problema_7

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1 Problema 7 - ANEXO

Abaixo temos o código responsável pela modelagem do exercício 7, escrito em linguagem de programação Python e que utiliza o CPLEX como solver.

O código resolve um TSP para diferentes tamanhos de problemas (com n entre 4 e 104 clientes) usando formulação de Danzig-Fulkerson-Johnson e abordagem por meio do problema de transbordo.

[ ]:

```python
from docplex.mp.model import Model
import random

# fix the seed (ensure the same results every time we run the code)
random.seed(0)


def create_distance_matrix(n: int) -> list[list[int]]:
    # create a NxN matrix filled with random distances. The diagonal is an␣
 ↪infinite distance
    c = [[random.randint(1, 100) for j in range(n)] for i in range(n)]
    for i in range(n):
        c[i][i] = 999999999
    return c


def modelo_danzig(distance_matrix):
    n = len(distance_matrix)
    mdl = Model(name="TSP_danzig")

    # Variables
    x = mdl.binary_var_matrix(range(n), range(n), name="x")

    # Objective function
    mdl.minimize(
        mdl.sum(distance_matrix[i][j] * x[i, j] for i in range(n) for j in␣
 ↪range(n))
    )
    # Constraints
    ## Each city is visited exactly once
    for i in range(n):
        mdl.add_constraint(mdl.sum(x[i, j] for j in range(n)) == 1)

    ## Each city is left exactly once
    for j in range(n):
        mdl.add_constraint(mdl.sum(x[i, j] for i in range(n)) == 1)

    ## Avoid subtours
    Q = mdl.integer_var_list(n, 0, n - 1, name="Q")
    for i in range(1, n):
        for j in range(1, n):
            if i != j:
                mdl.add_constraint(Q[i] - Q[j] + n * x[i, j] <= n - 1)

    # Solve
    mdl.solve()

    return {
        "exec_time": mdl.solve_details.time,
        "num_constraints": int(mdl.number_of_constraints),
        "solution": mdl.solution,
    }


def modelo_mtz(distance_matrix):
    n = len(distance_matrix)
    M = 100000000

    mdl = Model(name="TSP_mtz")

    # Variables
    x = mdl.binary_var_matrix(range(n), range(n), name="x")

    # Objective function
    mdl.minimize(
        mdl.sum(distance_matrix[i][j] * x[i, j] for i in range(n) for j in␣
 ↪range(n))
    )

    # Constraints
    ## Each city is visited exactly once
    for i in range(n):
        mdl.add_constraint(mdl.sum(x[i, j] for j in range(n)) == 1)
    ## Each city is left exactly once
    for j in range(n):
        mdl.add_constraint(mdl.sum(x[i, j] for i in range(n)) == 1)

    ## Avoid subtours
    y = mdl.integer_var_matrix(range(n), range(n), name="y")

    ### 1
    mdl.add_constraint(mdl.sum(y[1, j] for j in range(1, n)) == n - 1)

    ### 2
    mdl.add_constraint(
        -1
        == mdl.sum(y[i, j] for j in range(1, n) for i in range(2, n) if i != j)
        - mdl.sum(y[k, i] for k in range(1, n) for i in range(2, n) if i != k)
    )

    ### 3
    for i in range(1, n):
        for j in range(1, n):
            if i != j:
                mdl.add_constraint(y[i, j] <= M * x[i, j])

    # Solve
    mdl.solve()

    return {
        "exec_time": mdl.solve_details.time,
        "num_constraints": int(mdl.number_of_constraints),
        "solution": mdl.solution,
    }


n_max = 102
numero_clientes = list(range(4, n_max, 5))

resultados_danzig = {}
resultados_mtz = {}

for n in numero_clientes:
    print(f"n = {n}/{n_max}")
    d_matrix = create_distance_matrix(n)

    resultados_danzig[n] = {}
    resultados_mtz[n] = {}

    res = modelo_danzig(d_matrix)
    resultados_danzig[n]["time"] = res["exec_time"]
    resultados_danzig[n]["num_restricoes"] = res["num_constraints"]

    res = modelo_mtz(d_matrix)
    resultados_mtz[n]["time"] = res["exec_time"]
    resultados_mtz[n]["num_restricoes"] = res["num_constraints"]




################################################################################
###### As próximas linhas servem apenas para visualização dos resultados #######
################################################################################

# create dataframes with resultados_danzig, resultados_mtz

import pandas as pd

df_danzig = pd.DataFrame(resultados_danzig).T
df_mtz = pd.DataFrame(resultados_mtz).T

# forca inteiros para os valores de num_restricoes
df_danzig["num_restricoes"] = df_danzig["num_restricoes"].astype(int)
df_mtz["num_restricoes"] = df_mtz["num_restricoes"].astype(int)

df_danzig.to_csv("resultados_danzig.csv")
df_mtz.to_csv("resultados_mtz.csv")

# display(df_danzig)
# display(df_mtz)

# merge dataframes with good columns names, same index
df = pd.concat([df_danzig, df_mtz], axis=1, keys=["danzig", "mtz"])
df.columns = df.columns.map(lambda x: "_".join(x))

df
```

```
n = 4/102
n = 9/102
n = 14/102
n = 19/102
n = 24/102
n = 29/102
n = 34/102
n = 39/102
n = 44/102
n = 49/102
n = 54/102
```

```
n = 59/102
n = 64/102
n = 69/102
n = 74/102
n = 79/102
n = 84/102
n = 89/102
n = 94/102
n = 99/102
```

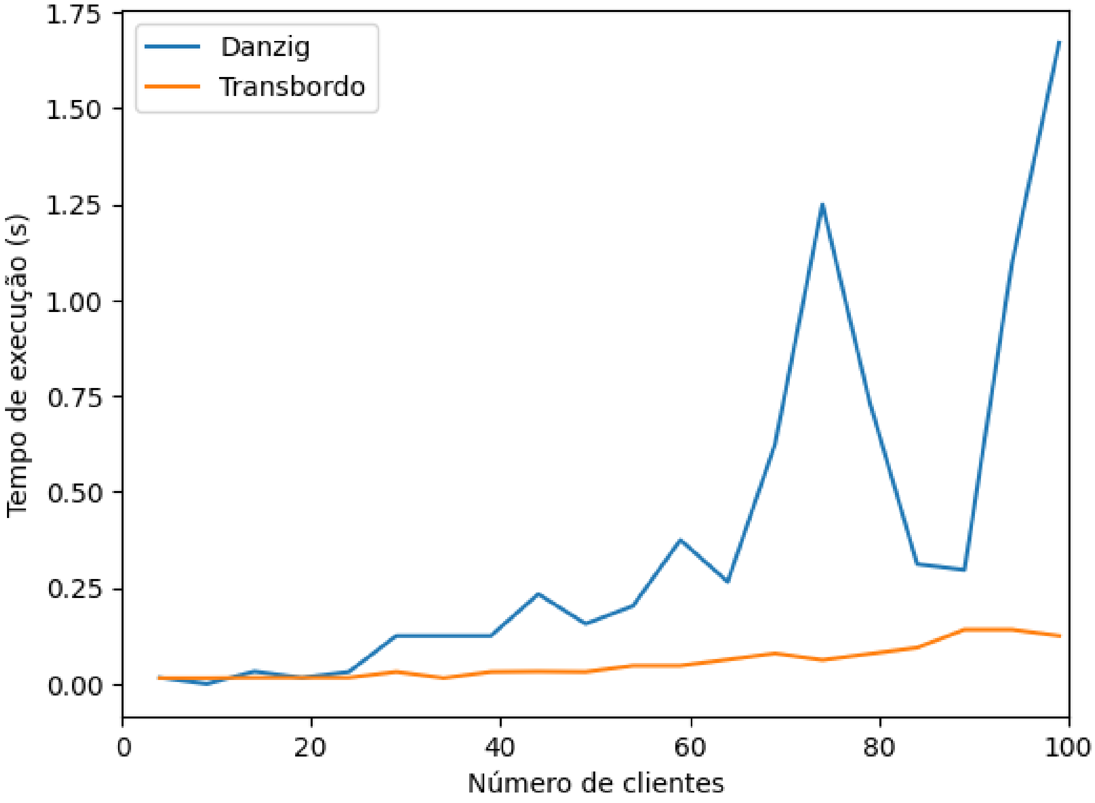
```
[ ]:     danzig_time  danzig_num_restricoes  mtz_time  mtz_num_restricoes
     4         0.016                     14     0.015                  16
     9         0.000                     74     0.015                  76
     14        0.032                    184     0.016                 186
     19        0.016                    344     0.016                 346
     24        0.031                    554     0.016                 556
     29        0.125                    814     0.031                 816
     34        0.125                   1124     0.015                1126
     39        0.125                   1484     0.031                1486
     44        0.234                   1894     0.032                1896
     49        0.156                   2354     0.031                2356
     54        0.203                   2864     0.047                2866
     59        0.375                   3424     0.047                3426
     64        0.266                   4034     0.063                4036
     69        0.625                   4694     0.079                4696
     74        1.250                   5404     0.062                5406
     79        0.734                   6164     0.078                6166
     84        0.312                   6974     0.094                6976
     89        0.297                   7834     0.141                7836
     94        1.094                   8744     0.141                8746
     99        1.672                   9704     0.125                9706
```

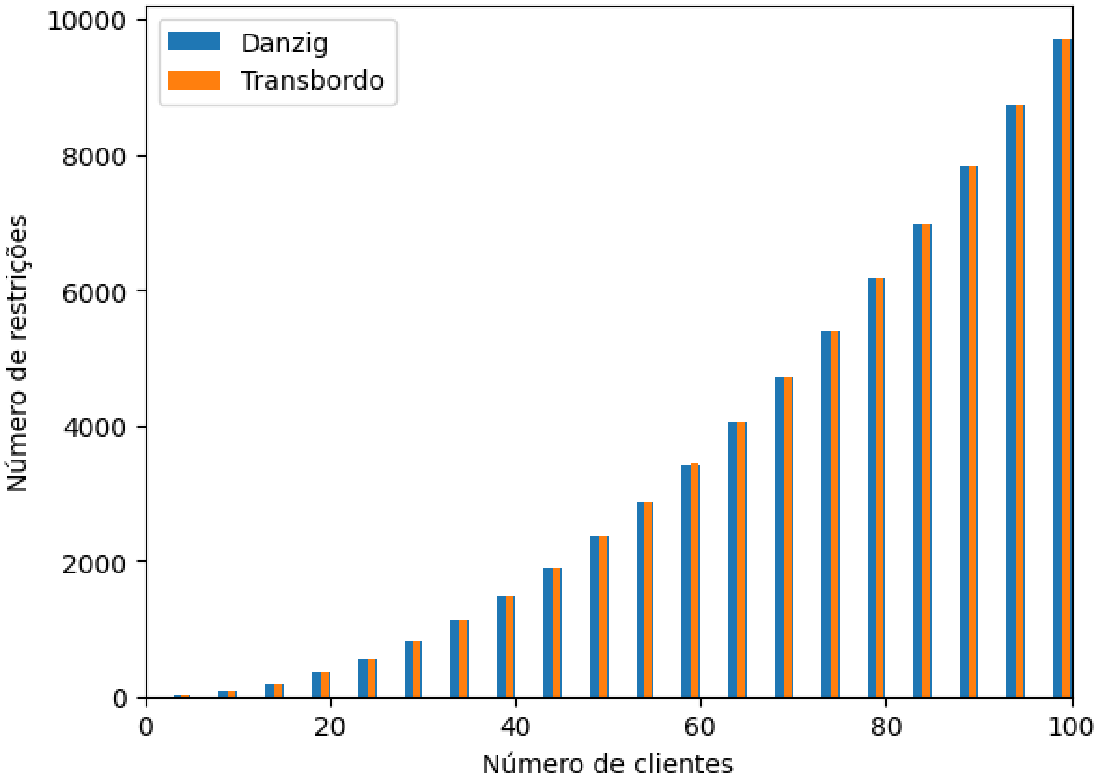
```
[ ]: import matplotlib.pyplot as plt


     # plot N x time with both plots
     plt.plot(df.index, df["danzig_time"], label="Danzig")
     plt.plot(df.index, df["mtz_time"], label="Transbordo")
     plt.xlabel("Número de clientes")
     plt.ylabel("Tempo de execução (s)")
     plt.legend()
     plt.xlim(0, 100)
     plt.show()

     # plot N x num_restricoes with both plots
     plt.bar(df.index, df["danzig_num_restricoes"], label="Danzig", align="center",␣
       ↪width=2)
```

```python
plt.bar(df.index, df["mtz_num_restricoes"], label="Transbordo", align="edge")
plt.xlabel("Número de clientes")
plt.ylabel("Número de restrições")
plt.legend()
plt.xlim(0, 100)
plt.show()
```




Danzig
Transbordo
Número de restrições
Número de clientes
0
2000
4000
6000
8000
10000
0
20
40
60
80
100