Usemore Soap Company: A Warehouse Location Case Study

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1 Enunciado

The Usemore Soap Company produces a line of cleaning compounds, used mainly for industrial and institutional purposes. Typical products include general cleaning compounds, dishwasher powders, rinse agents, hand soaps, motor vehicle washing compounds, and cleaning products for the food industry. The product line is composed of more than 200 products and nearly 800 individual product items. Package sizes range from 18-pound cases to large metal drums weighing 550 pounds.

Sales are generated throughout the 48 contiguous United States, with additional sales in Hawaii, Alaska, and Puerto Rico. Customers typically purchase in quantities less than 10,000 pounds, that is, less-than-truckload (LTL) quantities. A few customers purchase in truckload and bulk quantities. Annual LTL sales, which pass through the warehouses, are running at the 150 million pound level. Volume sales, which are served directly from plants, add another 75 million pounds. These sales represent approximately $160 million in revenue.

The primary marketing effort comes from a direct selling force operating under the incentive of a liberal sales commission structure. Salespeople look upon themselves as individual entrepreneurs and have a great deal of autonomy within the company. This marketing strategy has generally proved successful for the company, as the company has often been referred to as one of the most profitable divisions within its widely diversified parent organization.

In spite of the high profitability, company management is concerned about the costs of producing and distributing the product line to maintain its competitive edge. Growth and shifting demand patterns are straining the production capacity of the four existing plants. In addition, changing costs of distribution, as well as the fact that the distribution network has not been studied in 12 years, raise questions about the proper placement of the warehouses. What follows is a summary of the problem conditions being faced by

management. You are to suggest an improved distribution network that meets the stated customer service policy and minimizes total network production-distribution costs.

1.1 BACKGROUND

The current distribution network consists of four product line plants located at Covington, Kentucky, New York, New York, Arlington, Texas, and Long Beachlifornia. The plants are currently producing product for their low-volume customers at the level of 595,102 cwt., 390,876 cwt., 249, 662 cwt., and 241, 386 cwt., respectively. This output is shipped from plants either to field warehouses in the distribution network or to customers within the local areas of the plants. In the latter case, plants serve as field warehouses as well as producing centers.

Warehousing takes place at 18 public warehouses and at the four plant locations, as shown in Table 1. These warehouses are dispersed in such a fashion that the majority of the customers are within a one-day delivery time frame of a stocking point; that is, approximately 300 miles. Except for the plants serving as warehouses, the warehouses are supplied in full truckload quantities. Less-than-truckload shipments serve customers. Customer order processing takes place at each warehouse location. In addition, two potential plant sites are being considered at Chicago, Illinois, and Memphis, Tennessee.

Additional warehouse sites are considered at the locations shown in Table 2.

Potential warehouse sites are made based on sales personnel's suggestions, favorable warehousing rates, good warehousing service availability, proximity to demand concentrations, and filling out of the distribution network. Of the existing and potential warehouse sites, it is hoped that an improved mix of warehouses can be found. In addition, plant expansion, either at existing sites or at new sites, will be needed to meet future demand projections. Specifically, answers to the following questions are sought:

1. How many warehouses should be operated now and in the future?

2. Where should they be located?

3. Which customers and associated demand should be assigned to each warehouse and plant?

4. Which warehouses should be supplied from each plant?

5. Should production capacity be expanded? When, where, and by how much?

6. What level of customer service should be provided?

Table 1: Current Plant and Public Warehouse Locations

No	Location (city)	Location (state)
1	Covington	KY
2	New York	NY
3	Arlington	TX
4	Long Beach	CA
5	Atlanta	GA
6	Boston	MA
7	Buffalo	NY
8	Chicago	IL
9	Cleveland	OH
10	Devenport	IA
11	Detroit	MI
12	Grand Rapids	MI
13	Greensboro	NC
14	Kansas City	KS
15	Baltimore	MD
16	Memphis	TN
17	Milwaukee	WI
18	Orlando	SL
19	Pittsburgh	PA
20	Portland	OR
21	W Sacramento	CA
22	W Chester	PA

1.2 SALES DATA

Manufacturing of soap liquids and powders is an uncomplicated duplicated process, which contributes to substantial competition in the marketplace. The undifferentiated nature of soap products results in keen competition in both price and service. Customer service is of particular concern because it is directly affected by the choice of warehouses. No specific dollar figure can be placed on the total value of good distribution service, as it depends on customer attitudes about service and resulting patronage. The general feeling in the company is that service should be maintained at a high level so as not to jeopardize sales. A "high" level of service is taken to mean delivery time of 24 to 48 hours or less. This generally places customers somewhere between 300 and 600 miles of warehouses. Annual sales for the products that move through the warehousing network are 147 million pounds for annual revenue of slightly more than $100 million. Sales are distributed similarly to population centers with an average profit margin of 20 percent. Figure 1 shows the six major sales territories, with sales volume in pounds by state.

The company has more than 70,000 individual customer accounts, and these are aggregated into 191 active demand centers. A demand center is a grouping of zip code areas into a zip sectional center as the focus of the collected demand. These demand

Table 2: Possible Public Warehouse Locations

No	Location (city)	Location (state)
23	Albuquerque	NM
24	Biling	MT
25	Denver	CO
26	El Paso	TX
27	Camp Hill	PA
28	Houston	TX
29	Las Vegas	NV
30	Minneapolis	MN
31	New Orleans	LA
32	Phoenix	AZ
33	Richmond	VA
34	St Louis	MO
35	Salt Lake City	UT
36	San Antonio	TX
37	Seattle	WA
38	Spokane	WA
39	San Francisco	CA
40	Indianapolis	IN
41	Louisville	KY
42	Columbus	OH
43	New York	NY
44	Hartford	CT
45	Miami	FL
46	Mobile	AL
47	Memphis *	TN
48	Chicago *	IL

centers, along with how they are currently being served, are given in Table 6. In addition, the sales territory in which the demand center is grouped is shown. The five-year plan shows volume growth throughout the United States. This growth will not be uniform due to population and business migration patterns, competition, and varying promotional efforts. The changes in volume compared with current volume levels are projected by sales territory as follows (Table 3):

Table 3: Sales Territory Growth Factors

Region No.	Sales Territory	5-Year Growth Factor
1	Northeast	1,3
2	Southeast	1,45
3	Midwest	1,25
4	Northwest	1,2
5	Southwest	1,15
6	West	1,35
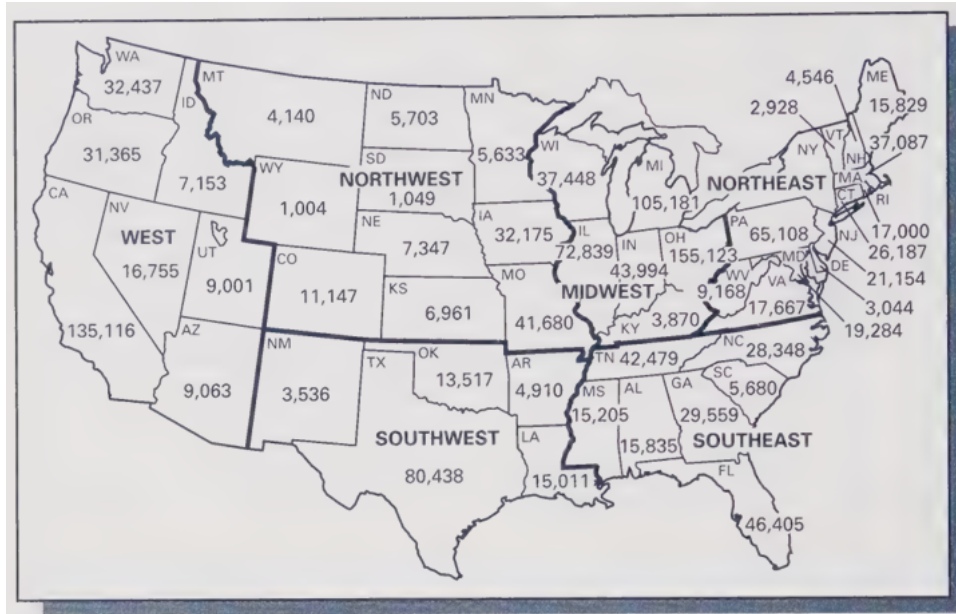
Figure 1: Usemore Soap Company Annual Sales in Cwt. by State, with Major Sales Districts Defined

1.3 PRODUCTION COSTS AND CAPACITIES

The production variable costs at existing plants vary by location. This variance results from labor rate differences, volume purchases of raw materials, and inbound transport cost differences due to the proximity of the plants to major raw materials sources. These costs are listed next (Table 4).

Table 4: Production Costs at Existing Plants

Plant	Variable production cost
Covington, KY	21,0
New York	19,9
Arlington	21,6
Long Beach	21,1

The potential plant at Chicago has an estimated cost of $21.0 per cwt.; and the Memphis plant has a cost of $20.6 per cwt. Expansion at any existing plant site would have the current variable cost. Fixed costs are not included for existing plants because these are sunk costs. However, to construct a new plant or expand an existing one would cost a minimum of $4 million. This cost would result in an output for the plant (or an output increase in the case of a plant addition) of up to 1 million cwt. per year for the near future.

According to current distribution patterns, the existing plants are producing, relative to throughput capacity (in cwt.), at the following rates (Table 5):

Current Production Rates

Plant	Current capacity (cwt.)	Current Production (cwt.)	Percent of Capacity
Covington, KY	620000	595102	96%
New York	430000	390876	91%
Arlington	300000	249662	83%
Long Beach	280000	241386	86%
Total	**1630000**	**1477026**	**91%**

1.4 WAREHOUSING RATES AND CAPACITIES

Company contracts with public warehousemen show that rates are categorized as storage, handling, and acessorial. Storage rates are quoted on a $/cwt./month basis of average inventory held. Handling charges are incurred whenever in or out movement of the product occurs and are assessed on a $/cwt. basis. Acessorial charges are for a number of services, such as bill of lading preparation, local delivery, and stock status reporting. Similar charges are estimated for the four plant warehouses as a fair share of production operations.

Also associated with warehousing are the stock replenishment costs. These are costs for preparing the paperwork for normal replenishment and the expediting of stock into the warehouse. Stock order costs as well as customer order costs are computed by multiplying the average cost per order by the average number of orders for the warehouse.

The warehouse-related costs and other associated information are given in Table 3. Costs for existing points are taken from company records. Those for potential warehouses are determined from quotas by warehousemen in the appropriate cities. Estimates are made of costs where such information is not otherwise available.

There are no effective capacity limits on public warehousing. Usemore's space need is a small fraction of a public warehouseman's total capacity. On the other hand, a throughput of at least 10,400 cwt. per year, or a replenishment truckload every two weeks, is the desired minimum throughput needed to open a warehouse. Available space is limited at the four current plant sites. The stocking limits in terms of throughput at Covington = 450,000 cwt., New York = 380,000 cwt., Arlington = 140,000 cwt., and Long Beach = 180,000 cwt.

1.5 TRANSPORTATION COSTS

Three general transportation cost types are important to Usemore: inbound, outbound, and local delivery transport charges. Inbound transportation costs to a warehouse depend on the volume shipped and the distance between plant and warehouse. A sampling of truck common carrier rates at various distances from the plants for full truckload shipments shows that the transport rate between a plant and warehouse (P-W) can be reasonably

approximated by a linear function. That is, the truckload rate is

$$\text{P-W rate (\$/cwt.)} = 0.92 + 0.0034 \cdot d \quad \text{(miles)} \tag{1}$$

where d is the distance between two points (For simplicity, one aggregated relationship is shown. In practice, several such relationships would be used to reflect the rate difference caused by geographic locations of the shipment origin points). Total inbound transport costs are determined by multiplying the P-W rate by the volume assigned to flow between the plant and warehouse.

Warehouse outbound transport costs depend on the distance that a customer is from the warehouse. If the customer is roughly within 30 miles of the warehouse, local cartage rates generally apply. These local delivery rates are shown by warehouse in Table 6 For distances greater than 30 miles, a linear function similar to that for the inbound rates can be developed. Given the average shipment size from the warehouses of approximately 1,000 pounds, the warehouse to customer (W-C) rate function is

$$\text{W-C rate (\$/cwt.)} = 5.45 + 0.0037 \cdot d \quad \text{(miles)} \tag{2}$$

Computation of total warehouse outbound transport costs is carried out in the same manner as for inbound transport costs.

1.6 INVENTORY COSTS

Inventory costs depend on the average inventory maintained at a warehouse and the inventory rate factors that apply to the inventory level. These rate factors include the cost of capital, personal property taxes, and insurance costs. The average inventory at a warehouse will vary by the demand on the warehouse and by the method used to control the inventory. A mathematical function to express inventory based on annual warehouse throughput is found by plotting the annual average inventory against annual throughput at each active stocking point. The resulting curve is shown in Figure 2.

Knowing that the annual cost-to-carry-inventory rate is approximately 12 percent of the average product value of \$26 per cwt., the total cost to carry inventory at each warehouse is given by

$$IC_i = (0.12)(26)(11.3d_i^{0.58}) = 35.3D_i^{0.58} \tag{3}$$

where

$$IC_i = \text{inventory carrying cost at warehouse i (\$)}$$
$$D_i = \text{annual throughput at warehouse i (cwt)}$$

Table 6: Stocking Point Rate and Order Size Information

Warehouse No.	Storage ($/cwt)	Handling ($/cwt)	Stock Order Processing ($/order)	Stock Order Size (cwt/order)	Customer Order Processing ($/order)	Customer Order Size (cwt/order)	Local delivery rate ($/cwt)
1	0,0672	0,46	18	400	1,79	9,05	1,9
2	0,0567	0,54	18	400	1,74	10,92	3,89
3	0,0755	0,38	18	400	2,71	11,59	2,02
4	0,0735	0,59	18	400	1,74	11,3	4,31
5	**0,0946**	**0,5**	**18**	401	0,83	9,31	1,89
6	0,1802	0,75	18	405	3,21	9	4,7
7	0,0946	0,74	18	405	1,23	8,37	1,55
8	0,2072	1,14	18	405	1,83	13,46	1,79
9	0,1802	1,62	18	409	4,83	9,69	4,92
10	0,1442	1,14	18	410	2,74	8,28	2,23
11	0,0946	1,04	18	409	3,93	10,2	1,81
12	0,1982	1,06	18	410	3,18	15	1
13	0,0766	1,06	18	400	1,08	9,07	1,63
14	0,1262	1,22	18	423	1,56	11,72	1,17
15	0,1126	0,82	18	426	1,2	9,35	1,73
16	0,991	0,64	18	433	1,78	8,7	0,5
17	0,1577	0,71	18	394	5,33	8,07	1,46
18	0,1307	0,79	18	398	0,91	7,66	2,29
19	0,1487	1,15	18	399	2,08	9,39	2,2
20	0,2253	0,8	18	490	1,1	7,31	1,49
21	0,137	1,39	18	655	1,7	9,31	2,72
22	0,0991	0,83	18	400	2,46	10,14	4,17
23	0,126	0,59	18	110	2,33	5,07	2,37
24	0,0637	0,45	18	134	1,88	6,8	1,36
25	0,0946	1,68	18	341	2,58	6,83	2,21
26	0,1216	0,88	18	149	1,83	14,32	0,8
27	0,0721	0,55	18	198	1,83	7,38	3,88
28	0,1532	0,8	18	420	1,58	9,7	2,14
29	0,1172	1,04	18	287	0,78	7,52	1,51
30	0,108	1,46	18	408	5,33	11,46	1,7
31	0,1487	0,95	18	340	1,36	10,48	1,63
32	0,1352	0,69	18	333	1,5	6,67	1,66
33	0,1126	0,64	18	277	2,33	11,98	1,54
34	0,1712	1,35	18	398	0,93	10,13	1,84
35	0,1261	0,79	18	434	2,08	6,81	1,58
36	0,1352	0,8	18	323	0,88	7,67	1,93
37	0,2704	0,96	18	423	0,89	8,57	3,08
38	0,225	0,8	18	425	2,88	7,61	1,43
39	0,1487	4,49	18	400	1,46	7,55	6,44
40	0,2073	1,14	18	400	2,75	10,13	2,83
41	0,2073	1,14	18	400	2,75	10,13	2,83
42	0,1802	1,62	18	400	2,75	10,13	4,81
43	0,2613	1,39	18	400	2,71	11,59	3,89
44	0,1396	0,71	18	400	2,04	9,37	3,89
45	0,1036	0,55	18	400	2,75	10,13	1,74
46	0,0946	0,55	18	400	1,74	9,31	1,89
47	0,0682	0,64	18	400	1,78	8,7	0,5
48	0,0682	1,22	18	400	1,79	9,05	1,55
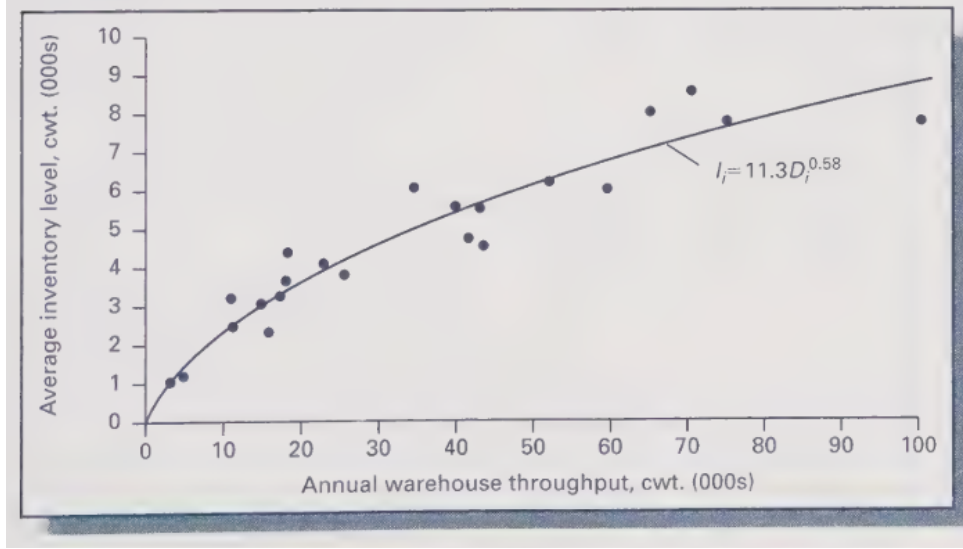
Figure 2: The Inventory-to-Warehouse Throughput Relationship for the Usemore Soap Company

1.7 WAREHOUSE OPERATING COSTS

Warehouse operating costs refer to the combination of storage and handling costs incurred resulting from assigning demand to warehouses. Storage costs are computed by taking the storage rate and multiplying it by an estimate of the average inventory in the warehouse. Mathematically, this can be expressed as

$$SC_i = SR_i \cdot (26) \cdot (11.3D_i^{0.58}) \tag{4}$$

where

$$SC_i = \text{annual cost of stock at warehouse i (\$)}$$
$$SR_i = \text{storage rate from warehouse i from Table 4}$$
$$D_i = \text{annual demand throughput at warehouse i (cwt)}$$

Handling costs are strictly a function of the warehouse throughput. They are determined by the handling rate multiplied by the throughput, or

$$HC_i = HR_i \cdot D_i \tag{5}$$

where

$$HC_i = \text{annual handling cost at warehouse i (\$)}$$

$$HR_i = \text{handling rate at warehouse i from Table 4}$$

1.8 ORDER-PROCESSING COSTS

Order-processing costs refer to the charges incurred in handling the paperwork associated with stock replenishment and customer orders. Both types of costs are computed for each warehouse in essentially the same way. That is, the order-processing rate is multiplied by the annual demand on the warehouse and the result divided by the order size.

1.9 TOTAL COSTS

The total costs for various production distribution configurations can be determined by summing all the relevant costs. For the Usemore Soap Company, these are production costs; warehouse operating costs (storage, handling, stock order processing, and customer order processing); transportation costs (warehouse inbound, outbound, and local delivery); and inventory-carrying costs. Changing the number and location of plants and warehouses will cause a change in the balance of these cost factors. For example, adding warehouses will typically reduce transportation costs but increase inventory costs, as well as affect customer service. Assessing the trade-offs between costs and customer service is at the heart of this problem type. The cost and customer service summaries for the current network design are shown in Tables 7 and 8. At present, Usemore Soap is able to place 93 percent of its demand within 300 miles of warehouses for a total annual cost of $42,112,463.

Table 7: Benchmark Customer Service Profile

Warehouse to customer distance	Percent of demand	Cumulative percent of demand	Total demand (cwt.)
0-100 mi.	56,40%	56,40%	833,043
101-200	21,30%	77,70%	314,607
201-300	15,70%	93,40%	231,893
301-400	2,10%	95,50%	31,018
401-500	1,50%	97%	22,155
501-600	0,50%	97,50%	7,385
601-700	2,00%	99,50%	29,541
701-800	0,50%	100%	7,384
801-900	0%	100%	0
901-1000	0%	100%	0
>1000	0%	100%	0
	100%		1,477,026

Table 8: Cost profile for the Current Distribution Network

Cost Category	Cost
Production	$ 30.761.520
Warehouse operations	$ 1.578.379
Order processing	$ 369.027
Inventory carrying	$ 457.290
Transportation - Inbound to warehouse	$ 2.050.367
Transportation - Outbound from warehouse	$ 6.895.880
Total cost	$ 42.112.463

1.10 A COMPUTER-ASSISTED ANALYSIS

Although enough data have been provided to carry out an analysis manually, a computer program (WARELOCA, a module in LOGWARE) accompanies this case study. Given a particular combination of plants, plant capacities, customer service constraints, and warehouses, the program optimally assigns demand centers to warehouses and warehouses to plants by means of linear programming. From the selected list of warehouses, the least expensive will be chosen if more than one choice is available within the prescribed service distance from a demand center. If a warehouse cannot be found within the service distance, the warehouse closest to the demand center will be selected. Only linear variable costs are used in the allocation of demand centers to warehouses. Storage and capital costs, which are nonlinear, are not used in the allocation process. They are included in the system costs for a particular configuration. Fixed costs are neither included in the allocation, nor are they shown in the total system costs. They must be externally added to system costs. WARELOCA is a program in which you provide the plant locations and capacities, warehouse locations, customer service distance, and demand and cost levels. Each run of the program represents an evaluation of a particular network configuration. The results of a sample WARELOCA run in which the current network is approximated (not the true benchmark) where the existing 4 plants and 22 warehouses are evaluated are given in Figure 3.

2 Solução

2.1 1st Question

The first question is to determine the optimal number of plants and warehouses, both for the current and future scenarios. To address this, we formulate a Mixed-Integer Linear Programming (MILP) model, which is a classical facility location problem with additional constraints and cost structures.

a

b

Conjuntos:

- $\mathcal{P}^a$: Conjunto de plantas atuais

- $\mathcal{P}^p$: Conjunto de plantas potenciais

- $\mathcal{P}$: Conjunto de plantas. $\mathcal{P} = \mathcal{P}^a \cup \mathcal{P}^p$

- $\mathcal{W}^a$: Conjunto de armazéns atual

- $\mathcal{W}^p$: Conjunto de armazéns potenciais

- $\mathcal{W}$: Conjunto de armazéns. $\mathcal{W} = \mathcal{W}^a \cup \mathcal{W}^p$

- $\mathcal{D}$: Conjunto de nós de demanda

Indices:

- p: índice para plantas

- w: índice para armazéns

- d: índice para nós de demanda

Variáveis de decisão:

- z_w: 1 se o armazém w será utilizado e 0 caso contrário, onde $w \in \mathcal{W}^a$

- x_{pw}: quantidade de produto que será enviada da planta p, $p \in \mathcal{P}^a$ para o armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$

- y_{wd}: quantidade de produto que será enviada do armazém (ou fábrica) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$ para o nó de demanda d, $d \in \mathcal{D}$

Table 9: Parâmetros

Símbolo	Unidades	Descrição
S_d	cwt	Demanda total do nó de demanda d, $d \in \mathcal{D}$
d_{pw}	milhas	Distância entre a planta p, $p \in \mathcal{P}^a$ e o armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
d_{wd}	milhas	Distância entre o armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$ e o nó de demanda d, $d \in \mathcal{D}$
ρ_p	\$/cwt	Custo variável de produção de cada planta p, $p \in \mathcal{P}^a$
C_p	cwt	Capacidade de produção da planta p, $p \in \mathcal{P}^a$

Table 9 – continued from previous page

Símbolo	Unidades	Descrição
C'_p	cwt	Capacidade de estocagem da planta p, $p \in \mathcal{P}^a$
τ_w	\$/cwt	Custo de estocagem do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
ϵ_w	\$/cwt	Custo de handling do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
γ_w	\$/order	Stock Order Processing do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
δ_w	cwt/order	Stock Order Size do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
ϕ_w	\$/order	Customer Order Processing do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$
ω_w	cwt/order	Customer Order size do armazém (ou planta) w, $w \in \mathcal{W}^a \cup \mathcal{P}^a$

$$
\begin{aligned}
\min \Big(&\text{Custo do movimento de produto das fábricas para os armazéns} \Big) \\
&+ \Big(\text{Custo do movimento de produto dos armazéns para os clientes} \Big) \\
&+ \text{Custo Variável de produção} \\
&+ \text{Custo de estocagem nos armazéns} \\
&+ \text{Custo de inventário} \\
&+ \text{Custo de Handling nos armazéns} \\
&+ \text{Custo de processamento do pedido de estoque} \\
&+ \text{Custo de processamento do pedido do cliente}
\end{aligned}
$$

(Custo do movimento de produto das fábricas para os armazéns - ou fábricas)

$$
\begin{cases}
\sum_{p \in \mathcal{P}^a} \sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} c_{pw}^{in} \cdot x_{pw}, \\
c_{pw}^{in} = 0.92 + 0.0034 \cdot d_{pw}, \quad \forall p \in \mathcal{P}^a, \forall w \in \mathcal{W}^a \cup \mathcal{P}^a
\end{cases}
\tag{6}
$$

(Custo do movimento de produto dos armazéns - ou fábricas - para os clientes)

$$
\begin{cases}
\sum_{p \in \mathcal{P}} \sum_{w \in \mathcal{W}} c_{wd}^{out} \cdot x_{wd}, \\
c_{wd}^{out} = \begin{cases} 5.45 + 0.0037 \cdot d_{wd}, & \text{se } d_{wd} > 30 \text{ milhas} \\ \text{Valor da tabela}, & \text{caso contrário} \end{cases}, \quad \forall w \in \mathcal{W}^a \cup \mathcal{P}^a, \forall d \in \mathcal{D}
\end{cases}
\tag{7}
$$

(Custo Variável de produção)

$$\sum_{p \in \mathcal{P}^a} \sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} \rho_p \cdot x_{pw}, \tag{8}$$

(Custo de estocagem nos armazéns)

$$\sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} 26 \cdot 11.3 \cdot (\sum_{d \in \mathcal{D}} y_{wd})^{0.58} \cdot \tau_w \tag{9}$$

(Custo de inventório)

$$\sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} 26 \cdot 0.12 \cdot 11.3 \cdot (\sum_{d \in \mathcal{D}} y_{wd})^{0.58} \tag{10}$$

(Custo de Handling nos armazéns - ou fábricas)

$$\sum_{p \in \mathcal{P}^a} \sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} 2 \cdot z_w \cdot \epsilon_w \cdot x_{pw} \tag{11}$$

(Custo de processamento do pedido de estoque)

$$\sum_{p \in \mathcal{P}^a} \sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} z_w \cdot x_{pw} \cdot \frac{\gamma_w}{\delta_w} \tag{12}$$

(Custo de processamento do pedido do cliente)

$$\sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} \sum_{d \in \mathcal{D}} z_w \cdot y_{wd} \cdot \frac{\phi_w}{\omega_w} \tag{13}$$

Sujeito a:

(Nível de serviço)

Seja $\mathcal{WP}$ um conjunto de pares ordenados de armazéns atuais (e plantas atuais) e nós de demanda, onde a distância entre o armazém (ou fábrica) e o nó de demanda é menor ou igual a 300 milhas.

$$\frac{\sum_{(w,p) \in \mathcal{WP}} y_{wd}}{\sum_{d \in \mathcal{D}} S_d} \geq 0.93 \tag{14}$$

(Satisfação da demanda)

$$\sum_{w \in \mathcal{W}^a \cup \mathcal{P}^a} y_{wd} = S_d, \quad \forall d \in \mathcal{D} \tag{15}$$

(Capacidade de produção das plantas)

$$\sum_{w \in \mathcal{W} \cup \mathcal{P}^a} x_{pw} \leq C_p, \quad \forall p \in \mathcal{P} \tag{16}$$

(Limite mínimo do uso de cada armazém)

$$\sum_{d \in \mathcal{D}} y_{wd} \geq 10400 \cdot z_w, \quad \forall w \in \mathcal{W}^a \cup \mathcal{P}^a \tag{17}$$

(Limite de estoque dos armazéns das plantas)

$$\sum_{p_2 \in \mathcal{P}^a} x_{p_1 p_2} \leq C'_p \quad \forall p_1 \in \mathcal{P}^a \tag{18}$$

(Balanço de massa das fábricas)

$$\sum_{p \in \mathcal{P}^a} x_{pw} = \sum_{d \in \mathcal{D}} y_{wd}, \quad \forall w \in \mathcal{W}^a \cup \mathcal{P}^a \tag{19}$$

(Restrições para dar o valor correto de z_w)

$$M = \sum_{d \in \mathcal{D}} S_d \tag{20}$$

$$x_{pw} \leq M \cdot z_w, \quad \forall w \in \mathcal{W}^a \cup \mathcal{P}^a, \forall p \in \mathcal{P}^a \tag{21}$$

$$y_{wd} \leq M \cdot z_w, \quad \forall w \in \mathcal{W}^a \cup \mathcal{P}^a, \forall d \in \mathcal{D} \tag{22}$$

Espaço das variáveis:

- $x_{pw} \in \mathbb{R}^+$

- $y_{wd} \in \mathbb{R}^+$

- $z_w \in \{0, 1\}$

2.2 2nd Question

[Centers of Population 2021] [Ballou, Rahardja e Sakai 2002]

Usar $\mathcal{W}$ em vez de $\mathcal{W}^a$

Usar $\mathcal{P}$ em vez de $\mathcal{P}^a$

Adicionar variável: $v_p, \forall p \in \mathcal{P}$: 1 se a planta p será for aumentada (ou construída) e 0 caso contrário

(Custos de expansão - ou construção)

$$4\,000\,000 \cdot \sum_{p \in \mathcal{P}} v_p$$

(Restrições para dar o valor correto de v_p)

$$\sum_{w \in \mathcal{W} \cup \mathcal{P}} x_{pw} \leq C_q + v_p \cdot 1\,000\,000, \quad \forall p \in \mathcal{P} \tag{23}$$

2.3 3rd Question

2.4 4th Question

2.5 5th Question

2.6 6th Question

References

[Ballou, Rahardja e Sakai 2002]BALLOU, R. H.; RAHARDJA, H.; SAKAI, N. Selected country circuity factors for road travel distance estimation. *Transportation Research Part A: Policy and Practice*, v. 36, n. 9, p. 843 – 848, 2002. Cited by: 69.

[Centers of Population 2021]CENTERS of Population. [S.l.], 2021. Disponível em: <https://www.census.gov/geographies/reference-files/time-series/geo/centers-population.html>.