

FAKULTET ELEKTROTEHNIKE, RAČUNARSTVA I INFORMACIJSKIH  
TEHNOLOGIJA OSIJEK

Diplomski studij – Robotika i umjetna inteligencija

## Osnove Robotike

Laboratorijska vježba 2

Inverzna kinematika robotskog manipulatora

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# 1. Cilj vježbe

Pobliže se upoznati s problemima inverzne kinematike i planiranja trajektorije robotskog manipulatora

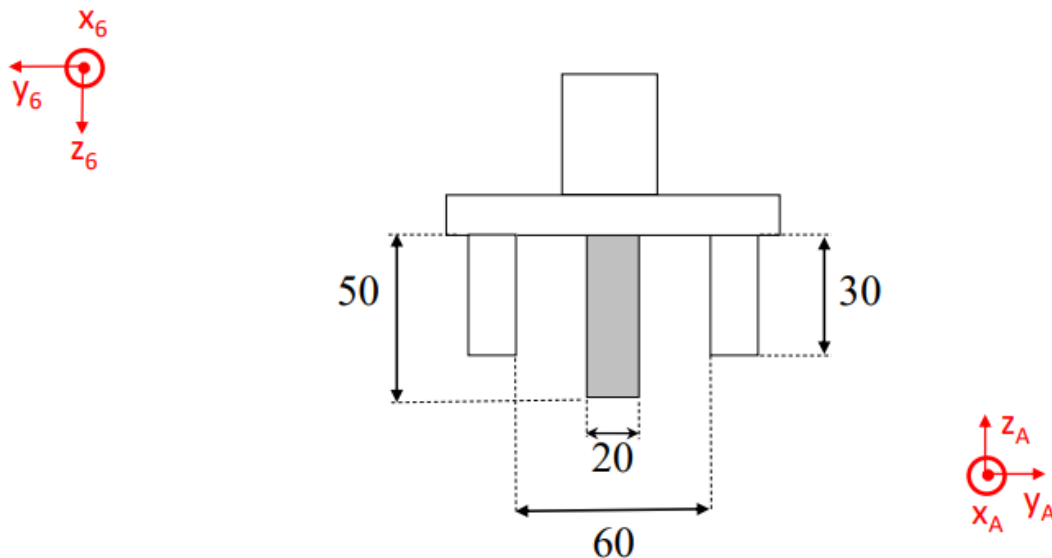
## 2. Opis vježbe

Šestoosni robotski manipulator razmatran u prvoj laboratorijskoj vježbi konstruiran je tako da mu se osi četvrtog, petog i šestog zgloba sijeku u jednoj točki. Problem inverzne kinematike za takav manipulator moguće je riješiti primjenom Pieperovog rješenja.

## 3. Rješenje

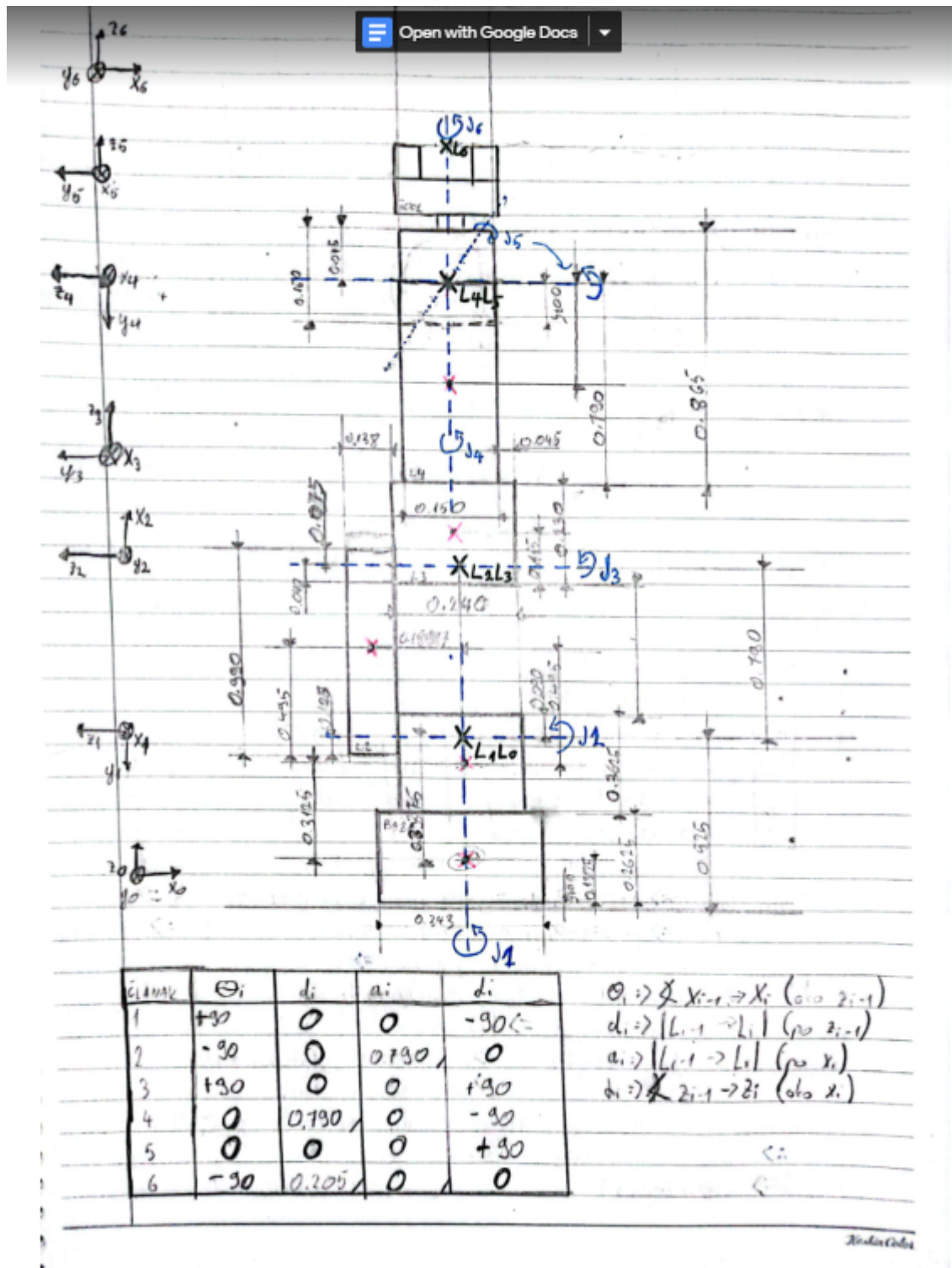
U rješenju je prikazat matematički postupak urađen u bilježnici koji nije točan za zadani zadatak uz veći broj ponavljanja računa.

1. Treba odrediti varijable zglobova koje omogućuju postavljanje alata u položaj prikladan za hvatanje kvadra A dimenzija  $40 \times 20 \times 50$ , položenog na površinu na kojoj je postavljen i robotski manipulator pri čemu su  $pA\ 1\ 0 = 300$   $pA\ 2\ 0 = -120$  koordinate središta kvadra u odnosu na bazni koordinatni sustav robota. Položaj alata prikladan za hvatanje kvadra A prikazan je na slici ispod



Sl. 2

Kinematički parametri razmatranog robotskog manipulatora određeni metodom DenavitHartenberga za položaj prikazan na slici ispod dani su u tablici ispod



Nadalje se slijede koraci iz prezentacije;

- Položaj alata u odnosu na kocku A
- Položaj alata u odnosu na bazni k.s

② Odczytaj pozycję  $p = [x \ y \ z]^T$  zgodnie z trzema zdaniami 5 iteracji  
( $d_0 = 0.205$ )

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = {}^0T_6 \begin{bmatrix} 0 \\ 0 \\ -d_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -0.205 \\ 0 & 0 & 0 & | & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -0.205 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$${}^0T_6 = \begin{bmatrix} 1 & 0 & 0 & | & 0.79 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0.62 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$${}^1T_6 = \begin{bmatrix} 1 & 0 & 0 & | & 0.79 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0.47 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$$\begin{aligned} p_1^0 &= 300 \\ p_1^1 &= 120 \\ p_1^2 &= 0.13125 \cdot 0.4 \end{aligned}$$

$${}^0T_A = \begin{bmatrix} 1 & 0 & 0 & | & 0.3 \\ 0 & 1 & 0 & | & -0.12 \\ 0 & 0 & 1 & | & -0.1125 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$${}^0T_6 = \begin{bmatrix} 1 & 0 & 0 & | & 0.3 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -0.1125 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$$A^{-1}T_6 = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & -1 & 0 & | & 0 \\ 0 & 0 & -1 & | & -5 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

Podaj aktualną wartość:  $p = [0.3, -0.12, -0.13125]^T$

$${}^0T_6 = {}^0T_A \cdot {}^1T_6 = \begin{bmatrix} 1 & 0 & 0 & | & 0.3 \\ 0 & -1 & 0 & | & -0.12 \\ 0 & 0 & -1 & | & -0.1125 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$$p = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & -1 & 0 & -0.12 \\ 0 & 0 & -1 & -0.13625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot X \begin{bmatrix} \cos \theta \\ 0 \\ -d_6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3 \\ -0.12 \\ 0.06875 \\ 1 \end{bmatrix}$$

$\begin{aligned} \sin \theta_0 &= 1 \\ \cos \theta_0 &= 0 \\ \sin 0 &= 0 \\ \cos 0 &= 1 \end{aligned}$

$$x = c_1 g_1 - s_1 g_2 \Rightarrow 0.3 = \cos \theta_1 g_1 - \sin \theta_1 g_2 \Rightarrow 0.3 = 0 \cdot g_1 - 1 \cdot g_2 \Rightarrow g_2 = 0.3$$

$$y = s_1 g_1 + c_1 g_2 \Rightarrow -0.12 = \sin \theta_1 g_1 + \cos \theta_1 g_2 \Rightarrow -0.12 = 1 \cdot g_1 + 0 \cdot g_2 \Rightarrow g_1 = -0.12$$

$$z = \sin d_1 (f_1 \sin \theta_2 + f_2 \cos \theta_2) + \cos d_1 (f_3 + d_2) + d_1$$

$$\Rightarrow 0.06875 = -1 \cdot (f_1 \cdot 1) + f_2 \cdot 0 + 0 \cdot (f_3 + d_2) + d_1$$

$$\Rightarrow 0.06875 = f_1 + 0 \Rightarrow f_1 = 0.06875$$

$$f_1 = d_4 \sin d_3 \sin \theta_3 + a_3 \cos \theta_3 + a_2 \Rightarrow f_1 = 0.790 \cdot 1 \cdot \sin 3^\circ + 0 + 0.7900$$

$$\Rightarrow f_1 = 0.79033 + 0.790$$

$$f_2 = a_3 \cos d_2 \sin \theta_2 - d_4 \cos d_2 \sin d_3 \cos \theta_3 - d_4 \sin d_2 \cos d_3 - \sin d_2 d_3$$

$$\Rightarrow f_2 = 0 + 0.790 \cdot \cos 3^\circ - 0 + 0$$

$$\Rightarrow f_2 = 0.7903$$

Pošto je  $a_1 = 0$ , do  $\theta_3$  se dolazi rješavanjem jednadžbe

$$x^2 + y^2 + z^2 = 2d_1 z + k_1,$$

gdje je

$$\begin{aligned} k_1 &= f_1^2 + f_2^2 + f_3^2 + a_1^2 - d_1^2 + d_2^2 + 2d_2 f_3 \\ &= f_1^2 + f_2^2 \end{aligned}$$

$$f_3 = a_3 \cdot \sin \theta_2 \cdot \sin \theta_3 - d_4 \sin \theta_2 \cdot \sin \theta_3 \cdot \cos \theta_3 + d_4 \cdot \cos \theta_2 \cdot \cos \theta_3 + \cos \theta_2 \cdot d_3$$

$$\Rightarrow f_3 = 0 - 0 + 0 + 0$$

$$\Rightarrow f_3 = 0$$

$$f_1 = 0.790 \sin \theta_3 + 0.790, \quad f_2 = -0.790 \cos \theta_3, \quad f_3 = 0$$

$$d_1 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 = 2d_1 z + k_1$$

$$\Rightarrow L_1: f_1^2 + f_2^2 + f_3^2 + a_1^2 - d_1^2 + d_1^2 + 2d_1 f_3 \Rightarrow f_1^2 + f_2^2 =$$

$$\Rightarrow \Rightarrow \boxed{-1.6439} \quad \boxed{-0.6477}$$

Umkehrung von gegeben oder gegeben  $\Rightarrow \theta_3 = 2 \arctan(\theta_3) \quad \theta_3 = -1.5439$

$$\theta_3 = -0.99603 \text{ rad}$$

$$\Rightarrow f_1 = 0.790 \cdot (-0.83931 + 0.790) \Rightarrow f_1 = 0.12694$$

$$\Rightarrow f_2 = -0.790 \cdot 0.54364 \Rightarrow f_2 = 0.429475$$

Umkehrung von gegeben

$$L_2 = \cos \theta_1 (f_1 + d_2) + d_1 \Rightarrow L_2 = 0 + 0 \quad L_2 = 0$$

$$z = \sin \theta_1 (f_1 \sin \theta_2 + f_2 \cos \theta_2) + L_2 \Rightarrow 0.06875 = -1 \cdot (0.12694 \sin \theta_2 - 0.429475 \cos \theta_2)$$

$$\Rightarrow 0.06875 = -0.12694 \sin \theta_2 + 0.429475 \cos \theta_2 \quad / : \frac{1}{\cos \theta_2}$$

$$0.12694 \frac{\sin \theta_2}{\cos \theta_2} + 0.06875 = \frac{\cos \theta_2}{\cos \theta_2}$$

$$0.12694 \tan \theta_2 = 1 - 0.06875$$

$$0.12694 \tan \theta_2 = 0.93125 \quad / : 0.12694$$

$$\tan \theta_2 = 7.33614$$

$$\theta_2 = 1.4353 \text{ rad} / 82.23^\circ$$

$$g_1 = \cos \theta_2 \cdot f_1 - \sin \theta_2 \cdot f_2 + a_1$$

$$\Rightarrow 0.13508 \cdot (0.790 \sin \theta_3 + 0.790) - 0.99608 \cdot (-0.790 \cos \theta_3) + 0$$

$$g_1 = 0.017147 + 0.425523$$

$$g_1 = 0.44267$$

$$g_2 = \cos \theta_1 (f_1 \sin \theta_2 + f_2 \cos \theta_2) - \sin \theta_1 (f_3 + d_2)$$

$$= 0 + 0$$

$$g_2 = 0$$

$$x = \cos \theta_1 \cdot g_1 - \sin \theta_1 \cdot g_2$$

$$0.3 = \cos \theta_1 \cdot 0.44267$$

$$\cos \theta_1 = 0.6777$$

$$\theta_1 = 0.82616 / 47.332^\circ$$

$$y = \sin \theta_1 \cdot g_1 + \cos \theta_1 \cdot g_2$$

$$-0.42 = 0.44267 \sin \theta_1$$

$$\sin \theta_1 = -0.2108$$

$$\theta_1 = -0.214517 / -15.728^\circ$$



$${}^0R_3 = \begin{bmatrix} c_1 - s_1 c d_1 & s_1 s d_1 & c_2 - s_2 c d_2 & s_2 s d_2 & c_3 - s_3 c d_3 & s_3 s d_3 \\ s_1 - c_1 c d_1 & -c_1 s d_1 & s_2 - c_2 c d_2 & -s_2 s d_2 & s_3 - c_3 c d_3 & -s_3 s d_3 \\ 0 & s d_1 & c d_1 & 0 & s d_3 & c d_3 \end{bmatrix}$$

$${}^0R_3 = \begin{bmatrix} 0.54363 & 0 & 0.83333 & 1.35082 & -0.99993 & 0 \\ -0.83333 & 0 & 0.54363 & 0.99993 & 1.35082 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} s d_1 = -1 & c d_1 = 0 & c_1 = 0.54363 & s_1 = 0.83333 \\ s d_2 = 0 & c d_2 = 1 & c_2 = 1.35082 & s_2 = 0.99993 \\ s d_3 = 1 & c d_3 = 0 & c_3 = 0.6774 & s_3 = 0.7353 \end{matrix}$$

$${}^0R_3 = \begin{bmatrix} 0.7353 & -0.5335 & 0.8333 & 0.6774 & 0 & 0.7353 \\ -1.337 & 0.7353 & 0.5436 & 0.7353 & 0.6774 & 0 \\ -0.8333 & -1.3508 & 0 & 0 & 1 & 0 \end{bmatrix}$$

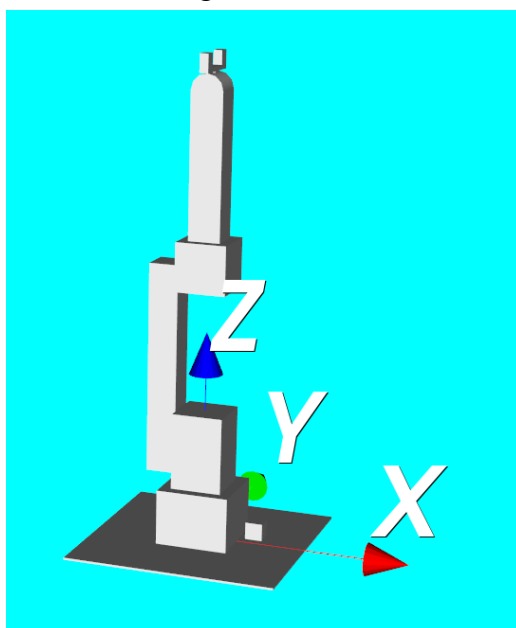
$${}^0R_3 = \begin{bmatrix} 0.1013 & 0.8333 & 0.9048 \\ -0.1565 & 0.5436 & -1.3869 \\ -1.6645 & 0 & 0.1864 \end{bmatrix}$$

Već pri računu  ${}^0R_3$  je uviđeno da robot neće ići u dobrom smjeru što se tiče zakretanja odnosno da nećemo dobiti dobre kuteve zakreta bez obzira na broj pokušaja i odabira vrijednosti za kuteve  $\Theta_1$ ,  $\Theta_2$ ,  $\Theta_3$

P.S. Rečeno je da pošaljem nepotpuno.

Također iz pokušaja koristeći se programskom podrškom dobijemo error odnosno dobijamo NaN vrijednosti za povratnu vrijednost kuta.

Dakle u položaju kao na slici.1 robot bi izgledao ovako:



Problem nastaje negdje pri slaganju matrica homogene transformacije te dolazi do lošeg množenja matrica. Program daje vrijednosti kuteva kao na slici ispod:

```
▼ q: array([nan, nan, nan, nan, nan,  0.])  
> special variables  
> [0:6] : [nan, nan, nan, nan, nan, 0.0]  
> dtype: dtype('float64')
```

Krajnji rezultat izvršenja programa je sljedeći:

