Assignment-01-19MAT101
SINGLE VARIABLE CALCULUS

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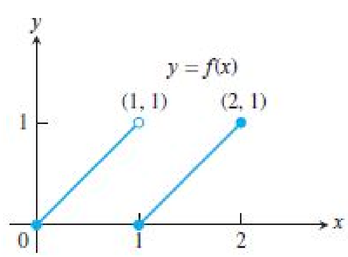
1 PROBLEMS ON FUNCTIONS AND GRAPHS

1. Draw the graph of the functions

   (a)
   $$f(x) = \begin{cases} -x & if\ x < 0 \\ x^2 & if\ 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

   (b)
   $$f(x) = \begin{cases} 4 - x^2 & if\ x \leq 1 \\ x^2 + 2x & if\ x > 1 \end{cases}$$

   (c) Write the piecewise defined function for the graph of a function given below



2 PROBLEMS ON LIMIT AND CONTINUITY OF FUNCTION

1. Find the right hand and the left hand limits of a function defined as follows:

$$f(x) = \begin{cases} \frac{|x-4|}{x-4}, & \text{if } x \neq 4 \\ 0, & \text{otherwise} \end{cases}$$

2. Prove that $Lim_{x\to 0} x\ sin(\frac{1}{x}) = 0$.

3. Show that $Lim_{x\to 3} \frac{1}{(x-3)^2} = \infty$, whereas $Lim_{x\to 3} \frac{1}{(x-3)}$ does not exist.

4. Find $Lim_{x\to 0}\ e^x\ sgn(x + [x])$, where the signum function is defined as:

$$sgn(x) = \begin{cases} 1 & if\ x > 0 \\ 1 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$$

5. We can observe that the inequality

$$1 - \frac{x^2}{6} < \frac{x\ sin(x)}{2 - 2\ cos(x)} < 1$$

holds for all the values of "x" close to zero, then what can you say about $Lim_{x\to 0} \frac{x\ sin(x)}{2-2\ cos(x)}$. Give reasons for your answer.

6. Let $Lim_{x\to 5} \sqrt{x-1} = 2$ be given, then find a value $\delta > 0$ such that it works for $\epsilon = 1$, that means , find a value $\delta > 0$ such that $|\sqrt{x-1} - 2| < 1$ whenever $0 < |x - 5| < \delta$.

7. Show that $Lim_{x\to 2}\ f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & \text{if } x \neq 2 \\ 1, & \text{otherwise} \end{cases}$$

8. Find the domain of the function $f(x) = |\frac{x\ sin\ (x)}{x^2+1}|$ and show that $f(x) = |\frac{x\ sin\ (x)}{x^2+1}|$ is a continuous function on the domain.

9. Find the values a, and b so that the following function

$$f(x) = \begin{cases} a\ x + 2\ b & x \leq 0 \\ x^2 + 3\ a - b & 0 < x \leq 2 \\ 6 - b\ x & -1 < x \leq 1 \\ 3\ x - 5 & x > 2 \end{cases}$$

is a continuous function.

10. Find the values a, b, and c so that the following function

$$f(x) = \begin{cases} 6 - 3\ b\ x & x \leq -2 \\ c\ x^2 - a\ x + 4 & -2 < x \leq -1 \\ 6 - b\ x & -1 < x \leq 1 \\ a\ x^2 + c & x > 1 \end{cases}$$

is a continuous function.

11. Check the continuity and differentiability of the function

$$f(x) = \begin{cases} x^n\ cos\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

where "n" is a natural number.

3 DERIVATIVES APPLICATIONS

1. Find the area of a triangle enclosed by an arbitrary tangent line to the curve $f(x) = \frac{1}{x}$, the x-axis and the y-axis, and what is your conclusion when you find the required area.

2. Find the absolute maximum and minimum values of $f(x) = x^2$ on the domains $D_0 = (-\infty,\ +\infty)$, $D_1 = [-2,\ 1]$, and $D_2 = (-2,\ 1]$ respectively.

3. Find the critical points of $f(x) = sin^2(x) - sin(x) - 1$ on the interval $[0,\ 2\pi]$, and identify the open intervals on which "f" is increasing and decreasing. Also, find the local maximum and local minimum of the function in the corresponding domain.

4. Let "c" be a point in the domain of the function "f", where $f'(c)$ exists and $f'(c) > 0$ that is "f" is positive, then show that function is increasing in the neighborhood of the corresponding point.

5. Examine the local maximum and the local minimum for the functions

   (a) $f(x) = (x - 3)^5 (x + 1)^4$
   (b) $f(x) = sin(x) + cos(x)$
   (c) $f(x) = x^5 - 5\ x^4 + 5\ x^3 - 1$

(d) $f(x) = x^3 - 6\ x^2 + 9\ x + 1$

6. Sketch a graph of the function $f(x) = x^4 - 4\ x^3 + 10$ using the following steps:

   (a) Observe where the the function "f" achieves the extremum.

   (b) Find the subintervals where the function is increasing or decreasing respectively.

   (c) Observe where the function "f" is concave up and where it is concave down.

   (d) Sketch the general shape of the curve for the function.

   (e) Plot some points such as local maximum, local minimum, points of inflection and intercepts respectively.