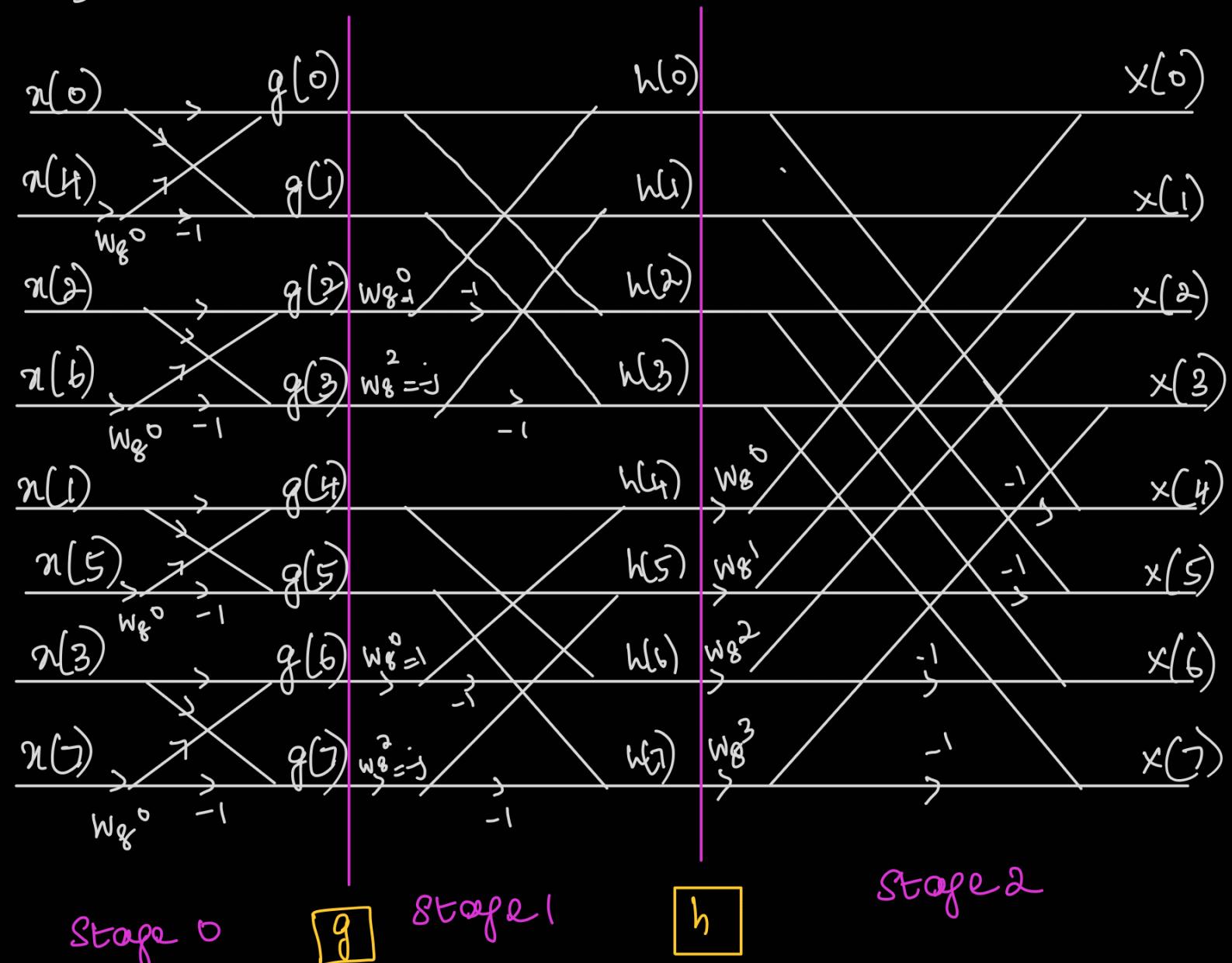


## Example i/p

$$\begin{aligned}x_0 &= 0 \cdot 0 + j 0 \cdot 0 \\x_1 &= 1 \cdot 0 + j 0 \cdot 0 \\x_2 &= 2 \cdot 0 + j 0 \cdot 0 \\x_3 &= 3 \cdot 0 + j 0 \cdot 0\end{aligned}$$

$$\begin{aligned}x_4 &= 4 \cdot 0 + j 0 \cdot 0 \\x_5 &= 5 \cdot 0 + j 0 \cdot 0 \\x_6 &= 6 \cdot 0 + j 0 \cdot 0 \\x_7 &= 7 \cdot 0 + j 0 \cdot 0\end{aligned}$$



$$w_N^n = \left( e^{-j \frac{2\pi n}{N}} \right)^n ; \quad w_8 = e^{\frac{-j 2\pi n}{8}} = e^{\frac{-j n\pi}{4}}$$

$g \Rightarrow$  o/p of stage 0 calculations

$h \Rightarrow$  o/p of stage 1 calculations

## Stage 1 Calculations :

$$g(0) = n(0) + n(4) = (0 \cdot 0 + j0 \cdot 0) + (4 \cdot 0 + j0 \cdot 0) = 4 \cdot 0 + j0 \cdot 0$$

$$g(1) = n(0) - n(4) = (0 \cdot 0 + j0 \cdot 0) - (4 \cdot 0 + j0 \cdot 0) = -4 \cdot 0 + j0 \cdot 0$$

$$g(2) = n(2) + n(6) = (2 \cdot 0 + j0 \cdot 0) + (6 \cdot 0 + j0 \cdot 0) = 8 \cdot 0 + j0 \cdot 0$$

$$g(3) = n(2) - n(6) = (2 \cdot 0 + j0 \cdot 0) - (6 \cdot 0 + j0 \cdot 0) = -4 \cdot 0 + j0 \cdot 0$$

$$g(4) = n(1) + n(5) = (1 \cdot 0 + j0 \cdot 0) + (5 \cdot 0 + j0 \cdot 0) = 6 \cdot 0 + j0 \cdot 0$$

$$g(5) = n(1) - n(5) = (1 \cdot 0 + j0 \cdot 0) - (5 \cdot 0 + j0 \cdot 0) = -4 \cdot 0 + j0 \cdot 0$$

$$g(6) = n(3) + n(7) = (3 \cdot 0 + j0 \cdot 0) + (7 \cdot 0 + j0 \cdot 0) = 10 \cdot 0 + j0 \cdot 0$$

$$g(7) = n(3) - n(7) = (3 \cdot 0 + j0 \cdot 0) - (7 \cdot 0 + j0 \cdot 0) = -4 \cdot 0 + j0 \cdot 0$$

$$\frac{n(0)}{\hookrightarrow} \text{in}_0\text{-real}[15:0], \quad \text{in}_0\text{-imag}[15:0]$$

$$\frac{n(4)}{\hookrightarrow} \text{in}_4\text{-real}[15:0], \quad \text{in}_4\text{-imag}[15:0]$$

$$g(0) = n(0) + n(4)$$

$$g_0\text{-real}[15:0] = \text{in}_0\text{-real}[15:0] + \text{in}_4\text{-real}[15:0]$$

$$g_0\text{-imag}[15:0] = \text{in}_0\text{-imag}[15:0] + \text{in}_4\text{-imag}[15:0]$$

## Stage 2 Calculations :

$$h(0) = g(0) + g(2)$$

$$h(1) = g(1) - j \cancel{g(3)}$$

$$h(2) = g(0) - g(2)$$

$$h(3) = g(1) + j g(3)$$

$$h(4) = g(4) + g(6)$$

$$h(5) = g(5) - j g(7)$$

$$h(6) = g(4) - g(6)$$

$$h(7) = g(5) + j g(7)$$

$$\begin{aligned} h(0) &= g(0) + g(2) \\ &= (4 \cdot 0 + j 0 \cdot 0) + (8 \cdot 0 + j 0 \cdot 0) \\ h(0) &= 12 \cdot 0 + j 0 \cdot 0 \end{aligned}$$

$$h(1) = g(1) - j g(3) = g(1) - g'(3)$$

$$\begin{aligned} g'(3) = j g(3) &= j(g(3)_{\text{real}} + j g(3)_{\text{imag}}) \\ &= j g(3)_{\text{real}} - g(3)_{\text{imag}} \end{aligned}$$

$$g'(3) = -g(3)_{\text{imag}} + j g(3)_{\text{real}}$$

$$\Rightarrow \begin{cases} g'(3)_{\text{real}} = -g(3)_{\text{imag}} \\ g'(3)_{\text{imag}} = g(3)_{\text{real}} \end{cases}$$

$$h(1) = \begin{pmatrix} -4+j0.0 \\ -4+j0.0 \end{pmatrix} - j \begin{pmatrix} -4+j0.0 \\ +j4+0.0 \end{pmatrix}$$

$$h(1) = -4+j4$$

$$h(2) = g(0) - g(2) = \begin{pmatrix} 4 \cdot 0 + j0 \cdot 0 \\ 8 \cdot 0 + j0 \cdot 0 \end{pmatrix} = -4 \cdot 0 + j0 \cdot 0$$

$$h(3) = g(1) + jg(3) = g(1) + g'(3)$$

$$g'(3) = j(g(3)-\text{real} + j g(3)-\text{imag})$$

$$= -g(3)-\text{imag} + j g(3)-\text{real}$$

$$\Rightarrow \begin{cases} g'(3)-\text{real} = -g(3)-\text{imag} \\ g'(3)-\text{imag} = g(3)-\text{real} \end{cases}$$

$$h(3) = \begin{pmatrix} 4 \cdot 0 + j0 \cdot 0 \\ -4 \cdot 0 + j0 \cdot 0 \end{pmatrix} + j \begin{pmatrix} -4 \cdot 0 + j0 \cdot 0 \\ -4 \cdot 0 - j4 \cdot 0 - 0 \cdot 0 \end{pmatrix}$$

$$= -4 \cdot 0 - j4 \cdot 0$$

$$h(4) = g(4) + g(6)$$

$$= \begin{pmatrix} 6+j0 \cdot 0 \\ 10+j0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 16 \cdot 0 + j0 \cdot 0 \\ 10+j0 \cdot 0 \end{pmatrix}$$

$$h(4) = 16 \cdot 0 + j0 \cdot 0$$

$$h(5) = g(5) - j g(5) = g(5) - g'(5)$$

$$g'(5) = j(g(5)-\text{real} + j g(5)-\text{imag})$$

$$= j g(5)-\text{real} - g(5)-\text{imag}$$

$$\Rightarrow \boxed{g'(5)-\text{real} = -g(5)-\text{imag}}$$

$$g'(5)-\text{imag} = g(5)-\text{real}$$

$$h(5) = g(5) - j g(5) = (-4+j0) - j(-4+j0-0)$$

$$= -4+j0 + j4 - 0$$

$$h(5) = -4+j4$$

$$h(6) = g(6) - g(6) = (6+j0-0) - (10-0+j0-0)$$

$$h(6) = -4-0 + j0-0$$

$$h(5) = g(5) + j g(5) = g(5) + g''(5)$$

$$g''(5) = j g(5) = j(g(5)-\text{real} + j g(5)-\text{imag})$$

$$= j g(5)-\text{real} - g(5)-\text{imag}$$

$$g''(5)-\text{real} = -g(5)-\text{imag}$$

$$g''(5)-\text{imag} = g(5)-\text{real}$$

$$h(7) = g(5) + jg(7) = (-4+j0 \cdot 0) + j(-4 \cdot 0 + j0 \cdot 0)$$

$$= -4 + j0 \cdot 0 - j4 \cdot 0 + 0$$

$$h(7) = -4 - j4$$

Stage 2 Results :

$$h(0) = 12 \cdot 0 + j0 \cdot 0$$

$$h(1) = -4 \cdot 0 + 4 \cdot 0$$

$$h(2) = -4 \cdot 0 + j0 \cdot 0$$

$$h(3) = -4 \cdot 0 - j4 \cdot 0$$

$$h(4) = 16 \cdot 0 + j0 \cdot 0$$

$$h(5) = -4 \cdot 0 + j4 \cdot 0$$

$$h(6) = -4 \cdot 0 + j0 \cdot 0$$

$$h(7) = -4 - j4$$

$$w_8^0 = 1$$

$$w_8^1 = e^{-j\frac{2\pi}{8}} = e^{-j\frac{\pi}{4}} = \cos \frac{\pi}{4} - j \sin \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = 0.707(1-j)$$

$$= 0.707 - j0.707$$

$$w_8^2 = e^{-j\frac{2\pi}{8} \times 2} = e^{-j\frac{\pi}{2}} = -j$$

$$w_8^3 = \left(e^{-j\frac{2\pi}{8}}\right)^3 = e^{-j\frac{6\pi}{8}} = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} = -\frac{(1+j)}{\sqrt{2}}$$

### Stage 3 Computations :

$i[ ] \rightarrow$  wires to denote o/p of stage 3.

$$i[0] = h(0) + h(4)$$

$$i[0] = (12 \cdot 0 + j0 \cdot 0) + (16 \cdot 0 + j0 \cdot 0) = 28 \cdot 0 + j0 \cdot 0$$

$$i[1] = h(1) + W_8^1 h(5)$$

$$= (-4 + j4 \cdot 0) + (-4 + j4 \cdot 0) \left( \frac{1-j}{\sqrt{2}} \right)$$

$$= (-4 + j4) + (-4 + j4)(0 \cdot 707 - j0 \cdot 707)$$

$$= -4 + j4 - 2 \cdot 828 + j2 \cdot 828 + 2 \cdot 828$$

$$= (-4 - 2 \cdot 828 + 2 \cdot 828) + j(4 + 2 \cdot 828 + 2 \cdot 828)$$

$$i[1] = -4 + j(9 \cdot 656)$$

$$i[2] = h[2] + W_8^2 h(6) = h[2] - j h[6]$$

$$= (-4 + j0) + -j(-4 + j0 \cdot 0)$$

$$i[2] = -4 + j0 \cdot 0 + j4 = -4 + j4$$

$$i[2] = h[2] - h'[6]$$

$$h'[6] = j h[6] = j [h[6]_{\text{real}} + j h[6]_{\text{imag}}]$$

$$= j h[6]_{\text{real}} - h[6]_{\text{imag}}$$

$$\begin{aligned} h'[6]_{\text{real}} &= -h[6]_{\text{imag}} \\ h'[6]_{\text{imag}} &= h[6]_{\text{real}} \end{aligned}$$

$$i[3] \rightarrow h[3] + w_8^3 h[7]$$

$$\begin{aligned} i[4] &= h[0] - h[4] \\ &= (12 \cdot 0 + j 0 \cdot 0) - (16 \cdot 0 + j 0 \cdot 0) \\ i[4] &= -4 \cdot 0 + j 0 \cdot 0 \end{aligned}$$

$$i[5] = h(1) - w_8^1 h(5)$$

$$\begin{aligned} i[6] &= h[2] - w_8^2 h[6] \\ &= h[2] - (-j) h[6] = h[2] + j h[6] \\ &= h[2] + h'[6] \\ h'[6] &= j (h[6]_{\text{real}} + j h[6]_{\text{imag}}) \\ &= j h[6]_{\text{real}} - h[6]_{\text{imag}} \end{aligned}$$

$$\begin{aligned} h'[6]_{\text{real}} &= -h[6]_{\text{imag}} \\ h'[6]_{\text{imag}} &= h[6]_{\text{real}} \end{aligned}$$

$$i[6] = (-4 \cdot 0 + j 0 \cdot 0) + j (-4 \cdot 0 + j 0 \cdot 0)$$

$$i[6] = -4.0 - j4.0$$

$$i[7] = h[3] - w_8^3 h[7]$$

D.  $w_8^1 h[5]$

$$(0 \cdot 707 - j 0 \cdot 707) (-4 + j 4)$$

$a$                              $b$

$$\begin{aligned} a_{\text{-real}} &= 0 \cdot 707 \\ a_{\text{-imag}} &= -0 \cdot 707 \end{aligned}$$

$$a \times b = c$$

$$b_{\text{-real}} = -4$$

$$b_{\text{-imag}} = 4$$

$$\begin{aligned} & (0 \cdot 707 - j 0 \cdot 707) (-4 + j 4) \\ &= (0 \cdot 707 \times -4) + j ((0 \cdot 707 \times 4) + (-0 \cdot 707 \times -4)) \\ & \quad - (-0 \cdot 707 \times 4) \end{aligned}$$

$$c_{\text{-real}} = (a_{\text{-real}} \times b_{\text{-real}}) - (a_{\text{-imag}} \times b_{\text{-imag}})$$

$$c_{\text{-imag}} = (a_{\text{-real}} \times b_{\text{-imag}}) + (a_{\text{-imag}} \times b_{\text{-real}})$$

$$c_{\text{real}} = \underbrace{(0 \cdot 707 \times b_{\text{real}})}_{= 0 \cdot 707 \times b_{\text{real}}} - \underbrace{(-0 \cdot 707 \times b_{\text{imag}})}_{= 0 \cdot 707 \times b_{\text{imag}}}$$

$$\boxed{c_{\text{real}} = 0 \cdot 707 (b_{\text{real}} + b_{\text{imag}})}$$

$$= 0 \cdot 707 (-4 + 4) = 0$$

$$c_{\text{imag}} = \underbrace{(0 \cdot 707 \times b_{\text{imag}})}_{= 0 \cdot 707 (b_{\text{imag}} - b_{\text{real}})} + \underbrace{(-0 \cdot 707 \times b_{\text{real}})}_{= 0 \cdot 707 (4 + 4)}$$

$$= 0 \cdot 707 \times 8$$

$$\boxed{c = 0 + j \underbrace{(5.65)}_{5.0} \quad \checkmark}$$

$$2). Wg^3 h(j) \rightarrow - \left( \frac{1+j}{\sqrt{2}} \right) h(j)$$

$$= - (0 \cdot 707 + j 0 \cdot 707) (h(j)_{\text{real}} + j h(j)_{\text{imag}})$$

$$= - \left( 0 \cdot 707 h(j)_{\text{real}} + j (0 \cdot 707 * h(j)_{\text{imag}} + 0 \cdot 707 h(j)_{\text{real}}) \right) - 0 \cdot 707 h(j)_{\text{imag}}$$

$$= \left( 0 \cdot 707 h(j)_{\text{real}} + 0 \cdot 707 h(j)_{\text{imag}} \right) + j \left( (-0 \cdot 707 * h(j)_{\text{imag}}) + (-0 \cdot 707 * h(j)_{\text{real}}) \right)$$

$$\begin{aligned}
 &= 0.707 (h(j)_\text{imag} - h(j)_\text{real}) + \\
 &\quad - j 0.707 (h(j)_\text{imag} + h(j)_\text{real}) \\
 &= 0.707 ((h(j)_\text{imag} - h(j)_\text{real}) + \\
 &\quad j(-h(j)_\text{real} + h(j)_\text{imag}))
 \end{aligned}$$

$$\begin{aligned}
 w_8^3 h(j) &= - (0.707 + j 0.707) (-4j) \\
 &= - (-2.828 - j 2.828 - j 2.828 + 2.828) \\
 &= -j 5.656
 \end{aligned}$$

### Verification of final results

1)  $x(0)$

- Expected :  $28 + j 0$
- Obtained :  $(1000 + j 0000)_H = (28 + j 0)_{10}$
- Error :  $|x_0)_\text{exp} - (x_0)_\text{obtained}| = 0$

2)  $x(1)$

- Expected :  $-4.0 + j 4.0 (1 + \sqrt{2})$
- $= -4.0 + j 4.0 (2 - 4j)$
- $= -4.0 + j 9.656$

$$\rightarrow \text{Obtained} = (fC.00 + j 09.00)_H$$

$$= -4 + j 9.625$$

$$(09.00)_H = (0000\ 1001 \cdot 1010\ 0000)_2$$

$$= 9.625$$

$$\text{Error : } |(-4 + j 9.625) - (-4 + j 9.625)|$$

$$\text{Error} = 0.031$$

3) x(2):

$$\begin{cases} \rightarrow \text{Expected} = -4 + j 4 \\ \rightarrow \text{Obtained} = (fC.00 + j 04.00)_H = (-4 + j 4)_D \\ \rightarrow \text{Error} = 0 \end{cases}$$

4) x(3):

$$\begin{cases} \rightarrow \text{Expected} = -4 + j 4 (0.414) = -4 + j 1.656 \\ \rightarrow \text{Obtained} = fC.00 + j (01.00) = -4 + j 1.625 \\ fC = -\left(2^{\text{comp}}(1111\ 1100\ 0000\ 0000)\right) = -\left(0000\ 0100\ 0000\ 0000\right) = -4.0 \\ (01.00) = 0000\ 0001 \cdot 1010\ 0000 = 1.625 \end{cases}$$

$$\rightarrow \text{Error} = |(-4 + j 1.656) - (-4 + j 1.625)|$$

$$\text{Error} = 0.031$$

5) x(4)

- ↳ Expected =  $-4 + j0$
- ↳ Obtained =  $(fC \cdot 00 + j fD \cdot 00)H = -4 + j0$
- ↳ Error = 0

6) x(5)

- ↳ Expected:  $-4 - j4 (0 \cdot 414) = -4 - j1 \cdot 656$
- ↳ Obtained:  $(fC \cdot 00 + j fE \cdot 60)H = -4 - j1 \cdot 402$

$$fE \cdot 60 = 1111 \ 1100 \cdot 1001 \ 1001$$

$$2's \text{ comp } (fE \cdot 60) = - \begin{pmatrix} 0000 & 0001 & 0111 & 0 & 0111 \end{pmatrix}$$

$$= -1 \cdot 402$$

$$\text{Error} = |1 \cdot 656 - 1 \cdot 402| = 0.254$$

7) x(6):

- ↳ Expected:  $-4^0 - j4^0$
- ↳ Actual:  $fC \cdot 00 + j fC \cdot 00 = -4 - j4$
- ↳ Error: 0

8) x(7)

- ↳ Expected:  $-4 - j4 (2 \cdot 414) = -4 - j9 \cdot 656$
- ↳ Actual:  $fC \cdot 00 + j fB \cdot 60$

$$fC \cdot 00 = 1111 \ 1100 \cdot 0000 \ 0000$$

$$2's \text{ comp } (1111 \ 1100 \cdot 0000 \ 0000) = - \begin{pmatrix} 0000 & 0100 & 0000 & 0000 \end{pmatrix}$$

$$= -4 \cdot 0$$

$$f_{b^0} \cdot b^0 = 1111 \quad 0110 \cdot \quad 0110 \quad 0000$$

$$2\Delta \text{ comp } (f_{b^0} \cdot b^0) = - \left( \begin{array}{cccc} 0000 & 1001 & 0100 & 0000 \end{array} \right)$$

$$= -9.625$$

$$\boxed{\text{Error} = |9.625 - 9.650| = 0.03}$$