

# LAB-3

## Exponential Smoothing

- Use this data for Exponential Smoothing (single, Double, Triple). Take the smoothing parameter as:  $\alpha=0.2$ ,  $\beta=0.1$ ,  $\gamma=0.4$

```
In [1]: import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline
```

```
In [2]: df=pd.read_csv("C:\\Users\\Lenovo\\OneDrive\\Desktop\\Timeseries_lab\\lab3\\Daily_Ter
df
```

```
Out[2]:
```

	Date	Temp
0	01-01-1981	20.7
1	02-01-1981	17.9
2	03-01-1981	18.8
3	04-01-1981	14.6
4	05-01-1981	15.8
...	...	...
3645	27-12-1990	14.0
3646	28-12-1990	13.6
3647	29-12-1990	13.5
3648	30-12-1990	15.7
3649	31-12-1990	13.0

3650 rows × 2 columns

```
In [3]: df.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3650 entries, 0 to 3649
Data columns (total 2 columns):
#   Column  Non-Null Count  Dtype
---  -
0   Date    3650 non-null    object
1   Temp    3650 non-null    float64
dtypes: float64(1), object(1)
```

memory usage: 57.21 KB

In [4]:

```
# Central tendency of the data
print(f'The Mean : {df.Temp.mean()}')
print(f'The standard deviation : {df.Temp.std()}')
```

The Mean : 11.177753424657539

The standard deviation : 4.07183689939719

- Converting date into time format

In [5]:

```
df['Date'] = pd.to_datetime(df.Date, format='%d-%m-%Y')
df['year'] = pd.DatetimeIndex(df['Date']).year
df['month'] = pd.DatetimeIndex(df['Date']).month
print(f'Min date : {df.Date.min()}')
print(f'Max date : {df.Date.max()}')
```

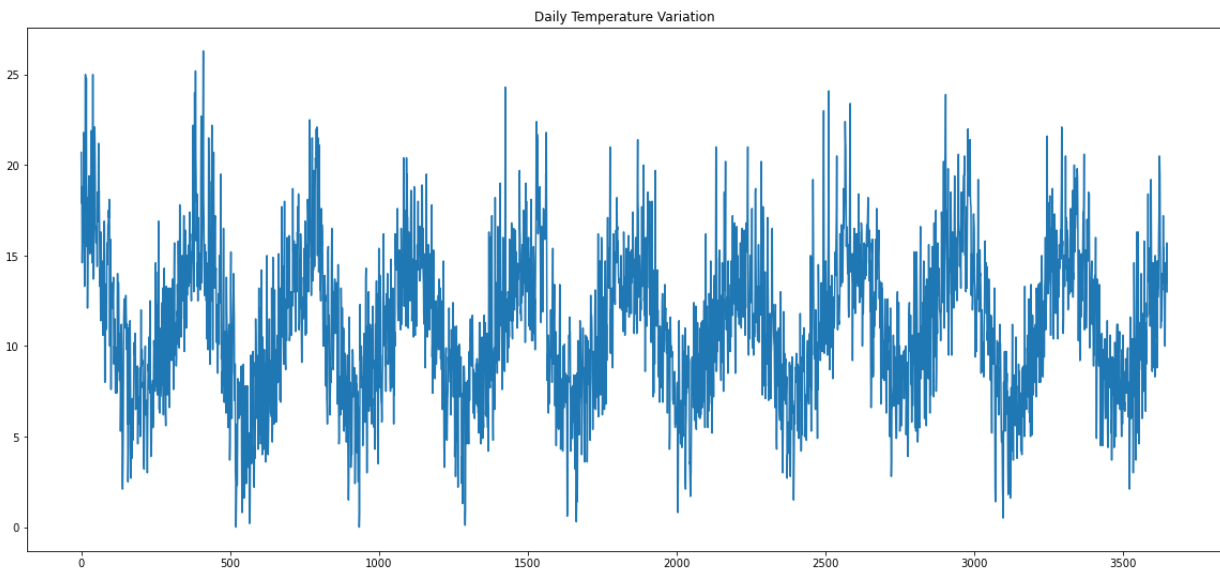
Min date : 1981-01-01 00:00:00

Max date : 1990-12-31 00:00:00

In [6]:

```
# Plot the value to get the characteristic of the serie
plt.figure(figsize=(20,9))
plt.title('Daily Temperature Variation')
plt.plot(range(len(df.index)), df.Temp)
```

Out[6]: [matplotlib.lines.Line2D at 0x21123d979d0]



In [7]:

df

Out[7]:

	Date	Temp	year	month
0	1981-01-01	20.7	1981	1
1	1981-01-02	17.9	1981	1
2	1981-01-03	18.8	1981	1
3	1981-01-04	14.6	1981	1

	Date	Temp	year	month
4	1981-01-05	15.8	1981	1
...	...	...	...	...
3645	1990-12-27	14.0	1990	12
3646	1990-12-28	13.6	1990	12
3647	1990-12-29	13.5	1990	12
3648	1990-12-30	15.7	1990	12
3649	1990-12-31	13.0	1990	12

- we can find yearly seasonality in the data

```
In [8]: # Yearly average temperature plot
plt.figure(figsize=(10, 6))
df1 = df.copy()
y_df = df1.groupby(by=['year', 'month']).mean() # This gives multilevel index
```

<Figure size 720x432 with 0 Axes>

```
In [9]: # Extract the monthly average temperature for each year
List = []
for i in range(df.year.min(), df.year.max()+1):
    List.append(y_df.loc[i, 'Temp'])
total = []
for i in List:
    total.extend(i)
total = zip(range(1, len(total)+1), total)
new_data = pd.DataFrame(total, columns=['Months', 'Average_Month_Temp'])
```

```
In [11]: total
```

```
Out[11]: <zip at 0x18cb0c5dec0>
```

```
In [12]: new_data
```

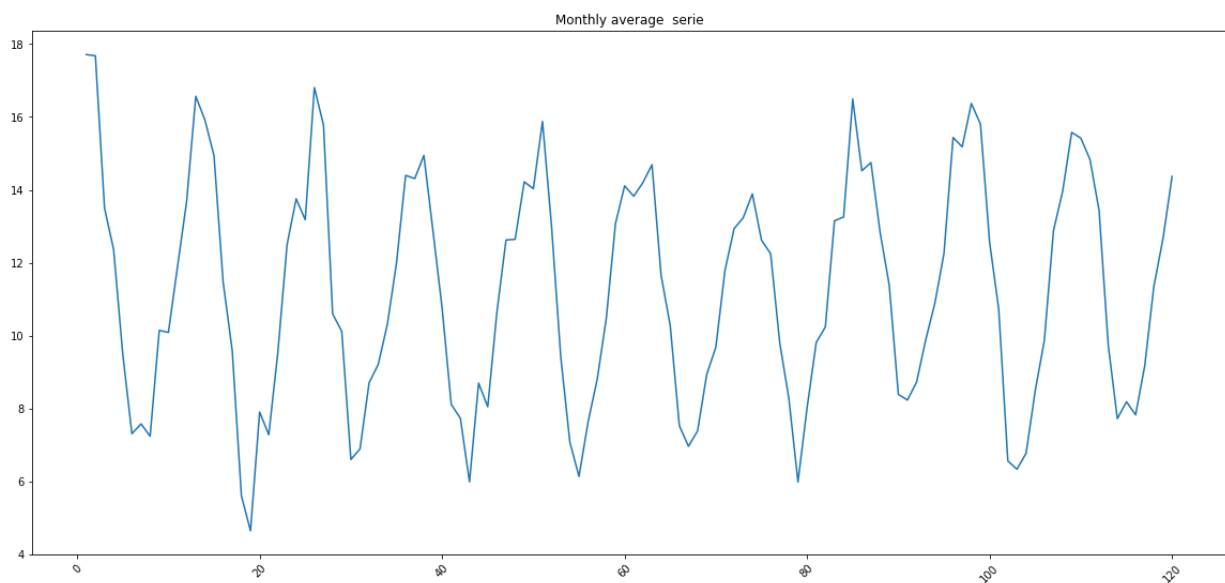
```
Out[12]:
```

	Months	Average_Month_Temp
0	1	17.712903
1	2	17.678571
2	3	13.500000
3	4	12.356667
4	5	9.490323
...	...	...
115	116	7.825806

	Months	Average_Month_Temp
<b>116</b>	117	9.166667
<b>117</b>	118	11.345161
<b>118</b>	119	12.656667
<b>119</b>	120	14.367742

In [10]:

```
# Plot the monthly average temperature for each year
plt.figure(figsize=(20,9))
plt.title('Monthly average serie')
plt.plot(new_data.Months, new_data.Average_Month_Temp)
plt.xticks(rotation=45)
plt.show()
```



In [11]:

```
# Set Date as index
df.set_index('Date', inplace=True)
df
```

Out[11]:

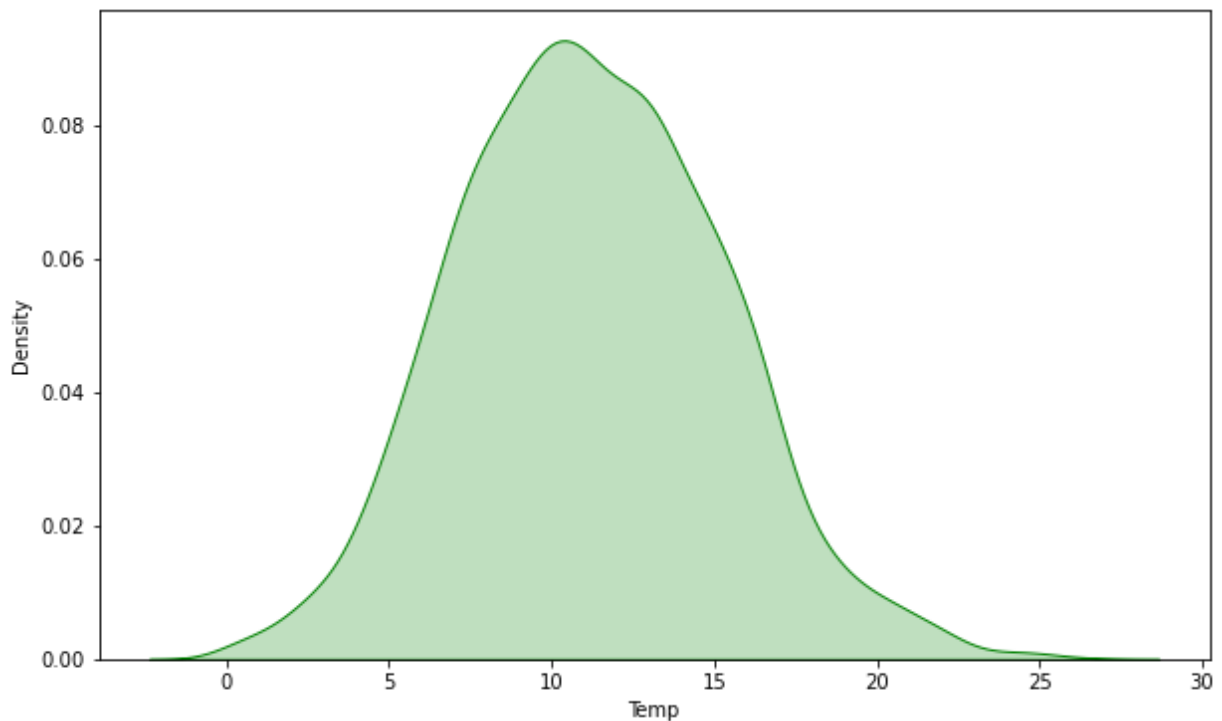
	Temp	year	month
Date			
<b>1981-01-01</b>	20.7	1981	1
<b>1981-01-02</b>	17.9	1981	1
<b>1981-01-03</b>	18.8	1981	1
<b>1981-01-04</b>	14.6	1981	1
<b>1981-01-05</b>	15.8	1981	1
...	...	...	...
<b>1990-12-27</b>	14.0	1990	12

	Temp	year	month
Date			
1990-12-28	13.6	1990	12
1990-12-29	13.5	1990	12
1990-12-30	15.7	1990	12

In [12]:

```
### distribution plot

plt.figure(figsize=(10, 6))
sns.kdeplot(df.Temp, shade=True, color='green')
plt.show()
```



- Average

In [14]:

```
def average(series):
    return np.mean(series)

series = df['Temp']
average(series)
```

Out[14]: 11.177753424657539

- Moving Average

```
In [15]: def moving_average(series, n):
          return np.mean(series[-n:])

          moving_average(series,1)
```

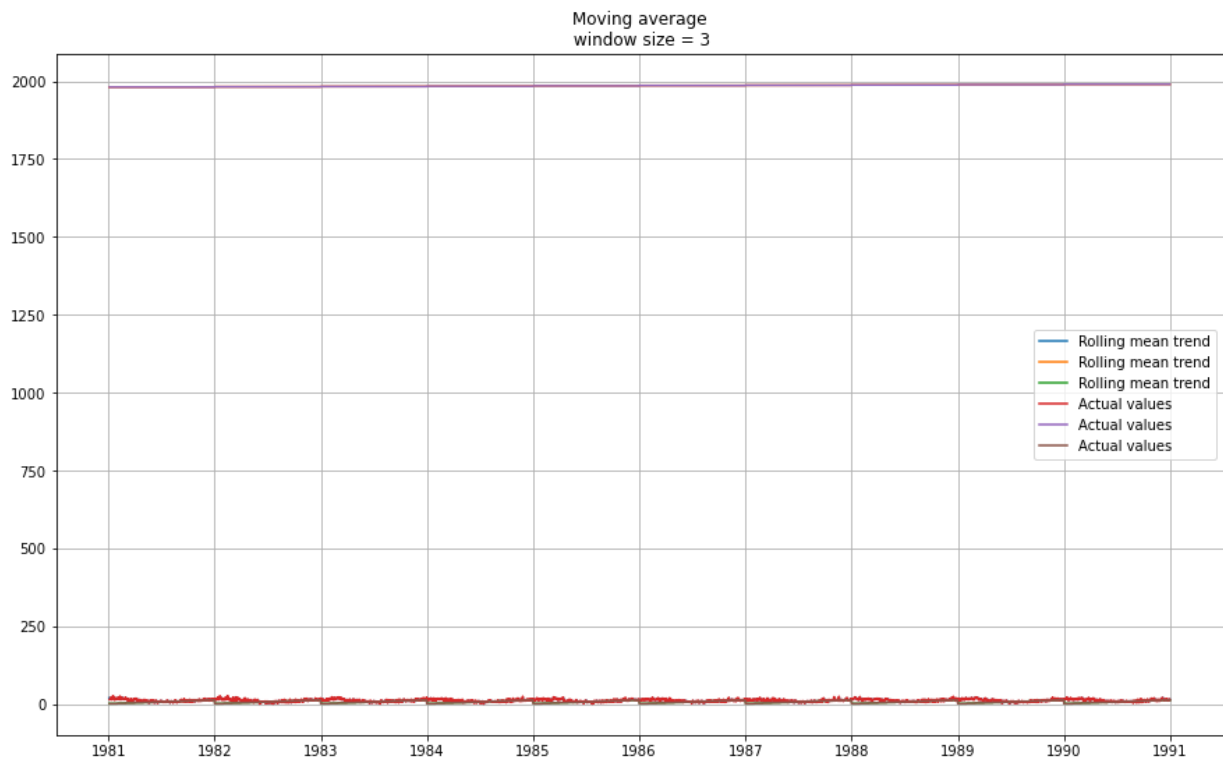
Out[15]: 13.0

```
In [16]: def plot_moving_avg(series, window):
          rolling_mean = series.rolling(window=window).mean()

          plt.figure(figsize=(15,9))
          plt.title('Moving average\n window size = {}'.format(window))
          plt.plot(rolling_mean, label='Rolling mean trend')

          plt.plot(series[window:], label='Actual values')
          plt.legend(loc='best')
          plt.grid(True)

          plot_moving_avg(df, window=3)
```



## Exponential Smoothing

### Simple Exponential Smoothing

- A weighted moving average is a moving average where within the sliding window values are given different weights, typically so that more recent points matter more. Instead of only weighting the time series' last  $k$  values, however, we could instead consider all of the data points, while assigning exponentially smaller weights as we go back in time. This method is

so called EXPONENTIAL SMOOTHING.

- $y^x = \alpha \cdot y_x + (1 - \alpha) \cdot y^{x-1}$
- We can think of  $\alpha$  as the smoothing factor or memory decay rate, it defines how quickly we will "forget" the last available true observation. The smaller  $\alpha$  is, the more influence the previous observations have and the smoother the series is. In other words, the higher the  $\alpha$ ,

In [46]:

```
# Here we have yearly seasonality
alpha= .2
single_data = new_data.copy()
forecast_values = [0]
for i in single_data.index:
    if i==0:
        forecast_values.append(single_data.Average_Month_Temp[0])
    else:
        temp = forecast_values[-1]
        forecast_values.append(alpha * single_data.Average_Month_Temp[i] + (1 - alp

# Assume k = 4, let find the forecast values for the next year.
k = 4
values = [forecast_values[-1]]
for i in range(k):
    temp = values[-1]
    values.append(alpha * single_data.Average_Month_Temp.values[-1] + (1 - alpha) *

forecast_values.extend(values[1:])

df = {'Months':np.arange(121, 126),
      'Average_Month_Temp':np.zeros(5)}
frame = pd.DataFrame(df, index=range(1,6))
single_data = pd.concat([single_data, frame])
single_data['Forecast'] = forecast_values

# Compute the error
single_data['Error'] = single_data.Forecast - single_data.Average_Month_Temp
```

In [47]:

```
# Compute the root mean square from scratch
var = sum(list(map(lambda x : x**2, single_data['Error'].values[1:121])))
single_RSME = np.sqrt(var/120)
print(f'The Root Mean Square error is : {np.round(single_RSME, 3)}')

# Using library to compute the root mean square
from sklearn.metrics import mean_squared_error
from math import sqrt
X = single_data['Forecast'].values[1:121]
Y = single_data['Average_Month_Temp'].values[1:121]
rms = sqrt(mean_squared_error(X, Y))
print('The Root mean square Using Library :', np.round(rms, 3))
```

The Root Mean Square error is : 3.424  
The Root mean square Using Library : 3.424

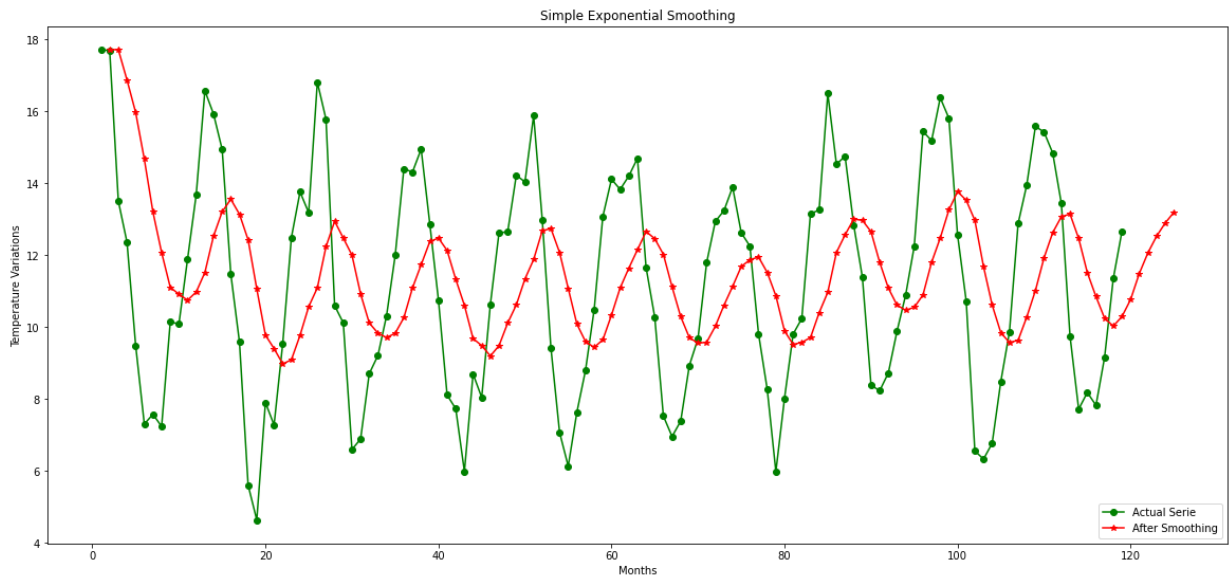
Graphical representation of Simple Exponential Smoothing

In [53]:

```

# Simple Exponential Smoothing
plt.figure(figsize=(20,9))
plt.title('Simple Exponential Smoothing')
plt.plot(single_data.Months.values[:119], single_data.Average_Month_Temp.values[:119])
plt.plot(single_data.Months.values[1:], single_data.Forecast.values[1:], 'r*-')
plt.legend(["Actual Serie", "After Smoothing"], loc ="lower right")
plt.xlabel('Months')
plt.ylabel('Temperature Variations')
plt.show()

```



Using predefined Model from sklearn to compare the result

In [55]:

```

from statsmodels.tsa.api import SimpleExpSmoothing
# Single Exponential Smoothing using alpha = 0.2
ins1 = SimpleExpSmoothing(new_data.Average_Month_Temp).fit(smoothing_level=0.2,optimize=True)
ins_cast1 = ins1.forecast(5).rename('alpha=0.2')

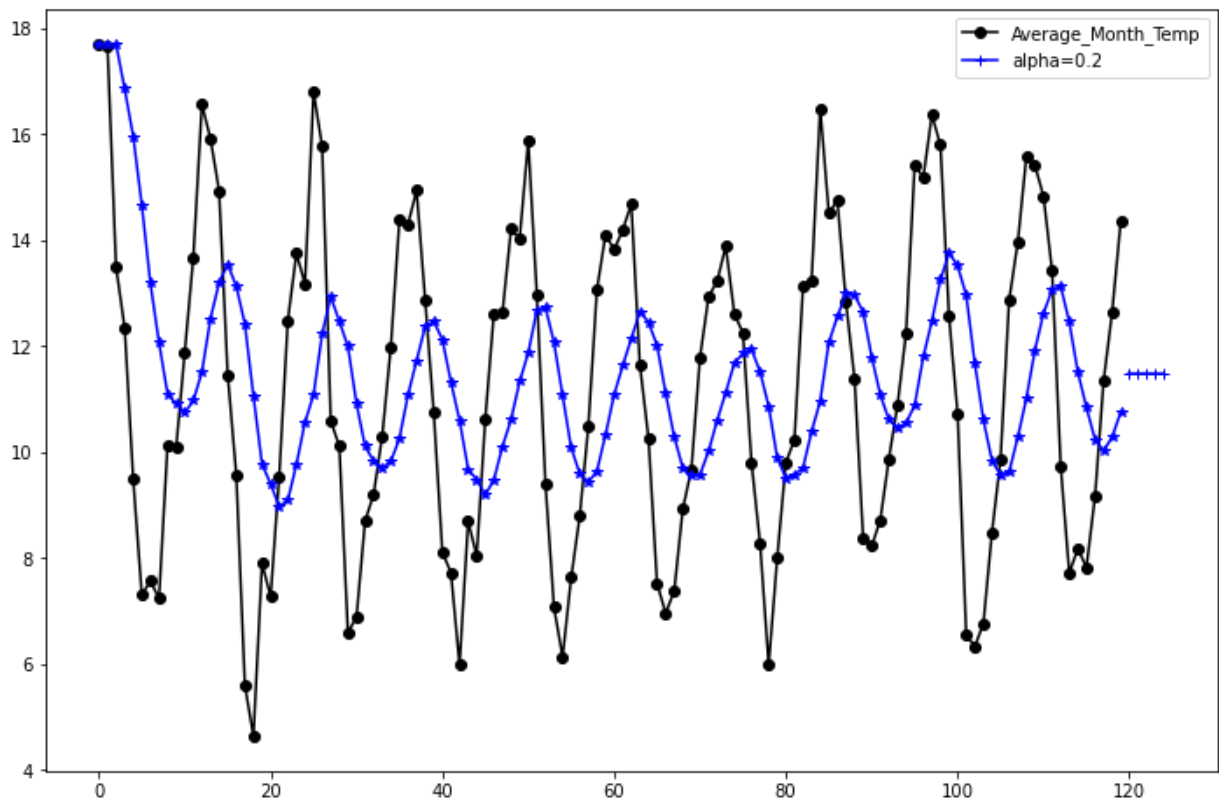
#After creating model we will visualize the plot
ax = new_data.Average_Month_Temp.plot(marker='o', color='black', figsize=(12,8), legend=True)

#Plot for alpha =0.2

ins_cast1.plot(marker='+', ax=ax, color='blue', legend=True)
ins1.fittedvalues.plot(marker='*', ax=ax, color='blue')
plt.show()

```





<Figure size 1440x648 with 0 Axes>

## Double Exponential Smoothing - Holt Method

- The idea behind Double Exponential Smoothing (a.k.a the Holt Method) is exponential smoothing applied to both level and trend. The basic idea is saying if our time series has a trend, we can incorporate that information to do better than just estimating the current level and using that to forecast the future observations. To achieve this, we will introduce two new notations: the current "trend", denoted by  $T$  (we can think of it as the slope of the time series), as well as the current "level", denoted by  $\ell$ .
- $\ell_x = \alpha y_x + (1 - \alpha)(\ell_{x-1} + T_{x-1})$  ----> Level
- $\ell$ , level is simply predicted point. But because now it's going to be only part of calculation of the forecast (our forecast is a combination of predicted point and trend), we can no longer refer to it as  $y^\wedge$
- $T_x = \beta(\ell_x - \ell_{x-1}) + (1 - \beta)T_{x-1}$  ----> Trend
- The second equation introduces  $0 < \beta < 1$ , the trend coefficient. As with  $\alpha$ , some values of  $\beta$  work better than others depending on the series. When  $\beta$  is big, we won't give too much weight to the past trends when estimating current trend
- $y^\wedge_{x+1} = \ell_x + T_x$  ----> 1 step forecast
- Similar to exponential smoothing, where we used the first observed value as the first expected value, we can use the first observed trend as the first expected trend, i.e. we'll use the first two points to compute the initial trend, i.e.  $(y_2 - y_1)/1$

```
In [32]: # Double Exponential Smoothing parameters
alpha=0.2
beta=0.1
```

```
In [33]: # Copy the original data into new dataset for double Exponential smoothing
double_data = new_data.copy()

level = []
trend = []
# Additive model
des_additive_forecast = []
# Multiplicative model
des_multiple_forecast = []
# Error when using additive model
des_additive_square_error = []
# Error when using multiplicative model
des_multiple_square_error = []

for i in double_data.index:
    if i == 0:
        level.append(0)
        trend.append(0)
        des_additive_forecast.append(0)
        des_multiple_forecast.append(0)
        des_additive_square_error.append(0)
        des_multiple_square_error.append(0)
    elif i == 1:
        level.append(double_data.Average_Month_Temp.values[0])
        trend.append(double_data.Average_Month_Temp.values[1]-double_data.Average_Month_Temp.values[0])
        des_additive_forecast.append(0)
        des_multiple_forecast.append(0)
        des_additive_square_error.append(0)
        des_multiple_square_error.append(0)
    else:
        temp = level[-1]
        level.append(alpha * double_data.Average_Month_Temp.values[i] + (1-alpha) * temp)
        temp1 = trend[-1]
        trend.append(beta * (level[i] - level[i-1]) + (1 - beta) * temp1)
        des_additive_forecast.append(level[i] + trend[i])
        des_multiple_forecast.append(level[i] * trend[i])
        des_additive_square_error.append(np.power((des_additive_forecast[i]-double_data.Average_Month_Temp.values[i]), 2))
        des_multiple_square_error.append(np.power((des_multiple_forecast[i]-double_data.Average_Month_Temp.values[i]), 2))

# Add Level, Trend, AdditiveForecast, AdditiveError, MultiplicativeForecast, MultiplicativeError
double_data['Level'] = level
double_data['Trend'] = trend
double_data['AdditiveForecast'] = des_additive_forecast
double_data['AdditiveError'] = des_additive_square_error
double_data['MultiplicativeForecast'] = des_multiple_forecast
double_data['MultiplicativeError'] = des_multiple_square_error

double_data.head()
```

```
Out[33]:
```

Months	Average_Month_Temp	Level	Trend	AdditiveForecast	AdditiveError	MultiplicativeForecast	MultiplicativeError
0	...	0	0	0	0	0	0
1	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...

	Months	Average_Month_Temp	Level	Trend	AdditiveForecast	AdditiveError	Multiplicative
0	1	17.712903	0.000000	0.000000	0.000000	0.000000	0
1	2	17.678571	17.712903	-0.034332	0.000000	0.000000	0
2	3	13.500000	16.897788	-0.112410	16.785378	10.793708	-1
3	4	13.356667	16.878403	-0.103000	16.806403	13.530371	0

In [34]: `double_data.MultiplicativeForecast`

Out[34]:

```
0    0.000000
1    0.000000
2   -1.899483
3   -2.942527
4   -4.201722
...
115  -1.426031
116  -1.367484
117  -0.901541
118  -0.293642
119   0.536952
Name: MultiplicativeForecast, Length: 120, dtype: float64
```

In [35]:

```
# Additive Root Square Mean Error
print(f'Root Square Mean Error made using Additive model : {np.sqrt(double_data.AdditiveForecast)}')
# Multiplicative Root Square Mean Error
print(f'Root Square Mean Error made using Additive model : {np.sqrt(double_data.MultiplicativeForecast)}')
```

```
Root Square Mean Error made using Additive model : 2.463954245104217
Root Square Mean Error made using Additive model : 2.463954245104217
```

### Forecasting the next year Data

In [36]:

```
# Assume k = 4, let find the forecast values for the next year.
k = 4
des_additive_values = []
des_multiple_values = []
for i in range(1, k+1):
    des_additive_values.append(level[-1] + i * trend[-1])
    des_multiple_values.append(level[-1] + trend[-1]**i)

# Add the forecast values for the additive model
des_additive_forecast.extend(des_additive_values)
# Add the forecast values for the multiplicative model
des_multiple_forecast.extend(des_multiple_values)

df = {'Months': np.arange(121, 125),
      'Average_Month_Temp': np.zeros(k)}
frame = pd.DataFrame(df, index=range(1, 5))
double_data = pd.concat([double_data, frame])
double_data['AdditiveForecast'] = des_additive_forecast
double_data['MultiplicativeForecast'] = des_multiple_forecast
```

In [37]: `double_data`

Out[37]:

	Months	Average_Month_Temp	Level	Trend	AdditiveForecast	AdditiveError	Multiplicat
<b>0</b>	1	17.712903	0.000000	0.000000	0.000000	0.000000	
<b>1</b>	2	17.678571	17.712903	-0.034332	0.000000	0.000000	
<b>2</b>	3	13.500000	16.897788	-0.112410	16.785378	10.793708	
<b>3</b>	4	12.356667	16.079492	-0.182999	15.896493	12.530371	
<b>4</b>	5	9.490323	14.908057	-0.281842	14.626215	26.377387	
...	...	...	...	...	...	...	
<b>119</b>	120	14.367742	11.670844	0.046008	11.716852	7.027217	
<b>1</b>	121	0.000000	NaN	NaN	11.716852	NaN	
<b>2</b>	122	0.000000	NaN	NaN	11.762860	NaN	
<b>3</b>	123	0.000000	NaN	NaN	11.808868	NaN	
<b>4</b>	124	0.000000	NaN	NaN	11.854876	NaN	

124 rows × 8 columns

In [52]:

# Double Exponential Smoothing Ullring Additive model

plt.figure(figsize=(20,9))

plt.title('Double Exponential Smoothing Using Additive model')

plt.plot(double\_data.Months.values[:119], double\_data.Average\_Month\_Temp.values[:119])

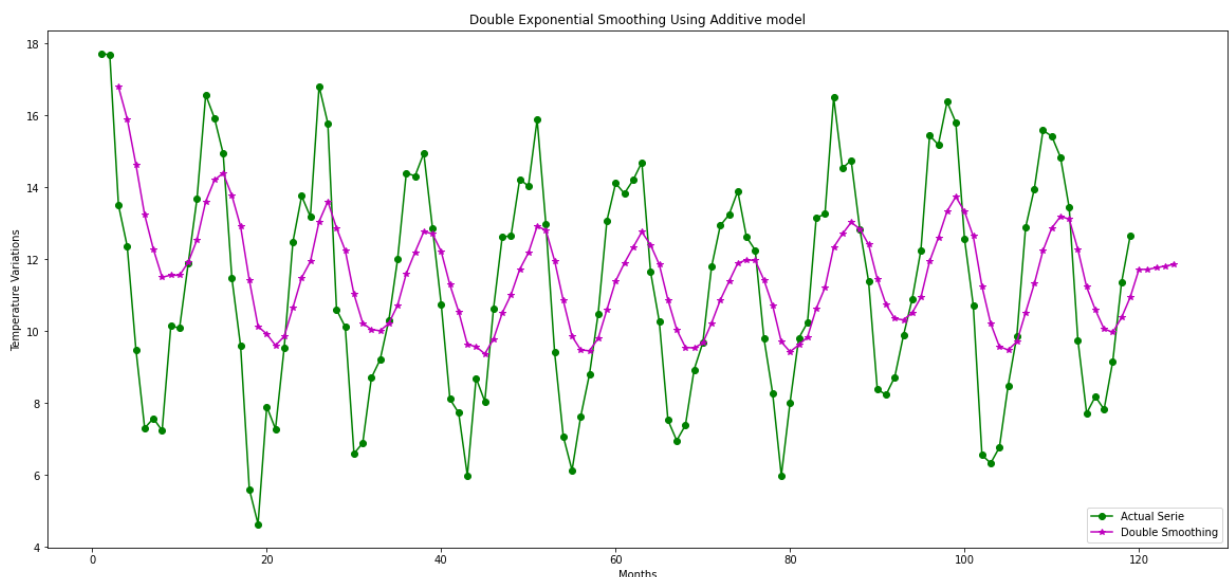
plt.plot(double\_data.Months.values[2:], double\_data.AdditiveForecast.values[2:], 'm\*')

plt.legend(["Actual Serie", "Double Smoothing"], loc="lower right")

plt.xlabel('Months')

plt.ylabel('Temperature Variations')

plt.show()



Triple Exponential Smoothing - Holt-Winters Method

- The idea behind triple exponential smoothing (a.k.a Holt-Winters Method) is to apply exponential smoothing to a third component - seasonality,  $S$ . This means we should not be using this method if our time series is not expected to have seasonality.
- $\ell_x = \alpha(y_x - S_{x-L}) + (1-\alpha)(\ell_{x-1} + T_{x-1})$  ---> level
- $T_x = \beta(\ell_x - \ell_{x-1}) + (1-\beta)T_{x-1}$  ---> Trend
- $S_x = \gamma(y_x - \ell_x) + (1-\gamma)s_{x-L}$  ---> seasonal
- $y^{x+m} = \ell_x + mT_x + S_{x-L+1+(m-1) \bmod L}$  ---> forecast
- Season length is the number of data points after which a new season begins. We will use  $L$  to denote season length.
- We now have a third coefficient,  $0 < \gamma < 1$ , which is the smoothing factor for the seasonal component.
- The index for the forecast,  $y^{x+m}$ , is  $x+m$  where  $m$  can be any integer. Meaning we can forecast any number of points into the future while accounting for previous value, trend and seasonality.
- The index of the seasonal component of the forecast  $S_{x-L+1+(m-1) \bmod L}$  may appear a little mind boggling, but as we'll soon see in the implementation, this is essentially offsetting into our observed data's list of seasonal components. e.g. if we are forecasting the 3rd point into the season, and we are 45 seasons into the future, we cannot use seasonal components from the 44th season in the future since that season is also generated by our forecasting procedure, we must use the last set of seasonal components from observed points, or from "the past" if you will.

In [42]:

```

# Triple Exponential Smoothing parametes
alpha = 0.2
beta = 0.1
gamma = 0.4

# Asume we have yearly seasonality
M = 4
triple_data = new_data.copy()

level = []
trend = []
seasonal = []
forecast = []

for i in range(M):
    seasonal.append(triple_data.Average_Month_Temp[i]/triple_data.Average_Month_Temp)
for j in triple_data.index.values[M:]:
    if j == 4:
        temp = triple_data.Average_Month_Temp.values[j-1]
        level.append(alpha * (triple_data.Average_Month_Temp[j] - seasonal[j-M]))
        trend.append(triple_data.Average_Month_Temp.values[j]-(temp))
        seasonal.append(gamma * (triple_data.Average_Month_Temp.values[j]-level[-1]))
    else:
        level.append(alpha * (triple_data.Average_Month_Temp[j] - seasonal[j-M]) + (
        trend.append(beta * (level[-1] - level[-2]) + (1-beta)*trend[-1]))
        seasonal.append(gamma * (triple_data.Average_Month_Temp.values[j]-level[-1]))
        forecast.append((level[j-M-1] + trend[j-M-1])*seasonal[j-M])

#Standarize the values obtain for the forecasting

from sklearn import preprocessing
scaler = preprocessing.MinMaxScaler(feature_range=(4.641935483870967, 18))
names = pd.DataFrame(forecast)
forecast_value = scaler.fit_transform(names)

```

In [51]:

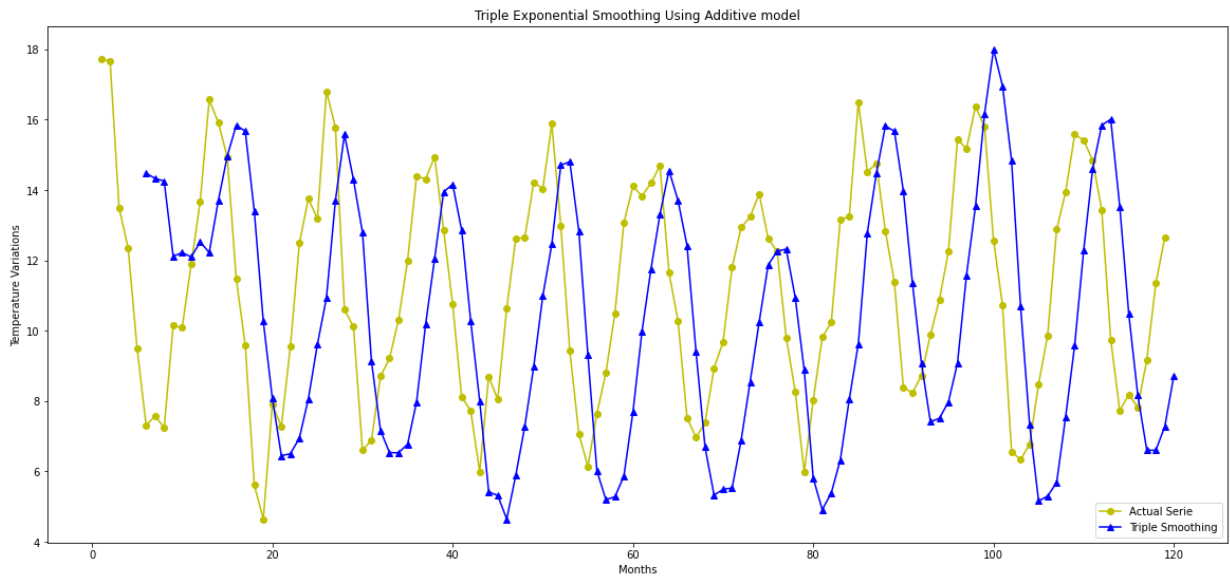
```

plt.figure(figsize=(20,9))
plt.title('Triple Exponential Smoothing Using Additive model')

plt.plot(triple_data.Months.values[:119], triple_data.Average_Month_Temp.values[:119])
plt.plot(triple_data.Months.values[5:], forecast_value, 'b^-')

plt.legend(["Actual Serie", "Triple Smoothing"], loc = "lower right")
plt.xlabel('Months')
plt.ylabel('Temperature Variations')
plt.show()

```



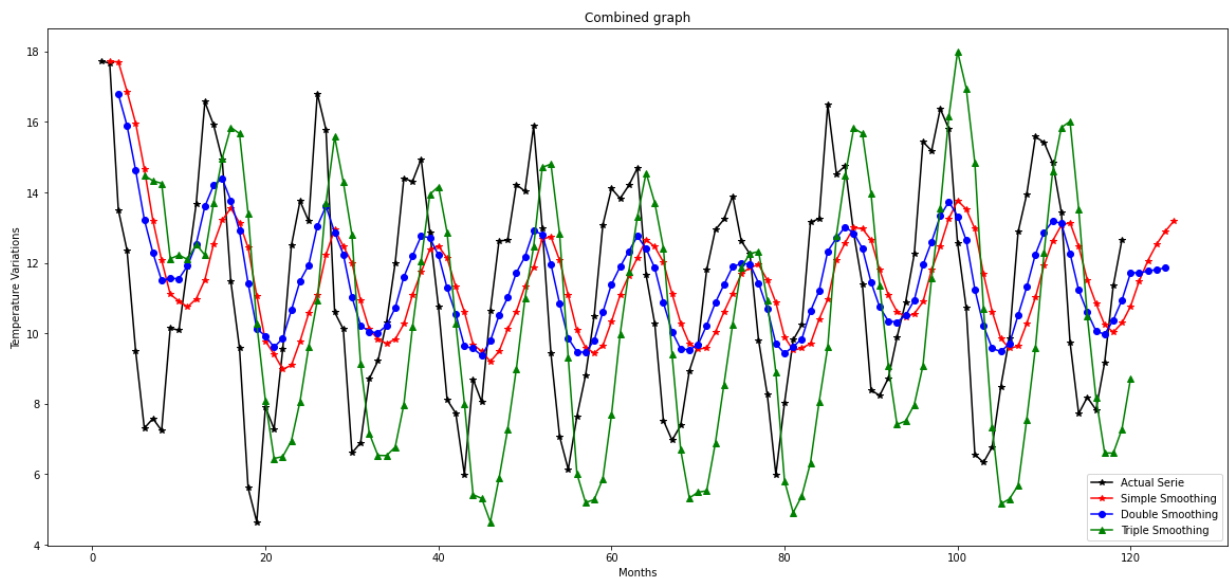
All The three graphs combined

In [50]:

```
plt.figure(figsize=(20,9))
plt.title('Combined graph')

plt.plot(single_data.Months.values[:119], single_data.Average_Month_Temp.values[:119])
plt.plot(single_data.Months.values[1:], single_data.Forecast.values[1:], 'r*-')
plt.plot(double_data.Months.values[2:], double_data.AdditiveForecast.values[2:], 'bo-')
plt.plot(triple_data.Months.values[5:], forecast_value, 'g^--')

plt.legend(["Actual Serie", "Simple Smoothing", "Double Smoothing", "Triple Smoothing"])
plt.xlabel('Months')
plt.ylabel('Temperature Variations')
plt.show()
```



In [ ]: