Diquark Model Implementation in FeynRules

Flynn Linton

Supervised by Prof. Andreas Crivellin

Fall Semester 2023

1 Introduction

Searches for physics beyond the Standard Model (SM) are necessary to uncover solutions to long-standing problems in modern physics, such as the flavour puzzle and electroweak heirachy problem. Scalar diquarks, a coloured scalar field coupled to two quarks, are of particular interest to flavour physics as they allow for flavour non-diagonal couplings, hence contributing to flavour-changing processes. This motivates diquarks to be candidates for explaining discrepancies between $b \to sl^+l^-$ data and SM predictions [1] and the asymmetry observed in top quark production [2], for instance. Given the lack of evidence for scalar diquarks at or below the electroweak scale ($m_Z \approx 91\,\text{GeV}$) at present colliders, we consider the possibility of diquarks with mass much greater than m_Z. This allows them to be integrated out in an effective field theory (EFT) at energy scales we can probe experimentally, giving rise to effective vertices proportional to the diquark mass and couplings which are constrained by measurements.

In this work, we minimally extend the Standard Model by considering all scalar fields that couple to two quarks such that the UV Lagrangian transforms as a singlet under $SU(3)_c \times SU(2)_L \times U(1)_Y$. The Feynman rules will then be extracted by implementing the model in the FeynRules Mathematica package [3]. In a continuation of this work, we would then integrate out the diquarks at the scale of the Standard Model and perform one-loop matching to the Standard Model effective field theory (SMEFT) to determine their low-energy effects:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{SM} + \sum_{d>4} \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \tag{1}$$

where d is the mass dimension of the Wilson operator $\mathcal{O}_i^{(d)}$ with Wilson coefficient $C_i^{(d)}$ in the operator product expansion, n_d is the multiplicity of operators of dimension d and we choose the matching scale Λ to be the mass of the diquark of interest. It will also be useful to perform matching to the low-energy effective field theory (LEFT) for calculating diquark contributions in B-physics. Cross-sections and decay widths can then be computed within SMEFT (or LEFT) to identify evidence of new heavy physics.

2 Constructing diquarks interactions

Within the Standard Model Lagrangian, we have the left-handed quark doublet Q_L^i and right-handed singlets u_R^i and d_R^i and their charge conjugates $q^c = i\sigma_2 q^*$ to couple to diquark fields Φ, where i runs over the three flavours. The representations each field transforms under in the Standard Model gauge group are given in Table 1. To preserve symmetry under the Lorentz group $SO(1,3)$ we must couple left-handed fermionic fields with right-handed ones.

	$SO(1,3)$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	$(1/2,0)$	3	2	$+1/6$
u_R	$(0,1/2)$	3	1	$+2/3$
d_R	$(0,1/2)$	3	1	$-1/3$

Table 1: Representations that quark fields transform under with respect to the Lorentz group and the Standard Model gauge group.

To construct colour and weak isospin singlets we must decompose tensor products of irreducible $SU(N)$ representations. These are just the representations which the two quark fields and one diquark field transform under. For $SU(3)_c$, the following products of three representations produce a singlet:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$
$$3 \otimes 3 \otimes \bar{6} = 27 \oplus \bar{10} \oplus 8 \oplus 8 \oplus 1$$
$$\bar{3} \otimes 3 \otimes 8 = 27 \oplus \bar{10} \oplus 8 \oplus 10 \oplus 1$$

For $SU(2)_L$:

$$2 \otimes 2 \otimes 1 = 3 \oplus 1$$
$$2 \otimes 2 \otimes 3 = 5 \oplus 3 \oplus 3 \oplus 1$$
$$\bar{2} \otimes 1 \otimes 2 = 3 \oplus 1$$
$$1 \otimes 1 \otimes 1 = 1$$

The possible interaction terms and the representations they transform under that form an $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet are given in Table 2a. Note that the charge conjugation operator also flips the hypercharge of the field. This gives rise to nine different diquark fields, given in Table 2b. It should be noted that the Clebsch-Gordan coefficient connecting two $SU(2)_L$ doublets is the antisymmetric matrix $\epsilon = i\sigma^2$, which transforms the doublet to its conjugate representation to form isospin singlets (and therefore conserving electric charge at the vertex) in the broken phase. Hence, the left-handed quark doublets $\bar{Q}_L^c$ and Q_L transform in the $\bar{2} \otimes 2$ representation when their $SU(2)_L$ indices are contracted.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$		
$\bar{Q}_L^c Q_L \Phi$	$3 \otimes 3 \otimes 3, \bar{6}$	$\bar{2} \otimes 2 \otimes 1, 3$	$-1/6$	$-1/6$	$+1/3$
$\bar{u}_R^c u_R \Phi$	$3 \otimes 3 \otimes 3, \bar{6}$	$1 \otimes 1 \otimes 1$	$-2/3$	$-2/3$	$+4/3$
$\bar{d}_R^c d_R \Phi$	$3 \otimes 3 \otimes 3, \bar{6}$	$1 \otimes 1 \otimes 1$	$+1/3$	$+1/3$	$-2/3$
$\bar{u}_R^c d_R \Phi$	$3 \otimes 3 \otimes 3, \bar{6}$	$1 \otimes 1 \otimes 1$	$-2/3$	$+1/3$	$+1/3$
$\bar{Q}_L u_R \Phi$	$\bar{3} \otimes 3 \otimes 8$	$\bar{2} \otimes 1 \otimes 2$	$-1/6$	$+2/3$	$-1/2$
$\bar{Q}_L d_R \Phi$	$\bar{3} \otimes 3 \otimes 8$	$\bar{2} \otimes 1 \otimes 2$	$-1/6$	$-1/3$	$+1/2$

(a)

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Φ_{ST}	$\bar{6}$	3	$-1/3$
Φ_{TT}	3	3	$-1/3$
Φ_{SS}	$\bar{6}$	1	$-1/3$
Φ_{TS}	3	1	$-1/3$
Φ_{SSu_R}	$\bar{6}$	1	$-4/3$
Φ_{TSu_R}	3	1	$-4/3$
Φ_{SSd_R}	$\bar{6}$	1	$+2/3$
Φ_{TSd_R}	3	1	$+2/3$
Φ_{OD}	8	2	$+1/2$

(b)

Table 2: (a) Possible diquark interaction terms whose representations form singlets under the Standard Model gauge group. (b) The nine diquark fields.

2.1 Diquark quantum numbers

The diquark Lagrangian must preserve electric charge to be invariant under $U(1)_{\text{em}}$ after spontaneous symmetry breaking (SSB) of the electroweak gauge group $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$. In the Weinberg-Salam model the electric charge is $Q = T^3 + Y$, where T^3 and Y are the weak isospin and hypercharge quantum numbers, respectively. For $SU(2)_L$ singlets, we simply have $Q = Y$. For triplets, we must diagonalise the $SU(2)_L$ generators T^1, T^2, T^3 to construct isospin eigenstates:

$$T^{\pm} \coloneqq \frac{1}{\sqrt{2}}(T^1 \pm T^2), \quad T^0 \coloneqq T^3, \tag{2}$$

which allows us to construct electric charge eigenstates and defines the components of an isospin triplet diquark transforming in the fundamental representation in the diagonal (charged) basis as

$$\Phi_{\text{trip}} = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}, \tag{3}$$

where $\phi^{\pm} \coloneqq \frac{1}{\sqrt{2}}(\phi_1 \mp i\phi_2)$ and $\phi^0 \coloneqq \phi_3$ are the physical components of the diquark field observed in broken $SU(2)_L$ with weak isospin ± 1 and zero, respectively. Here, ϕ_i are the components of Φ before diagonalising the generators. The sign convention for $\phi^{\pm}$ is chosen for the purpose of charge conservation. Similarly, an isospin doublet transforming in the fundamental representation in the charged basis is

$$\Phi_{\text{doub}} = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tag{4}$$

where $\varphi^+ \coloneqq \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$ and $\varphi^0 \coloneqq \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$ are the charged and neutral components which have isospin $\pm 1/2$, respectively. Now, φ_1 and φ_2 are the two components of the isospin *doublet* before diagonalising the generators. In broken $SU(2)_L$, the components of Φ_{trip} and Φ_{doub} are the physical diquark fields, which have charge eigenvalues given by

$$Q_{\text{trip}} = \begin{pmatrix} 2/3 & & \\ & -1/3 & \\ & & -4/3 \end{pmatrix}, \quad Q_{\text{doub}} = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}. \tag{5}$$

Since $SU(3)_c$ remains unbroken, the coloured components of the diquarks do not need to be expanded, as we observe in all other Standard Model fields.

3 The diquark Lagrangian

Given the interactions in Table 2 that form singlets under the Standard Model gauge group and imposing charge/hypercharge conservation at each vertex, we can construct the Lagrangian for each diquark field:

$$\begin{aligned}
&\mathcal{L}_A = \lambda_A \bar{Q}_L^c \epsilon \Phi_A Q_L, && A \in \{ST, TT\}, \\
&\mathcal{L}_B = \lambda_B \bar{Q}_L^c \epsilon \Phi_B Q_L + \tilde{\lambda}_B \bar{u}_R^c \Phi_B d_R, && B \in \{SS, TS\}, \\
&\mathcal{L}_C = \lambda_C \bar{u}_R^c \Phi_C u_R, && C \in \{SSu_R, TSu_R\}, \\
&\mathcal{L}_D = \lambda_D \bar{d}_R^c \Phi_D d_R, && D \in \{SSd_R, TSd_R\}, \\
&\mathcal{L}_{OD} = \lambda_{OD} \bar{Q}_L \Phi_{OD} d_R + \tilde{\lambda}_{OD} \bar{Q}_L \tilde{\Phi}_{OD} u_R,
\end{aligned} \tag{6}$$

where ϵ is the antisymmetric matrix, λ are the diquark couplings and $\tilde{\Phi}_{OD}$ denotes charge conjugation applied to *only* the $SU(2)_L$ indices. The diquark fields expanded in terms of their $SU(3)_c$ and $SU(2)_L$ indices are

$$\begin{aligned} \Phi_{ST} &= \bar{K_6}^r \Phi_{ST}^{r,\alpha} \frac{\sigma^\alpha}{2}, \qquad \Phi_X = \bar{K_6}^r \Phi_X^r, \qquad \Phi_{OD} = T^a \Phi_{OD}^a, \\ \Phi_{TT} &= K_3^m \Phi_{ST}^{m,\alpha} \frac{\sigma^\alpha}{2}, \qquad \Phi_Y = K_3^m \Phi_Y^m, \end{aligned} \tag{7}$$

with $X \in \{SS, SSu_R, SSd_R\}$ and $Y \in \{TS, TSu_R, TSd_R\}$. Here, $\bar{K_6}$ and K_3 are Clebsch-Gordan coefficients for the $\bar{6}$ and 3 representations of $SU(3)_c$, respectively, with $r = 1, \dots 6$ (sextet) and $m = 1, 2, 3$ (triplet). The $SU(3)_c$ generators are T^a with $a = 1, \dots, 8$ and σ^α are the Pauli matrices. The weak isospin indices for Φ_{OD} have been omitted in Eq (7) since they are directly contracted with the components of the left-handed quark doublet Q_L. The full diquark Lagrangian is therefore

$$\mathcal{L}_{dq} = \sum_k \left[(D_\mu \Phi_k)^\dagger (D^\mu \Phi_k) - m_k^2 \Phi_k^2 + \mathcal{L}_k \right] + \text{h.c.}, \tag{8}$$

where the index k sums over the nine diquark fields with mass m_k, assumed to be much heavier than the LHC scale (13 TeV) such that they can be integrated out. The covariant derivative is

$$D_\mu = \partial_\mu - i g_s G_\mu^a T^a - i \frac{g_2}{2} W_\mu^\alpha \frac{\sigma^\alpha}{2} - i \frac{g_1}{2} Y B_\mu, \tag{9}$$

which defines the interaction between Standard Model gauge fields and diquarks before electroweak symmetry breaking.

3.1 A remark on charge conservation

Expanding the $SU(2)_L$ indices in $\mathcal{L}_{dq}$ gives us the physical interaction vertices after spontaneous symmetry breaking in the electroweak sector of the Standard Model. For vertices of the type $\bar{Q}_L^c \Phi Q_L$, we must contract the indices in such a way that isospin and electric charge are conserved. Let us consider, for instance, Φ_{trip}, which transforms under the adjoint representation of $SU(2)_L$, so we can write

$$\Phi_{\text{trip}} = \Phi_{\text{trip}}^\alpha \frac{\sigma^\alpha}{2} = \frac{1}{2} \begin{pmatrix} \phi_3 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & -\phi_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -\phi^0 \end{pmatrix}. \tag{10}$$

To conserve isospin and electric charge at vertices $\mathcal{O} \coloneqq \bar{Q}_L^c \Phi_{\text{trip}} Q_L$, we contract the $SU(2)_L$ indices such that

$$\begin{aligned} \mathcal{O} &= (\bar{Q}_L^c)_\rho \epsilon_{\rho\eta} (\Phi_{\text{trip}})_{\eta\sigma} (Q_L)_\sigma \\ &= -\frac{1}{2} (\bar{d}_L^c u_L \phi^0 - \sqrt{2} \bar{u}_L^c u_L \phi^- + \sqrt{2} \bar{d}_L^c d_L \phi^+ + \bar{u}_L^c d_L \phi^0). \end{aligned} \tag{11}$$

with $\rho, \sigma, \eta = 1, 2$. For isospin singlets Φ_{sin}, the contraction is simpler:

$$(\bar{Q}_L^c)_\rho \epsilon_{\rho\sigma} \Phi_{\text{sin}} (Q_L)_\sigma = -\bar{d}_L^c u_L \Phi_{\text{sin}} + \bar{u}_L^c d_L \Phi_{\text{sin}}. \tag{12}$$

3.2 Flavour antisymmetrisation of the couplings

The anti-symmetric Clebsch-Gordan coefficient ϵ that appears in diquark vertices $\bar{Q}_L^c \Phi Q_L$ imposes constraints on the symmetry of the couplings. Naturally, an anti-symmetric representation of $SU(2)_L$ becomes symmetric and vice versa. Hence, the operator's symmetry under the exchange of flavour, $SU(3)_c$ and $SU(2)_L$ indices is reversed. Consider $\mathcal{L}_{ST}$, for instance. For convenience, we can expand all of the indices on the operator:

$$\mathcal{L}_{ST} = \lambda_{ST}^{ij} \mathcal{O}_{ST}^{ij}, \quad \mathcal{O}_{ST}^{ij} \coloneqq (\bar{Q}_L^c)_m^{i,a} \epsilon_{\rho\eta} (\Phi_{ST})_{\eta\sigma}^{ab} (Q_L)_\sigma^{j,b}, \tag{13}$$

where $a, b = 1, \ldots, 8$ are colour indices and the rest are defined as in Eq (7). $\mathcal{O}_{ST}$ transforms as a $3 \otimes 3 \otimes \bar{6}$ representation of $SU(3)_c$ and a $\bar{2} \otimes 2 \otimes 3$ representation of $SU(2)_L$. The 3 and $\bar{6}$ representations of $SU(3)_c$ are anti-symmetric and symmetric, respectively, so $\mathcal{O}_{ST}$ is symmetric under exchange of a, b. For $SU(2)_L$, the 3-dimensional representation and $\bar{2} \otimes 2$ are both anti-symmetric, so $\mathcal{O}_{ST}$ is symmetric under exchange of ρ, σ and η.

For $\mathcal{L}_{ST}$ to be non-vanishing, the coupling λ_{ST}^{ij} must be symmetric in its flavour indices. A similar argument holds for the corresponding operator in $\mathcal{L}_{TS}$. However, for $\mathcal{L}_{TT}$ and $\mathcal{L}_{SS}$, the corresponding operators are anti-symmetric in exactly one of $SU(3)_c$ and $SU(2)_L$ and symmetric in the other, constraining the couplings to be anti-symmetric in flavour space. Since the fundamental representation of $SU(3)_c$ is anti-symmetric, we also have anti-symmetric couplings for Φ_{TSu_R} and Φ_{TSd_R}, which is summarised in Table 3.

Vertex	$SU(3)$	$SU(2)$	Coupling
$\bar{Q}_L^c \Phi_{ST} Q_L$	S	S	S
$\bar{Q}_L^c \Phi_{TT} Q_L$	A	S	A
$\bar{Q}_L^c \Phi_{SS} Q_L$	S	A	A
$\bar{Q}_L^c \Phi_{TS} Q_L$	A	A	S
$\bar{u}_R^c \Phi_{TSu_R} u_R$	A	S	A
$\bar{d}_R^c \Phi_{TSd_R} d_R$	A	S	A

Table 3: Specification of symmetric (S) and anti-symmetric (A) diquark vertices under exchange of $SU(3)_c$ and $SU(2)_L$ indices and the constraint on the symmetry of the coupling. The flavour symmetry for all other couplings is unconstrained.

4 Next Steps

So far, we have constructed the diquark Lagrangian and determined the flavour structure for each coupling. To extract useful information about decays in B-physics, one can perform matching to $\mathcal{L}_{dq}$ from SMEFT and calculate the renormalisation group equations to determine the physical parameters of the EFT at the electroweak scale. Then, matching to LEFT can be performed and cross-sections and decay widths can be computed below the electroweak scale.

5 Appendix

The following conventions are used for group theory indices of diquarks and Standard Model fields:

$SU(2)$ triplet:	$\alpha, \beta, \cdots = 1, 2, 3$
$SU(2)$ doublet	$\rho, \sigma, \cdots = 1, 2, 3$
$SU(3)$ octet:	$a, b, \cdots = 1, \ldots, 8$
$SU(3)$ anti-sextet:	$r, s, \cdots = 1, \ldots, 6$
$SU(3)$ triplet:	$m, n, \cdots = 1, 2, 3$

References

[1] Andreas Crivellin and Matthew Kirk. "Diquark explanation of b→sℓ+ℓ-". In: *Phys. Rev. D* 108.11 (2023), p. L111701. DOI: 10.1103/PhysRevD.108.L111701.

[2] Gian Francesco Giudice, Ben Gripaios, and Raman Sundrum. "Flavourful production at hadron colliders". In: *Journal of High Energy Physics* 2011 (8 2011). ISSN: 10298479. DOI: 10.1007/JHEP08(2011)055.

[3] Adam Alloul et al. "FeynRules 2.0 - A complete toolbox for tree-level phenomenology". In: *Comput. Phys. Commun.* 185 (2014), pp. 2250–2300. DOI: 10.1016/j.cpc.2014.04.012. arXiv: 1310.1921 [hep-ph].