Homework 2 (score 8)

Deadline: Oct. 21, Monday, 2024

I. Problem Description

Problem 1 (score 2): MATLAB file c2p3.mat contains the response of a cat LGN cell to two-dimensional visual images (these data are described in [1] and were kindly provided by Clay Reid). In the file, counts is a vector containing the number of spikes in each 15.6 ms bin, and stim contains the 32767, 16×16 images that were presented at the corresponding times. Specifically, stim(x,y,t) is the stimulus presented at the coordinate (x,y) at time-step t. Note that stim is an int8 array that must to be converted into double using the command stim=double(stim) in order to be manipulated within MATLAB.

(a) Calculate the spike-triggered average images for each of the 12 time steps before each spike and show them all (using the imagesc command). Note that in this example, the time bins can contain more than one spike, so the spike-triggered average must be computed by weighting each stimulus by the number of spikes in the corresponding time bin, rather than weighting it by either 1 or 0 depending on whether a spike is present or not. In the averaged images, you should see a central receptive field that reversed sign over time. **(Score 1)**

(b) By summing up the images across one spatial dimension, produce a figure like that of Figure 2.25C in [2]. **(Score 1)**

Hint: Some starter MATLAB codes are provided.

Problem 2 (score 2): Show that if an infinite number of unit vectors $\vec{c}_a$ is chosen uniformly from a probability distribution that is independent of direction, $\sum_{a=1}^{N}(\vec{v}\cdot\vec{c}_a)\vec{c}_a \propto \vec{v}$ for any vector $\vec{c}_a$. This conclusion is in the textbook [2], page 101.

Problem 3 (score 2): Assume that the tuning curves of a set of neurons are the Gaussian functions

$$f_a(s) = r_{\max}\exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_a}\right)^2\right),$$

and that these curves are evenly and densely distributed across the range of s values. We have obtained the maximum likelihood (ML) estimation of the stimulus (see page 46 in the lecture slides)

$$s_{\text{ML}} = \frac{\sum_a r_a s_a/\sigma_a^2}{\sum_a r_a/\sigma_a^2}.$$

If the prior distribution of the stimulus is itself a Gaussian function with mean s_{prior} and variance σ_{prior}^2, what's the maximum a posteriori (MAP) of the stimulus?

Problem 4 (score 2): Simulate the responses of four interneurons in the cercal system of the cricket and check the accuracy of a vector decoding scheme. For a true wind direction θ, the average firing rates of the four interneurons should be generated as

$$\langle r_i\rangle = [50\text{Hz}\cos(\theta-\theta_i)]_+,$$

where $[\cdot]_+$ indicates half-wave rectification, and $\theta_i = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ for $i = 1, 2, 3, 4$. The actual rates, r_i, are then obtained by adding to these mean rates a random number chosen from a Gaussian distribution with zero mean and a standard devation of 5 Hz (set any rates of the come out negative to zero). From these rates, construct the x and y components of the population vector

$$x = \sum_{i=1}^{4} r_i\cos(\theta_i) \quad \text{and} \quad y = \sum_{i=1}^{4} r_i\sin(\theta_i)$$

and, from the direction of this vector, compute an estimate θ_{est} of the wind direction. Here we use angle to represent the direction, which is different from equation 3.22 in the textbook [2]. Average the squared difference $(\theta - \theta_{\text{est}})^2$ over 1000 trials. The square root of this quantity is the error. Plot the error as a function of θ over the range $-90^\circ \leq \theta \leq 90^\circ$.

Hint: Some starter MATLAB codes are provided.

II. Files to be Submitted

1) A brief report about the results in either Chinese or English.
2) For problems 2 and 3, if you write the answers in a paper by hand, please scan or take a photo to change it to e-version. Please make sure every detail is clear in the e-version.
3) Complete source codes.

References

[1] Kara, Reinagel, and Reid (2000) *Neuron* 30: 803-817.

[2] Dayan and Abbott (2001) *Theoretical Neuroscience*. The MIT Press.