

## CORDIC in vectoring mode

$$d_i = -\text{sign}(x^i y^i) \Rightarrow \boxed{y^i \rightarrow 0} \quad \text{With each iteration } y^i \text{ goes to 0}$$

CORDIC in vectoring mode to compute  $\tan^{-1}\left(\frac{y}{x}\right)$  and  $\sqrt{x^2+y^2}$

Q).  $x=1; y=2$ ; Compute  $\tan^{-1}\left(\frac{y}{x}\right)$  and  $\sqrt{x^2+y^2}$

→ we take i/p  $x(0)$  and  $y(0)$ . And we set  $z(0)=0$

→  $\boxed{x(0)=1; y(0)=2} \rightarrow$  I/p given by user.

$$\boxed{y(i) \rightarrow 0}$$

→ with each passing iteration, our goal is to make

→ The Shift Add algorithm of CORDIC is given by:

## Shift-Add Algorithm

- Hence, the original algorithm has now been reduced to an iterative **shift-add** algorithm for pseudo-rotations of a vector:

$$x^{(i+1)} = (x^{(i)} - d_i(2^{-i} y^{(i)}))$$

$$y^{(i+1)} = (y^{(i)} + d_i(2^{-i} x^{(i)}))$$

$$z^{(i+1)} = z^{(i)} - d_i \theta^{(i)}$$

- Thus each iteration requires:

- 2 shifts
- 1 table lookup ( $\theta^{(i)}$  values)
- 3 additions

→ The Shift-Add algorithm remains the same for both Vectoring Mode and Rotation Mode. The differentiating factor is the computation of  $d_i$ .  
 → only differentiating factor is the computation of  $d_i$ .  
 → Here  $\theta_i$  values denote the look up table of angles which we have already used in normal  $\cos \theta$ ,  $\sin \theta$  computation.

Look up values of  $\theta_i$

i	0	1	2	3	4	5	6	7	8
$\theta_i$	45	26.6	14	7.1	3.6	1.8	0.9	0.4	0.2

Step 0:  $x_0 = 1; y_0 = 2; z_0 = 0; \theta_0 = 45^\circ$

$$\hookrightarrow d_0 = -[\text{sign}(x_0 y_0)] = -\text{sign}(1 \times 2) = -\text{sign}(2)$$

$d_0 = -(-1) = -1$

signum function

$$\rightarrow \theta_0 = 45^\circ \text{ [Value from look up table]}$$

Step 1:

$$\begin{aligned} \rightarrow x_1 &= x_0 - d_0 2^0 y_0 = 1 - [(-1) \times 2] = 1 - (-2) = 3 \\ \rightarrow y_1 &= y_0 + d_0 2^0 x_0 = 2 + ((-1) \times 1 \times 1) = 2 - 1 = 1 \\ \rightarrow z_1 &= z_0 - d_0 \theta_0 = 0 - (-1 \times 45) = 45 \\ \rightarrow d_1 &= -\text{sign}(x_1 y_1) = -\text{sign}(3) = -1 \end{aligned}$$

$$\rightarrow \theta_1 = 26.6^\circ \text{ [Value from look up table]}$$

Step 2:

$$\rightarrow x_2 = x_1 - d_1 2^{-1} y_1 = x_1 - \left(-1 \times \frac{y_1}{2}\right) = x_1 + \frac{y_1}{2} = 3 + 0.5 = 3.5$$

$$\rightarrow y_2 = y_1 + d_1 2^{-1} x_1 = 1 + \left(-1 \times \frac{x_1}{2}\right) = 1 + \left(\frac{-3}{2}\right) = 1 - 1.5 = -0.5$$

$$\rightarrow z_2 = z_1 - d_1 \theta_1 = 45 - (-1 \times 26.6) = 71.6$$

$$\rightarrow \theta_2 = 14^\circ \text{ [Value from look up table]}$$

$$\rightarrow d_2 = -\text{sign}(x_2 y_2) = -\left(3.5 \times -0.5\right) = +1$$

**Step 3**

$$\begin{aligned} \rightarrow x_3 &= x_2 - d_2 \left( 2^{-2} y_2 \right) = 3.5 - \left( 1 \times \frac{y_2}{4} \right) = 3.5 - \left( \frac{-0.5}{4} \right) = 3.625 \\ \rightarrow y_3 &= y_2 + d_2 \left( 2^{-2} x_2 \right) = -0.5 + \left( 1 \times \frac{x_2}{4} \right) = -0.5 + \frac{3.5}{4} = 0.375 \\ \rightarrow z_3 &= z_2 - d_2 \theta_2 = 71.6 - \left( 1 \times 14 \right) = 57.6 \\ \rightarrow d_3 &= -\text{sgn}(x_3 y_3) = -\text{sgn}(3.625 \times 0.375) = -1 \\ \rightarrow \theta_3 &= 7.1^\circ \end{aligned}$$

**Step 4**:

$$\begin{aligned} \rightarrow x_4 &= x_3 - d_3 \left( 2^{-3} y_3 \right) = 3.625 - \left( -1 \times \frac{0.375}{8} \right) = 3.671 \\ \rightarrow y_4 &= y_3 + d_3 \left( 2^{-3} x_3 \right) = 0.375 + \left( -1 \times \frac{x_3}{8} \right) = -0.0781 \\ \rightarrow z_4 &= z_3 - d_3 \theta_3 = 57.6 - \left[ (-1) \times 7.1 \right] = 64.7^\circ \\ \rightarrow \theta_4 &= 3.6^\circ \\ \rightarrow d_4 &= -\text{sgn}(x_4 y_4) = -\text{sgn}(3.671 \times -0.0781) = 1 \end{aligned}$$

↳ **sign()** or **sgn()**  
 denotes signum function.

**Step 5**

$$\begin{aligned} \rightarrow x_5 &= x_4 - d_4 \left( 2^{-4} y_4 \right) = 3.671 - \left( 1 \times \frac{1}{16} \times (-0.0781) \right) = 3.675 \\ \rightarrow y_5 &= y_4 + \left( d_4 2^{-4} x_4 \right) = -0.0781 + \left( 1 \times \frac{1}{16} \times 3.671 \right) = 0.1513 \\ \rightarrow z_5 &= z_4 - d_4 \theta_4 = 64.7 - (1 \times 3.6) = 61.1 \\ \rightarrow \theta_5 &= 1.8 \\ \rightarrow d_5 &= -\text{sgn}(x_5 y_5) = -\text{sgn}(3.675 \times 0.1513) = -1 \end{aligned}$$

Step 6

$$\begin{aligned} \rightarrow x_6 &= x_5 - \left( d_5 \begin{pmatrix} -5 \\ 2 \\ y_5 \end{pmatrix} \right) = x_5 - \left( \frac{-y_5}{32} \right) = 3.675 - \left( \frac{-0.1513}{32} \right) = 3.679 \\ \rightarrow y_6 &= y_5 + \left( d_5 \begin{pmatrix} -5 \\ 2 \\ x_5 \end{pmatrix} \right) = 0.1513 + \left( \frac{-1 \times 3.675}{32} \right) = 0.036456 \\ \rightarrow z_6 &= z_5 - d_5 \theta_5 = 61.1 - (-1 \times 1.8) = 62.9 \\ \rightarrow \theta_6 &= 0.9 \\ \rightarrow d_6 &= -\operatorname{sgn}(x_6 y_6) = -\operatorname{sgn}(3.679 \times 0.036456) = -1 \end{aligned}$$

Step 7

$$\begin{aligned} \rightarrow x_7 &= x_6 - \left( d_6 \begin{pmatrix} -5 \\ 2 \\ y_6 \end{pmatrix} \right) = 3.679 - \left( \frac{-1 \times 0.036456}{64} \right) = 3.67956 \\ \rightarrow y_7 &= y_6 + \left( d_6 \begin{pmatrix} -5 \\ 2 \\ x_6 \end{pmatrix} \right) = 0.036456 + \left( \frac{-1 \times 3.679}{64} \right) = -0.02102 \\ \rightarrow z_7 &= z_6 - d_6 \theta_6 = 62.9 - (-1 \times 0.9) = 63.8 \\ \rightarrow \theta_7 &= 0.4 \\ \rightarrow d_7 &= -\operatorname{sgn}(x_7 y_7) = -\operatorname{sgn}(3.679 \times -0.021) = +1 \end{aligned}$$

$$\left( \frac{\sqrt{x^2 + y^2}}{0.6072} \right)$$

Step 8

$$\begin{aligned} \rightarrow x_8 &= x_7 - \left( d_7 \begin{pmatrix} -7 \\ 2 \\ y_7 \end{pmatrix} \right) = 3.679 - \left( 1 \times \begin{pmatrix} -7 \\ 2 \\ -0.02102 \end{pmatrix} \right) = 3.679 \\ \rightarrow y_8 &= y_7 + \left( d_7 \begin{pmatrix} -7 \\ 2 \\ x_7 \end{pmatrix} \right) = -0.02102 + \left( 1 \times \frac{1}{128} \times 3.67956 \right) = 0.0077 \\ \rightarrow z_8 &= z_7 - d_7 \theta_7 = 63.8 - (1 \times 0.4) = 63.4 \\ \rightarrow \theta_8 &= 0.2 \\ \rightarrow d_8 &= -\operatorname{sgn}(x_8 y_8) = -\operatorname{sgn}(3.679 \times 0.0077) = -1 \end{aligned}$$

$$\rightarrow \tan^{-1} \left( \frac{2}{1} \right) = 63.434$$