

---

# APPENDIX B: Equation List

---

## B.1 I-V Model

### B.1.1 Threshold Voltage

$$\begin{aligned} V_{th} = & V_{th0ox} + K_{1ox} \cdot \sqrt{\Phi_s - V_{bseff}} - K_{2ox} V_{bseff} \\ & + K_{1ox} \left( \sqrt{1 + \frac{Nl_x}{L_{eff}}} - 1 \right) \sqrt{\Phi_s} + (K_3 + K_{3b} V_{bseff}) \frac{T_{ox}}{W_{eff} + W_0} \Phi_s \\ & - D_{VT0w} \left( \exp \left( -D_{VT1w} \frac{W_{eff} \cdot L_{eff}}{2l_{tw}} \right) + 2 \exp \left( -D_{VT1w} \frac{W_{eff} \cdot L_{eff}}{l_{tw}} \right) \right) (V_{bi} - \Phi_s) \\ & - D_{VT0} \left( \exp \left( -D_{VT1} \frac{L_{eff}}{2l_t} \right) + 2 \exp \left( -D_{VT1} \frac{L_{eff}}{l_t} \right) \right) (V_{bi} - \Phi_s) \\ & - \left( \exp \left( -D_{sub} \frac{L_{eff}}{2l_{io}} \right) + 2 \exp \left( -D_{sub} \frac{L_{eff}}{l_{io}} \right) \right) (E_{tao} + E_{tab} V_{bseff}) \mathcal{N}_{ds} \end{aligned}$$

$$V_{th0ox} = V_{th0} - K_1 \cdot \sqrt{\Phi_s}$$

$$K_{1ox} = K_1 \cdot \frac{T_{ox}}{T_{oxm}}$$

$$K_{2ox} = K_2 \cdot \frac{T_{ox}}{T_{oxm}}$$

$$l_t = \sqrt{\epsilon_{si} X_{dep} / C_{ox}} (1 + D_{VT2} V_{bseff})$$

$$l_{tw} = \sqrt{\epsilon_{si} X_{dep} / C_{ox}} (1 + D_{VT2w} V_{bseff})$$

$$l_{to} = \sqrt{\epsilon_{si} X_{dep0} / C_{ox}}$$

$$X_{dep} = \sqrt{\frac{2\epsilon_{si}(\Phi_s - V_{bseff})}{qN_{ch}}}$$

$$X_{dep0} = \sqrt{\frac{2\epsilon_{si}\Phi_s}{qN_{ch}}}$$

$$(\delta_1=0.001)$$

$$V_{bseff} = V_{bc} + 0.5[V_{bs} - V_{bc} - \delta_1 + \sqrt{(V_{bs} - V_{bc} - \delta_1)^2 - 4\delta_1 V_{bc}}]$$

$$V_{bc} = 0.9 \left( \Phi_s - \frac{K_1^2}{4K_2^2} \right)$$

$$V_{bi} = v_t \ln\left(\frac{N_{ch}N_{DS}}{n_i^2}\right)$$

### B.1.2 Effective ( $V_{gs}-V_{th}$ )

$$V_{gseff} = \frac{2 n v_t \ln \left[ 1 + \exp\left(\frac{V_{gs} - V_{th}}{2 n v_t}\right) \right]}{1 + 2 n C_{ox} \sqrt{\frac{2\Phi_s}{q\epsilon_{si}N_{ch}}} \exp\left(-\frac{V_{gs} - V_{th} - 2V_{off}}{2 n v_t}\right)}$$

$$n = 1 + N_{factor} \frac{C_d}{C_{ox}} + \frac{(C_{dsc} + C_{dscd} V_{ds} + C_{dscb} V_{bseff}) \left( \exp(-D_{VT1} \frac{L_{eff}}{2l_t}) + 2 \exp(-D_{VT1} \frac{L_{eff}}{l_t}) \right)}{C_{ox}} + \frac{C_{it}}{C_{ox}}$$

$$C_d = \frac{\epsilon_{si}}{X_{dep}}$$

### B.1.3 Mobility

For mobMod=1

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff}) \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right)^2}$$

For mobMod=2

$$\mu_{eff} = \frac{\mu_o}{1 + (U_a + U_c V_{bseff}) \left( \frac{V_{gsteff}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff}}{T_{OX}} \right)^2}$$

For mobMod=3

$$\mu_{eff} = \frac{\mu_o}{1 + [U_a \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right) + U_b \left( \frac{V_{gsteff} + 2V_{th}}{T_{OX}} \right)^2] (1 + U_c V_{bseff})}$$

**B.1.4 Drain Saturation Voltage**

For  $R_{ds} > 0$  or  $\lambda \neq 1$ :

$$V_{dsat} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$a = A_{bulk}^2 W_{eff} V_{sat} C_{ox} R_{DS} + \left(\frac{1}{\lambda} - 1\right) A_{bulk}$$

$$b = -\left( (V_{gsteff} + 2V_t) \left(\frac{2}{\lambda} - 1\right) + A_{bulk} E_{sat} L_{eff} + 3A_{bulk} (V_{gsteff} + 2V_t) W_{eff} V_{sat} C_{ox} R_{DS} \right)$$

$$c = (V_{gsteff} + 2V_t) E_{sat} L_{eff} + 2(V_{gsteff} + 2V_t)^2 W_{eff} V_{sat} C_{ox} R_{DS}$$

$$\lambda = A_1 V_{gsteff} + A_2$$

For  $R_{ds} = 0$  and  $\lambda = 1$ :

$$V_{dsat} = \frac{E_{sat} L_{eff} (V_{gsteff} + 2V_t)}{A_{bulk} E_{sat} L_{eff} + (V_{gsteff} + 2V_t)}$$

$$A_{bulk} = \left( 1 + \frac{K_{lox}}{2\sqrt{\Phi_s - V_{bs eff}}} \left( \frac{A_0 L_{eff}}{L_{eff} + 2\sqrt{X_J X_{dep}}} \left( 1 - A_{gs} V_{gsteff} \left( \frac{L_{eff}}{L_{eff} + 2\sqrt{X_J X_{dep}}} \right)^2 \right) + \frac{B_0}{W_{eff}' + B_1} \right) \right) \cdot \frac{1}{1 + K_{eta} V_{bs eff}}$$

$$E_{sat} = \frac{2v_{sat}}{\mu_{eff}}$$

### B.1.5 Effective $V_{ds}$

$$V_{dseff} = V_{dsat} - \frac{1}{2} \left( V_{dsat} - V_{ds} - \delta + \sqrt{(V_{dsat} - V_{ds} - \delta)^2 + 4\delta V_{dsat}} \right)$$

### B.1.6 Drain Current Expression

$$I_{ds} = \frac{I_{dso}(V_{dseff})}{1 + \frac{R_{ds}I_{dso}(V_{dseff})}{V_{dseff}}} \left( 1 + \frac{V_{ds} - V_{dseff}}{V_A} \right) \left( 1 + \frac{V_{ds} - V_{dseff}}{V_{ASCBE}} \right)$$

$$I_{dso} = \frac{W_{eff}\mu_{eff}C_{ox}V_{gsteff} \left( 1 - A_{bulk} \frac{V_{dseff}}{2(V_{gsteff} + 2v_t)} \right) V_{dseff}}{L_{eff}[1 + V_{dseff} / (E_{sat}L_{eff})]}$$

$$V_A = V_{Asat} + \left( 1 + \frac{P_{vag}V_{gsteff}}{E_{sat}L_{eff}} \right) \left( \frac{1}{V_{ACLM}} + \frac{1}{V_{ADIBLC}} \right)^{-1}$$

$$V_{ACLM} = \frac{A_{bulk}E_{sat}L_{eff} + V_{gsteff}}{P_{CLMA_{bulk}E_{sat} \text{ litl}}} (V_{ds} - V_{dseff})$$

$$V_{ADIBLC} = \frac{(V_{gsteff} + 2V_t)}{\theta_{ROUT}(1 + P_{DIBLCB}V_{bseff})} \left( 1 - \frac{A_{bulk}V_{dsat}}{A_{bulk}V_{dsat} + V_{gsteff} + 2V_t} \right)$$

$$\theta_{ROUT} = P_{DIBLC1} \left[ \exp(-D_{ROUT} \frac{L_{eff}}{2l_{t0}}) + 2 \exp(-D_{ROUT} \frac{L_{eff}}{l_{t0}}) \right] + P_{DIBLC2}$$

$$\frac{1}{V_{ASCBE}} = \frac{P_{scbe2}}{L_{eff}} \exp\left(\frac{-P_{scbe1} \text{litl}}{V_{ds} - V_{dseff}}\right)$$

$$V_{Asat} = \frac{E_{sat}L_{eff} + V_{dsat} + 2R_{DS}V_{sat}C_{ox}W_{eff}V_{gsteff} \left[ 1 - \frac{A_{bulk}V_{dsat}}{2(V_{gsteff} + 2V_t)} \right]}{2/\lambda - 1 + R_{DS}V_{sat}C_{ox}W_{eff}A_{bulk}}$$

$$\text{litl} = \sqrt{\frac{\epsilon_{si}T_{ox}X_j}{\epsilon_{ox}}}$$

### B.1.7 Substrate Current

$$I_{sub} = \frac{\alpha_0 + \alpha_1 \cdot L_{eff}}{L_{eff}} (V_{ds} - V_{dseff}) \exp\left(-\frac{\beta_0}{V_{ds} - V_{dseff}}\right) \frac{I_{ds0}}{1 + \frac{R_{ds}I_{ds0}}{V_{dseff}}} \left( 1 + \frac{V_{ds} - V_{dseff}}{V_A} \right)$$

**B.1.8 Polysilicon Depletion Effect**

$$V_{poly} = \frac{1}{2} X_{poly} E_{poly} = \frac{q N_{gate} X_{poly}^2}{2 \epsilon_{si}}$$

$$\epsilon_{ox} E_{ox} = \epsilon_{si} E_{poly} = \sqrt{2 q \epsilon_{si} N_{gate} V_{poly}}$$

$$V_{gs} - V_{FB} - \Phi_s = V_{poly} + V_{ox}$$

$$a(V_{gs} - V_{FB} - \Phi_s - V_{poly})^2 - V_{poly} = 0$$

$$a = \frac{\epsilon_{ox}^2}{2 q \epsilon_{si} N_{gate} T_{ox}^2}$$

$$V_{gs\_eff} = V_{FB} + \Phi_s + \frac{q \epsilon_{si} N_{gate} T_{ox}^2}{\epsilon_{ox}^2} \left( \sqrt{1 + \frac{2 \epsilon_{ox}^2 (V_{gs} - V_{FB} - \Phi_s)}{q \epsilon_{si} N_{gate} T_{ox}^2}} - 1 \right)$$

**B.1.9 Effective Channel Length and Width**

$$L_{eff} = L_{drawn} - 2dL$$

$$W_{eff} = W_{drawn} - 2dW$$

$$W_{eff}' = W_{drawn} - 2dW'$$

$$dW = dW' + dW_g V_{gseff} + dW_b \left( \sqrt{\Phi_s - V_{bseff}} - \sqrt{\Phi_s} \right)$$

$$dW' = W_{int} + \frac{W_l}{L^{Wln}} + \frac{W_w}{W^{Wwn}} + \frac{W_{wl}}{L^{Wln} W^{Wwn}}$$

$$dL = L_{int} + \frac{L_l}{L^{Lln}} + \frac{L_w}{W^{Lwn}} + \frac{L_{wl}}{L^{Lln} W^{Lwn}}$$

### B.1.10 Source/Drain Resistance

$$R_{ds} = \frac{R_{dsw} \left( 1 + P_{rwg} V_{gseff} + P_{rwb} \left( \sqrt{\Phi_s - V_{bseff}} - \sqrt{\Phi_s} \right) \right)}{(10^6 W_{eff}')^{W_r}}$$

### B.1.11 Temperature Effects

$$V_{th}(T) = V_{th}(T_{norm}) + (K_{T1} + K_{T1l} / L_{eff} + K_{T2} V_{bseff})(T / T_{norm} - 1)$$

$$\mu_o(T) = \mu_o(T_{norm}) \left( \frac{T}{T_{norm}} \right)^{\mu_{te}}$$

$$V_{sat}(T) = V_{sat}(T_{norm}) - A_T(T / T_{norm} - 1)$$



## Capacitance Model Equations

---

$$R_{dsw}(T) = R_{dsw}(T_{norm}) + P_{rt} \left( \frac{T}{T_{norm}} - 1 \right)$$

$$U_a(T) = U_a(T_{norm}) + U_{a1}(T / T_{norm} - 1)$$

$$U_b(T) = U_b(T_{norm}) + U_{b1}(T / T_{norm} - 1)$$

$$U_c(T) = U_c(T_{norm}) + U_{c1}(T / T_{norm} - 1)$$

## B.2 Capacitance Model Equations

### B.2.1 Dimension Dependence

$$L_{active} = L_{drawn} - 2\delta L_{eff}$$

$$W_{active} = W_{drawn} - 2\delta W_{eff}$$

$$\delta L_{eff} = DLC + \frac{Llc}{L^{Lln}} + \frac{Lwc}{W^{Lwn}} + \frac{Lwlc}{L^{Lln}W^{Lwn}}$$

$$\delta W_{eff} = DWC + \frac{Wlc}{L^{Wln}} + \frac{Wwc}{W^{Wwn}} + \frac{Wwlc}{L^{Wln}W^{Wwn}}$$