CORDIC in Vectoring Mode

$d_i = -\text{sign}(x^i y^i) \Rightarrow \boxed{y^i \to 0} \rightarrow$ With each iteration y^i goes to 0

CORDIC in vectoring mode to compute $\tan^{-1}\left(\frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$

Q). $x = 1$; $y = 2$; Compute $\tan^{-1}\left(\frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$

→ We take i/p $x(0)$ and $y(0)$. And we set $z(0) = 0$

→ $\boxed{x(0) = 1;\ y(0) = 2} \rightarrow$ I/p given by user.

→ With each passing iteration, our goal is to make $\boxed{y(i) \to 0}$

→ The Shift Add algorithm of CORDIC is given by:

Shift-Add Algorithm

- Hence, the original algorithm has now been reduced to an iterative shift-add algorithm for pseudo-rotations of a vector:

$$x^{(i+1)} = (x^{(i)} - d_i(2^{-i}y^{(i)}))$$

$$y^{(i+1)} = (y^{(i)} + d_i(2^{-i}x^{(i)}))$$

$$z^{(i+1)} = z^{(i)} - d_i\theta^{(i)}$$

- Thus each iteration requires:
    - 2 shifts
    - 1 table lookup ($\theta^{(i)}$ values)
    - 3 additions

↳ The Shift-Add algorithm remains the same for both Vectoring Mode and Rotation Mode.

→ Only differentiating factor is the computation of d_i

→ Here θ_i values denote the look up table of angles which we have already used in normal $\cos\theta$, $\sin\theta$ computation.

Look up values of θ_i

i	0	1	2	3	4	5	6	7	8
θ_i	45	26.6	14	7.1	3.6	1.8	0.9	0.4	0.2

Step 0 : $x_0 = 1;\ y_0 = 2;\ z_0 = 0;\ \theta_0 = 45^\circ$

$\rightarrow d_0 = -\left[\operatorname{sign}(x_0 y_0)\right] = -\operatorname{sign}(1\times 2) = -\underbrace{\operatorname{sign}}_{\text{signum function}}(2)$

$$\boxed{d_0 = -(1) = -1}$$

$\rightarrow \theta_0 = 45^\circ$ (value from look up table)

step 1 :

$\rightarrow x_1 = x_0 - d_0\, 2^{-0} y_0 = 1 - \left[(-1)\times 2\right] = 1-(-2) = 3$

$\rightarrow y_1 = y_0 + d_0\, 2^{-0} x_0 = 2 + ((-1)\times 1\times 1) = 2-1 = 1$

$\rightarrow z_1 = z_0 - d_0\theta_0 = 0 - (-1\times 45) = 45$

$\rightarrow d_1 = -\operatorname{sgn}(x_1 y_1) = -\operatorname{sgn}(3) = -1$

$\rightarrow \theta_1 = 26.6^\circ$ [Value from look up table]

Step 2 :

$\rightarrow x_2 = x_1 - d_1(2^{-1} y_1) = x_1 - \left(-1\times \frac{y_1}{2}\right) = x_1 + \frac{y_1}{2} = 3+0.5 = 3.5$

$\rightarrow y_2 = y_1 + d_1\, 2^{-1} x_1 = 1 + \left(-1\times \frac{x_1}{2}\right) = 1 + \left(\frac{-3}{2}\right) = 1 - 1.5 = -0.5$

$\rightarrow z_2 = z_1 - d_1\theta_1 = 45 - (-1\times 26.6) = 71.6$

$\rightarrow \theta_2 = 14^\circ$ [Value from look up table]

$\rightarrow d_2 = -\operatorname{sgn}(x_2 y_2) = -(3.5\times -0.5) = +1$

Step 3

$\hookrightarrow x_3 = x_2 - d_2\left(2^{-2} y_2\right) = 3.5 - \left(1 \times \frac{y_2}{4}\right) = 3.5 - \left(\frac{-0.5}{4}\right) = 3.625$

$\hookrightarrow y_3 = y_2 + d_2\left(2^{-2} x_2\right) = -0.5 + \left(1 \times \frac{x_2}{4}\right) = -0.5 + \frac{3.5}{4} = 0.375$

$\hookrightarrow z_3 = z_2 - d_2 \theta_2 = 71.6 - (1 \times 14) = 57.6$

$\hookrightarrow d_3 = -sgn\,(x_3 y_3) = -sgn\,(3.375 \times 0.375) = -1$

$\hookrightarrow \theta_3 = 7.1^\circ$

Step 4 :

$\hookrightarrow x_4 = x_3 - d_3\left(2^{-3} y_3\right) = 3.625 - \left(-1 \times \frac{0.375}{8}\right) = 3.671$

$\hookrightarrow y_4 = y_3 + d_3\left(2^{-3} x_3\right) = 0.375 + \left(-1 \times \frac{x_3}{8}\right) = -0.0781$

$\hookrightarrow z_4 = z_3 - d_3 \theta_3 = 57.6 - \left[(-1) \times 7.1\right] = 64.7^\circ$

$\hookrightarrow \theta_4 = 3.6^\circ$

$\hookrightarrow d_4 = -sgn\,(x_4 y_4) = -sgn\left(3.671 \times -0.0781\right) = 1$

$\hookrightarrow$ sign( ) or sgn( ) denotes signum function.

Step 5

$\hookrightarrow x_5 = x_4 - d_4\left(2^{-4} y_4\right) = 3.671 - \left(1 \times \frac{1}{24} \times (-0.0781)\right) = 3.675$

$\hookrightarrow y_5 = y_4 + \left(d_4\, 2^{-4} x_4\right) = -0.0781 + \left(1 \times \frac{1}{16} \times 3.671\right) = 0.1513$

$\hookrightarrow z_5 = z_4 - d_4 \theta_4 = 64.7 - (1 \times 3.6) = 61.1$

$\hookrightarrow \theta_5 = 1.8$

$\hookrightarrow d_5 = -sgn\,(x_5 y_5) = -sgn\left(3.675 \times 0.1513\right) = -1$

Step 6

$\hookrightarrow x_6 = x_5 - \left(d_5\, 2^{-5} y_5\right) = x_5 - \left(\frac{-y_5}{32}\right) = 3.675 - \left(\frac{-0.1513}{32}\right) = 3.679$

$\hookrightarrow y_6 = y_5 + \left(d_5 2^{-5} x_5\right) = 0.1513 + \left(\frac{-1 \times 3.675}{32}\right) = 0.036456$

$\hookrightarrow z_6 = z_5 - d_5 \theta_5 = 61.1 - (-1 \times 1.8) = 62.9$

$\hookrightarrow \theta_6 = 0.9$

$\hookrightarrow d_6 = -\operatorname{sgn}(x_6 y_6) = -\operatorname{sgn}(3.679 \times 0.036456) = -1$

Step 7

$\hookrightarrow x_7 = x_6 - \left(d_6 2^{-6} y_6\right) = 3.679 - \left(\frac{-1 \times 0.036456}{64}\right) = 3.67956$

$\hookrightarrow y_7 = y_6 + \left(d_6 2^{-6} x_6\right) = 0.036456 + \left(\frac{-1 \times 3.679}{64}\right) = -0.02102$

$\hookrightarrow z_7 = z_6 - d_6 \theta_6 = 62.9 - (-1 \times 0.9) = 63.8$

$\hookrightarrow \theta_7 = 0.4$

$\hookrightarrow d_7 = -\operatorname{sgn}(x_7 y_7) = -\operatorname{sgn}(3.679 \times -0.021) = +1$

$\hookrightarrow \left(\frac{\sqrt{x^2+y^2}}{0.6072}\right)$

Step 8

$\hookrightarrow x_8 = x_7 - \left(d_7 2^{-7} y_7\right) = 3.679 - \left(1 \times 2^{-7} \times -0.02102\right) = \boxed{3.679}$

$\hookrightarrow y_8 = y_7 + \left(d_7 2^{-7} x_7\right) = -0.02102 + \left(1 \times \frac{1}{128} \times 3.67956\right) = 0.0077$

$\hookrightarrow \boxed{z_8 = z_7 - d_7 \theta_7 = 63.8 - (1 \times 0.4) = 63.4}$

$\hookrightarrow \tan^{-1}\left(\frac{2}{1}\right) = 63.434$

$\hookrightarrow \theta_8 = 0.2$

$\hookrightarrow d_8 = -\operatorname{sgn}(x_8 y_8) = -\operatorname{sgn}(3.679 \times 0.0077) = -1$