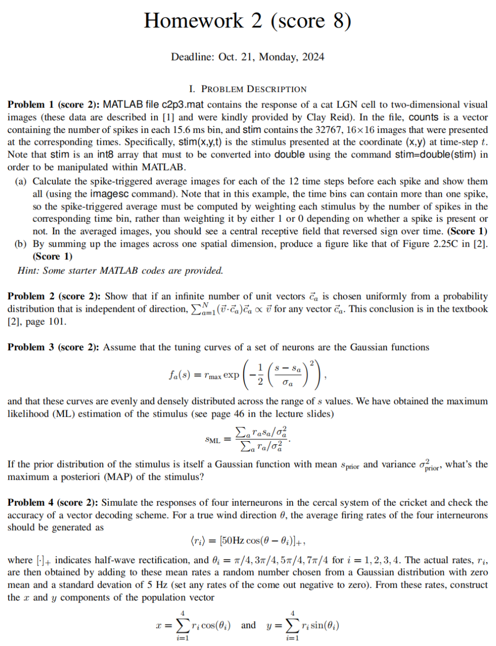
# 作业要求

## 原文件

[homework2.pdf]



Homework 2 (score 8)

Deadline: Oct. 21, Monday, 2024

I. PROBLEM DESCRIPTION

**Problem 1 (score 2):** MATLAB file c2p3.mat contains the response of a cat LGN cell to two-dimensional visual images (these data are described in [1] and were kindly provided by Clay Reid). In the file, counts is a vector containing the number of spikes in each 15.6 ms bin, and stim contains the 32767, $16\times16$ images that were presented at the corresponding times. Specifically, stim(x,y,t) is the stimulus presented at the coordinate (x,y) at time-step $t$. Note that stim is an int8 array that must to be converted into double using the command stim=double(stim) in order to be manipulated within MATLAB.

(a) Calculate the spike-triggered average images for each of the 12 time steps before each spike and show them all (using the imagesc command). Note that in this example, the time bins can contain more than one spike, so the spike-triggered average must be computed by weighting each stimulus by the number of spikes in the corresponding time bin, rather than weighting it by either 1 or 0 depending on whether a spike is present or not. In the averaged images, you should see a central receptive field that reversed sign over time. **(Score 1)**

(b) By summing up the images across one spatial dimension, produce a figure like that of Figure 2.25C in [2]. **(Score 1)**

*Hint: Some starter MATLAB codes are provided.*

**Problem 2 (score 2):** Show that if an infinite number of unit vectors $\vec{c}_a$ is chosen uniformly from a probability distribution that is independent of direction, $\sum_{a=1}^{N}(\vec{v}\cdot\vec{c}_a)\vec{c}_a \propto \vec{v}$ for any vector $\vec{c}_a$. This conclusion is in the textbook [2], page 101.

**Problem 3 (score 2):** Assume that the tuning curves of a set of neurons are the Gaussian functions

$$f_a(s) = r_{\max}\exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_a}\right)^2\right),$$

and that these curves are evenly and densely distributed across the range of $s$ values. We have obtained the maximum likelihood (ML) estimation of the stimulus (see page 46 in the lecture slides)

$$s_{\mathrm{ML}} = \frac{\sum_a r_a s_a/\sigma_a^2}{\sum_a r_a/\sigma_a^2}.$$

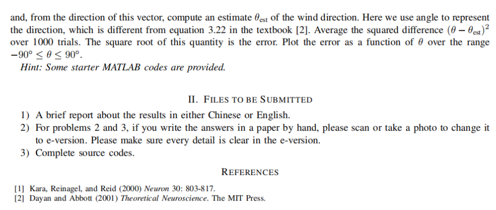
If the prior distribution of the stimulus is itself a Gaussian function with mean $s_{\mathrm{prior}}$ and variance $\sigma_{\mathrm{prior}}^2$, what's the maximum a posteriori (MAP) of the stimulus?

**Problem 4 (score 2):** Simulate the responses of four interneurons in the cercal system of the cricket and check the accuracy of a vector decoding scheme. For a true wind direction $\theta$, the average firing rates of the four interneurons should be generated as

$$\langle r_i\rangle = [50\mathrm{Hz}\cos(\theta-\theta_i)]_+,$$

where $[\cdot]_+$ indicates half-wave rectification, and $\theta_i = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ for $i = 1,2,3,4$. The actual rates, $r_i$, are then obtained by adding to these mean rates a random number chosen from a Gaussian distribution with zero mean and a standard devation of 5 Hz (set any rates of the come out negative to zero). From these rates, construct the $x$ and $y$ components of the population vector

$$x = \sum_{i=1}^{4} r_i\cos(\theta_i) \quad \text{and} \quad y = \sum_{i=1}^{4} r_i\sin(\theta_i)$$



and, from the direction of this vector, compute an estimate $\theta_{\mathrm{est}}$ of the wind direction. Here we use angle to represent the direction, which is different from equation 3.22 in the textbook [2]. Average the squared difference $(\theta-\theta_{\mathrm{est}})^2$ over 1000 trials. The square root of this quantity is the error. Plot the error as a function of $\theta$ over the range $-90^\circ \le \theta \le 90^\circ$.

*Hint: Some starter MATLAB codes are provided.*

II. FILES TO BE SUBMITTED

1) A brief report about the results in either Chinese or English.
2) For problems 2 and 3, if you write the answers in a paper by hand, please scan or take a photo to change it to e-version. Please make sure every detail is clear in the e-version.
3) Complete source codes.

REFERENCES


[1] Kara, Reinagel, and Reid (2000) *Neuron* 30: 803-817.
[2] Dayan and Abbott (2001) *Theoretical Neuroscience*. The MIT Press.


# Homework 2 (Total Score: 8)

### Deadline: Oct. 21, Monday, 2024

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## I. PROBLEM DESCRIPTION

### Problem 1 (Score: 2)

The MATLAB file `c2p3.mat` contains the response of a cat LGN (Lateral Geniculate Nucleus) cell to two-dimensional visual images. These data are described in [1] and were kindly provided by Clay Reid. In the file, `counts` is a vector containing the number of spikes in each 15.6 ms bin, and `stim` contains the 32,767, 16×16 images that were presented at the corresponding times. Specifically, `stim(x,y,t)` is the stimulus presented at coordinate (x,y) at time-step t. Note that `stim` is an `int8` array, so you need to convert it to `double` using the command `stim=double(stim)` to manipulate it in MATLAB.

1. **(a)** Calculate the spike-triggered average images for each of the 12 time steps before each spike and display all of them using the `imagesc` command. Remember, the time bins can contain more than one spike, so the spike-triggered average must be computed by weighting each stimulus by the number of spikes in the corresponding time bin. In the averaged images, you should observe a central receptive field that reverses sign over time.

   *(Score: 1)*

2. **(b)** By summing up the images across one spatial dimension, generate a figure similar to Figure 2.25C in [2].

   *(Score: 1)*

   *Hint: Starter MATLAB codes are provided.*

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## Problem 2 (Score: 2)

Show that if an infinite number of unit vectors$\vec{c}_a$ are chosen uniformly from a probability distribution that is independent of direction:

$$\sum_{a=1}^{N}(\vec{v}\cdot\vec{c}_a)\vec{c}_a \propto \vec{v}$$

for any vector \( \vec{v} \). This result can be found in the textbook [2], page 101.

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## Problem 3 (Score: 2)

Assume the tuning curves of a set of neurons are Gaussian functions:

$$f_a(s) = r_{\max}\exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_a}\right)^2\right),$$

where the curves are evenly and densely distributed across the range of s values. Using the Maximum Likelihood (ML) estimation of the stimulus, as seen on page 46 of the lecture slides, the estimated stimulus \( $s_{\mathrm{ML}}$ \) is given by:

$$s_{\mathrm{ML}} = \frac{\sum_a r_a s_a / \sigma_a^2}{\sum_a r_a / \sigma_a^2}.$$

If the prior distribution of the stimulus is a Gaussian function with mean \($s_{\mathrm{prior}}$ \) and variance \( $\sigma^2_{\mathrm{prior}}$\), what is the Maximum A Posteriori (MAP) estimate of the stimulus?

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## Problem 4 (Score: 2)

Simulate the responses of four interneurons in the cercal system of a cricket, and check the accuracy of a vector decoding scheme. For a true wind direction \( \theta \), the average firing rates of the four interneurons should be:

$$\langle r_i \rangle = [50\,\mathrm{Hz} \cdot \cos(\theta - \theta_i)]_+,$$

where \($[\cdot]_+$ \) denotes half-wave rectification, and $\theta_i$takes values \( $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$\) for \( $i = 1, 2, 3, 4$\). The actual firing rates \($r_i$\) are obtained by adding to these mean rates a random number from a Gaussian distribution with zero mean and standard deviation of 5 Hz (set any negative rates to zero). From these rates, calculate the x and y components of the population vector:

$$x = \sum_{i=1}^{4} r_i \cos(\theta_i), \quad y = \sum_{i=1}^{4} r_i \sin(\theta_i),$$

and from the direction of this vector, compute an estimate \( $\theta_{\mathrm{est}}$ \) of the wind direction. The direction here is represented differently from equation 3.22 in the textbook [2]. Average the squared difference \($(\theta - \theta_{\mathrm{est}})^2$\) over 1000 trials. The square root of this value gives the error. Plot the error as a function of \( $\theta$ \) over the range \($-90^\circ \le \theta \le 90^\circ$\).

*Hint: Starter MATLAB codes are provided.*

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# II. FILES TO BE SUBMITTED

1. A brief report of the results in either Chinese or English.

2. For problems 2 and 3, if handwritten, please scan or take a photo to create an electronic version. Ensure the details are clear.
3. Complete source codes.

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## REFERENCES

[1] Kara, Reinagel, and Reid (2000) Neuron 30: 803-817.

[2] Dayan and Abbott (2001) Theoretical Neuroscience. The MIT Press.

# 中文版

## 作业2 (总分：8分)

**截止日期：2024年10月21日，周一**

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### I. 问题描述

### 问题 1 (2分)

MATLAB 文件 `c2p3.mat` 包含了猫的 LGN（外侧膝状体）细胞对二维视觉图像的响应数据。这些数据描述见文献 [1]，由 Clay Reid 提供。在文件中，`counts` 是一个向量，包含每个 15.6 ms 时间段内的脉冲数量，而 `stim` 包含了 32767 个 16×16 图像，这些图像在相应的时间点上呈现。具体来说，`stim(x,y,t)` 表示在时间步 t 时，在坐标 (x,y) 处呈现的刺激。注意，`stim` 是一个 `int8` 数组，因此需要使用命令 `stim=double(stim)` 将其转换为 `double` 以便在 MATLAB 中操作。

1. **(a)** 计算每个脉冲之前 12 个时间步的脉冲触发平均图像，并使用 `imagesc` 命令显示所有图像。注意，在这个例子中，时间段可能包含多个脉冲，因此脉冲触发平均值必须通过按每个时间段内的脉冲数对每个刺激加权来计算，而不是根据是否存在脉冲按 1 或 0 进行加权。在平均图像中，你应该能看到一个中央感受野，随时间反转符号。

   **(1分)**

2. **(b)** 通过在一个空间维度上对图像求和，生成类似于文献 [2] 中图 2.25C 的图像。

   **(1分)**

   **提示：提供了一些初始的 MATLAB 代码。**

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## 问题 2 (2分)

证明如果从与方向无关的概率分布中均匀地选择无限多个单位向量 $\vec{c}_a$，则有：

$$\sum_{a=1}^{N}(\vec{v}\cdot\vec{c}_a)\vec{c}_a \propto \vec{v}$$

对于任意向量 $\vec{v}$。该结果可以在课本 [2] 第 101 页找到。

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## 问题 3 (2分)

假设一组神经元的调谐曲线是高斯函数：

$$f_a(s) = r_{\max}\exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_a}\right)^2\right),$$

并且这些曲线在 s 值的范围内均匀且密集地分布。我们获得了刺激的最大似然（ML）估计（见讲义第 46 页）：

$$s_{\mathrm{ML}} = \frac{\sum_a r_a s_a/\sigma_a^2}{\sum_a r_a/\sigma_a^2}.$$

如果刺激的先验分布本身是均值为 $s_{\mathrm{prior}}$，方差为 $\sigma_{\mathrm{prior}}^2$ 的高斯函数，那么刺激的最大后验概率（MAP）是多少?

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## 问题 4 (2分)

模拟蟋蟀尾毛系统中四个中间神经元的响应，并检查向量解码方案的准确性。对于一个真实的风向 $\theta$，四个中间神经元的平均脉冲频率应为：

**D**

其中 $[\cdot]_+$ 表示半波整流，$\theta_i$对于$i = 1, 2, 3, 4$ 分别取值 $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$。实际的脉冲频率 $r_i$ 是在这些平均频率上添加一个从均值为 0，标准差为 5 Hz 的高斯分布中选取的随机数得到的（将任何负值频率设为 0）。根据这些频率，计算群体向量的 x 和 y 分量：

$$x = \sum_{i=1}^{4} r_i\cos(\theta_i), \quad y = \sum_{i=1}^{4} r_i\sin(\theta_i),$$

并根据该向量的方向，计算风向的估计值 $\theta_{\text{est}}$。这里使用的方向与课本 [2] 中的方程 3.22 不同。将平方差 $(\theta - \theta_{\text{est}})^2$ 在 1000 次实验中取平均，取该值的平方根作为误差。绘制误差随 $\theta$ 变化的曲线，范围为 $-90^\circ \leq \theta \leq 90^\circ$。

**提示：提供了一些初始的 MATLAB 代码。**

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## II. 需提交的文件

1. 关于结果的简短报告，可以用中文或英文书写。
2. 对于问题 2 和 3，如果手写作答，请扫描或拍照将其转换为电子版。请确保电子版中的每个细节都清晰可见。
3. 完整的源代码。

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## 参考文献

[1] Kara, Reinagel, and Reid (2000) Neuron 30: 803-817.

[2] Dayan and Abbott (2001) Theoretical Neuroscience. The MIT Press.