# **kafe Documentation**

Release 0.5.2

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**kafe** is a data fitting framework designed for use in undergraduate physics lab courses. It provides a basic Python toolkit for fitting models to data as well as visualisation of the data and the model function. It relies on Python packages such as *NumPy* and *matplotlib*, and uses the python interface to the minimizer *Minuit* contained in the data analysis framework *ROOT*.

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# KAFE OVERVIEW

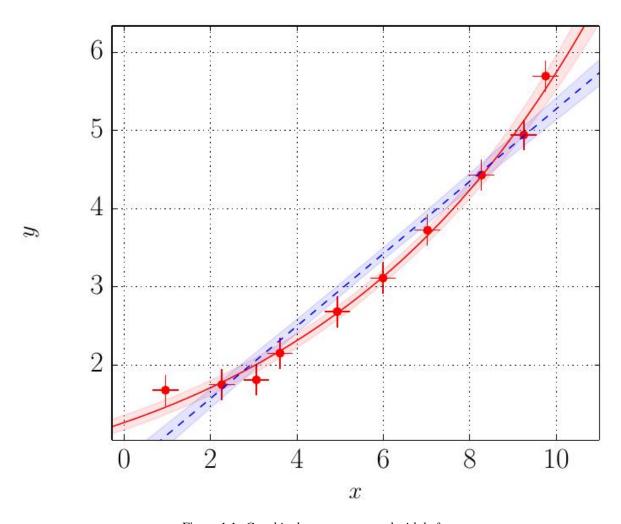


Figure 1.1: Graphical output generated with kafe.

The kafe package provides a rather general approach to fitting of a model function to two-dimensional data points with correlated uncertainties in both dimensions. A typical use-case would be measurements of two quantities, an x- and a y-value, with both uncorrelated (statistical) uncertainties and correlated systematic uncertainties.

Use cases range from performing a simple average of measurements to complex situations with correlated uncertainties on the measurements of the x and y values. The python-API guarantees full flexibility for data input. Helper functions, which also serve as examples for own implementations, are available to handle file-based examples.

The model function describes the y values as a function of the x-values and a set of model parameters  $\{p\}$ ,  $y=f(x; \{p\})$ . Full flexibility exists as model functions are implemented as *python* code. Again, examples are provided,

but user implementations are supported as well.

Fitting is based on the  $\chi^2$ -method, assuming Gaussian errors and correlations described by covariance matrices. The level of agreement between data points and the fit model is expressed in terms of the  $\chi^2$  probability, i. e. the probability to find less agreement between data and model than actually observed. Full access to the covariance matrix of the - typically correlated - model parameters is provided.

The graphical output visualises the data and the fit model at the best-fit-point of the parameters and also shows the uncertainty of the fit model as a light band surrounding the line representing the model function.

#### 1.1 Code Structure

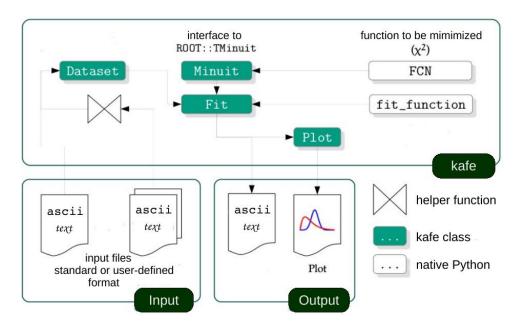


Figure 1.2: Code structure of the kafe package

The code of cafe is centred around very few classes to handle Data input, fitting and plotting, as illustrated in the figure on the right-hand side.

Data, their uncertainties, and, optionally, the correlations of the uncertainties - are passed through the interface of the cafe class <code>Dataset</code>. Input can be included in the Python code or is read from files in standardised or user-defined formats. The representation of the data within the <code>Dataset</code> class is minimalistic, consisting of the x and y values and the full covariance matrices of their uncertainties. Correlated errors between x and y values are not supported yet, as such use cases are extremely rare.

A helper function, build\_dataset(), is available to transform various error models, like a combination of independent and correlated errors or common absolute or relative errors, to this basic format.

Adding a model function, taken either from a prepared set of fit functions within kafe or from a user's own *python* implementation, results in a Fit object, which controls the minimizer Minuit and provides the results through access methods.

One or multiple fit objects, i. e. the input data and model functions(s) at the best-fit point in parameter-space, are visualised by the class Plot with the help of *matplotlib* functionality. The plot module also contains functionality to display the model uncertainty by surrounding the model function at the best-fit values of the parameters by a light band, the one- $\sigma$  uncertainty band, which is obtained by propagation of the uncertainties of the fit parameters, taking into account their correlations.

# 1.2 Example

Only very few lines of Python code are needed to perform fits with kafe. The sniplet of code shown below performs a fit of a quadratic function to some data points with uncertainties:

```
from kafe import *
from kafe.function_library import quadratic_3par
#### build a Dataset instance:
myDataset = build_dataset(
    [0.05, 0.36, 0.68, 0.80, 1.09, 1.46, 1.71, 1.83, 2.44, 2.09, 3.72, 4.36, 4.60],
    [0.35, 0.26, 0.52, 0.44, 0.48, 0.55, 0.66, 0.48, 0.75, 0.70, 0.75, 0.80, 0.90],
    yabserr=[0.06,0.07,0.05,0.05,0.07,0.07,0.09,0.1,0.11,0.1,0.11,0.12,0.1],
    title='some data',
    axis_labels=['$x$', '$y=f(x)$'])
#### Create the Fit object
myFit = Fit (myDataset, quadratic_3par)
# Set initial values and error estimates
myFit.set_parameters((0., 1., 0.2), (0.5, 0.5, 0.5))
# Do the Fit
myFit.do_fit()
#### Create the plots and output them
myPlot = Plot(myFit)
myPlot.plot_all()
myPlot.save('kafe_example0.pdf') # to file
myPlot.show()
                                   # to screen
```

The output in text form (also available via various get\_...() methods of the Fit class) contains the values of the parameters at the best-fit point, their correlation matrix and the fit probability. The example produces the following graphical output:

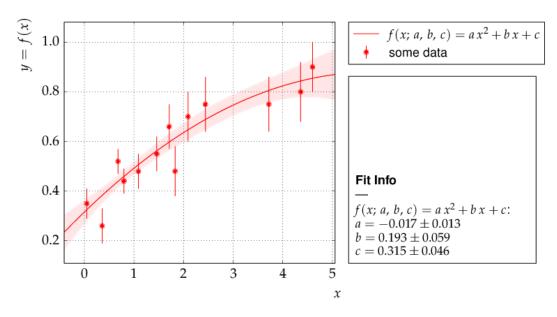


Figure 1.3: Example: Data points with one-dimensional error bars compared to a quadratic model function with **kafe**.

More and advanced examples - like fitting different models to one data set, comparison of different data sets with model functions, averaging of correlated measurements or multi-parameter fits - are provided as part of the *kafe* distribution and are described in the section *Examples* below. They may serve as a starting point for own applications.

1.2. Example 5

#### 1.3 Installation

To install kafe, unpack the archive kafe-<version>.tgz, change to the sub-directory kafe-<versoin>/src/ and follow the installation instructions below.

1.) Install using pip:

To install kafe using the *Pip* installer (http://www.pip-installer.org/), simply run the helper script as root:

```
./install.sh
```

If you don't have Pip installed, use:

```
easy_install pip
```

To remove kafe using pip, just run the helper script:

```
./uninstall.sh
```

#### 2. Install using setuptools:

Installing using Python's *setup* tools works, but does not provide a clean uninstall. Use this method if installing with *Pip* is not possible:

```
python setup.py install
```

**Kafe needs a working version of the CERN data analysis framework root,** freely available at http://root.cern.ch

### 1.3.1 Dependencies

The recommended versions of external packages for kafe are as follows, the version numbers in parentheses refer to the minimum requirements:

Be sure that the version of *ROOT* you use is compiled with *PyROOT* support. For *Python* to see the *PyROOT* bindings, the following environment variables must be set correctly:

```
export ROOTSYS=<directory where ROOT is installed>
export LD_LIBRARY_PATH=${LD_LIBRARY_PATH}:$ROOTSYS/lib
export PYTHONPATH=$ROOTSYS/lib:$PYTHONPATH
```

For more info, refer to [http://root.cern.ch/drupal/content/pyroot].

Qt is needed because it is the supported interactive front-end for matplotlib. Other front-ends are not supported and can cause weird behaviour.

LaTeX is used by matplotlib for displaying labels and mathematical expressions on graphs.

# FIT EXAMPLES, UTILITIES, TIPS AND TRICKS

A wide range of applications of the kafe core and the usage of the helper functions is exemplified here. All of them are contained in the sub-directory examples/ of the cafe distribution and are intended to serve as a basis for user projects.

# 2.1 Example 1 - model comparison

To decide whether a model is adequate to describe a given set of data, typically several models have to be fit to the same data. Here is the code for a comparison of a data set to two models, namely a linear and an exponential function:

```
# import everything we need from kafe
from kafe import *
# additionally, import the two model functions we want to fit:
from kafe.function_library import linear_2par, exp_2par
############
# Load the Dataset from the file
my_dataset = Dataset(input_file='dataset.dat', title="Example Dataset")
### Create the Fits
my_fits = [Fit(my_dataset, exp_2par),
          Fit(my_dataset, linear_2par)]
### Do the Fits
for fit in my_fits:
 fit.do_fit()
### Create the plots, save and show output
my_plot = Plot(my_fits[0], my_fits[1])
my_plot.plot_all(show_data_for=0) # show data only once (it's the same data)
my_plot.save('plot.pdf')
my_plot.show()
```

The file *dataset.dat* contains x and y data in the standard *kafe* data format, where values and errors (and optionally also correlation coefficients) are given for each axis separately. # indicates a comment line, which is ignored when reading the data:

```
# axis 0: x
# datapoints uncor. err.
0.957426  3.0e-01
2.262212  3.0e-01
3.061167  3.0e-01
3.607280  3.0e-01
4.933100  3.0e-01
5.992332  3.0e-01
7.021234  3.0e-01
```

```
8.272489 3.0e-01
9.250817 3.0e-01
9.757758 3.0e-01
# axis 1: y
# datapoints uncor. err.
1.672481 2.0e-01
1.743410 2.0e-01
1.805217 2.0e-01
2.147802 2.0e-01
2.679615 2.0e-01
3.110055 2.0e-01
3.723173 2.0e-01
4.430122
         2.0e-01
4.944116
         2.0e-01
5.698063 2.0e-01
```

The resulting output is shown below. As can be seen already from the graph, the exponential model better describes the data. The  $\chi^2$  probability in the printed output shows, however, that the linear model would be marginally acceptable as well:

```
linear_2par
chi2prob 0.052
HYPTEST accepted (CL 5%)
exp_2par
chi2prob 0.96
HYPTEST accepted (CL 5%)
```

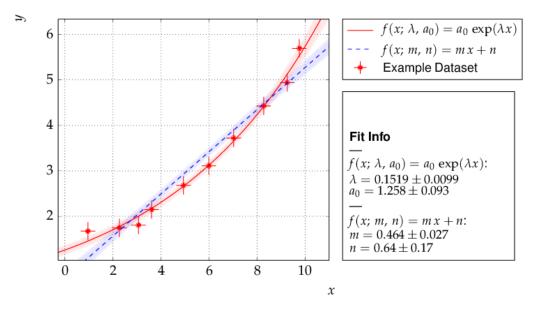


Figure 2.1: Output of example 1 - compare two models

# 2.2 Example 2 - two fits and models

Another typical use case consists of comparing two sets of measurements and the models derived from them. This is very similar to the previous example with minor modifications:

############

This results in the following output:

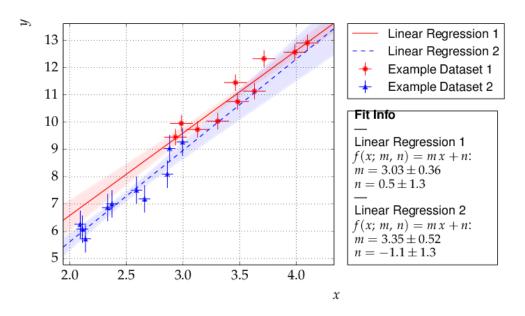


Figure 2.2: Output of example 2 - comparison of two linear fits.

Although the parameters extracted from the two data sets agree within errors, the uncertainty bands of the two functions do not overlap in the region where the data of Dataset 2 are located, so the data are most probably incompatible with the assumption of an underlying single linear model.

# 2.3 Example 3 - properties of a Gauss curve

This example creates a dummy Dataset object whose points lie exactly on a Gaussian curve. The Fit will then converge toward that very same Gaussian. When plotting, the data points used to "support" the curve can be omitted.

This example shows how to access the kafe plot objects to annotate plots with matplotlib functionality.

# 2.4 Example 4 - average of correlated measurements

The average of a set of measurements can be considered as a fit of a constant to a set of input data. This example illustrates how correlated errors are handled in *kafe*. Measurements can have a common error, which may be absolute or relative, i. e. depend on the input value. In more complicated cases the full covariance matrix must be constructed.

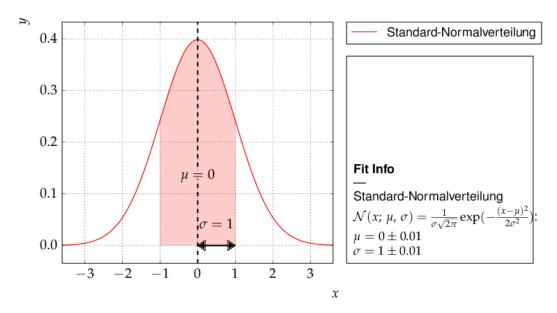


Figure 2.3: *Output of example 3 - properties of a Gauss curve.* 

kafe has a helper function, build\_dataset in module fit, which aids in setting up the covariance matrix and transforming the input to the default format used by the Dataset and Fit classes. Two further helper functions in module file\_tools aid in reading the appropriate information from data files.

- 1. The function parse\_column\_data reads the input values and their independent errors from one file, and optionally covariance matrices for the x and y axes from additional files. The field ordering is defined by a control string.
- 2. Another helper function, buildDataset\_fromFile, specifies input values or blocks of input data from a single file with keywords.

The second version needs only very minimal additional user code, as illustrated here:

The input file is necessarily more complicated, but holds the full information on the data set in one place. Refer to the documentation of the module parse\_general\_inputfile in module file\_tools for a full description of the presently implemented keywords. The input file for the averaging example is here:

```
# Measurements of W boson mass (combined LEP2, 2013)
# ------
# example to use parse_general_inputfile from kafe;
# covariance matrix build from common errors
# --
# Meta data for plotting
*TITLE measurements of the W boson mass
*xLabel number of measurement
```

```
*yLabel $m_\matrhm{W}$
*yUnit GeV/$c^2$
# x data need not be given for averaging
# -----
 Measurements of W mass by ALEPH, DELPI, L3 and OPAL
                             from from LEP2 Report Feb. 2013
 common errors within channels
                   2q21: 0.021 GeV,
                      4q: 0.044 GeV,
    and between channels: 0.025 GeV
*yData_SCOV
                syst sqrt of the off-diagonal
# W_mass err
# 2q21 channel
                                     elements of the
               0.021
 80.429 0.055
                               #
                                        covariance matrix
               0.021 0.021
 80.339 0.073
 80.217 0.068 0.021 0.021 0.021
                       0.021 0.021 0.021
 80.449 0.058 0.021
# 4q channel
 80.477 0.069 0.044 0.025 0.025 0.025 0.025 0.044
 80.310 0.091 0.044 0.025 0.025 0.025 0.025 0.044 0.044
 80.324 0.078 0.044 0.025 0.025 0.025 0.025 0.044 0.044 0.044
 80.353 \quad 0.068 \quad 0.044 \quad 0.025 \quad 0.025 \quad 0.025 \quad 0.025 \quad 0.044 \quad 0.044 \quad 0.044 \quad 0.044
```

# 2.5 Example 5 - multi-parameter fit (damped oscillation)

This example shows the fitting of a more complicated model function to data collected from a damped harmonic oscillator. In such non-linear fits, stetting the initial values is sometimes crucial to let the fit converge at the global minimum. The Fit object provides the method set\_parameters for this purpose. As the fit function for this problem is not a standard one, it is defined explicitly making use of the decorator functions available in *kafe* to provide nice type setting of the parameters. This time, the function parse\_colmn\_data is used to read the input, which is given as separate columns with the fields

```
<time> <Amplitude> <error on time> <error on Amplitude>
. . . .
```

Here is the example code:

```
from kafe import *
from numpy import exp, cos
# Model function definition #
# Set an ASCII expression for this function
@ASCII(x_name="t", expression="A0*exp(-t/tau)*cos(omega*t+phi)")
# Set some LaTeX-related parameters for this function
@LaTeX(name='A', x_name="t",
    parameter_names=('a_0', '\\tau{}', '\\omega{}', '\\varphi{{}'),
    expression="a_0\/\,\end{t} {\tau}\/\,
               "\cos(\\omega{}\\,t+\\varphi{})")
@FitFunction
def damped_oscillator(t, a0=1, tau=1, omega=1, phi=0):
 return a0 * exp(-t/tau) * cos(omega*t + phi)
# ---- Workflow #
# load the experimental data from a file
my_dataset = parse_column_data('damped_oscillation.dat',
 field_order="x,y,xabserr,yabserr", title="Damped Oscillator",
 axis_labels=['Time t','Amplitude'])
# --- Create the Fit
```

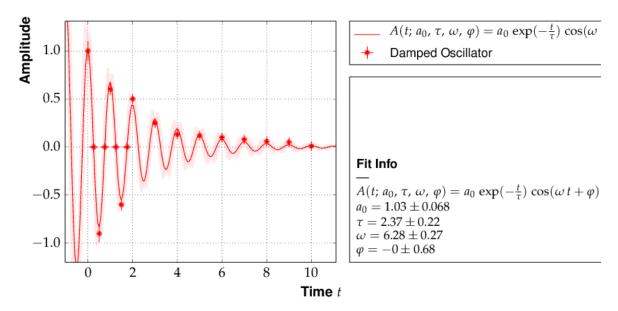


Figure 2.4: Example 5 - fit of the time dependence of the amplitude of a damped harmonic oscillator.

# 2.6 Example 6 - another multi-parameter fit

This example is not much different from the previous one, except that the fit function, a standard fourth-degree polynomial from the module function\_library, is modified to reflect the names of the problem given:

```
from kafe.function_library import poly4
# modify function's independent variable name to reflect its nature:
poly4.x_name = 'x=cos(t)'
poly4.latex_x_name = 'x=\\cos(\\theta)'
...
# Set the axis labels appropriately
my_plot.axis_labels = ['$\\cos(\\theta)$', 'counting rate']
```

This is the resulting output:

# 2.7 Example 7 - non-linear multi-parameter fit

Again, not much new in this example, except that the model is now very non-linear, the intensity distribution of light after passing through a double-slit. The non-standard model definition again makes use of the decorator mechanism to provide nice output - the decorators (expressions beginning with '@') can safely be omitted if

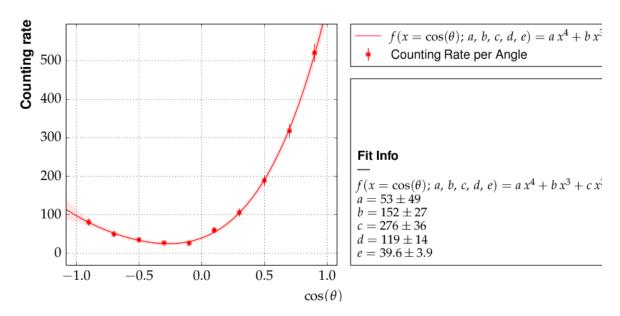


Figure 2.5: *Output of example 6 - counting rate.* 

LaTeX output is not needed. Setting of appropriate initial conditions is absolutely mandatory for this example, because there exist many local minima of the  $\chi^2$  function.

Another problem becomes obvious when carefully inspecting the fit function definition: only two of the three parameters g, b or k can be determined, and therefore one must be kept fixed, or an external constraint must be applied. Failing to do so will result in large, correlated errors on the parameters g, b and k as an indication of the problem.

Fixing parameters of a model function is achieved by the method fix\_parameters(), and a constraint within a given uncertainty is achieved by the method constrain\_parameters() of the Fit class.

Here are the interesting pieces of code:

```
# Model function definition #
# Set an ASCII expression for this function
@ASCII(x_name="x", expression="10*(\sin(k/2*b*\sin(x))/(k/2*b*\sin(x))"
                          "*\cos(k/2*g*\sin(x)))^2")
# Set some LaTeX-related parameters for this function
@LaTeX(name='I', x_name="\\alpha{}",
 parameter_names=('I_0', 'b', 'g', 'k'),
 "{\\frac{k}{2}\\,b\\,\\sin{\\alpha}}"
               "\\\cos(\frac{k}{2}\\,g\\,\sin{{\alpha}})\\right)^2")
@FitFunction
def double_slit(alpha, I0=1, b=10e-6, g=20e-6, k=1.e7):
 k_half_sine_alpha = k/2*sin(alpha) # helper variable
 k_b = k_half_sine_alpha * b
 k_g = k_half_sine_alpha * g
 return I0 \star (sin(k_b)/(k_b) \star cos(k_g)) \star \star 2
. . .
# Set the initial values for the fit
my_fit.set_parameters((1., 20e-6, 50e-6, 9.67e6))
# fix one of the (redundant) parameters, here 'k'
my_fit.fix_parameters('k')
. . .
```

If the parameter k in the example above has a (known) uncertainty, is is more appropriate to constrain it within its uncertainty (which may be known from an indepedent measurement of from the secifications of the laser used in the experiment). To take into account a wave number k known with a precision of 10'000, the last line in the example above should be replaced by:

```
my_fit.constrain_parameters(['k'],[9.67e6], [1.e4])
...
```

This is the resulting output:

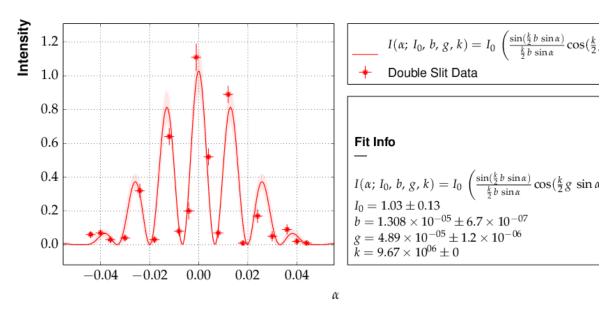


Figure 2.6: Example 7 - fit of the intensity distribution of light behind a double slit with fixed or constrained wave length.

# 2.8 Example 8 - fit of a Breit-Wigner Resonance to data with correlated errors

This example illustrates how to define the data and the fit function in a single file - provided by the helper function <code>buildFit\_fromFile</code> in module <code>file\_tools</code>. Parsing of the input file is done by the function <code>parse\_general\_inputfile</code>, which had already been introduced in Example 4. The definition of the fit function as python code including the *kafe* decorators in the input file, however, is new. Note: because spaces are used to to separate data fields in the input file, spaces needed for proper python indentation have to be replaced by '~'. The last key in the file defines the start values of the parameters and their initial ranges.

The advantage of this approach is the location of all data and the fit model in one place, which is strictly separated from the python code. The python code below is thus very general and can handle a large large variety of problems without modification (except for the file name, which could easily be passed on the command line):

```
BWplot.save("plot.pdf")
BWplot.show()
```

The magic happens in the input file, which now has to provide all the information needed to perform the fit:

```
# Fit of a Breit-Wigern function to
     measruements of hadronic Z cross sections a LEP
# Meta-data for plotting
*TITLE LEP Hadronic Cross Section ($\sigma^0_\mathrm{had}$)
*xLabel $E_CM$
*xUnit $\mathrm{GeV}$
*yLabel $\sigma^0_{\mathrm{had}}$
*yUnit $\mathrm{nb}$
#-----
# DATA: average of hadronic cross sections measured by
# ALEPH, DELPHI, L3 and OPAL around 7 energy points at the Z resonance
# CMenergy E err
*xData
88.387 0.005
89.437 0.0015
90.223 0.005
91.238 0.003
92.059 0.005
93.004 0.0015
93.916 0.005
# Centre-of-mass energy has a common uncertainty
*xAbsCor 0.0017
# sig^0_h sig err
                   # rad.cor sig_h measured
*yData
6.803 0.036
                # 1.7915 5.0114
13.965 0.013
                # 4.0213 9.9442
26.113 0.075
                # 7.867
                            18.2460
41.364 0.010
                # 10.8617 30.5022
27.535 0.088
                # 3.9164 23.6187
13.362 0.015
                # -0.6933
                           14.0552
 7.302 0.045
                 # -1.8181
                             9.1196
# cross-sections have a common relative error
*yRelCor 0.0007
*FITLABEL Breit-Wigner-Fit {\large{( with s-dependent width )}}
*FitFunction
# Breit-Wigner with s-dependent width
@ASCII(expression='s0*E^2*G^2/[(E^2-M^2)^2+(E^4*G^2/M^2)]')
@LaTeX(name='f', parameter_names=('\\sigma^0', 'M_Z','\\Gamma_Z'),
expression='\\frac{\\sigma^0\\, M_Z^2\\Gamma^2}'
             '{((E^2-M_Z^2)^2+(E^4\\Gamma^2 / M_Z^2))}')
@FitFunction
def fitf(E, M=91.2, G=2.5, s0=41.0):
~~return s0*E*E*G*G/((E*E-M*M)**2+(E**4*G*G/(M*M)))
*InitialParameters  # set initial values and ranges
91.2 0.1
2.5 0.1
41. 0.5
```

Here is the output:

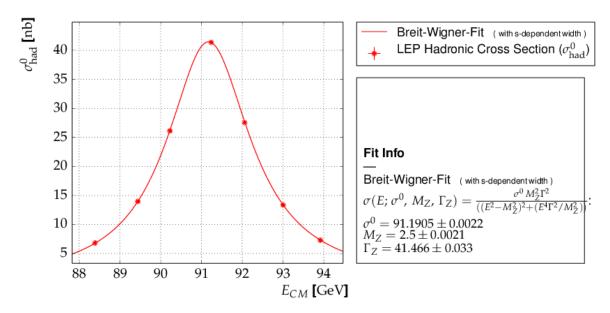


Figure 2.7: Output of example 8 - Fit of a Breit-Wigner function.

# 2.9 Example 9 - fit of a function to histogram data

This example brings us to the limit of what is presently possible with kafe. Here, the data represent the centre of a histogram bins ad the number of entries, :math:: $n_i$ , in each bin. The (statistical) error is typically estimated as the square root of the (observed) number of entries in each bin. For large numbers of entries, this is not a problem, but for small numbers, especially for bins with 0 entries, the correlation between the observed number of entries and the error derived from it leads to a bias when fitting functions to the histogram data. In particular, bins with zero entries cannot be handled in the  $\chi^2$ -function, and are typically omitted to cure the problem. However, the bias remains, as bins with downward fluctuations of the observed numbers of events have get assigned smaller errors and hence larger weights in the fitting procedure - leading to the aforementioned bias.

These problems are avoided by using a likelihood method for such use cases, where Poission distribution of the uncertainties and their dependence on the values of the fit model is properly taken into accout. However, the  $\chi^2$ -method can be saved to some extend if the fitting procedure is iterated. In a pre-fit, a first approximation of the model function is determined, where the error in bins with zero entries is set to one. The model function determined from the pre-fit is then used to calculate the expected errors for each bin, and the original errors are replaced before performing the final fit. Note that the numbers of entries in the bins must be sufficiently large to justify a replacement of the (asymmetric) Poission uncertainties by the symmetric uncertainties implied by the  $\chi^2$ -method.

The implementation of this procedure needs accesses some more fundamental methods of the *Dataset*, *Fit* and *FitFunction* classes. The code shown below demonstrates how to get a grip on such more complex procedures with *kafe*:

```
# Load Dataset from file
hdataset = Dataset(input_file='hdataset.dat', title="Data for example 9")

# error for bins with zero contents is set to 1.
covmat=hdataset.get_cov_mat('y')
for i in range(0,len(covmat)):
   if covmat[i,i]==0.: covmat[i,i]=1.
hdataset.set_cov_mat('y',covmat) # write it back

# Create the Fit instance
hfit = Fit(hdataset,gauss,fit_label="Fit of a Gaussian to histogram data")
# perform an initial fit with temporary errors (minimial output)
```

```
hfit.call_minimizer(final_fit=False, verbose=False)
#
#re-set errors using model at pre-fit parameter values:
# sigma_i^2=cov[i,i]=n(x_i)
fdata=hfit.fit_function.evaluate(hfit.xdata, hfit.current_parameter_values)
np.fill_diagonal(covmat, fdata)
hfit.current_cov_mat=covmat # write back new covariance matrix
#
# now do final fit with full output
hfit.do_fit()
# and create, draw, save and show plot
...
```

Here is the output, which shows that the parameters of the normal distribution, from which the data were generated, are well reproduced within the uncertainties by the fit result:

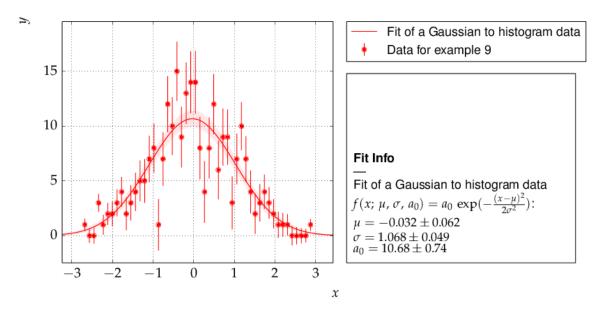


Figure 2.8: Output of example 9 - Fit of a Gaussian distribution to histogram data

# KAFE DOCUMENTATION – MODULE DESCRIPTIONS

The following documentation of functions and methods of relevance to the user interface was generated from the *DocStrings* contained in the python code of the *kafe* package. For further information or if in doubt about the exact functionality, users are invited to consult the source code.

### 3.1 kafe. init Module

**kafe** – a Python package for fitting and plotting for use in physics lab courses.

This Python package allows fitting of user-defined functions to data. A dataset is represented by a *Dataset* object which stores measurement data as *NumPy* arrays. The uncertainties of the data are also stored in the *Dataset* as an *error matrix*, allowing for both correlated and uncorrelated errors to be accurately represented.

The constructor of a *Dataset* object accepts several keyword arguments and can be used to construct a *Dataset* from input data which has been loaded into *Python* as *NumPy* arrays. Alternatively, a plain-text representations of a *Dataset* can be read from a file.

Also provided are helper functions which construct a *Dataset* object from a file containing column data (one measurement per row, column order can be specified), or from a keyword-driven input format.

# 3.2 \_version\_info Module

```
kafe._version_info.get_version()
    kafe version 0.5.2
```

#### 3.3 dataset Module

The *Dataset* object is a data structure for storing measurement and error data. In this implementation, the *Dataset* has the compulsory field *data*, which is used for storing the measurement data, and another field *cov\_mats*, used for storing the covariance matrix for each axis.

There are two ways a *Dataset* can be constructed. The most straightforward way is to specify an input file containing a plain-text representation of the dataset:

```
>>> my_dataset = Dataset(input_file='/path/to/file')
```

or

```
>>> my_dataset = Dataset(input_file=my_file_object)
```

If an *input\_file* argument is provided, the *data* and *cov\_mats* arguments are ignored. The *Dataset* plain-text representation format is as follows:

```
# x data
x_1 sigma_x_1
x_2 sigma_x_2 cor_x_12
... ... ...
x_N sigma_x_N cor_x_1N ... cor_x_NN
# y data
y_1 sigma_y_1
y_2 sigma_y_2 cor_y_12
... ...
y_N sigma_y_N cor_y_1N ... cor_y_NN
```

Here, the *sigma\_...* represents the fully uncorrelated error of the data point and *cor\_...\_ij* is the correlation coefficient between the *i*-th and *j*-th data point.

Alternatively, field data can be set by passing iterables as arguments. Available arguments for this purpose are:

data: tuple/list of tuples/lists/arrays of floats

a tuple/list of measurement data. Each element of the tuple/list must be iterable and be of the same length. The first element of the **data** tuple/list is assumed to be the *x* data, and the second to be the *y* data:

```
>>> my_dataset = Dataset(data=([0., 1., 2.], [1.23, 3.45, 5.62]))
```

Alternatively, x-y value pairs can also be passed as **data**. The following is equivalent to the above:

```
>>> my_dataset = Dataset(data=([0.0, 1.23], [1.0, 3.45], [2.0, 5.62]))
```

In case the *Dataset* contains two data points, the ordering is ambiguous. In this case, the first ordering (x data first, then y data) is assumed.

cov\_mats : tuple/list of numpy.matrix (optional)

a tuple/list of two-dimensional iterables containing the covariance matrices for *x* and *y*, in that order. Covariance matrices can be any sort of two-dimensional NxN iterables, assuming N is the number of data points.

This keyword argument can be omitted, in which case covariance matrices of zero are assumed. To specify a covariance matrix for a single axis, replace the other with None.

```
>>> my_dataset = Dataset(data=([0., 1., 2.], [1.23, 3.45, 5.62]), cov_mats=(None, my_cov_
```

>>> my\_dataset = Dataset(data=([0., 1., 2.], [1.23, 3.45, 5.62]), cov\_mats=(my\_cov\_mat\_x,

title: string (optional)

the name of the *Dataset*. If omitted, the *Dataset* will be given the generic name 'Untitled Dataset'.

axis\_labels: list of strings (optional)

labels for the x and y axes. If omitted, these will be set to 'x' and 'y', respectively.

```
axis_units: list of strings (optional)
```

units for the *x* and *y* axes. If omitted, these will be assumed to be dimensionless, i.e. the unit will be an empty string.

#### cov\_mat\_is\_regular (axis)

Returns *True* if the covariance matrix for an axis is regular and False if it is singular.

axis ['x' or 'y'] Axis for which to check for regularity of the covariance matrix.

#### cov mats = None

list of covariance matrices

```
get_axis (axis_alias)
```

Get axis id from an alias.

**axis\_alias** [string or int] Alias of the axis whose id should be returned. This is for example either '0' or 'x' for the x-axis (id 0).

```
get_cov_mat (axis, fallback_on_singular=None)
```

Get the error matrix for an axis.

```
axis ['x' or 'y'] Axis for which to load the error matrix.
```

fallback\_on\_singular [numpy.matrix or string (optional)] What to return if the matrix is singular. If this is None (default), the matrix is returned anyway. If this is a numpy.matrix object or similar, that is returned istead. Alternatively, the shortcuts 'identity' or 1 and 'zero' or 0 can be used to return the identity and zero matrix respectively.

```
get_data(axis)
```

Get the measurement data for an axis.

axis [string] Axis for which to get the measurement data. Can be 'x' or 'y'.

```
get_data_span (axis, include_error_bars=False)
```

Get the data span for an axis. The data span is a tuple (min, max) containing the smallest and highest coordinates for an axis.

```
axis ['x' or 'y'] Axis for which to get the data span.
```

include\_error\_bars [boolean (optional)] True if the returned span should be enlarged to contain the
error bars of the smallest and largest datapoints (default: False)

```
get_formatted (format_string='.06e', delimiter='t')
```

Returns the dataset in a plain-text format which is human-readable and can later be used as an input file for the creation of a new *Dataset*. The format is as follows:

```
# x data
x_1 sigma_x_1
x_2 sigma_x_2 cor_x_12
... ... ...
x_N sigma_x_N cor_x_1N ... cor_x_NN

# y data
y_1 sigma_y_1
y_2 sigma_y_2 cor_y_12
... ...
y_N sigma_y_N cor_y_1N ... cor_y_NN
```

Here, the  $x_i$  and  $y_i$  represent the measurement data, the  $sigma_?_i$  are the statistical uncertainties of each data point, and the  $cor_?_i$  are the correlation coefficients between the i-th and j-th data point.

If the x or y errors are not correlated, then the entire correlation coefficient matrix can be omitted. If there are no statistical uncertainties for an axis, the second column can also be omitted. A blank line is required at the end of each data block!

**format\_string** [string (optional)] A format string with which each entry will be rendered. Default is '.06e', which means the numbers are represented in scientific notation with six significant digits.

delimiter [string (optional)] A delimiter used to separate columns in the output.

3.3. dataset Module 21

#### get\_size()

Get the size of the *Dataset*. This is equivalent to the length of the *x*-axis data.

#### has correlations (axis)

Returns *True* if the specified axis has correlation data, False if not.

axis ['x' or 'y'] Axis for which to check for correlations.

#### has errors (axis)

Returns True if the specified axis has statistical error data.

axis ['x' or 'y'] Axis for which to check for error data.

#### n axes = None

dimensionality of the Dataset. Currently, only 2D Datasets are supported

#### n\_datapoints = None

number of data points in the Dataset

#### read\_from\_file(input\_file)

Reads the *Dataset* object from a file.

returns [boolean] True if the read succeeded, False if not.

#### set\_cov\_mat (axis, mat)

Set the error matrix for an axis.

axis ['x' or 'y'] Axis for which to load the error matrix.

mat [numpy.matrix or None] Error matrix for the axis. Passing None unsets the error matrix.

#### set\_data (axis, data)

Set the measurement data for an axis.

axis ['x' or 'y'] Axis for which to set the measurement data.

data [iterable] Measurement data for axis.

#### write\_formatted (file\_path, format\_string='.06e', delimiter='t')

Writes the dataset to a plain-text file. For details on the format, see get\_formatted.

file\_path [string] Path of the file object to write. WARNING: overwrites existing files!

**format\_string** [string (optional)] A format string with which each entry will be rendered. Default is '.06e', which means the numbers are represented in scientific notation with six significant digits.

delimiter [string (optional)] A delimiter used to separate columns in the output.

```
kafe.dataset.build_dataset(xdata, ydata, cov_mats=None, xabserr=0.0, xrelerr=0.0, xab-
scor=0.0, xrelcor=0.0, yabserr=0.0, yrelerr=0.0, yabscor=0.0,
yrelcor=0.0, title=None, basename=None, axis_labels=None,
axis_units=None)
```

This helper function creates a *Dataset* from a series of keyword arguments.

Valid keyword arguments are:

- **xdata and ydata** [list/tuple/*np.array* of floats] These keyword arguments are mandatory and should be iterables containing the measurement data.
- **cov\_mats** [None or 2-tuple (optional)] This argument defaults to None, which means no covariance matrices are used. If covariance matrices are needed, a tuple with two entries (the first for x covariance matrices, the second for y) must be passed.

Each element of this tuple may be either None or a NumPy matrix object containing a covariance matrix for the respective axis.

*error specification keywords* [iterable or numeric (see below)] In addition to covariance matrices, errors can be specified for each axis (x or y) according to a simplified error model.

In this respect, a valid keyword is composed of an axis, an error relativity specification (*abs* or *rel*) and error correlation type (*err* or *cor*). The errors are then set as follows:

#### 1. For totally uncorrelated errors (err):

- if keyword argument is iterable, the error list is set to that
- if keyword argument is a number, an error list with identical entries is generated

#### 2. For fully correlated errors (cor):

• keyword argument *must* be a single number. The global correlated error for the axis is then set to that

So, for example:

```
>>> myDataset = build_dataset(..., yabserr=0.3, yrelcor=0.1)
```

creates a Dataset with an uncorrelated error of 0.3 for each y coordinate and a fully correlated (systematic) error of y of 0.1.

title [string (optional)] The title of the Dataset.

**basename** [string or None (optional)] A basename for the *Dataset*. All output files related to this dataset will use this as a basename. If this is None (default), the basename will be inferred from the filename.

axis\_labels [2-tuple of strings (optional)] a 2-tuple containing the axis labels for the *Dataset*. This is relevant when plotting *Fits* of the *Dataset*, but is ignored when plotting more than one *Fit* in the same *Plot*.

*axis\_units* [2-tuple of strings (optional)] a 2-tuple containing the axis units for the *Dataset*. This is relevant when plotting *Fits* of the *Dataset*, but is ignored when plotting more than one *Fit* in the same *Plot*.

### 3.4 file tools Module

```
kafe.file_tools.buildDataset_fromFile(file_to_parse)
```

build a kafe Dataset object from input file with key words and file format defined in  $\texttt{parse\_general\_inputfile}$ 

file\_to\_parse: file-like object or string containing a file path The file to parse.

returns an instance of the Dataset class, constructed with the help of the method
 Dataset.build\_dataset()

```
kafe.file_tools.buildFit_fromFile(file_to_parse)
```

build a kafe Fit object from input file with keywords and file format defined in  $parse\_general\_input\_file$ 

file\_to\_parse: file-like object or string containing a file path The file to parse.

returns an instance of the Fit class, constructed with the help of the methods
 Dataset.build\_dataset() and Fit.build\_fit()

Parses a file which contains measurement data in a one-measurement-per-row format. The field (column) order can be specified. It defaults to x,y'. Valid field names are 'x, y, xabserr, yabserr, xrelerr, yrelerr. Another valid field name is ignore which can be used to skip a field.

A certain type of field can appear several times. If this is the case, all specified errors are added in quadrature:

$$\sigma_{\text{tot}} = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 + \dots}$$

Every valid measurement data file *must* have an x and a y field.

For more complex error models, errors and correlations may be specified as covariance matrices. If this is desired, then any number of covariance matrices (stored in separate files) may be specified for an axis by using the *cov\_mat\_files* argument.

Additionally, a delimiter can be specified. If this is a whitespace character or omitted, any sequence of whitespace characters is assumed to separate the data.

**file to parse** [file-like object or string containing a file path] The file to parse.

**field\_order** [string (optional)] A string of comma-separated field names giving the order of the columns in the file. Defaults to 'x, y'.

delimiter [string (optional)] The field delimiter used in the file. Defaults to any whitespace.

**cov\_mat\_files** [several (see below, optional)] This argument defaults to None, which means no covariance matrices are used. If covariance matrices are needed, a tuple with two entries (the first for x covariance matrices, the second for y) must be passed.

Each element of this tuple may be either None, a file or file-like object, or an iterable containing files and file-like objects. Each file should contain a covariance matrix for the respective axis.

When creating the *Dataset*, all given matrices are summed over.

title [string (optional)] The title of the Dataset.

**basename** [string or None (optional)] A basename for the *Dataset*. All output files related to this dataset will use this as a basename. If this is None (default), the basename will be inferred from the filename.

axis\_labels [2-tuple of strings (optional)] a 2-tuple containing the axis labels for the *Dataset*. This is relevant when plotting *Fits* of the *Dataset*, but is ignored when plotting more than one *Fit* in the same *Plot*.

*axis\_units* [2-tuple of strings (optional)] a 2-tuple containing the axis units for the *Dataset*. This is relevant when plotting *Fits* of the *Dataset*, but is ignored when plotting more than one *Fit* in the same *Plot*.

return [Dataset] A Dataset built from the parsed file.

```
kafe.file_tools.parse_general_inputfile(file_to_parse)
```

This function can be used to specify *kafe* Dataset or Fit objects in a single input file, thus requiring minimal Python code. Keywords as specified in a dictionary tokens specify all objects and parameters needed by the functions build\_dataset() in module dataset and build\_fit() in module fit.

file\_to\_parse: file-like object or string containing a file path The file to parse.

return [dataset\_kwargs, fit\_kwargs] keyword lists to build a kafe Dataset or Fit ' object with
 the helper functions ''build\_dataset or build\_fit

Description of the format of the input file

The interpretation of the input data is driven by keywords. All data following a key must be of the same kind, a block of data ends when a new key is specified.

Some keys only expect a single float or string-tpye value, given on the same line, separated by a space (' '):

```
<key> <value>
```

For multiple input, i.e. data, uncertainties and covariance or correlation matrices, the format is:

```
<key> <xval> <xerr> [<xsyst> <elements of cov/cor matrix>]
...
<xval> <xerr> [<xsyst> <elements of cov/cor matrix>]
```

The field separator is space (' '). Note that the number of input values in each line must correspond to the specified format of the (correlated) ucertainties.

The currently implemented keys are:

•for metadata:

- -\*TITLE <name of the data set>
- -\*BASENAME <name from which output file names are derived>
- -\*xLabel <x axis label>
- -\*yLabel <y axis label>
- -xUnit <x axis unit>
- -yUnit <y axis unit>
- •for input data:
  - -\*xData x data and, optionally, uncertainties

```
<xval> [<x-uncert.>] ...
```

-\*yData y data and uncertainties

```
<yval> <y uncert.> ...
```

•x or y data, independent and correlated uncertainties and elements of correlation matrix, given as as a lower triangular matrix with no diagonal:

```
-*xData_COR
```

-\*yData\_COR

```
<x/y val> <indep. x/y uncert.> <x/y syst> <elements of cor matrix> ...
```

•x or y data, independent and correlated uncertainties and sqrt of elements of covariance matrix, given as as a lower triangular matrix with no diagonal:

```
-*xData SCOV
```

-\*yData\_SCOV

```
<x/y val> <idep. x/y uncert.> <x/y syst> <sqrt of elements of cov matrix> ...
```

•x or y data, independent uncertainties and full covariance matrix (note that the correlated uncertainties are contained in the diagonal of the matrix in this case, i.e. the field <xsyst> is to be omitted):

```
-*xData_COV
```

-\*yData\_COV

```
<x/y val> <indep. x/y ucert.> <elements of cov matrix> ...
```

•Additional keys allow to specify correlated absolute or relative uncertainties:

```
-*xAbsCOR <common abs. x uncert.>
-*yAbsCOR <common abs. y uncert.>
-*xRelCor <common rel. x uncert.>
-*yRelCor <common rel. y uncert.>
```

•To specify the fit function, the defined keywords are:

-\*FitFunction followed by python code (note: blanks for line indent must be replaced by '~'):

```
def fitf(x, ...):
    ~~~return ...
```

The name  $\mathit{fitf}$  is mandatory. The kafe decorator functions <code>@ASCII</code>, <code>@LATEX</code> and <code>@FitFunction</code> are suppoted.

- -\*FITLABEL <the name for the fit>
- -\*InitialParameters followed by two columns of float values for the initial values of the parameters and their range, one line per fit parameter is mandatory

<initial value> <range>

•Model parameters can be constrained within their uncertainties, if prior knowledge on the value(s) and uncertainty(ies) of parameters are to be accounted for in the fit. This option is specified via the keyword:

-\*ConstrainedParameters followed by one or more lines with the fields

<parameter name> <parameter value> <parameter uncert.>,

where *parameter name* is the name of the parameter in the fit function specification.

Here is an example of an input file to calculate the average of correlated measurements:

```
# Meta data for plotting
*TITLE Higgs-mass measurements
*xLabel number of measurement
*yLabel $m_\mathrm{H}$
*yUnit GeV/$c^2$
#*xData # commented out, as not needed for simple average
\star yData\_SCOV # assume that minimum of syst. errors is a common error
# mH
        err
                 syst as sqrt(cov)
124.51
        0.52
                 0.06
125.60
        0.40
                 0.20 0.06
125.98
        0.42
                 0.28 0. 0.
124.70
        0.31
                 0.15 0.
                            0.
                                0.15
# set Python code of fit function
### there are some restrictions:
       function name must be 'fitf'
##
##
       blanks must be replaced by '\,{\scriptstyle^\sim}{}'
# kafe fit function decorators are supported
*FitFunction
@ASCII(expression='av')
@LaTeX(name='f', parameter_names=('av'), expression='av')
@FitFunction
def fitf(x, av=1.): # fit an average
~~~return av
*FITLABEL Average
*InitialParameters
120.1.
```

#### 3.5 fit Module

Object representing a fit. This object references the fitted *Dataset*, the fit function and the resulting fit parameters.

Necessary arguments are a *Dataset* object and a fit function (which should be fitted to the *Dataset*). Optionally, an external function FCN (the minimum of which should be located to find the best fit) can be specified. If not given, the FCN function defaults to  $\chi^2$ .

dataset [Dataset] A Dataset object containing all information about the data

**fit\_function** [function] A user-defined Python function to be fitted to the data. This function's first argument must be the independent variable *x*. All other arguments *must* be named and have default values given. These defaults are used as a starting point for the actual minimization. For example, a simple linear function would be defined like:

```
>>> def linear_2par(x, slope=1., y_intercept=0.):
... return slope * x + y_intercept
```

Be aware that choosing sensible initial values for the parameters is often crucial for a successful fit, particularly for functions of many parameters.

*external\_fcn* [function (optional)] An external *FCN* (function to minimize). This function must have the following call signature:

```
>>> FCN(xdata, ydata, cov_mat, fit_function, parameter_values)
```

It should return a float. If not specified, the default  $\chi^2$  FCN is used. This should be sufficient for most fits.

**fit\_label** [LaTeX-formatted string (optional)] A name/label/short description of the fit function. This appears in the legend describing the fitter curve. If omitted, this defaults to the fit function's LaTeX expression.

```
call_external_fcn (*parameter_values)
```

Wrapper for the external FCN. Since the actual fit process depends on finding the right parameter values and keeping everything else constant we can use the *Dataset* object to pass known, fixed information to the external FCN, varying only the parameter values.

parameter\_values [sequence of values] the parameter values at which FCN is to be evaluated

```
call_minimizer (final_fit=True, verbose=False)
```

Instructs the minimizer to do a minimization.

#### constrain\_parameters (parameters, parvals, parerrs)

Constrain the parameter with the given name to  $c \pm \sigma$ .

This is achieved by adding an appropriate *penalty term* to the  $\chi^2$  function, see function chi2.

parameters list of paramter id's or names to constrain

parvals list of parameter values

parerrs list of errors on parameters

#### current\_cov\_mat = None

the current covariance matrix used for the Fit

#### dataset = None

this Fit instance's child Dataset

```
do_fit (quiet=False, verbose=False)
```

Runs the fit algorithm for this Fit object.

First, the *Dataset* is fitted considering only uncertainties in the y direction. If the *Dataset* has no uncertainties in the y direction, they are assumed to be equal to 1.0 for this preliminary fit, as there is no better information available.

Next, the fit errors in the x direction (if they exist) are taken into account by projecting the covariance matrix for the x errors onto the y covariance matrix. This is done by taking the first derivative of the fit function in each point and "projecting" the x error onto the resulting tangent to the curve.

This last step is repeated until the change in the error matrix caused by the projection becomes negligible.

quiet [boolean (optional)] Set to True if no output should be printed.

verbose [boolean (optional)] Set to True if more output should be printed.

#### external\_fcn = None

the (external) function to be minimized for this Fit

#### fit function = None

the fit function used for this Fit

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#### fix\_parameters (\*parameters\_to\_fix)

Fix the given parameters so that the minimizer works without them when *do\_fit* is called next. Parameters can be given by their names or by their IDs.

#### get\_current\_fit\_function()

This method returns a function object corresponding to the fit function for the current parameter values. The returned function is a function of a single variable.

**returns** [function] A function of a single variable corresponding to the fit function at the current parameter values.

#### get\_error\_matrix()

This method returns the covariance matrix of the fit parameters which is obtained by querying the minimizer object for this *Fit* 

returns [numpy.matrix] The covariance matrix of the parameters.

#### get\_function\_error(x)

This method uses the parameter error matrix of the fit to calculate a symmetric (parabolic) error on the function value itself. Note that this method takes the entire parameter error matrix into account, so that it also accounts for correlations.

The method is useful if, e.g., you want to draw a confidence band around the function in your plot routine.

x [float or sequence of float] the values at which the function error is to be estimated

**returns** [float or sequence of float] the estimated error at the given point(s)

#### get\_parameter\_errors (rounding=False)

Get the current parameter uncertainties from the minimizer.

**rounding** [boolean (optional)] Whether or not to round the returned values to significance.

returns [tuple] A tuple of the parameter uncertainties

#### get\_parameter\_values (rounding=False)

Get the current parameter values from the minimizer.

**rounding** [boolean (optional)] Whether or not to round the returned values to significance.

returns [tuple] A tuple of the parameter values

#### latex\_parameter\_names = None

LaTeX parameter names

#### minimizer = None

this Fit's minimizer (Minuit)

#### number\_of\_parameters = None

the total number of parameters

#### parameter\_names = None

the names of the parameters

#### print fit details()

prints some fit goodness details

#### print\_fit\_results()

prints fit results

#### print\_rounded\_fit\_parameters()

prints the fit parameters

#### project\_x\_covariance\_matrix()

Project elements of the x covariance matrix onto the total matrix.

This is done element-wise, according to the formula:

$$C_{\text{tot},ij} = C_{y,ij} + C_{x,ij} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j}$$

#### release\_parameters (\*parameters\_to\_release)

Release the given parameters so that the minimizer begins to work with them when *do\_fit* is called next. Parameters can be given by their names or by their IDs. If no arguments are provied, then release all parameters.

#### set\_parameters (\*args, \*\*kwargs)

Sets the parameter values (and optionally errors) for this fit. This is usually called just before the fit is done, to establish the initial parameters. If a parameter error is omitted, it is set to 1/1000th of the parameter values themselves. If the default value of the parameter is 0, it is set, by exception, to 0.001.

This method accepts up to two positional arguments and several keyword arguments.

args[0] [tuple/list of floats (optional)] The first positional argument is expected to be a tuple/list containing the parameter values.

args[1] [tuple/list of floats (optional)] The second positional argument is expected to be a tuple/list of parameter errors, which can also be set as an approximate estimate of the problem's uncertainty.

**no\_warning** [boolean (optional)] Whether to issue warnings (False) or not (True) when communicating with the minimizer fails. Defaults to False.

Valid keyword argument names are parameter names. The keyword arguments themselves may be floats (parameter values) or 2-tuples containing the parameter values and the parameter error in that order:

<parameter\_name> [float or 2-tuple of floats (optional)] Set the parameter with the name <'parameter\_name'> to the value given. If a 2-tuple is given, the first element is understood to be the value and the second to be the parameter error.

#### xdata = None

the x coordinates of the data points used for this Fit

#### ydata = None

the y coordinates of the data points used for this Fit

kafe.fit.build\_fit (dataset, fitfunc, fitlabel='untitled', initial\_fit\_parameters=None, constrained parameters=None)

This helper fuction creates a Fit from a series of keyword arguments.

Valid keywords are:

dataset: a kafe Dataset

fitfunc [a python function, eventually with] @FitFunction, @LATEX and @FitFunction decorators

fitlabel: name for this fit

**fitparameters** [None or 2-tuple of list, tuple/np.array of floats] specifying initial parameter values and errors

**constrained\_parameters:** None or 3-tuple of list, tuple/np.array' of one string and 2 floats specifiying the names, values and uncertainties of constraints to apply to model parameters

returns Fit object

kafe.fit.chi2(xdata, ydata, cov\_mat, fit\_function, parameter\_values, constrained\_parameters=None)

The  $\chi^2$  implementation. Calculates  $\chi^2$  according to the formula:

$$\chi^2 = \lambda^T C^{-1} \lambda$$

Here,  $\lambda$  is the residual vector  $\lambda = \vec{y} - \vec{f}(\vec{x})$  and C is the covariance matrix.

3.5. fit Module

If a constraint  $c_i \pm \sigma_i$  is applied to a parameter i, a *penalty term* is added for each constrained parameter according to:

$$\chi^2 + = \left(\frac{v_i - c_i}{\sigma_i}\right)^2$$

**xdata** [iterable] The x measurement data

ydata [iterable] The y measurement data

**cov\_mat** [numpy.matrix] The total covariance matrix

**fit\_function** [function] The fit function f(x)

**constrained\_parameters**: None or list of two iterables with a length equal to the number of parameters. An uncertainty of 0 means that a parameter remains unconstrained.

**parameter\_values** [list/tuple] The values of the parameters at which f(x) should be evaluated.

#### kafe.fit.round\_to\_significance(value, error, significance=2)

Rounds the error to the established number of significant digits, then rounds the value to the same order of magnitude as the error.

value [float] value to round to significance

error [float] uncertainty of the value

significance [int (optional)] number of significant digits of the error to consider

### 3.6 function\_library Module

Collection of model functions

# 3.7 function\_tools Module

kafe.function\_tools.ASCII(\*\*kwargs)

Optional decorator for fit functions. This overrides a FitFunction's plain-text (ASCII) attributes. The new values for these attributes must be passed as keyword arguments to the decorator. Possible arguments:

name [string] Plain-text representation of the function name.

**parameter\_names** [list of strings] List of plain-text representations of the function's arguments. The length of this list must be equal to the function's argument number. The argument names should be in the same order as in the function definition.

**x\_name** [string] Plain-text representation of the independent variable's name.

expression [string] Plain-text-formatted expression representing the function's formula.

 ${f class}$  kafe.function\_tools.FitFunction(f)

Decorator class for fit functions. If a function definition is decorated using this class, some information is collected about the function which is relevant to the fitting process, such as the number of parameters, their names and default values. Some details pertaining to display and representation are also set, such as LaTeX representations of the parameter names and the function name. Other decorators can be applied to a function object to specify things such as a LaTeX or plain-text expression for the fit function.

derive\_by\_parameters (x\_0, precision\_spec, parameter\_list)

Returns the gradient of *func* with respect to its parameters, i.e. with respect to every variable of *func* except the first one.

precision\_spec [float or iterable of floats] An array of floats indicating the initial point spacing for numerically evaluating the derivative. Can be a single float value to use the same spacing for every derivation.

#### derive\_by\_x (x\_0, precision\_list, parameter\_list)

If  $x\_0$  is iterable, gives the array of derivatives of a function  $f(x, par_1, par_2, ...)$  around  $x = x_i$  at every  $x_i$  in  $\vec{x}$ . If  $x\_0$  is not iterable, gives the derivative of a function  $f(x, par_1, par_2, ...)$  around  $x = x\_0$ .

#### evaluate (x\_0, parameter\_list)

Evaluate the fit function at an x-value or at an array of x-values for the parameter values in *prarameter list*.

x 0 float or array of floats

parameter\_list values of function parameters

returns function value(s)

#### expression = None

a math expression (string) representing the function's result

#### get\_function\_equation (equation\_format='latex', equation\_type='full', ensuremath=True)

Returns a string representing the function equation. Supported formats are LaTeX and ASCII inline math. Note that LaTeX math is wrapped by default in an \ensuremath{} expression. If this is not desired behaviour, the flag ensuremath can be set to False.

equation\_format [string (optional)] Can be either "latex" (default) or "ascii".

*equation\_type* [string (optional)] Can be either "full" (default), "short" or "name". A "name"-type equation returns a representation of the function name:

f

A "short"-type equation limits itself to the function name and variables:

```
f(x, par1, par2)
```

A "full"-type equation includes the expression which the function calculates:

```
f(x, par1, par2) = par1 * x + par2
```

**ensuremath** [boolean (optional)] If a LaTeX math equation is requested, True (default) will wrap the resulting expression in an  $\ensuremath{}$  tag. Otherwise, no wrapping is done.

#### latex expression = None

a LaTeX math expression, the function's result

#### latex\_name = None

The function's name in LaTeX

#### latex\_parameter\_names = None

A list of parameter names in LaTeX

#### latex x name = None

A LaTeX symbol for the independent variable.

#### name = None

The name of the function

#### number\_of\_parameters = None

The number of parameters

#### parameter\_defaults = None

The default values of the parameters

#### parameter\_names = None

The names of the parameters

#### $x_name = None$

The name given to the independent variable

```
kafe.function_tools.LaTeX(**kwargs)
```

Optional decorator for fit functions. This overrides a FitFunction's *latex\_* attributes. The new values for the *latex\_* attributes must be passed as keyword arguments to the decorator. Possible arguments:

**name** [string] LaTeX representation of the function name.

**parameter\_names** [list of strings] List of LaTeX representations of the function's arguments. The length of this list must be equal to the function's argument number. The argument names should be in the same order as in the function definition.

 $x_name$  [string] LaTeX representation of the independent variable's name.

*expression* [string] LaTeX-formatted expression representing the function's formula.

kafe.function\_tools.**derivative** (func, derive\_by\_index, variables\_tuple, derivative\_spacing) Gives  $\frac{\partial f}{\partial x_k}$  for  $f = f(x_0, x_1, \ldots)$ . func is f, variables\_tuple is  $\{x_i\}$  and derive\_by\_index is k.

kafe.function\_tools.outer\_product(input\_array)

Takes a *NumPy* array and returns the outer (dyadic, Kronecker) product with itself. If  $input\_array$  is a vector  $\mathbf{x}$ , this returns  $\mathbf{x}\mathbf{x}^T$ .

#### 3.8 minuit Module

kafe.minuit.D\_MATRIX\_ERROR = {0: 'Error matrix not calculated', 1: 'Error matrix approximate!', 2: 'Error matrix Error matrix status codes

**FCN\_wrapper** (number\_of\_parameters, derivatives, f, parameters, internal\_flag)

This is actually a function called in *ROOT* and acting as a C wrapper for our *FCN*, which is implemented in Python.

This function is called by *Minuit* several times during a fit. It doesn't return anything but modifies one of its arguments (f). This is ugly, but it's how ROOT's TMinuit works. Its argument structure is fixed and determined by Minuit:

**number\_of\_parameters** [int] The number of parameters of the current fit

**derivatives** [C array] If the user chooses to calculate the first derivative of the function inside the *FCN*, this value should be written here. This interface to *Minuit* ignores this derivative, however, so calculating this inside the *FCN* has no effect (yet).

**f** [C array] The desired function value is in f[0] after execution.

parameters [C array] A C array of parameters. Is cast to a Python list

**internal\_flag** [int] A flag allowing for different behaviour of the function. Can be any integer from 1 (initial run) to 4(normal run). See *Minuit*'s specification.

fix\_parameter (parameter\_number)

Fix parameter number rameter\_number>.

**parameter\_number** [int] Number of the parameter to fix.

function to minimize = None

the actual FCN called in FCN\_wrapper

get\_chi2\_probability(n\_deg\_of\_freedom)

Returns the probability that an observed  $\chi^2$  exceeds the calculated value of  $\chi^2$  for this fit by chance, even for a correct model. In other words, returns the probability that a worse fit of the model to the data exists. If this is a small value (typically <5%), this means the fit is pretty bad. For values below this threshold, the model very probably does not fit the data.

**n\_def\_of\_freedom** [int] The number of degrees of freedom. This is typically  $n_ext data points - n_ext parameters$ .

#### get\_contour (parameter1, parameter2, n\_points=20)

Returns a list of points (2-tuples) representing a sampling of the  $1\sigma$  contour of the TMinuit fit. The FCN has to be minimized before calling this.

**parameter1** [int] ID of the parameter to be displayed on the *x*-axis.

parameter2 [int] ID of the parameter to be displayed on the y-axis.

**n\_points** [int (optional)] number of points used to draw the contour. Default is 20.

**returns** [2-tuple of tuples] a 2-tuple (x, y) containing n\_points+1 points sampled along the contour. The first point is repeated at the end of the list to generate a closed contour.

#### get\_error\_matrix()

Retrieves the parameter error matrix from TMinuit.

return: numpy.matrix

#### get\_fit\_info(info)

Retrieves other info from Minuit.

info [string] Information about the fit to retrieve. This can be any of the following:

- 'fcn': FCN value at minimum,
- 'edm': estimated distance to minimum
- 'err\_def': Minuit error matrix status code
- 'status\_code': Minuit general status code

#### get\_parameter\_errors()

Retrieves the parameter errors from TMinuit.

return [tuple] Current Minuit parameter errors

#### get\_parameter\_info()

Retrieves parameter information from TMinuit.

```
return [list of tuples] (parameter_name, parameter_val, parameter_error)
```

#### get\_parameter\_name (parameter\_nr)

Gets the name of parameter number parameter\_nr

parameter\_nr [int] Number of the parameter whose name to get.

#### get\_parameter\_values()

Retrieves the parameter values from TMinuit.

return [tuple] Current Minuit parameter values

#### $max\_iterations = None$

maximum number of iterations until TMinuit gives up

#### minimize (final\_fit=True, log\_print\_level=2)

Do the minimization. This calls *Minuit*'s algorithms MIGRAD for minimization and, if *final\_fit* is *True*, also HESSE for computing/checking the parameter error matrix.

#### number\_of\_parameters = None

number of parameters to minimize for

#### release\_parameter (parameter\_number)

Release parameter number <parameter\_number>.

parameter\_number [int] Number of the parameter to release.

#### reset()

Execute TMinuit's mnrset method.

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#### set\_err (up\_value=1.0)

Sets the UP value for Minuit.

*up\_value* [float (optional, default: 1.0)] This is the value by which *FCN* is expected to change.

#### set\_parameter\_errors (parameter\_errors=None)

Sets the fit parameter errors. If parameter\_values='None', sets the error to 10% of the parameter value.

#### set\_parameter\_names (parameter\_names)

Sets the fit parameters. If parameter\_values='None', tries to infer defaults from the function\_to\_minimize.

#### set\_parameter\_values (parameter\_values)

Sets the fit parameters. If parameter\_values='None', tries to infer defaults from the function\_to\_minimize.

#### set\_print\_level (print\_level=1)

Sets the print level for Minuit.

print\_level [int (optional, default: 1 (frugal output))] Tells TMinuit how much output to generate.
The higher this value, the more output it generates.

#### set\_strategy (strategy\_id=1)

Sets the strategy Minuit.

*strategy\_id* [int (optional, default: 1 (optimized))] Tells TMinuit to use a certain strategy. Refer to TMinuit's documentation for available strategies.

#### tolerance = None

TMinuit tolerance

#### update\_parameter\_data(show\_warnings=False)

(Re-)Sets the parameter names, values and step size on the C++ side of Minuit.

#### kafe.minuit.P\_DETAIL\_LEVEL = 1

default level of detail for TMinuit's output (typical range: -1 to 3, default: 1)

#### 3.9 constants Module

# 3.10 plot Module

```
class kafe.plot.Plot (*fits, **kwargs)
```

Bases: object

The constuctor accepts a series of *Fit* objects as positional arguments. Some keyword arguments can be provided to override the defaults.

#### axis\_labels = None

axis labels

#### compute plot range(include error bars=True)

Compute the span of all child datasets and sets the plot range to that

#### draw\_fit\_parameters\_box (plot\_spec=0)

Draw the parameter box to the canvas

plot\_spec [int, list of ints, string or None (optional, default: 0)] Specify the plot id of the plot for which to draw the parameters. Passing 0 will only draw the parameter box for the first plot, and so on. Passing a list of ints will only draw the parameters for plot ids inside the list. Passing 'all' will print parameters for all plots. Passing None will return immediately doing nothing.

#### draw\_legend()

Draw the plot legend to the canvas

```
extend_span (axis, new_span)
Expand the span of the current plot.

This method extends the current plot span to include new_span
```

#### fits = None

list of 'Fit's to plot

#### init\_plots()

Initialize the plots for each fit.

#### on draw(event)

Function to call when a draw event occurs.

#### plot (p id, show data=True, show function=True)

Plot the *Fit* object with the number  $p_{id}$  to its figure.

plot\_all (show\_info\_for='all', show\_data\_for='all', show\_function\_for='all')
 Plot every Fit object to its figure.

#### plot\_range = None

plot range

#### plot\_style = None

plot style

#### save (output file)

Save the *Plot* to a file.

#### show()

Show the *Plot* in a matplotlib interactive window.

#### show legend = None

whether to show the plot legend (True) or not (False)

#### class kafe.plot.PlotStyle

Class for specifying a style for a specific plot. This object stores a progression of marker and line types and colors, as well as preferences relating to point size and label size. These can be overriden by overwriting the instance variables directly. A series of *get\_...* methods are provided which go through these lists cyclically.

#### get\_line(idm)

Get a specific line type. This runs cyclically through the defined defaults.

#### get\_linecolor(idm)

Get a specific line color. This runs cyclically through the defined defaults.

#### get\_marker(idm)

Get a specific marker type. This runs cyclically through the defined defaults.

#### get\_markercolor(idm)

Get a specific marker color. This runs cyclically through the defined defaults.

#### get pointsize(idm)

Get a specific point size. This runs cyclically through the defined defaults.

#### kafe.plot.label\_to\_latex(label)

Generates a simple LaTeX-formatted label from a plain-text label. This treats isolated characters and words beginning with a backslash as mathematical expressions and surround them with \$ signs accordingly.

**label** [string] Plain-text string to convert to LaTeX.

#### kafe.plot.pad\_span (span, pad\_coeff=1, additional\_pad=None)

Enlarges the interval *span* (list of two floats) symmetrically around its center to length *pad\_coeff*. Optionally, an *additional\_pad* argument can be specified. The returned span is then additionally enlarged by that amount.

additional\_pad can also be a list of two floats which specifies an asymmetric amount by which to enlarge the span. Note that in this case, positive entries in additional\_pad will enlarge the span (move the interval

3.10. plot Module 35

end away from the interval center) and negative amounts will shorten it (move the interval end towards the interval center).

### 3.11 latex\_tools Module

kafe.latex\_tools.ascii\_to\_latex\_math(str\_ascii, monospace=True, ensuremath=True)

Escapes certain characters in an ASCII input string so that the result can be included in math mode without error.

**str\_ascii** [string] A plain-text string containing characters to be escaped for LaTeX math mode.

monospace [boolean (optional)] Whether to render the whole expression as monospace. Defaults to True.

ensuremath [boolean (optional)] If this is True, the resulting formula is wrapped in an \ensuremath{} ensuremath{} { } }
tag. Defaults to True.

### 3.12 numeric\_tools Module

kafe.numeric\_tools.MinuitCov\_to\_cor(cov\_mat)

Converts a covariance matrix as returned by Minuit to the corresponding correlation matrix; note that the Minuit covariance matrix may contain lines/rows with zeroes if parameters are fixed

**cov\_mat** [numpy.matrix] The Minuit covariance matrix to convert.

kafe.numeric\_tools.cor\_to\_cov(cor\_mat, error\_list)

Converts a correlation matrix to a covariance matrix according to the formula

$$Cov_{ij} = Cor_{ij} \sigma_i \sigma_j$$

cor\_mat [numpy.matrix] The correlation matrix to convert.

**error\_list** [sequence of floats] A sequence of statistical errors. Must be of the same length as the diagonal of *cor\_mat*.

kafe.numeric\_tools.cov\_to\_cor(cov\_mat)

Converts a covariance matrix to a correlation matrix according to the formula

$$Cor_{ij} = \frac{Cov_{ij}}{\sqrt{Cov_{ii} Cov_{jj}}}$$

**cov mat** [numpy.matrix] The covariance matrix to convert.

kafe.numeric\_tools.extract\_statistical\_errors(cov\_mat)

Extracts the statistical errors from a covariance matrix. This means it returns the (elementwise) square root of the diagonal entries

cov\_mat The covariance matrix to extract errors from. Type: numpy.matrix

kafe.numeric\_tools.make\_symmetric\_lower(mat)

Copies the matrix entries below the main diagonal to the upper triangle half of the matrix. Leaves the diagonal unchanged. Returns a *NumPy* matrix object.

mat [numpy.matrix] A lower diagonal matrix.

**returns** [numpy.matrix] The lower triangle matrix.

kafe.numeric\_tools.zero\_pad\_lower\_triangle(triangle\_list)

Converts a list of lists into a lower triangle matrix. The list members should be lists of increasing length from 1 to N, N being the dimension of the resulting lower triangle matrix. Returns a *NumPy* matrix object.

For example:

triangle\_list [list] A list containing lists of increasing length.

**returns** [numpy.matrix] The lower triangle matrix.

#### 3.13 stream Module

```
class kafe.stream.StreamDup(out_file, suppress_stdout=False)
    Bases: object
```

Object for simultaneous logging to stdout and files. This object provides a file/like object for the outout to be written to. Writing to this object will write to stdout (usually the console) and to a file.

**out\_file** [file path or file-like object or list of file paths ...] File(s) to which to log the output, along with stdout. If a file exists on disk, it will be appended to.

```
fileno()
```

Returns the file handler id of the main (first) output file.

```
flush()
write(message)
write_timestamp(prefix)
write_to_file(message)
write_to_stdout(message, check_if_suppressed=False)
```

Explicitly write to stdout. This method will not check by default whether  $suppress\_stdout$  is set for this StreamDup. If  $check\_if\_suppressed$  is explicitly set to True, then this check occurs.

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