Gráfico de superfícies cônicas e quádricas

em 20-05

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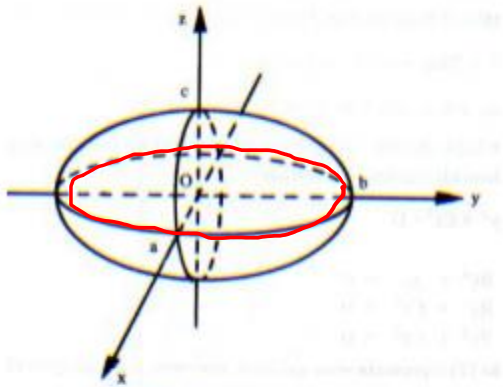
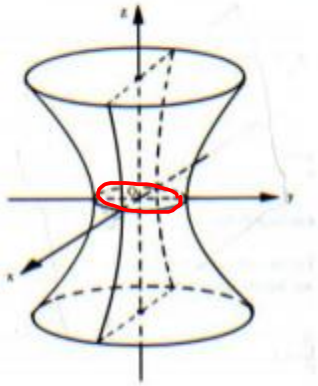
FURB

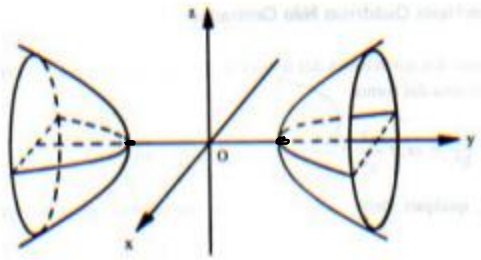
Superfícies Quádricas Centradas



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

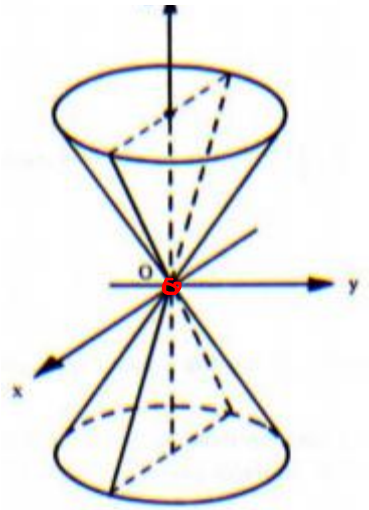


$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$



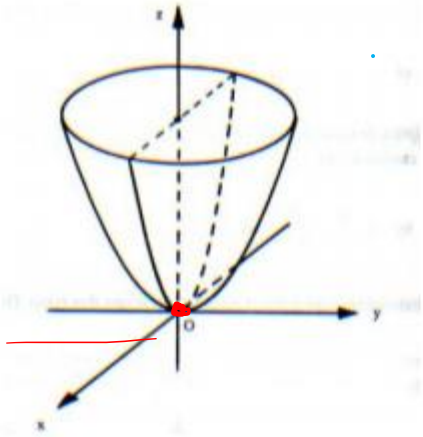
$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

superfície cônica

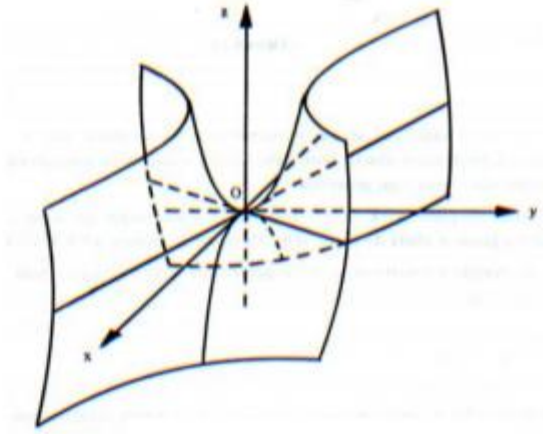


$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

Superfícies Quádricas Não Centradas



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=cz$$



$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=cz$$

Obs.: Para graficar as quádricas centradas e não centradas faremos um estudo de seus traços.

O traço de uma superfície é a curva obtida da intersecção de uma superfície com um dos planos coordenados. Logo, as superfícies quádricas terão 3 traços:

- **traço no plano xy ou z = 0**
- **traço no plano xz ou y = 0**
- **traço no plano yz ou z = 0**

Exercícios

b) $2x^2 + 4y^2 + z^2 - 16 = 0$ elipsóide

c) $x^2 - 4y^2 + 2z^2 = 8$

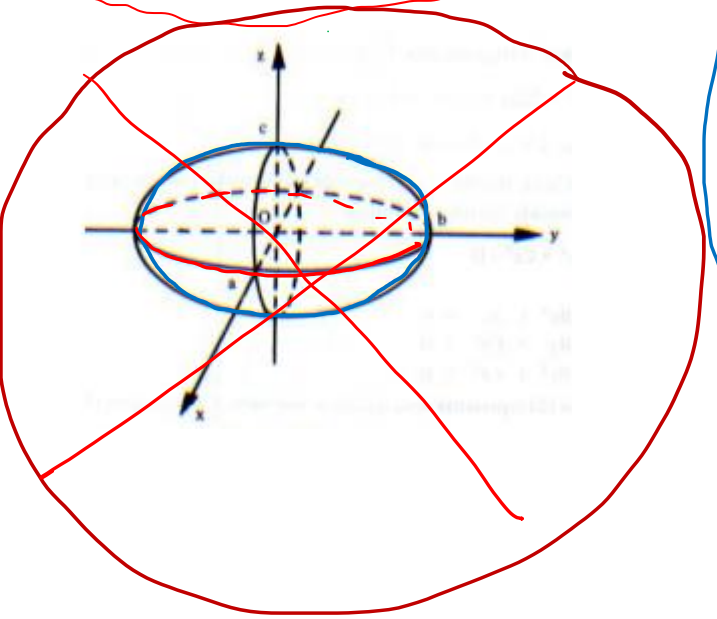
d) $z^2 - 4x^2 - 4y^2 = 4$

e) $x^2 + z^2 - 4y = 0$

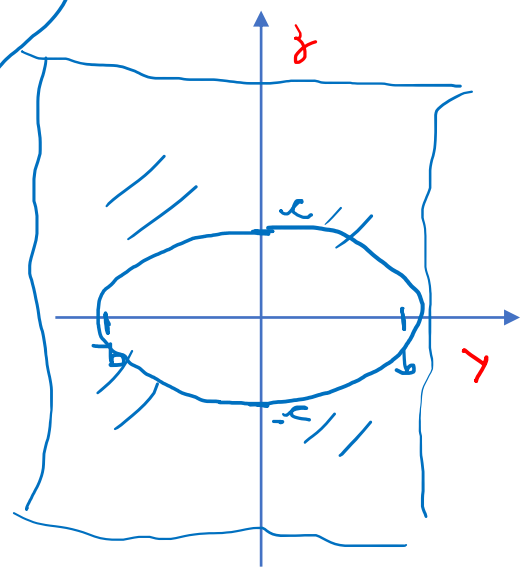
r) $2y^2 + 3z^2 - x^2 = 0$

O que são traços de uma superfície?

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



Traço no plano yz

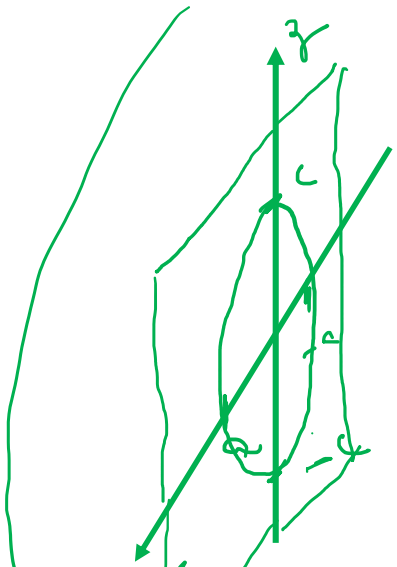


$x=0$

é uma elipse

$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Traço no plano xz

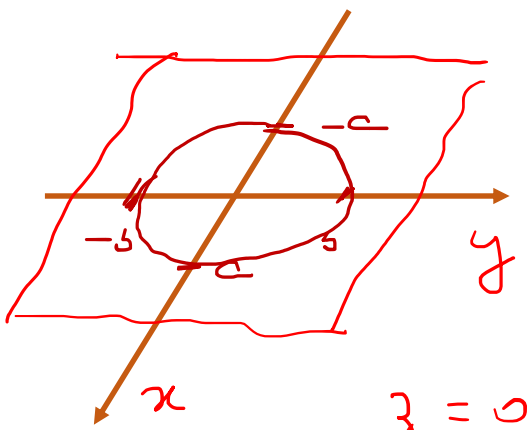


$y=0$

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$$

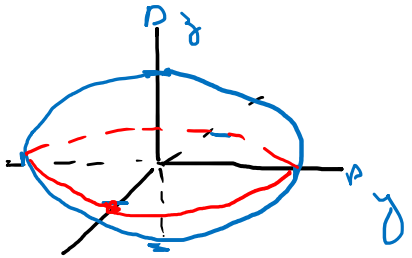
é uma elipse

Traço no plano xy

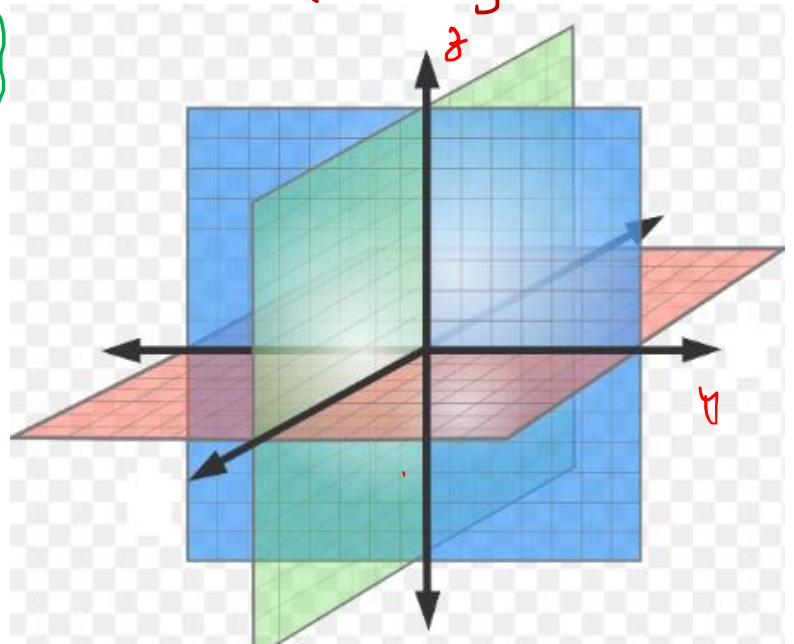


$z=0$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



Elipsóide em $\mathbb{R}^3$



d) $z^2 - 4x^2 - 4y^2 = 4$

e) $x^2 + z^2 - 4y = 0$

Tarefa

Estudo das traços e a representação em $\mathbb{R}^3$

d) $z^2 - 4x^2 - 4y^2 = 4$ ÷ 4

$\frac{z^2}{4} - x^2 - y^2 = 1$

Traço no plano $xy \Rightarrow z = 0$

$\frac{0^2}{4} - x^2 - y^2 = 1$

$-x^2 - y^2 = 1$ (-1)

$x^2 + y^2 = -1$

Não traço no plano xy

a parábola não toca o plano xy.

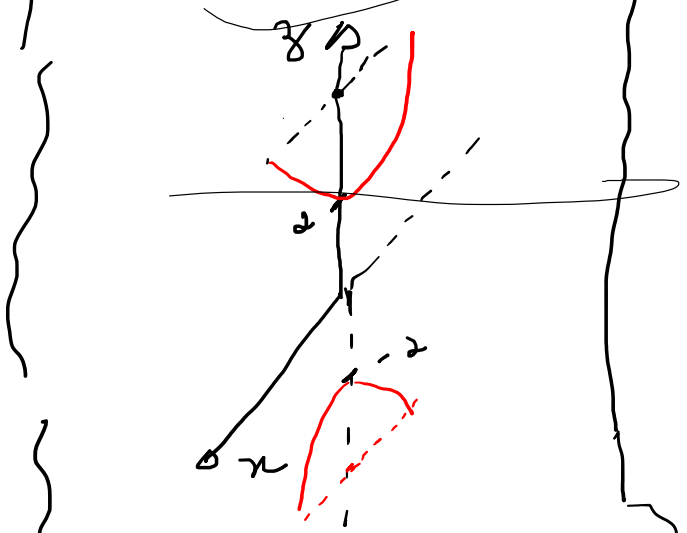
Traço no plano $xz \Rightarrow y = 0$

$\frac{z^2}{4} - x^2 - 0^2 = 1$

$\frac{z^2}{4} - x^2 = 1$ $a^2 = 4$, $a = \pm 2$

hipérbole (0,0) x z

toca o eixo z.



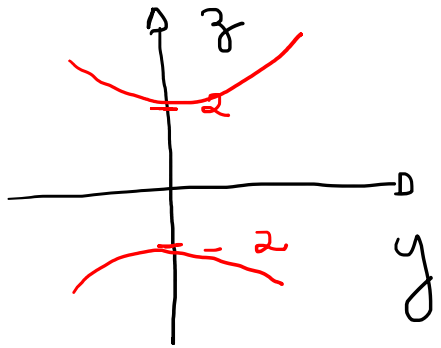
traço no plano xz é uma hipérbole

Traço no plano $yz \Rightarrow x = 0$

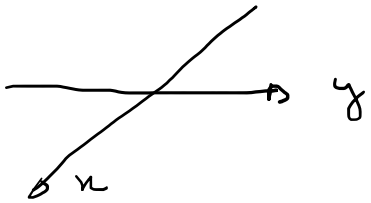
$\frac{z^2}{4} - y^2 = 1$

hipérbole C(0,0) $a^2 = 4$, $a = \pm 2$ eixo z

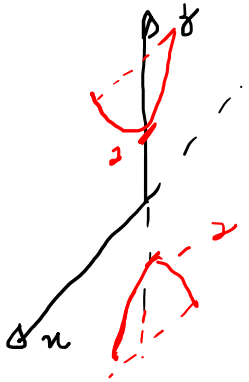
toca o eixo z.



traço no plano yz é uma hipérbole

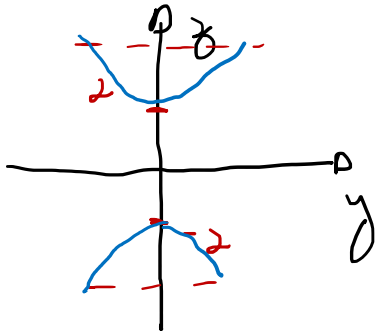


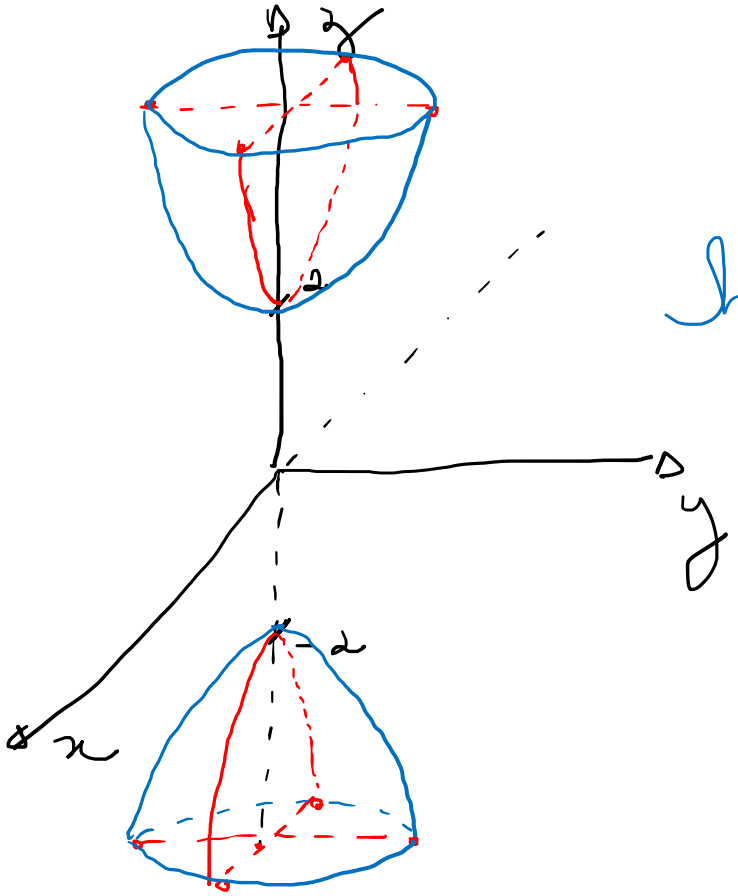
$\neq$ Foco



traço é uma hipérbole

$\Rightarrow$ $\mathbb{R}^3$





hiperboloide de duas folhas

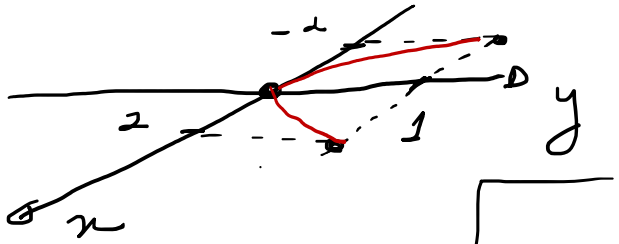
e) $x^2 + z^2 - 4y = 0$

Traço no plano $xy \Rightarrow z = 0$

$x^2 - 4y = 0$

$y = \frac{x^2}{4}$

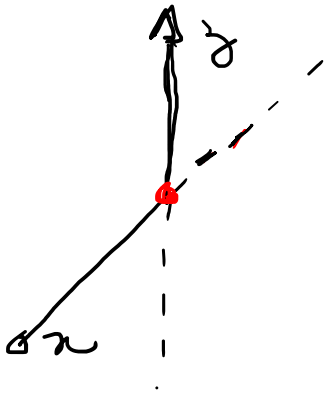
x	y
0	0
2	1
-2	1



Traço no plano xy é uma parábola

Traço no plano $yz \Rightarrow x = 0$

$x^2 + z^2 = 0 \Rightarrow$ 1 pto



Traço no plano xz é um pto

Traço no plano $xz \Rightarrow y = 0$

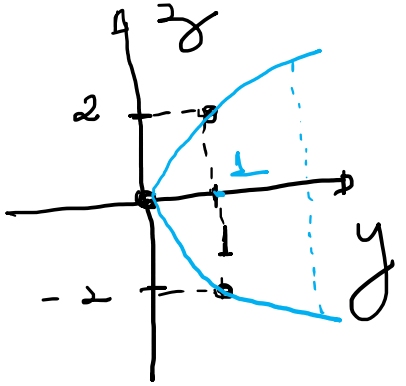
$x^2 + z^2 = 0 \Rightarrow$ 1 pto

Traço no plano $yz \Rightarrow x = 0$

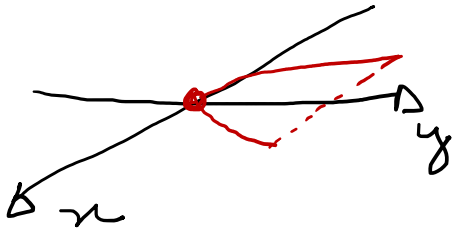
$z^2 - 4y = 0$

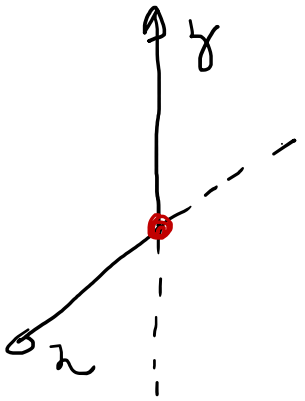
$y = \frac{z^2}{4}$ parábola

z	y
0	0
2	1
-2	1

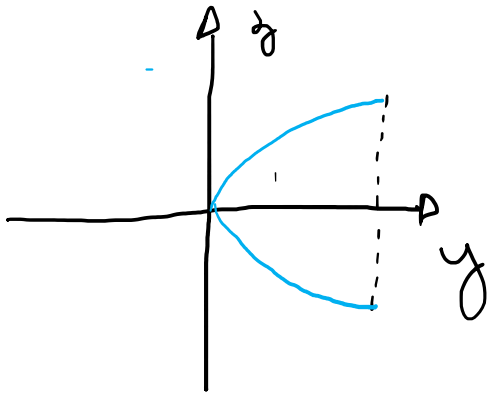


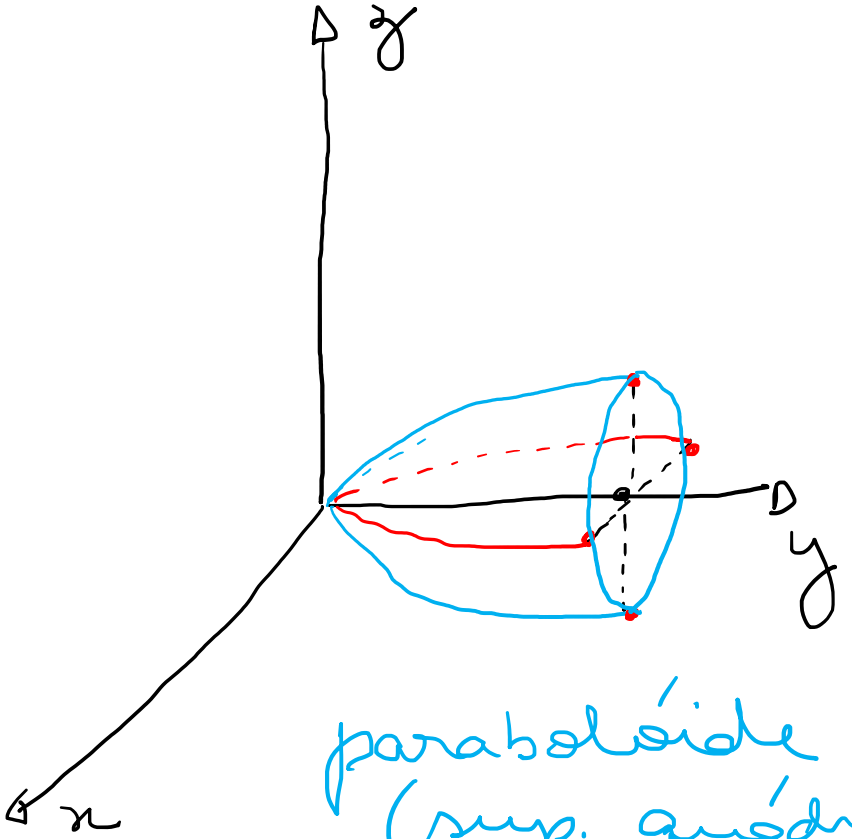
Traço no plano yz é uma parábola





$\mathbb{R}^3$





paraboloide
(sup. quádrica
não centrada)

EXERCÍCIOS:

1) Identificar a superfície e fazer a sua representação gráfica.

a) $x^2 + y^2 = 9$

b) $x^2 = 4y$

c) $x = 4$

d) $2x + 3y - 6 = 0$

e) $y = 6$

f) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

g) $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{16} = 1$

h) $x^2 + y^2 + z^2 = 16$

i) $4x + 2y + 3z - 12 = 0$

j) $y^2 - x^2 + z^2 = 0$

l) $4x^2 + 9y^2 - z = 0$

m) $\frac{x^2}{1} - \frac{y^2}{4} - \frac{z^2}{4} = 1$

n) $\frac{y^2}{4} + x^2 - \frac{z^2}{9} = 1$

2) Identificar as quádricas representadas pelas equações e fazer a representação gráfica:

a) $x^2 + y^2 + z^2 = 25$

b) $2x^2 + 4y^2 + z^2 - 16 = 0$

c) $x^2 - 4y^2 + 2z^2 = 8$

d) $z^2 - 4x^2 - 4y^2 = 4$

e) $x^2 + z^2 - 4y = 0$

f) $x^2 + y^2 + 4z = 0$

g) $4x^2 - y^2 = z$

h) $z^2 = x^2 + y^2$

i) $z = x^2 + y^2$

j) $x^2 + y^2 = 9$

l) $y^2 = 4z$

m) $x^2 - 4y^2 = 16$

n) $4y^2 + z^2 - 4x = 0$

o) $-x^2 + 4y^2 + z^2 = 0$

p) $16x^2 + 9y^2 - z^2 = 144$

q) $16x^2 - 9y^2 - z^2 = 144$

r) $2y^2 + 3z^2 - x^2 = 0$

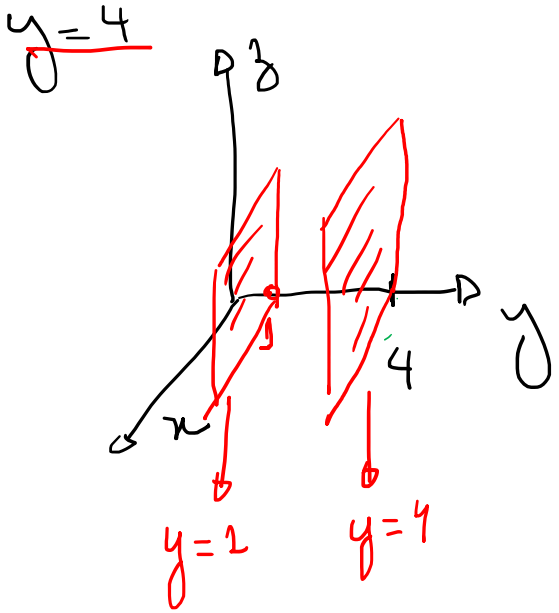
s) $4x^2 + 9y^2 = 36z$

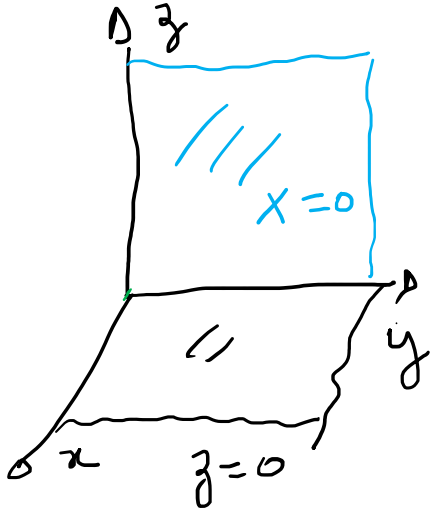
Faça a representação gráfica do figura tridimensional limitada pelas seguintes superfícies:

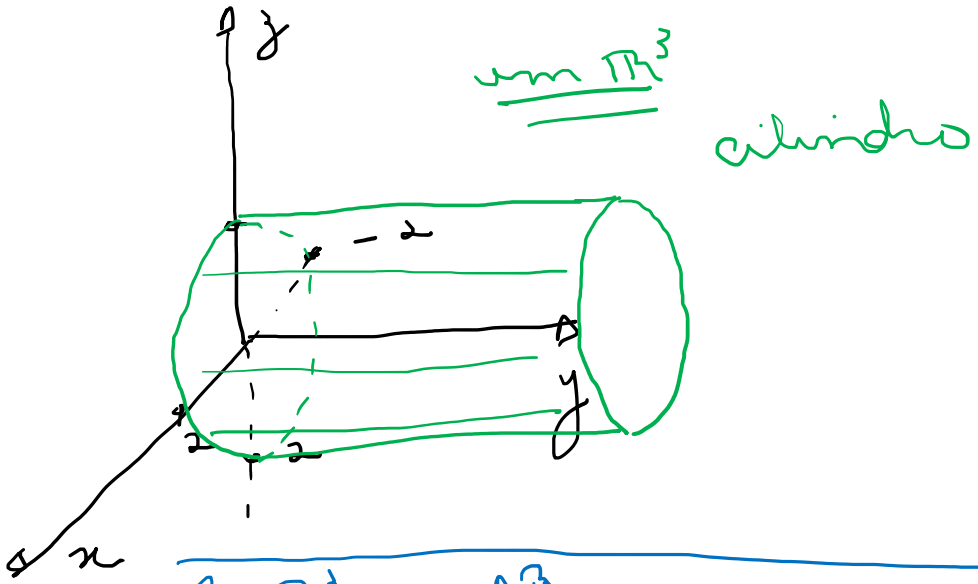
a) $x^2 + z^2 = 4$, $y = 4$, $x = 0$ e $z = 0$.

$y = 1$

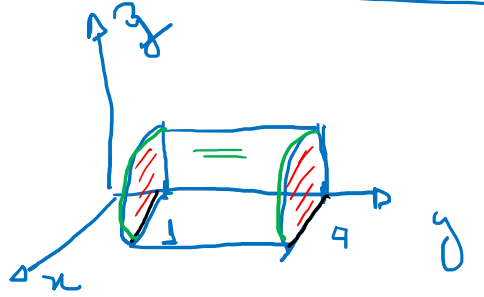
$x^2 + z^2 = 4$ no plano xz circunf.



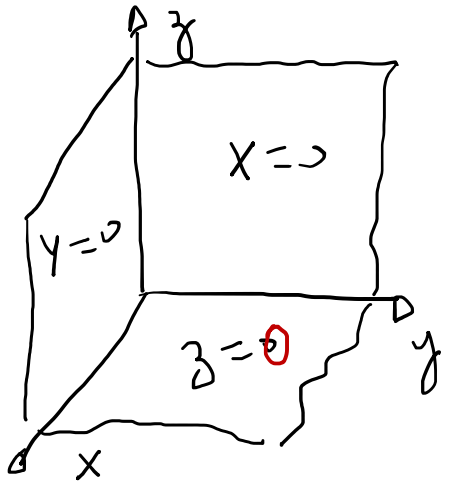


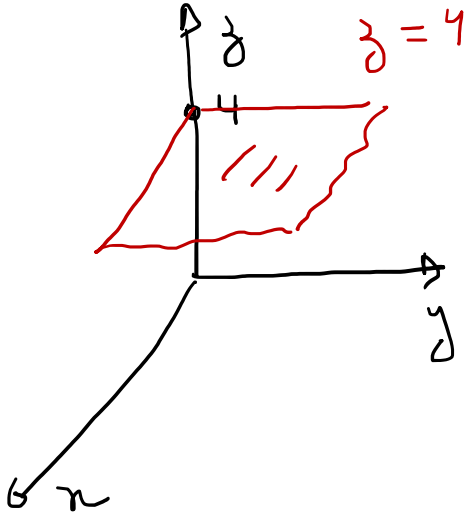


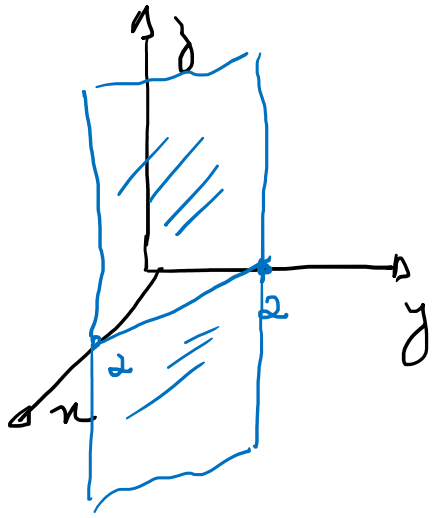
em $\mathbb{R}^3$



b) $x+y-2=0$, $z=0$, $z=4$, $x=0$ e $y=0$







$x+y-2=0$

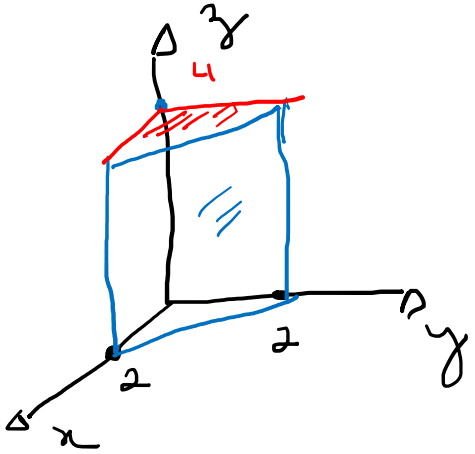
em $\mathbb{R}^3$

plano.

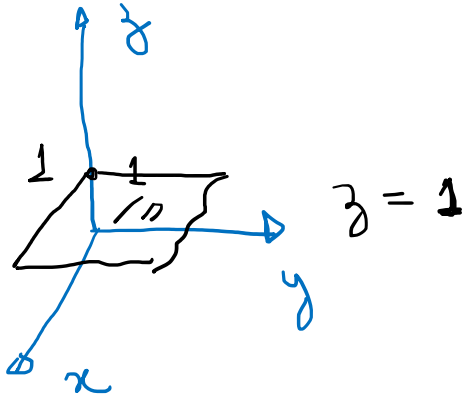
$P_y\ (0,2,0)$

$P_x\ (2,0,0)$

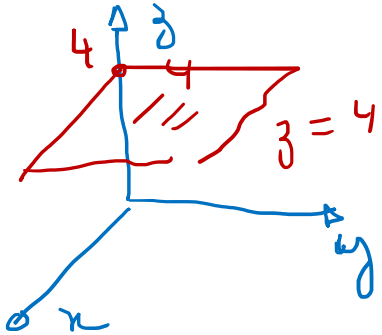
figura Tridimensional resultante de 1 dos superfícies

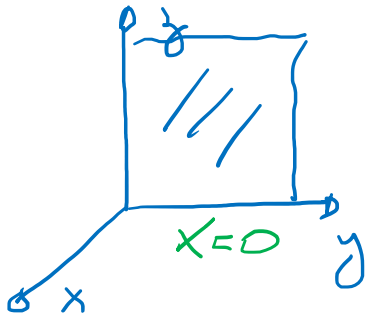


c) $x^2 + y^2 - z^2 = 0$, $z = 1$, $z = 4$ e $x = 0$.



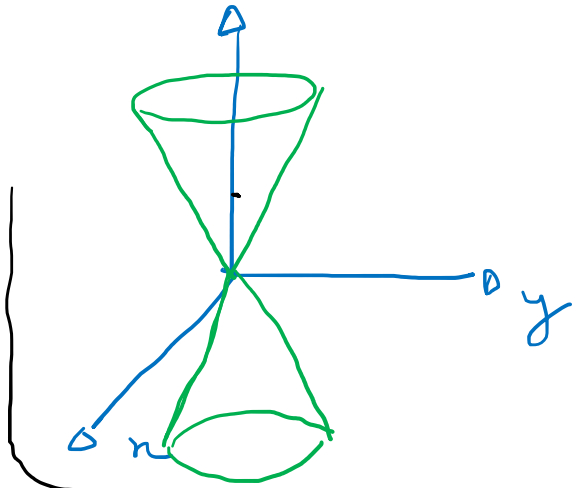
$z = 1$



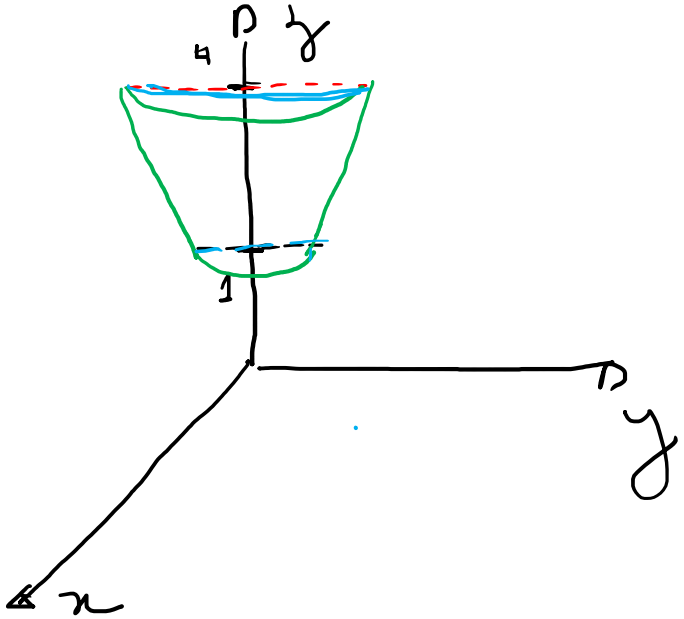


$x^2 + y^2 - z^2 = 0$

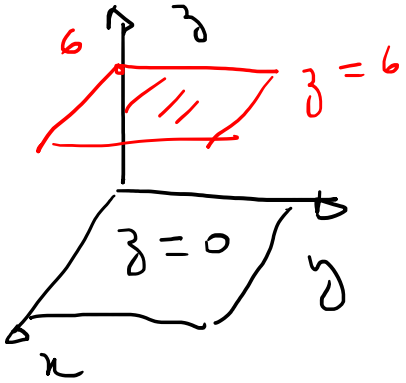
Cone

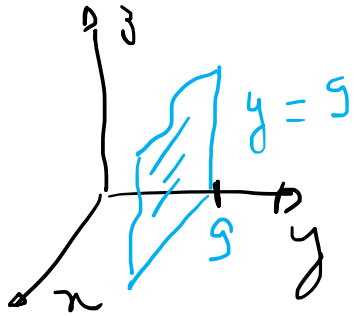


Represente da Fig. Tridimensional resultante da ∩ das sup.



d) $y = x^2$, $z = 0$, $z = 6$ e $y = 9$.

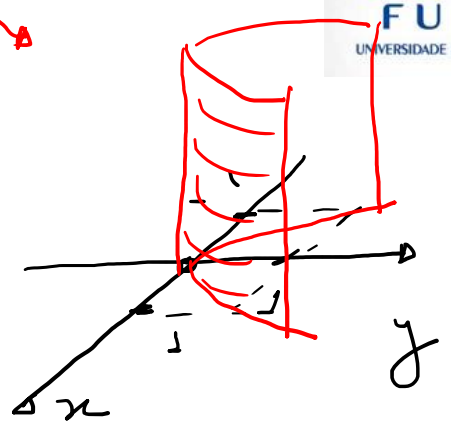




$y = x^2$

x	y
0	0
1	1
-1	1

falta z



Em $\mathbb{R}^3$ $\Rightarrow$ Π dos sup.