
---	----	---	----	----	----	----	----	----	----

**PIVOT = 12**

**$i = 3$   $j = 2$**

**$i < j$  (False) NO SWAP**

**Return  $j = 2$**

**Q = Partisi = 2**

QuickSort(0,9)



```
graph TD; A[QuickSort(0,9)] --> B[QuickSort(0,2)]; A --> C[QuickSort(3,9)];
```

QuickSort(0,2)

QuickSort(3,9)

QuickSort(0,2)



3	11	9
---	----	---

35	12	17	23	15	31	20
----	----	----	----	----	----	----

**PIVOT = 3**

**i = 0 j = 0**

**i < j (False) NO SWAP**

**Return j = 0**

**Q = Partisi = 0**

QuickSort(0,0)

QuickSort(1,2)

QuickSort(1,2)

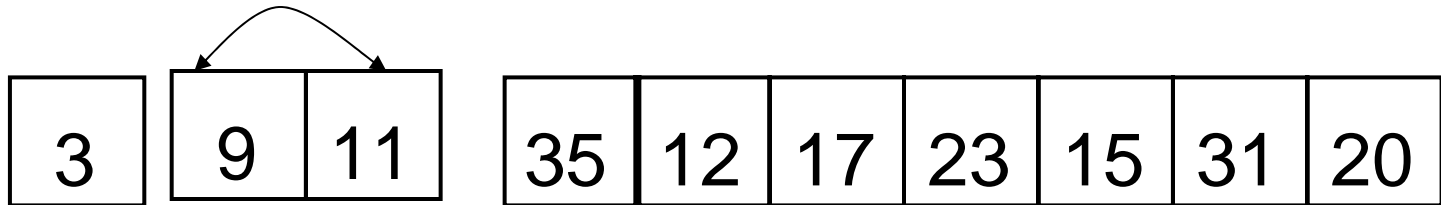


PIVOT = 11

$i = 1$   $j = 2$

$i < j$  SWAP

SWAP



PIVOT = 11

$i = 2$   $j = 1$

$i < j$  NO SWAP

Return  $j = 1$

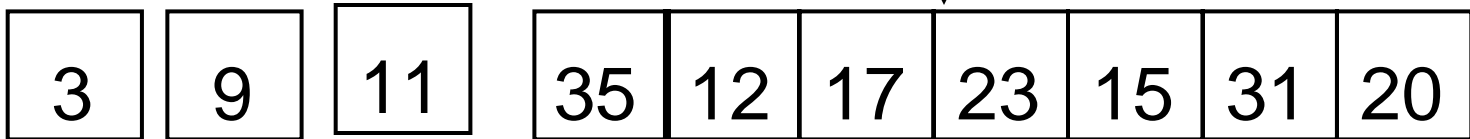
**Q = Partisi = 1**

QuickSort(1,2)

QuickSort(1,1)

QuickSort(2,2)

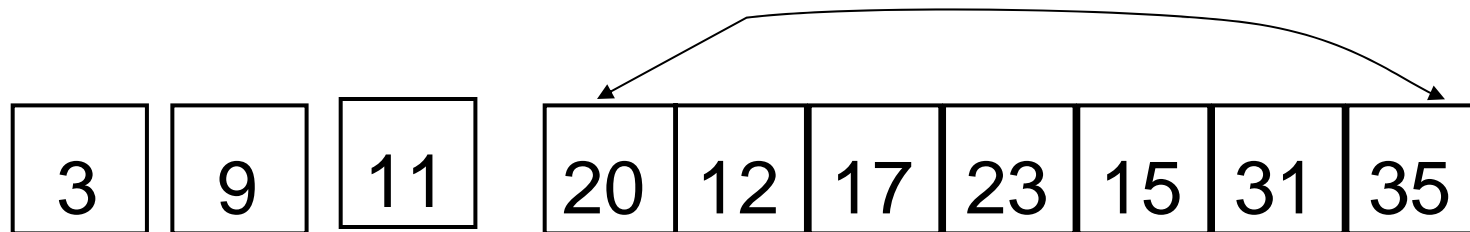
QuickSort(3,9)



PIVOT = 35

i = 3 j = 9

i < j SWAP



**PIVOT = 35**

**$i = 9$   $j = 8$**

**$i < j$  NO SWAP**

**Return  $j = 8$**

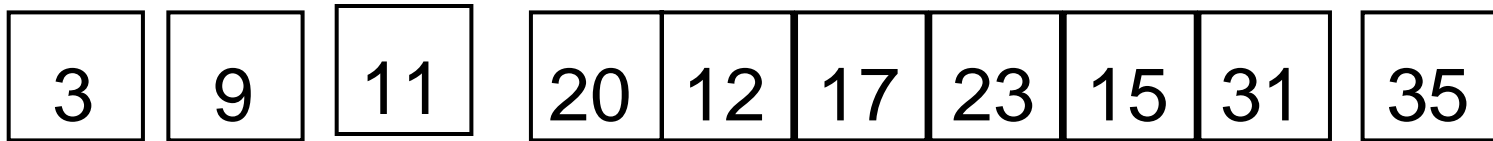
**$Q = \text{Partisi} = 8$**

**QuickSort(3,9)**

**QuickSort(3,8)**

**QuickSort(9,9)**





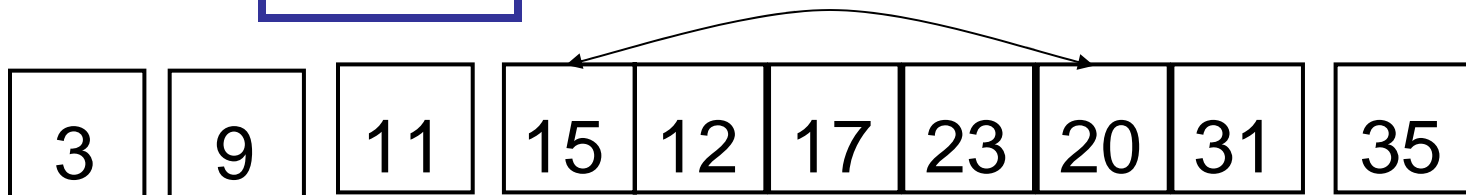
QuickSort(3,8)

PIVOT = 20

$i = 3$   $j = 7$

$i < j$  SWAP

SWAP



PIVOT = 20

$i = 6$   $j = 5$

$i < j$  NO SWAP

Return  $j = 5$

**Q = Partisi = 5**

QuickSort(3,8)

QuickSort(3,5)

QuickSort(6,8)

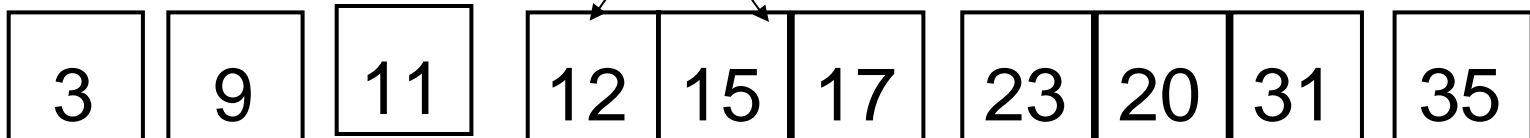


**PIVOT = 15**

**$i = 3$   $j = 4$**

**$i < j$  SWAP**

**SWAP**



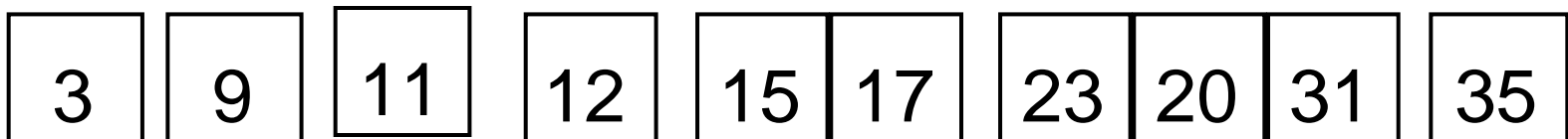
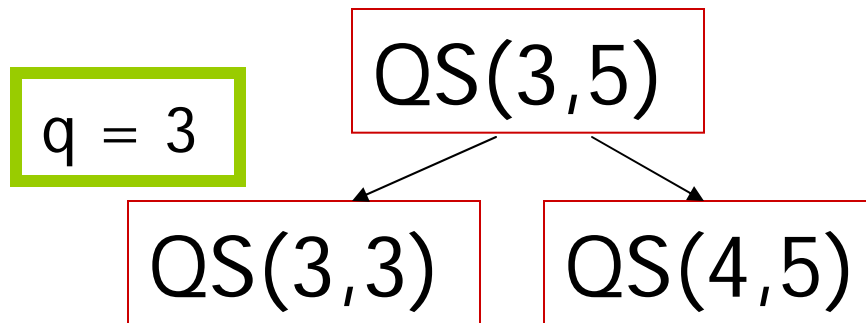
PIVOT = 15

$i = 4$   $j = 3$

$i < j$  NO SWAP

Return  $j = 3$

**Q = Partisi = 3**



QS(4,5)

PIVOT = 15

$i = 4$   $j = 4$

$i < j$  NO SWAP

Return  $j = 4$

**Q = Partisi = 4**

$q = 4$

QS(4,5)

QS(4,4)

QS(5,5)

3	9	11	12	15	17	23	20	31	35
---	---	----	----	----	----	----	----	----	----

QuickSort(6,8)

PIVOT = 23

$i = 6$   $j = 7$

$i < j$  SWAP

3	9	11	12	15	17	20	23	31	35
---	---	----	----	----	----	----	----	----	----

PIVOT = 23

$i = 7$   $j = 6$

$i < j$  NO SWAP

Return  $j = 6$

**Q = Partisi = 6**

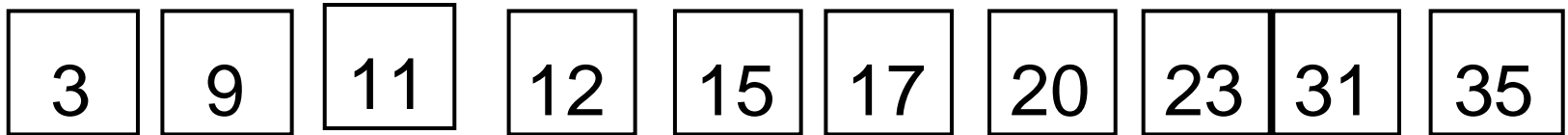


QS(6,8)

q = 6

QS(6,6)

QS(7,8)



QS(7,8)

PIVOT = 23

$i = 7$   $j = 7$

$i < j$  NO SWAP

Return  $j = 7$

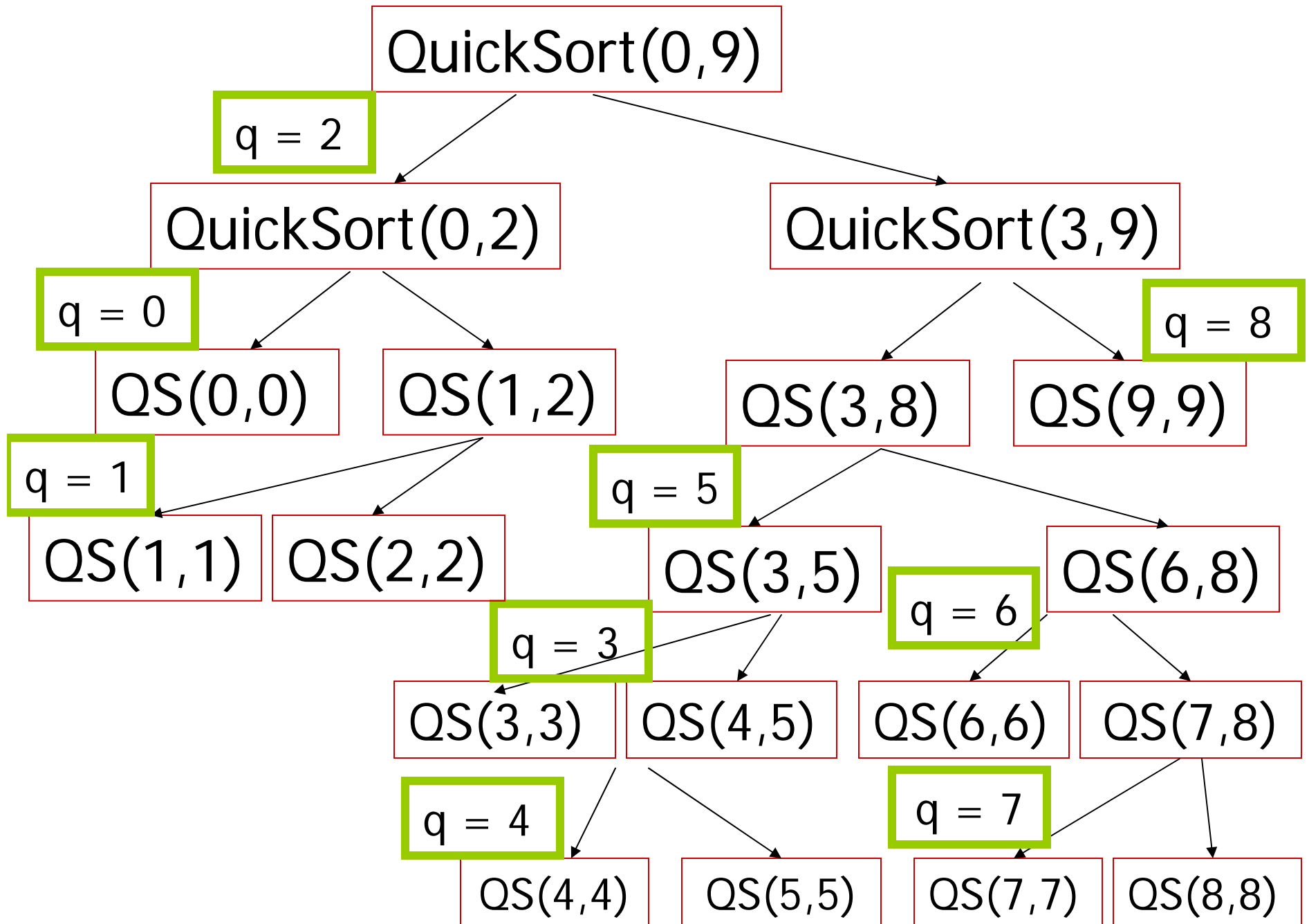
**Q = Partisi = 7**



QS(7,8)

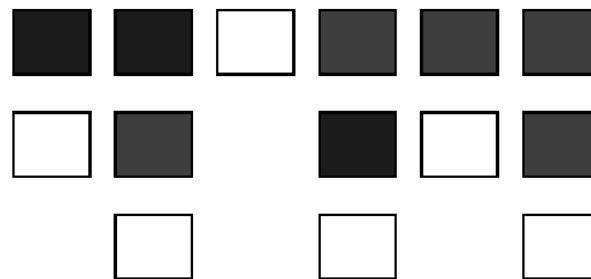
QS(7,7)

QS(8,8)



# Quicksort Analysis

- Jika diasumsikan pivot diambil secara random, terdistribusi secara uniform
- **Best case running time:  $O(n \log_2 n)$** 
  - Pada setiap pemanggilan rekursif, posisi elemen pivot selalu persis di tengah, array dipecah menjadi dua bagian yang sama, elemen-elemen yang lebih kecil dan yang lebih besar dari pivot



# Quicksort Analysis

## **Worst case: $O(N^2)$**

- Pada setiap pemanggilan rekursif, pivot selalu merupakan elemen terbesar (atau terkecil); array dipecah menjadi satu bagian yang semua elemennya lebih kecil dari pivot, pivot, dan sebuah bagian lagi array yang empty



# Summary of Sorting Algorithms

Algorithm	Time	Notes
selection-sort	$O(n^2)$	<ul style="list-style-type: none"><li>• in-place</li><li>• slow (good for small inputs)</li></ul>
insertion-sort	$O(n^2)$	<ul style="list-style-type: none"><li>• in-place</li><li>• slow (good for small inputs)</li></ul>
quick-sort	$O(n \log n)$ expected	<ul style="list-style-type: none"><li>• in-place, randomized</li><li>• fastest (good for large inputs)</li></ul>
merge-sort	$O(n \log n)$	<ul style="list-style-type: none"><li>• sequential data access</li><li>• fast (good for huge inputs)</li></ul>

# Mergesort X Quicksort

- **Mergesort**

- Splits partitions in half
- Merges smaller lists back into larger list
- Requires overhead when sorting arrays

- **Quicksort**

- Relies on a pivot point for sorting
- Smaller sets are sorted based on pivot point
- Can perform slowly if a bad pivot point is used

# Mergesort X Quicksort

- **Mergesort**
  - Use extra space
  - Is guaranteed to have  $O(n \log n)$  performance in the worst case
- **Quicksort**
  - Does **not** use extra space
  - Is **not** guaranteed to have  $O(n \log n)$  performance in the worst case, unlike merge sort

# Mergesort X Quicksort

- **Advantages of quicksort over mergesort**
  - Quicksort doesn't require an extra array
  - the hidden constant in the average case for quicksort is smaller than the hidden constant for merge sort.
- **Advantage of mergesort over quicksort:**
  - better worst case behavior
- **Quicksort and mergesort are optimal, in the sense that a general sorting algorithm cannot do better than average case  $O(n \log n)$ .**

# Mergesort X Quicksort

- Both QuickSort and MergeSort are  $O(n \log n)$  for their average cases.
- However, the characteristic of 'O' notation is that you drop all constant factors.
- Therefore, one  $O(n)$  algorithm could take 1000 times as long as another  $O(n)$  algorithm, and as long as the complexity isn't dependant on the size on 'n', they're both  $O(n)$  algorithms.

# Mergesort X Quicksort

- The advantage of QSort over MergeSort is that it's constant factor is smaller than MergeSort's, and so therefore, on the average case, it is faster (**but not less complex**).
- Empirically, QSort is best, especially if the 'pivot' value has been properly selected