CHAPTER 3: Unified I-V Model

The development of separate model expressions for such device operation regimes as subthreshold and strong inversion were discussed in Chapter 2. Although these expressions can accurately describe device behavior within their own respective region of operation, problems are likely to occur between two well-described regions or within transition regions. In order to circumvent this issue, a unified model should be synthesized to not only preserve region-specific expressions but also to ensure the continuities of current and conductance *and* their derivatives in *all* transition regions as well. Such high standards are kept in BSIM3v3.2.1 . As a result, convergence and simulation efficiency are much improved.

This chapter will describe the unified I-V model equations. While most of the parameter symbols in this chapter are explained in the following text, a complete description of all I-V model parameters can be found in Appendix A.

3.1 Unified Channel Charge Density Expression

Separate expressions for channel charge density are shown below for subthreshold (Eq. (3.1.1a) and (3.1.1b)) and strong inversion (Eq. (3.1.2)). Both expressions are valid for small V_{ds}.

$$Q_{chsubs0} = Q_0 \exp\left(\frac{V_{gs} - V_{th}}{n v_t}\right) \tag{3.1.1a}$$

where Q_0 is

$$Q_0 = \sqrt{\frac{q \varepsilon_{si} N_{ch}}{2\phi_s}} v_t \exp\left(-\frac{V_{off}}{n v_t}\right) \tag{3.1.1b}$$

$$Q_{chs0} = C_{ox}(V_{gs} - V_{th}) \tag{3.1.2}$$

In both Eqs. (3.1.1a) and (3.1.2), the parameters $Q_{chsubs0}$ and Q_{chs0} are the channel charge densities at the source for very small Vds. To form a unified expression, an effective (V_{gs}-V_{th}) function named V_{gsteff} is introduced to describe the channel charge characteristics from subthreshold to strong inversion

$$V_{gsteff} = \frac{2 n v_t \ln\left[1 + \exp\left(\frac{V_{gs} - V_{th}}{2 n v_t}\right)\right]}{1 + 2 n C_{OX} \sqrt{\frac{2\Phi_s}{q \varepsilon_{si} N_{ch}}} \exp\left(-\frac{V_{gs} - V_{th} - 2V_{off}}{2 n v_t}\right)} \tag{3.1.3}$$

The unified channel charge density at the source end for both subthreshold and inversion region can therefore be written as

$$Q_{chs0} = C_{ox} V_{gsteff} \tag{3.1.4}$$

Figures 3-1 and 3-2 show the smoothness of Eq. (3.1.4) from subthreshold to strong inversion regions. The V_{gsteff} expression will be used again in subsequent sections of this chapter to model the drain current.

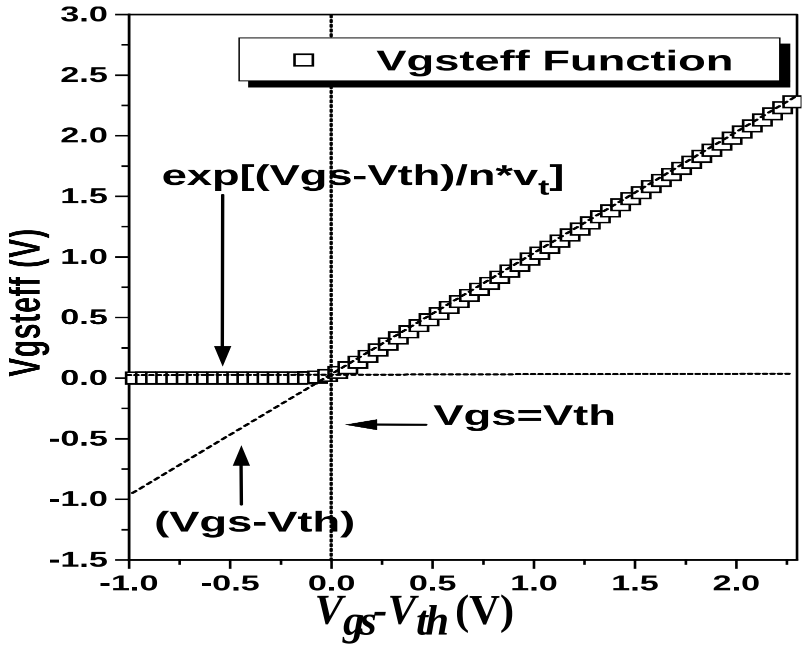

Figure 3-1. The V_{gsteff} function vs. (V_{gs}-V_{th}) in linear scale.

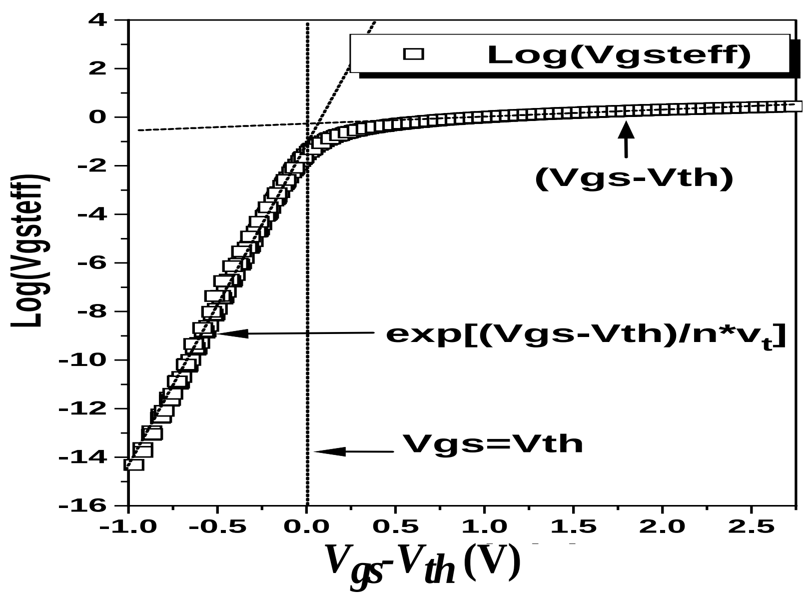

Figure 3-2. V_{gsteff} function vs. (V_{gs}-V_{th}) in log scale.

Eq. (3.1.4) serves as the cornerstone of the unified channel charge expression at the source for small V_{ds}. To account for the influence of V_{ds}, the V_{gsteff} function must keep track of the change in channel potential from the source to the drain. In other words, Eq. (3.1.4) will have to include a y dependence. To initiate this formulation, consider first the re-formulation of channel charge density for the case of **strong inversion**

$$Q_{chs(y)} = C_{ox}(V_{gs} - V_{th} - A_{bulk}V_{F(y)}) \tag{3.1.5}$$

The parameter $V_F(y)$ stands for the quasi-Fermi potential at any given point y, along the channel with respect to the source. This equation can also be written as

$$Q_{chs(y)} = Q_{chs0} + \Delta Q_{chs(y)} \tag{3.1.6}$$

The term $\Delta Q_{chs}(y)$ is the incremental channel charge density induced by the drain voltage at point y. It can be expressed as

$$\Delta Q_{chs(y)} = -CoxA_{bulk}V_{F(y)} \tag{3.1.7}$$

For the **subthreshold region** (V_{gs}<<V_{th}), the channel charge density along the channel from source to drain can be written as

$$\begin{aligned} Q_{chsubs(y)} &= Q_0 \exp\left(\frac{V_{gs} - V_{th} - A_{bulk}V_{F(y)}}{nv_t}\right) \\ &= Q_{chsubs0} \exp\left(-\frac{A_{bulk}V_{F(y)}}{nv_t}\right) \end{aligned} \tag{3.1.8}$$

A Taylor series expansion of the right-hand side of Eq. (3.1.8) yields the following (keeping only the first two terms)

$$Q_{chsubs(y)} = Q_{chsubs0}\left(1 - \frac{A_{bulk} V_{F(y)}}{n v_t}\right) \tag{3.1.9}$$

Analogous to Eq. (3.1.6), Eq. (3.1.9) can also be written as

$$Q_{chsubs(y)} = Q_{chsubs0} + \Delta Q_{chsubs(y)} \tag{3.1.10}$$

The parameter $\Delta Q_{chsubs}(y)$ is the incremental channel charge density induced by the drain voltage in the subthreshold region. It can be written as

$$\Delta Q_{chsubs(y)} = -\frac{A_{bulk} V_{F(y)}}{n v_t} Q_{chsubs0} \tag{3.1.11}$$

Note that Eq. (3.1.9) is valid only when $V_F(y)$ is very small, which is maintained fortunately, due to the fact that Eq. (3.1.9) is only used in the linear regime (i.e. $V_{ds} \le 2v_t$).

Eqs. (3.1.6) and (3.1.10) both have drain voltage dependencies. However, they are decupled and a unified expression for $Q_{ch}(y)$ is needed. To obtain a unified expression along the channel, we first assume

$$\Delta Q_{ch(y)} = \frac{\Delta Q_{chs(y)} \Delta Q_{chsubs(y)}}{\Delta Q_{chs(y)} + \Delta Q_{chsubs(y)}} \tag{3.1.12}$$

Here, $\Delta Q_{ch}(y)$ is the incremental channel charge density induced by the drain voltage. Substituting Eq. (3.1.7) and (3.1.11) into Eq. (3.1.12), we obtain

$$\Delta Q_{ch(y)} = \frac{V_{F(y)}}{Vb} Q_{chs0} \tag{3.1.13}$$

where $V_b = (V_{gsteff} + n^*v_t)/A_{bulk}$. In order to remove any association between the variable n and bias dependencies (V_{gsteff}) as well as to ensure more precise modeling of Eq. (3.1.8) for linear regimes (under subthreshold conditions), n is replaced by 2. The expression for V_b now becomes

$$V_b = \frac{V_{gsteff} + 2v_t}{A_{bulk}} \tag{3.1.14}$$

A unified expression for $Q_{ch}(y)$ from subthreshold to strong inversion regimes is now at hand

$$Q_{ch(y)} = Q_{chs0}(1 - \frac{V_{F(y)}}{V_b}) \tag{3.1.15}$$

The variable Q_{chs0} is given by Eq. (3.1.4).

3.2 Unified Mobility Expression

Unified mobility model based on the V_{gsteff} expression of Eq. 3.1.3 is described in the following.

(mobMod = 1) (3.2.1)

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff})(\frac{V_{gsteff} + 2V_{th}}{T_{OX}}) + U_b(\frac{V_{gsteff} + 2V_{th}}{T_{OX}})^2}$$

To account for depletion mode devices, another mobility model option is given by the following

(mobMod = 2) (3.2.2)

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff})(\frac{V_{gsteff}}{T_{OX}}) + U_b(\frac{V_{gsteff}}{T_{OX}})^2}$$

To consider the body bias dependence of Eq. 3.2.1 further, we have introduced the following expression

(For mobMod = 3) (3.2.3)

$$\mu_{eff} = \frac{\mu_o}{1 + [U_a(\frac{V_{gsteff} + 2V_{th}}{T_{OX}}) + U_b(\frac{V_{gsteff} + 2V_{th}}{T_{OX}})^2](1 + U_c V_{bseff})}$$

3.3 Unified Linear Current Expression

3.3.1 Intrinsic case (R_{ds}=0)

Generally, the following expression [2] is used to account for both drift and diffusion current

$$I_{d(y)} = WQ_{ch(y)}\mu_{ne(y)}\frac{dV_{F(y)}}{dy} \tag{3.3.1}$$

where the parameter $u_{ne}(y)$ can be written as

$$\mu_{ne(y)} = \frac{\mu_{eff}}{1+\frac{E_y}{E_{sat}}} \tag{3.3.2}$$

Substituting Eq. (3.3.2) in Eq. (3.3.1) we get

$$I_{d(y)} = WQ_{chso}(1-\frac{V_{F(y)}}{V_b})\frac{\mu_{eff}}{1+\frac{E_y}{E_{sat}}}\frac{dV_{F(y)}}{dy} \tag{3.3.3}$$

Eq. (3.3.3) resembles the equation used to model drain current in the strong inversion regime. However, it can now be used to describe the current characteristics in the subthreshold regime when V_{ds} is very small ($V_{ds}<2v_t$). Eq. (3.3.3) can now be integrated from the source to drain to get the expression for linear drain current in the channel. This expression is valid from the subthreshold regime to the strong inversion regime

$$I_{ds0} = \frac{W\mu_{eff}Q_{chs0}V_{ds}\left(1-\frac{V_{ds}}{2V_b}\right)}{L\left(1+\frac{V_{ds}}{E_{sat}L}\right)} \tag{3.3.4}$$

3.3.2 Extrinsic Case (R_{ds} > 0)

The current expression when R_{ds} > 0 can be obtained based on Eq. (2.5.9) and Eq. (3.3.4). The expression for linear drain current from subthreshold to strong inversion is:

$$I_{ds} = \frac{I_{dso}}{1 + \frac{R_{ds} I_{dso}}{V_{ds}}} \tag{3.3.5}$$

3.4 Unified V_{dsat} Expression

3.4.1 Intrinsic case (R_{ds}=0)

To get an expression for the electric field as a function of y along the channel, we integrate Eq. (3.3.1) from 0 to an arbitrary point y. The result is as follows

$$E_y = \frac{I_{dso}}{\sqrt{(W Q_{chs0} \mu_{eff} - \frac{I_{dso}}{E_{sat}})^2 - \frac{2 I_{ds0} W Q_{chs0} \mu_{eff}\, y}{V_b}}} \tag{3.4.1}$$

If we assume that drift velocity saturates when $Ey=Esat$, we get the following expression for I_{dsat}

$$I_{dsat} = \frac{W \mu_{eff} Q_{chs0} E_{sat} L V_b}{2L(E_{sat} L + V_b)} \tag{3.4.2}$$

Let V_{ds}=V_{dsat} in Eq. (3.3.4) and set this equal to Eq. (3.4.2), we get the following expression for V_{dsat}

$$V_{dsat} = \frac{E_{sat}L(V_{gsteff} + 2v_t)}{A_{bulk}E_{sat}L + V_{gsteff} + 2v_t} \tag{3.4.3}$$

3.4.2 Extrinsic Case (R_{ds}>0)

The V_{dsat} expression for the extrinsic case is formulated from Eq. (3.4.3) and Eq. (2.5.10) to be the following

$$V_{dsat} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{3.4.4a}$$

where

$$a = A_{bulk}{}^2 W_{eff} v_{sat} C_{ox} R_{DS} + (\frac{1}{\lambda} - 1) A_{bulk} \tag{3.4.4b}$$

$$b = -\left((V_{gsteff} + 2v_t)(\frac{2}{\lambda} - 1) + A_{bulk}E_{sat}L_{eff} + 3A_{bulk}(V_{gsteff} + 2v_t)W_{eff}v_{sat}C_{ox}R_{DS} \right) \tag{3.4.4c}$$

$$c = (V_{gsteff} + 2v_t)E_{sat}L_{eff} + 2(V_{gsteff} + 2v_t)^2 W_{eff}v_{sat}C_{ox}R_{DS} \tag{3.4.4d}$$

$$\lambda = A_1 V_{gsteff} + A_2 \tag{3.4.4e}$$

The parameter λ is introduced to account for non-saturation effects. Parameters A_1 and A_2 can be extracted.

3.5 Unified Saturation Current Expression

A unified expression for the saturation current from the subthreshold to the strong inversion regime can be formulated by introducing the V_{gsteff} function into Eq. (2.6.15). The resulting equations are the following

$$I_{ds} = \frac{I_{dso(Vdsat)}}{1 + \frac{R_{ds} I_{dso(Vdsat)}}{V_{dsat}}} \left(1 + \frac{V_{ds} - V_{dsat}}{V_A}\right)\left(1 + \frac{V_{ds} - V_{dsat}}{V_{ASCBE}}\right) \tag{3.5.1}$$

where

$$V_A = V_{Asat} + \left(1 + \frac{P_{vag} V_{gsteff}}{E_{sat} L_{eff}}\right)\left(\frac{1}{V_{ACLM}} + \frac{1}{V_{ADIBLC}}\right)^{-1} \tag{3.5.2}$$

$$V_{Asat} = \frac{E_{sat} L_{eff} + V_{dsat} + 2 R_{DS} v_{sat} C_{ox} W_{eff} V_{gsteff} \left[1 - \frac{A_{bulk} V_{dsat}}{2(V_{gsteff} + 2 v_t)}\right]}{2/\lambda - 1 + R_{DS} v_{sat} C_{ox} W_{eff} A_{bulk}} \tag{3.5.3}$$

$$V_{ACLM} = \frac{A_{bulk} E_{sat} L_{eff} + V_{gsteff}}{P_{CLM} A_{bulk} E_{sat}\, litl} (V_{ds} - V_{dsat}) \tag{3.5.4}$$

$$V_{ADIBLC} = \frac{(V_{gsteff} + 2v_t)}{\theta_{rout}(1 + P_{DIBLCB}V_{bseff})}\left(1 - \frac{A_{bulk}V_{dsat}}{A_{bulk}V_{dsat} + V_{gsteff} + 2v_t}\right) \tag{3.5.5}$$

$$\theta_{rout} = P_{DIBLC1}\left[\exp(-D_{ROUT}\frac{L_{eff}}{2l_{t0}}) + 2\exp(-D_{ROUT}\frac{L_{eff}}{l_{t0}})\right] + P_{DIBLC2} \tag{3.5.6}$$

$$\frac{1}{V_{ASCBE}} = \frac{P_{scbe2}}{L_{eff}}\exp\left(\frac{-P_{scbe1}\, litl}{V_{ds} - V_{dsat}}\right) \tag{3.5.7}$$

3.6 Single Current Expression for All Operating Regimes of V_{gs} and V_{ds}

The V_{gsteff} function introduced in Chapter 2 gave a unified expression for the linear drain current from subthreshold to strong inversion as well as for the saturation drain current from subthreshold to strong inversion, *separately*. In order to link the continuous linear current with that of the continuous saturation current, a smooth function for V_{ds} is introduced. In the past, several smoothing functions have been proposed for MOSFET modeling [22-24]. The smoothing function used in BSIM3 is similar to that proposed in [24]. The final current equation for both linear and saturation current now becomes

$$I_{ds} = \frac{I_{dso(Vdseff)}}{1 + \frac{R_{ds}I_{dso(Vdseff)}}{V_{dseff}}}\left(1 + \frac{V_{ds} - V_{dseff}}{V_A}\right)\left(1 + \frac{V_{ds} - V_{dseff}}{V_{ASCBE}}\right) \tag{3.6.1}$$

Most of the previous equations which contain V_{ds} and V_{dsat} dependencies are now substituted with the V_{dseff} function. For example, Eq. (3.5.4) now becomes

(3.6.2)

$$V_{ACLM} = \frac{A_{bulk} E_{sat} L_{eff} + V_{gsteff}}{P_{CLM} A_{bulk} E_{sat}\ litl} (V_{ds} - V_{dseff})$$

Similarly, Eq. (3.5.7) now becomes

(3.6.3)

$$\frac{1}{V_{ASCBE}} = \frac{P_{scbe2}}{L_{eff}} \exp\left(\frac{-P_{scbe1}\ litl}{V_{ds} - V_{dseff}}\right)$$

The V_{dseff} expression is written as

(3.6.4)

$$V_{dseff} = V_{dsat} - \frac{1}{2}\left(V_{dsat} - V_{ds} - \delta + \sqrt{(V_{dsat} - V_{ds} - \delta)^2 + 4\delta V_{dsat}}\right)$$

The expression for V_{dsat} is that given under Section 3.4. The parameter δ in the unit of volts can be extracted. The dependence of V_{dseff} on V_{ds} is given in Figure 3-3. The V_{dseff} function follows V_{ds} in the linear region and tends to V_{dsat} in the saturation region. Figure 3-4 shows the effect of δ on the transition region between linear and saturation regimes.

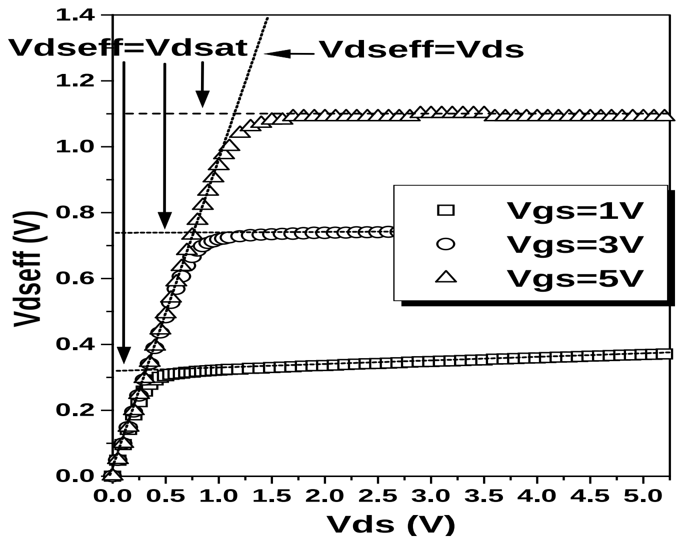

Figure 3-3. V_{dseff} **vs.** V_{ds} **for** δ**=0.01 and different** V_{gs}.

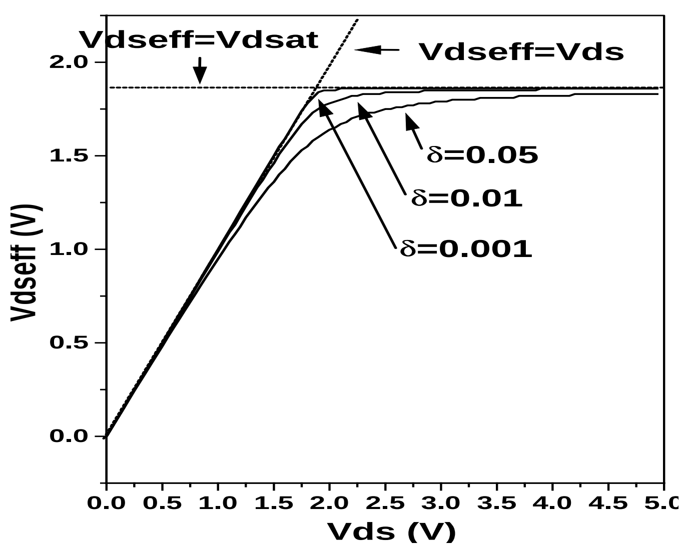

Figure 3-4. V_{dseff} **vs.** V_{ds} **for** V_{gs}**=3V and different** δ **values.**

3.7 Substrate Current

The substrate current in BSIM3v3.2.1 is modeled by

$$I_{sub} = \frac{\alpha_0 + \alpha_1 \cdot L_{eff}}{L_{eff}} \left(V_{ds} - V_{dseff}\right) \exp\left(-\frac{\beta_0}{V_{ds} - V_{dseff}}\right) \frac{I_{ds0}}{1 + \frac{R_{ds} I_{ds0}}{V_{dseff}}} \left(1 + \frac{V_{ds} - V_{dseff}}{V_A}\right) \tag{3.7.1}$$

where parameters α_0 and β_0 are impact ionization coefficients; parameter α_1 improves the I_{sub} scalability.

3.8 A Note on V_{bs}

All V_{bs} terms have been substituted with a V_{bseff} expression as shown in Eq. (3.8.1). This is done in order to set an upper bound for the body bias value during simulations. Unreasonable values can occur if this expression is not introduced.

$$V_{bseff} = V_{bc} + 0.5[V_{bs} - V_{bc} - \delta_1 + \sqrt{(V_{bs} - V_{bc} - \delta_1)^2 - 4\delta_1 V_{bc}}\ ] \tag{3.8.1}$$

where δ_1=0.001V.

Parameter V_{bc} is the maximum allowable V_{bs} value and is obtained based on the condition of $dV_{th}/dV_{bs} = 0$ for the V_{th} expression of 2.1.4.