

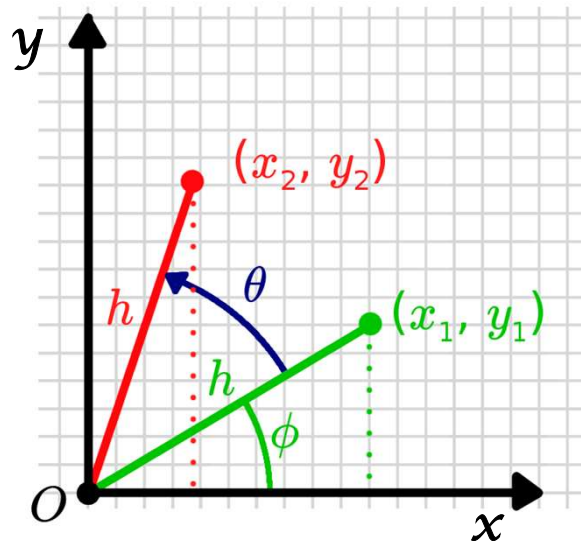


Why is a Matrix4 used in Transform widget ?

```
Transform(  
  alignment: Alignment.center,  
  transform: Matrix4.rotationZ(angleZ),  
  child: Transform(  
    alignment: Alignment.center,  
    transform: Matrix4.rotationY(angleY),  
    child: Transform(  
      alignment: Alignment.center,  
      transform: Matrix4.rotationX(angleX),  
      child: ClipRRect(  
        borderRadius:  
          const BorderRadius.all(Radius.circular(20)),  
        child: Image.asset(  
          "assets/images/snoopy_laptop.jpg",  
          width: 230)), // Image.asset // ClipRRect  
      )  
    )  
  )  
)
```



Rotation in two dimensions



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} h \cos(\phi) \\ h \sin(\phi) \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} h \cos(\phi + \theta) \\ h \sin(\phi + \theta) \end{bmatrix}$$

Additionstheoreme
Trigonometrie

$$= \begin{bmatrix} h \cos(\phi) \cos(\theta) - h \sin(\phi) \sin(\theta) \\ h \sin(\phi) \cos(\theta) + h \cos(\phi) \sin(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} x_1 \cos(\theta) - y_1 \sin(\theta) \\ x_1 \sin(\theta) + y_1 \cos(\theta) \end{bmatrix}$$

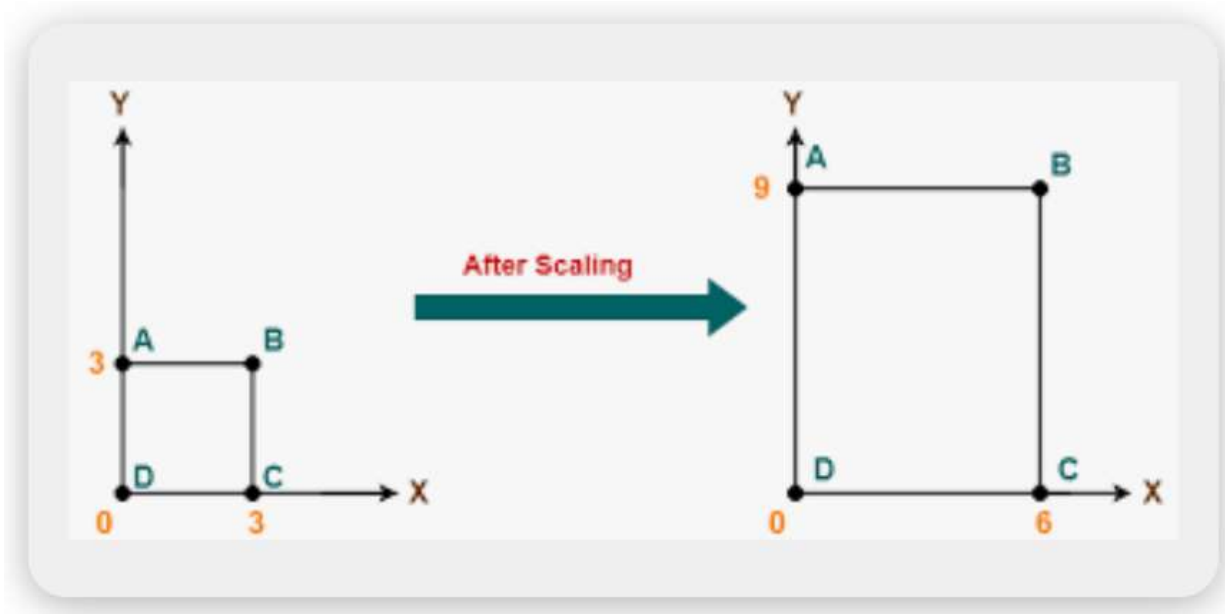
$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_{11}b_1 + a_{12}b_2 + \dots + a_{1n}b_n \\ a_{21}b_1 + a_{22}b_2 + \dots + a_{2n}b_n \\ \vdots \\ a_{m1}b_1 + a_{m2}b_2 + \dots + a_{mn}b_n \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Copied from <https://articulatedrobotics.xyz/tutorials/coordinate-transforms/rotation-matrices-2d/>
and <https://www.geeksforgeeks.org/parallel-matrix-vector-multiplication-in-numpy/>



Scaling in two dimensions



$$x' = 2 * x \quad y' = 3 * y$$

In general:

$$x' = s_x * x \quad y' = s_y * y$$

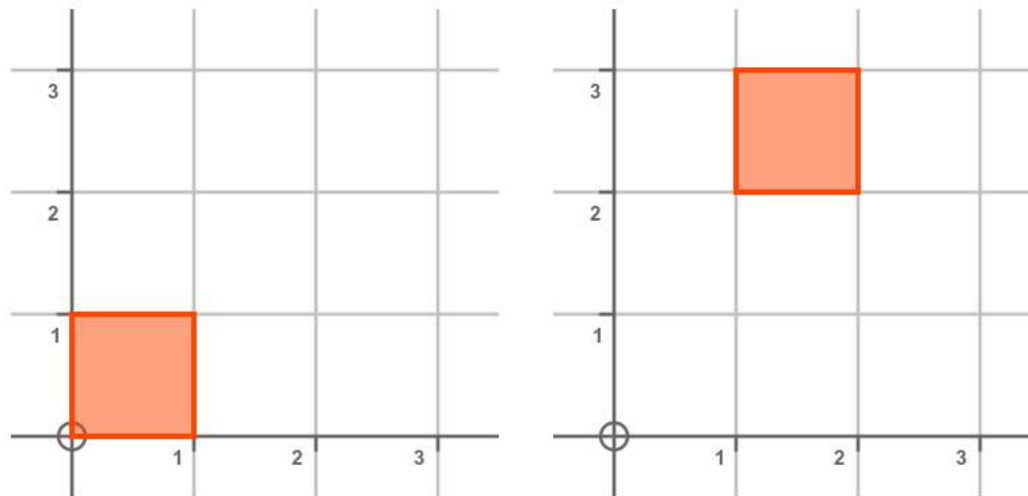
Matrix Representation of Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$



Translation

Translating a shape means moving it to a different position, without changing its shape or orientation in any way. This diagram illustrates a translation by $(1, 2)$. The square moves by 1 unit in the x-direction and 2 units in the y-direction:



We can represent this by adding 2 vectors, the original position (x, y) and a displacement vector (u, v) :

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x + u \\ y + v \end{pmatrix}$$



Translation using matrix multiplication

But this isn't ideal. In all the other cases, we used matrix multiplication to represent the transform. It is a bit inconvenient to have to use a different calculation for translation. And in a future article, we will see that we can combine multiple transforms into a single matrix. To do that, we need a consistent way to represent all transformations.

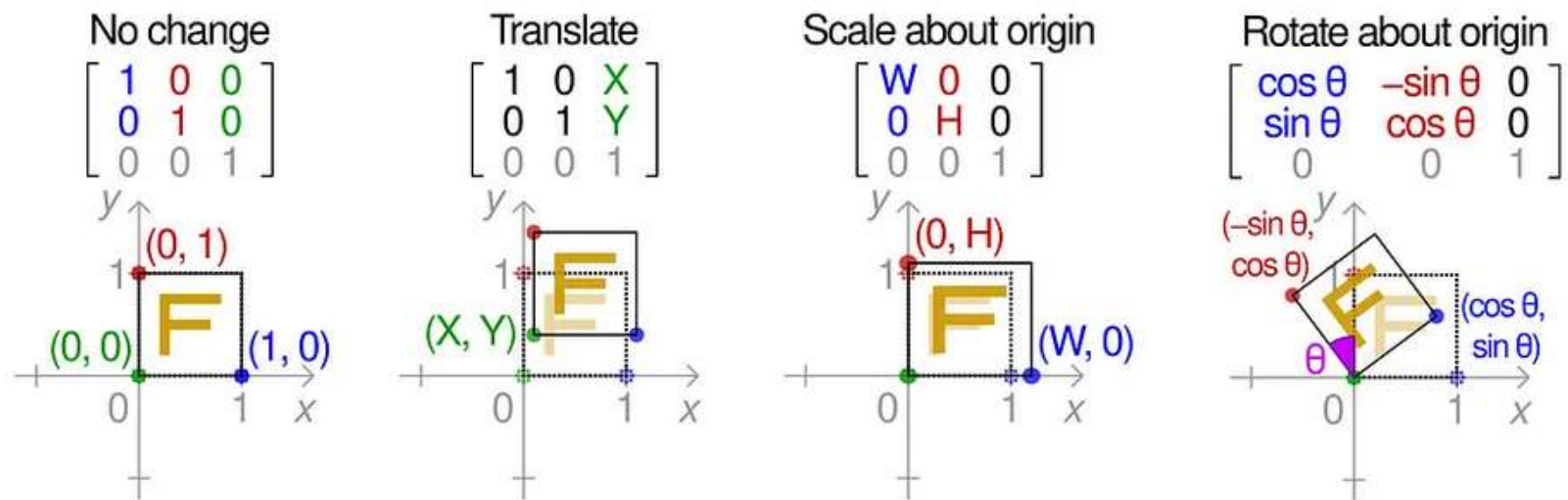
It turns out that we can do this by using a 3 by 3 matrix. We extend our transformation matrix by adding an extra column to the right, containing the transformation values u and v . To keep the matrix square, we add an extra row that always contains 0, 0, 1:

We also need to extend our position vector to be 3 elements long. We do this by adding an extra row element that is always equal to 1.

$$\begin{pmatrix} 1 & 0 & u \\ 0 & 1 & v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + u \\ y + v \\ 1 \end{pmatrix}$$



2D Transformations with 3x3 Matrices

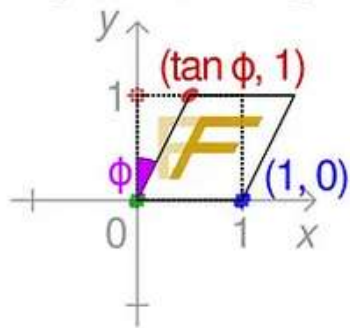




2D Transformations with 3x3 Matrices

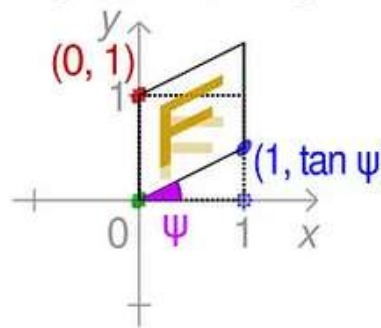
Shear in x direction

$$\begin{bmatrix} 1 & \tan \phi & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



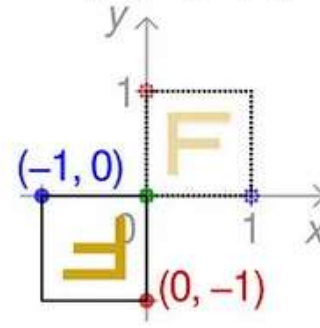
Shear in y direction

$$\begin{bmatrix} 1 & 0 & 0 \\ \tan \psi & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



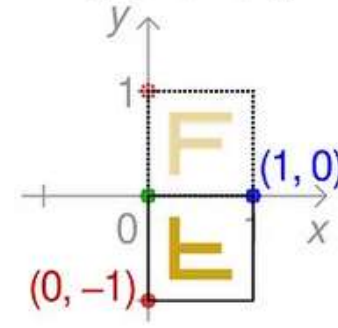
Reflect about origin

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



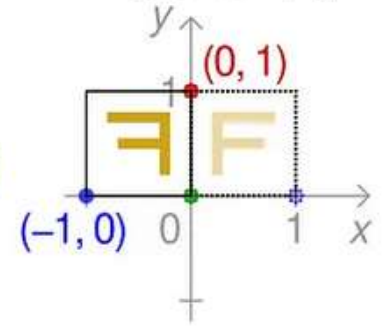
Reflect about x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflect about y-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





3D Transformations with 4x4 Matrices

➤ Translation Transformation-

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

➤ Scaling Transformation-

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about X-

$$\begin{aligned} y' &= y \cos \gamma - z \sin \gamma \\ z' &= y \sin \gamma + z \cos \gamma \\ x' &= x \end{aligned}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma & 0 \\ 0 & \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Y-

$$\begin{aligned} z' &= z \cos \beta - x \sin \beta \\ x' &= z \sin \beta + x \cos \beta \\ y' &= y \end{aligned}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \beta & 0 & \sin \beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation about Z-

$$\begin{aligned} x' &= x \cos \alpha - y \sin \alpha \\ y' &= x \sin \alpha + y \cos \alpha \\ z' &= z \end{aligned}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$



Implementation in Flutter

Rotation about X-

$$y' = y \cos \gamma - z \sin \gamma$$

$$z' = y \sin \gamma + z \cos \gamma$$

$$x' = x$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma & 0 \\ 0 & \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
/// Rotation of [radians] around X.  
factory Matrix4.rotationX(double radians) => Matrix4.zero()  
  .._m4storage[15] = 1.0  
  ..setRotationX(radians);
```

```
/// Sets the upper 3x3 to a rotation of [radians] around X  
void setRotationX(double radians) {  
  final c = math.cos(radians);  
  final s = math.sin(radians);  
  _m4storage[0] = 1.0;  
  _m4storage[1] = 0.0;  
  _m4storage[2] = 0.0;  
  _m4storage[4] = 0.0;  
  _m4storage[5] = c;  
  _m4storage[6] = s;  
  _m4storage[8] = 0.0;  
  _m4storage[9] = -s;  
  _m4storage[10] = c;  
  _m4storage[3] = 0.0;  
  _m4storage[7] = 0.0;  
  _m4storage[11] = 0.0;  
}
```