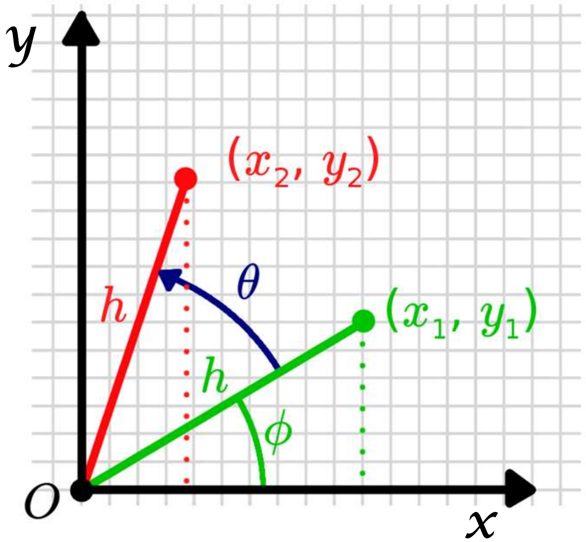
Why is a Matrix4 used in Transform widget ?

```
Transform(
  alignment: Alignment.center,
  transform: Matrix4.rotationZ(angleZ),
  child: Transform(
      alignment: Alignment.center,
      transform: Matrix4.rotationY(angleY),
      child: Transform(
        alignment: Alignment.center,
        transform: Matrix4.rotationX(angleX),
        child: ClipRRect(
            borderRadius:
                const BorderRadius.all(Radius.circular(20)),
            child: Image.asset(
                "assets/images/snoopy_laptop.jpg",
                width: 230)), // Image.asset // ClipRRect
```

Rotation in two dimensions



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} h\cos(\phi) \\ h\sin(\phi) \end{bmatrix}$$

Additionstheoreme Trigonometrie

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} h\cos(\phi+\theta) \\ h\sin(\phi+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} h\cos(\phi)\cos(\theta) - h\sin(\phi)\sin(\theta) \\ h\sin(\phi)\cos(\theta) + h\cos(\phi)\sin(\theta) \end{bmatrix}$$

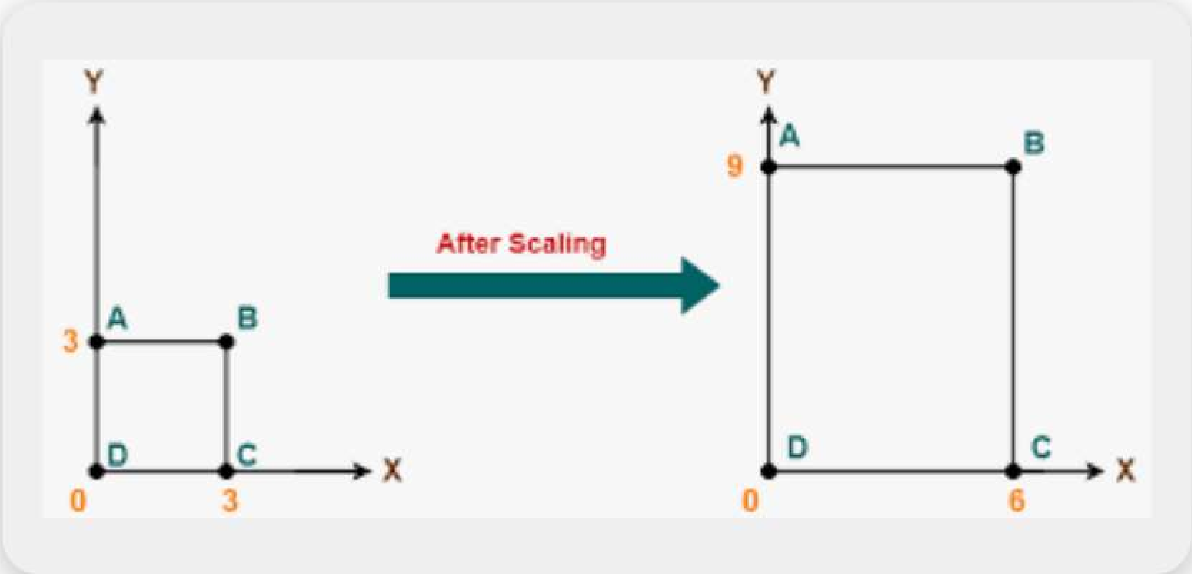
$$= \begin{bmatrix} x_1\cos(\theta) - y_1\sin(\theta) \\ x_1\sin(\theta) + y_1\cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_{11}b_1 + a_{12}b_2 + \dots \; a_{1n}b_n \\ a_{21}b_1 + a_{22}b_2 + \dots \; a_{2n}b_n \\ \vdots \\ a_{m1}b_1 + a_{m2}b_2 + \dots \; a_{mn}b_n \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}}$$

Copied from https://articulatedrobotics.xyz/tutorials/coordinate-transforms/rotation-matrices-2d/
and https://www.geeksforgeeks.org/parallel-matrix-vector-multiplication-in-numpy/

Scaling in two dimensions



$x' = 2 * x \qquad y' = 3 * y$

In general:

$x' = sx * x \qquad y' = sy * y$

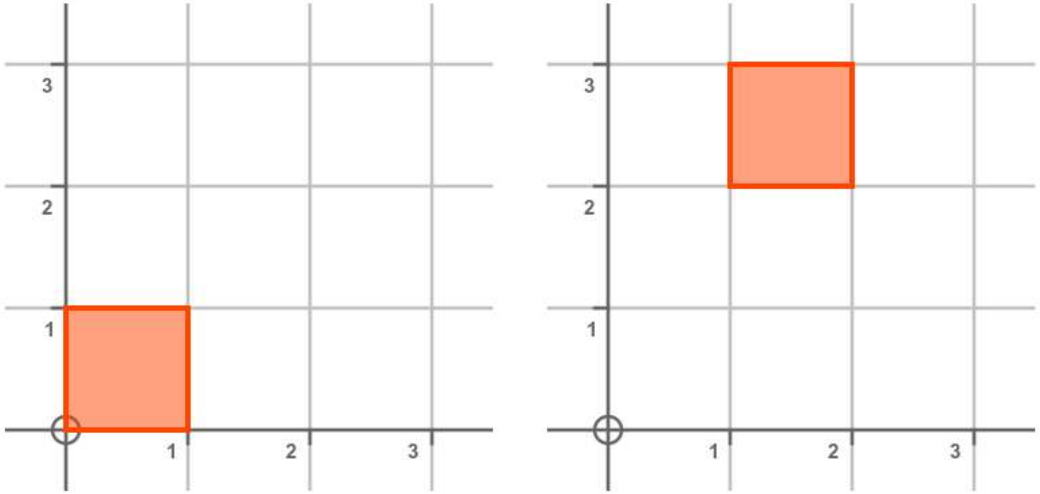
Matrix Representation of Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Copied from https://bcalabs.org/subject/2d-transformation-in-computer-graphics and https://www.gatevidyalay.com/scaling-in-computer-graphics-definition-examples/

Translation

Translating a shape means moving it to a different position, without changing its shape or orientation in any way. This diagram illustrates a translation by (1, 2). The square moves by 1 unit in the x-direction and 2 units in the y-direction:



We can represent this by adding 2 vectors, the original position *(x, y)* and a displacement vector *(u, v)*:

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x+u \\ y+v \end{pmatrix}$$

Copied from https://bcalabs.org/subject/2d-transformation-in-computer-graphics

Translation using matrix multiplication

But this isn't ideal. In all the other cases, we used matrix multiplication to represent the transform. It is a bit inconvenient to have to use a different calculation for translation. And in a future article, we will see that we can combine multiple transforms into a single matrix. To do that, we need a consistent way to represent all transformations.

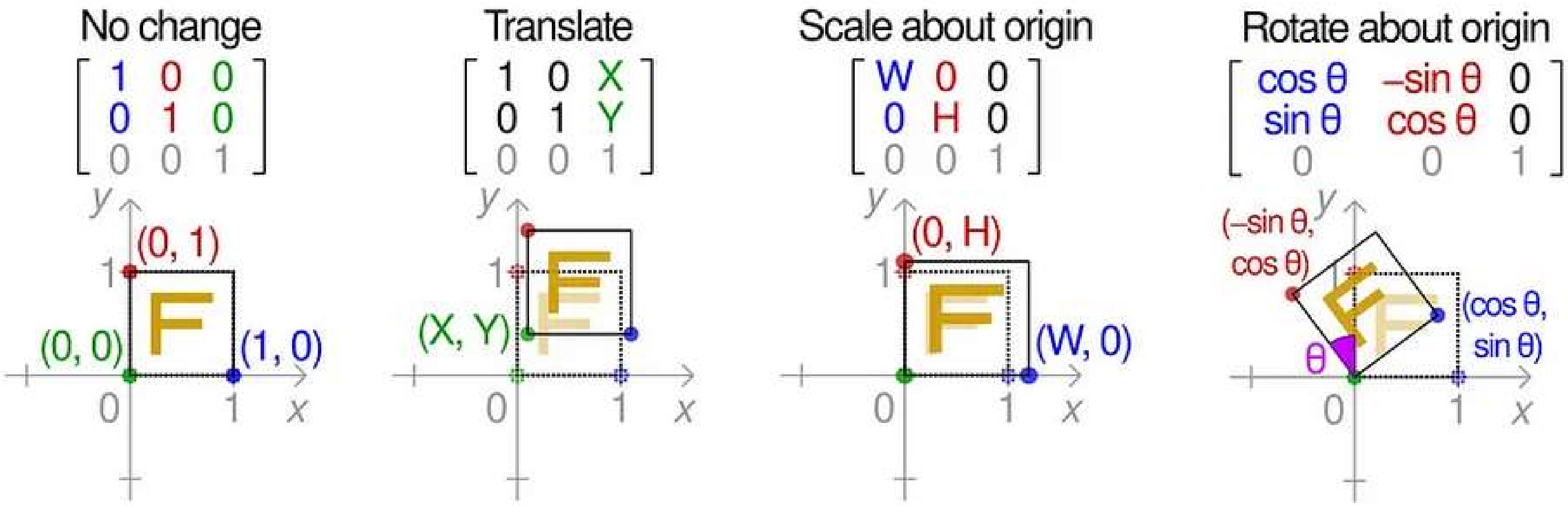
It turns out that we can do this by using a 3 by 3 matrix. We extend our transformation matrix by adding an extra column to the right, containing the transformation values *u* and *v*. To keep the matrix square, we add an extra row that always contains 0, 0, 1:

We also need to extend our position vector to be 3 elements long. We do this by adding an extra row element that is always equal to 1.

$$\begin{pmatrix} 1 & 0 & u \\ 0 & 1 & v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + u \\ y + v \\ 1 \end{pmatrix}$$

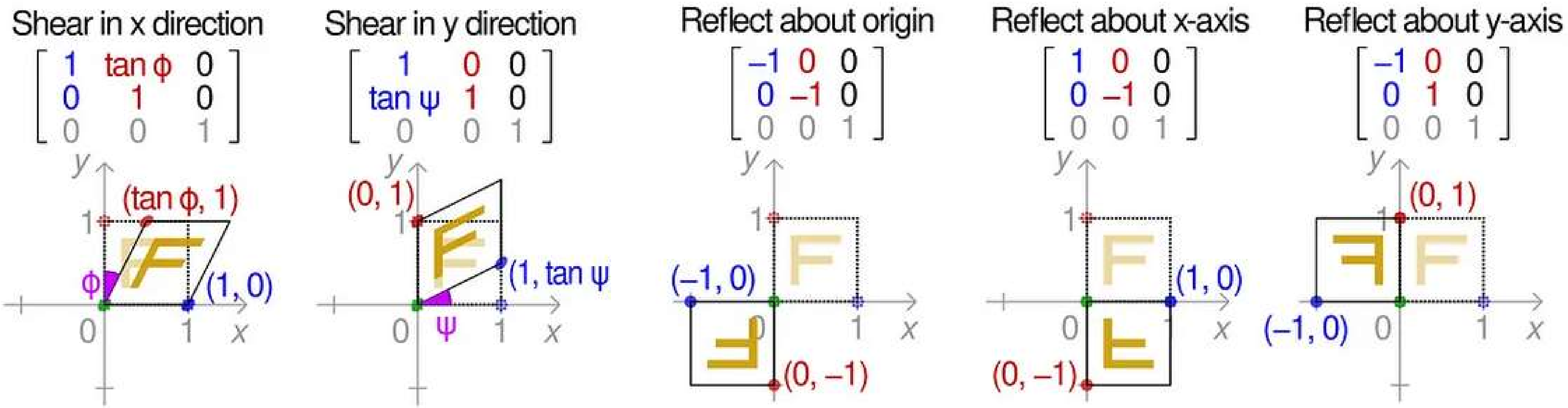
Copied from https://bcalabs.org/subject/2d-transformation-in-computer-graphics

2D Transformations with 3x3 Matrices



Copied from https://medium.com/@junfeng142857/affine-transformation-why-3d-matrix-for-a-2d-transformation-8922b08bce75

2D Transformations with 3x3 Matrices



Copied from https://medium.com/@junfeng142857/affine-transformation-why-3d-matrix-for-a-2d-transformation-8922b08bce75

3D Transformations with 4x4 Matrices

➢ **Translation Transformation-**

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_x \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

➢ **Scaling Transformation-**

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about X-

$$y' = y\cos\gamma - z\sin\gamma$$
$$z' = y\sin\gamma + z\cos\gamma$$
$$x' = x$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Y-

$$z' = z\cos\beta - x\sin\beta$$
$$x' = z\sin\beta + x\cos\beta$$
$$y' = y$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Z-

$$x' = x\cos\alpha - y\sin\alpha$$
$$y' = x\sin\alpha + y\cos\alpha$$
$$z' = z$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Copied from https://medium.com/@junfeng142857/affine-transformation-why-3d-matrix-for-a-2d-transformation-8922b08bce75

Implementation in Flutter

Rotation about X-

$$y' = y\cos\gamma - z\sin\gamma$$
$$z' = y\sin\gamma + z\cos\gamma$$
$$x' = x$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
/// Rotation of [radians] around X.
factory Matrix4.rotationX(double radians) => Matrix4.zero()
  .._m4storage[15] = 1.0
  ..setRotationX(radians);
```

```
/// Sets the upper 3x3 to a rotation of [radians] around X
void setRotationX(double radians) {
  final c = math.cos(radians);
  final s = math.sin(radians);
  _m4storage[0] = 1.0;
  _m4storage[1] = 0.0;
  _m4storage[2] = 0.0;
  _m4storage[4] = 0.0;
  _m4storage[5] = c;
  _m4storage[6] = s;
  _m4storage[8] = 0.0;
  _m4storage[9] = -s;
  _m4storage[10] = c;
  _m4storage[3] = 0.0;
  _m4storage[7] = 0.0;
  _m4storage[11] = 0.0;
}
```

Combined transformations

➢ **Translation Transformation-**

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_x \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about X-

$$y' = y\cos\gamma - z\sin\gamma$$
$$z' = y\sin\gamma + z\cos\gamma$$
$$x' = x$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Y-

$$z' = z\cos\beta - x\sin\beta$$
$$x' = z\sin\beta + x\cos\beta$$
$$y' = y$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_x \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Copied from https://medium.com/@junfeng142857/affine-transformation-why-3d-matrix-for-a-2d-transformation-8922b08bce75

Matrix Multiplication is not commutative

Let A and B be matrices. Then in general,

$A \times B \neq B \times A$. (not commutative.)

$$\mathbf{A} \cdot \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\mathbf{B} \cdot \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

Copied from https://pvgisours.tistory.com/44 and https://studyflix.de/mathematik/matrizen-multiplizieren-1521