CORDIC [Coordinate Rotation Digital Computer]

Rotation of a vector :

→ Assume the origin to be the center of rotation.

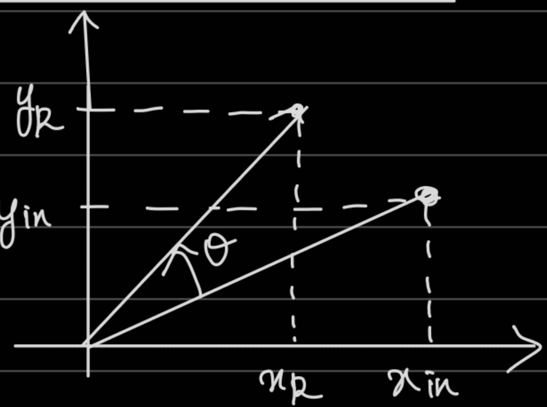
→ Let us say we a vector given by coordinates

→ Let us say we rotate the vector by an arbitrary angle θ.

→ The rotated vector is given by :

$$\boxed{\begin{aligned} x_R &= x_{in}\cos\theta - y_{in}\sin\theta \\ y_R &= x_{in}\sin\theta + y_{in}\sin\theta \end{aligned} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}}$$

→ Pictorial Representation of the rotation] =



→ If we choose $y_{in} = 0$ and rotate (x_{in}, θ) in anti-clockwise direction by an angle of θ, we would get

$$\boxed{\begin{pmatrix} x_R \\ y_R \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}}$$

↳ If we rotate $(1, 0)$ in anti-clockwise direction by an angle of θ, the x and y coordinates of the rotated vector denote the cos and the sine of our arbitrary angle of rotation.

→ List of functions that can be computed through rotations is very long.

List of functions that can be implemented through rotations:

Interestingly, the list of the functions that can be calculated from rotation is relatively long. Inverse trigonometric functions such as arctan, arcsin, arccos, hyperbolic and logarithmic functions, polar to rectangular transform, Cartesian to polar transform, multiplication, and division are some of the most important operations that can be obtained from variants of rotation.

AIM of CORDIC

↳ CORDIC uses rotations to compute all these complex functionalities in a hardware friendly manner [i.e, by avoiding the use of multipliers and using only shifting, additions, subtrations]

Basics of CORDIC

$$\begin{pmatrix} x_R \\ y_R \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix} = \cos\theta \begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}$$

↳ This is the equation for one rotation.

↳ Involves 4-5 multiplications and some addition/subtraction operations as well.

↳ So how do we avoid these multiplications?

Two fundamental ideas in CORDIC:

1). Rotating an input vector by an arbitrary angle θ_d is equivalent to rotating the same vector through angles θ_i $(i = 0, 1, 2 \cdots n)$ provided $\boxed{\theta_d = \sum_{i=0}^{n} \theta_i}$

Eg). Rotating a vector (x_{in}, y_{in}) by an angle 57.535° can be done by rotating the vector $45^\circ, 26.565^\circ, -14.03^\circ$

(-ve angles are also included).

2). Second idea is to choose the smaller elementary angles θ_i in such a way that $\boxed{\tan\theta_i = 2^{-i}}$

$(i = 0, 1, 2 \ldots n)$. In this way multiplication by $\tan(\theta_i)$ reduces to a bitwise shift operation which is less hardware intensive and is also faster than conventional multiplication.

Drawback: We may lose out on accuracy a little bit.

→ We still need to multiply by $\cos\theta$.
→ This can be incorporated as a scaling factor.

Example: Let us say we have an input vector $\begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}$ that we want to rotate by 57.535°.

$$\rightarrow \quad 57.535^\circ = \underset{\tan 45 = 1}{\underset{\downarrow}{45^\circ}} + \underset{\tan(26.565) = \frac{1}{2}}{\underset{\downarrow}{26.565^\circ}} - \underset{\tan(-14.03) = \frac{-1}{4}}{\underset{\downarrow}{14.03^\circ}}$$

First Rotation:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \cos 45 \begin{pmatrix} 1 & -\tan 45 \\ \tan 45 & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \cos 45 \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}}$$

Second Rotation : $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \cos 26.565 \begin{pmatrix} 1 & -\tan 26.565 \\ +\tan 26.565 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \cos(26.565) \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$= \cos(26.565) \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \cos 45 \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \cos(26.565)\cos(45) \begin{pmatrix} 1 & -1/2 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}}$$

Third Rotation:

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \cos(-14.03) \begin{bmatrix} 1 & -\tan(-14.03) \\ \tan(-14.03) & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$\cos(-14.03) = \cos(14.03)$ [cos is an Even function]

$$\boxed{\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \underbrace{\cos(45)\cos(26.565)\cos(14.03)} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{2^2} \\ -\frac{1}{2^2} & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ y_{in} \end{pmatrix}}$$

↳ If we fix the number of steps in our algorithm, then these terms can be grouped together as a scaling factor

→ Secondly, as number of steps increases, magnitude of θ keeps on reducing and $\cos\theta$ tends to 1.

→ For $i=6$; $\theta_i = \tan^{-1}\left(\frac{1}{2^6}\right) = 0.8951 \Rightarrow \boxed{\cos(0.8951) = 0.9987}$

→ For $i=7$; $\theta_i = \tan^{-1}\left(\frac{1}{2^7}\right) \Rightarrow \cos\left[\tan^{-1}\left(\frac{1}{2^7}\right)\right] = 0.99996$

→ So we can neglect $\cos\theta$ terms for $i>6$
[of course if we want greater accuracy, then we need to include them. This is a designer's choice]

$$K = \cos\left[\tan^{-1}\left(\frac{1}{2^0}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^1}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^2}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^3}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^4}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^5}\right)\right] \cdot \cos\left[\tan^{-1}\left(\frac{1}{2^6}\right)\right]$$

$$K = \cos(45) \cdot \cos(26.565) \cdot \cos(14.036) \cos(7.125) \cos(3.576) \cos(1.7899) \cos(0.8951)$$

$$K = 0.707 \times 0.8944 \times 0.9701 \times 0.9922 \times 0.9980 \times 0.9995 \times 0.9998$$

$$\boxed{K = 0.607}$$

→ This scaling factor would be a constant regardless of what the angle of rotation is.

→ For greater accuracy, we can include more digits after the decimal point in the scaling factor.

→ We can store this scaling factor somewhere else in the system without using a multiplier.

→ Omitting the scaling factor, CORDIC algorithm can be summarized as:

$$\boxed{\begin{matrix} x[i+1] \\ y[i+1] \end{matrix} = \begin{pmatrix} x[i] - \sigma_i 2^{-i} y[i] \\ \sigma_i 2^{-i} x[i] + y[i] \end{pmatrix} = \begin{pmatrix} 1 & -\sigma_i 2^{-i} \\ \sigma_i 2^{-i} & 1 \end{pmatrix} \begin{pmatrix} x[i] \\ y[i] \end{pmatrix}}$$

→ $\sigma_i \in \{-1, 1\}$ decides the sign of the smaller elementary angle.

→ CORIC algorithm always performs a certain number of rotations with pre-defined angle [say 12]

→ Only thing that the algorithm decides is whether the rotation would be clockwise or counter-clockwise.

Example :

Let angle of rotation $= 70°$. Let $n = 12$.

→ Let us start with $K\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0.607\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.607 \\ 0 \end{pmatrix}$

$\boxed{\text{→ We get rid of the multiplication at the end by including it at the very beginning.}}$

→ This ensures that there are no multiplications needed.

Step 0 :

$\rightarrow \theta_{current} = 0^\circ < 70^\circ \Rightarrow \sigma_0 = +1$

$\rightarrow \theta_{next} = +45$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 & -\tan 0 \\ \tan 0 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.607 \\ 0 \end{pmatrix}$$

Step 1.

Accumulated angle of rotation

Current angle of rotation.

$$\rightarrow \boxed{\theta_{current}} = \theta_{prev} + \boxed{(\sigma_{prev} \cdot 45)}$$

$$\theta_{current} = 0 + 45 = 45$$

$$\theta_{current} < \theta_{target} \Rightarrow \boxed{\sigma_1 = +1}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.607 \\ 0.607 \end{pmatrix}$$

Step 2 :

$$\theta_{current} = \theta_{prev} + [\sigma_{prev}(26.565)]$$

$$\theta_{current} = 45 + (1 \times 26.565) = 71.565$$

$$\theta_{current} > 71.565 \Rightarrow \boxed{\sigma_2 = -1}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(26.565) \\ +\tan(26.565) & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0.607 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0.607 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.3035 \\ 0.9105 \end{pmatrix}}$$

Step 3 : $\underbrace{\theta_{current}}_{\Downarrow \text{Accumulated angle of rotation}} = \theta_{prev} + \underbrace{(\sigma_{prev} \cdot 14.036)}_{\text{Current angle of rotation}}$

$$\theta_{current} = 71.565 + (-1 \times 14.036)$$

$$\theta_{current} = 71.565 - 14.036 = 57.529 < \theta_{target} = \boxed{\sigma_3 = +1}$$

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(-14.036) \\ \tan(-14.036) & 1 \end{pmatrix} \begin{pmatrix} 0.3035 \\ 0.9105 \end{pmatrix} = \begin{pmatrix} 1 & 1/4 \\ -1/4 & 1 \end{pmatrix} \begin{pmatrix} 0.3035 \\ 0.9105 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.5311 \\ 0.8346 \end{pmatrix}}$$

Step 4 :

$\hookrightarrow$ $\theta_{current} = \theta_{prev} + (\sigma_{prev} \cdot 7.125)$

$$\theta_{current} = 57.529 + 7.125 = 64.654 < 70 \Rightarrow \boxed{\sigma_4 = +1}$$

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(7.125) \\ \tan(7.125 & 1 \end{pmatrix} \begin{pmatrix} 0.5311 \\ 0.8346 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{8} \\ \frac{1}{8} & 1 \end{pmatrix} \begin{pmatrix} 0.5311 \\ 0.8346 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0.4267 \\ 0.9009 \end{pmatrix}}$$

Step 5 :

$\hookrightarrow$ $\theta_{current} = \theta_{prev} + (\sigma_{prev} \cdot 3.576^\circ) = \theta_{prev} + 3.576^\circ$

$$= 64.654 + 3.576 = 68.23 < 70$$

$$\Rightarrow \boxed{\sigma_5 = +1}$$

$$\begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(3.576^\circ) \\ \tan(3.576^\circ) & 1 \end{pmatrix} \begin{pmatrix} 0.4267 \\ 0.9009 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{16} \\ \frac{1}{16} & 1 \end{pmatrix} \begin{pmatrix} 0.4267 \\ 0.9009 \end{pmatrix} = \begin{pmatrix} 0.3703 \\ 0.9275 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} 0.3703 \\ 0.9275 \end{pmatrix}}$$

Step 6 :

$$\theta_{current} = (\theta_{prev} + \sigma_{prev} \cdot 1.7899) = 68.23 + 1.7899$$

$$\theta_{current} = 70.019 \geq 70 \Rightarrow \boxed{\sigma_6 = -1}$$

$$\begin{pmatrix} x_6 \\ y_6 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(1.7899) \\ \tan(1.7899) & 1 \end{pmatrix} \begin{pmatrix} 0.3703 \\ 0.9275 \end{pmatrix} = \begin{pmatrix} 1 & -1/32 \\ 1/32 & 1 \end{pmatrix} \begin{pmatrix} 0.3703 \\ 0.9275 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_6 \\ y_6 \end{pmatrix} = \begin{pmatrix} 0.3399 \\ 0.9840 \end{pmatrix}}$$

→ Stopping with 6 iterations.

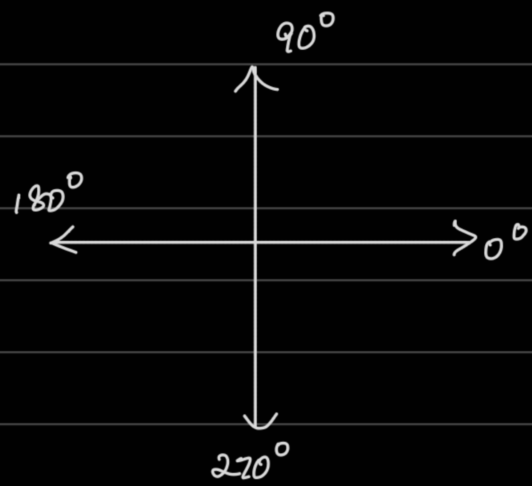
	Actual	Obtained
→ $\cos 70 =$	0.342	0.3399
$\sin 70 =$	0.9396	0.9840

⟹ Applicable for angles in Quadrants I

NOTE : Irrespective of the target angle, CORDIC will go through all the iterations. Even if error between θ_{actual} and θ_{target} is 0, the algorithm would still go through the pre-defined number of iterations.

→ We lose a bit of accuracy in the process.

Target angle in Quadrant 4



→ Let $\theta_{target} = 285^{\circ}$

→ Convert it such that magnitude of angle of rotation is within $(0^{\circ}, 90^{\circ})$

$$\boxed{285^{\circ} = 285 - 360 = -75^{\circ}}$$

Step 0 :

↳ $\theta_{current} = 0$; $\theta_{target} = -75^{\circ}$

↳ $\theta_{current} > \theta_{target} \Rightarrow \boxed{\sigma_0 = -1}$

↳ $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.607 \\ 0 \end{pmatrix}$

Step 1 :

↳ $\theta_{current} = \theta_{previous} + (\sigma_0 . \tan^{-1}(1))$

$$= \underbrace{0}_{\text{Accumulated}} + \underbrace{(-45)}_{\text{current angle of rotation.}} = -45^{\circ}$$

$\theta_{current} > \theta_{target} \Rightarrow \boxed{\sigma_1 = -1}$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(-45) \\ \tan(-45) & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.607 \\ -0.607 \end{pmatrix}$$

Step 2 :

↳ $\theta_{current} = \theta_{previous} + \left(\sigma_{prev} . \tan^{-1}\left(\frac{1}{2}\right)\right)$

$$= -45 + (-1 . 26.565)$$

$$= -45 - 26.565 = -71.565 \geq -75 \Rightarrow \boxed{\sigma_2 = -1}$$

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(-26.565) \\ \tan(-26.565) & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ -0.607 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0.5 \\ -0.5 & 1 \end{pmatrix} \begin{pmatrix} 0.607 \\ -0.607 \end{pmatrix} = \begin{pmatrix} 0.3035 \\ -0.9105 \end{pmatrix}}$$

Step 3 : $\theta_{current} = \theta_{previous} + \left[\sigma_{prev} \cdot \tan^{-1}\left(\frac{1}{2^2}\right)\right]$

$$= -75 + (-1 \times 14.036) = -89.036 < -75$$

$$\Rightarrow \boxed{\sigma_3 = +1}$$

$$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & -\tan(-14.036) \\ \tan(-14.036) & 1 \end{pmatrix} \begin{pmatrix} 0.3035 \\ -0.9105 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_3 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & +\frac{1}{4} \\ -\frac{1}{4} & 1 \end{pmatrix} \begin{pmatrix} 0.3035 \\ -0.9105 \end{pmatrix} = \begin{pmatrix} 0.0758 \\ -0.9863 \end{pmatrix}}$$

Step 4 : $\theta_{current} = \theta_{prev} + \left(\sigma_{prev} \cdot \left(\tan^{-1}\left(\frac{1}{2^3}\right)\right)\right)$

$$= -89.036 + (1 \cdot 7.125)$$

$$= -81.91 < -75^0 \Rightarrow \boxed{\sigma_4 = +1}$$

$$\boxed{\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{8} \\ \frac{1}{8} & 1 \end{pmatrix} \begin{pmatrix} 0.0758 \\ -0.9863 \end{pmatrix} = \begin{pmatrix} 0.1990 \\ -0.9105 \end{pmatrix}}$$

Step5 : $\theta_{current} = \theta_{prev} + \left[\sigma_{prev} \cdot \left(\tan^{-1}\left(\frac{1}{2^4}\right)\right)\right]$

$= -81.91 + \left[1 \times \tan^{-1}\left(\frac{1}{16}\right)\right]$

$= -78.33 < -75 \Rightarrow \boxed{\sigma_5 = +1}$

$$\begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{16} \\ \frac{1}{16} & 1 \end{pmatrix} \begin{pmatrix} 0.1990 \\ -0.9105 \end{pmatrix} = \begin{pmatrix} 0.2559 \\ -0.8980 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_5 \\ y_5 \end{pmatrix} = \begin{pmatrix} 0.2559 \\ -0.7115 \end{pmatrix}}$$

Stopping with 5 iterations for simplicity

	Actual	Obtained
cos (285)	0.2588	0.2559
sin (285)	-0.9659	-0.8980

Method to solve if angle is in Quadrant 4

→ Subtract 360° from the angle so that the magnitude of the angle lies within 0° to 90°

→ (We still need to figure out what to if angle number is beyond (0° - 360°) range).

$\boxed{\theta_{conv} = \theta_{target} + 360°}$ → Logic for solving if target angle is in Quadrant 4.

$$\boxed{\begin{aligned} \sin(\theta_{conv}) &= \sin(\theta_{target}) \\ \cos(\theta_{conv}) &= \cos(\theta_{target}) \end{aligned}}$$

As $\theta = \theta + 360°$

→ $x \rightarrow \cos(\theta_{target})$

$y \rightarrow \sin(\theta_{target})$

Target Angle in 2^{nd} Quadrant

- $90^\circ \le \theta_{target} \le 180^\circ$
- Let $\theta_{conv} = \theta_{target} - 90^\circ \Rightarrow \boxed{0^\circ \le \theta_{conv} \le 90^\circ}$
- Solve for $\sin(\theta_{conv})$ & $\cos(\theta_{conv})$ using CORDIC

$$\cos(\theta_{conv}) = \cos(\theta_{target} + 90^\circ) = \cos\theta_{target}\cos 90 - \sin\theta_{target}\sin 90$$
$$= -\sin\theta_{target}$$

$$\boxed{\cos(\theta_{conv}) = -\sin(\theta_{target})}$$

$$\sin(\theta_{conv}) = \sin(\theta_{target} + 90^\circ) = \sin\theta_{target}\cos 90 + \sin 90\cos\theta_{target}$$
$$= \cos\theta_{target}$$

$$\sin(\theta_{conv}) = \cos\theta_{target}$$

$$\Rightarrow \boxed{\begin{array}{l} x_r = \cos(\theta_{conv}) = -\sin(\theta_{target}) \\ y_r \quad \sin(\theta_{conv}) = \cos(\theta_{target}) \end{array}} \quad \Rightarrow \boxed{\begin{array}{l} \sin\theta_{target} = -x_r \\ \cos\theta_{target} = y_r \end{array}}$$

→ In this method, -ve of x coordinate result gives us $\sin(\theta_{target})$ and the (y coordinate) gives us $\cos(\theta_{target})$

Target angle in Quadrant 3

$180^\circ \le \theta_{target} \le 270^\circ$

→ $\theta_{conv} = \theta_{target} - 180^\circ \quad (0 \le \theta_{conv} \le 90^\circ)$

→ Solve for $\sin(\theta_{conv})$ and $\cos(\theta_{conv})$ using CORDIC.

→ $x_r = \cos(\theta_{conv}) = \cos(\theta_{target} - 180^\circ) = \cos\theta_{target}\cos 180 + \sin\theta_{target}\sin 180$

$$\boxed{x_r = -\cos\theta_{target}}$$

$\rightarrow$ $y_r = \sin(\theta_{conv}) = \sin(\theta_{target} - 180^\circ) = \sin\theta_{target}\cos 180 - \cos\theta_{target}\sin 180$

$$y_r = -\sin(\theta_{target})$$

$$\boxed{\begin{aligned} x_r &= -\cos\theta_{target} \Rightarrow \cos(\theta_{target}) = -x_r \\ y_r &= -\sin\theta_{target} \Rightarrow \sin(\theta_{target}) = -y_r \end{aligned}}$$

$\hookrightarrow$ Negating x, y of the resultant values gives us the cos & sin of the target angle of rotation.